BFKL, BK and the infrared

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Diffraction 2012

color screening

From the fits to lattice data on field strength correlators the propagation length of perturbative gluons is

$$R_c \sim 0.3\,\mathrm{fermi}$$

Hence, Yukawa screened transverse chromoelectric field of the relativistic quark, $\vec{\mathcal{E}}(\vec{\rho}) \sim g_S(\rho) K_1(\rho/R_c)\vec{\rho}/\rho$

$$\mathcal{E}(\rho \gtrsim R_c) \sim g_S \exp\left[-\rho/R_c\right]$$

to be discussed:

- BFKL, BK phenomenolgy of DIS in presence of finite correlation length of perturbative gluons.
- Sharp sensitivity of non-linear effects to R_c:

$$\sim \sigma^2(\rho) \sim \rho^4$$

DGLAP density of glue in the dipole $\vec{r}$:

$$|\Psi_{q\bar{q}g}|^2 \sim \alpha_S \frac{r^2}{\rho^4}$$

Hence, the non-linear correction:

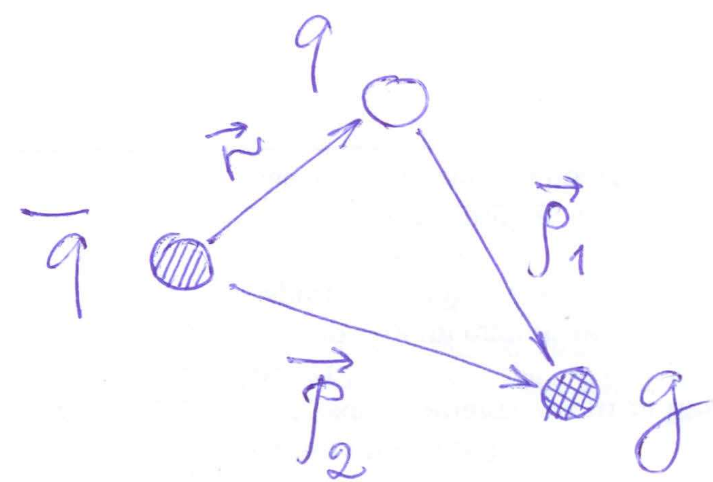
$$\delta\sigma \sim \int^{R_c^2} |\Psi_{q\bar{q}g}|^2 \sigma^2 \propto R_c^2$$

BFKL, BK and partial waves

$N_c \gg 1$ - the profile function Γ obeys the equation

$$\frac{\partial \Gamma(x, r, \mathbf{b})}{\partial \log(1/x)} = \int d^2 \boldsymbol{\rho}_1 \left| \vec{\mathcal{E}}(\vec{\rho}_1) - \vec{\mathcal{E}}(\vec{\rho}_2) \right|^2 \times$$

$$\times \left[\Gamma_1 + \Gamma_2 - \Gamma - \Gamma_1 \times \Gamma_2 \right]$$



$$\Gamma(x, r, \mathbf{b}) = \frac{\sigma}{4\pi B} \exp\left[- \frac{b^2}{2B} \right]$$

$$\frac{d\sigma_{el}}{dt} \sim \exp\left[B t \right]$$

B- the diffraction cone slope

B - diffraction cone slope

Normalization is such that the unitarity limit is $\sigma = 8\pi B$

- $$B(x,r) = \frac{1}{8}r^2 + \frac{1}{3}R_N^2 + 2\alpha'_{I\!P}\log(1/x)$$

- dimensionful $\alpha'_{I\!P}$ connected to the infrared parameter R_c

$$\alpha'_{I\!P} \sim \frac{3}{16\pi}\alpha_S(R_c)R_c^2 \sim 0.1\,\mathrm{GeV}^{-2}$$

- leading pomeron singularity – moving pole in j-plane

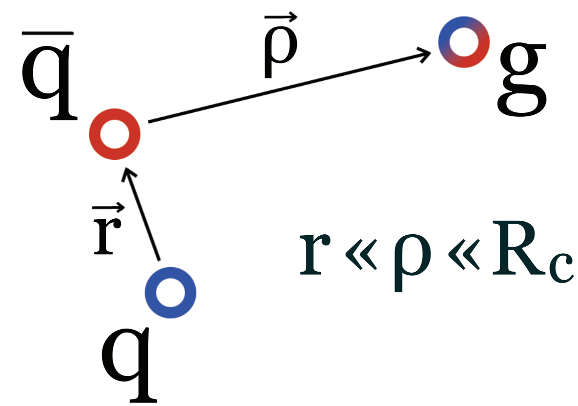
$$j = \alpha_{I\!P}(t) = 1 + \Delta_{I\!P} + \alpha'_{I\!P}t$$

GLR-MQ

$$r^2 \ll \rho^2 \ll R_c^2$$

$$\partial_\xi \sigma(\xi, r) = \frac{C_F}{\pi} \alpha_S(r) r^2 \int_{r^2}^{R_c^2} \frac{d\rho^2}{\rho^4} \times$$

$$\times \left[2\sigma(\xi, \rho) - \frac{\sigma(\xi, \rho)^2}{8\pi B} \right].$$



The function $\rho^{-2}\sigma(\xi, \rho) \sim \alpha_S(\rho) G(x, \rho)$ is flat in ρ^2 and the non-linear term is dominated by $\rho \sim R_c$,

$$\frac{1}{8B} \int_{r^2}^{R_c^2} \frac{d\rho^2}{\rho^4} \sigma(\xi, \rho)^2 \simeq \frac{R_c^2}{8B} \left(\frac{\pi^2}{N_c} \alpha_S(R_c) G(x, R_c) \right)^2$$

the unitarity limit - $\sigma \leq 8\pi B$

GLR-MQ

$$\partial_\xi \partial_\eta G(\xi, \eta) = cG(\xi, \eta) - \omega(\eta) G^2(\xi, \eta),$$

$$\xi = \log(1/x)$$

$$\eta = \log\left(\alpha_S(R_c)/\alpha_S(\rho)\right)$$

$$\omega(\eta) = \omega(0) \exp[-\eta - \lambda(e^\eta - 1)] \sim \exp\left[-e^\eta\right]$$

$$\omega(0) \propto \frac{R_c^2}{8B}$$

GLR-MQ

The factor $\omega(\eta)$ is a steeply falling function of η concentrated at

$$\eta \lesssim 1$$

Substitute one steeply falling function (s.f.f.) by another s.f.f. with properly ajusted slope

$$\omega(\eta) \to \omega(0) \exp\left[-\gamma\eta\right]$$

$$\partial_\xi \partial_\eta G(\xi, \eta) = G(\xi, \eta) - \omega e^{-\eta} G^2(\xi, \eta),$$

partial solution

$$\partial_\xi \partial_\eta G(\xi,\eta) = G(\xi,\eta) - \omega e^{-\eta} G^2(\xi,\eta).$$

$$G(\xi,\eta) = \frac{e^{\xi+\eta}}{\omega e^{\xi} + const}$$

after recovering hidden coefficients

$$G(\xi,\eta) = \frac{e^{\Delta\xi+\gamma\eta}}{\omega e^{\Delta\xi} + const}$$

with

$$\omega = \frac{R_c^2}{8B} \cdot \frac{\pi \alpha_S(R_c)}{2N_c}$$

some properties of G

- The gluon fusion mechanism tames the exponential ξ-growth of $G(\xi,\eta)$ but fails to stop the accumulation of large log's $\eta = \log(1/\alpha_S(r))$ at very small $r^2 \ll R_c^2$.
- For vanishing non-linearity, $\omega \sim R_c^2/B \to 0$,

$$G \sim \exp[\Delta\xi + \gamma\eta] \sim \left(\frac{1}{x}\right)^{\Delta} \left[\frac{1}{\alpha_S(r)}\right]^{\gamma}$$

more properties: saturation.

$$\xi = \log(1/x) \to \infty$$

$$G \sim \omega^{-1} e^{\eta}$$

$$\sigma(r) = 8\pi B \cdot \frac{r^2}{R_c^2} \cdot 2 \left[\frac{\alpha_S(R_c)}{\alpha_S(r)} \right]^{\gamma}$$

$$r^2 \ll R_c^2$$

Remind, the unitarity limit:

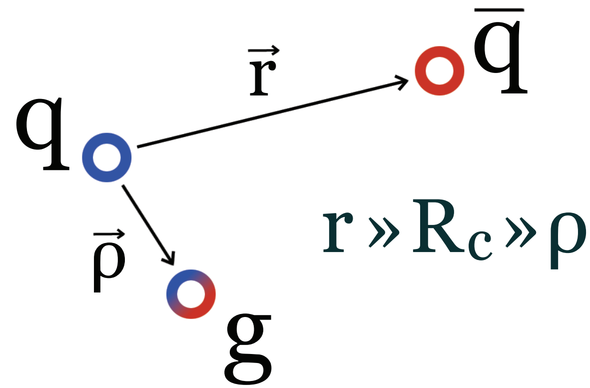
$$\sigma = 8\pi B$$

BFKL, BK and constituent quarks

$$r^2 \gg R_c^2 \gg \rho^2$$

$$\partial_\xi \sigma(\xi, r) = \frac{\alpha_S C_F}{\pi^2} \int d^2\rho_1 R_c^{-2} K_1^2(\rho_1/R_c)$$

$$\times \left\{ \sigma(\xi,\rho_1) + \sigma(\xi,\rho_2) - \sigma(\xi,r) - \frac{\sigma(\xi,\rho_1)\sigma(\xi,\rho_2)}{4\pi(B_1+B_2)} \right\},$$



where $B_i = B(\xi, \rho_i)$.

constituent quarks and R_c

$$\delta\sigma \sim R_c^{-2} \int^{R_c^2} d\rho^2 K_1^2(\rho/R_c) \frac{\sigma(\rho)\sigma(r)}{8\pi B}$$

$$\sim \int^{R_c^2} \frac{d\rho^2}{\rho^2} \frac{\sigma(\rho)\sigma(r)}{8\pi B}$$

$$\sim \frac{\sigma(R_c)\sigma(r)}{8\pi B}$$

$$\propto \frac{R_c^2}{8\pi B}$$

$\sigma(\xi, r)$ varies slowly for $r \gg R_c$. Then,

$$c^{-1}\partial_\xi \sigma(\xi, r) = \sigma(\xi, R_c) + R_c^2 \partial_{r^2} \sigma(\xi, r) - \sigma(\xi, R_c)\sigma(\xi, r)/8\pi B,$$

Its partial solution is

$$\sigma(\xi, r) = \frac{\sigma_0(r^2 + c\xi R_c^2) + v(\xi)}{1 + v(\xi)/8\pi B},$$

where

$$v(\xi) = e^{c\xi} \int_0^{c\xi} \sigma_0(R_c^2 + zR_c^2)e^{-z} dz \sim$$

$$\sim e^{c\xi}\sigma_0(R_c^2 + c\xi R_c^2)$$

weak non-linearity regime

for large dipoles

$$r^2 \gtrsim R_c^2$$

$$\sigma \sim e^{c\xi}\sigma_0(R_c^2(1+c\xi))\left(1 - \frac{\sigma_0(r^2 + c\xi R_c^2)}{8\pi B}\right).$$

$\sigma_0(r^2)$ is the Born approximation, the 2g-exchange.

Related to the phenomenon of decoupling of subleading vacuum singularities for dipole sizes $r \sim R_c$.

BFKL spectrum and solutions

The spectrum of the running BFKL equation is a series of moving poles in the complex j-plane (L 1986). Hence, the BFKL-Regge expansion:

$$\sigma(x,r) = \sigma_0(r)x^{-\Delta_0} + \sigma_1(r)x^{-\Delta_1} + \sigma_2(r)x^{-\Delta_2} + ...$$

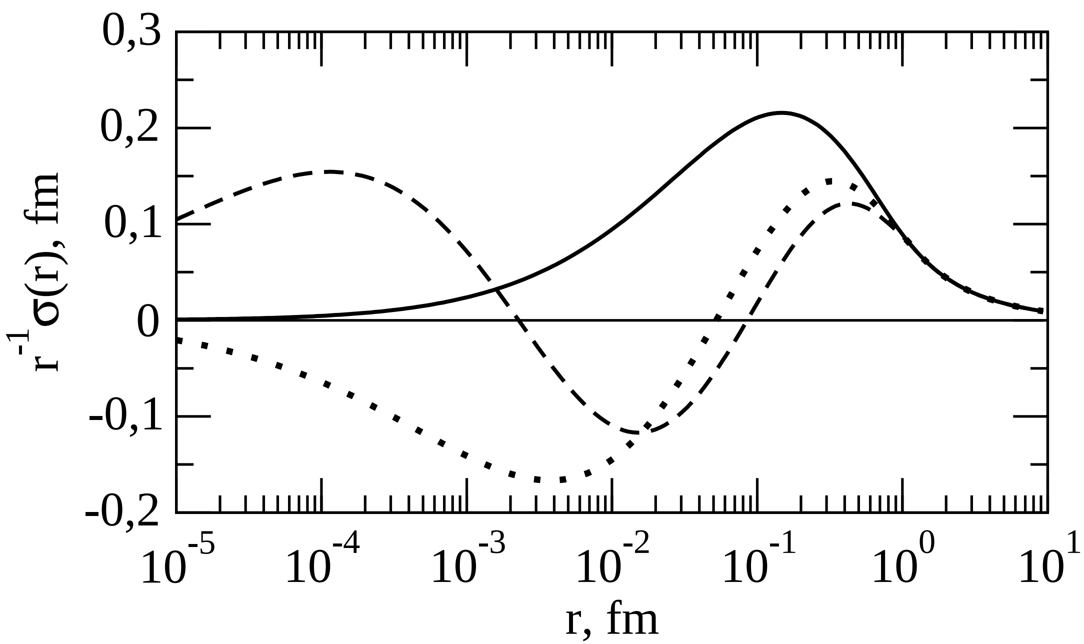
$$\sigma_n(r)$$

a solution of the eigenvalue problem

$$\mathcal{K} \otimes \sigma_n = \Delta_n \sigma_n(r).$$

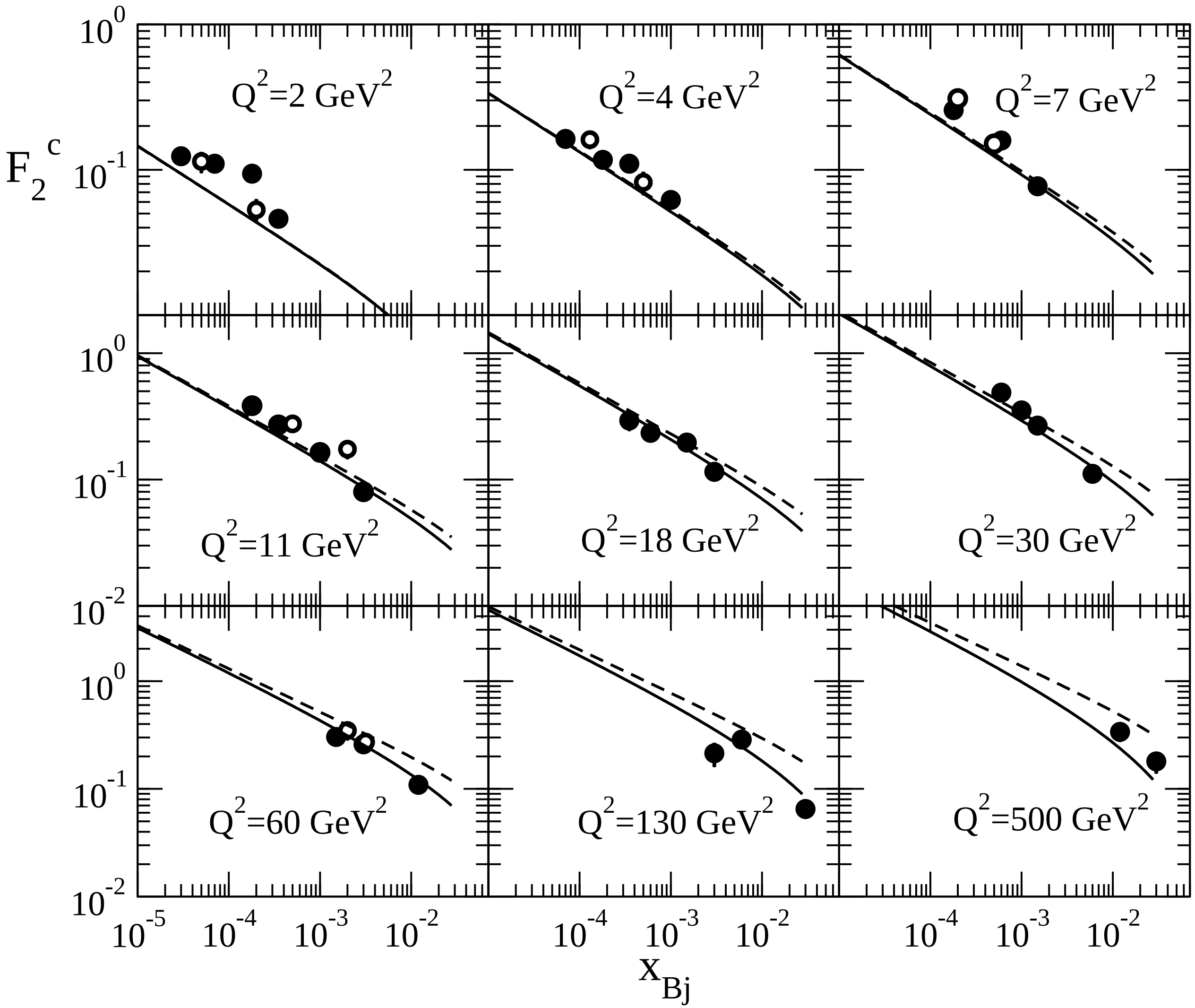
with the pomeron intercept Δ_n as the eigenvalue.

BFKL eigenfunctions $\sigma_n(r)$



- With our IR regularization the node of $\sigma_1(r)$ is at $r_1 \sim 0.1\,\mathrm{fm}$
- for larger n its position r_1 moves to larger $r \sim R_c$.
- the first nodes for large n accumulate at $r \sim R_c$ - accumulation point

F_2^c: decoupling of sub-pomerons



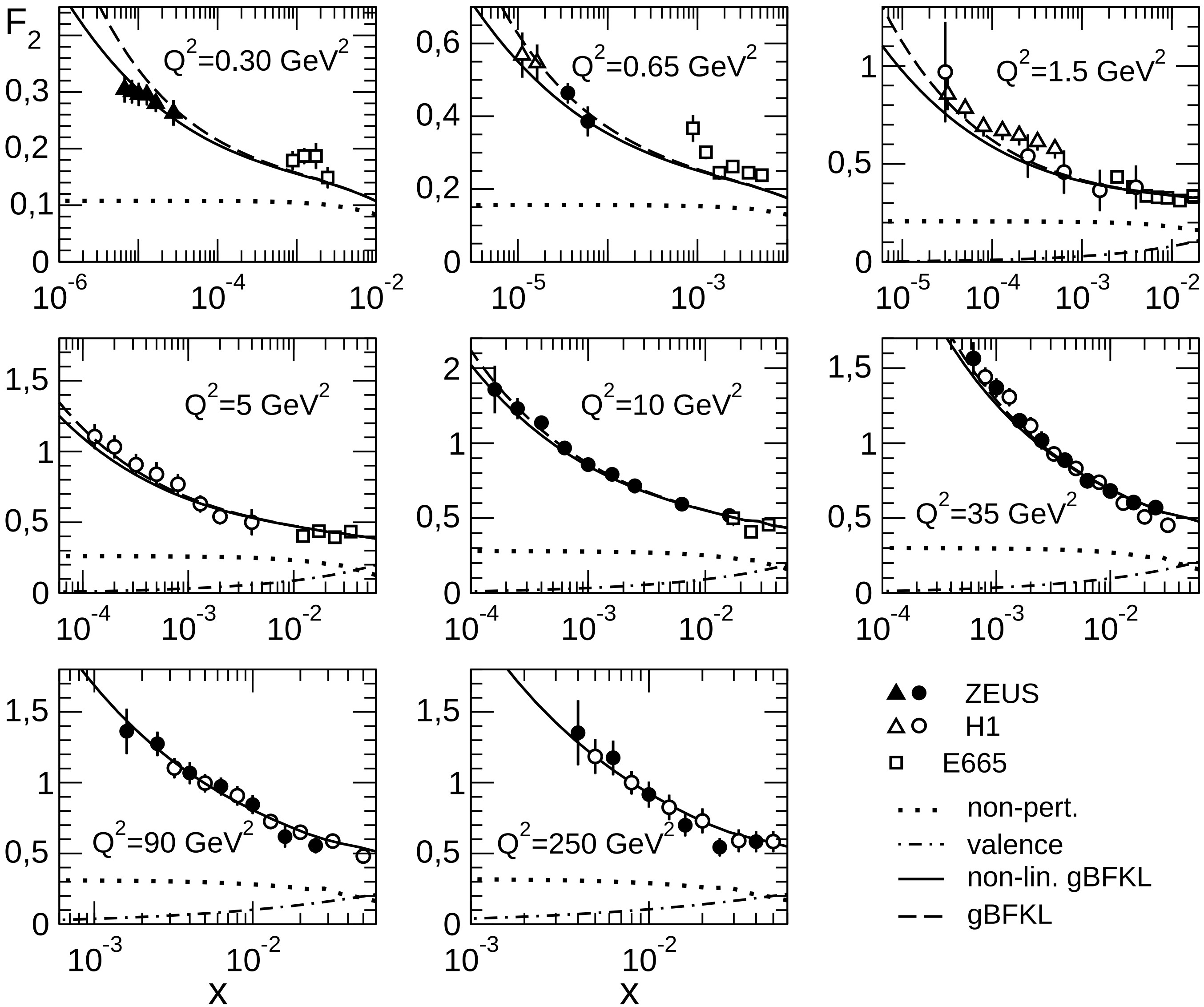
Data: H1 Collab. Eur.Phys.J. C45, 23 (2006)

strong non-linearity

$$\xi = \log(1/x) \to \infty$$

$$\sigma = 8\pi B$$

back to reality: HERA data



Boundary conditions

The BFKL cross section $\sigma(\xi, r)$ sums the Leading-Log$(1/x)$ multi-gluon production cross sections within the perturbation theory. Consequently, realistic boundary condition for the BFKL dynamics - the lowest order $q\bar{q}$-nucleon cross section at some $x = x_0$. The Yukawa screened two-gluon exchange

$$\sigma(0,r) = 4C_F \int \frac{d^2\mathbf{k}}{(k^2+\mu_G^2)^2} \alpha_S(k^2) \alpha_S(\kappa^2) \times$$

$$\times \left[1 - J_0(kr)\right] \left[1 - F_2(\mathbf{k}, -\mathbf{k})\right],$$

where $\mu_G = 1/R_c$,

Perturbative - non-perturbative

Because the propagation radius is short compared to the typical range of strong interactions the dipole cross section would miss the interaction strength. This missing strength is modeled by the x-independent dipole cross section and it has been assumed that the perturbative, $\sigma(\xi, r)$, and non-perturbative, $\sigma_{npt}(r)$, cross sections are additive,

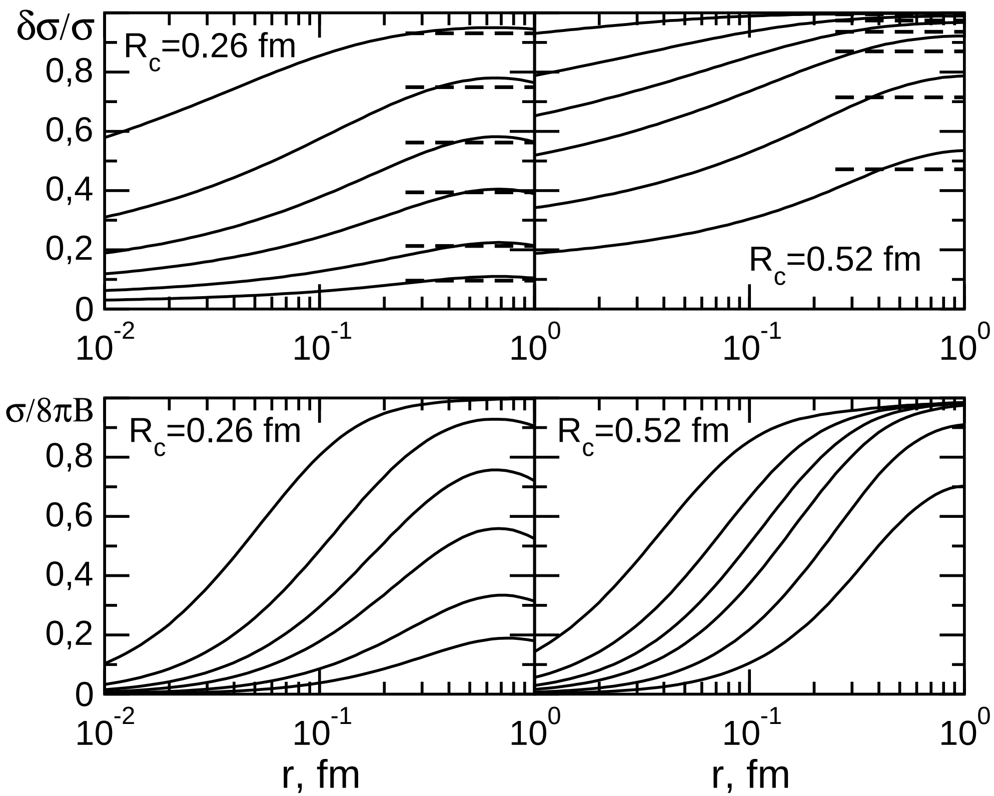
$$\sigma_{tot}(\xi, r) = \sigma(\xi, r) + \sigma_{npt}(r).$$

The principal point about the non-perturbative component of $\sigma_{tot}(\xi, r)$ is that it must not be subjected to the perturbative BFKL evolution.

Specific form of $\sigma_{npt}(r)$ motivated by the QCD string picture and used in the present paper is as follows:

$$\sigma_{npt}(r) = a\alpha_S^2(r)r^2/(r+d).$$

Here $d = 0.5$ fm is close to the radius of freezing of the running QCD coupling r_f and $a = 5.$ fm.



The dipole size dependence of the non-linear corr. to the linear CD BFKL cross section for $\xi = \log(0.03/x) = 6$, 8.5, 11, 13, 15.5, 20.

summary

It is not surprizing that introducing small propagation length for perturbative gluons kills nonlinear effects. More surprizing is that this small $R_c = 0.26$ fermi gives correct value of the BFKL pomeron intercept $\Delta \approx 0.4$ in agreement with data. Bigger value of R_c leads to bigger Δ in conflict with data.