NLO BFKL kernel for the adjoint representation of the gauge group

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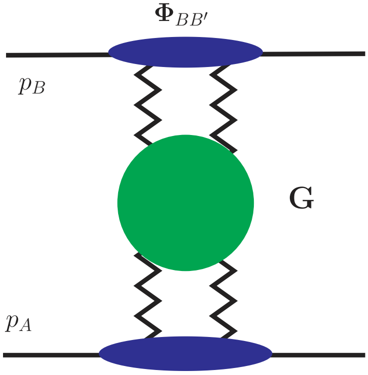
Introduction

The BFKL (Balitsky-Fadin-Kuraev-Lipatov) approach is based on the remarkable property of QCD – gluon reggeization. The scattering amplitudes are represented by the convolution

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}$$



Introduction

The universal (process independent) Greens's function G can be presented as

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ is the BFKL kernel, Y is the total rapidity $(Y = \ln(s/s_0))$.
Talking about the BFKL approach, one usually means BFKL Pomeron, that is, a colourless state in the t -channel. But the approach is applicable for any colour state, which two gluons can form. For QCD, that is for tree colours, there are 6 irreducible representations:

$$\underline{1}, \underline{8_a}, \underline{8_s}, \underline{10}, \underline{\overline{10}}, \underline{27}.$$

For $N_c > 3$ there are 7 possible representations.
Now the kernel is known in the NLO both for forward scattering, i.e. for $t = 0$ and the colour singlet in the t–channel,
V.S. F., L.N. Lipatov, 1998
M. Ciafaloni, G. Camici, 1998

and for arbitrary t and any possible colour state in the t–channel
V. S. F., D. A. Gorbachev, 2000
V. S. F., R. Fiore, 2005
For phenomenological applications, the most interesting is the Pomeron. But from theoretical point of view the gluon channel (antisymmetric colour octet, or adjoint representation, in the t-channel) is even more important, first of all because of the gluon reggeization. It requires fulfillment of the bootstap relations

$$\langle AA'|e^{Y\hat{\mathcal{K}}}|BB'\rangle = \langle AA'|BB'\rangle\ e^{Y\omega(t)},$$

in the antisymmetric adjoint representation; $\omega(t)$ is the gluon trajectory.
Now fulfillment of these relations is proved in the NLO.

Introduction

But there are at least two other reasons for significance of the kernel of the BFKL equation for the adjoint representation.
One is related to the BKP equation
J. Bartels,1980
J. Kwiecinski, M. Praszalowicz, 1980
-the generalization of the BFKL equation to bound states consisting of three and more reggeized gluons, in particular the C-odd three gluon system — Odderon. The colour octet BFKL kernel appears in the BKP equation for the odderon because any pair of the three reggeized gluons are in the colour octet state.
Recently, another application of the BFKL approach, related to the BDS ansatz
Z. Bern, L. J. Dixon and V. A. Smirnov, 2005
was extensively developed

Introduction

Verification of the BDS ansatz for the inelastic amplitudes in $N = 4$ SUSY and calculation of the remainder factor —
J. Bartels, L. N. Lipatov, A. Sabio Vera, 2009
L. N. Lipatov and A. Prygarin, 2011
It was demonstrated that the BDS amplitude $M_{2\to4}^{BDS}$ should be multiplied by the factor containing the contribution of the Mandelstam cuts, and this contribution was found in the LLA.

Infrared safety of the kernels for odderon and remainder factor

Proper BFKL kernel for the adjoint representation is not infrared safe.

Remind tat the kernel is given by the sum

$$\widehat{\mathcal{K}} = \widehat{\omega}_1 + \widehat{\omega}_2 + \widehat{\mathcal{K}}_r$$

where the trajectories $\widehat{\omega}_i$ and "real" part $\widehat{\mathcal{K}}_r$ separately are infrared singular. In the LO $\widehat{\mathcal{K}}_r$ for singlet and adjoint kernels differ by the coefficient $1/2$.

Cancelation of infrared divergencies in the singlet channel implies the singularity in the adjoint one.

But the kernels for odderon and remainder factor do not coinside neither with the proper BKKL kernel, nor with each other.

Infrared safety of the kernels for odderon and remainder factor

In the octet case trajectories must be taken with the coefficient $1/2$ (because there are three reggeized gluons, each with its trajectory, and three paired interactions between them), so that the odderon pair interaction kernel is

$$\widehat{\mathcal{K}}_{12} = \frac{1}{2}\left(\widehat{\omega}_1 + \widehat{\omega}_2\right) + \widehat{\mathcal{K}}_r$$

In the LO this kernel differs from the colour singlet kernel only by the coefficient $1/2$. The kernel for the remainder factors is

$$\widehat{\mathcal{K}} = \left(\widehat{\omega}_1 + \widehat{\omega}_2 - \widehat{\omega}\right) + \widehat{\mathcal{K}}_r$$

where $\widehat{\omega}_{12}$ is the gluon trajectory for the total momenta, and is infrared safe also.

It is important that the singular part of the trajectory does not depend on momenta, and the singularities of the real parts of the singlet and octet kernels differ by the coefficient $1/2$, both in the leading and in the next-to leading orders.

Infrared safety of the kernels for odderon and remainder factor

It means that the kernels for odderon and remainder factor remain infrared safe in the NLO.

It occurs possible to perform explicit cancelation of the infrared singularities and to write the kernels in physical 2-dimensional space of transverse momenta.

V. S. F., L. N. Lipatov, 2011

J. Bartels, L. N. Lipatov and G. P. Vacca, 2012

Important property of the LO kernels is conformal invariance

–in coordinate space for odderon kernel,

–in momentum space for the remainder factor kernel.

Supposing conformal invariance of the NLO kernels one can find easily their eigenvalues.

For remainder factor kernel the limit of large momentum momentum transfer is suitable for this purpose.

V.S. F., L.N. Lipatov, 2011

Eigenvalues of the kernel for remainder factor

In the limit

$$|q_1| \sim |q_1'| \ll |q| \approx |q_2| \approx |q_2'| \,,$$

with the denotation $\vec{q}_1$ and $\vec{q}_1{}'$ by $\vec{p}$ and $\vec{p}\,'$, respectively, the kernel for the reminder factor in $N=4$ SUSY takes the form

$$K(\vec{p},\vec{p}\,') = -\delta^2(\vec{p}-\vec{p}\,')\,|p|^2\,\frac{\alpha N_c}{4\pi^2}\,\left(\left(1-\frac{\alpha N_c}{2\pi}\zeta(2)\right)\right.$$

$$\left.\times\int d^2p\,'\,\left(\frac{2}{|p\,'|^2}+\frac{2(\vec{p}\,',\vec{p}-\vec{p}\,')}{|p\,'|^2|p-p\,'|^2}\right)-3\alpha\,\zeta(3)\right)$$

$$+\frac{\alpha N_c}{4\pi^2}\,\left(1-\frac{\alpha N_c}{2\pi}\zeta(2)\right)\,\left(\frac{|p|^2+|p\,'|^2}{|p-p\,'|^2}-1\right)+\frac{\alpha^2 N_c^2}{32\,\pi^3}\,R(\vec{p},\vec{p}\,')\,,$$

where

Eigenvalues of the kernel for remainder factor

$$R(\vec{p},\vec{p}\,') =$$

$$\left(\frac{1}{2} - \frac{|p|^2+|p\,'|^2}{|p-p\,'|^2}\right)\ln^2\frac{|p|^2}{|p\,'|^2} - \frac{|p|^2-|p\,'|^2}{2|p-p\,'|^2}\ln\frac{|p|^2}{|p\,'|^2}\ln\frac{|p|^2|p\,'|^2}{|p-p\,'|^4}$$

$$+4\frac{[\vec{p}\times\vec{p}\,']^2}{(\vec{p}-\vec{p}\,')^2}\int_0^1 dx\,\frac{1}{|(1-x)p+xp\,'|^2}\ln\frac{(1-x)|p|^2+x|p\,'|^2}{x(1-x)|p-p\,'|^2}\,.$$

Due to the rotational and dilatational invariance of the kernel its eigenfunctions have the simple form

$$\Phi_{\nu n}(\vec{p}) = |p|^{2i\nu}e^{i\phi n}\,, \tag{1}$$

where ϕ is the angle of the transverse vector $\overrightarrow{p}$ with respect to the axis x. Note, that ν is real and n is integer.
Corresponding eigenvalues are

Eigenvalues of the kernel for remainder factor

$$\omega(\nu,n) = -a\left(E_{\nu n} + a\,\epsilon_{\nu n}\right)\;,\;\; a = \frac{\alpha N_c}{2\pi}\;,$$

where $E_{\nu n}$ is the "energy" in the leading approximation

$$E_{\nu n} = -\frac{1}{2}\,\frac{|n|}{\nu^2+\frac{n^2}{4}} + \psi(1+i\nu+\frac{|n|}{2}) + \psi(1-i\nu+\frac{|n|}{2}) - 2\psi(1)$$

and the next-to-leading correction $\epsilon_{\nu n}$ can be written as follows

$$\epsilon_{\nu n} = -\frac{1}{4}\left(\psi''(1+i\nu+\frac{|n|}{2}) + \psi''(1-i\nu+\frac{|n|}{2}) + \frac{2i\nu\left(\psi'(1-i\nu+\frac{|n|}{2}) - \psi'(1+i\nu+\frac{|n|}{2})\right)}{\nu^2+\frac{n^2}{4}}\right)$$

$$-\zeta(2)\,E_{\nu n} - 3\zeta(3) - \frac{1}{4}\,\frac{|n|\left(\nu^2-\frac{n^2}{4}\right)}{\left(\nu^2+\frac{n^2}{4}\right)^3}\;,\;\; \psi(x) = (\ln\Gamma(x))'.$$

Eigenvalues of the kernel for remainder factor

Here the ζ-functions are expressed in terms of polylogarithms

$$Li_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n}\,, \;\; \zeta(n) = Li_n(1)\,. \tag{2}$$

Note, that $\omega(\nu, n)$ has the important property

$$\omega(0,0) = 0\,. \tag{3}$$

It is in an agreement with the existence of the eigenfunction $\Phi = 1$ with a vanishing eigenvalue, which is a consequence of the bootstrap relation.

Corrections to the reminder factor

Using obtained eigenfunctions one can easily construct the Green function. This Green function allows to calculate the remainder functions R_n for an arbitrary number of external legs in the regions, where there are Mandelstam's cuts corresponding to the composite states of two reggeized gluons. In particular, the remainder function R_6 for the gluon transition $2 \to 4$ was found
V.S. F., L.N. Lipatov, 2011
with the next-to-leading accuracy. The obtained result in three loops is in an agreement with the recently suggested anzatz
L. J. Dixon, J. M. Drummond and J. M. Henn, 2011
for the remainder function. This anzatz allows to construct the product of corresponding impact-factors in the next-to-next-to-leading approximation.
The obtained result allowed also to calculate the collinear anomalous dimension in the Mandelstam region explicitly in one loop and its leading and next-to-leading singularities in all

Check of the conformal invariance of the kernel for remainder factor

All these results were obtained supposing conformal invariance of the kernel for remainder factor. But such kernel obtained from the ”standard” BFKL kernel is not conformal invariant. In fact, it was supposed that the ”standard” BFKL kernel is defined in ”a bad scheme”, and there is ”a good scheme”, which gives a confirmal invariant kernel. Remind that in the NLO there is an ambiguity, analogous to the well known ambiguity of the NLO anomalous dimensions, because it is possible to redistribute radiative corrections between the kernel and the impact factors. It permits to make transformations

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} - \alpha_s[\hat{\mathcal{K}}^{(B)}, \hat{U}]$$

conserving the LO kernel $\hat{\mathcal{K}}^{(B)}$ (which is fixed in our case by the requirement of conformal invariance) and changing the NLO part of the kernel.

Check of the conformal invariance of the kernel for remainder factor

Therefore the "standard" and conformal invariant kernels has to be connected by such transformation. In principle, one can write a formal expression for the operator $\hat{U}$, since the difference between these kernels is known. Indeed, let us denote the diffetence as $\hat{\Delta}$ and the Born kernel $\hat{\mathcal{K}}^B$ eigenstates $|\mu\rangle$, and corresponding eigenvalues ω_μ^B. Then, if $\hat{\Delta} = \alpha_s \left[\hat{\mathcal{K}}^B, \hat{U}\right]$,

$$\left(\omega_{\mu'}^B - \omega_\mu^B\right) \langle\mu'|\alpha_s\hat{U}|\mu\rangle = \langle\mu'|\hat{\Delta}|\mu\rangle.$$

It is seen from here that the operator $\hat{U}$ exists only if the operator $\hat{\Delta}$ has zero matrix elements between states of equal energies. If so, supposing that the states $|\mu\rangle$ form a complete set,

$$\langle\mu'|\alpha_s\hat{U}|\mu\rangle = \sum_{\mu,\mu'} \frac{|\mu'\rangle\langle\mu'|\hat{\Delta}|\mu\rangle\langle\mu|}{\omega_{\mu'}^B - \omega_{\mu'}^B}.$$

Check of the above relation is in progress.

Summary

- The kernel of the BFKL equation for the adjoint representation has a lot of applications.
- For odderon and remainder factor this kernel is infrared stable.
- The LO kernel for reminder factor is conformal invariant with respect to Möbius transformations in transverse momentum space.
- Assuming conformal invariance of the NLO kernel its eigenfunctions were calculated, remainder function R_6 for the gluon transition $2 \to 4$ was found with the next-to-leading accuracy and the collinear anomalous dimension was calculated.
- Check of this assumption is in progress.