Partonic Orbital Angular Momentum

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The Sum Rule

Nucleon Spin

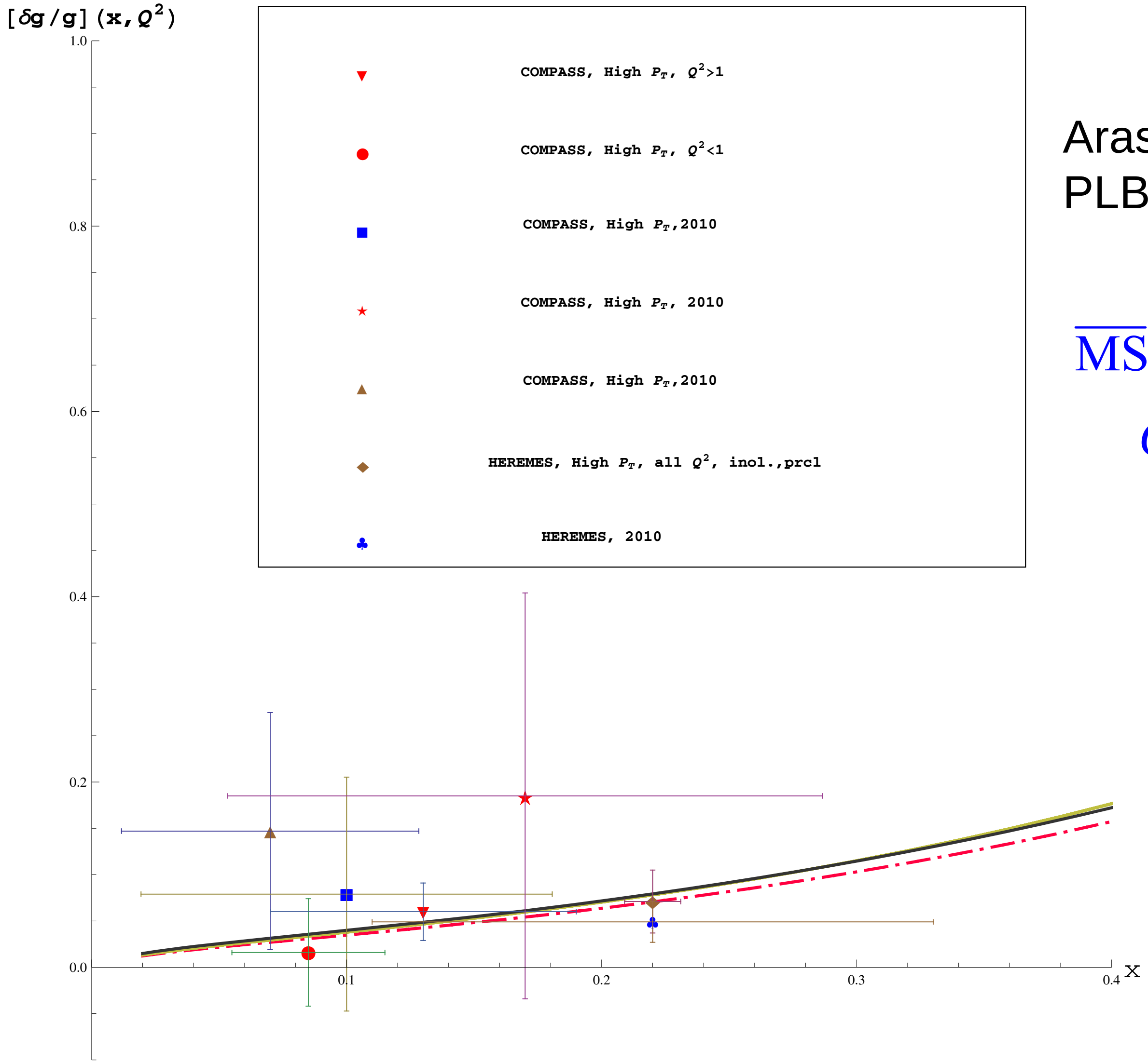
$$\frac{1}{2}=\frac{1}{2}\Delta\Sigma+\Delta G+L_{q,g}$$

$\Delta\Sigma$: The Quark contribution to the nucleon spin. Accurately known: about 0.4

ΔG: Less accurate, few measurements, mostly on $\frac{\delta g\left(x,Q^2\right)}{g\left(x,Q^2\right)}$

Large Error bars. $\delta g\left(x,Q^2\right)$ turns out to be small

But that does not imply, by itself, that ΔG Is small



Arash, Taghavi, Shahve,
PLB **691**, 2010

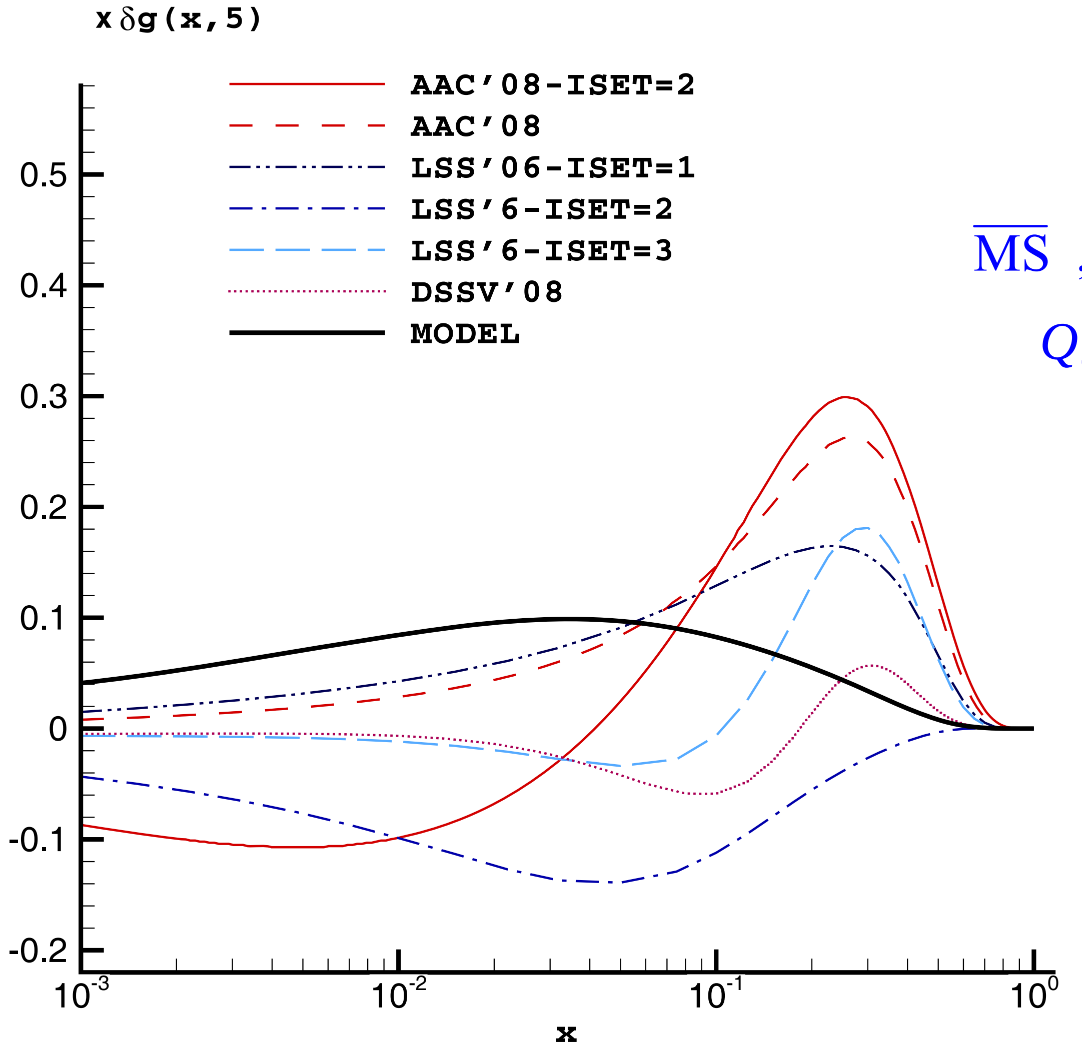
$\overline{\text{MS}}$, $\Lambda_{QCD} = 0.22$ GeV

$Q_0^2 = 0.283\, GeV^{\,2}$

$$\begin{pmatrix} \delta M_S(n,Q^2) \\ \delta M_G(n,Q^2) \end{pmatrix} = \{\mathbf{L}^{-(\frac{2}{\beta_0})\delta\hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi}\hat{\mathbf{U}}\mathbf{L}^{-(\frac{2}{\beta_0})\delta\hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi}L^{-(\frac{2}{\beta_0})\delta\hat{P}^{(0)n}}\hat{\mathbf{U}}\} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where $\mathbf{L} \equiv \alpha_s(Q^2)/\alpha_s(Q_0^2)$, and $\delta\hat{P}^{(0)n}$ is 2×2 singlet matrix of splitting functions, given

$$\delta\hat{P}^{(0)n} = \begin{pmatrix} \delta P_{qq}^{(0)n} & 2f\delta P_{qg}^{(0)n} \\ \delta P_{gq}^{(0)n} & \delta P_{gg}^{(0)n} \end{pmatrix},$$



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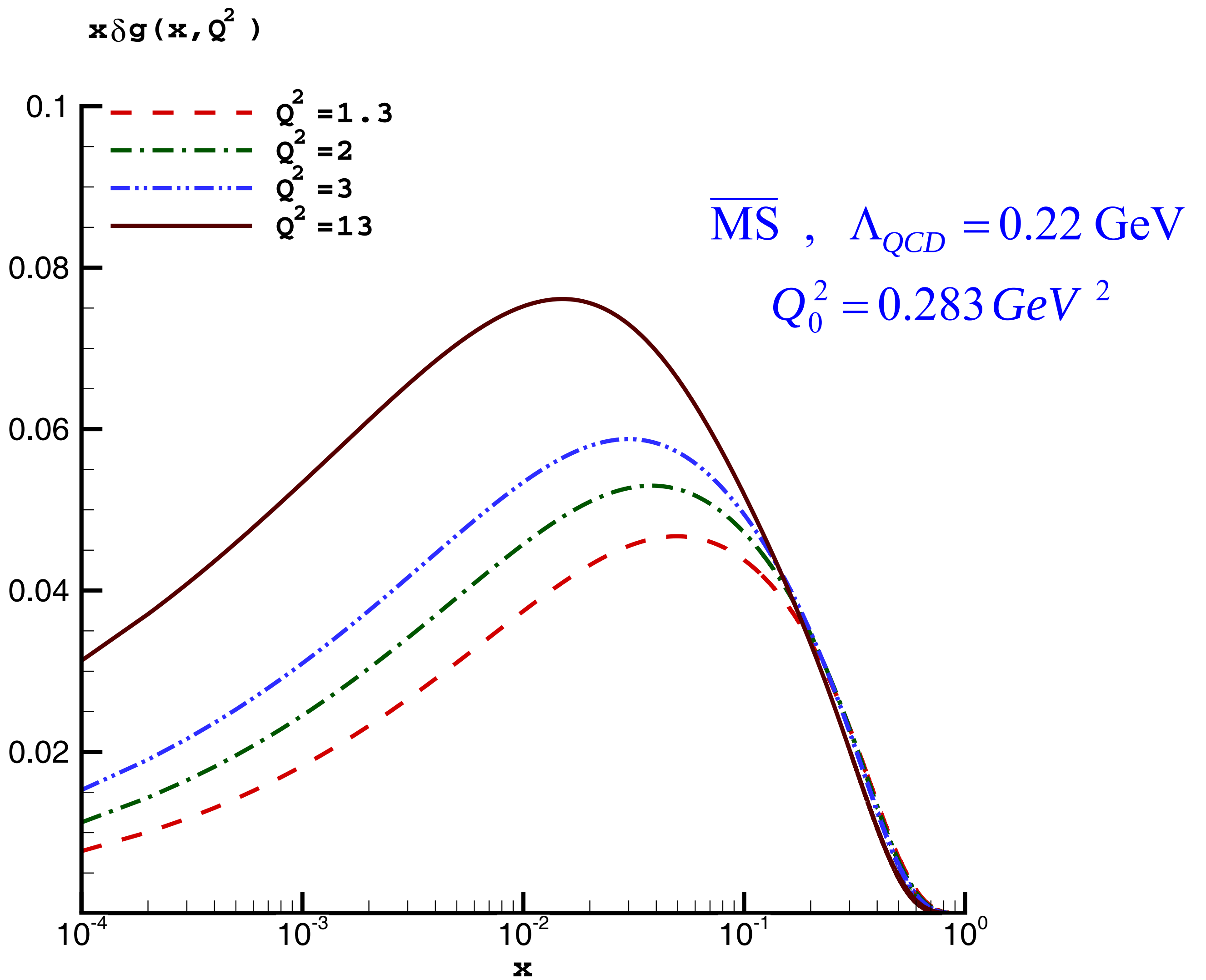
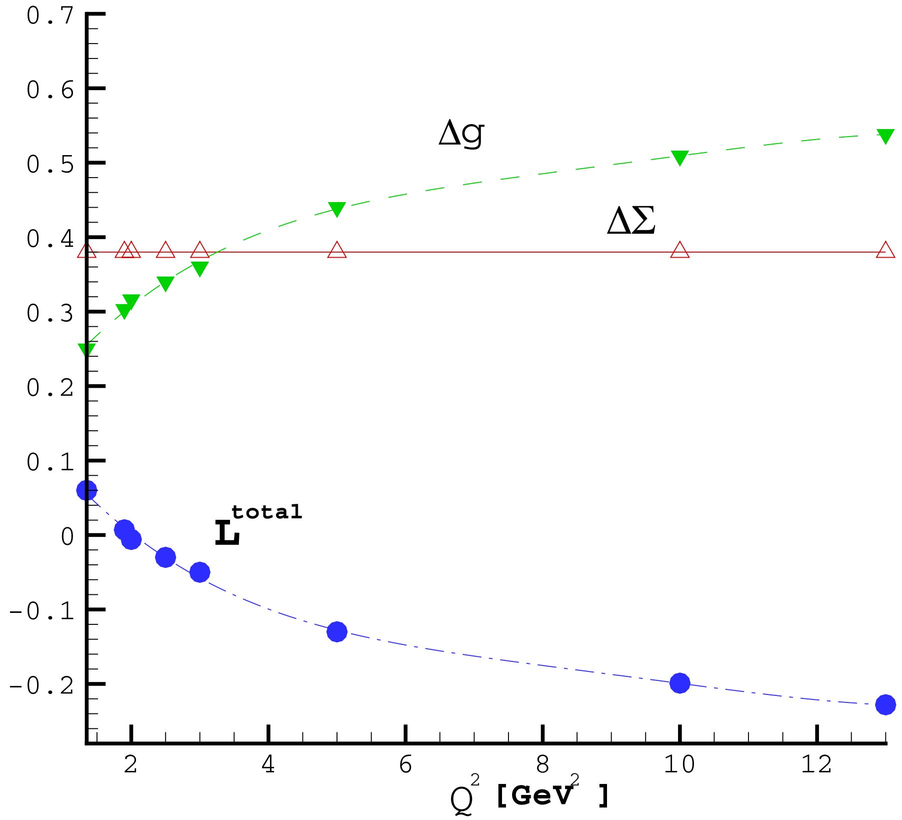

xδg(x,Q²)
0.1
0.08
0.06
0.04
0.02
Q² =1.3
Q² =2
Q² =3
Q² =13
$\overline{\mathrm{MS}}$, $\Lambda_{QCD} = 0.22$ GeV
$Q_0^2 = 0.283\, GeV^2$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

The Picture That Emerges



L^{total}

Not a real calculation. It only implied using the sum Rule. It does, however, showes a distinct feature: It is Negative and grows with Q^2

Pointed out long time ago:

P. Ratcliffe, Phys.Lett. **B192**,180 (1987).

Can we say something about L

One could write the nucleon spin sum Rule as:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q + \mathcal{L}_g$$

Jaffe and Manohar,
Nucl.Phys. *B* **337**(1990)

And use a light cone framework and light cone gauge. For a spin up state the terms are defined as

$$\begin{cases} \Delta\Sigma = \left\langle P\uparrow \right| \int d^3x\, \bar{\psi}\gamma^3\gamma_5\psi \left| P\uparrow \right\rangle \\ \Delta G = \left\langle P\uparrow \right| \int d^3x \left(E^1A^2 - E^2A^1\right) \left| P\uparrow \right\rangle \\ \mathcal{L}_q = \left\langle P\uparrow \right| i\int d^3x\, \psi^\dagger \left(x^1\partial^2 - x^2\partial^1\right)\psi \left| P\uparrow \right\rangle \\ \mathcal{L}_g = \left\langle P\uparrow \right| \int d^3x E^i \left(x^2\partial^1 - x^1\partial^2\right) A^i \left| P\uparrow \right\rangle \end{cases}$$

ψ : quark field
E^i : Gluon Electric Fiels
A^μ : gauge potential

summed over quark flavors
for flavor singlet quantities

But, Except for $\Delta\Sigma$, Other terms are not gauge invariant

Ji's Decomposition

Replace: $\partial^{\mu} \rightarrow D^{\mu}$ gives L_q and define

$$J_g = \left\langle P \uparrow \middle| \int d^3x \left[\mathbf{x} \times \left(\mathbf{E} \times \mathbf{B} \right) \right]_3 \middle| P \uparrow \right\rangle$$

x.D.Ji,*etal* Phys.Rev.Lett.78(1997)

$$\Rightarrow \quad \frac{1}{2}\Delta\Sigma + L_q + J_g = \frac{1}{2}$$ Ji's decomposition

All terms are gauge invariant,but gluon's total angular momentum is not further decomposed

The key quantities for experimental determination of J_q and J_g are the Generalized Parton Distributions. They show up in the cross section of DVCS and are central to Ji's quark angualr momentum sum rule.

$$L^q = \int dx \int d^2b \left(xH^q(x,b) + xE^q(x,b) - \tilde{H}^q(x,b) \right)$$

In the local limit GPD's reduce to Form Factors

$\Rightarrow$ Two FF's: spin Flip $B\left(q^2\right)$ – analog to Pauli FF

A measure of Orbital Angular Momentum of quark and gluon

Spin conserving FF, $A\left(q^2\right)$ analog analog to Dirac FF

gives the momentum fraction

They are related to matrix element of Energy – momentum tensor

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\left[A^{q,g}_{20}(t)\gamma^{(\mu\bar{P}^{\nu})} + B^{q,g}_{20}(t)\frac{P^{(\gamma i\sigma^{\nu})_{\alpha}}\Delta_{\alpha}}{2M}\right]U(P) + \cdots$$

Substitute into Nucleon matrix element, one finds

$$J_{q,g} = \frac{1}{2}\left[A^{q,g}_{20}(0) + B^{q,g}_{20}(0)\right]$$

$A^{q,g}_{20}(0)$ are related to the unpolarized quark and gluon PDF as

$$A^{q}_{20} = \int_0^1 x \sum_{q,\bar{q}} q(x)dx \equiv \langle x\rangle^{q} \qquad A^{g}_{20} = \int_0^1 xg(x) = \langle x\rangle^{g}$$

B's are the second moments of unpolarized spin-flip GPD's in the forward limit

Constraints:

$A_{20}^{q}(0)+A_{20}^{g}(0)=1$, $\boxed{B_{20}^{q}(0)+B_{20}^{g}(0)=0}$ ← Nontrivial

$$\Rightarrow \text{3 cases}:\left\{B_{20}^{q}=B_{20}^{g}=0,\quad B_{20}^{q}=-B_{20}^{g},\quad B_{20}^{u}=B_{20}^{d}=B_{20}^{g}\right.$$

Lattice calculations support the second LHPC Hagler, et. al PRD 77, 094502 (2008)

Also, See K.F.Liu, χQCD Coll. arXiv: 1203.6388 (2012)

$$B_{20}^{q}=0.064\pm0.022 \quad , \quad B_{20}^{g}=-0.059\pm0.052$$

This All boils down to:

$\Rightarrow$ the total angular momenta J_q and J_g evolve exactly as unpolarized parton distributions do

These parton distributions are determined by many groups, including in the valon model, which is essentially reduces to determination of the internal structure of a constituent quark

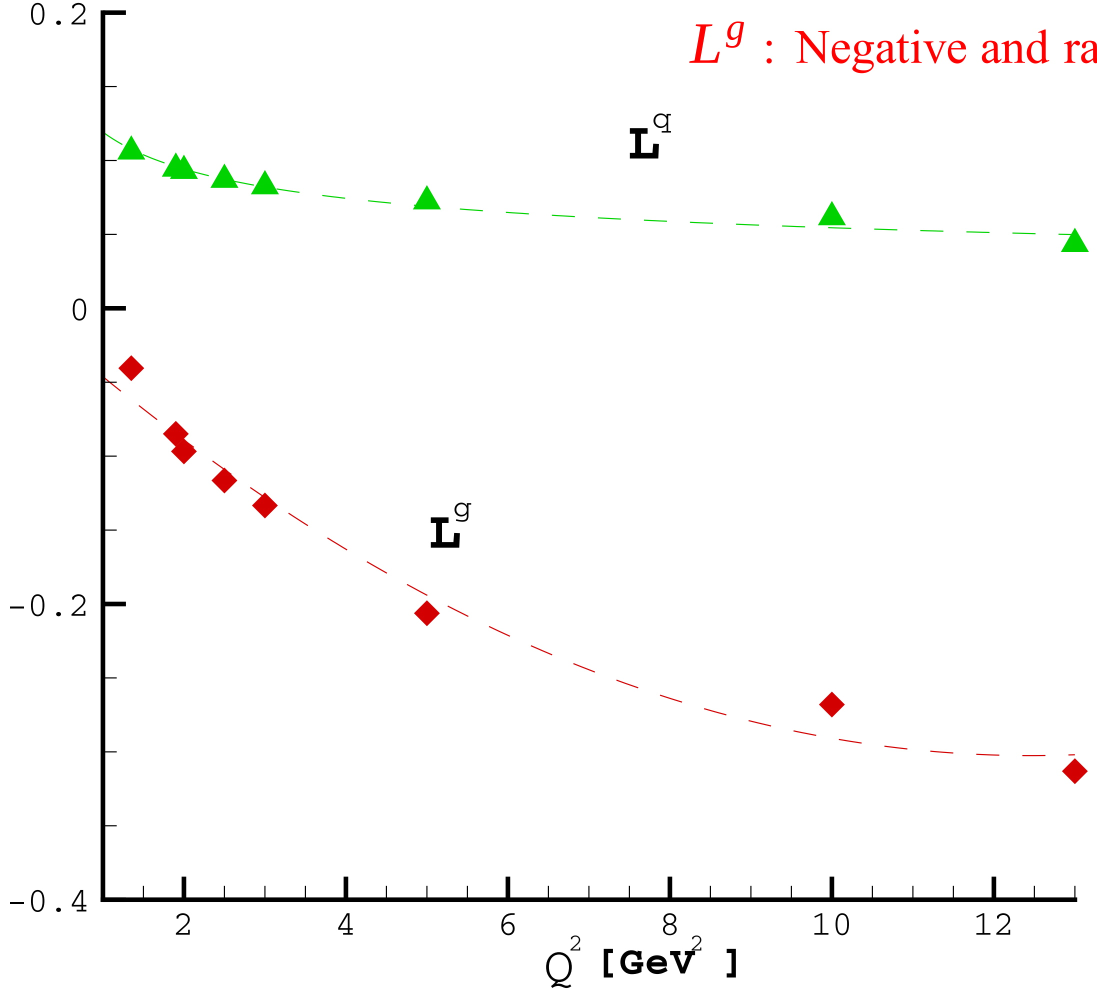
We have utilized them *Arash, Taghavi, JHEP* 07 (2007), *PLB* 668 (2008)
Shahveh, Taghavi, Arash, PLB 691 (2010)

Using $L_g = J_g - \Delta G$

We do have both $\delta g\left(x, Q^2\right)$ and ΔG from our model.

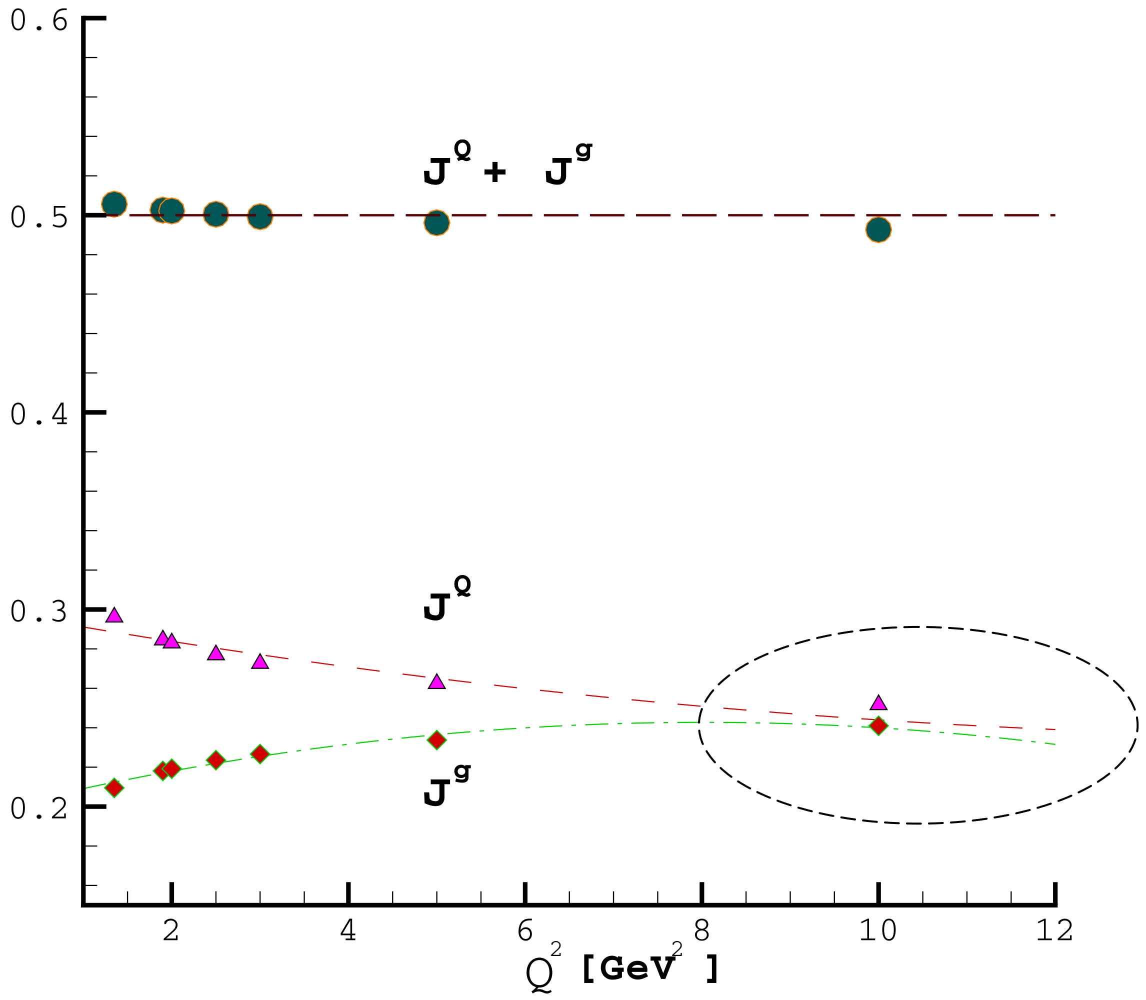
L^q : while being positive, it is small, ≤ 0.1

L^g : Negative and rapidly decreases

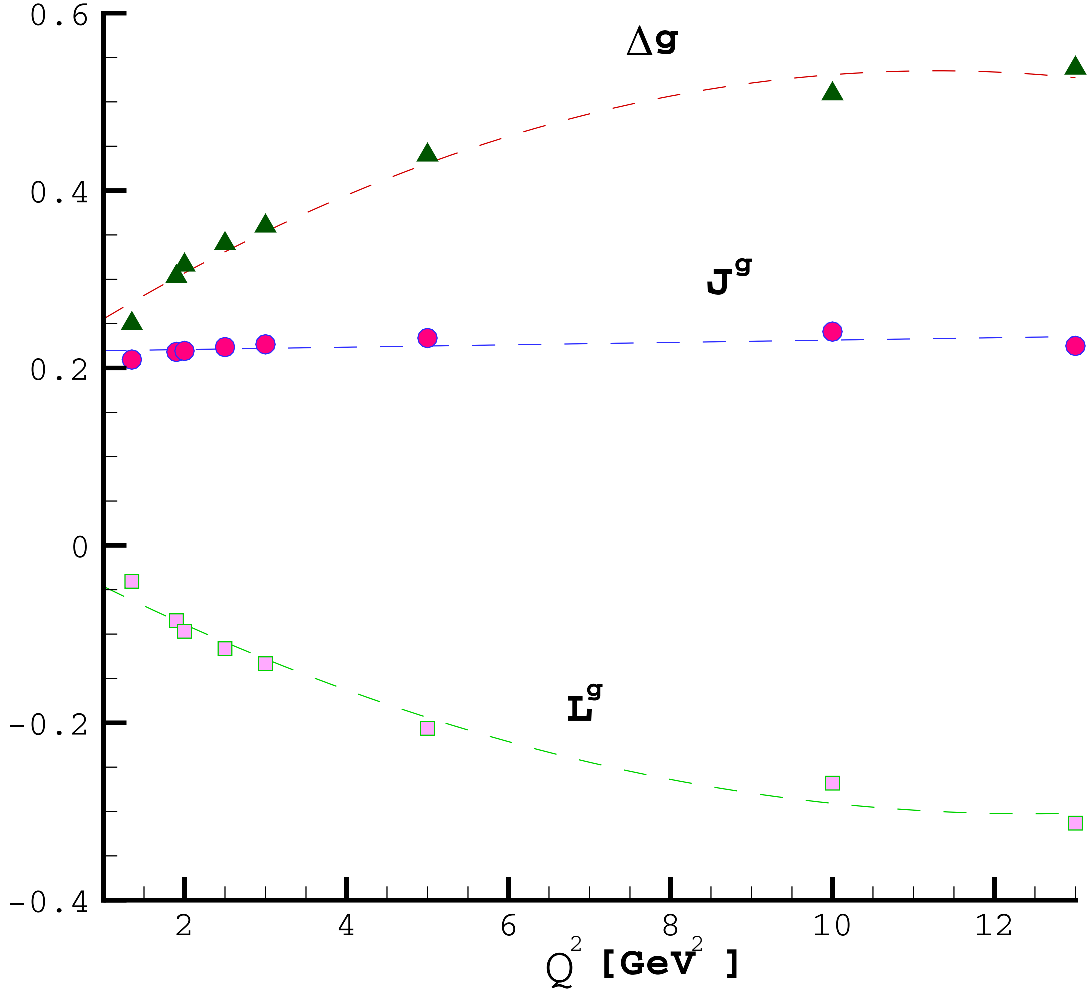


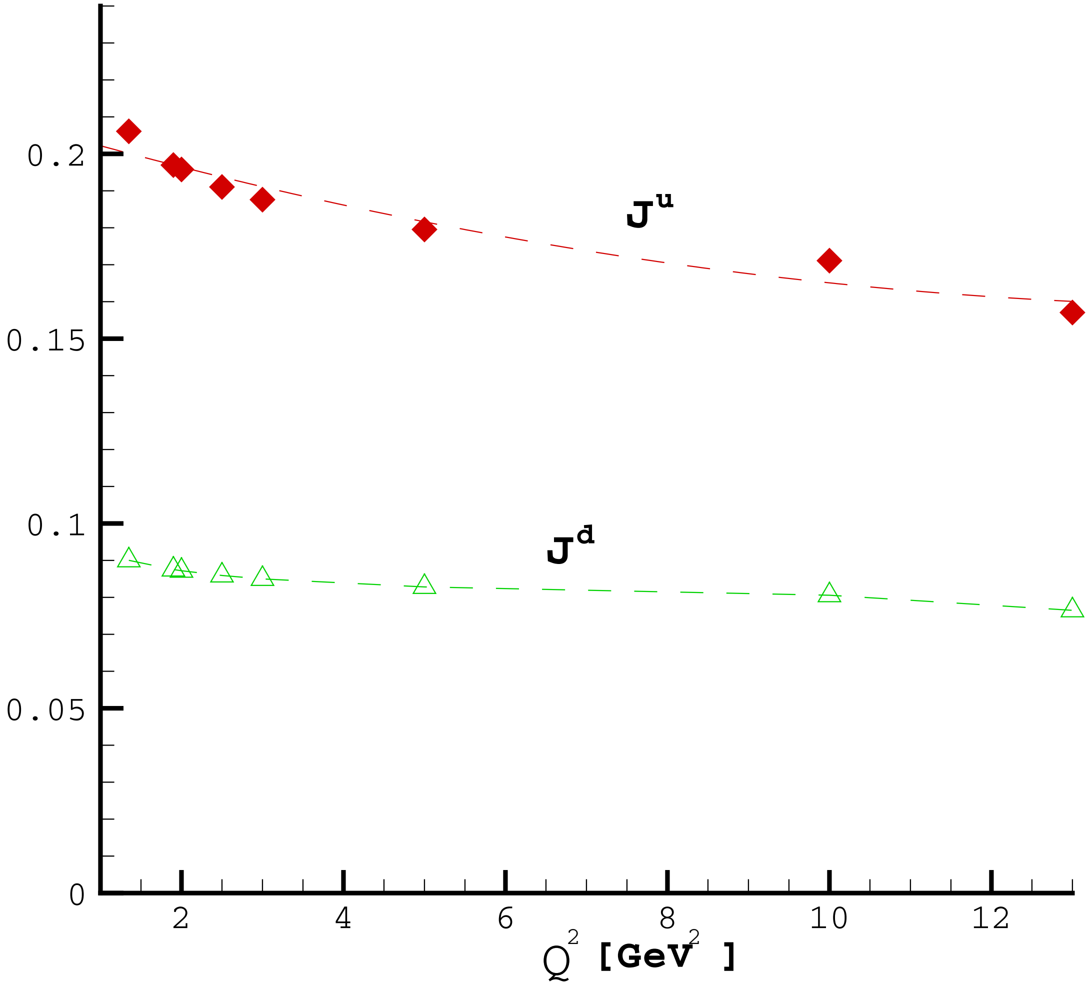
So, With these results for $L^{q,g}$ do we still satisfy proton spin Sum Rule

$$\frac{1}{2} = J^{q} + J^{g} \,?$$ *yes! we have checked.*



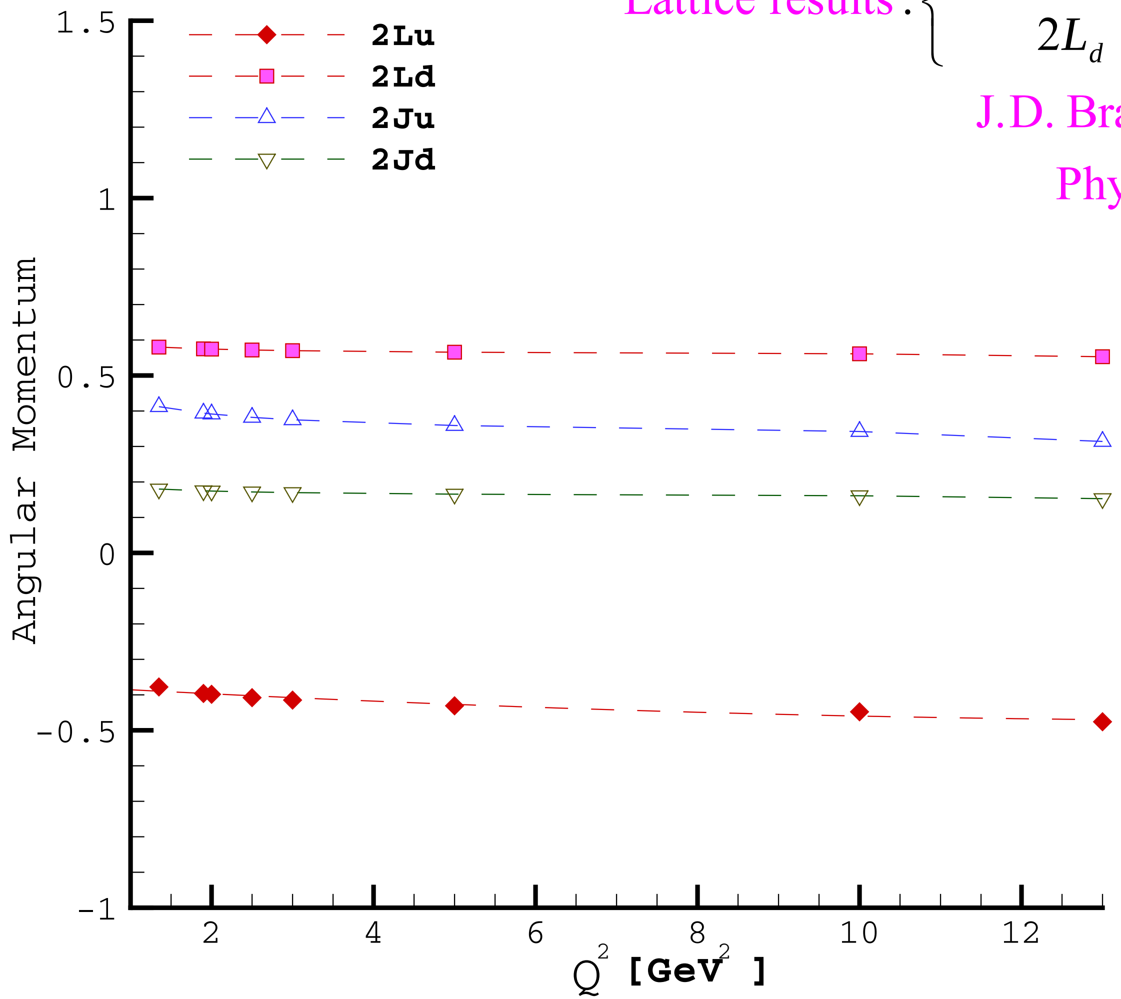
The emerging picture looks like this

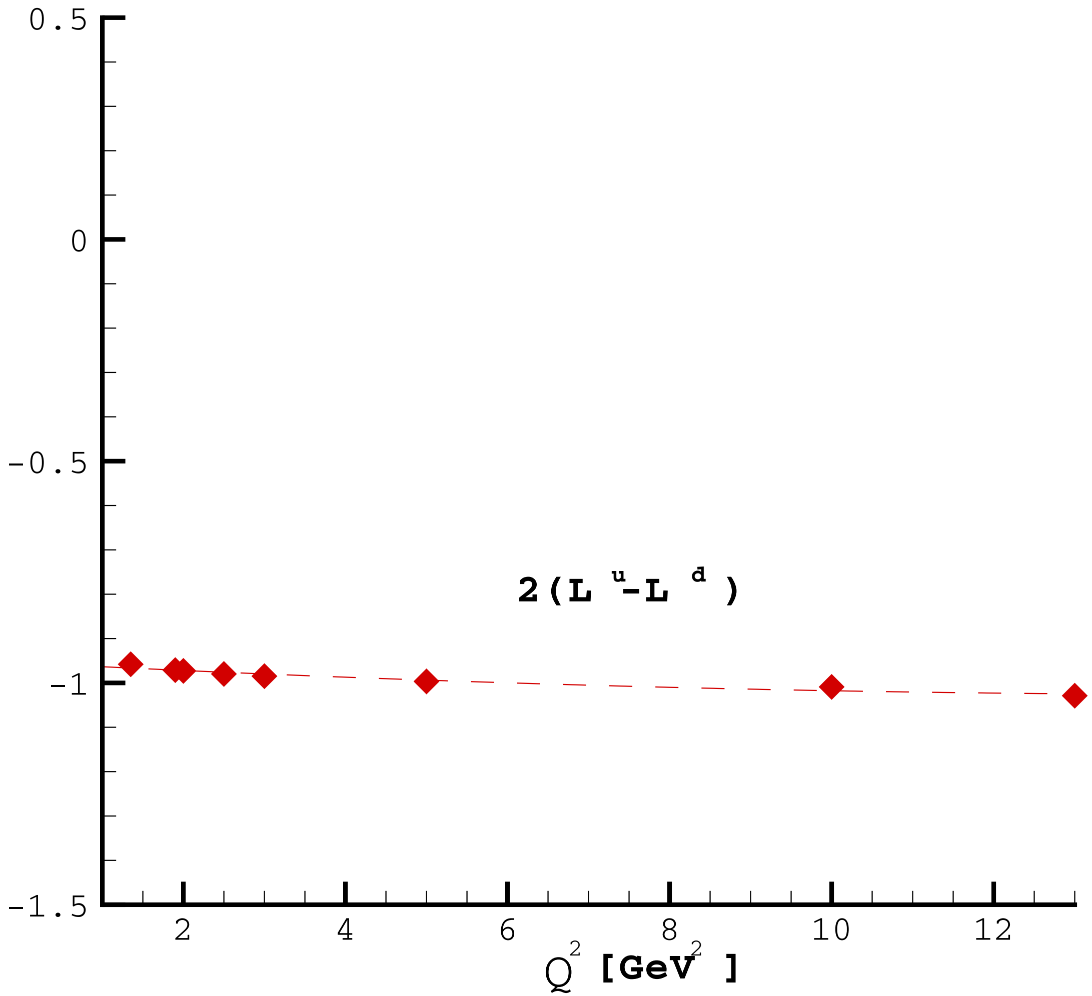



0.2
0.15
0.1
0.05
0
2
4
6
8
10
12
J^u
J^d
Q^2 [GeV^2]

Lattice results : $\begin{cases} 2L_u = -0.350 \pm 0.036 \pm 0.017 \\ 2L_d = 0.410 \pm 0.035 \end{cases}$

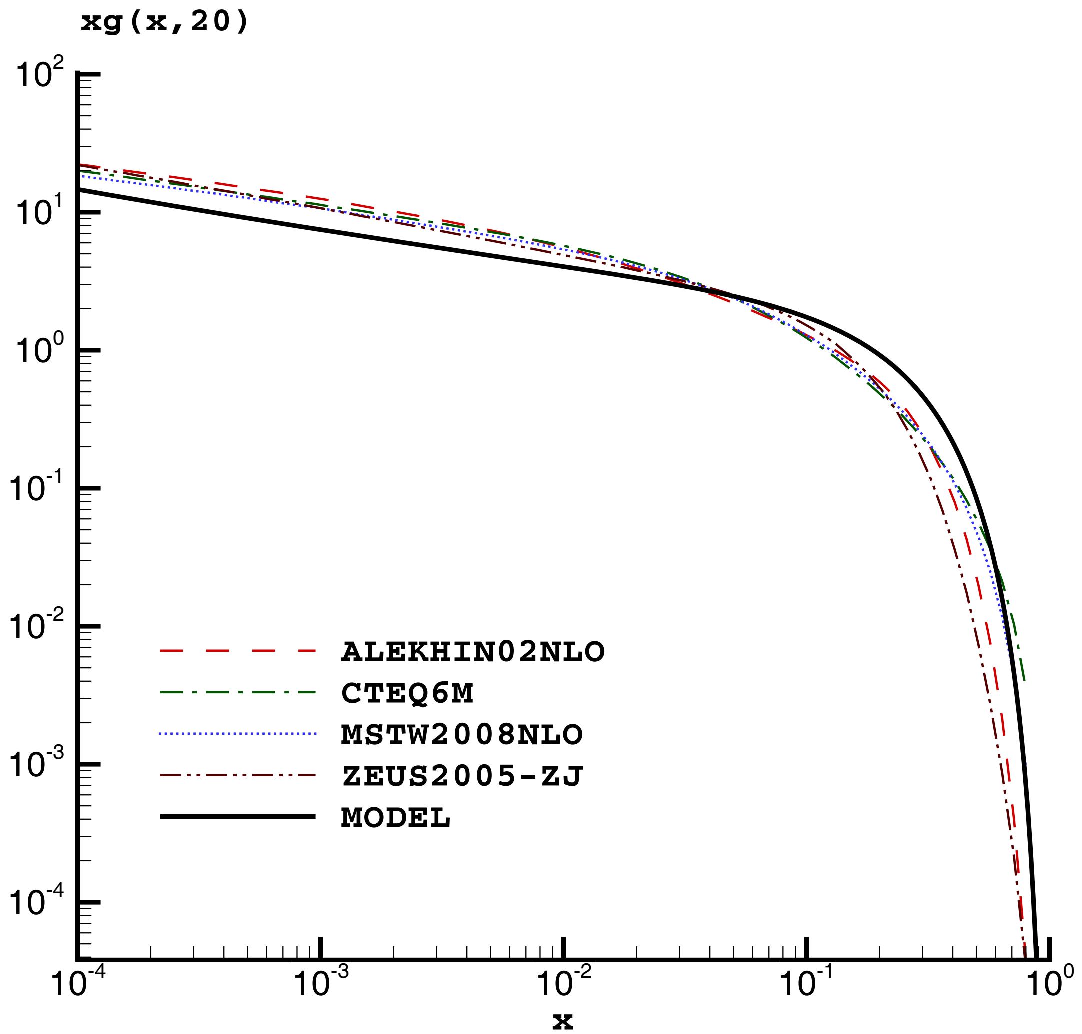
J.D. Bratt *et.al*., LHPC, Phys. Rev. D82 (2010) 094502



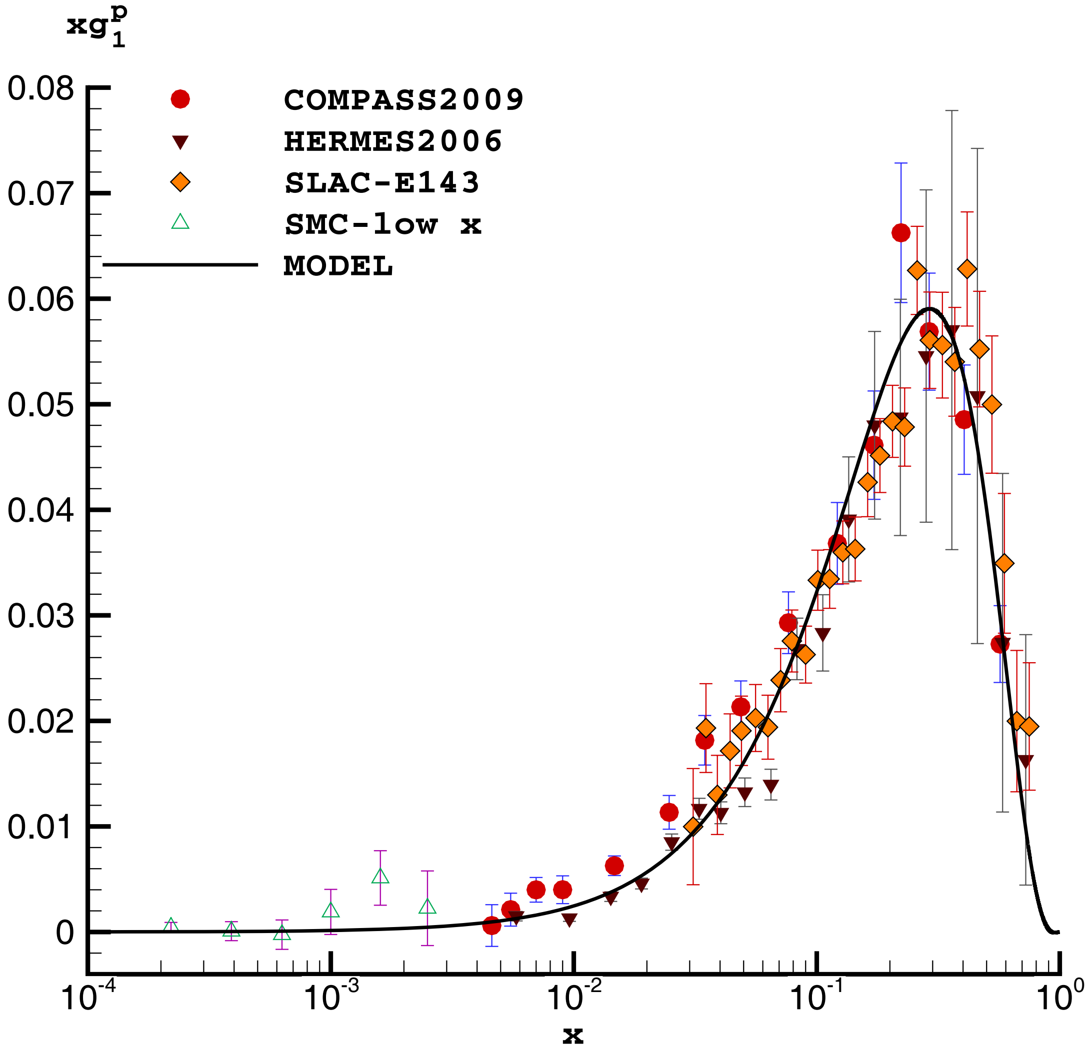

0.5
0
-0.5
-1
-1.5
2(L^u-L^d)
2
4
6
8
10
12
Q^2 [GeV^2]

Conclusions:

- Quark OAM contribute positively to the nucleon spin content, but it is small. ≈ 0.1
- Gluon OAM is about is negative and decreases as Q^2 increases. This behavior is compensated by increase in $\Delta G\left(Q^2\right)$ as a result J^g is independent of Q^2.
- It is also evident that above Q^2=5 GeV^2 the total angular momentum of quark and gluon approach to a same number.



xg(x,20)
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
ALEKHIN02NLO
CTEQ6M
MSTW2008NLO
ZEUS2005-ZJ
MODEL
10^-4
10^-3
10^-2
10^-1
10^0
x


xg_1^p
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
COMPASS2009
HERMES2006
SLAC-E143
SMC-low x
MODEL
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
x