Extracting $|V_{ub}|$and $B \to X_s\gamma$ from global fits

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Inclusive $|V_{ub}|$ at BABAR and Belle

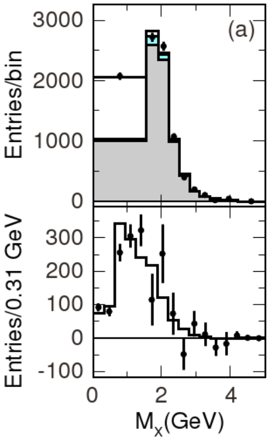
Very rough scetch of an inclusive $|V_{ub}|$ measurement

1. Measurement of partial branching fraction $\Delta\mathcal{B}(B \to X_u \ell\nu)$
   - ⋆ Select phase space regions more-or-less enriched with $B \to X_u \ell\nu$

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\tau_B \Delta\Gamma_{\text{theory}}}}$$

2. Input to $\Delta\Gamma_{\text{theory}}$
   - ⋆ m_b from $B \to X_c \ell\nu$ or elsewhere
   - ⋆ Shape function model (tested against $B \to X_s\gamma$)

[arXiv:1112.0702 (hep-ex)]



Inclusive $|V_{cb}|$ follows a different strategy:

- Global fit to kinematic moments measured in $B \to X_c \ell\nu$ to extract $|V_{cb}|$, m_b and nonperturbative parameters

Global Fit Approach to $|V_{ub}|$ and $B \to X_s\gamma$

Employ strategy that proved successful for $|V_{cb}|$

- Determine $|V_{ub}|$, m_b and shape function (SF) simultaneously
- Combine different decay modes, measurements and experiments
  - ⋆ Different $B \to X_s\gamma$ spectra
    - Information about shape function, m_b and C_7
  - ⋆ Different $B \to X_u\ell\nu$ partial BFs (or spectra)
    - Information about $|V_{ub}|$, shape function and m_b
    - Differential spectra would be more powerful
  - ⋆ External constraints on m_b and shape function moments (from $B \to X_c\ell\nu$ or other) could also be incorporated

What we gain from a global fit

- Minimize uncertainties by making maximal use of all available data
- Consistent treatment of correlated uncertainties (experimental, theoretical, input parameters)

Master Formula for Differential Spectra

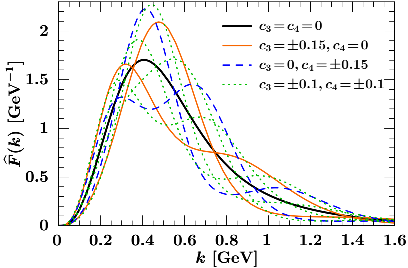
$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2 \left|C_7^{\mathrm{incl}}\right|^2 \int \mathrm{d}k\, \widehat{W}_{77}(E_\gamma; k)\, \widehat{F}(m_B - 2E_\gamma - k) + \cdots$$

$$\mathrm{d}\Gamma_u = |V_{ub}|^2 \int \mathrm{d}k\, \widehat{W}_u(p_X^-, p_X^+, E_\ell; k)\, \widehat{F}(p_X^+ - k) + \cdots$$

Normalization of spectra

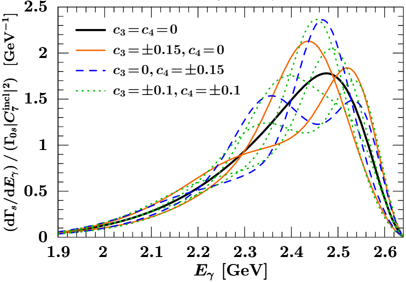
Differential shape (SF)

Different $\widehat{F}(k)$



$\Rightarrow$

Resulting $\mathrm{d}\Gamma_s/\mathrm{d}E_\gamma$ (leading power)



Regions of Phase Space

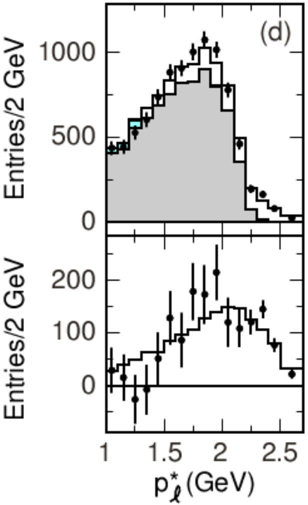
Different measurements probe different phase-space regions

- SF region: large E_γ, E_ℓ (near peak/endpoint)
  - ⋆ Experimentally clean(er), theoretically more difficult
- Local OPE region: small E_γ, E_ℓ, large q^2
  - ⋆ Suffers from large backgrounds, theoretically easier
- Something in between: $m_X \sim m_D$, moderately large E_γ, E_ℓ

⇒ Include as wide region as possible since there is no "golden" region

⇒ Need combination of optimal theory descriptions for each region

[arXiv:1112.0702 (hep-ex)]



Theory Inputs for $B \to X_s\gamma$

$$\Gamma_s \propto |V_{tb}V_{ts}^*|^2 m_b^2 \Big\{ |C_7^{\rm incl}|^2 \Big[\big(\widehat{W}_{77}^{\rm sing} + \widehat{W}_{77}^{\rm nons}\big) \otimes \widehat{F} + \sum_n W_{77,n} F_n^{\rm subl} \Big]$$

$$+ \sum_{i,j\neq 7} \Big[{\rm Re}(C_7^{\rm incl}) 2C_i \widehat{W}_{7i}^{\rm nons} + C_i C_j \widehat{W}_{ij}^{\rm nons} \Big] \otimes \widehat{F} + \cdots \Big\}$$

Leading C_7^2 contribution

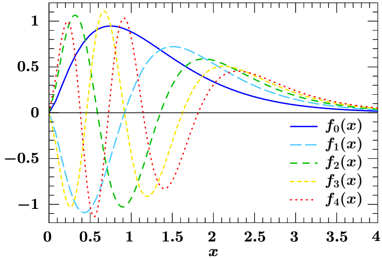
- Included at full NNLL+NNLO (in short-distance scheme, e.g. $1S$, kin, ...)
- $1/m_b$ subleading shape functions (can be) absorbed into leading one
  - ⋆ Have large impact on extracted value of m_b ($\sim 70\,\mathrm{MeV}$)

Contributions from other operators $\sim C_i C_7, C_i C_j$

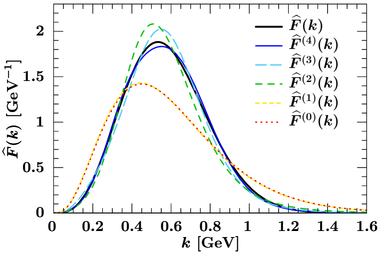
- Largest effects come from virtual corrections, are absorbed into $C_7^{\rm incl}$
  - ⋆ Important charm-mass effects only enter via SM prediction for $C_7^{\rm incl}$
- Remaining perturbative contributions included at NLO
  - ⋆ Some NNLO are known, but NLO already have very small effect on fit
- $C_{i\neq 7}$ fixed to SM values

Modeling the Shape Function

Basis



Expansion of Gaussian $\hat{F}(k)$



Design suitable orthonormal basis for $\hat{F}(k)$ (formally model independent)

$$\hat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} c_n f_n(x)\right]^2 \quad \text{with} \quad \int \mathrm{d}k\, \hat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

- Builds an orthonormal basis $f_n(x)$ on top of any given model function
- Keep terms up to $n \leq N$ as required by precision of data
- Experimental uncertainties and correlations can be properly captured by uncertainties and correlations in basis coefficients c_n

[Ligeti, Stewart, F Tackmann (2008)]

Residual Basis Dependence from Truncation

$$\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_{n=0}^{N} c_n f_n(x)\right]^2$$

In practice, series must be truncated

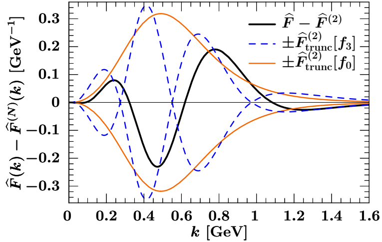
- Induces residual basis (model) dependence
- Truncation error scales as $1 - \sum_{n=0}^{N} c_n^2$

Optimal N and λ are determined from data

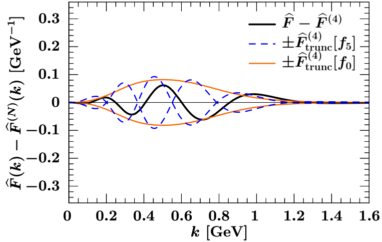
- Choose λ so series converges quickly
- Choose N so truncation error is small compared to exp. uncertainties
- Add more terms with more precise data

$\Rightarrow$ Must be careful not to "overtune"

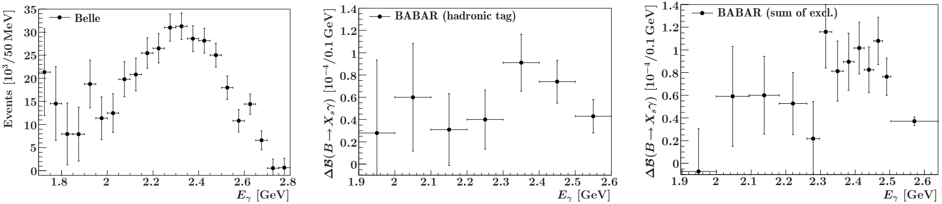
Truncation error at $N = 2$



Truncation error at $N = 4$



Current Inputs for $B \to X_s\gamma$ Fit



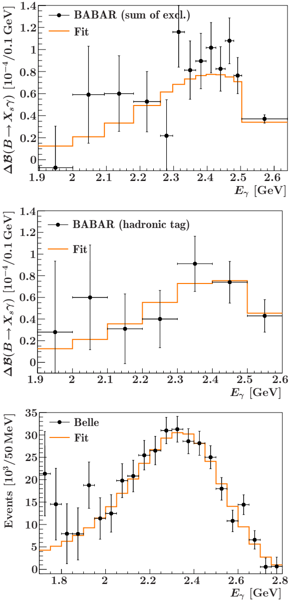
Belle $605\,\mathrm{fb}^{-1}$

- Thanks to Belle, especially Antonio Limosani, for providing the covariance matrix, experimental efficiency and resolution!
- Efficiency and resolution effects folded into theory predictions

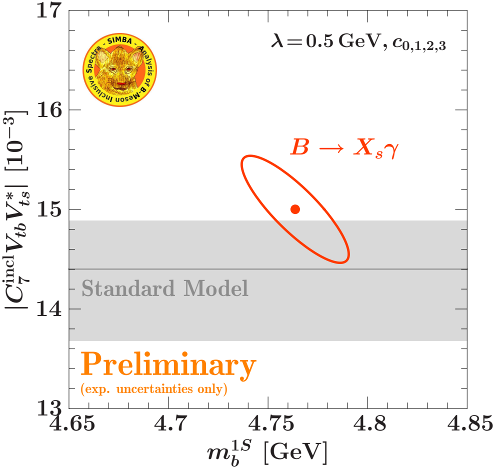
$B\!A\!B\!AR$ sum-over-exclusive-modes ($80\,\mathrm{fb}^{-1}$), hadronic tag ($210\,\mathrm{fb}^{-1}$)

- Correlations are available
- Spectra efficiency corrected, resolution not an issue
- Thanks to Francesca Di Lodovico and Henning Flächer

$B \to X_s\gamma$ Fit Results

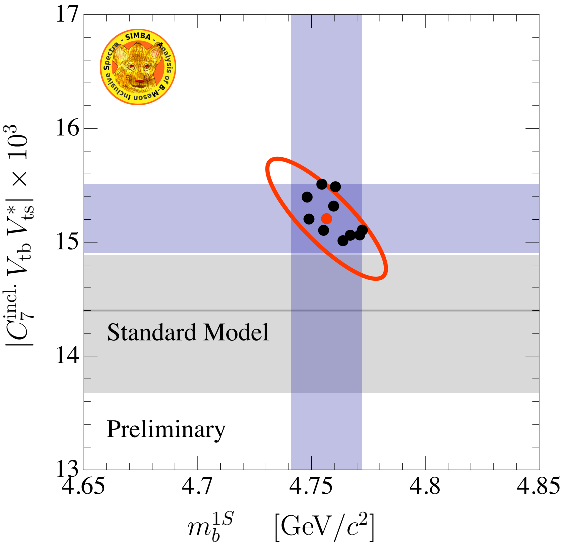


Fitting four coefficients c_0 to c_3



Data slightly above SM
(same pattern as Misiak vs HFAG)

Theoretical Uncertainties

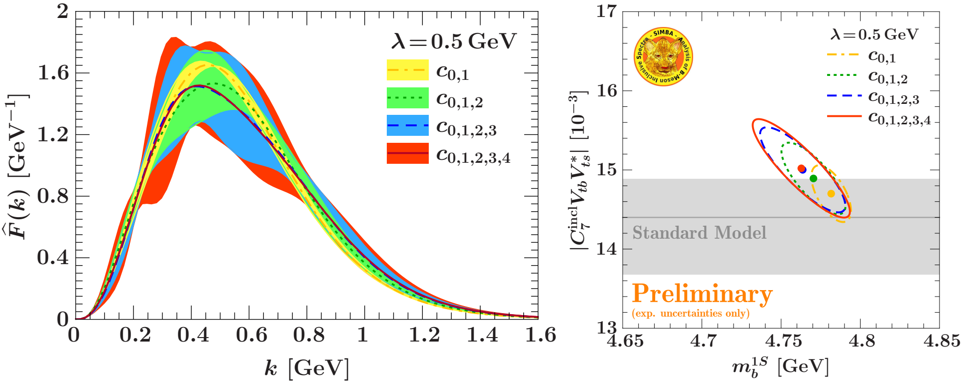


Dominant theoretical uncertainties from variation of soft, jet and hard scale

Complete evaluation of theoretical uncertainties still ongoing

Direct fit to spectrum eliminates need to extrapolate rate to $E_\gamma > \mathbf{1.6\ GeV}$ to compare data to predictions

Convergence of the Basis Expansion



- Uncertainties underestimated with too few coefficients ($c_{0,1}$)
  - ⋆ Would need to include additional uncertainty due to truncation
- Very little change from including 5th coefficient (c_4)
  - ⋆ Truncation uncertainty negligible compared to other uncertainties

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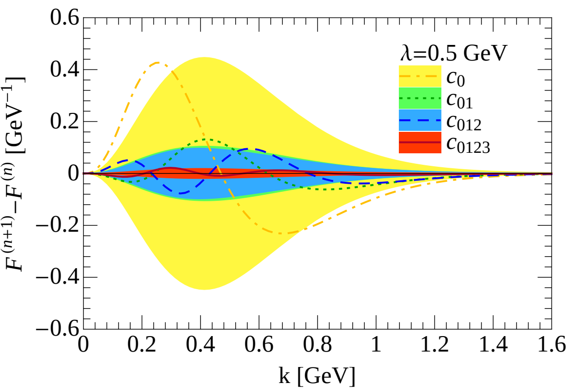

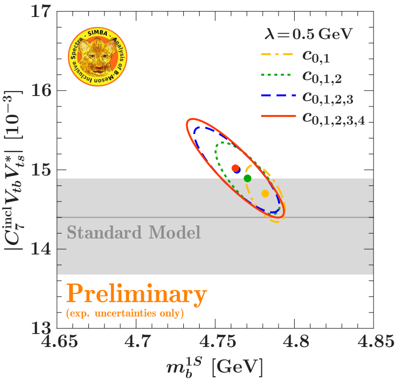

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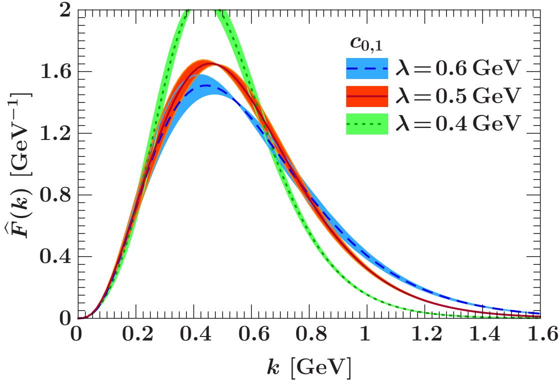
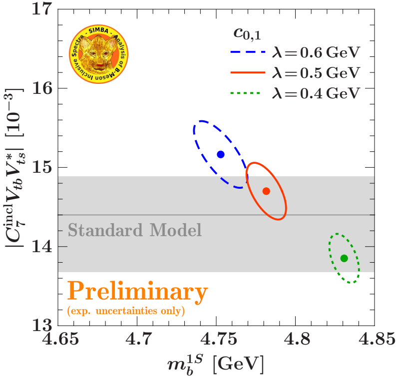
Basis Independence

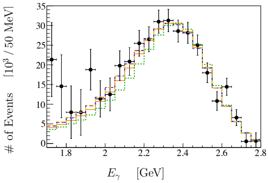




Fit with only two basis functions

- Equivalent to fixed model with fitted 1st moment
- All with good χ^2/ndf: $37.5/40$, $28.8/40$, $27.8/40$

$\Rightarrow$ Uncertainties underestimate model dependence



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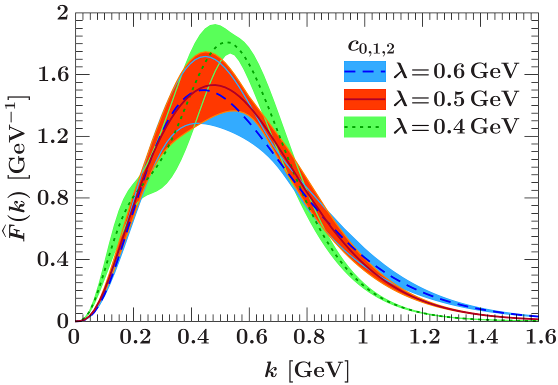

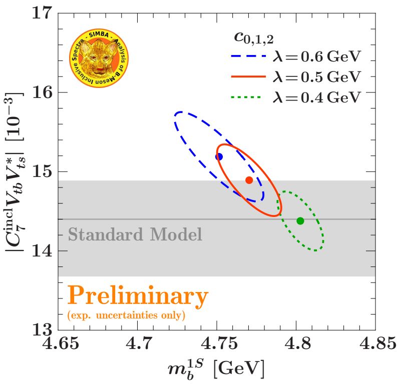

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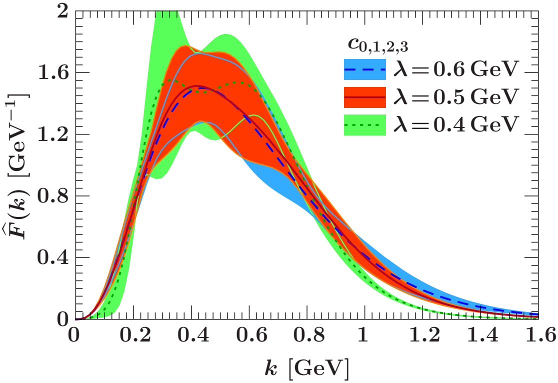

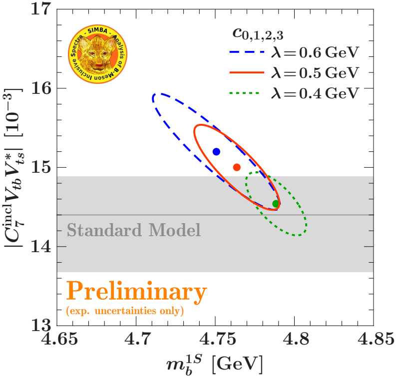

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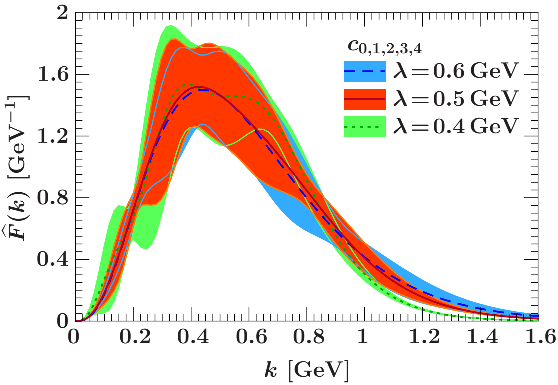

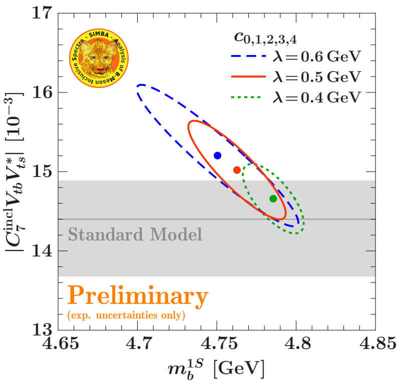

$\Rightarrow$ With enough coefficients results agree within uncertainties and become basis (model) independent

Simba @ SuperB

Global fit approach can be very powerful with high statistics

- Measure spectra in addition to (partial) BFs to maximize the available shape information, especially for $B \to X_u \ell \nu$
  - ⋆ Shape information is key to constraining subleading corrections
- Large dataset can be taken advantage of to agressively reject backgrounds at the cost of efficiency and to maximize resolution
  - ⋆ My personal favorite is a super-clean B_{reco} sample

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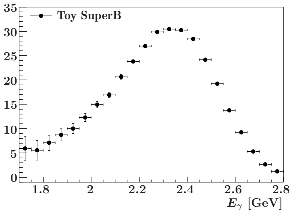
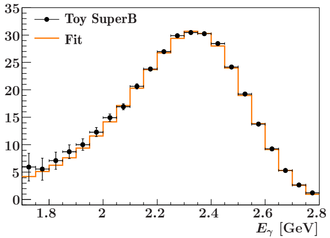
Toy $B \to X_s \gamma$ @ $75\,\mathrm{ab}^{-1}$

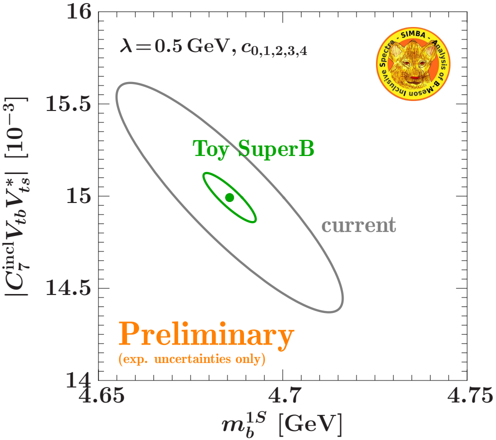
- Spectrum generated with $\lambda = 0.6\,\mathrm{GeV}$, $c_0 = 1$
- Uncertainties and correlations obtained from inclusive Belle spectrum:
  - ⋆ Statistical uncertainties scaled by lumi
  - ⋆ Systematic uncertainties scaled by $1/3$
  - ⋆ Correlations and detector resolution assumed to be the same



$B \to X_s\gamma$ @ SuperB



Fitting five coefficients c_0 to c_4



Theoretical uncertainties will dominate

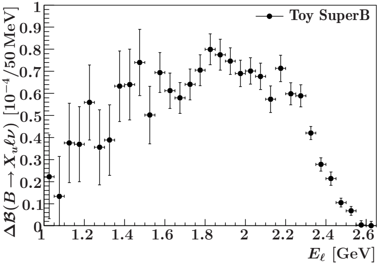
High precision data can be used to fit for more c_n and for subleading effects

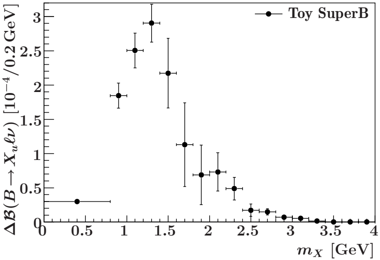
Note: NLL+NLO since we will also include $B \to X_u \ell \nu$, for simplicity ignoring subleading SF

Including $B \to X_u \ell \nu$ @ SuperB

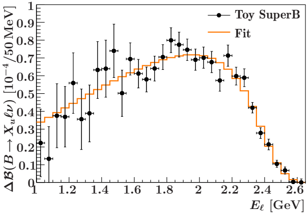
Toy $B \to X_u \ell \nu$ @ $75\,\mathrm{ab}^{-1}$

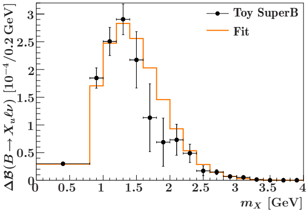
- m_X and E_ℓ spectra generated with $\lambda = 0.6\,\mathrm{GeV}$, $c_0 = 1$
- Uncertainties and correlations inspired by current *BABAR* hadronic tag analysis
  - ⋆ Assuming main uncertainties and correlations due to $B \to X_c \ell \nu$ background
  - ⋆ Aimed at being (too?) conservative
  - ⋆ Caveat: no resolution effects considered here



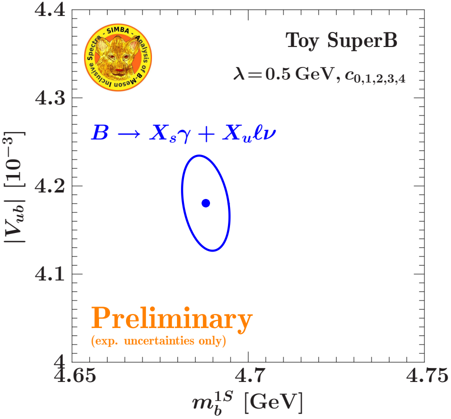


Including $B \to X_u \ell \nu$ @ SuperB



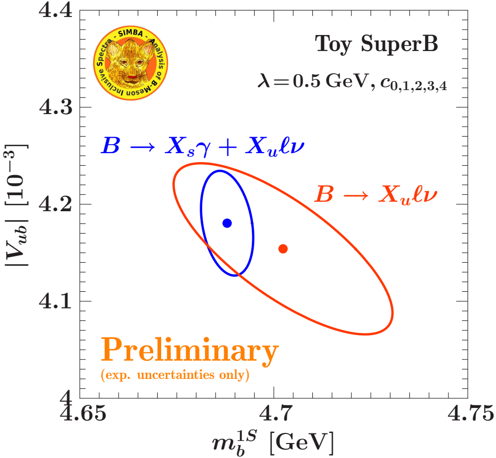


Fitting five coefficients c_0 to c_4



- Large amount of data can be used to push analyses to the limits, on the experimental as well as the theoretical side
- High precision data should be used to disentangle subleading effects between $B \to X_s\gamma$ and $B \to X_u \ell \nu$ (no attempt here!)

$B \to X_u \ell \nu$ Standalone @ SuperB



Fitting five coefficients c_0 to c_4

Use $B \to X_u \ell \nu$ alone to determine m_b and shape function along with $|V_{ub}|$

- Eliminates sensitivity to different subleading effects in $B \to X_s \gamma$ and $B \to X_u \ell \nu$

Summary

Global fit to $B \to X_s\gamma$ and $B \to X_u\ell\nu$
with a model-independent treatment of the shape function

- Global fit approach minimizes uncertainties by making maximal use of available data
- SF and its uncertainties are determined by the data
  - ⋆ Convergence behavior can be studied on data to estimate truncation uncertainty

⇒ Study of $B \to X_s\gamma$ without need to extrapolate to lower E_γ

⇒ Combine $B \to X_s\gamma$ and $B \to X_u\ell\nu$ for $|V_{ub}|$ determination and constrain m_b and shape function at the same time

Global fit approach well-suited for SuperB

- High luminosity can be used to measure high-quality differential spectra

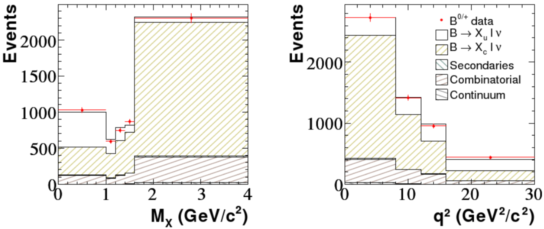
→ Can be used for more sophisticated analyses than current B factory data

- Precise study of $B \to X_s\gamma$ including subleading effects from data
- Precision $|V_{ub}|$ without external inputs

Backup

Inclusive $|V_{ub}|$ with Hadronic Tags at Belle

[arXiv:0907.0379 (hep-ex)]



$p_\ell^{*B} > 1.0$ GeV	$\Delta\mathcal{B}/\mathcal{B}$ (%)
$\mathcal{B}(D^{(*)}\ell\nu)$	1.2
$(D^{(*)}\ell\nu)$ form factors	1.2
$\mathcal{B}(D^{**}e\nu)$ & form factors	0.2
$B \to X_u\ell\nu$ (SF)	3.6
$B \to X_u\ell\nu$ $(g \to s\bar{s})$	1.5
$\mathcal{B}(B \to \pi/\rho/\omega\ell\nu)$	2.3
$\mathcal{B}(B \to \eta,\ \eta'\ell\nu)$	3.2
$\mathcal{B}(B \to X_u\ell\nu)$ un-meas.	2.9
Cont./Comb.	1.8
Sec./Fakes/Fit.	1.0
PID/Reconstruction	3.1
BDT	3.1
Systematics	8.1
Statistics	8.8

- 90% of $B \to X_u\ell\nu$ phase space included
- But much of the gained phase space comes with large backgrounds and corresponding systematics
  - ⋆ Main sensitivity still from low-M_X region

Formerly "theoretical" uncertainties now moved into "experimental" uncertainties (SF), where they are even harder to get rid of later

Master formula for binned spectra

Shape function basis $\widehat{F}(\lambda x) = \frac{1}{\lambda}\Big[\sum_n c_n f_n(x)\Big]^2$

Insert expansion for $\widehat{F}(k)$ into Master formula

$$\mathrm{d}\Gamma_s = |V_{tb}V_{ts}^*|^2 m_b^2 \, |C_7^{\mathrm{incl}}|^2 \sum_{m,n} c_m c_n \mathrm{d}\Gamma_{mn}^{77} \qquad \mathrm{d}\Gamma_u = |V_{ub}|^2 \sum_{m,n} c_m c_n \mathrm{d}\Gamma_{mn}^{u}$$

- Default basis parameter: $\lambda = 0.5\,\mathrm{GeV}$
- Include up to 5 basis coefficients (c_0 to c_4)
- Fix $\sum_n c_n^2 = 1$ to enforce correct normalization $\int k\widehat{F}(k) = 1$

Fit Setup

χ^2 Fit

- Includes all experimental correlations
- Extensively validated with pseudo experiments
  - ⋆ Just having a good χ^2/ndf is not enough

Shape function basis $\widehat{F}(\lambda x) = \frac{1}{\lambda}\left[\sum_n c_n f_n(x)\right]^2$

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Disclaimer: What I am showing is active work in progress

- Numbers still subject to change
- Theoretical uncertainties very preliminary

Global Fit to $B \to X_s\gamma$

Current status of experiment to theory comparison

- HFAG extrapolation down to $E_\gamma^{\rm cut} = 1.6\,\rm GeV$
  (adds model dependence)

$$\mathcal{B}(E_\gamma > 1.6\,\rm GeV) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}$$

- Fixed-order NNLO estimate by Misiak et al. (2006)

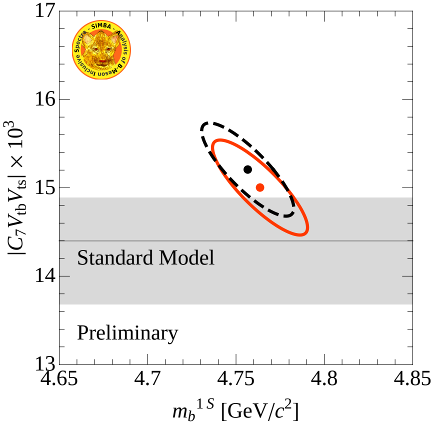
$$\mathcal{B}(E_\gamma > 1.6\,\rm GeV) = (3.15 \pm 0.23) \times 10^{-4}$$

Sensitivity to new physics lies in normalization, parametrized by $|V_{tb}V_{ts}^* C_7^{\rm incl}|$

- Most sensitivity in data comes from large E_γ
- Fit determines both $|V_{tb}V_{ts}^* C_7^{\rm incl}|$ (normalization) and $\widehat{F}(k)$ (shape)
  - ⋆ Can directly compare $|V_{tb}V_{ts}^* C_7^{\rm incl}|$ to its SM prediction
  - ⋆ Avoids any extrapolation

Theoretical Effects and Uncertainties

Effect of non-singular terms



Theoretical uncertainties from variation of soft, jet and hard scale