Naturalness of the Higgs boson mass. Impact of gravity

Riccardo Gandolfo

University of Catania and INFN - Italy

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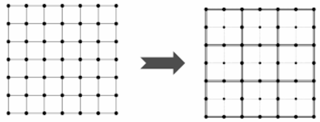
(iii) arXiv:2505.07628 ; (iv) Phys.Rev.D 112 (2025) 4, 045002 ; (v) arXiv:2507.13832

Renormalization - Wilson's Lesson

Any QFT is an Effective Field Theory



Theory at Λ $\rightarrow$ Theory at $\Lambda/2$ $\rightarrow$ ...

$S_\Lambda \rightarrow S_{\Lambda/2} \rightarrow \ldots$

Inclusion of fluctuations, physical running scale $\Lambda \rightarrow \Lambda/2 \rightarrow \Lambda/4 \rightarrow \Lambda/8 \rightarrow \ldots$

Piling up of fluctuations $\rightarrow$ Evolution of parameters (couplings/masses)

Renormalization - Higgs mass (Standard Model)

Standard Model - **EFT** valid up to a **high scale** Λ $(M_P,\, M_{GUT},\, ...)$

Λ **physical cut-off** - SM effective Lagrangian $\mathcal{L}_{SM}^{(\Lambda)}$ processes for momenta $p \lesssim \Lambda$

Above Λ the SM **has to** be replaced by its **UV completion**

Naturalness/Hierarchy (NH) problem

1. Unsuppressed quantum fluctuations $\rightarrow$ $m_H^2 \propto \Lambda^2$ **(Big Hierarchy)**

"quadratic sensitivity" to Λ

If Λ **too large** $\rightarrow$ $m_H^2(\Lambda)$ **"unnaturally" large**

$\rightarrow$ problem of "hierarchy" with the Fermi scale μ_F, where $m_H(\mu_F) \sim 125$ GeV

... But also ...

2. Heavy particles (BSM) coupled to Higgs $\rightarrow$ $m_H^2 \propto M^2$ **(Little Hierarchy)**

Higgs Mass - Hierarchy/Naturalness Problem

Old days attitude

Great importance to (Big) **Hierarchy Problem** i.e. to Λ^2 sensitivity of m_H^2

Supersymmetry possible and **very popular** "solution"

More recent attitude

Since no sign of Supersymmetry up to now

Lot of activity on **non-supersymmetric models**

... scale invariant models (Meissner, Nicolai, Shaposhnikov, ...) ... newly proposed symmetries (Lindner, ...) ... non-perturbative instanton effects (Shaposhnikov, ...) ...

Important to understand what is the general attitude in all these cases:

Quadratic sensitivity of m_H^2 to Λ **not a problem** ... **technical issue** ... just go on and subtract this quadratic divergence ... Moreover, if dimensional regularization used ...

(Big) **Hierarchy Problem** downgraded to a **technical issue** (not an issue at all!)

... But ... remember ... **Wilson's lesson ...**

What about dimensional regularization?

Very good technique to go "directly" to the renormalized theory i.e. to the "critical region" in the parameter space (*)

Fine-tuning *automatically* encoded in the calculations, although in a *hidden* manner

Needless to say, DR cannot provide a solution to the problem (despite claims to the contrary)

(*) C. Branchina, V. Branchina, F. Contino and N. Darvishi, *Dimensional regularization, Wilsonian RG, and the naturalness and hierarchy problem*, Phys. Rev. D **106** (2022) no.6, 065007, arXiv:2204.10582

Lesson

According to Wilson's lesson - and contrary to what is sometimes stated -

we cannot ignore the (Big) Hierarchy Problem

- The mass m_H^2 at the scale Λ is "naturally" expected to be $m_H^2(\Lambda) \sim \Lambda^2$
- What takes the SM to the critical regime $m_H^2 \ll \Lambda^2$ (renormalized theory)?

Physical mechanism usually invoked to cope with (Big) Hierarchy:

Supersymmetry

This talk: gravity leads the theory to the critical regime $m_H^2 \ll \Lambda^2$

- no SUSY (**physical** cancellation mechanism)
- No **technical** cancellation (dim.reg., ζ-function regularization)

Pure Gravity: recap

Pure Gravity - One-loop

Einstein-Hilbert Action : $S_{\rm grav} = \frac{1}{16\pi G}\int \mathrm{d}^4x \sqrt{g}\,(-R+2\Lambda)$

One-loop correction $\delta S^{1l}_{\rm grav}$

$$e^{-\delta S^{1l}_{\rm grav}} = \lim_{\xi\to 0}\int [\mathcal{D}\mu]\, e^{-\delta S^{(2)}} \quad ; \quad \delta S^{(2)} \equiv S_2 + S_{\rm gf} + S_{\rm ghost}$$

Fradkin-Vilkovisky measure

$$\left[\mathcal{D}\mu\right] \equiv \prod_x \left[g^{(a)\,00}(x)\,\left(g^{(a)}(x)\right)^{-1}\left(\prod_{\alpha\le\beta}\mathrm{d}h_{\alpha\beta}(x)\right)\left(\prod_\rho \mathrm{d}v^*_\rho(x)\right)\left(\prod_\sigma \mathrm{d}v_\sigma(x)\right)\right]$$

$$\rho_{vac} = \frac{\Lambda^{1l}_{\rm cc}}{8\pi G^{1l}} = \frac{\Lambda_{\rm cc}}{8\pi G}\left(1 - \frac{3G\Lambda_{\rm cc}}{\pi}\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right)$$

only logarithmic corrections to ρ_{vac}

No Naturalness Problem in pure gravity ; No need for Bare $\Lambda_{\rm cc} \sim M_P^2$

We may naturally have $\Lambda_{\rm cc} \ll M_P^2$

$$\Lambda^{1l}_{\rm cc} \sim \Lambda_{\rm cc}$$

Pure gravity - RG

$$\boxed{k\frac{\partial\Lambda_k}{\partial k} = \frac{3G_k}{\pi}\,\frac{\Lambda_k\left(k^2-\frac{2}{3}\Lambda_k\right)}{1+\frac{3G_k}{2\pi}\left(k^2-\frac{2}{3}\Lambda_k\right)} \quad ; \quad k\frac{\partial G_k}{\partial k} = \frac{3G_k^2}{\pi}\,\frac{k^2-\frac{8}{3}\Lambda_k}{1+\frac{3G_k}{2\pi}\left(k^2-\frac{2}{3}\Lambda_k\right)}}$$

rather than

$$k\frac{\partial}{\partial k}\Lambda_k = \frac{G_k}{\pi}\left[k^4 + 6\Lambda_k\left(k^2+\Lambda_k\right) - \Lambda_k\frac{34k^2+48\,\Lambda_k}{6}\right] \quad ; \quad k\frac{\partial}{\partial k}G_k = -G_k^2\,\frac{34k^2+48\,\Lambda_k}{6\pi}$$

NO sign of **physical UV-attractive** fixed point (**AS**)

- identification of the physical running scale properly done
- truly diffeomorphism invariant measure (FV) used

$\Longrightarrow$ **No** Asymptotic Safety scenario

i.e. **No** UV-attractive Fixed point (as in QCD)

Gravity is an Effective Field Theory (only Gaussian fixed point as in QED and ϕ^4)

The Higgs mass

Scalar theory non minimally coupled to gravity

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{g}\,\left(-R+2\Lambda_{cc}\right) + \int \mathrm{d}^4x\,\sqrt{g}\left[\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{\xi}{2}R\phi^2 + V(\phi)\right]$$

Taking the metric $g^{(a)}_{\mu\nu}$ of a sphere of radius a

$$S^{(a)}[\phi] = \frac{\pi\Lambda_{cc}}{3G}a^4 - \frac{2\pi}{G}a^2 + \int \mathrm{d}^4x\,\sqrt{g^{(a)}}\left[\frac{1}{2}\,g^{(a)\,\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \frac{\xi}{2}\,\frac{12}{a^2}\,\phi^2 + V(\phi)\right]$$

One-loop correction δS^{1l} $\quad(\phi = \Phi + \eta \;\;;\;\;$ expand S up to $\eta^2)$

$$e^{-\delta S^{1l}} = \int \left[\mathcal{D}\mu(\eta)\right]\, e^{-S_2}$$

where $\left[\mathcal{D}\mu\right]$ Fradkin-Vilkovisky measure (arXiv:2506.05100)

$$S_2 \equiv \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g^{(a)}}\;\eta\left[-\Box_a + \frac{12\,\xi}{a^2} + V''(\Phi)\right]\eta \quad ; \quad \left[\mathcal{D}\mu(\eta)\right] = \prod_x\left[\left(g^{(a)\,00}(x)\right)^{\frac{1}{2}}\left(g^{(a)}(x)\right)^{\frac{1}{4}}\mathrm{d}\eta(x)\right]$$

Dimensionless operators

$g^{(a)}_{\mu\nu} = a^2\,\widetilde{g}_{\mu\nu}$ where components of $\widetilde{g}_{\mu\nu}$ are dimensionless and a-independent

$$\left(g^{(a)\,00}(x)\right)^{1/2}\left(g^{(a)}(x)\right)^{1/4} = a\left(\widetilde{g}^{\,00}(x)\right)^{1/2}\left(\widetilde{g}(x)\right)^{1/4}$$

Convenient redefinition: $\widehat{\eta} \equiv a\eta$

$$S_2 = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{\widetilde{g}}\;\widehat{\eta}\left[-\widetilde{\Box} + 12\,\xi + a^2\,V''(\Phi)\right]\widehat{\eta}$$

$-\widetilde{\Box}$ dimensionless spin-0 Laplace-Beltrami operator $\qquad -\widetilde{\Box} \equiv -a^2\,\Box_a$

Since $\qquad \mathrm{d}\eta(x) = a^{-1}\mathrm{d}\widehat{\eta}(x)$

$\left[\mathcal{D}\mu(\eta)\right]$ can be written as $\qquad \left[\mathcal{D}\mu(\eta)\right] = \mathcal{A}\prod_x\left[\mathrm{d}\widehat{\eta}(x)\right]$

where a-independent terms such as $\prod_x\left(\widetilde{g}^{\,00}(x)\right)^{1/2}\left(\widetilde{g}(x)\right)^{1/4}$ are included in the factor $\mathcal{A}$

Dimensionless operators

Eigenvalues λ_n of $-\widetilde{\Box}$ and degeneracies D_n

$$\lambda_n = n^2 + 3n \qquad ; \qquad D_n = \frac{1}{3}\left(n + \frac{3}{2}\right)^3 - \frac{1}{12}\left(n + \frac{3}{2}\right)$$

Quantum correction to the action from Gaussian integration

$$e^{-\delta S} = \int \left[\mathcal{D}\mu(\eta)\right] e^{-S_2} \qquad \Longrightarrow \qquad \Gamma = S^{(a)}[\Phi] + \frac{1}{2}\log\left[\det\left(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)\right)\right] + \mathcal{C}$$

Had we missed $\left(g^{(a)\,00}(x)\right)^{1/2}\left(g^{(a)}(x)\right)^{1/4} \Longrightarrow$ a-dependence of the determinant altered $\Longrightarrow$ determinant dimensionful $\Longrightarrow$ arbitrary scale μ needed to make argument of Log dimensionless

Calculation of $\quad \log\det\left(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)\right)$

As for the pure gravity case

- Product of eigenvalues
- Proper time

1 - Product of eigenvalues

Finite number N ($\gg 1$) of eigenvalues λ_n. Largest eigenvalue: $\lambda_N \sim N^2$ ($N-2$ for convenience)

$$\delta S^{1l} = \frac{1}{2}\sum_{n=0}^{N-2}\left[D_n \log\left(\lambda_n + 12\xi + a^2 V''(\Phi)\right)\right] + \mathcal{C}$$

N = numerical UV cutoff

Expanding for $N \gg 1$

δS^{1l} in terms of the numerical UV cutoff N:

$$\delta S^{1l} = \frac{8\pi^2}{3}a^4\left[-\frac{\left(V''(\Phi)\right)^2}{64\pi^2}\log N^2 + \frac{12}{a^2}\frac{V''(\Phi)}{384\pi^2}\left(N^2 + 2(1-6\xi)\log N^2\right)\right]$$
$$+\frac{N^4}{48}\left(-1 + 2\log N^2\right) - \frac{N^2}{72}\left(13 - 72\xi + 3\log N^2\right) + \left(2\xi(1-3\xi) - \frac{29}{180}\right)\log N^2 + \mathcal{C}$$

Consider Φ^4- theory : $V(\Phi) = \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4!}\Phi^4$

One-loop effective action $\Gamma^{1l} = S^{(a)}[\Phi] + \delta S^{1l}$

$$\begin{aligned}
\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Bigg\{ &- \frac{1}{16\pi G}\left[1 - \frac{G\, m^2}{24\pi}\left(N^2 + 2(1-6\xi)\log N^2\right)\right]\frac{12}{a^2} \\
&+ \frac{\Lambda_{cc}}{8\pi G}\left[1 - \frac{G\, m^4}{8\pi\Lambda_{cc}}\log N^2\right] + \frac{\xi}{2}\left[1 + \frac{\lambda}{384\pi^2\,\xi}\Big(N^2 + 2\,(1-6\xi)\log N^2\Big)\right]\frac{12}{a^2}\Phi^2 \\
&+ \frac{m^2}{2}\left[1 - \frac{\lambda}{32\pi^2}\log N^2\right]\Phi^2 + \frac{\lambda}{4!}\left[1 - \frac{3\lambda}{32\pi^2}\log N^2\right]\Phi^4 \Bigg\} \\
&+ \frac{N^4}{48}\left(-1 + 2\log N^2\right) - \frac{N^2}{72}\left(13 - 72\xi + 3\log N^2\right) + \left(2\xi\,(1-3\xi) - \frac{29}{180}\right)\log N^2
\end{aligned}$$

2 - Proper time

Since $(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi))$ dimensionless $\Longrightarrow$ regularize the determinant with dimensionless proper-time τ (lower cut $N \gg 1$). (λ_n and D_n eigenvalues and degeneracies)

$$\det(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)) = e^{-\int_{1/N^2}^{+\infty} \frac{\mathrm{d}\tau}{\tau}\,\mathrm{K}(\tau)} \quad ; \quad \mathrm{K}(\tau) = \sum_{n=0}^{+\infty} D_n\, e^{-\tau\left(\lambda_n + 12\xi + a^2 V''(\Phi)\right)}$$

Perform first integration over τ and then sum over n with EML

$$\Big[\ \sum_{n=n_i}^{n_f} f(n) = \int_{n_i}^{n_f} \mathrm{d}x\, f(x) + \frac{f(n_f)+f(n_i)}{2} + \sum_{k=1}^{p} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i)\right) + R_{2p}$$

p is an integer ; B_m are Bernoulli numbers ; R_{2p} is the rest

$$R_{2p} = \sum_{k=p+1}^{\infty} \frac{B_{2k}}{(2k)!}\left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i)\right) = \frac{(-1)^{2p+1}}{(2p)!}\int_{n_i}^{n_f} \mathrm{d}x\, f^{(2p)}(x) B_{2p}(x-[x])$$

$B_n(x)$ Bernoulli polynomials ; $[x]$ integer part of x ; $f^{(i)}$ i-th derivative of f $\Big]$

Expanding the resulting expression of δS^{1l} for $N \gg 1$

$$\delta S^{1l} = \frac{8\pi^2}{3} a^4 \Bigg[-\frac{\big(V''(\Phi)\big)^2}{64\pi^2} \log N^2 + \frac{12}{a^2} \frac{V''(\Phi)}{384\pi^2} \left(N^2 + 2\,(1-6\xi) \log N^2 \right) \Bigg]$$
$$- \frac{N^4}{24} - \frac{1-6\xi}{6} N^2 + \left(2\xi\,(1-3\xi) - \frac{29}{180} \right) \log N^2 + \mathcal{C}$$

Taking $V(\Phi) = \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4!}\Phi^4 \quad \Longrightarrow \quad \Gamma^{1l}\ (= S^{(a)} + \delta S^{1l})$ is

$$\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Bigg\{ -\frac{1}{16\pi G} \left[1 - \frac{G\,m^2}{24\pi} \left(N^2 + 2(1-6\xi) \log N^2 \right) \right] \frac{12}{a^2} + \frac{\Lambda_{cc}}{8\pi G} \left[1 - \frac{G\,m^4}{8\pi \Lambda_{cc}} \log N^2 \right]$$
$$+ \frac{\xi}{2} \left[1 + \frac{\lambda}{384\pi^2\,\xi} \left(N^2 + 2\,(1-6\xi) \log N^2 \right) \right] \frac{12}{a^2} \Phi^2$$
$$+ \frac{m^2}{2} \left[1 - \frac{\lambda}{32\pi^2} \log N^2 \right] \Phi^2 + \frac{\lambda}{4!} \left[1 - \frac{3\lambda}{32\pi^2} \log N^2 \right] \Phi^4 \Bigg\}$$
$$- \frac{N^4}{24} - \frac{1-6\xi}{6} N^2 + \left(2\xi\,(1-3\xi) - \frac{29}{180} \right) \log N^2$$

Apart from irrelevant a- and Φ-independent terms, the two results for Γ^{1l} coincide

One-loop corrections to $\frac{\Lambda_{cc}}{G}$, $\frac{1}{G}$, ξ, m^2 and λ

Comparing Γ^{1l} with $S^{(a)}[\Phi]$ we read the corrections to $\frac{\Lambda_{cc}}{G}$, $\frac{1}{G}$, ξ, m^2 and λ in terms of N

$$
\begin{aligned}
\frac{1}{G^{1l}} &= \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(N^2 + 2(1-6\xi)\log N^2\right)\right] \\
\frac{\Lambda_{cc}^{1l}}{G^{1l}} &= \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log N^2\right] \\
m^2_{1l} &= m^2\left[1 - \frac{\lambda}{32\pi^2}\log N^2\right] \\
\lambda^{1l} &= \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log N^2\right] \\
\xi^{1l} &= \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(N^2 + 2\,(1-6\xi)\log N^2\right)\right]
\end{aligned}
$$

Now connect N with Λ ($\sim M_P$ or string scale M_s) $\Lambda = \frac{N}{a_{\rm dS}} = N\sqrt{\frac{\Lambda_{cc}}{3}}$

... Let's read how the couplings get modified ...

$$\frac{\Lambda_{cc}^{1l}}{8\pi G^{1l}} = \frac{\Lambda_{cc}}{8\pi G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1l}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1l}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1l} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1l} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Quartic self-coupling - only mild logarithmic correction (coincides with flat space-time result)

Scalar mass $\delta m^2 \sim \log\Lambda$ rather than $\sim \Lambda^2$: **no quadratic divergence**

Usual result $\delta m^2 \sim \Lambda^2$: enormus fine-tuning

Present result we may well have $m^2(\Lambda) \ll \Lambda^2$

No (Big) Naturalness Problem for the scalar mass

If SM embedded in SUSY, GUT, ..., fields of heavy mass M coupled to Higgs $\Rightarrow \delta m^2 \propto M^2$.
Physical mechanism that disposes of these contributions and makes $m_{\rm H}^2 \sim (125\,\text{GeV})^2$ needed !!

... work in progress ...

Still this is an important physical result, obtained within the Wilsonian framework, where **physical cutoff built-in**: Absence of quadratic divergence not due to "technical tricks" (dimensional, zeta function regularization, ...) nor to physical cancellations (SUSY, ...)

… still reading the couplings …

$$\frac{\Lambda_{cc}^{1l}}{G^{1l}} = \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1l}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1l}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1l} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1l} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Non-minimal coupling ξ : besides a mild logarithmic correction (present in previous literature)

ξ receives a quadratically divergent contribution

UV sensitivity of m^2 and ξ inverted : $m^2 \sim \log\Lambda \quad ; \quad \xi \sim \Lambda^2$.

Phenomenological remarks

Higgs boson mass confronted with measured $m_{\rm H}^2 \sim (125\,\text{GeV})^2 \Longrightarrow$ quadratic sensitivity to Λ gives rise to severe naturalness problem. Much less is known on the experimental value of ξ.

... still reading the couplings ...

$$\frac{\Lambda_{cc}^{1l}}{G^{1l}} = \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1l}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1l}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1l} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1l} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Vacuum energy $\rho_{\rm vac} = \frac{\Lambda_{cc}}{8\pi G}$. Radiative correction $\sim \log\Lambda$: *no quartic/quadratic divergence*

Note : $\log\Lambda$ correction multiplied by $m^4 \Longrightarrow$ for SM masses $m \sim \mu_{\rm F}$ still left with (at least) 50 orders of magnitude discrepancy with measured vacuum energy. Absence of Λ^4 and Λ^2 corrections sheds some light on the CC problem, and makes it less severe

Inverse Newton constant - Quadratically UV-sensitive contribution as usual

Comparison with the literature

Calculation usually performed within the heat kernel expansion. Quadratically divergent contribution to the mass typically found (obviously dimensional regularization excluded from these considerations: trick to cancel powerlike divergences)

Let us go back to Γ^{1l} : temporarily connect N and Λ through

$$\Lambda = \frac{N}{a} \quad \text{rather than} \quad \Lambda = \frac{N}{a_{\mathrm{dS}}}$$

$$\begin{aligned}
\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Bigg\{ & - \frac{1}{16\pi G}\left[1 + \frac{1-6\xi}{12\pi} G\Lambda^2 - \frac{G\,m^2}{24\pi}\left(2(1-6\xi)\log\left(a^2\Lambda^2\right)\right)\right]\frac{12}{a^2} \\
& + \frac{\Lambda_{\mathrm{cc}}}{8\pi G}\left[1 - \frac{G}{8\pi\Lambda_{\mathrm{cc}}}\Lambda^4 + \frac{m^2 G}{4\pi\Lambda_{\mathrm{cc}}}\Lambda^2 - \frac{G\,m^4}{8\pi\Lambda_{\mathrm{cc}}}\log\left(a^2\Lambda^2\right)\right] \\
& + \frac{\xi}{2}\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(2\,(1-6\xi)\log\left(a^2\Lambda^2\right)\right)\right]\frac{12}{a^2}\Phi^2 \\
& + \frac{m^2}{2}\left[1 + \frac{\lambda\,\Lambda^2}{32\pi^2\,m^2} - \frac{\lambda}{32\pi^2}\log\left(a^2\Lambda^2\right)\right]\Phi^2 + \frac{\lambda}{4!}\left[1 - \frac{3\lambda}{32\pi^2}\log\left(a^2\Lambda^2\right)\right]\Phi^4 \Bigg\} \\
& + \left(2\xi\,(1-3\xi) - \frac{29}{180}\right)\log\left(a^2\Lambda^2\right)
\end{aligned}$$

From $\frac{N}{a_{\rm dS}}$ to $\frac{N}{a}$

$$\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Big\{ - \Big[1 - \frac{G\, m^2}{24\pi} \Big(N^2 + 2(1-6\xi) \log N^2 \Big) \Big] \frac{1}{16\pi G} \frac{12}{a^2} + \Big[1 - \frac{G\, m^4}{8\pi \Lambda_{cc}} \log N^2 \Big] \frac{\Lambda_{cc}}{8\pi G}$$
$$+ \Big[1 + \frac{\lambda}{384\pi^2\, \xi} \Big(N^2 + 2\,(1-6\xi) \log N^2 \Big) \Big] \frac{\xi}{2} \frac{12}{a^2} \phi^2 + \Big[1 - \frac{\lambda}{32\pi^2} \log N^2 \Big] \frac{m^2}{2} \phi^2 + \Big[1 - \frac{3\lambda}{32\pi^2} \log N^2 \Big] \frac{\lambda}{4!} \phi^4 \Big\}$$
$$- \frac{N^4}{24} - \frac{1-6\xi}{6} N^2 + \Big(2\xi\,(1-3\xi) - \frac{29}{180} \Big) \log N^2$$

Legenda: **Green** $\to \frac{\Lambda_{cc}}{8\pi G}$; **Red** $\to m^2$; **Blue** $\to \frac{1}{16\pi G}$

$$\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Big\{ - \Big[1 + \frac{1-6\xi}{12\pi} G\Lambda^2 - \frac{G\, m^2}{24\pi} \Big(2(1-6\xi) \log \big(a^2\Lambda^2\big) \Big) \Big] \frac{1}{16\pi G} \frac{12}{a^2}$$
$$+ \Big[1 - \frac{G}{8\pi\Lambda_{cc}} \Lambda^4 + \frac{m^2\, G}{4\pi\Lambda_{cc}} \Lambda^2 - \frac{G\, m^4}{8\pi\Lambda_{cc}} \log \big(a^2\Lambda^2\big) \Big] \frac{\Lambda_{cc}}{8\pi G}$$
$$+ \Big[1 + \frac{\lambda}{384\pi^2\, \xi} \Big(2\,(1-6\xi) \log \big(a^2\Lambda^2\big) \Big) \Big] \frac{\xi}{2} \frac{12}{a^2} \phi^2$$
$$+ \Big[1 + \frac{\lambda\, \Lambda^2}{32\pi^2\, m^2} - \frac{\lambda}{32\pi^2} \log \big(a^2\Lambda^2\big) \Big] \frac{m^2}{2} \phi^2 + \Big[1 - \frac{3\lambda}{32\pi^2} \log \big(a^2\Lambda^2\big) \Big] \frac{\lambda}{4!} \phi^4 \Big\}$$
$$+ \Big(2\xi\,(1-3\xi) - \frac{29}{180} \Big) \log \big(a^2\Lambda^2\big)$$

Speculations / Conclusions

Curved vs flat spacetime

Radiative corrections to m_H^2 / flat spacetime /quadratic sensitivity ... However ...

Evidence for positive vacuum energy $\frac{\Lambda_{cc}}{8\pi G} > 0$: Flat spacetime not suitable cosmological description

Radiative corrections should be computed on curved background

Flat spacetime as limit ... different from flat ab initio ...

Our calculations indicate that usual methods may fail ...

Quadratic divergence in ξ instead of m^2 cannot be detected in flat spacetime computations, since in this case $R\phi^2 \equiv 0$

BACK-UP SLIDES

More on Wilson's lesson

Great progress in our understanding of (renormalisation in) QFTs

Connection between **Renormalization in QFT** and **Theory of Critical Phenomena**

Critical Phenomena: Critical regime (ferromagnet: transition to ferromagnetic phase) reached when the correlation length ξ among statistical fluctuations becomes $>>$ inter-atomic distance a

Ferromagnet: $\xi >> a$ when T approaches the critical temperature T_c. For T close to T_c we have $\xi \sim |T - T_c|^{-\nu}$, with ν **critical exponent**

Connection QFTs/critical phenomena: $m_H \to \frac{1}{\xi}$, $\Lambda \to \frac{1}{a}$

$m_H^2(\Lambda) \ll \Lambda^2$ nothing but **Higgs system** in the **critical regime (renormalized regime)**

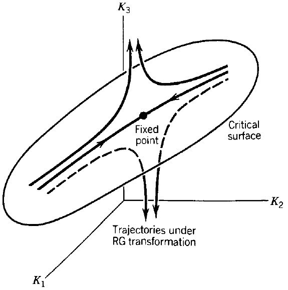
While **we know** what drives the **Ferromagnetic system** towards the critical regime namely the **tuning** of T towards T_c

We do not know why **Higgs system** $\to$ "critical regime" $m_H^2 \ll \Lambda^2$ $(\frac{1}{\xi} << \frac{1}{a})$

i.e. **what triggers** the **Higgs system** $\to$ **renormalized theory**

Renormalization of a QFT **is not** a matter of **"cancellation of divergences"**

RG flow

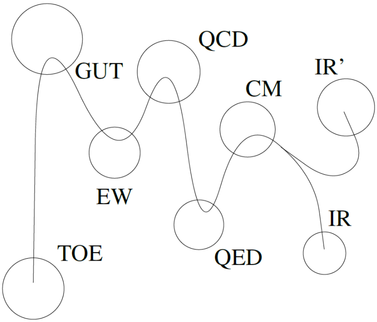


Renormalized theory - defined **around** a **fixed point** (close to critical surface)

When RG flow close to critical surface we have $m_H^2 \ll \Lambda^2$ $\quad (\frac{1}{\xi} << \frac{1}{a})$

The question is: what *drives* the RG flow close to the critical surface?

The Global Picture



All operators present from the beginning - UV - then intergrate out degrees of freedom

Classification **relevant / irrelevant / marginal** operators different in different regions of the parameter space

Pauli term: $\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$
4-Fermion inter.: $\bar{\psi}\psi\bar{\psi}\psi$
SM operators dim > 4
... (*)

(*) J. Alexandre, V. Branchina and J. Polonyi, *Global renormalization group*, Phys. Rev. D **58** (1998), 016002, arXiv:hep-th/9712147

Big Hierarchy

1. **A popular approach** - Assumes that UV completion of SM provides at Λ [**?**, **?**, **?**]

$$m_H^2(\Lambda) \ll \Lambda^2 \tag{1}$$

Sometimes viewed as a **"quantum gravity miracle"** [**?**] : realizes a conspiracy among the SM couplings at the scale Λ (Example: **Veltman condition**). In such a scenario:

(i) **Naturalness Problem** "solved" from physics "outside" the SM realm : left-over of its UV completion

(ii) **Hierarchy Problem** solved "inside" the SM, by considering the perturbative RG equation for $m_H^2(\mu)$ ($\gamma \ll 1$: perturbative anomalous mass dimension)

$$\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma \; m_H^2(\mu) \tag{2}$$

$m_H^2(\mu_F)$ and $m_H^2(\Lambda) \sim$ same order: no problem of hierarchy.

This way of framing the problem relies on two ingredients :

(a) **quantum gravity miracle**

(b) Above: the **correct RG equation** for $m_H^2(\mu)$ in the **whole range** $[\mu_F, \Lambda]$

Big Hierarchy

2. **Self-Tuning approach** - Assumes again equation

$$\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma \ m_H^2(\mu)$$

as the **correct RG equation** for $m_H^2(\mu)$ in the **whole range** $[\mu_F, \Lambda]$ and assumes that gravity could provide a non-perturbative value for γ (~ 2). In this case, the large hierarchy between the Fermi scale μ_F and the UV scale Λ can be accommodated [**?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**]: NH problem would disappear

Approaches 1 and 2 do not adderess correctly the problem

The ultimate reason is that **any Effective Field Theory**, including the **Standard Model**, is **necessarily** defined in the **Wilsonian framework**

3. Some authors suggest/hope/assume that **dimensional regularization** is endowed with special properties that make it the correct "physical" way to calculate the radiative corrections in quantum field theory. The Big Hierarcchy problem then seems to be absent from the beginning
[**?**, **?**].

These approaches do not solve the Big Hierarchy problem

The ultimate reason is that **any Effective Field Theory**, including the **Standard Model**, is **necessarily** defined in the **Wilsonian framework**

... More on Wilson's lesson ...

(i) parameters (masses/couplings) $g_i(\Lambda)$ in the effective Lagrangian $\mathcal{L}_{SM}^{(\Lambda)}$ result from integrating out higher energy modes $k > \Lambda$ related to the UV completion of the SM
(ii) the same parameters $g_i(\mu)$ at a lower scale $\mu < \Lambda$ result from integrating out the modes of the fields that appear in $\mathcal{L}_{SM}^{(\Lambda)}$ in the range $[\mu, \Lambda]$.

RG Equation for m_H (subtracted)

Equation $\mu \frac{d}{d\mu} m_H^2(\mu) = \gamma \; m_H^2(\mu)$. This equation is obtained when the "critical value" $m_{cr}^2(\mu)$ is subtracted to $m^2(\mu)$. In other words, $m_H^2(\mu)$ in (2) is not the Wilsonian mass $m^2(\mu)$. It is rather: $m_H^2(\mu) \equiv m^2(\mu) - m_{cr}^2(\mu)$. Equation (2) then incorporates the fine-tuning, and cannot be invoked to solve the NH problem.

Calculation with EML in proper time regularization

$(-\widetilde{\Box}^{(s)} - \alpha)$ dimensionless $\Longrightarrow$ determinants regularized in terms of a **dimensionless** proper-time τ (lower cut: **number** $N \gg 1$)

$$\det_i(-\widetilde{\Box}^{(s)} - \alpha) = e^{-\int_{1/N^2}^{+\infty} \frac{d\tau}{\tau} \mathrm{K}_i^{(s)}(\tau)} \quad ; \quad \mathrm{K}_i^{(s)}(\tau) = \sum_{n=s+i}^{+\infty} D_n^{(s)}\, e^{-\tau\left(\lambda_n^{(s)} - \alpha\right)}$$

After integration over τ, sum over n performed with EML sum formula

$$\sum_{n=n_i}^{n_f} f(n) = \int_{n_i}^{n_f} \mathrm{d}x\, f(x) + \frac{f(n_f) + f(n_i)}{2} + \sum_{k=1}^{p} \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i)\right) + R_{2p}$$

p is an integer, B_m are Bernoulli numbers, R_{2p} is the rest given by

$$R_{2p} = \sum_{k=p+1}^{\infty} \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i)\right) = \frac{(-1)^{2p+1}}{(2p)!} \int_{n_i}^{n_f} \mathrm{d}x\, f^{(2p)}(x) B_{2p}(x - [x])$$

$B_n(x)$ are the Bernoulli polynomials, $[x]$ the integer part of x, and $f^{(i)}$ the i-th derivative of f with respect to its argument

Proper time RG

Wilsonian RG strategy implemented introducing an IR regulator k in the one-loop result

$$\int_{1/\Lambda^2_{\rm cut}}^{+\infty} \mathrm{d}s \quad \longrightarrow \quad \int_{1/\Lambda^2_{\rm cut}}^{1/k^2} \mathrm{d}s \ ,$$

taking the derivative with respect to k, and finally realizing the RG improvement of the one-loop result

Equivalently introducing a smooth function $f_k(s)$ that interpolate between $f_k(s) \approx 0$ for $s \gg k^{-2}$ and $f_k(s) \approx 1$ for $s \ll k^{-2}$ $\Rightarrow$ RG equation for the action

$$\partial_t \widehat{S}_k[g, \bar{g}] = -\frac{1}{2} \operatorname{Tr} \int_0^{+\infty} \frac{\mathrm{d}s}{s} \, \partial_t f_k(s) \left[e^{-s \, \widehat{S}_k^{(2)}} - 2 \, e^{-s \, S^{(2)}_{\rm ghost}} \right] .$$

where $\widehat{S}\,[\bar{h}; g] \equiv S[\bar{g} + \bar{h}] \, + \, S_{\rm gf}[\bar{h}; \bar{g}]$

Background metric $\bar{g}_{\mu\nu} = g^{(a)}_{\mu\nu}$, Einstein-Hilbert truncation for $\widehat{S}_k \;\Rightarrow\; \widehat{S}_k^{(2)}$ contains dimensionful Laplace-Beltrami operators $-\Box$ for the sphere of radius a (and different spins $0, 1, 2$) whose eigenvalues $\widehat{\lambda}_n$ go like $\widehat{\lambda}_n \sim \frac{n^2}{a^2}$

The term $\partial_t f_k(s)$ effectively selects the eigenmodes of $-\Box$ whose corresponding eigenvalues lie in a narrow range ("infinitesimal shell") around k^2, i.e. $\widehat{\lambda}_n \sim k^2$

As for the effective average action formalism, here the running scale k is identified through the relation $k = L/a$, and the **same conclusions on the UV-attractive fixed point of the asymptotic safety scenario hold true**