FIELD QUANTIZATION AND NEUTRINO MIXING IN EINSTEIN-CARTAN SPACETIMES

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INTRODUCTION

We present the formulation of the Riemann-Cartan theory

- Affine connections and metricity conditions
- Generalisation of field equations in U_4 theory via the variational principle
- New neutrino oscillation formulas, in the presence of torsion

It follows directly from the transformation laws that the sum of two affine connections is not a tensor, while the antisymmetric part of the affine connections

$$S^a_{bc} \equiv \frac{1}{2}(\Gamma^a_{bc} - \Gamma^a_{cb}) \equiv \Gamma^a_{[bc]}$$

trasforms as a tensor. S^a_{bc} is called Cartan tensor
where the coefficients Γ^a_{bc} are called affine connection coefficients.

A generalization of the Einstein-Cartan theory is obtained by considering the non-metricity equation: $$\nabla_a g_{bc} = -Q_{abc}$$

where ∇_a is the covariant derivative depending on the affine connections, and Q_{abc} is the non-metricity tensor.

We shall analyze a theory such that the condition $Q_{abc} = 0$ is satisfied, ensuring the conservation of length units and angles under parallel transport.

Affine connections can be expressed in the form

$$\Gamma_{ij}^{k} = \{{}^{k}_{ij}\} - K_{ij}^{\cdot\cdot k}$$

$$K_{ij}^{\cdot\cdot k} \equiv -S_{ij}^{\cdot\cdot k} + S_{j\cdot i}^{\cdot k} - S^{k}_{\cdot ij} = -K_{i\cdot j}^{\cdot k}$$

where $\{{}^{k}_{ij}\}$ denote the Christoffel symbols, while $K_{ij}^{\cdot\cdot k}$ represents the **contortion tensor**, which a priori depends both on the metric and on the torsion.

The linearly connected space L_4, endowed with the metric g, reduces to the Riemann–Cartan space-time U_4 in the case where $Q_{abc} = 0$.

In turn, U_4 reduces to the Riemann space-time V_4 characteristic of General Relativity in the absence of torsion.

Finally, V_4 reduces to the Minkowski space-time R_4 in the absence of curvature.

$$(L_4, g) \overset{Q=0}{\rightarrow} U_4 \overset{S=0}{\rightarrow} V_4 \overset{R=0}{\rightarrow} R_4$$

The action of the matter field with the gravitational field is of the form:

$$W = \int d^4x\,(-g)^{\frac{1}{2}}\mathscr{L}(\Psi,\partial\Psi,g,\partial g,S) + \left(\frac{1}{2k}\right)\int d^4x\,(-g)^{\frac{1}{2}}R(g,\partial g,S,\partial S)$$

Varying the action with respect to the independent variables

$$-\frac{\delta(eR)}{\delta g_{ij}} = ke\sigma^{ij} \qquad -\frac{\delta(eR)}{\delta S_{ij}^{\cdot\cdot k}} = 2ke\mu_k^{\cdot ji}$$

and using the modified Palatini equation

$$\delta R_{ij} = \nabla_k(\delta\Gamma_{ij}^k) - \nabla_i(\delta\Gamma_{kj}^k) + 2S_{ki}^{\cdot\cdot m}\delta\Gamma_{mj}^k$$

FIELD EQUATIONS Theory U_4

$$G^{ij} = k\Sigma^{ij} \qquad T^{ijk} = k\tau^{ijk}$$

The second equation is an algebraic relationship between spin and torsion, while the first is a generalisation of Einstein's equation.

By expressing Einstein's tensor in its Riemannian part $G(\{\})$ and it's non-Riemannian part, we find

$$G^{ij}(\{\}) = k\tilde{\sigma}^{ij}$$

In this equation, $\tilde{\sigma}^{ij}$ has been defined as the combined energy-momentum tensor, expressed as:

$$\boxed{\tilde{\sigma}^{ij} = \sigma^{ij} + k[-4\tau^{ik}{}_{..[l}\tau^{jl}{}_{..k]} - 2\tau^{ikl}\tau^{j}{}_{.kl} + \tau^{kli}\tau_{kl}{}^{..j} + \frac{1}{2}g^{ij}(4\tau_{m.[l}{}^{.k}\tau^{ml}{}_{..k]} + \tau^{mkl}\tau_{mkl})]}$$

$\tilde{\sigma}^{ij}$ is symmetrical by definition and satisfies the law of conservation

$$\boxed{\mathfrak{D}_j\tilde{\sigma}^{ij} = 0}$$

In the absence of torsion, the field equations reduce to those of general relativity. The geometric concept of torsion is intrinsically linked to local transformations of the Poincaré group.

The local Poincaré group leads to Riemann-Cartan geometry

Friedrich W. Hehl, Paul von der Heyde, G. David Kerlick, General relativity with spin and torsion: Foundations and prospects, Rev. Mod. Phys. 1976.

MIXING IN QUANTUM MECHANICS

For three flavour states, we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

The oscillation formulae for $\nu_e \to \nu_e$ is

$$\begin{aligned} P(\nu_e \to \nu_e) = 1 &- \cos^4\theta_{13}\,\sin^2(2\theta_{12})\,\sin^2\left(\frac{\Delta m^2_{21}\,L}{4E}\right) \\ &- \sin^2(2\theta_{13})\left[\cos^2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{31}\,L}{4E}\right) + \sin^2\theta_{12}\,\sin^2\left(\frac{\Delta m^2_{32}\,L}{4E}\right)\right] \end{aligned}$$

S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41 (1978) 225

Mixing in quantum field theory

Pontecorvo's formalism has been generalised to QFT and corrections have been made to the formulas we have seen. The generalisation to QFT considers the mixing between fields [1].

The generalisation in QFT was also made for 3 flavours [2]

$$\begin{aligned}
\mathcal{Q}^{e}_{\mathbf{k},e}(t) \quad = \quad & 1 - \sin^2(2\theta_{12})\cos^4\theta_{13}\left[|U^{\mathbf{k}}_{12}|^2 \sin^2\left(\Delta^{\mathbf{k}}_{12}t\right) + |V^{\mathbf{k}}_{12}|^2 \sin^2\left(\Omega^{\mathbf{k}}_{12}t\right)\right] \\
& - \sin^2(2\theta_{13})\cos^2\theta_{12}\left[|U^{\mathbf{k}}_{13}|^2 \sin^2\left(\Delta^{\mathbf{k}}_{13}t\right) + |V^{\mathbf{k}}_{13}|^2 \sin^2\left(\Omega^{\mathbf{k}}_{13}t\right)\right] \\
& - \sin^2(2\theta_{13})\sin^2\theta_{12}\left[|U^{\mathbf{k}}_{23}|^2 \sin^2\left(\Delta^{\mathbf{k}}_{23}t\right) + |V^{\mathbf{k}}_{23}|^2 \sin^2\left(\Omega^{\mathbf{k}}_{23}t\right)\right]
\end{aligned}$$

[1] M.Blasone and G.Vitiello, Annals Phys. 244: 283-311, (1995)
[2] M. Blasone, A. Capolupo, and G. Vitiello, Phys. Rev. D 66, 025033 (2002)

Neutrino oscillation formulas in the precence of costant torsion

The field satisfied the equation $$i\hbar\gamma^\mu\partial_\mu\Psi - m\Psi = -\frac{3\hbar}{2}\breve{T}^\lambda(r)\gamma_\lambda\gamma^5\Psi$$

The solutions for free particles with spin aligned or opposite to the axial torsion are given by:

$$u^{\uparrow}_{\vec{k}} = N^{+}\begin{pmatrix} 1 \\ 0 \\ \frac{k_3}{E^{+}_{\vec{k}}+\widetilde{m}^{+}} \\ \frac{k_1+ik_2}{E^{+}_{\vec{k}}+\widetilde{m}^{+}} \end{pmatrix} \qquad u^{\downarrow}_{\vec{k}} = N^{-}\begin{pmatrix} 0 \\ 1 \\ \frac{k_1-ik_2}{E^{-}_{\vec{k}}+\widetilde{m}^{-}} \\ -\frac{k_3}{E^{-}_{\vec{k}}+\widetilde{m}^{-}} \end{pmatrix}$$

$$v^{\uparrow}_{\vec{k}} = N^{+}\begin{pmatrix} \frac{k_3}{E^{+}_{\vec{k}}+\widetilde{m}^{+}} \\ \frac{k_1+ik_2}{E^{+}_{\vec{k}}+\widetilde{m}^{+}} \\ 1 \\ 0 \end{pmatrix} \qquad v^{\downarrow}_{\vec{k}} = N^{-}\begin{pmatrix} \frac{k_1-ik_2}{E^{-}_{\vec{k}}+\widetilde{m}^{-}} \\ -\frac{k_3}{E^{-}_{\vec{k}}+\widetilde{m}^{-}} \\ 0 \\ 1 \end{pmatrix}.$$

$$N^{\pm} = \sqrt{\frac{E^{\pm}+\widetilde{m}^{\pm}}{2E^{\pm}}}.$$

$N_i^{\pm}$ are obtained from the normalisation condition

$$\Psi^\dagger\Psi = 1$$

$$\widetilde{m}_i^{\pm} \equiv m_i \pm \frac{3}{2}h\breve{T}^3$$

$$E^{\pm}_{\vec{k}} = \sqrt{\vec{k}^2 + \widetilde{m}^{\pm 2}}.$$

A. Capolupo, G. D. Maria, S. Monda, A. Quaranta, R. Serao, *Universe* 10(4), 170 (**2024**).

The linearity of the generator allows us to define the flavour annihilators of the fields and assuming that: $\vec{k} = (0, 0, |\vec{k}|)$

$$\boxed{\begin{aligned}\alpha^r_{\vec{k},\nu_e}(t) = c_{12}c_{13}\alpha^r_{\vec{k},1} + s_{12}c_{13}\left(\left(\Gamma^{rr}_{12;\vec{k}}(t)\right)^*\alpha^r_{\vec{k},2} + \varepsilon^r\left(\Sigma^{rr}_{12;\vec{k}}(t)\right)\beta^{r\dagger}_{-\vec{k},2}\right) \\ + e^{-i\delta}s_{13}\left(\left(\Gamma^{rr}_{13;\vec{k}}(t)\right)^*\alpha^r_{\vec{k},3} + \varepsilon^r\left(\Sigma^{rr}_{13;\vec{k}}(t)\right)\beta^{r\dagger}_{-\vec{k},3}\right),\end{aligned}}$$

$$\Gamma^{rr}_{ij;\vec{k}} = \Xi^{rr}_{ij;\vec{k}} \text{ and } \Sigma^{rr}_{ij;\vec{k}} = \chi^{rr}_{ij;\vec{k}} \text{ for constant torsion}$$

These equations for annihilators have a structure similar to that in QFT, but the Bogoliubov coefficients are different.

$$\Xi^{\pm\pm}_{ij;\vec{k}} = N^{\pm}_i N^{\pm}_j \left[1 + \frac{k^2}{\left(E^{\pm}_{\vec{k},i} + \widetilde{m}^{\pm}_i\right)\left(E^{\pm}_{\vec{k},j} + \widetilde{m}^{\pm}_j\right)}\right] = \cos(\xi^{\pm\pm}_{ij;\vec{k}}),$$

$$\chi^{\pm\pm}_{ij;\vec{k}} = N^{\pm}_i N^{+}_j \left[\frac{k_3}{E^{\pm}_{\vec{k},j} + \widetilde{m}^{\pm}_j} - \frac{k_3}{E^{\pm}_{\vec{k},i} + \widetilde{m}^{\pm}_i}\right] = \sin(\xi^{\pm\pm}_{ij;\vec{k}}),$$

Neutrino oscillation formulas in the presence of constant torsion for three flavour generations

$$\mathcal{Q}^{\uparrow \vec{k}}_{\nu_\rho \to \nu_\sigma}(t) \equiv \left\langle \nu^{\uparrow}_{\vec{k},\rho}(t) \right| :: Q_{\nu_\sigma} :: \left| \nu^{\uparrow}_{\vec{k},\rho}(t) \right\rangle - {}_f\langle 0| :: Q_{\nu_\sigma} :: |0\rangle_f$$

The following relationship is satisfied:

$$\boxed{\mathcal{Q}^{\uparrow \vec{k}}_{\nu_\rho \to \nu_e}(t) + \mathcal{Q}^{\uparrow \vec{k}}_{\nu_\rho \to \nu_\mu}(t) + \mathcal{Q}^{\uparrow \vec{k}}_{\nu_\rho \to \nu_\tau}(t) = 1}$$

From the analysis of this structure, we can retrace the calculations made in field theory to include torsion, since the entire effect is contained in the Bogoliubov coefficients.

$$\begin{aligned}
\mathcal{Q}^{r,\vec{k}}_{\nu_e \to \nu_e}(t) = 1 &- \sin^2(2\theta_{12}) \cos^4(\theta_{13}) \left[\left|\Gamma^{rr}_{12;\vec{k}}\right|^2 \sin^2\left(\Delta^r_{12;\vec{k}} t\right) + \left|\Sigma^{rr}_{12;\vec{k}}\right|^2 \sin^2\left(\Omega^r_{12;\vec{k}} t\right) \right] \\
&- \sin^2(2\theta_{13}) \cos^2(\theta_{12}) \left[\left|\Gamma^{rr}_{13;\vec{k}}\right|^2 \sin^2\left(\Delta^r_{13;\vec{k}} t\right) + \left|\Sigma^{rr}_{13;\vec{k}}\right|^2 \sin^2\left(\Omega^r_{13;\vec{k}} t\right) \right] \\
&- \sin^2(2\theta_{13}) \sin^2(\theta_{12}) \left[\left|\Gamma^{rr}_{23;\vec{k}}\right|^2 \sin^2\left(\Delta^r_{23;\vec{k}} t\right) + \left|\Sigma^{rr}_{23;\vec{k}}\right|^2 \sin^2\left(\Omega^r_{23;\vec{k}} t\right) \right],
\end{aligned}$$

where $r = \pm$, $\Delta^r_{ij;\vec{k}} \equiv \frac{E^r_{j;\vec{k}} - E^r_{i;\vec{k}}}{2}$, $\Omega^r_{ij;\vec{k}} \equiv \frac{E^r_{j;\vec{k}} + E^r_{i;\vec{k}}}{2}$. $\Gamma^{rr}_{ij;\vec{k}} = \Xi^{rr}_{ij;\vec{k}}$ and $\Sigma^{rr}_{ij;\vec{k}} = \chi^{rr}_{ij;\vec{k}}$ for constant torsion

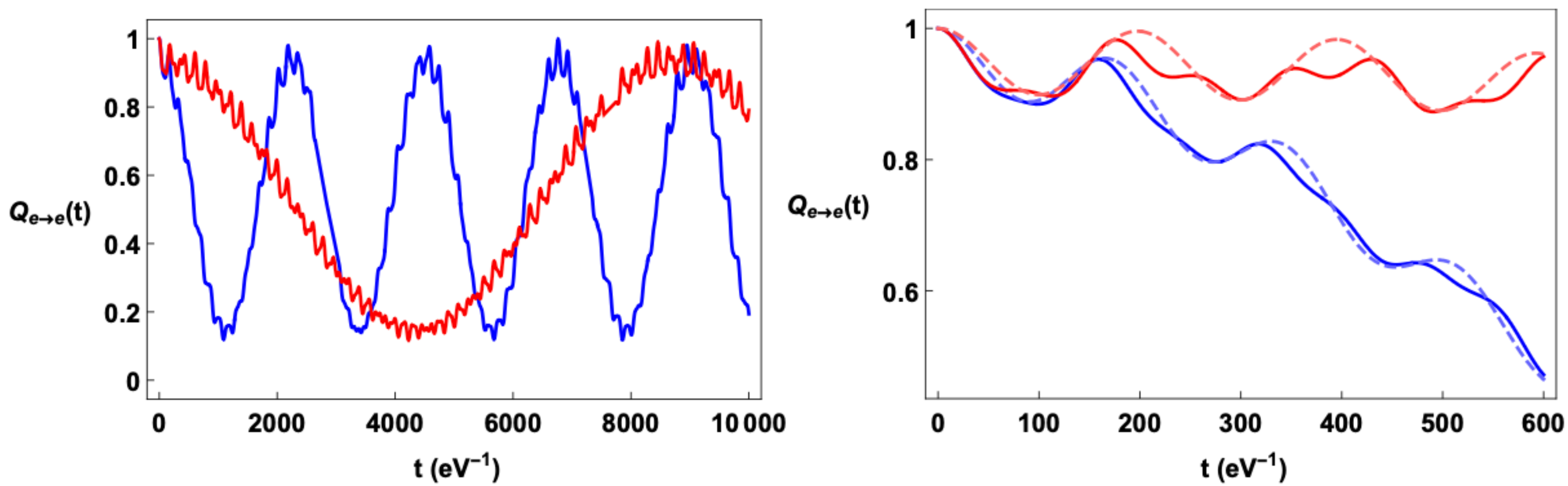


Figure 1. Color on line. Plots of the oscillation formulae in a constant torsion background: in the left-hand panel $\mathcal{Q}^{\uparrow\vec{k}}_{\nu_e\to\nu_e}(t)$ (blue line) and $\mathcal{Q}^{\downarrow\vec{k}}_{\nu_e\to\nu_e}(t)$ (red line) as a function of time. Torsion was picked to be comparable to the momentum as $T^3 = 2\times10^{-4}$ eV. In the right panel, the detail of the same formulae and the comparison with the corresponding quantum mechanics oscillation formulae (dashed line) are reported.

$$\mathcal{Q}^{\uparrow\vec{k}}_{\nu_e\to\nu_e} \neq \mathcal{Q}^{\downarrow\vec{k}}_{\nu_e\to\nu_e}$$

Neutrino oscillation formulas in the precence of time-dependent torsion

The field satisfied the equation $$(i\gamma^{\mu}\partial_{\mu} - m)\Psi(x) = \eta \breve{T}^{\rho}(t)\gamma_{\rho}\gamma^{5}\Psi(x)$$

In order to derive the solution of the Dirac equation with torsion, we write the spinor in the following form

$$\psi(\vec{x},t) = \sum_{r}\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\left(u^{r}_{\vec{k}}(t)\alpha^{r}_{\vec{k}} + v^{r}_{-\vec{k}}(t)\beta^{r\dagger}_{-\vec{k}}\right)e^{i\vec{k}\cdot\vec{x}}$$

We use ansatz

$$u_{\vec{p},\lambda}(t,\boldsymbol{x}) = e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\begin{pmatrix} f_p(t)\xi_{\lambda}(\hat{p}) \\ g_p(t)\lambda\xi_{\lambda}(\hat{p}) \end{pmatrix}$$ for positive energy and

$$v_{\vec{p},\lambda}(t,\boldsymbol{x}) = e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\begin{pmatrix} g^{*}_p(t)\xi_{\lambda}(\hat{p}) \\ -f^{*}_p(t)\lambda\xi_{\lambda}(\hat{p}) \end{pmatrix}$$ for negative energy.

Dirac Field Quantization with Time-Dependent Torsion

$$\begin{cases} f_{\vec{p},\lambda}(t) = \exp\left\{-i\frac{t^2}{2}\eta\lambda\alpha^i\hat{p}^i\right\}\exp\{-i\omega_{p,\lambda}t\}C_{\vec{p},\lambda} \\ g_{\vec{p},\lambda}(t) = \frac{p+\eta\lambda\check{T}^0}{(\omega_{p,\lambda}+m)}\exp\left\{-i\frac{t^2}{2}\eta\lambda\alpha^i\hat{p}^i\right\}\exp\{-i\omega_{p,\lambda}t\}C_{\vec{p},\lambda} \end{cases}.$$

where $\omega_{p,\lambda} = \sqrt{m^2 + (p + \eta\lambda\check{T}^0)^2}.$

By imposing normalisation condition

$$\boxed{\left|f_{\vec{p},\lambda}(t)\right|^2 + \left|g_{\vec{p},\lambda}(t)\right|^2 = \frac{1}{(2\pi)^3}}$$

we determine

$$C_{\vec{p},\lambda} = \frac{\omega_{p,\lambda}+m}{(2\pi)^{\frac{3}{2}}\sqrt{(\omega_{p,\lambda}+m)^2+(p+\eta\lambda\check{T}^0)^2}}.$$

The linearity of the generator allows us to define the flavour annihilators of the fields and assuming that: $\vec{k} = (0, 0, |\vec{k}|)$

$$\boxed{\begin{aligned}\alpha^r_{\vec{k},\nu_e}(t) = c_{12}c_{13}\alpha^r_{\vec{k},1} + s_{12}c_{13}\Big(\Big(\Gamma^{rr}_{12;\vec{k}}(t)\Big)^*\alpha^r_{\vec{k},2} + \varepsilon^r\Big(\Sigma^{rr}_{12;\vec{k}}(t)\Big)\beta^{r\dagger}_{-\vec{k},2}\Big)\\ + e^{-i\delta}s_{13}\Big(\Big(\Gamma^{rr}_{13;\vec{k}}(t)\Big)^*\alpha^r_{\vec{k},3} + \varepsilon^r\Big(\Sigma^{rr}_{13;\vec{k}}(t)\Big)\beta^{r\dagger}_{-\vec{k},3}\Big),\end{aligned}}$$

$\Gamma^{rr}_{ij;\vec{k}} = \Pi^{rr}_{ij;\vec{k}}$ and $\Sigma^{rr}_{ij;\vec{k}} = \Upsilon^{rr}_{ij;\vec{k}}$ for time-dependent torsion

In this case, the Bogoliubov coefficients are

$$\Pi^{++}_{ij;\vec{p}}(t) = (2\pi)^3 \exp\Big\{-i\Big(\omega^j_{p,+} - \omega^i_{p,+}\Big)t\Big\}\Big(C^+_{\vec{p},i}\Big)^*\Big(C^+_{\vec{p},j}\Big)\left[1 + \frac{|p+\eta\check{T}^0|^2}{\Big(\omega^i_{p,+}+m_i\Big)\Big(\omega^j_{p,+}+m_j\Big)}\right],$$

$$\Pi^{--}_{ij;\vec{p}}(t) = (2\pi)^3 \exp\Big\{-i\Big(\omega^j_{p,-} - \omega^i_{p,-}\Big)t\Big\}\Big(C^-_{\vec{p},i}\Big)^*\Big(C^-_{\vec{p},j}\Big)\left[1 + \frac{|p-\eta\check{T}^0|^2}{\Big(\omega^i_{p,-}+m_i\Big)\Big(\omega^j_{p,-}+m_j\Big)}\right],$$

$$\Upsilon^{++}_{ij;\vec{p}}(t) = (2\pi)^3 \exp\Big\{+it^2\eta\alpha^i\hat{p}^i\Big\}\exp\Big\{+i\Big(\omega^j_{p,+} + \omega^i_{p,+}\Big)t\Big\}\Big(C^+_{\vec{p},i}\Big)^*\Big(C^+_{\vec{p},j}\Big)^*\Big(p+\eta\check{T}^0\Big)\left[\frac{1}{\omega^j_{p,+}+m_j} - \frac{1}{\omega^i_{p,+}+m_i}\right],$$

$$\Upsilon^{--}_{ij;\vec{p}}(t) = (2\pi)^3 \exp\Big\{+it^2\eta\alpha^i\hat{p}^i\Big\}\exp\Big\{+i\Big(\omega^j_{p,+} + \omega^i_{p,+}\Big)t\Big\}\Big(C^+_{\vec{p},i}\Big)^*\Big(C^+_{\vec{p},j}\Big)^*\Big(p-\eta\check{T}^0\Big)\left[\frac{1}{\omega^j_{p,+}+m_j} - \frac{1}{\omega^i_{p,+}+m_i}\right],$$

where $i, j = 1, 2, 3$ and $j > i$.

From the analysis of this structure, we can retrace the calculations made for the case of constant torsion, since the entire effect is contained in the Bogoliubov coefficients.

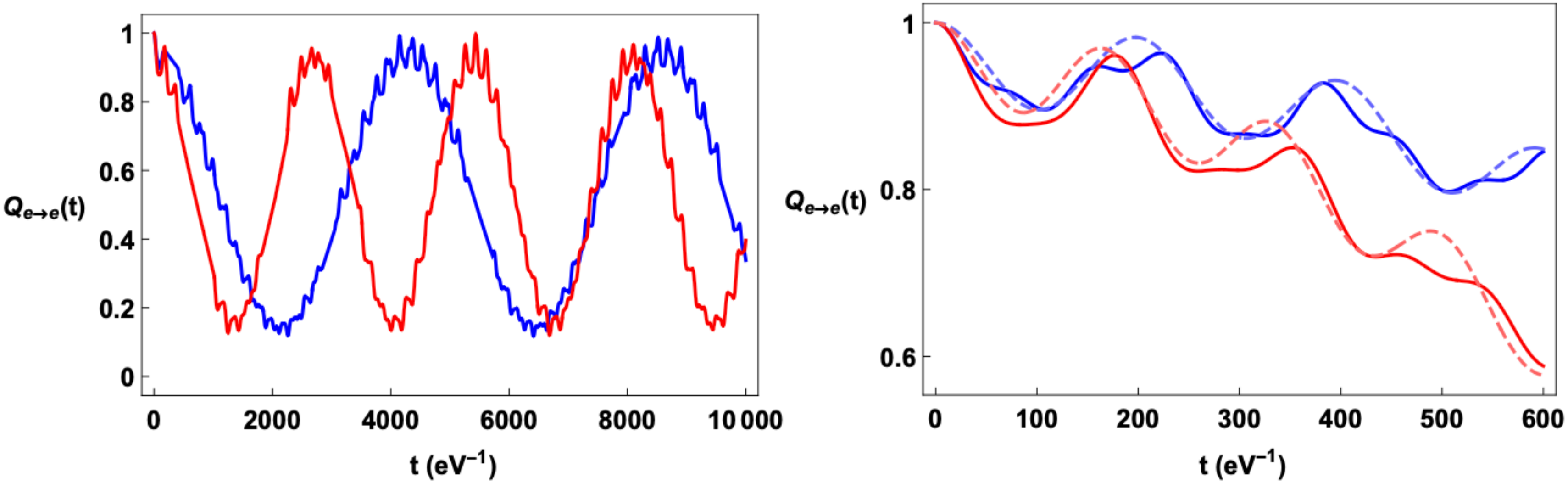


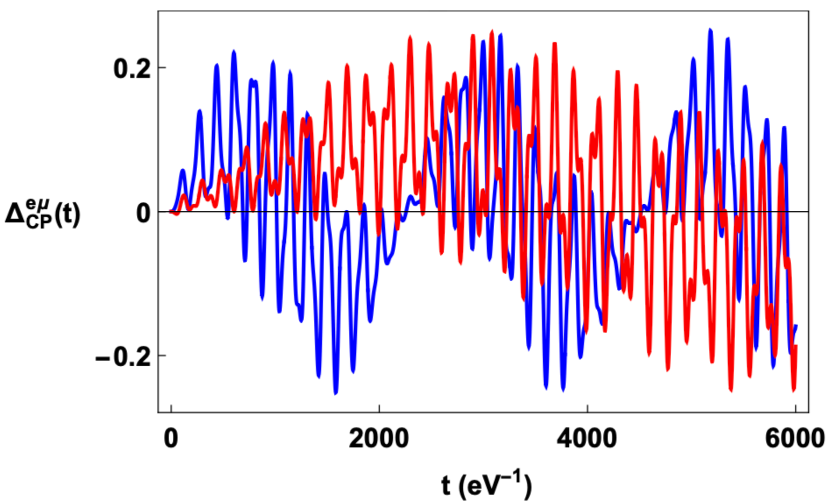
Figure 4. Color on line. Plots of the oscillation formulae in a time-dependent torsion: in the left-hand panel, $\mathcal{Q}^{\uparrow\vec{k}}_{\nu_e\to\nu_e}(t)$ (blue line) and $\mathcal{Q}^{\downarrow\vec{k}}_{\nu_e\to\nu_e}(t)$ (red line) are plotted as a function of time. In the right panel, the details of the same formulae and the comparison with the corresponding quantum mechanics oscillation formulae (dashed line) are reported. We consider $\eta \check{T}^0 = 5\times 10^{-3}$ eV.

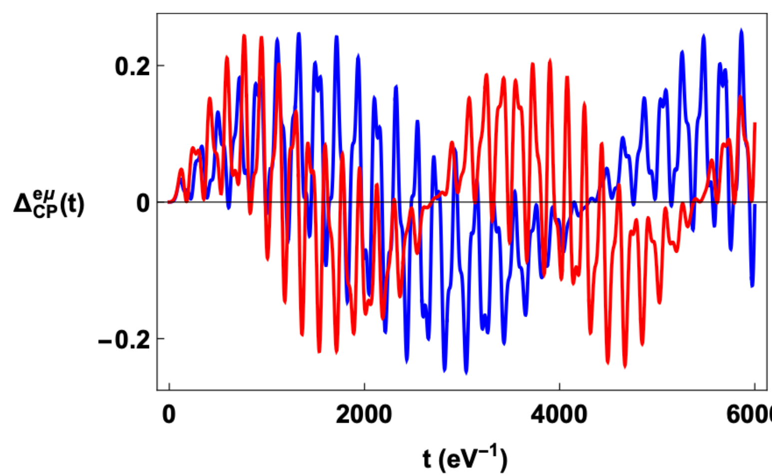
$$\mathcal{Q}^{\uparrow\vec{k}}_{\nu_e\to\nu_e} \neq \mathcal{Q}^{\downarrow\vec{k}}_{\nu_e\to\nu_e}$$

CP Violation and Flavor Vacuum

We study the impact of torsion on the CP violation in neutrino oscillation due to the presence of Dirac phase in the mixing matrix.

$$\Delta^{e\mu}_{r;CP}(t) = 4J_{CP}\Big[\left|\Gamma^{\pm\pm}_{12;\vec{k}}\right|^2 \sin\left(2\Delta^{\pm}_{12;\vec{k}}t\right) - \left|\Sigma^{\pm\pm}_{12;\vec{k}}\right|^2 \sin\left(2\Omega^{\pm}_{12;\vec{k}}t\right) + + \left(\left|\Gamma^{\pm\pm}_{12;\vec{k}}\right|^2 - \left|\Sigma^{\pm\pm}_{13;\vec{k}}\right|^2\right)\sin\left(2\Delta^{\pm}_{23;\vec{k}}t\right)$$
$$+ \left(\left|\Sigma^{\pm\pm}_{12;\vec{k}}\right|^2 - \left|\Sigma^{\pm\pm}_{13;\vec{k}}\right|^2\right)\sin\left(2\Omega^{\pm}_{23;\vec{k}}t\right) - \left|\Gamma^{\pm\pm}_{13;\vec{k}}\right|^2 \sin\left(2\Delta^{\pm}_{13;\vec{k}}t\right) + \left|\Sigma^{\pm\pm}_{13;\vec{k}}\right|^2 \sin\left(2\Omega^{\pm}_{13;\vec{k}}t\right)\Big],$$





Plot of $\Delta^{e\mu}_{\uparrow;CP}(t)$ (blue line) and $\Delta^{e\mu}_{\downarrow;CP}(t)$ (red line) for costant torsion (left panel) and for Time-dependent torsion (right panel)

Expectation values of the number operators

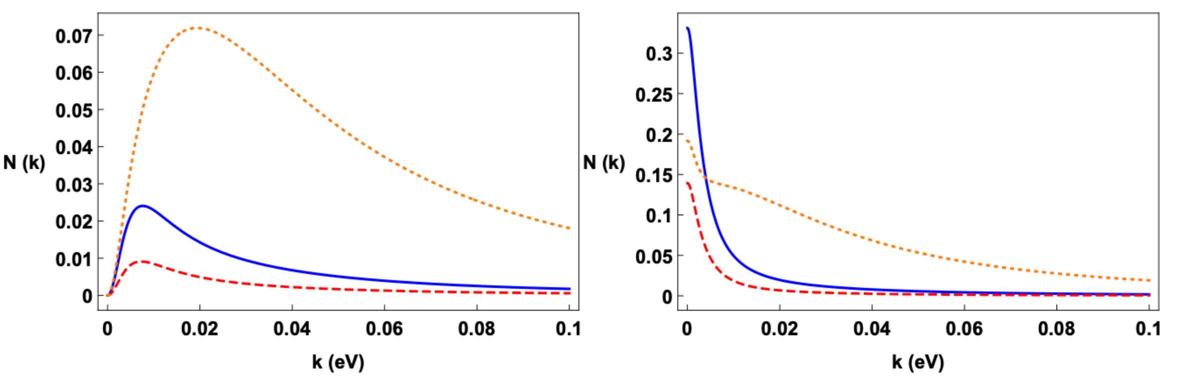
$$\mathcal{N}^r_{1;\vec{k}} ={}_f \langle 0(t)|N^r_{\alpha_1,\vec{k}}|0(t)\rangle_f = {}_f\langle 0(t)|N^r_{\beta_1,\vec{k}}|0(t)\rangle_f = s_{12}^2 c_{13}^2 \left|\Sigma^{\pm\pm}_{12;\vec{k}}\right|^2 + s_{13}^2 \left|\Sigma^{\pm\pm}_{13;\vec{k}}\right|^2,$$

$$\mathcal{N}^r_{2;\vec{k}} ={}_f \langle 0(t)|N^r_{\alpha_2,\vec{k}}|0(t)\rangle_f = {}_f\langle 0(t)|N^r_{\beta_2,\vec{k}}|0(t)\rangle_f = \left|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}\right|^2 \left|\Sigma^{\pm\pm}_{12;\vec{k}}\right|^2 + s_{23}^2 c_{13}^2 \left|\Sigma^{\pm\pm}_{23;\vec{k}}\right|^2,$$

$$\mathcal{N}^r_{3;\vec{k}} ={}_f \langle 0(t)|N^r_{\alpha_3,\vec{k}}|0(t)\rangle_f = {}_f\langle 0(t)|N^r_{\beta_3,\vec{k}}|0(t)\rangle_f$$

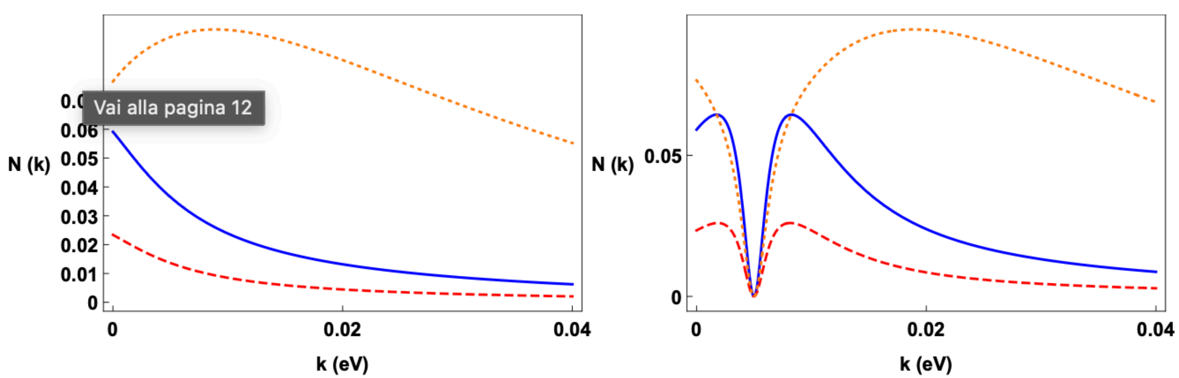
$$= \left|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}\right|^2 \left|\Sigma^{\pm\pm}_{23;\vec{k}}\right|^2 + \left|s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}\right|^2 \left|\Sigma^{\pm\pm}_{13;\vec{k}}\right|^2 ,$$

where, $r = \uparrow, \downarrow$.



Plots of $\mathcal{N}^{\uparrow}_{i;\vec{k}}$ as a function of $|\vec{k}|$
N1 (blue solid), N2 (red dashed line) and N3 (orange dotted line).
(Right panel) Plots of $\mathcal{N}^{\downarrow}_{i;\vec{k}}$ as a function of $|\vec{k}|$

for costant torsion



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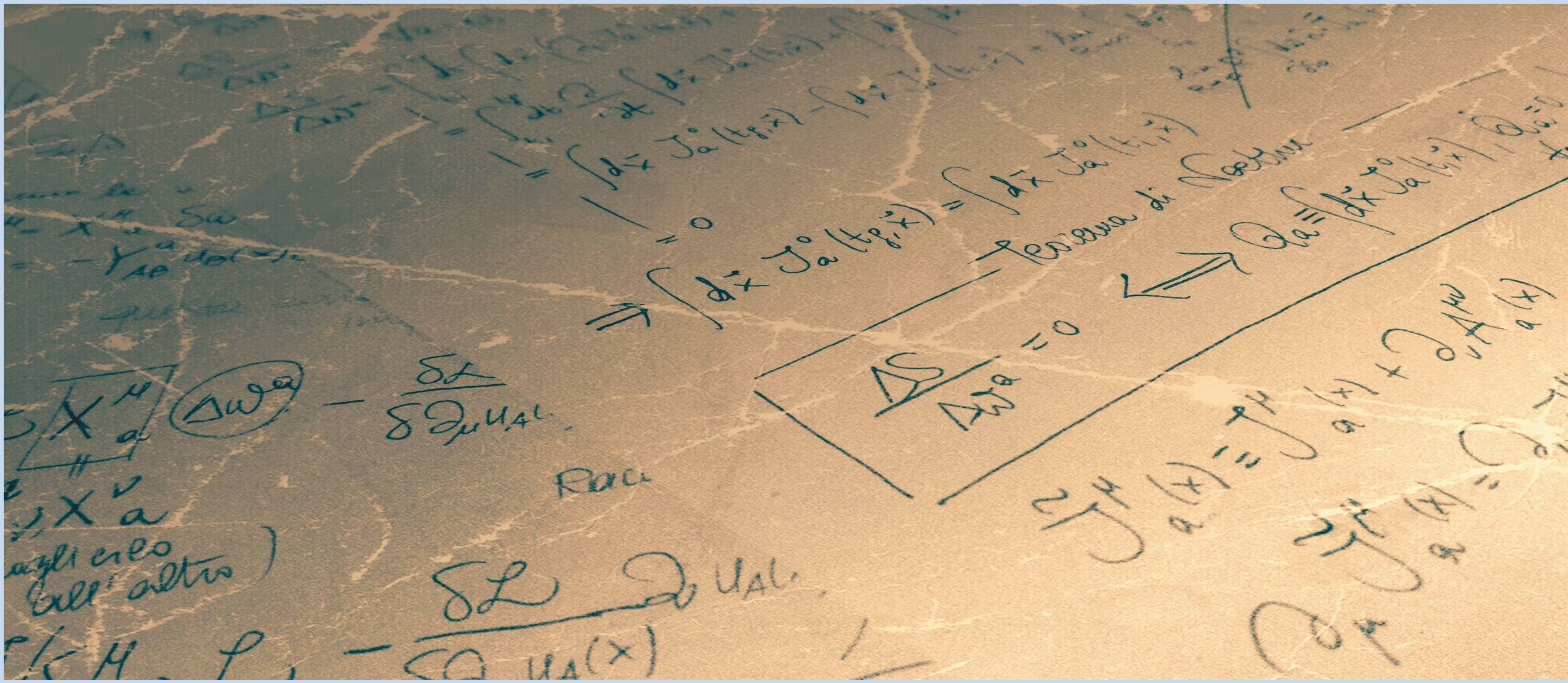
for Time-dependent torsion

CONCLUSIONS

The Einstein-Cartan theory is a modification of general relativity.

- Since spin and torsion are intrinsically linked, it has been shown how torsion couples to Dirac fermions.
- We derived new oscillation formulae which are dependent on the spin orientations of the neutrino fields.
- We showed that the energy splitting induced by the torsion term affects the oscillation frequencies and the Bogoliubov coefficients which represent the amplitudes of the oscillation formulae.
- The torsion effects are relevant on neutrino oscillations in non-relativistic regimes. Therefore, experiments studying neutrinos with very low momenta, such as PTOLEMY, could provide verification of such results in the future.



…Thank you for your attention

14 Novembre 2025

Il caso in esame del mixing tra tre sapori del neutrino risulta particolarmente interessante poiché include la violazione CP. A fissato spin si consideri la quantità $\Delta^{\rho\sigma}_{\uparrow;CP}$ definita come segue:

$$\begin{aligned}
\Delta^{\rho\sigma}_{\uparrow;CP}(t) &\equiv \mathcal{Q}^{\uparrow\vec{k}}_{\nu_\rho\to\nu_\sigma}(t) + \mathcal{Q}^{\uparrow\vec{k}}_{\overline{\nu}_\rho\to\overline{\nu}_\sigma}(t) \\
&= \left|\left\{\alpha^{\uparrow}_{\vec{k},\nu_\sigma}(t), \alpha^{\uparrow\dagger}_{\vec{k},\nu_\rho}(0)\right\}\right|^2 + \left|\left\{\beta^{\uparrow\dagger}_{-\vec{k},\nu_\sigma}(t), \alpha^{\uparrow\dagger}_{\vec{k},\nu_\rho}(0)\right\}\right|^2 + \\
&\quad - \left|\left\{\alpha^{\uparrow\dagger}_{-\vec{k},\nu_\sigma}(t), \beta^{\uparrow\dagger}_{\vec{k},\nu_\rho}(0)\right\}\right|^2 - \left|\left\{\beta^{\uparrow}_{\vec{k},\nu_\sigma}(t), \beta^{\uparrow\dagger}_{\vec{k},\nu_\rho}(0)\right\}\right|^2 .
\end{aligned}$$

Ricordando che risultano soddisfatte le seguenti relazioni:

$$\sum_\sigma Q_{\nu_\sigma}(t) = Q\,, \qquad \langle\nu_\rho|\, Q\,|\nu_\rho\rangle = 1 \qquad e \qquad \langle\overline{\nu}_\rho|\, Q\,|\overline{\nu}_\rho\rangle = -1\,,$$

si mostra la validità della seguente identità:

$$\sum_\sigma \Delta^{\rho\sigma}_{\uparrow;CP} = 0 \quad , \qquad \rho,\sigma = e,\mu,\tau\,.$$

Nel caso della transizione $\nu_e \to \nu_\mu$ si mostra che la quantità $\Delta^{\rho\sigma}_{\uparrow;CP}$ assume la seguente forma:

$$\begin{aligned}
\Delta^{e\mu}_{\uparrow;CP}(t) = 4J_{CP}\Big[&\left|U^{++}_{12;\vec{k}}\right|^2 \sin\left(2\Delta^{+}_{12;\vec{k}}t\right) - \left|V^{++}_{12;\vec{k}}\right|^2 \sin\left(2\Omega^{+}_{13;\vec{k}}t\right) + \\
&+ \left(\left|U^{++}_{12;\vec{k}}\right|^2 - \left|V^{++}_{13;\vec{k}}\right|^2\right)\sin\left(2\Delta^{+}_{23;\vec{k}}t\right) + \left(\left|V^{++}_{12;\vec{k}}\right|^2 - \left|V^{++}_{13;\vec{k}}\right|^2\right)\sin\left(2\Omega^{+}_{23;\vec{k}}t\right) + \\
&- \left|U^{++}_{13;\vec{k}}\right|^2 \sin\left(2\Delta^{+}_{13;\vec{k}}t\right) + \left|V^{++}_{13;\vec{k}}\right|^2 \sin\left(2\Omega^{+}_{13;\vec{k}}t\right)\Big] .
\end{aligned}$$

Inoltre $\Delta^{e\tau}_{r;CP}(t) = -\Delta^{e\mu}_{r;CP}(t)$ con $r = \uparrow, \downarrow$.

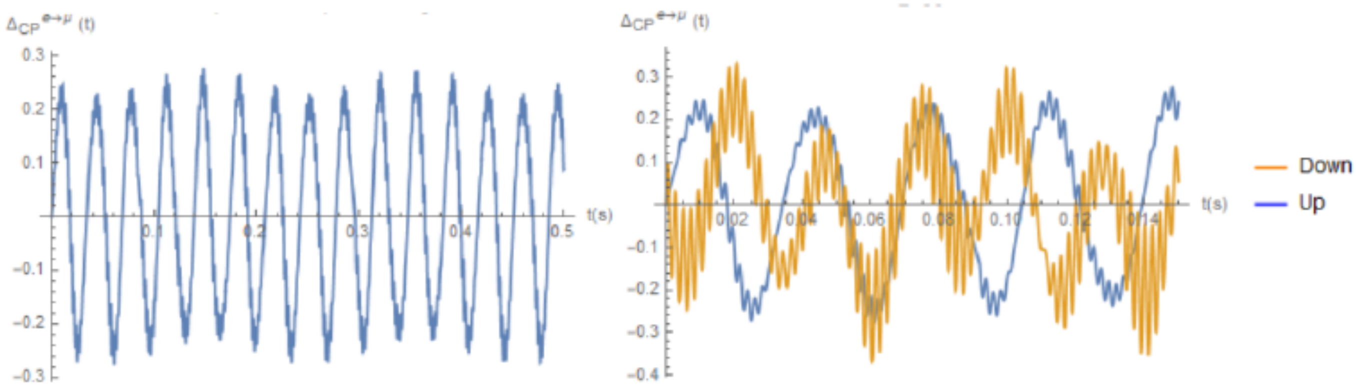


Figura 3.10: Nel pannello di sinistra è stato riportato l'andamento dell'asimmetria CP a fissato spin up $\uparrow$ in funzione del tempo per i valori dei parametri riportati in Tab.(3.1) in cui è stato scelto il valore di torsione tale per cui $\frac{3}{2}h\check{T}^3 = 50eV$. Nel pannello di destra è stato confrontato l'andamento dell'asimmetria CP a fissato spin up $\uparrow$ con quello a spin down $\downarrow$ in funzione del tempo per gli stessi valori dei parametri.

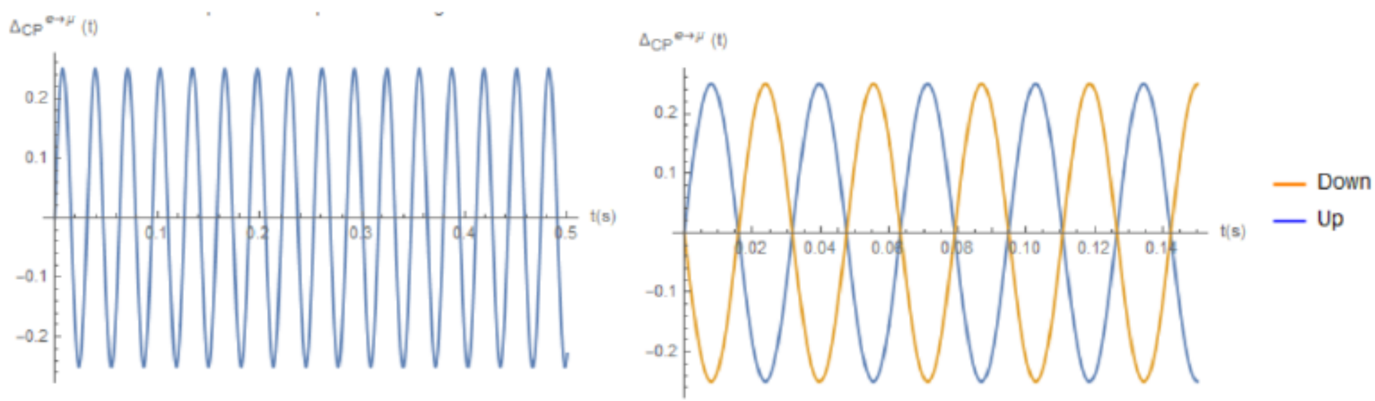


Figura 3.11: Nel pannello di sinistra è stato riportato l'andamento dell'asimmetria CP a fissato spin up $\uparrow$ in funzione del tempo per i valori dei parametri riportati in Tab.(3.1) in cui è stato scelto il valore di torsione tale per cui $\frac{3}{2}h\check{T}^3 = 1000eV$. Nel pannello di destra è stato confrontato l'andamento dell'asimmetria CP a fissato spin up $\uparrow$ con quello a spin down $\downarrow$ in funzione del tempo per gli stessi valori dei parametri.

La generalizzazione a tre sapori, oltre alla violazione CP, induce, a differenza del caso di due generazioni, un differente numero di particelle condensate a seconda della massa in analisi. Si mostra infatti tramite le relazioni che legano gli operatori di creazione e distruzione a sapore definito con quelle a massa definita le seguenti identità:

$$\mathcal{N}^{\uparrow}_{1;\vec{k}} = {}_f \langle 0(t)| N^{\uparrow}_{\alpha_1,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\uparrow}_{\beta_1,\vec{k}} |0(t)\rangle_f = s_{12}^2 c_{13}^2 \left|V^{++}_{12;\vec{k}}\right|^2 + s_{13}^2 \left|V^{++}_{13;\vec{k}}\right|^2 ,$$

$$\mathcal{N}^{\downarrow}_{1;\vec{k}} = {}_f \langle 0(t)| N^{\downarrow}_{\alpha_1,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\downarrow}_{\beta_1,\vec{k}} |0(t)\rangle_f = s_{12}^2 c_{13}^2 \left|V^{--}_{12;\vec{k}}\right|^2 + s_{13}^2 \left|V^{--}_{13;\vec{k}}\right|^2 ,$$

$$\mathcal{N}^{\uparrow}_{2;\vec{k}} = {}_f \langle 0(t)| N^{\uparrow}_{\alpha_2,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\uparrow}_{\beta_2,\vec{k}} |0(t)\rangle_f = \left|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}\right|^2 \left|V^{++}_{12;\vec{k}}\right|^2 + s_{23}^2 c_{13}^2 c_{13}^2 \left|V^{++}_{23;\vec{k}}\right|^2 ,$$

$$\mathcal{N}^{\downarrow}_{2;\vec{k}} = {}_f \langle 0(t)| N^{\downarrow}_{\alpha_2,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\downarrow}_{\beta_2,\vec{k}} |0(t)\rangle_f = \left|-s_{12}c_{23} + e^{i\delta} c_{12}s_{23}s_{13}\right|^2 \left|V^{--}_{12;\vec{k}}\right|^2 + s_{23}^2 c_{13}^2 c_{13}^2 \left|V^{--}_{23;\vec{k}}\right|^2 ,$$

$$\mathcal{N}^{\uparrow}_{3;\vec{k}} = {}_f \langle 0(t)| N^{\uparrow}_{\alpha_3,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\uparrow}_{\beta_3,\vec{k}} |0(t)\rangle_f = \left|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}\right|^2 \left|V^{++}_{23;\vec{k}}\right|^2 + \left|s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}\right|^2 \left|V^{++}_{13;\vec{k}}\right|^2 .$$

$$\mathcal{N}^{\downarrow}_{3;\vec{k}} = {}_f \langle 0(t)| N^{\downarrow}_{\alpha_3,\vec{k}} |0(t)\rangle_f = {}_f \langle 0(t)| N^{\downarrow}_{\beta_3,\vec{k}} |0(t)\rangle_f = \left|-c_{12}s_{23} + e^{i\delta} s_{12}c_{23}s_{13}\right|^2 \left|V^{--}_{23;\vec{k}}\right|^2 + \left|s_{12}s_{23} + e^{i\delta} c_{12}c_{23}s_{13}\right|^2 \left|V^{--}_{13;\vec{k}}\right|^2 .$$

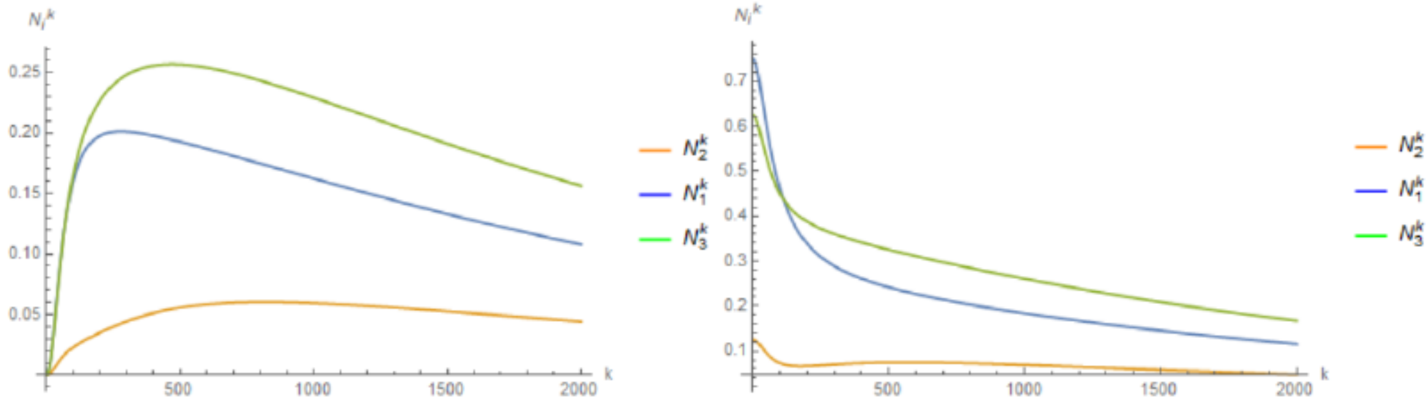


Figura 3.12: Nel pannello di sinistra sono riportati gli andamenti di $\mathcal{N}^{\uparrow}_{i;\vec{k}}$ a fissato spin up $\uparrow$ in funzione di $\left|\vec{k}\right|$ per i valori dei parametri riportati in Tab.(3.1) in cui è stato scelto il valore di torsione tale per cui $\frac{3}{2}h\breve{T}^3 = 50eV$. Nel pannello di destra sono riportati gli andamenti di $\mathcal{N}^{\downarrow}_{i;\vec{k}}$ a fissato spin down $\downarrow$ in funzione di $\left|\vec{k}\right|$ per gli stessi valori dei parametri.

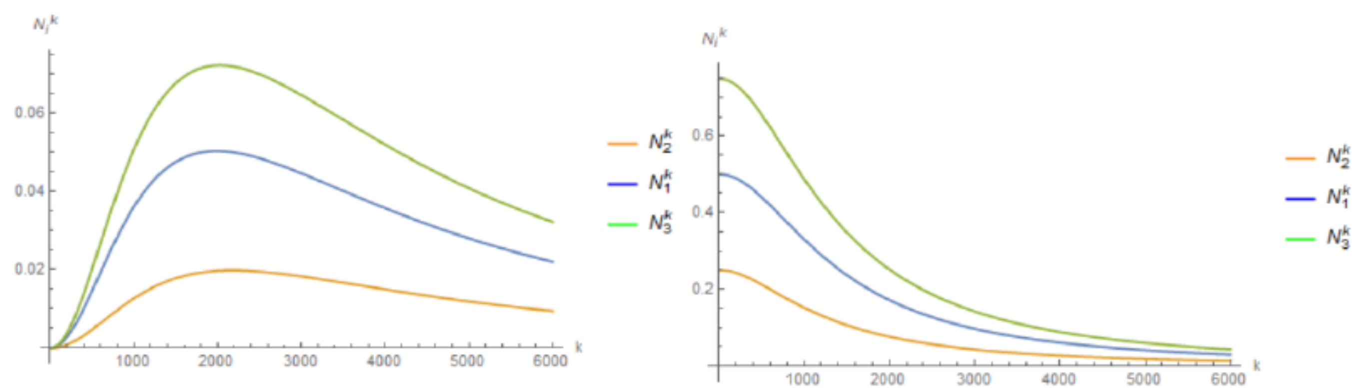


Figura 3.13: Nel pannello di sinistra sono riportati gli andamenti di $\mathcal{N}^{\uparrow}_{i;\vec{k}}$ a fissato spin up $\uparrow$ in funzione di $\left|\vec{k}\right|$ per i valori dei parametri riportati in Tab.(3.1) in cui è stato scelto il valore di torsione tale per cui $\frac{3}{2}h\breve{T}^3 = 1000eV$. Nel pannello di destra sono riportati gli andamenti di $\mathcal{N}^{\downarrow}_{i;\vec{k}}$ a fissato spin down $\downarrow$ in funzione di $\left|\vec{k}\right|$ per gli stessi valori dei parametri.