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Recent progress in the Pre-Big Bang Scenario

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Different types of “Inflation”

I: $a \sim t^\beta,\ \beta > 1,\ t \to +\infty$
standard inflation, accelerated expansion and decreasing curvature:

$$\dot{a} > 0,\ \ddot{a} > 0,\ \dot{H} < 0$$

II: $a \sim (-t)^\beta,\ \beta < 1,\ t \to 0_-$

- **IIa** : **super inflation**: $\beta < 0$, accelerated expansion with growing curvature:

$$\dot{a} > 0,\ \ddot{a} > 0,\ H > 0,\ \dot{H} > 0$$

- **IIb** : **accelerated contraction**: $0 < \beta < 1$, accelerated contraction with growing curvature:

$$\dot{a} < 0,\ \ddot{a} < 0,\ H < 0,\ \dot{H} < 0$$

Pre-Big Bang Scenario

It is a scenario obtained from string theory

$$g_s \ll 1\,, \quad H^2\alpha' \ll 1\,, \quad \alpha' = l_s^2/2\pi$$

$$\boxed{S = -\frac{1}{2l_s^{d-1}} \int d^{d+1}\sqrt{|g|}e^{-\phi}\Big[R + (\boldsymbol{\nabla}\phi)^2 - \frac{1}{12}\mathcal{H}^2_{\mu\nu\rho}\Big]}$$

R Ricci Scalar, ϕ Dilaton

$$\mathcal{H}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} \qquad \text{K-R field}$$

$$\mathcal{H}^{\mu\nu\rho} = \frac{e^\phi}{\sqrt{|g|}}\epsilon^{\mu\nu\rho\lambda}\partial_\lambda\sigma \qquad \text{in 4-D}$$

T-duality: given a universe with radius R, the theory is invariant under the transformation $R \to l_s^2/R$.

M. Gasperini, G. Veneziano, *"The pre-big bang scenario in string cosmology"*, Physics Reports, Volume 373, 2003.

- Equations are invariant under **scale-factor duality**:

$$a_i \to \tilde{a}_i = a_i^{-1}\,, \qquad \tilde{\phi} \to \phi - 2\sum_i \ln a_i \ ,$$

Thus we have 4 related solutions:

$$\{a(t),\ a(-t),\ a^{-1}(t),\ a^{-1}(-t)\}$$

.

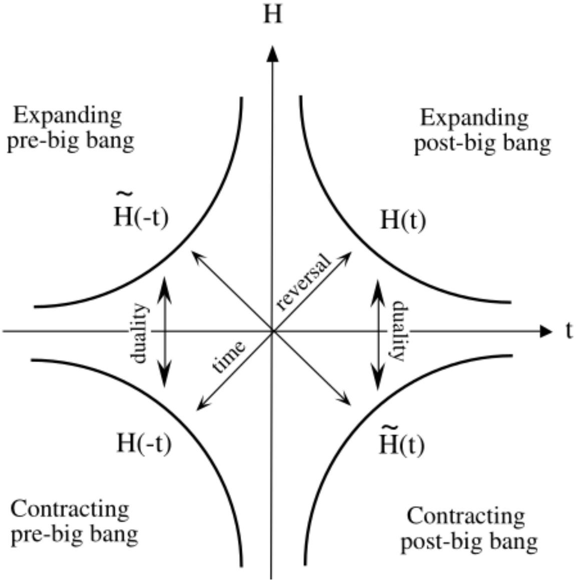
- We can move from the S-frame to the E-frame by a conformal transformation:

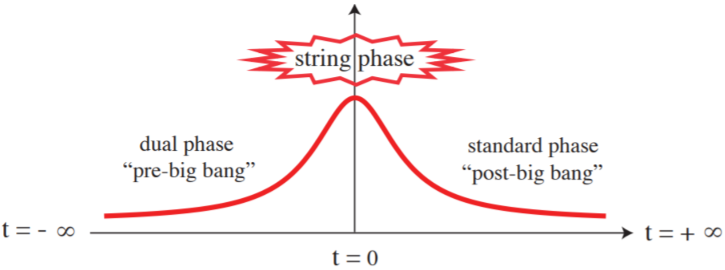
$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}\ , \quad \Omega = e^{-\frac{\phi}{d-1}}\ , \quad \tilde{\phi} = \sqrt{\frac{2}{d-1}}\phi$$

in which the action takes the form:

$$S(\tilde{g},\tilde{\phi}) = -\frac{1}{2\lambda_s^{d-1}}\int d^{d+1}x\sqrt{|\tilde{g}|}\Big[\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}\Big]\ .$$

Expanding pre-big bang $a(t)$ **in S-frame** $\Longleftrightarrow$ **contracting pre-big bang** $\tilde{a(t)}$ **in E-frame**


H
Expanding
pre-big bang
Expanding
post-big bang
$\tilde{H}(-t)$
H(t)
reversal
duality
duality
t
time
H(-t)
$\tilde{H}(t)$
Contracting
pre-big bang
Contracting
post-big bang



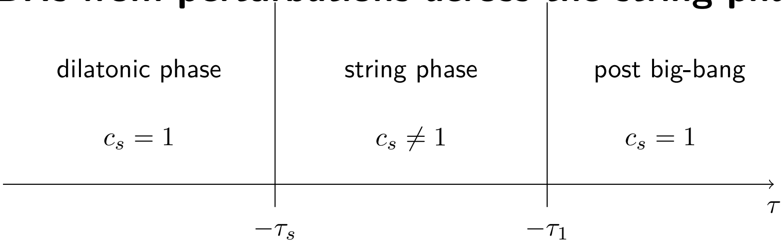
The Pre-Big Bang solution has a singularity in the future ($t \to 0_-$); $H, g_s \to \infty$. The action needs to be corrected:

$$S = S_0 + S_{\alpha'} + S_{loop} \ ,$$

- α' corrections in the action, high derivative terms.
- loop corrections, as correction in powers of g_s.

Such corrections generate a non-trivial sound speed on perturbations.

PBHs from perturbations across the string phase



We can evaluate the power spectrum at re-entry $k = aH$:

$$\mathcal{P}(k) = \frac{k^3}{2\pi^2} \left|\mathcal{R}_k^3\right|^2_{|k\tau|=1}$$

$$\mathcal{P}(k) \sim \left(\frac{H_1}{M_\mathrm{P}}\right)^2 \left(\frac{k}{k_1}\right)^{3-2|\nu_2|} c_s^{-1-2|\nu_2|}\,, \qquad k_s/c_s < k < k_1/c_s,$$

sound speed c_s dependence $\implies$ enhancement of the spectrum.

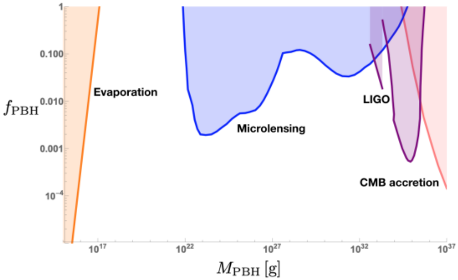
PBHs production

PBHs production

A fluctuation with frequency ω_M, which re-enters at the scale H_M

$$M_{\mathsf{PBH}} \sim \frac{M_{\mathrm{P}}^2}{H_M}. \qquad \longrightarrow \qquad \beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}\bigg|_{at formation} \supset \mathcal{P}$$

- Natural candidates for dark matter $\qquad f_{pbh} \equiv \frac{\Omega_{pbh}}{\Omega_{cdm}} \sim \beta$

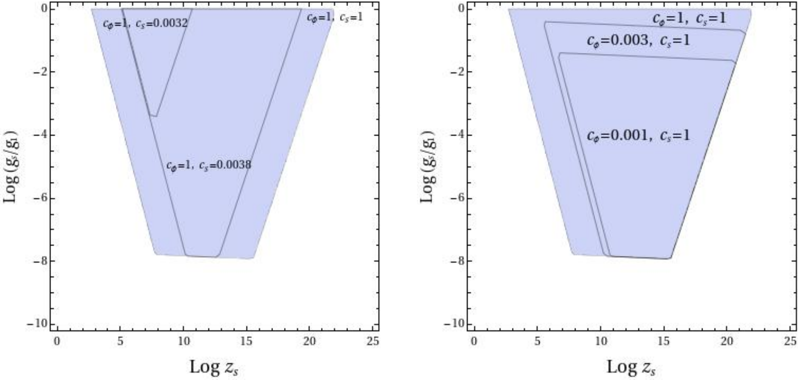


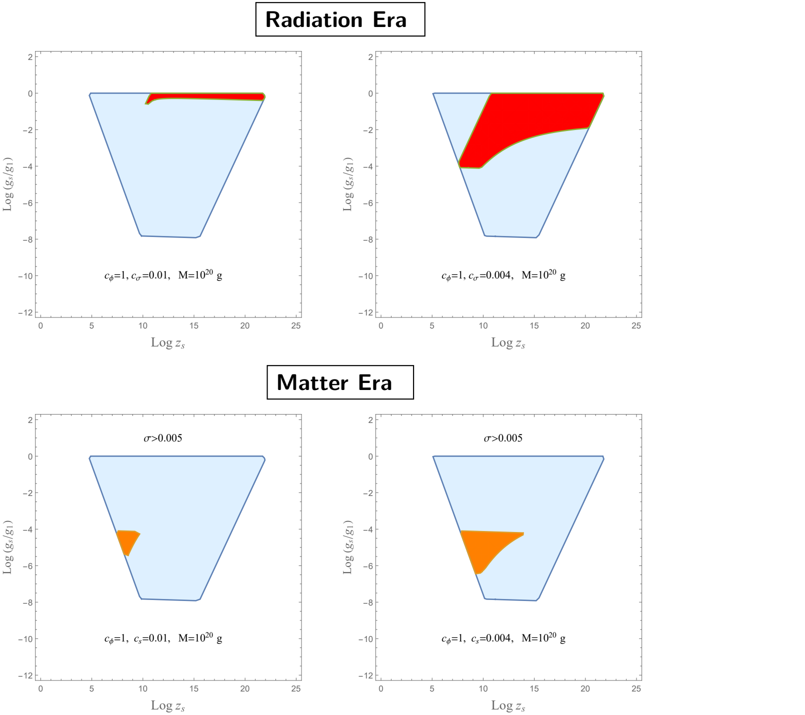
Parameter space of the model

Our parameter space can be expressed in terms of 2 parameters:

$$z_s = \frac{\tau_s}{\tau_1} = \frac{a_1}{a_s} = \frac{\omega_1}{\omega_s} \qquad \text{time scale of string phase}$$

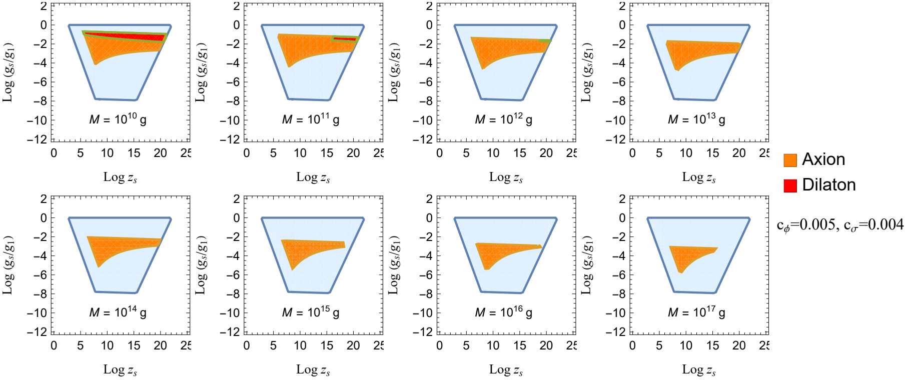
$$\frac{g_s}{g_1} = \left(\frac{\tau_s}{\tau_1}\right)^{-\beta} = z_s^{-\beta} \qquad \text{evolution of dilaton field}$$





Radiation Era
Matter Era
c_ϕ=1, c_σ=0.01, M=10^{20} g
c_ϕ=1, c_σ=0.004, M=10^{20} g
σ>0.005
σ>0.005
c_ϕ=1, c_s=0.01, M=10^{20} g
c_ϕ=1, c_s=0.004, M=10^{20} g
Log z_s
Log (g_s/g_1)

Light PBHs



M = 10^10 g
M = 10^11 g
M = 10^12 g
M = 10^13 g
M = 10^14 g
M = 10^15 g
M = 10^16 g
M = 10^17 g
Log (g_s/g_1)
Log z_s
Axion
Dilaton
c_ϕ=0.005, c_σ=0.004

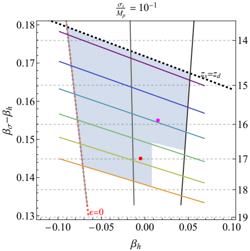
Gravitational Waves production

$$\frac{\Omega_{\rm GW}(f,t_0)}{\Omega_{\rm GW}(f_1,t_0)} = \begin{cases} \left(\dfrac{f}{f_1}\right)^{3-|3-2\beta_h|}, & f_\sigma \lesssim f \lesssim f_1 \\[2ex] \left(\dfrac{f_\sigma}{f_1}\right)^{3-|3-2\beta_h|} \left(\dfrac{f}{f_\sigma}\right)^{1-|3-2\beta_h|}, & f_d \lesssim f \lesssim f_\sigma \\[2ex] \left(\dfrac{f_\sigma}{f_1}\right)^{3-|3-2\beta_h|} \left(\dfrac{f_d}{f_\sigma}\right)^{1-|3-2\beta_h|} \left(\dfrac{f}{f_d}\right)^{3-|3-2\beta_h|}, & f_s \lesssim f \lesssim f_d \\[2ex] \left(\dfrac{f_\sigma}{f_1}\right)^{3-|3-2\beta_h|} \left(\dfrac{f_d}{f_\sigma}\right)^{1-|3-2\beta_h|} \left(\dfrac{f_s}{f_d}\right)^{3-|3-2\beta_h|} \left(\dfrac{f}{f_s}\right)^{3}, & f \lesssim f_s \ . \end{cases}$$

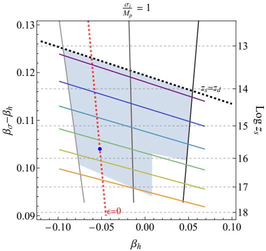
Gravitational Waves production

$$\Omega_{\rm GW}(f_s) \simeq 2.9^{+5.4}_{-2.3} \times 10^{-8},$$

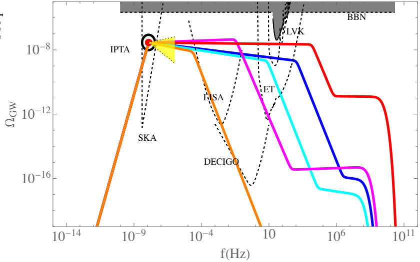
$$f_s \simeq (1.2 \pm 0.6) \times 10^{-8}\ \mathrm{Hz}$$



Log $\frac{H_1}{M_p}$	Log $\frac{m}{M_p}$	Log $\frac{z_s}{z_\sigma}$	Log $\frac{z_s}{z_d}$	Log $\frac{z_d}{z_\sigma}$
−1	−12	7.4	0.7	6.7
−2	−10	9.0	3.7	5.3
−3	−8	10.7	6.7	4.0
−3.00	−6	12.4	9.7	2.7
	−4	14.0	12.7	1.3
	−2	15.7	15.7	0.0



Log $\frac{H_1}{M_p}$	Log $\frac{m}{M_p}$	Log $\frac{z_s}{z_\sigma}$	Log $\frac{z_s}{z_d}$	Log $\frac{z_d}{z_\sigma}$
−1	−12	8.7	0.7	8.0
−2	−10	10.4	3.7	6.7
−3	−8	12.0	6.7	5.3
−2.60	−6	13.7	9.7	4.0
	−4	15.4	12.7	2.7
	−2	17.0	15.7	1.3



O(d,d) formulation up to all orders in α'

- when we approach string-curvature scales $H^{-1} \sim l_S$, all higher-order corrections in α' become comparable to each other.

With a purely time-dependent (d+1)-dimensional string background the most general O(d,d)-invariant action can be written as

$$S[\Phi, n, S] = \frac{1}{2} \int d^d x dt n e^{-\overline{\phi}} \Big[-(\mathcal{D}\overline{\phi})^2 + X(\mathcal{D}\mathcal{S}) \Big] ,$$

$$\overline{\phi} = \phi - \log\sqrt{g_s} , \qquad \mathcal{S}_{2d\times 2d} = \begin{pmatrix} bg^{-1} & g - bg^{-1}b \\ g^{-1} & -g^{-1}b \end{pmatrix} ,$$

The case of a flat FLRW background

$$n(t) = 1 , \qquad g_{ij}(t) = -a(t)^2 \delta_{ij} \qquad b_{ij} = 0 ,$$

$$\boxed{F(H) = 2d \sum_{k=1}^{\infty} (-\alpha')^{k-1} c_k 2^{2k} H^{2k}}$$

encodes all $\alpha'-$corrections

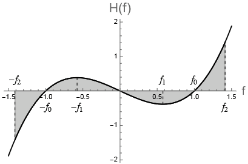
O. Hohm and B. Zwiebach, Duality invariant cosmology to all orders in α', Phys. Rev. D 100 (2019) no.12, 126011 [arXiv:1905.06963 [hep-th]]

Cosmology up to all orders

- α' correction up to all orders encoded in the "hamiltonian" h

$$\boxed{h(z) = F - HF'\,, \qquad \frac{\partial h}{\partial z_i} = H_i\,, \qquad F(H_i) = 2d\sum_{k=1}^{\infty}(-\alpha')^{k-1}c_k 2^{2k} H_i^{2k}}$$

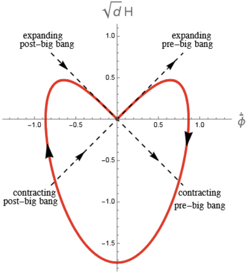
Regular solutions



- $h(z)$ exhibits a second zero at $z = z_2$.

$\Rightarrow$ $H(z)$ vanishes at (at least) one point $z_0 < z_2$.

$\Rightarrow$ $H(z)$ has local extrema at various points $(z_1, \dots) \in (0, |z_2|)$.



Dilation Stabilization

At late times the coupling blows up $\longrightarrow$ String loop corrections are needed

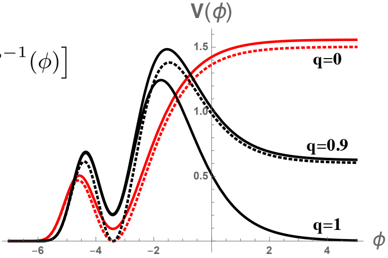
$$\Downarrow$$

Generation of a non-perturbative dilaton potential

- breaking of the duality symmetry: non-trivial final evolution
- Istanton-like suppression in early time: $V(\phi) \xrightarrow[t\to-\infty]{g_s\to 0} e^{-\mathsf{const}/g_s}$

$$V(\phi) = A\, e^{-B(\phi)/\beta} \left[\left(c^2 - B(\phi)\right)^2 + \delta B(\phi)\right] \left[1 - q\, B^{-1}(\phi)\right]$$

$$B(\phi) = \frac{1+\alpha\, g_s^2}{\alpha\, g_s^2} = \frac{1+\alpha\, e^{\phi}}{\alpha\, e^{\phi}}$$



Possible late-time evolutions

- The potential is unable to substantially modify the overall dilaton evolution: we recover a **regular transition** as in the cases with **no potential**.
- The potential bounces back the dilaton towards the small coupling regime: time-reversed regular bounce from **expanding** pre- to **contracting** post-bang regimes.

♦ **Dilaton** gets **trapped** in the local minimum of the potential.

I : **Stabilisation** of the dilaton

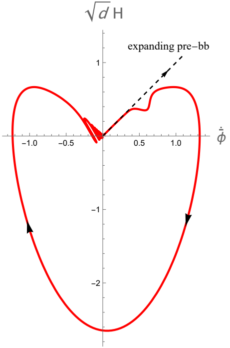
II : **FLRW** or **de Sitter** final geometry

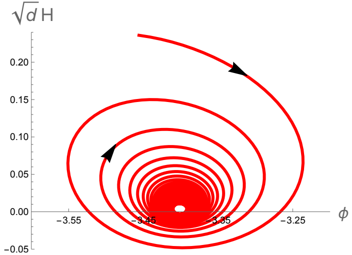
III : **Isotropisation** of the final geometry if we start from anisotropic initial conditions

Dilatonic Stabilization and FLRW attractor

$$h(z) = \frac{d}{2}\left(z^2 - \alpha' \frac{z^4}{2}\right)$$

$$V_0 \equiv V(\phi_m) = 0$$



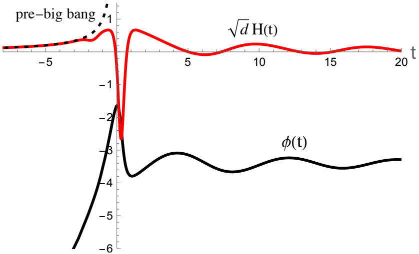


$$\phi(t) = \phi_0 + \frac{2(d-1)}{t\,\omega\sqrt{d}}\sin(\omega t + \theta)$$

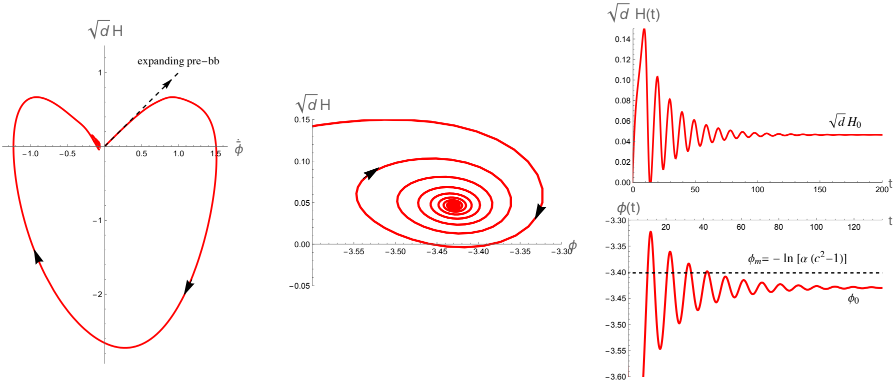
$$H(t) = \frac{2}{t\,d}\left[1 + \sqrt{d}\cos(\omega t + \theta)\right]$$

$$\omega = m\sqrt{d-1}$$

$$m = V''(\phi_m)$$



$V_0 > 0$: de Sitter attractor



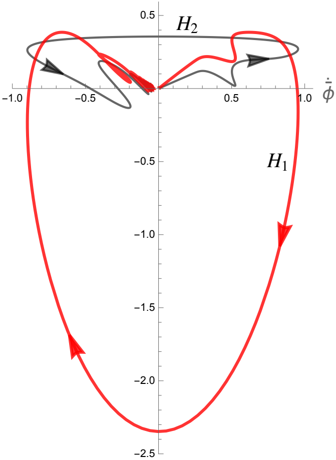
$$\left[\frac{\partial}{\partial\phi}\left(V\,e^{\frac{2\,\phi}{d-1}}\right)\right]_{\phi=\phi_0} = 0\,, \qquad \phi_0 \simeq \phi_m - \frac{2\,V_0}{m^2(d-1)}\,, \qquad H_0 \simeq \left[\frac{2\,V_0}{d\,(d-1)}\right]^{1/2}$$

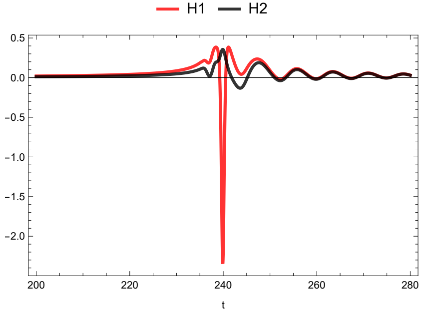
E-frame potential

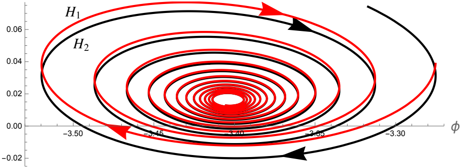
Dilaton Stabilization and Isotropization

Anisotropic Bianchi-I type background & $V_0 > 0$

$$h(z_i) = \frac{1}{2}\sum_i z_i^2 - c_2\frac{\alpha'}{8}\sum_i z_i^4 \ , \qquad \frac{\partial h}{\partial z_i} = H_i$$







Take-home message

♦ Phenomenological constraints

$\mathbf{I}$: **PBHs** production

$\mathbf{II}$: **GWs** production

$\mathbf{III}$: ...

♦ Regular solutions

$\mathbf{I}$: Regular bouncing if $\exists\, \mathbf{z_2} : \mathbf{h(z_2) = 0}$

$\mathbf{II}$: Non-perturbative potential $\mathbf{V}(\boldsymbol{\phi})$ to stabilize the dilaton

♦ Dilaton gets trapped

$\mathbf{I}$: **Stabilisation** of the dilaton

$\mathbf{II}$: **FLRW** or **de Sitter** final geometry

$\mathbf{III}$: **Isotropisation** of the final geometry

Thank you for the attention