Equivalence Principle Violation in Metric-Affine Gravity

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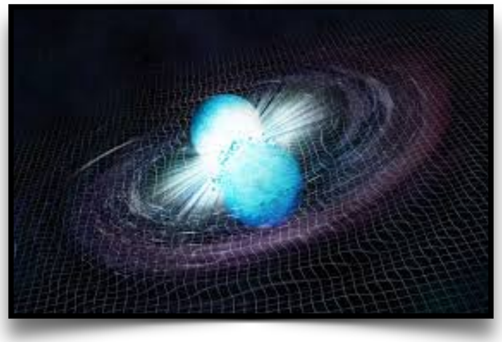


OUTLINE

1. **DIFFERENT FORMULATIONS OF THE EQUIVALENCE PRINCIPLE**

2. **METRIC-AFFINE GEOMETRY**

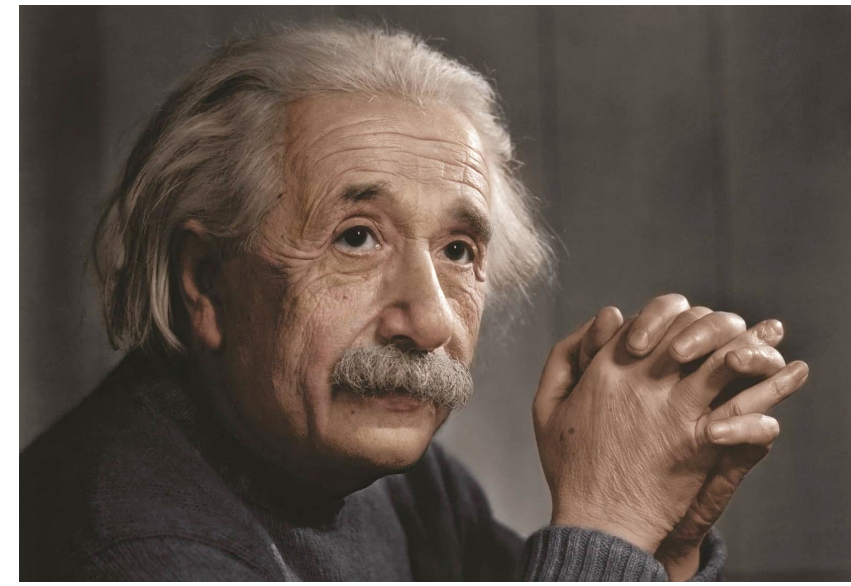
3. **GRAVITY AT FINITE TEMPERATURE**

4. **EQUIVALENCE PRINCIPLE VIOLATION IN METRIC-AFFINE GRAVITY (*will appear in a forthcoming paper!*)**

5. **CONCLUSIONS**

THE EQUIVALENCE PRINCIPLE (1)

Einstein formulation of the Equivalence Principle:



" There then occurred to me the
'glückischste Gedanke meines Lebens'
[Happiest thought of my life]
in the following form.
The gravitational field has only a relative existence…Because for an observer falling freely from the roof of a house there exists, at least in his immediate surroundings, no gravitational field. Indeed, if the observer drops some bodies then these remain relative to him in a state of rest or uniform motion, independent of their particular chemical or physical nature.
The observer has the right to interpret his state as 'at rest' ".

(A. Einstein, as quoted in Pais, *Subtle is the Lord: The Science and the Life of Albert Einstein,* (1982))

THE EQUIVALENCE PRINCIPLE (2)

"For an observer falling freely from the roof of a house there exists, at least in his immediate surroundings, no gravitational field."

This describes a local inertial frame in free fall, where the effects of gravity are not felt: objects appear to float or move uniformly

Geometric description: Let $g_{\alpha\beta}(x)$ be the metric tensor in one coordinate system.

At each point P of the spacetime it is possible to introduce new coordinates x'^{α} such that:

$$g'_{\alpha\beta}(x'_P) = \eta_{\alpha\beta}$$

$$\left.\frac{\partial g'_{\alpha\beta}}{\partial x'^{\gamma}}\right|_P = 0$$

Local Inertial Frame

THE EQUIVALENCE PRINCIPLE (3)

Different formulations of Equivalence Principle (EP):

-*Newtonian* EP: equality of inertial and gravitational mass (universality of free fall)

-*Weak* EP: All test bodies, regardless of their composition or internal structure, fall with the same acceleration in a given gravitational field

-*Einstein* EP: In a small enough region of spacetime, the laws of physics reduce to those of special relativity for a freely falling observer

-*Strong* EP: The outcome of any local experiment (gravitational or non-gravitational) is independent of the velocity of the free-falling apparatus and its location in spacetime (extends the Einstein EP by including gravitational experiments)

METRIC-AFFINE GEOMETRY (1)

Metric-affine geometry (MAG) extends the Riemannian structure of GR by relaxing the metric-compatibility constraint and allowing the affine connection and the metric to be treated as independent variables

Riemannian geometry

- **Metric-compatibility condition**

$$\overset{\circ}{\nabla}_\lambda g_{\mu\nu} = 0$$

- **Torsionless connection**

$$\overset{\circ}{\Gamma}{}^\lambda{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^\lambda{}_{\nu\mu}$$

$\downarrow$

- **Levi-Civita connection**

$$\overset{\circ}{\Gamma}{}^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\lambda} \left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu} \right)$$

MAG

- **Non-metricity**

$$Q_{\lambda\mu\nu} = \nabla_\lambda g_{\mu\nu} \neq 0$$

- **Torsion**

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}$$

- **General connection**

$$\Gamma^\lambda{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^\lambda{}_{\mu\nu} + M^\lambda{}_{\mu\nu}$$

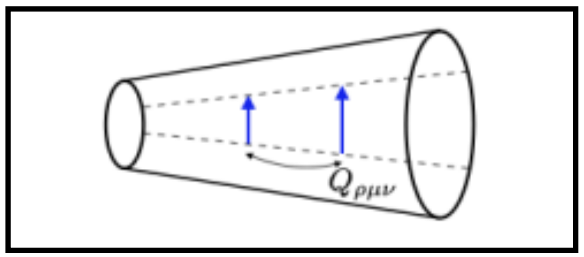
$$M_{\alpha[\mu\nu]} = -\frac{1}{2} T_{\alpha\mu\nu} \qquad M_{(\alpha\mu)\nu} = -\frac{1}{2} Q_{\nu\alpha\mu}$$

METRIC-AFFINE GEOMETRY (2)

Key features of MAG:

1. **Lengths and angles are not preserved during the parallel transport of vectors**

Consider the tangent vector (four-velocity) $\dot{x}^\mu := dx^\mu/ds$ **along an autoparallel curve, where** $\dot{x}^\lambda \nabla_\lambda \dot{x}^\mu = 0$**. The norm of** $\dot{x}^\lambda$ **is not conserved**



$$
\begin{aligned}
\frac{D}{ds}(g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu) &= 2\dot{x}^\lambda(\nabla_\lambda\dot{x}^\mu)\dot{x}_\mu + \dot{x}^\lambda Q_{\lambda\mu\nu}\dot{x}^\mu\dot{x}^\nu \\
&= \dot{x}^\lambda Q_{\lambda\mu\nu}\dot{x}^\mu\dot{x}^\nu\,,
\end{aligned}
$$

$$\downarrow$$

Even if one considers two vectors parallelly transported along a curve, their scalar product is not conserved

$$\downarrow$$

Vectors cannot be normalized globally

METRIC-AFFINE GEOMETRY (3)

2. Autoparallels and geodesics no longer coincide

Autoparallel curve

The tangent vector to the curve $x^\mu(s)$ is parallelly transported along the curve

$$\dot{x}^\lambda \nabla_\lambda \dot{x}^\mu = 0 \longrightarrow \frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{(\alpha\beta)} \dot{x}^\alpha \dot{x}^\beta = 0$$

Geodesic curve

Extremal curve (shortest or longest lines); curve which is of extremal length with respect to the metric of the manifold

$$\delta \mathcal{S} = 0$$

$$\mathcal{S} = \int ds \sqrt{-g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}} \longrightarrow \frac{d^2 x^\mu}{ds^2} + \mathring{\Gamma}^\mu{}_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 0$$

METRIC-AFFINE GEOMETRY (4)

3. Nonvanishing acceleration even along autoparallels (anomalous acceleration)

Two independent types of acceleration can be defined

Proper acceleration — $A^{\mu} := \dot{x}^{\lambda} \nabla_{\lambda} \dot{x}^{\mu}$

Anomalous acceleration — $\tilde{a}_{\mu} := \dot{x}^{\lambda} \nabla_{\lambda} \dot{x}_{\mu}$

$$\begin{aligned} \tilde{a}_{\mu} &= \dot{x}^{\lambda} \nabla_{\lambda} (g_{\mu\nu} \dot{x}^{\nu}) = g_{\mu\nu} A^{\nu} + Q_{\lambda\mu\nu} \dot{x}^{\lambda} \dot{x}^{\nu} \\ &= A_{\mu} + Q_{\lambda\mu\nu} \dot{x}^{\lambda} \dot{x}^{\nu} \,, \end{aligned}$$

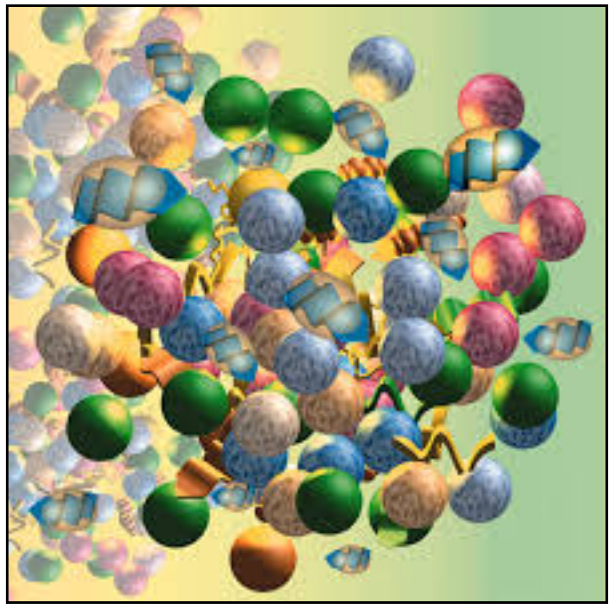
In MAG, autoparallels, i.e., curves for which $A^{\mu} = 0$, have in general a non-zero acceleration term, which is represented by the anomalous term $\tilde{a}^{\mu}$

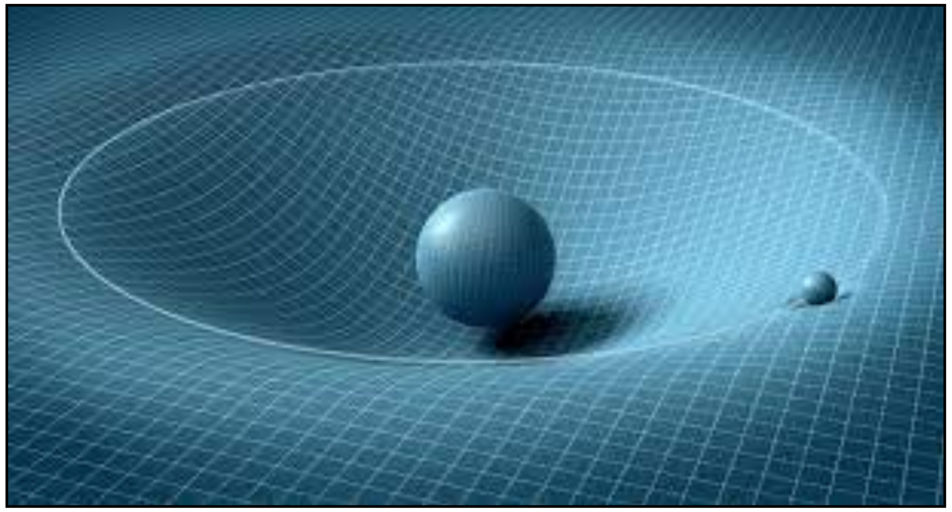
METRIC-AFFINE GEOMETRY (5)

Additional key difference with respect to Riemannian geometry: both anomalous and proper accelerations are not orthogonal to the four-velocity

$$A^\mu \dot{x}_\mu = \frac{1}{2}\left(-\dot{\phi} - Q_{\lambda\mu\nu}\dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu\right)$$
$$\tilde{a}^\mu \dot{x}_\mu = \frac{1}{2}\left(-\dot{\phi} + Q_{\lambda\mu\nu}\dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu\right)$$

$$\dot{x}_\mu \dot{x}^\mu \equiv \phi(x^\rho)$$





FINITE-TEMPERATURE GRAVITY (1)

- In its standard formulation, GR does not account for temperature

- $T \neq 0$ $\longrightarrow$ Matter no longer exists in vacuum but interacts with a heat bath, typically modeled as a background of photons or other relativistic particles

$\downarrow$

EP violations are expected to arise

Let us introduce a set of tetrad fields $\{e_\mu{}^a\}$ (with $a = \hat{0}, \hat{1}, \hat{2}, \hat{3}$ anholonomic Lorentz index) spanning the local Minkowski tangent space at each point of the spacetime manifold.

In this framework, $e_{\hat{0}}{}^{\mu}$ picks out the heat-bath rest frame

FINITE-TEMPERATURE GRAVITY (2)

We adopt a context where Riemannian geometry is valid

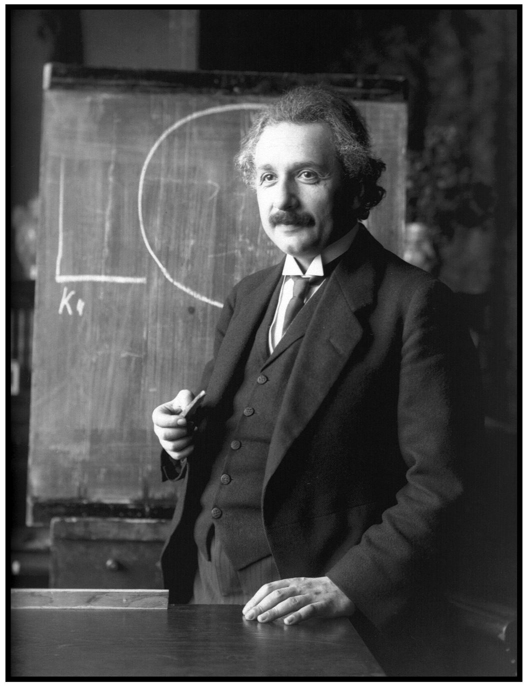
$$\overset{\circ}{G}{}^{\mu\nu} = 8\pi\Theta^{\mu\nu}$$

Ordinary Einstein equations

$$\overset{\circ}{\nabla}_{\mu}\overset{\circ}{G}{}^{\mu\nu} = 0$$

Contracted Bianchi identities

We can describe the motion of a test particle in thermal equilibrium with a photon heat bath starting from the conservation law:



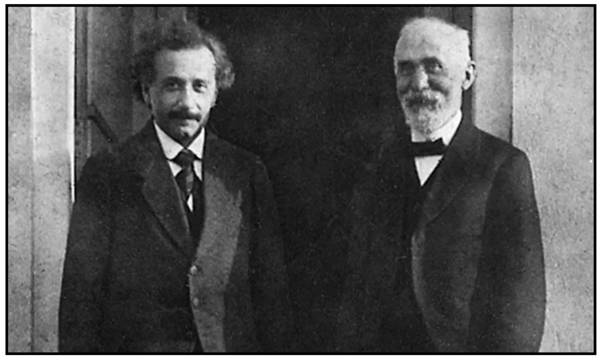
$$\overset{\circ}{\nabla}_{\mu}\Theta^{\mu\nu} = 0$$

FINITE-TEMPERATURE GRAVITY (3)

$$\Theta^{\mu\nu} = T^{\mu\nu} - \frac{2}{3}\alpha\pi\frac{T^2}{E^2}e^{\mu}{}_{\hat{0}}e^{\nu}{}_{\hat{0}}T^{\hat{0}\hat{0}}$$

$\Theta^{\mu\nu}$: generalized energy-momentum tensor describing the effective source of gravity at finite temperature

$T^{\mu\nu}$: Standard energy-momentum tensor associated with the test particle



$$E = \sqrt{m_0^2 + |\vec{p}|^2 + \frac{2}{3}\alpha\pi T^2}$$

Generalized dispersion relation

m_0: particle mass at $T = 0$
$\vec{p}$: momentum three-vector
α: fine-structure constant

$\Theta^{\mu\nu}$ characterizes a matter distribution coupled to gravity in a generally covariant but not locally Lorentz-invariant way: it transforms as a tensor under diffeomorphisms, but is does not transform as a scalar under local Lorentz transformations

FINITE-TEMPERATURE GRAVITY (4)

Modified worldline equation for the test particle

$$m := \sqrt{m_0^2 + 2\alpha\pi T^2/3}$$

$$\ddot{x}^\lambda + \mathring{\Gamma}^\lambda{}_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \frac{d}{ds}\left(\frac{2}{3}\alpha\pi\frac{T^2}{mE}e^\lambda{}_{\hat{0}}\right) + \frac{2}{3}\alpha\pi\frac{T^2}{m^2}\mathring{\Gamma}^\lambda{}_{\mu\nu}e^\mu{}_{\hat{0}}e^\nu{}_{\hat{0}}$$

Lowest order in T^2/m^2

Deviation from geodesic motion that indicates a breakdown of the universality of free fall

The presence of the introduces a preferred frame (i.e., the frame where the heat bath is at rest) that breaks explicitly the Lorentz invariance of the finite-temperature vacuum.

The lack of Lorentz invariance of the finite-temperature vacuum gives rise to a gravity model in which local Lorentz symmetry is broken in the tangent space spanned by the tetrad field, while general covariance remains intact on the spacetime manifold

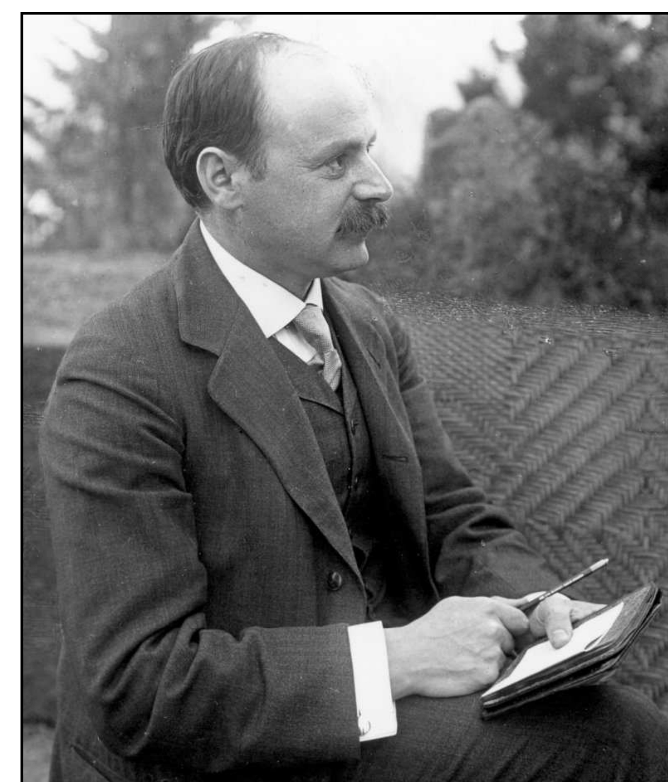
FINITE-TEMPERATURE GRAVITY (5)

Application: test particle in Schwarzschild background sourced by a mass M

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Nonvanishing tetrads components

$$e_t^{\ \hat{0}} = \sqrt{1 - \frac{2M}{r}}$$
$$e_r^{\ \hat{1}} = \left(\sqrt{1 - \frac{2M}{r}}\right)^{-1}$$
$$e_\theta^{\ \hat{2}} = r$$
$$e_\phi^{\ \hat{3}} = r \sin\theta$$

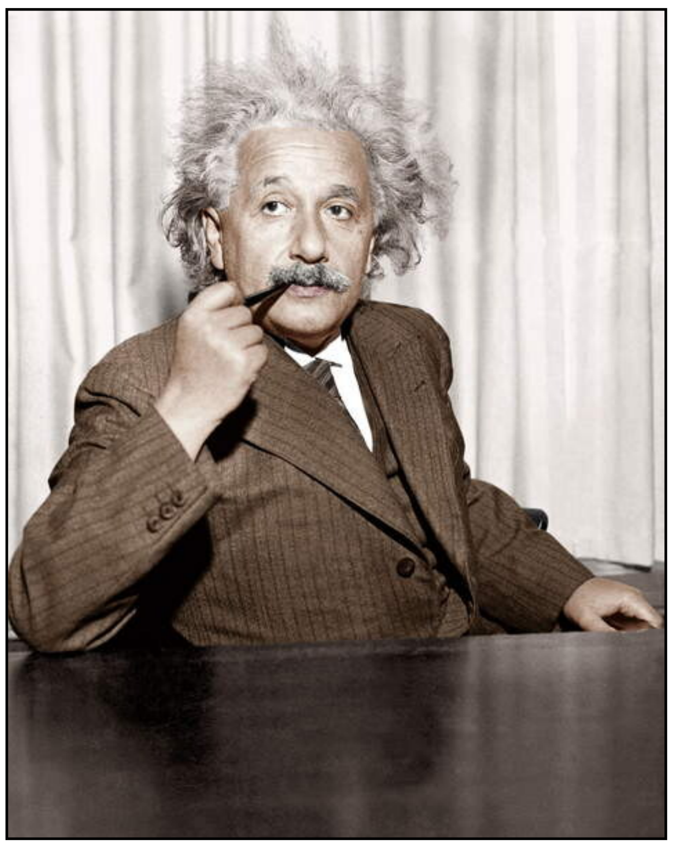


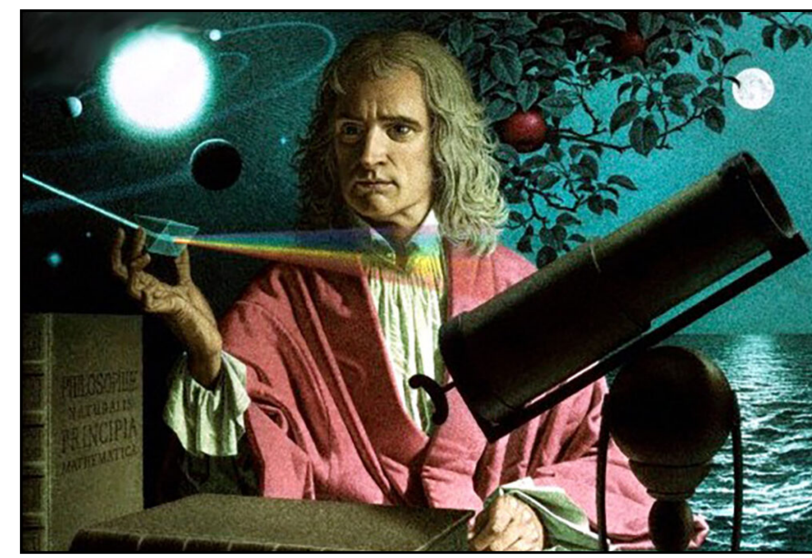
FINITE-TEMPERATURE GRAVITY (6)

By employing the weak-field approximation, one obtains

$$\ddot{r} = -\frac{M}{r^2}\left(1 - \frac{2}{3}\alpha\pi\frac{T^2}{m^2}\right)$$

$$m := \sqrt{m_0^2 + 2\alpha\pi T^2/3}$$





$$\frac{m_g}{m_i} = 1 - \frac{2\alpha\pi T^2}{3m_0^2},$$

$$m_i = m_0 + \frac{\alpha\pi T^2}{3m_0}$$

$$m_g = m_0 - \frac{\alpha\pi T^2}{3m_0}$$

EP VIOLATION IN MAG (1)

Consider a MAG with

Vanishing torsion tensor $T^{\lambda}_{\ \mu\nu}$

Nonvanishing non-metricity tensor $Q_{\mu\nu\alpha}$

Spherically symmetric and static geometry

$$ds^2 = -A(r)dt^2 + \frac{1}{B(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Worldline of a test particle

$$\ddot{r} + \frac{B}{2}\left[\frac{A'}{A} + \frac{\dot{r}^2}{B}\left(\frac{A'}{A} - \frac{B'}{B}\right) + \frac{\left(Q^t\right)'}{2m_i}\sqrt{A + \frac{\dot{r}^2}{B}A}\right] = 0$$

$Q^{\lambda} := g^{\mu\nu}Q^{\lambda}_{\ \ \mu\nu}$

Weyl vector

EP VIOLATION IN MAG (2)

Weak-field-slow-motion regime

Real gravitational field sourced by some mass M

The metric $g_{\mu\nu}$ is asymptotically flat and when $r \to \infty$ we have

$$A = 1 + 2\Phi,$$

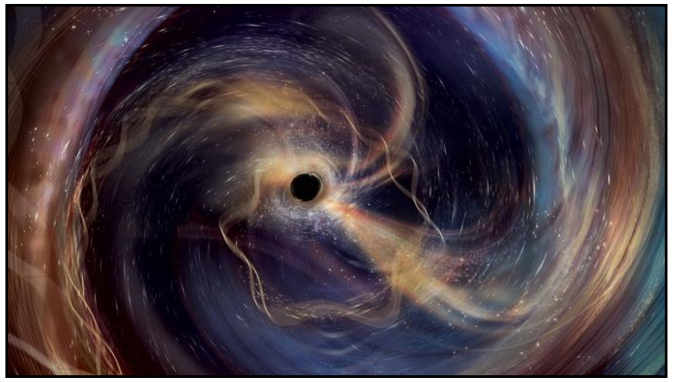
$$B^{-1} = 1 - 2\Phi,$$

$$\Phi = -\frac{M}{r}$$

Newtonian potential

We further suppose that

$$Q^t = \chi\Phi$$



EP VIOLATION IN MAG (3)

The dynamics of the test particle is ruled by

$$\boxed{\ddot{r} = -\frac{M}{r^2}\left(1 + \frac{\chi}{4m_i}\right)}$$

$$\boxed{\ddot{r} = -\frac{M}{r^2}\left(1 - \frac{2}{3}\alpha\pi\frac{T^2}{m^2}\right)}$$

$$\boxed{\frac{m_g}{m_i} = 1 + \frac{\chi}{4m_i}}$$

$$\boxed{\frac{m_g}{m_i} = 1 - \frac{2\alpha\pi T^2}{3m_0^2},}$$

MAG

Riemann geometry

The resemblance between the two sets of equations is remarkable

CONCLUSIONS

-EP violations naturally emerge in finite-temperature gravity

-We have shown that these violations can be described within the context of MAG

- **Non-metricity ↔ Lorentz symmetry breaking (formal analogy)**

- **Future goal: Providing a purely geometric description of EP violations in MAG**

...Stay tuned!