

MOND: An alternative to particle dark matter

Federico Lelli

INAF - Arcetri Astrophysical Observatory



MOND = MOdified Newtonian Dynamics

MOND = MOdified Newtonian Dynamics

MOND = NO DM

MOND = MOdified Newtonian Dynamics

MOND = NO DM

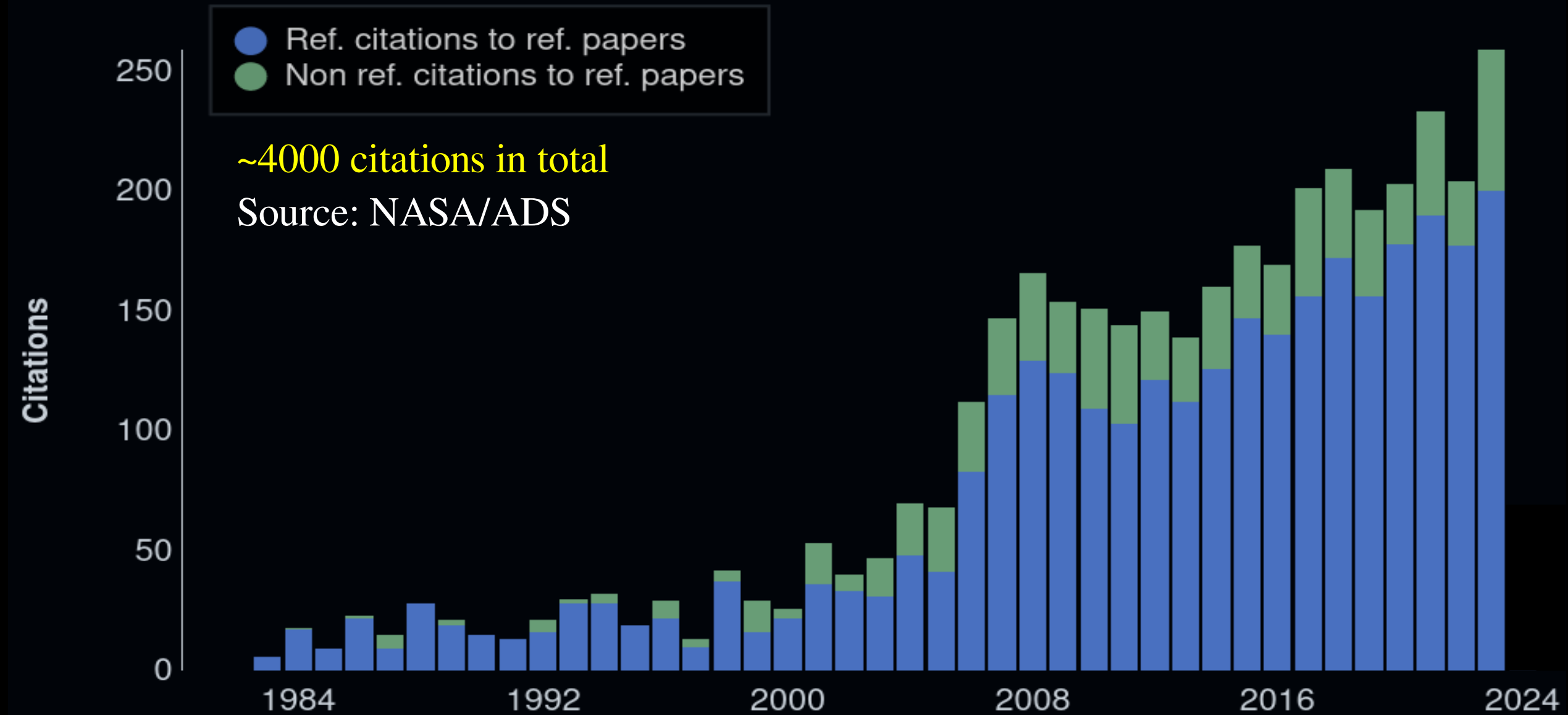
MOND = MilgrOmiaN Dynamics



Proposed by Moti Milgrom (1983a, b, c)

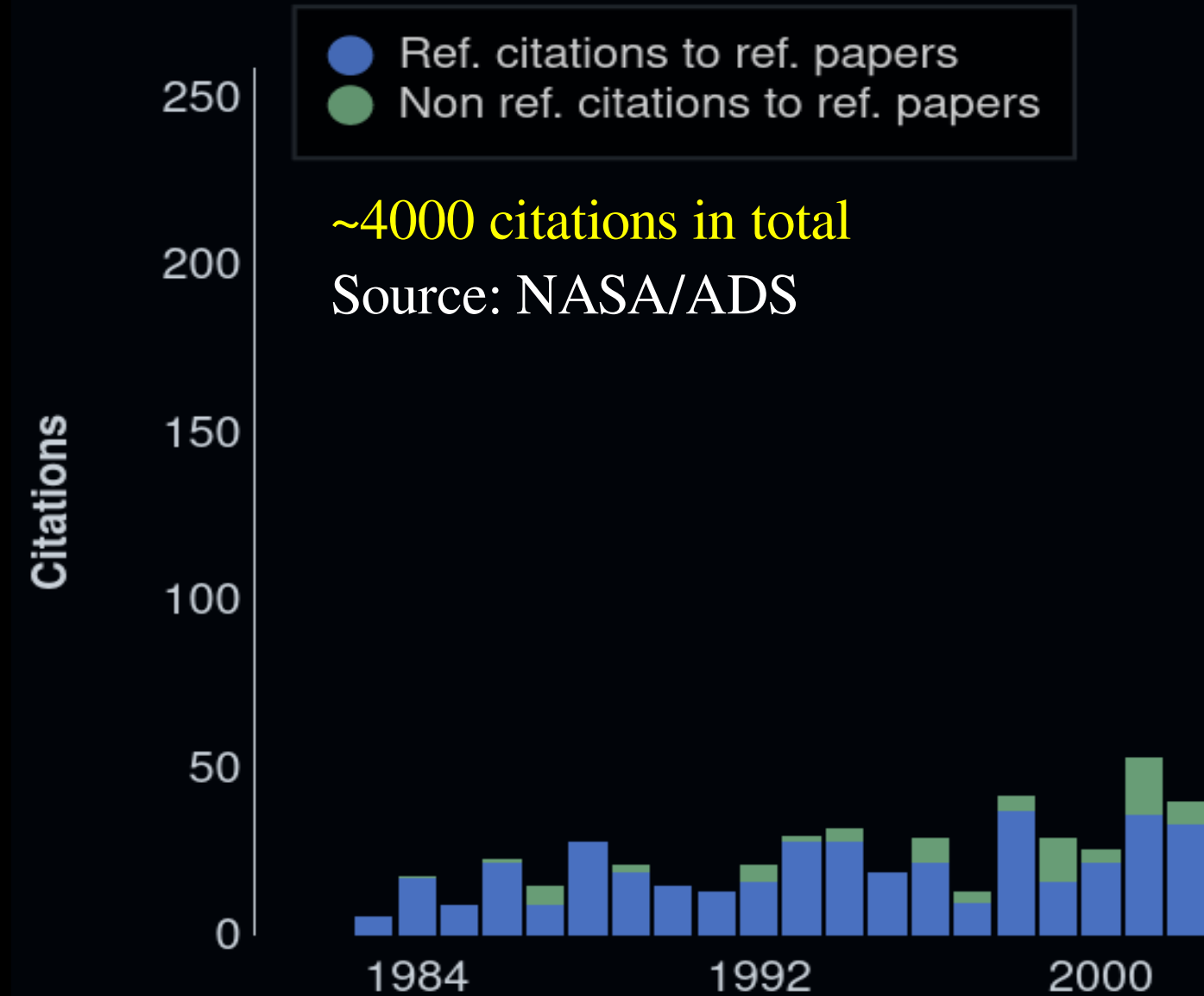
MOND is not a new idea. But it has gained
“momentum” over the past 40+ years.

Citations to the original MOND trilogy (Milgrom 1983a, b, c)



Citations to the original MOND trilogy (Milgrom 1983a, b, c)

People working on MOND in the late 90s



Citations to the original MOND trilogy (Milgrom 1983a, b, c)

(Fraction of the) people working on MOND today



Citations to the original MOND trilogy (Milgrom 1983a, b, c)

(Fraction of the) people working on MOND today



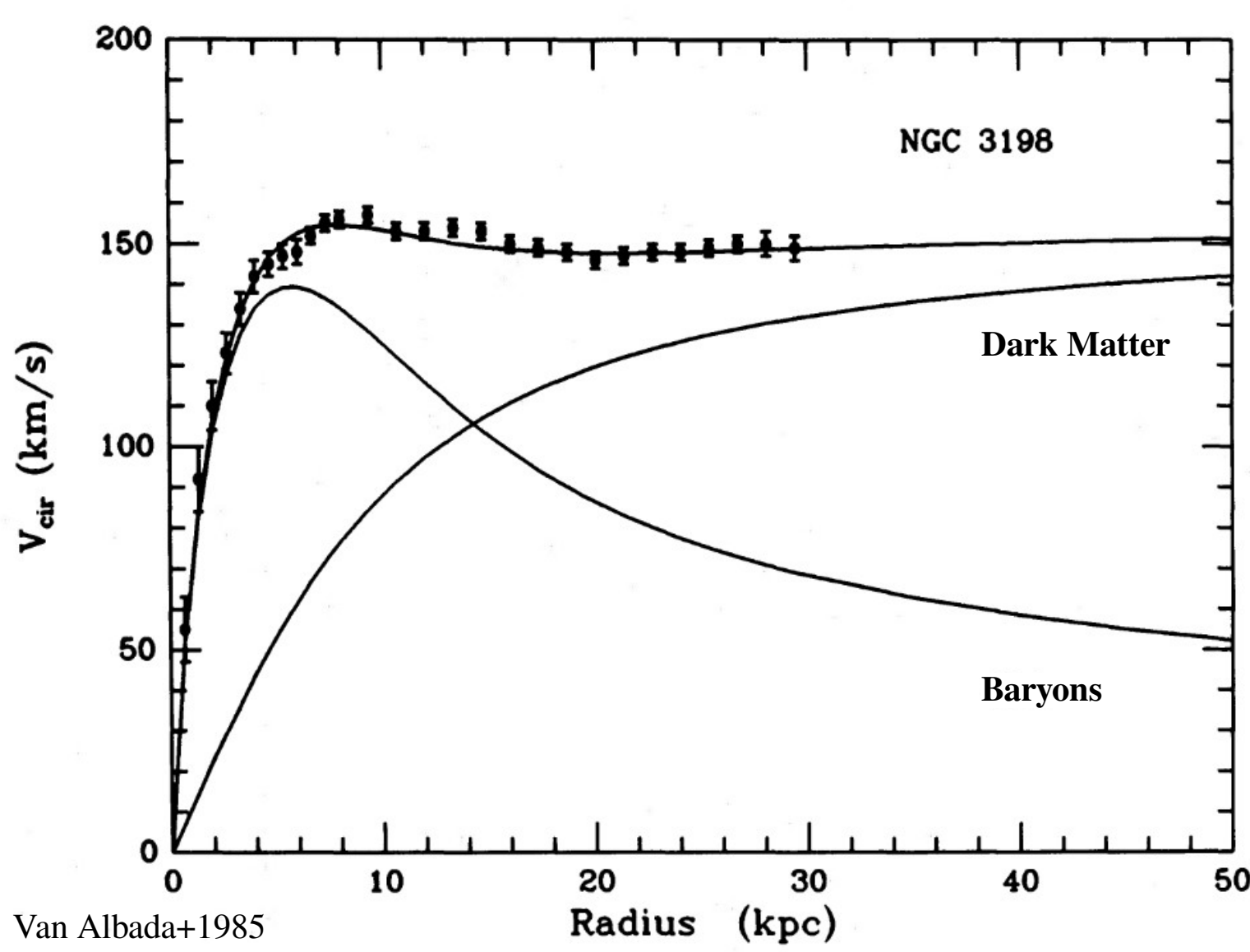
Talk Outline

1. General MOND paradigm
2. Tests on galaxy scales
3. Tests on groups/clusters scales
4. MOND theories & cosmology

Talk Outline

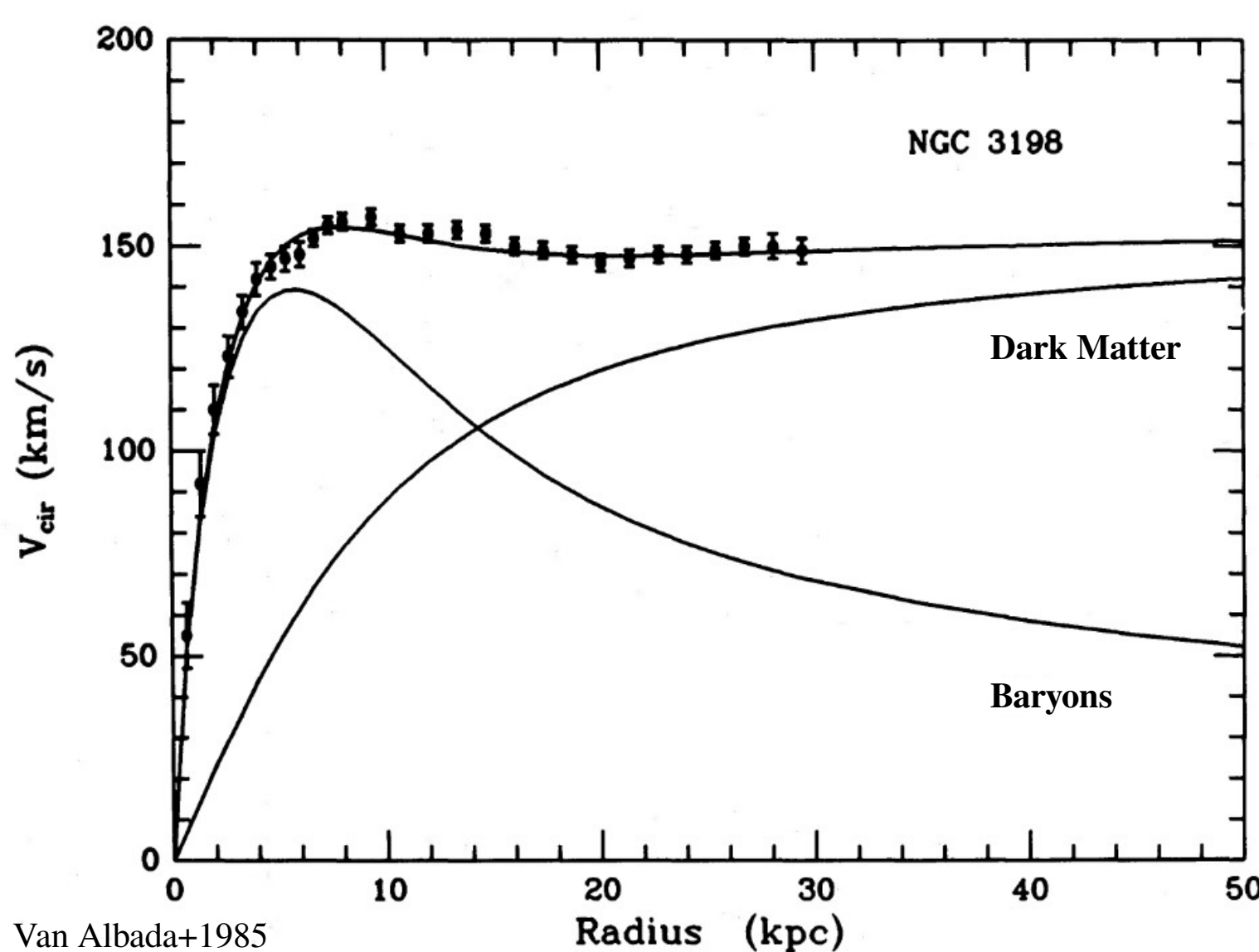
1. General MOND paradigm
2. Tests on galaxy scales
3. Tests on groups/clusters scales
4. MOND theories & cosmology

Rotation curves of disk galaxies: baryon-halo conspiracy



$$V_c^2(R) = V_{\text{bary}}^2(R) + V_{\text{DM}}^2(R) = \text{const}$$

Rotation curves of disk galaxies: baryon-halo conspiracy



$$V_c^2(R) = V_{\text{bary}}^2(R) + V_{\text{DM}}^2(R) = \text{const}$$

“Luminous and dark matter seem to conspire to produce the flat observed rotation curves in the outer regions. It seems unlikely that this coupling [...] results from the large-scale gravitational interaction between the two components.”

Van Albada & Sancisi (1986, RSPTA)

Still a problem for Λ CDM!

See Desmond (2017, MNRAS)

Flat rotation curves = Law of Nature. What would Newton do?

Kinematic acceleration along a circular orbit: $a = \left| \frac{d^2 \vec{x}}{dt^2} \right| = \frac{V_c^2}{R} \propto \frac{1}{R}$

Flat rotation curves = Law of Nature. What would Newton do?

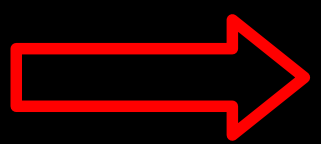
Kinematic acceleration along a circular orbit: $a = \left| \frac{d^2 \vec{x}}{dt^2} \right| = \frac{V_c^2}{R} \propto \frac{1}{R}$

Newtonian gravitational field at large radii: $g_{N,b} = - \left| \vec{\nabla} \phi_{N,b} \right| \simeq \frac{G_N M_b}{R^2} \propto \frac{1}{R^2}$

Flat rotation curves = Law of Nature. What would Newton do?

Kinematic acceleration along a circular orbit: $a = \left| \frac{d^2 \vec{x}}{dt^2} \right| = \frac{V_c^2}{R} \propto \frac{1}{R}$

Newtonian gravitational field at large radii: $g_{N,b} = - \left| \vec{\nabla} \phi_{N,b} \right| \simeq \frac{G_N M_b}{R^2} \propto \frac{1}{R^2}$



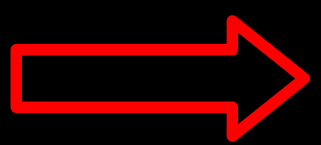
$$a = \sqrt{a_0 g_{N,b}}$$

a_0 constant with dimensions of acceleration

Flat rotation curves = Law of Nature. What would Newton do?

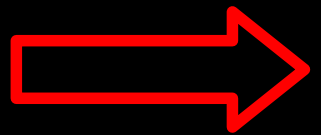
Kinematic acceleration along a circular orbit: $a = \left| \frac{d^2 \vec{x}}{dt^2} \right| = \frac{V_c^2}{R} \propto \frac{1}{R}$

Newtonian gravitational field at large radii: $g_{N,b} = - \left| \vec{\nabla} \phi_{N,b} \right| \simeq \frac{G_N M_b}{R^2} \propto \frac{1}{R^2}$



$$a = \sqrt{a_0 g_{N,b}}$$

a_0 constant with dimensions of acceleration



$$\frac{V_c^2}{\cancel{R}} = \sqrt{\frac{a_0 G_N M_b}{\cancel{R^2}}}$$

Flat rotation curve at large radii

MOND postulates at the non-relativistic level (Milgrom 2009, ApJ)

1) **New constant of Physics:** $a_0 \simeq 10^{-10} \text{ m/s}^2$

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

MOND postulates at the non-relativistic level (Milgrom 2009, ApJ)

1) **New constant of Physics:** $a_0 \simeq 10^{-10} \text{ m/s}^2$

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) **For $a \gg a_0 \rightarrow a = g_{\text{N,b}}$**

correspondence principle as in Quantum Mechanics (Newton recovered)

MOND postulates at the non-relativistic level (Milgrom 2009, ApJ)

1) **New constant of Physics:** $a_0 \simeq 10^{-10} \text{ m/s}^2$

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) **For $a \gg a_0 \rightarrow a = g_{\text{N,b}}$**

correspondence principle as in Quantum Mechanics (Newton recovered)

3) **For $a \ll a_0 \rightarrow$ Scale Invariance**

Equations of motion do not change under $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$

MOND postulates at the non-relativistic level (Milgrom 2009, ApJ)

1) **New constant of Physics:** $a_0 \simeq 10^{-10} \text{ m/s}^2$

similar role as c in Relativity and $\hbar$ in Quantum Mechanics

2) **For $a \gg a_0 \rightarrow a = g_{N,b}$**

correspondence principle as in Quantum Mechanics (Newton recovered)

3) **For $a \ll a_0 \rightarrow$ Scale Invariance**

Equations of motion do not change under $(\vec{x}, t) \rightarrow (\lambda \vec{x}, \lambda t)$


$$a = \sqrt{a_0 g_{N,b}} \quad V_c = \text{invariant}$$

A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXIES¹

M. MILGROM

Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study

Received 1982 February 4; accepted 1982 December 28

ABSTRACT

I use a modified form of the Newtonian dynamics (inertia and/or gravity) to describe the motion of bodies in the gravitational fields of galaxies, *assuming that galaxies contain no hidden mass*, with the following main results.

- ① The Keplerian, circular velocity around a finite galaxy becomes independent of r at large radii, thus resulting in asymptotically flat velocity curves.
- ② The asymptotic circular velocity (V_∞) is determined only by the total mass of the galaxy (M): $V_\infty^4 = a_0 GM$, where a_0 is an acceleration constant appearing in the modified dynamics. This relation is consistent with the observed Tully-Fisher relation if one uses a luminosity parameter which is proportional to the observable mass.
- ③ The discrepancy between the dynamically determined Oort density in the solar neighborhood and the density of observed matter disappears.
- ④ The rotation curve of a galaxy can remain flat down to very small radii, as observed, only if the galaxy's average surface density Σ falls in some narrow range of values which agrees with the Fish and Freeman laws. For smaller values of Σ , the velocity rises more slowly to the asymptotic value.
- ⑤ The value of the acceleration constant, a_0 , determined in a few independent ways is approximately $2 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1})^2 \text{ cm s}^{-2}$, which is of the order of $CH_0 = 5 \times 10^{-8} (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) \text{ cm s}^{-2}$.

The main predictions are:

- ① Rotation curves calculated on the basis of the *observed* mass distribution and the modified dynamics should agree with the observed velocity curves.
- ② The $V_\infty^4 = a_0 GM$ relation should hold exactly.
- ③ An analog of the Oort discrepancy should exist in all galaxies and become more severe with increasing r in a predictable way.



Talk Outline

~~1. General MOND paradigm~~

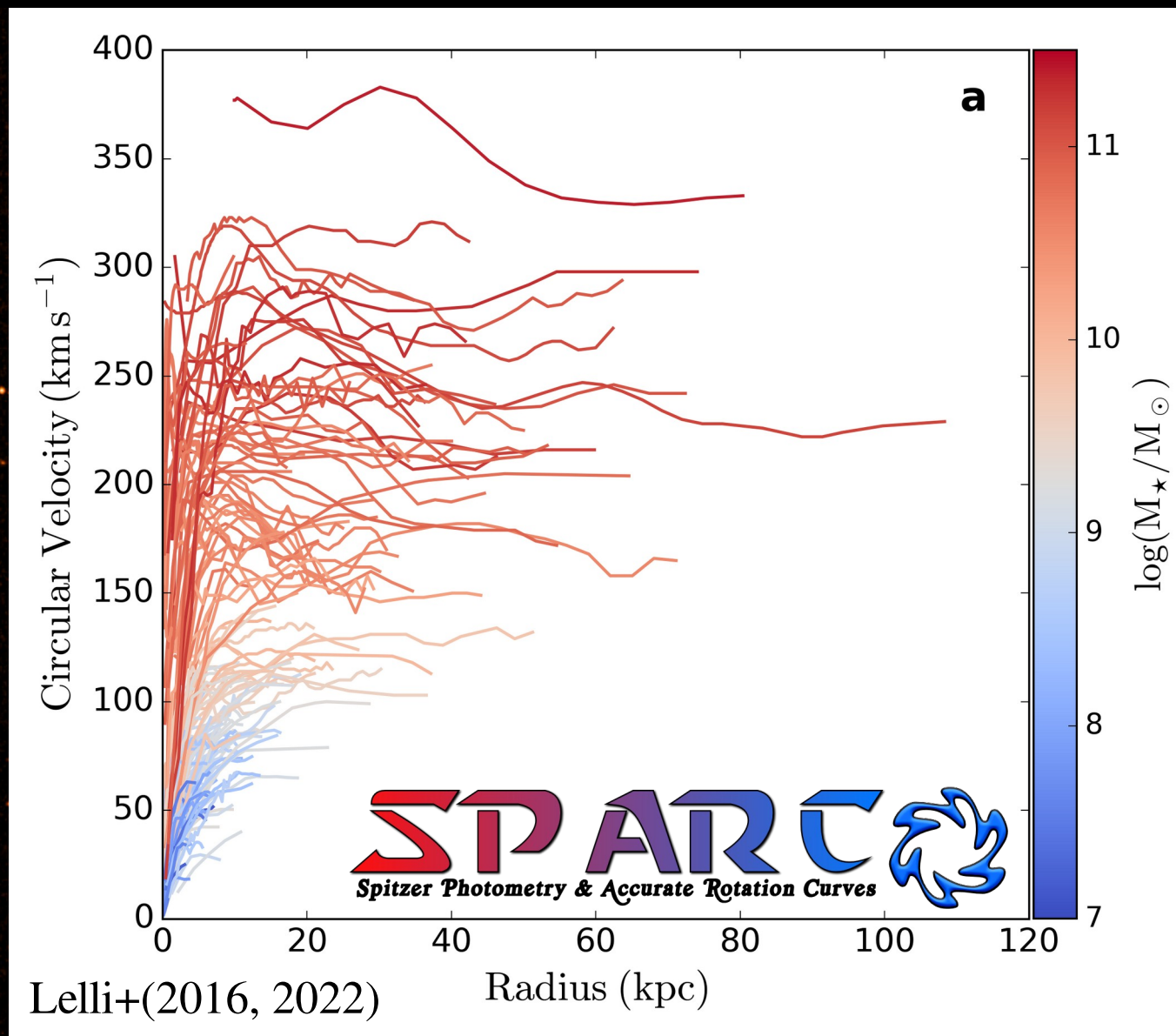
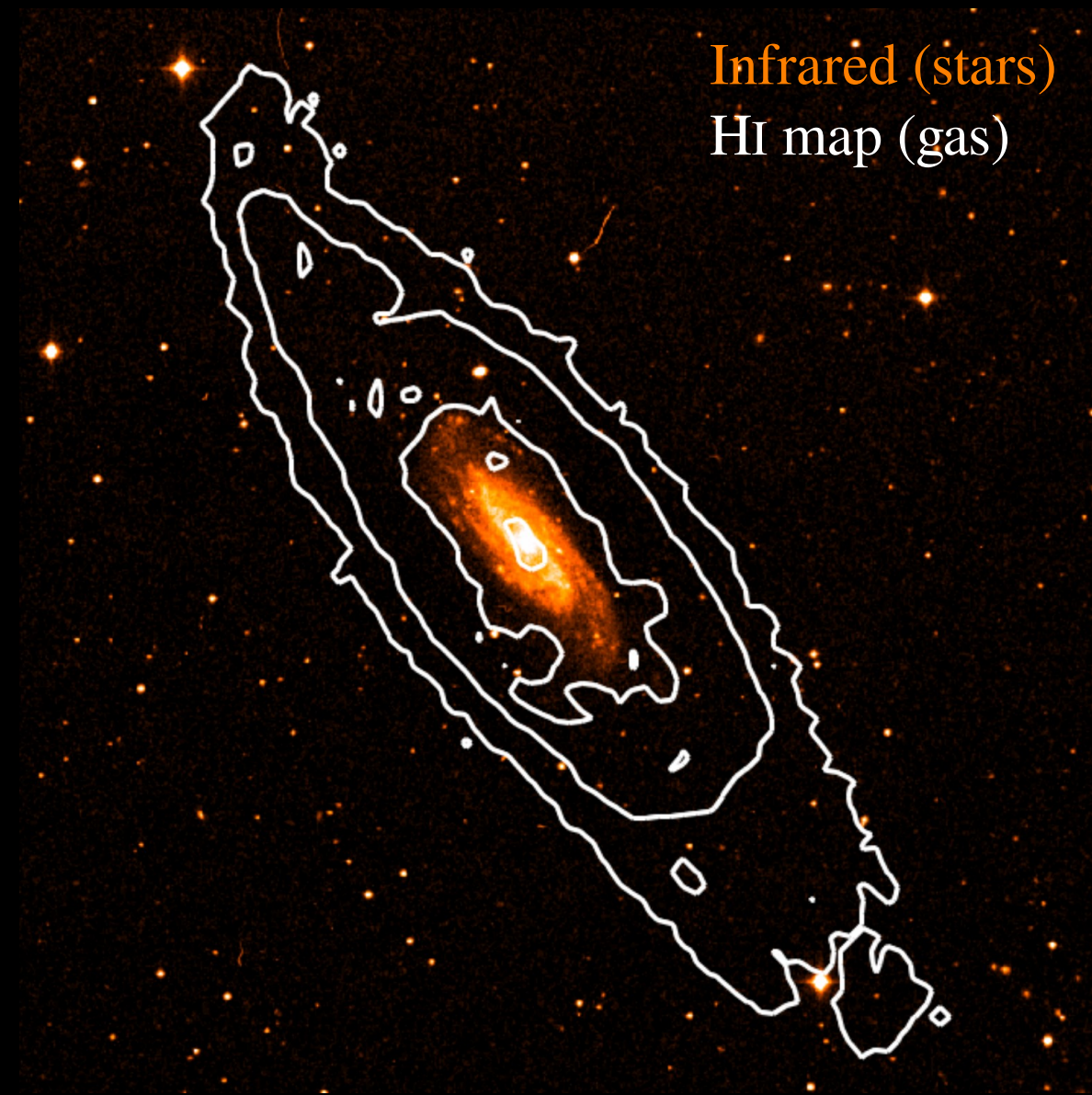
2. Tests on galaxy scales

3. Tests on groups/clusters scales

4. MOND theories & cosmology

(1) $V_c(R) = \text{const}$ at large R for isolated objects

(1) $V_c(R) = \text{const}$ at large R for isolated objects $\rightarrow$ HI line at 21 cm



(1) $V_c(R) = \text{const}$ at large R for isolated objects $\rightarrow$ Weak Lensing

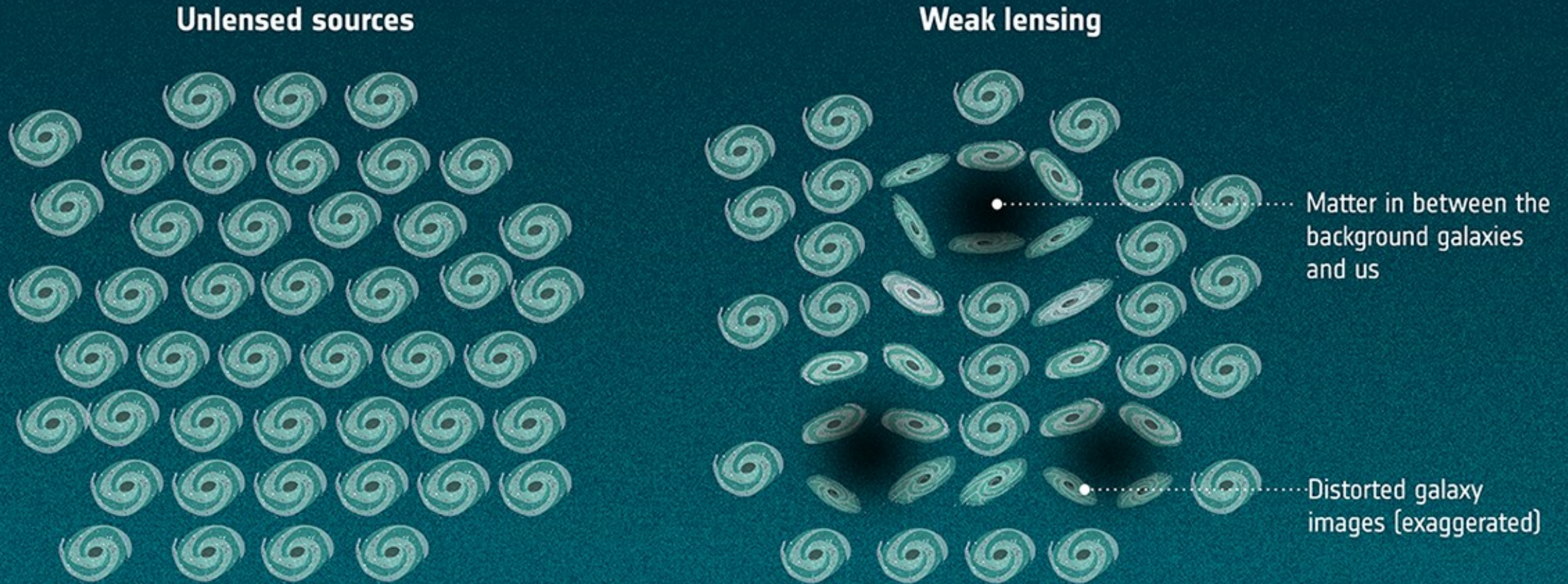


Image Credits: ESA

(1) $V_c(R) = \text{const}$ at large R for isolated objects $\rightarrow$ Weak Lensing

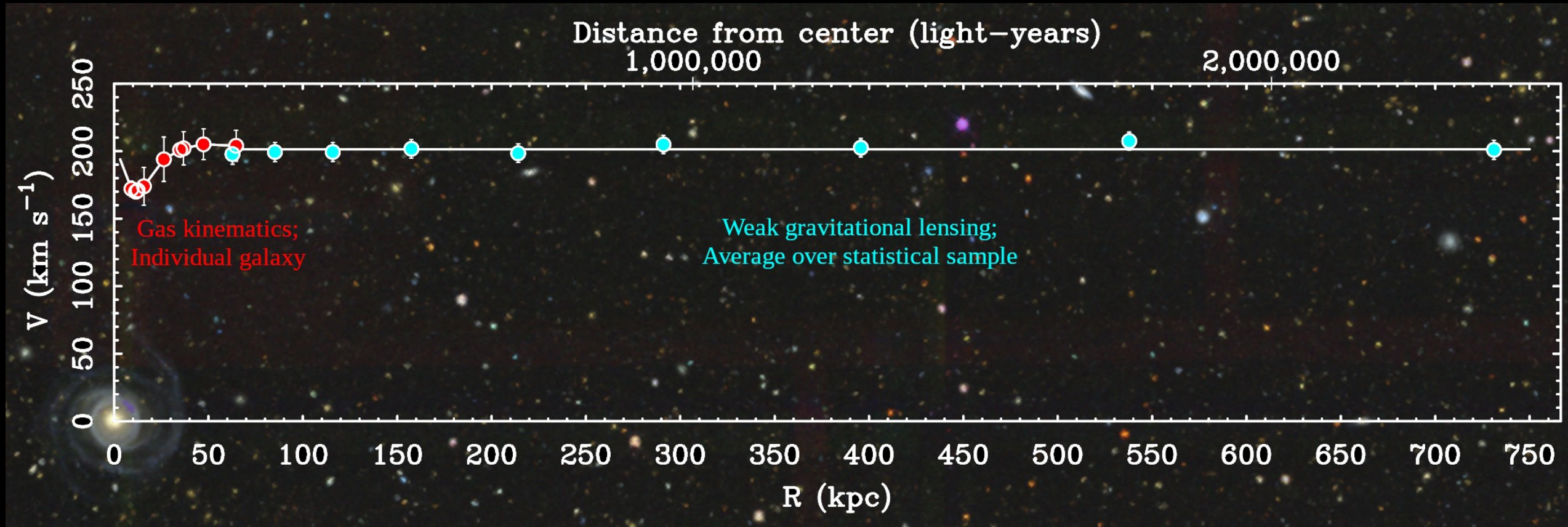
$$\frac{V_c^2}{r} = 4 G_N \int_0^{\pi/2} \Delta \Sigma \left| \frac{r}{\sin(\theta)} \right| d\theta$$

From weak lensing data ($\Delta \Sigma$) to gravitational acceleration ($g = V_c^2/r$) in spherical symmetry
 $\rightarrow$ Isolated galaxies from KiDS (Mistele+2024a, b)

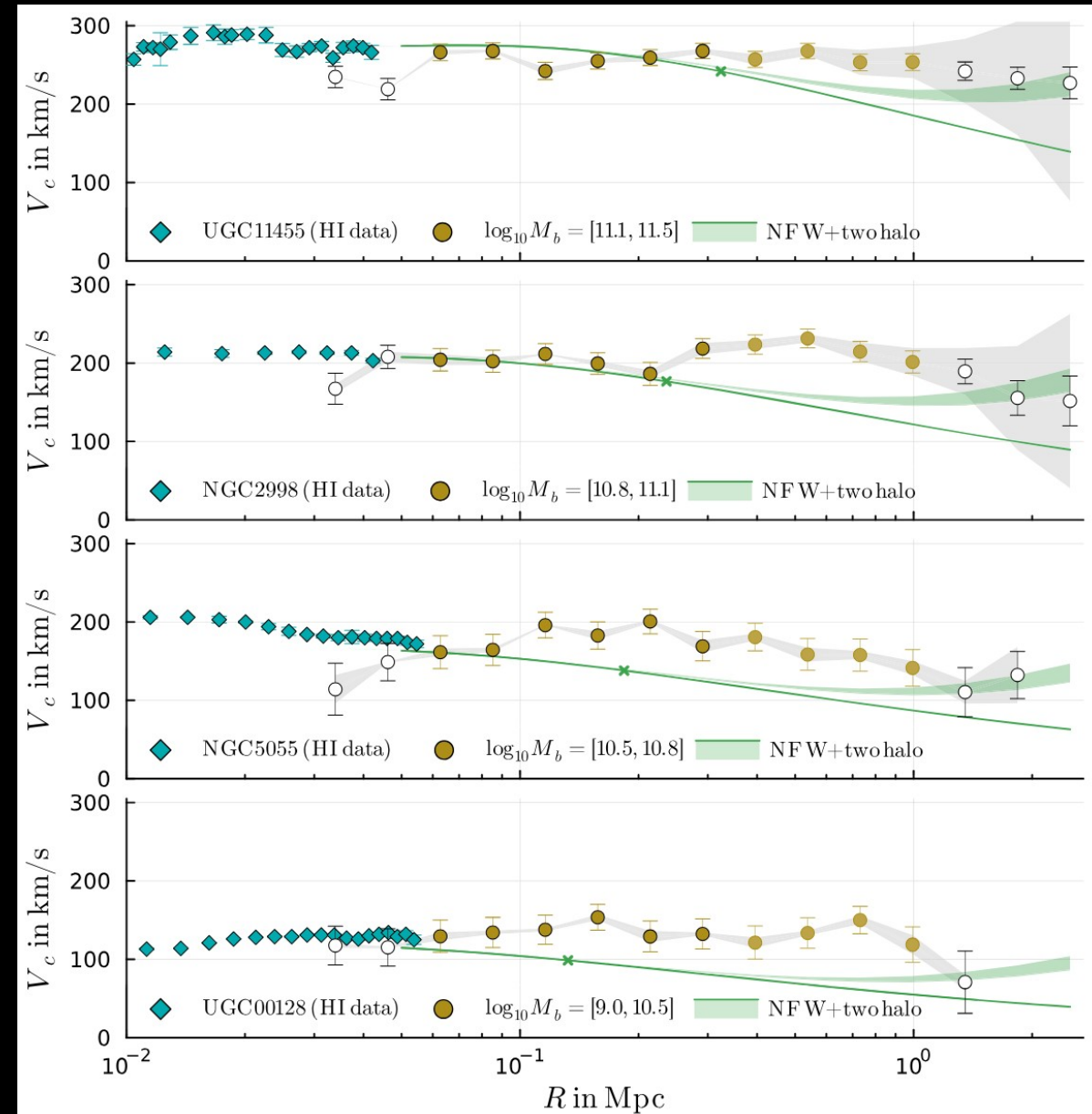
(1) $V_c(R) = \text{const}$ at large R for isolated objects $\rightarrow$ Weak Lensing

$$\frac{V_c^2}{r} = 4 G_N \int_0^{\pi/2} \Delta \Sigma \left| \frac{r}{\sin(\theta)} \right| d\theta$$

From weak lensing data ($\Delta \Sigma$) to gravitational acceleration ($g=V_c^2/r$) in spherical symmetry
 $\rightarrow$ Isolated galaxies from KiDS (Mistele+2024a, b)



(1) $V_c(R) = \text{const}$ at large R for isolated objects $\rightarrow$ Weak Lensing



Binning by baryonic mass:

Circular-velocity curve stays flat beyond the “virial radius” of the DM halo:

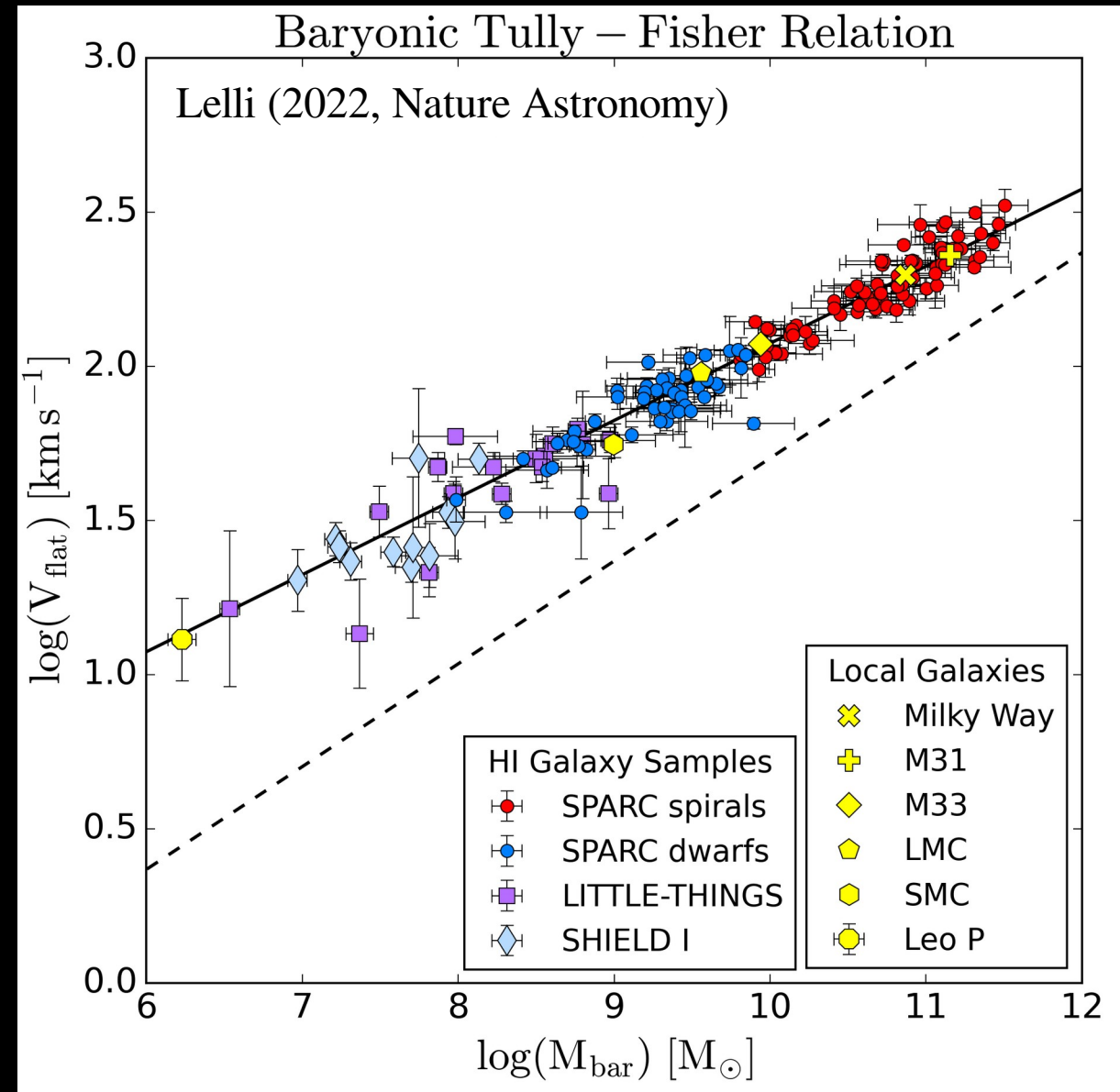
$\rightarrow$ In contradiction with Λ CDM predictions

$\rightarrow$ In agreement with MOND predictions

Mistele+(2024b, ApJ)

(2) $V_{\text{flat}}^4 = a_0 G_N M_b$ for isolated objects

(2) $V_{\text{flat}}^4 = a_0 G_N M_b$ for isolated objects $\rightarrow$ rotation supported gals

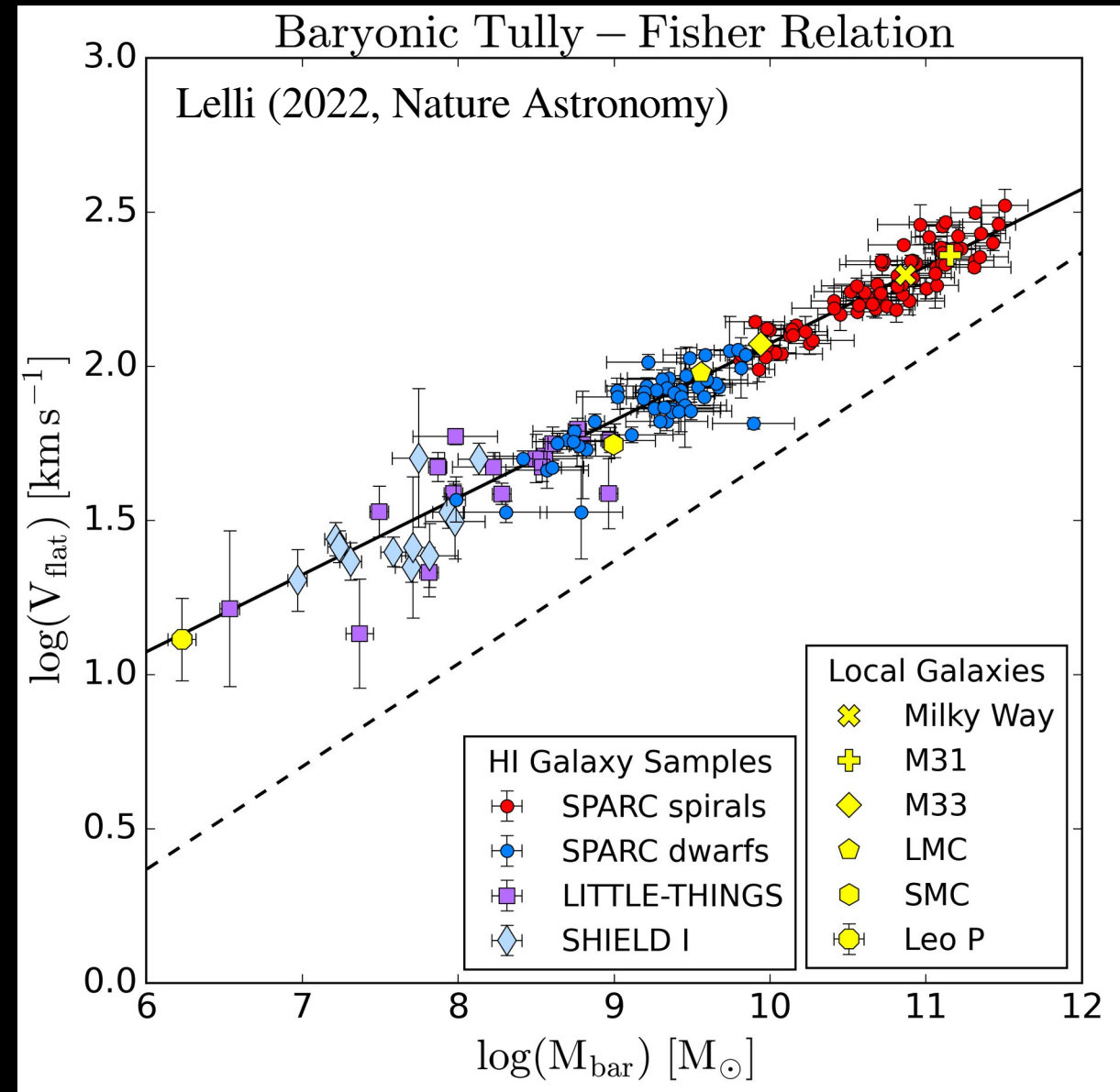


Four predictions in one equation:

✓ Key quantities are V_{flat} and M_b (stars+gas)

Some References: McGaugh+(2000, 2005, 2010); Verheijen (2001); Noordermeer & Verheijen (2005); Lelli+(2016, 2019, 2022, 2024); Ponomareva+(2018); Schombert+(2020), Di Teodoro+(2021, 2022).

(2) $V_{\text{flat}}^4 = a_0 G_N M_b$ for isolated objects $\rightarrow$ rotation supported gals



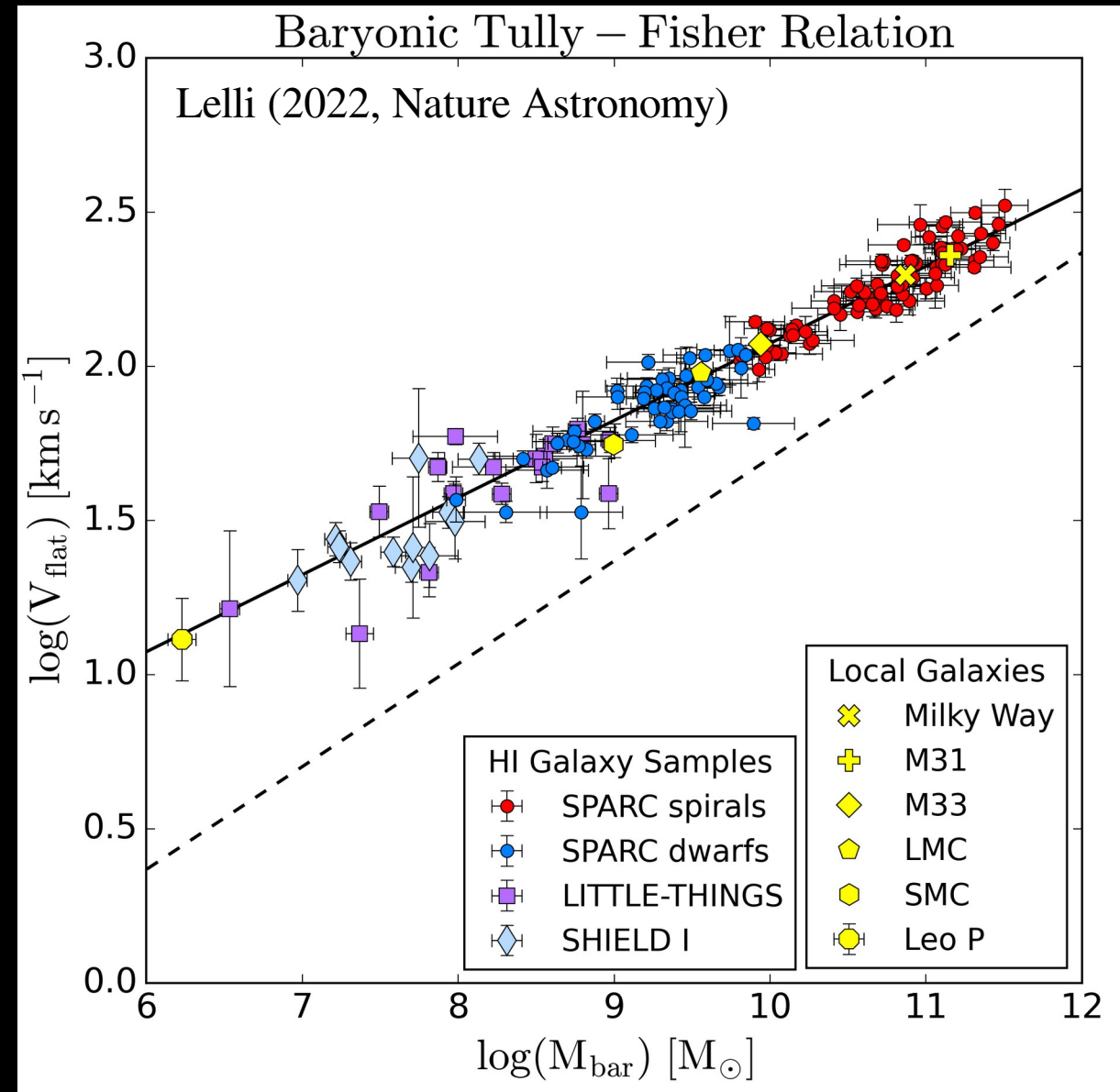
Four predictions in one equation:

✓ Key quantities are V_{flat} and M_b (stars+gas)

✓ Slope should be exactly 4

Some References: McGaugh+(2000, 2005, 2010); Verheijen (2001); Noordermeer & Verheijen (2005); Lelli+(2016, 2019, 2022, 2024); Ponomareva+(2018); Schombert+(2020), Di Teodoro+(2021, 2022).

(2) $V_{\text{flat}}^4 = a_0 G_N M_b$ for isolated objects $\rightarrow$ rotation supported gals



Four predictions in one equation:

✓ Key quantities are V_{flat} and M_b (stars+gas)

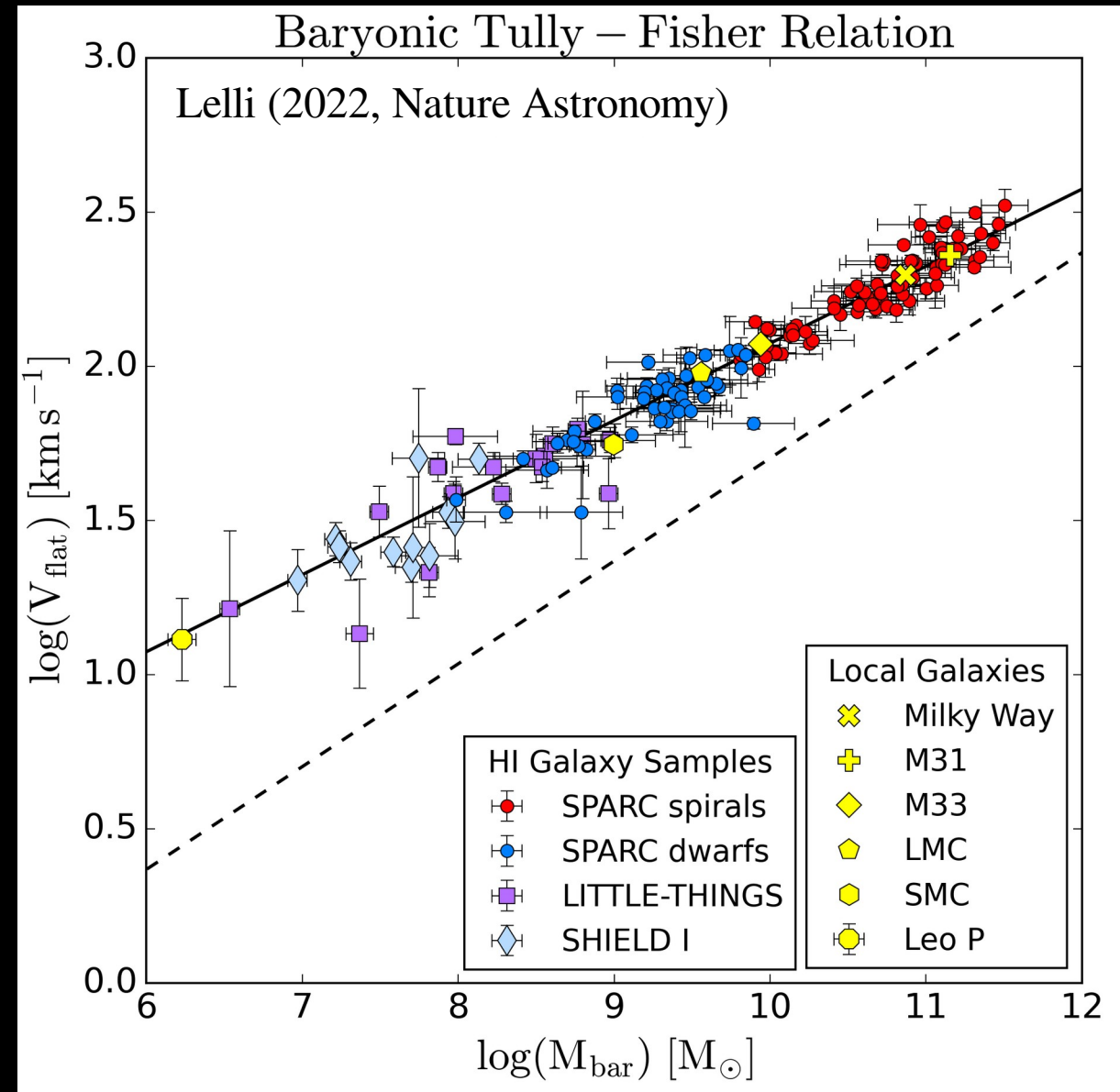
✓ Slope should be exactly 4

✓ Normalization is $a_0 G_N$

$\rightarrow$ OK with independent estimates

Some References: McGaugh+(2000, 2005, 2010); Verheijen (2001); Noordermeer & Verheijen (2005); Lelli+(2016, 2019, 2022, 2024); Ponomareva+(2018); Schombert+(2020), Di Teodoro+(2021, 2022).

(2) $V_{\text{flat}}^4 = a_0 G_N M_b$ for isolated objects $\rightarrow$ rotation supported gals



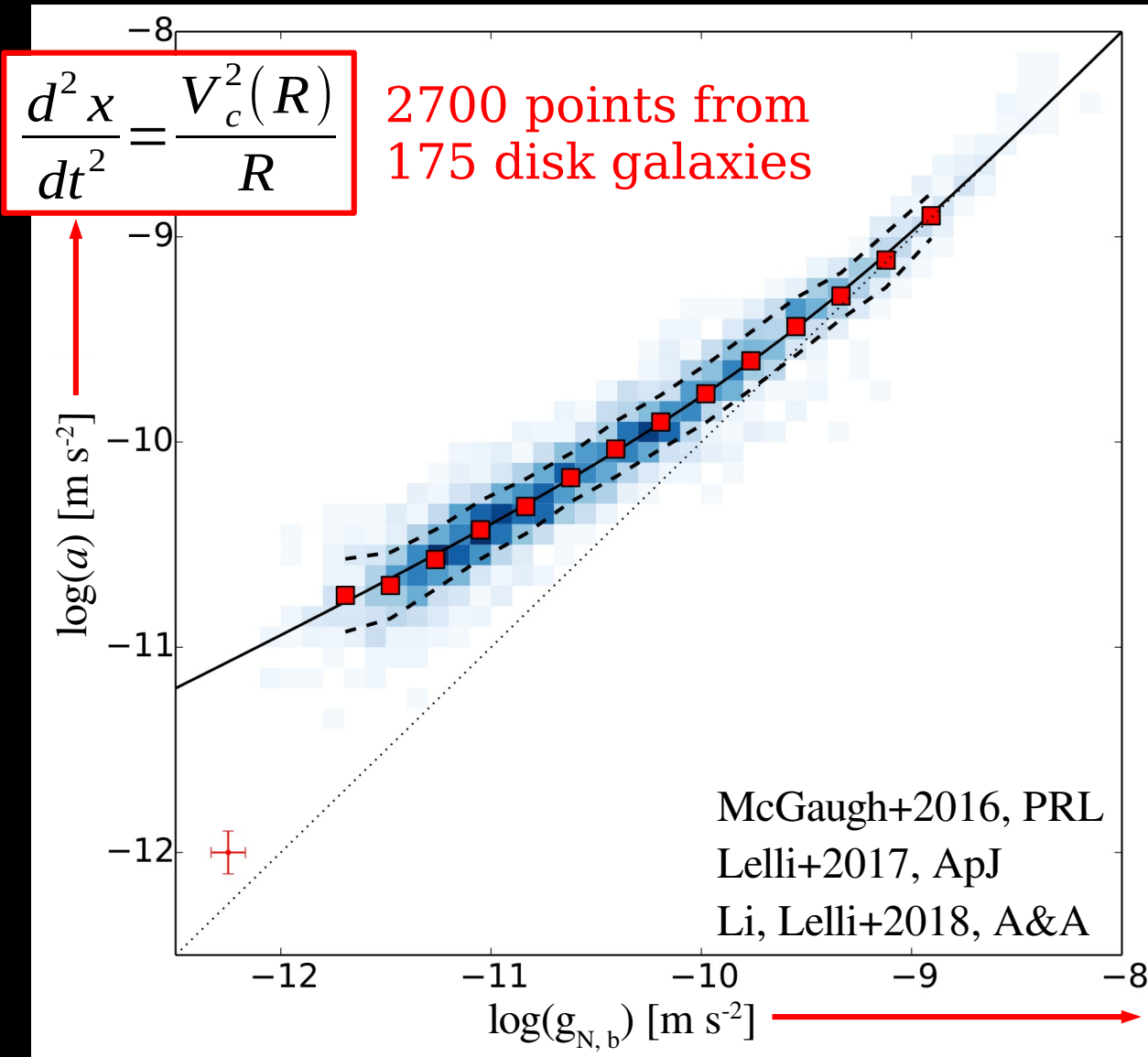
Four predictions in one equation:

- ✓ Key quantities are V_{flat} and M_b (stars+gas)
- ✓ Slope should be exactly 4
- ✓ Normalization is $a_0 G_N$
 - $\rightarrow$ OK with independent estimates
- ✓ No dependence on other galaxy properties (e.g., size, gas fraction, morphology)
 - $\rightarrow$ NO intrinsic scatter along the relation

Some References: McGaugh+(2000, 2005, 2010); Verheijen (2001); Noordermeer & Verheijen (2005); Lelli+(2016, 2019, 2022, 2024); Ponomareva+(2018); Schombert+(2020), Di Teodoro+(2021, 2022).

(3) **Rotation curves** can be predicted from the baryon distribution

(3) Rotation curves can be predicted from the baryon distribution

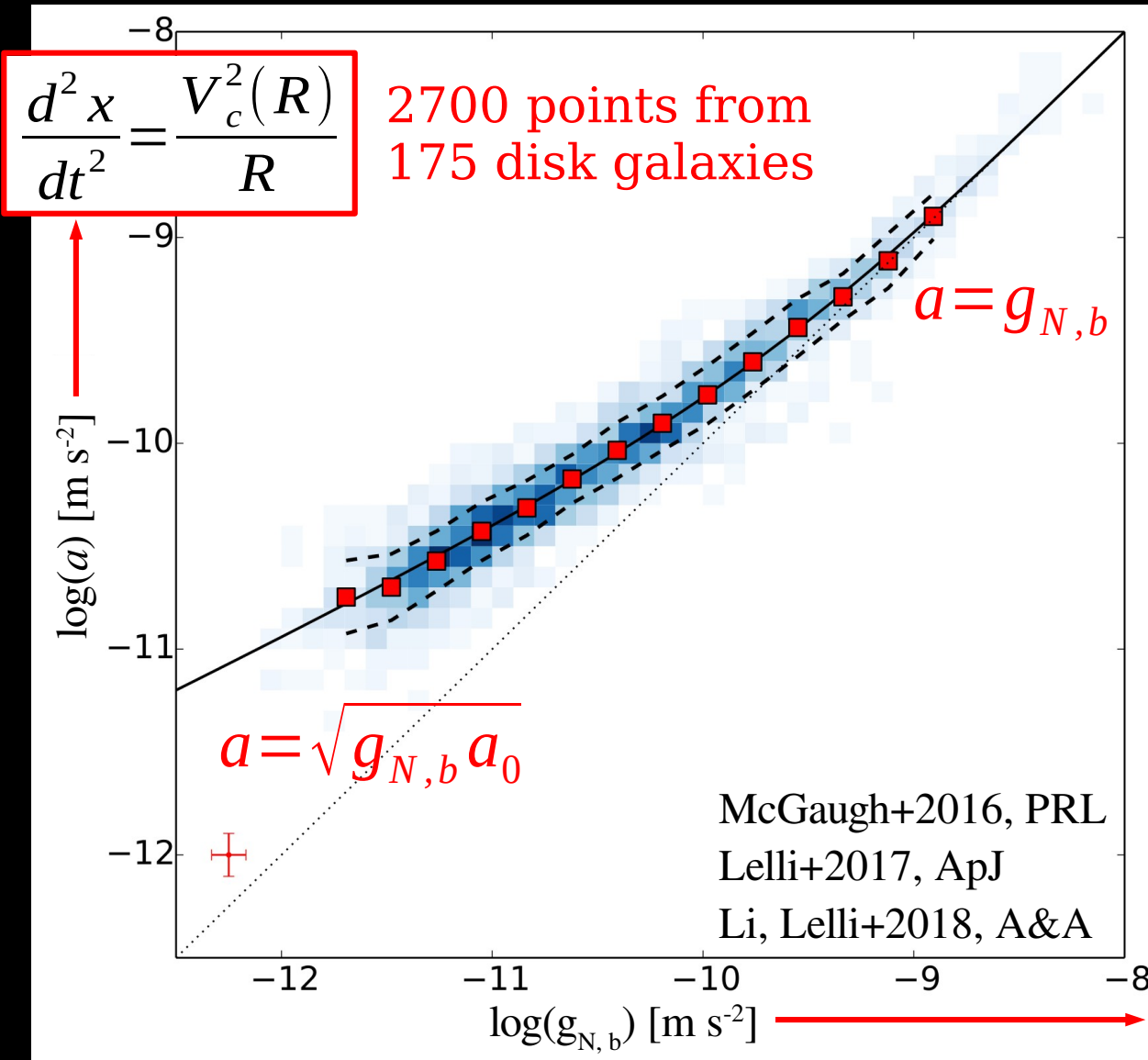


Radial Acceleration Relation (RAR)

(i) Fully empirical, independent of MOND

$$\nabla^2 \Phi_{N,b}(R, z) = 4\pi G \rho_b(R, z)$$
$$g_{N,b}(R, z=0) = -\nabla \Phi_{N,b}(R, z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



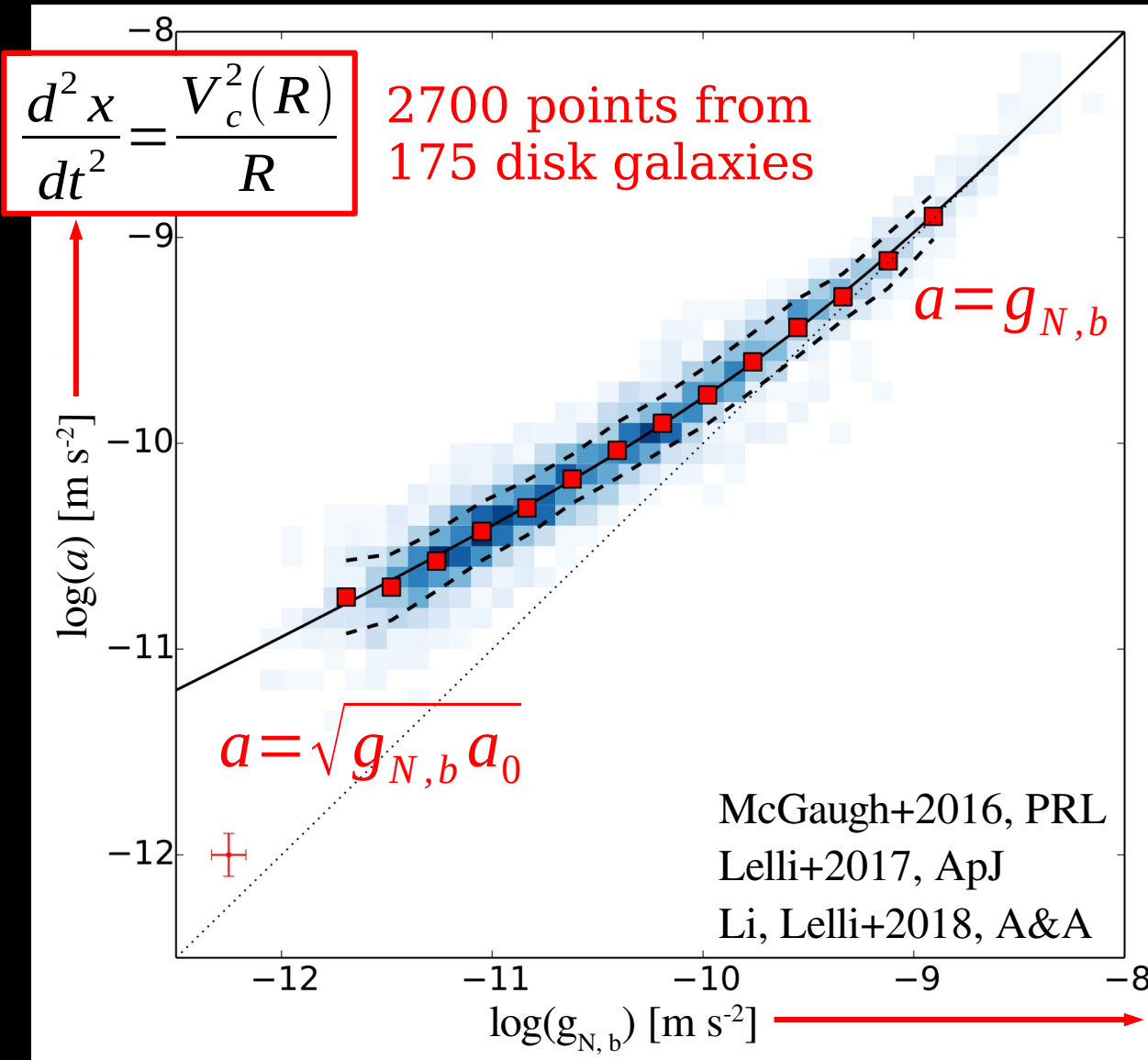
Radial Acceleration Relation (RAR)

- (i) Fully empirical, independent of MOND
- (ii) MOND asymptotic limits → OK

$$\nabla^2 \Phi_{N,b}(R, z) = 4\pi G \rho_b(R, z)$$

$$g_{N,b}(R, z=0) = -\nabla \Phi_{N,b}(R, z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



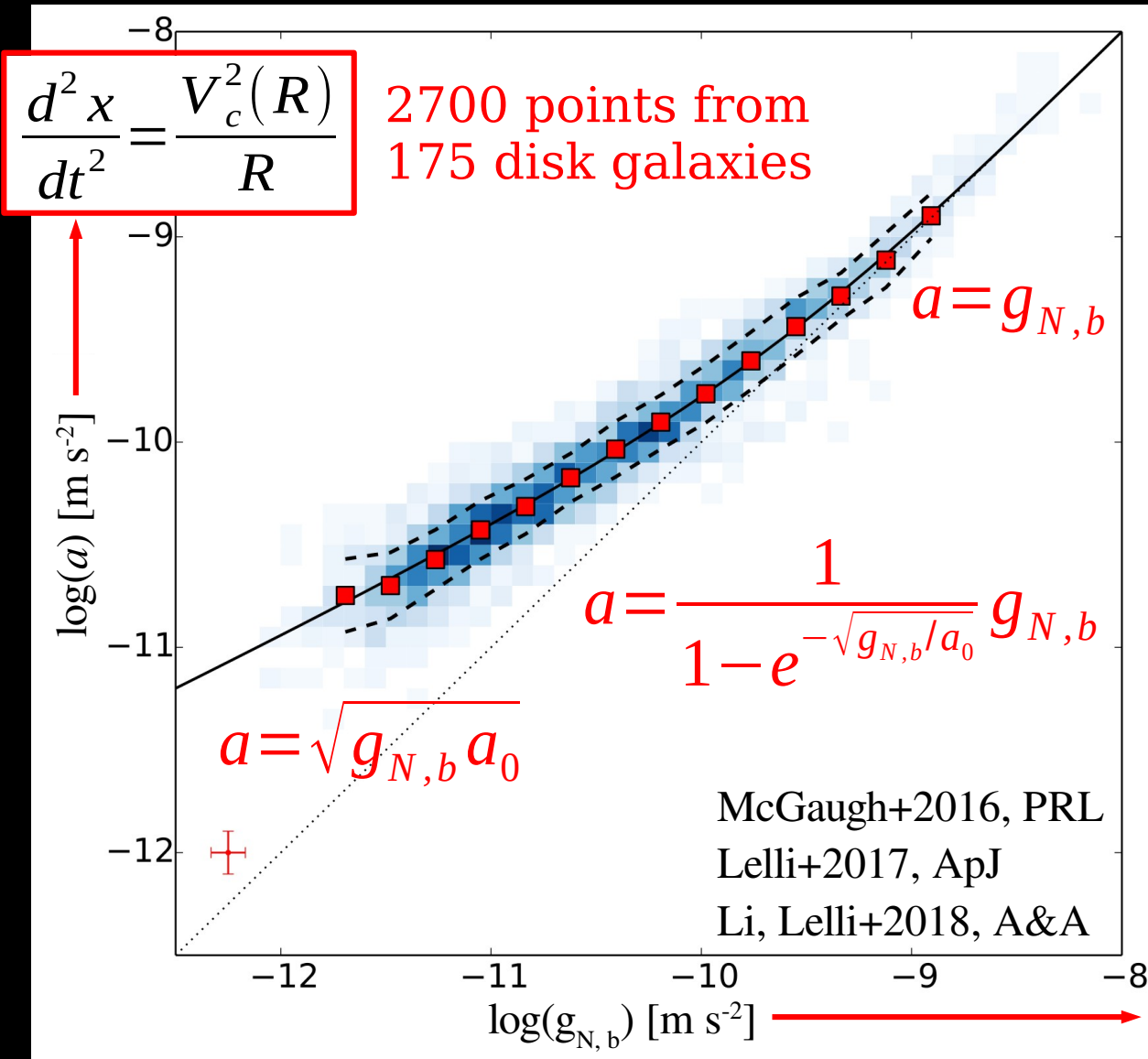
Radial Acceleration Relation (RAR)

- (i) Fully empirical, independent of MOND
- (ii) MOND asymptotic limits → OK
- (iii) Negligible scatter ($\sim$ errors) → OK

$$\nabla^2 \Phi_{N,b}(R, z) = 4\pi G \rho_b(R, z)$$

$$g_{N,b}(R, z=0) = -\nabla \Phi_{N,b}(R, z=0)$$

(3) Rotation curves can be predicted from the baryon distribution



Radial Acceleration Relation (RAR)

- (i) Fully empirical, independent of MOND
- (ii) MOND asymptotic limits → **OK**
- (iii) Negligible scatter ($\sim$ errors) → **OK**
- (iv) MOND interpolation function ν or μ

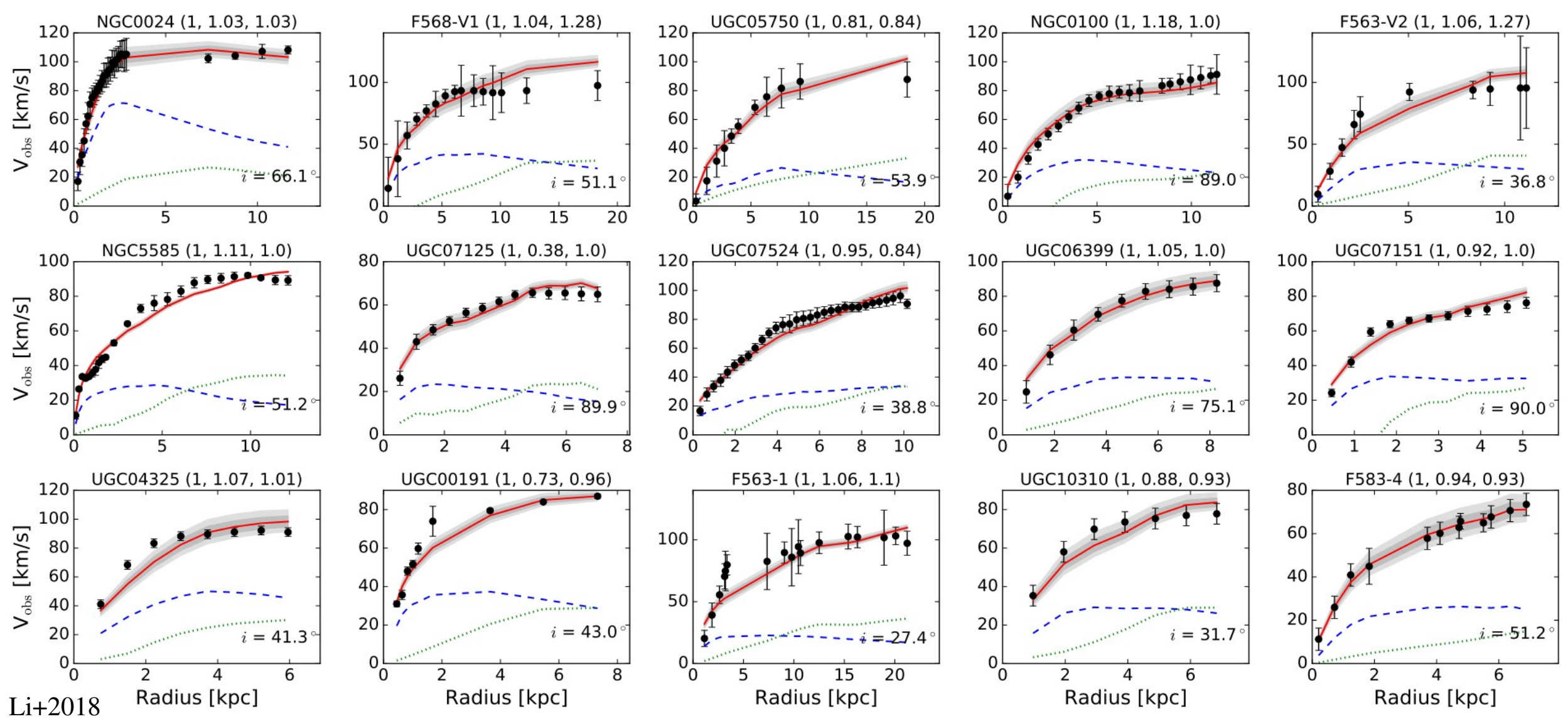
$$a = \nu\left(\frac{g_{N,b}}{a_0}\right) g_{N,b} \iff a \mu\left(\frac{a}{a_0}\right) = g_{N,b}$$

We can now assume $\nu(g_{N,b}/a_0)$ and predict rotation curves given ρ_b (within the errors)

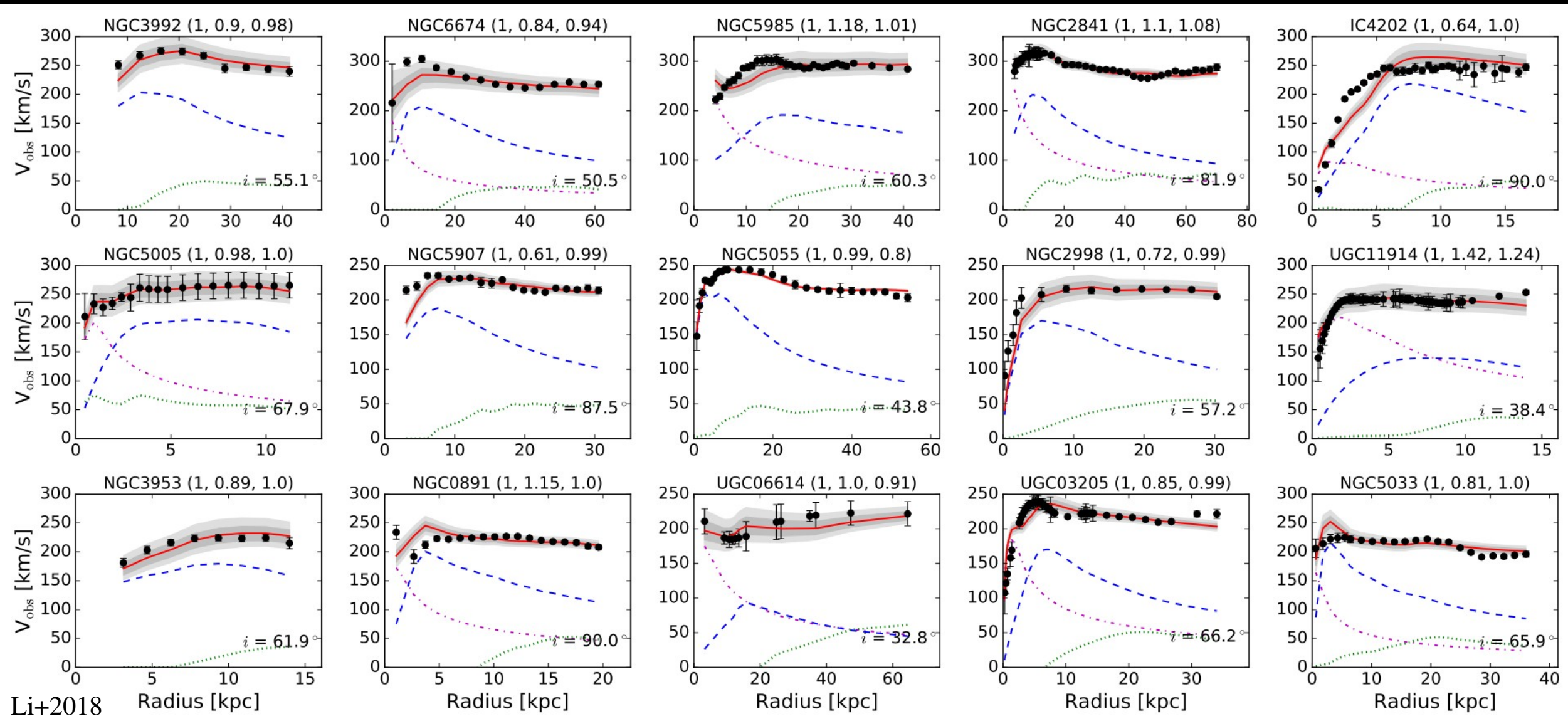
$$\nabla^2 \Phi_{N,b}(R, z) = 4\pi G \rho_b(R, z)$$

$$g_{N,b}(R, z=0) = -\nabla \Phi_{N,b}(R, z=0)$$

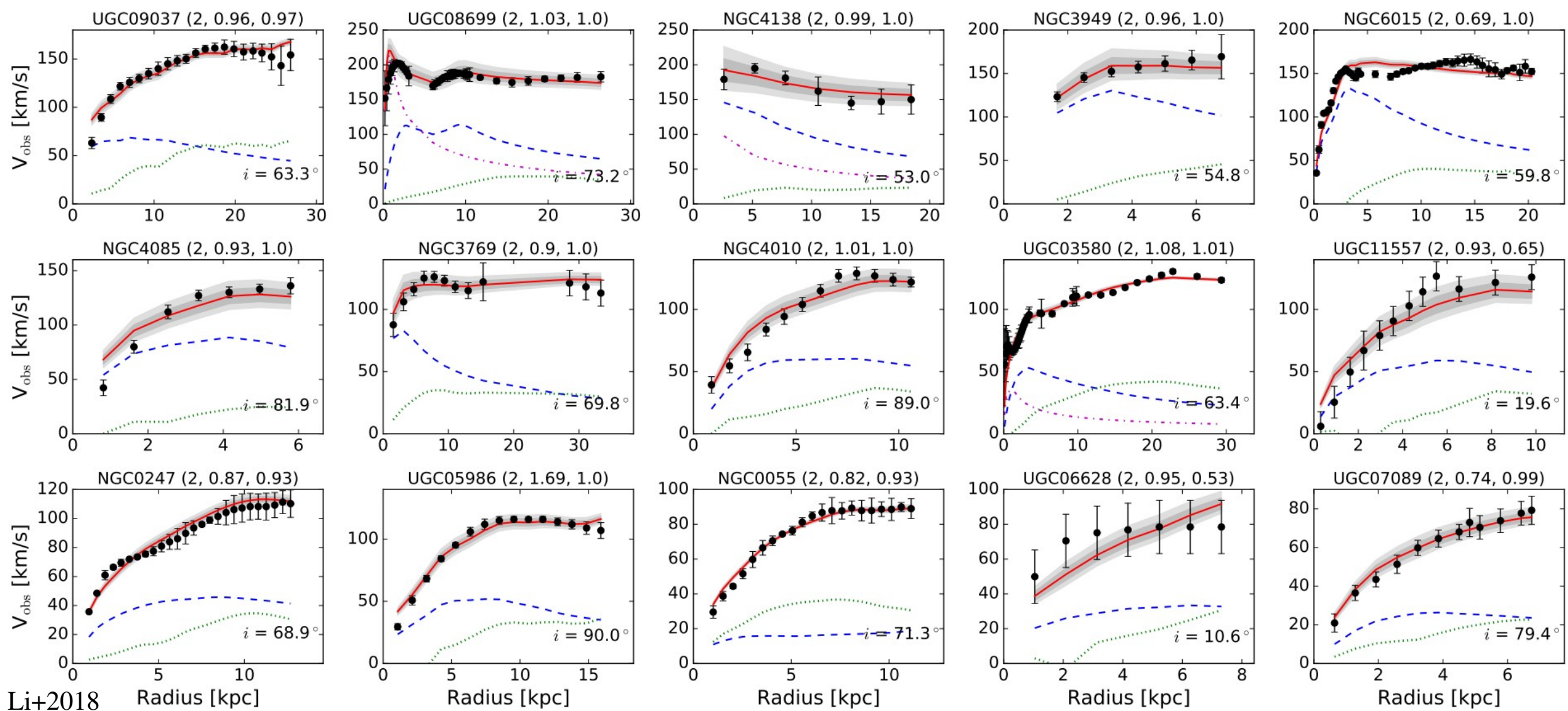
(3) Rotation curves can be predicted from the baryon distribution



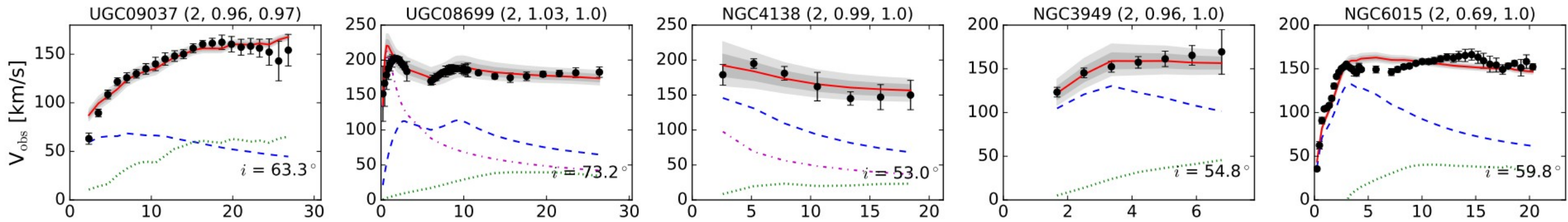
(3) Rotation curves can be predicted from the baryon distribution



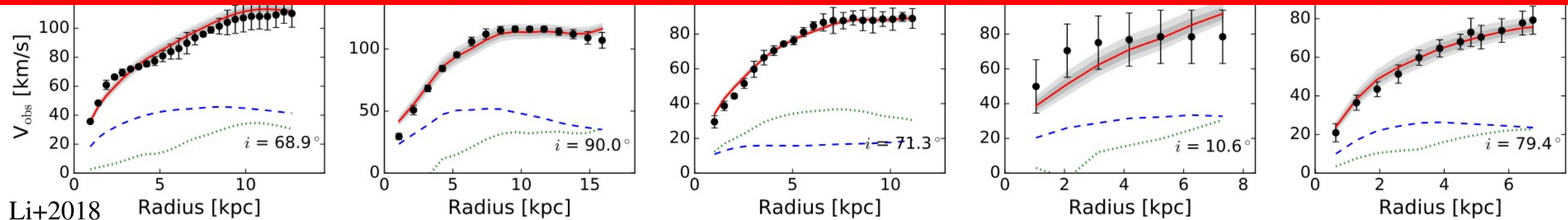
(3) **Rotation curves** can be predicted from the baryon distribution



(3) **Rotation curves** can be predicted from the baryon distribution



For the full sample of 175 galaxies, see Li, Lelli, McGaugh et al. (2018)
or the SPARC database (<http://astroweb.cwru.edu/SPARC/>)



Talk Outline

1. General MOND paradigm

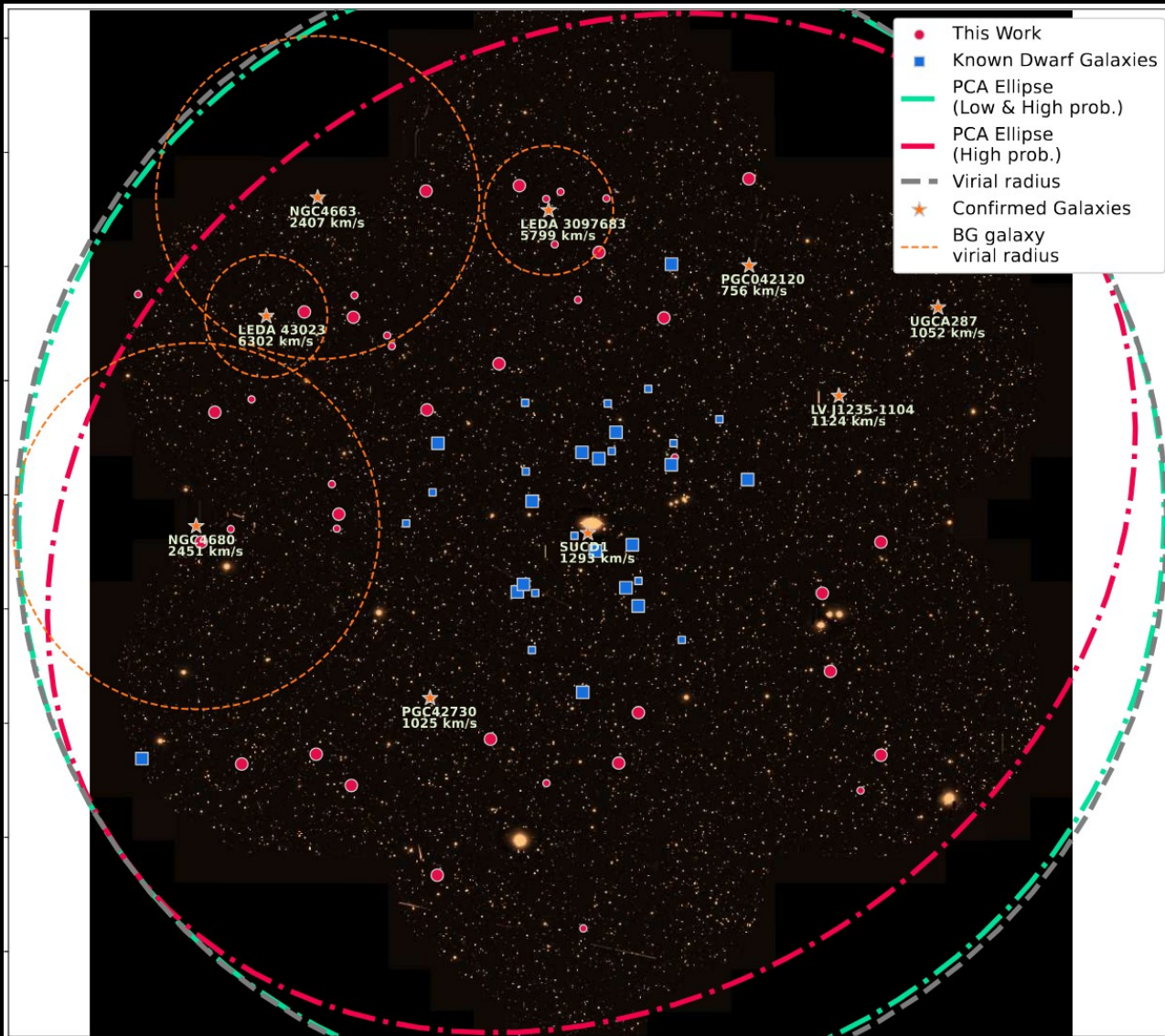
~~2. Tests on galaxy scales~~

3. Tests on groups/clusters scales

4. MOND theories & cosmology

Galaxy Groups: bound systems with ~10-100 galaxies

Sombrero (M104) galaxy group (Crosby et al. 2022)

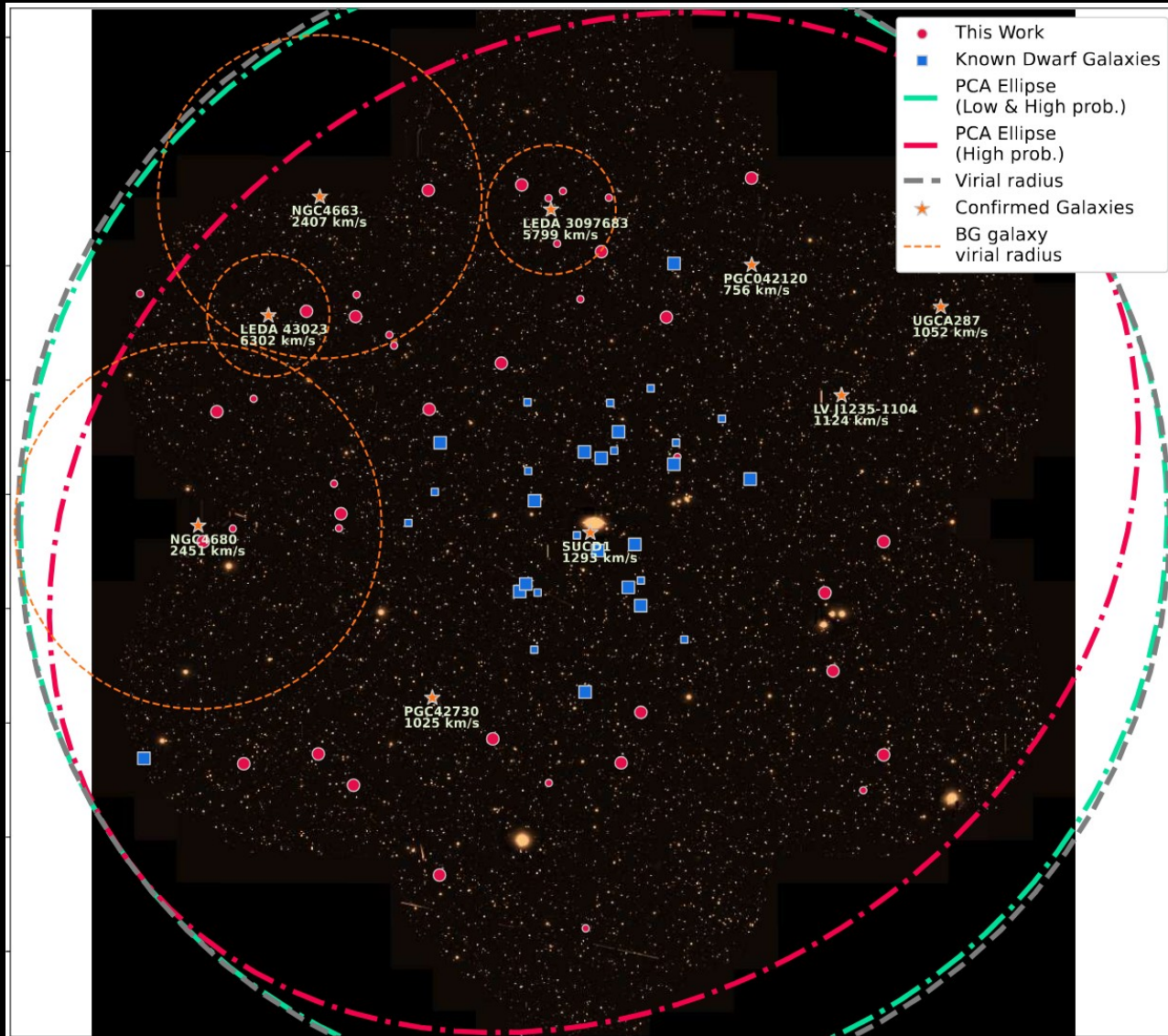


Velocity dispersion (“temperature”):

$$\sigma_V^2 = \frac{1}{M_b} \sum_i m_i V_i^2$$

Galaxy Groups: bound systems with ~10-100 galaxies

Sombrero (M104) galaxy group (Crosby et al. 2022)



Velocity dispersion (“temperature”):

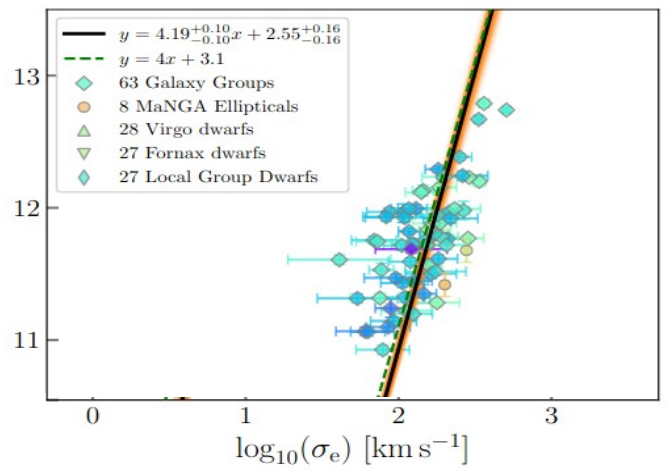
$$\sigma_V^2 = \frac{1}{M_b} \sum_i m_i V_i^2$$

MOND virial theorem (Milgrom 2014):

$$\sigma_V^4 = \frac{a_0 G_N}{c} M_b$$

- It applies only to objects for which $a \ll a_0$ everywhere (fully in the deep MOND limit)
- c is theory-dependent but is expected $\sim O(1)$
- $c = 9/4$ for a broad class of MOND theories

Galaxy Groups: bound systems with ~ 10 -100 galaxies



$$\sigma_V^4 = \frac{a_0 G_N}{c} M_b \rightarrow \text{confirmed in galaxy groups!}$$

Galaxy Groups: bound systems with ~10-100 galaxies

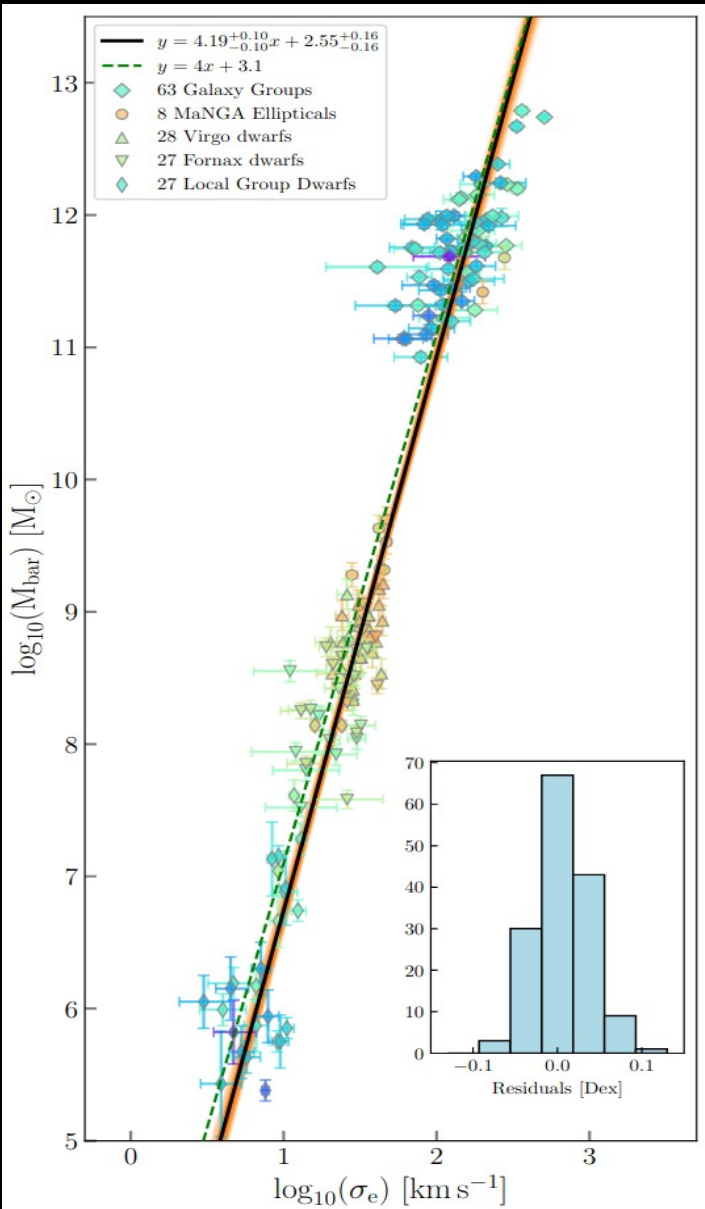
$$\sigma_V^4 = \frac{a_0 G_N}{c} M_b \rightarrow \text{confirmed in galaxy groups!}$$

The same relation is followed by pressure-supported galaxies in the deep-MOND limit (Tian et al. in prep.):

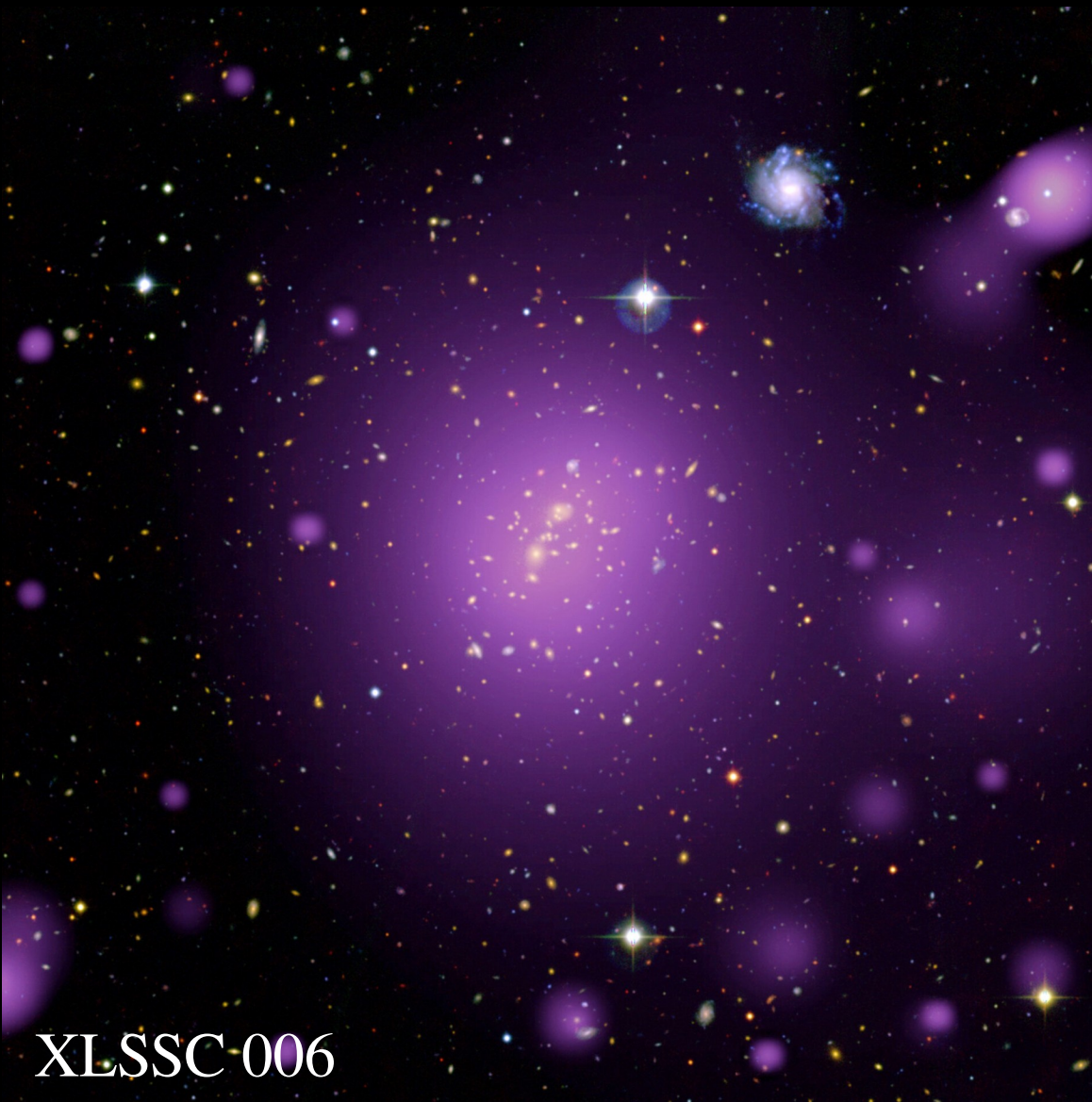
- Dwarf ellipticals in galaxy clusters
- Dwarf spheroidals in the Local Group

MOND virial theorem confirmed over ~8 dex in M_b !

Same value of a_0 from BTFR, RAR, and this relation!



Galaxy Clusters: bound systems with ~ 100 -1000 galaxies



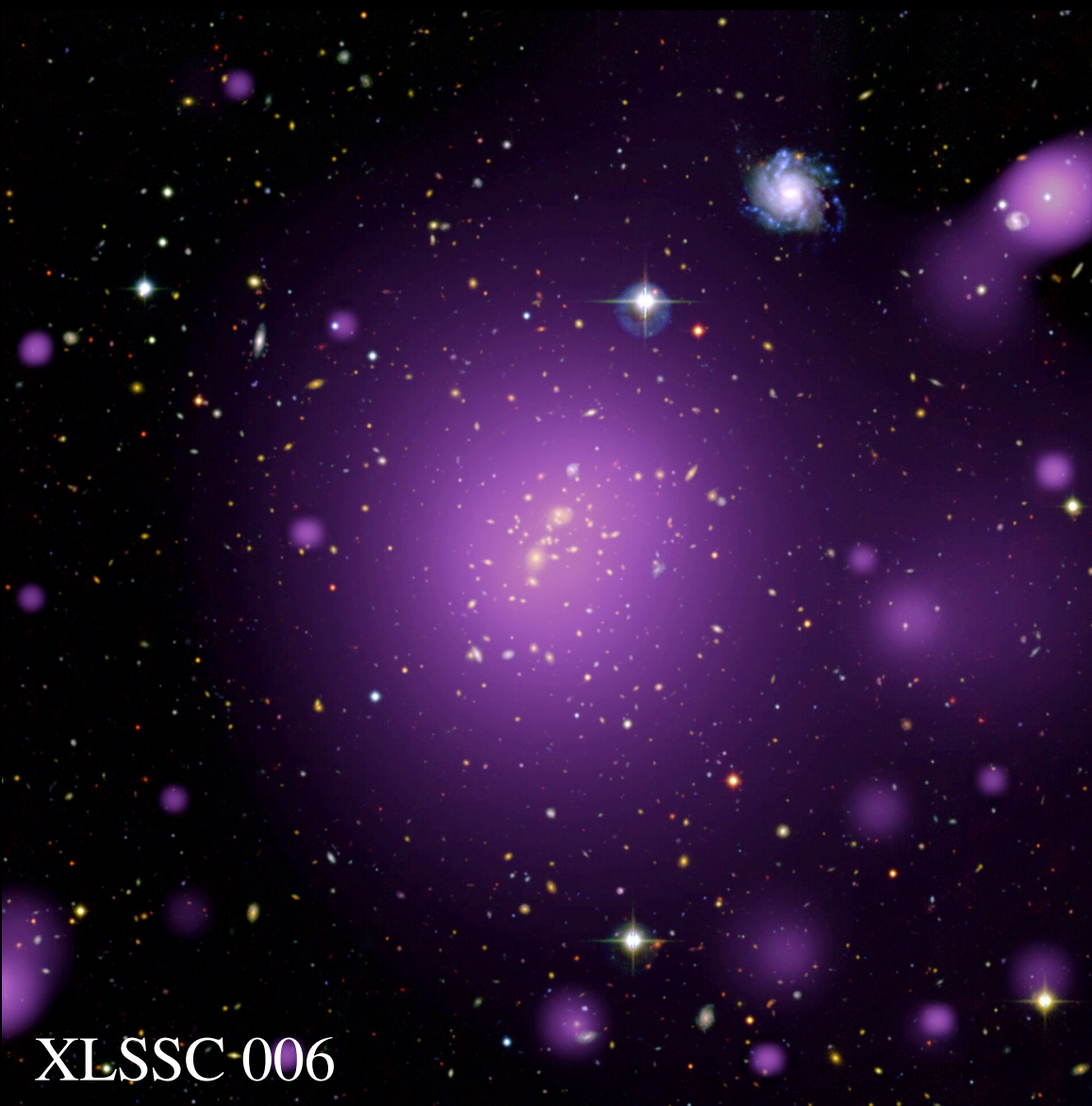
Observed baryon budget:

$\sim 10\%$ galaxies (optical & NIR)

$\sim 90\%$ hot ionized gas (X rays)

XLSSC 006

Galaxy Clusters: bound systems with ~ 100 -1000 galaxies



XLSSC 006

Observed baryon budget:

$\sim 10\%$ galaxies (optical & NIR)

$\sim 90\%$ hot ionized gas (X rays)

Sphere in Hydrostatic Equilibrium

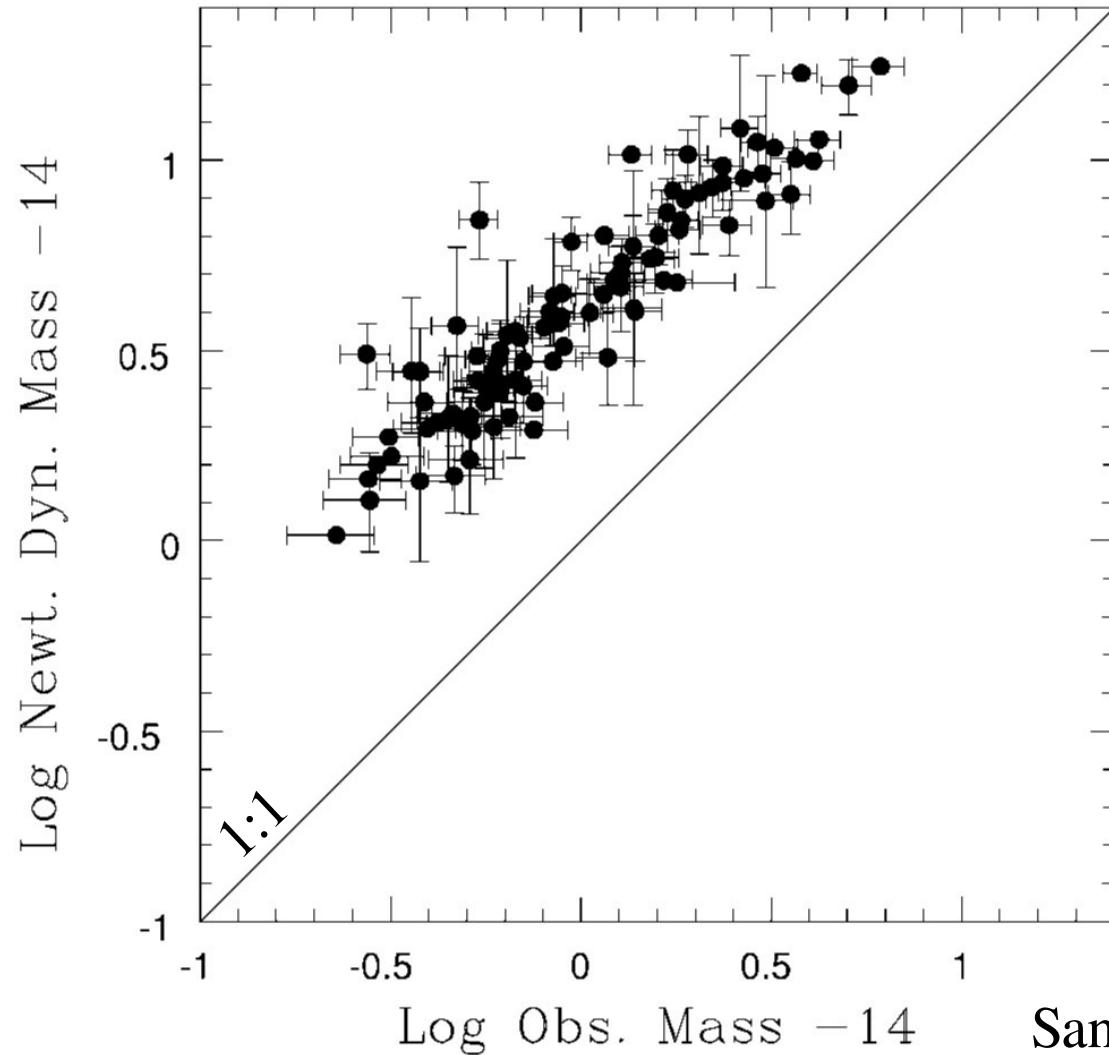
$$\frac{\partial P_{gas}}{\partial r} = \rho_{gas} \frac{\partial \Phi}{\partial r} \quad P_{gas} = \frac{k_B T_{gas}}{w m_p}$$

$$\Rightarrow \frac{\partial \Phi}{\partial r} = \frac{k_B}{w m_p} \frac{1}{\rho_{gas}} \frac{\partial}{\partial r} (\rho_{gas} T_{gas})$$

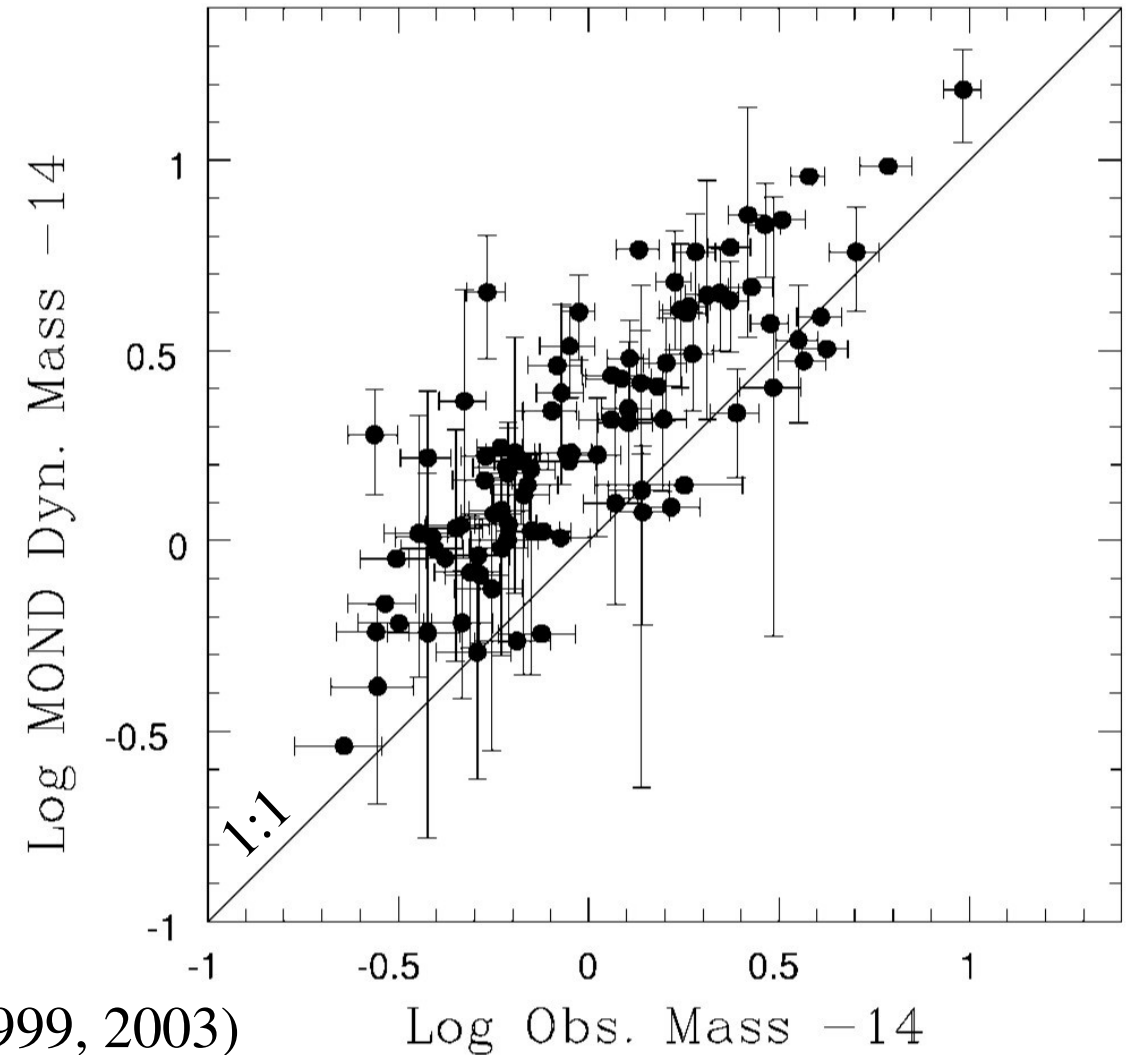
Galaxy Clusters: Long-standing challenge for MOND

Newtonian analysis: $M_{\text{dyn}}/M_{\text{bar}} \simeq 5$

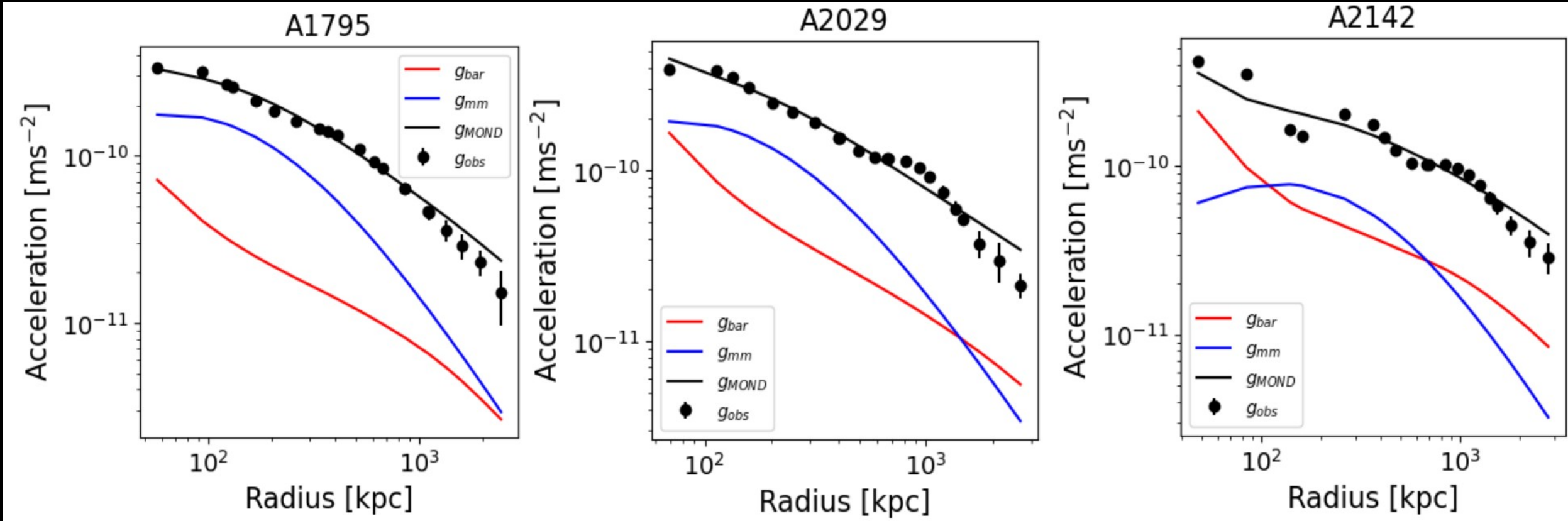
MOND analysis: $M_{\text{dyn}}/M_{\text{bar}} \simeq 2$



Sanders (1999, 2003)



MOND fits with a missing mass component (Kelleher & Lelli 2024)



Density profile of missing mass:

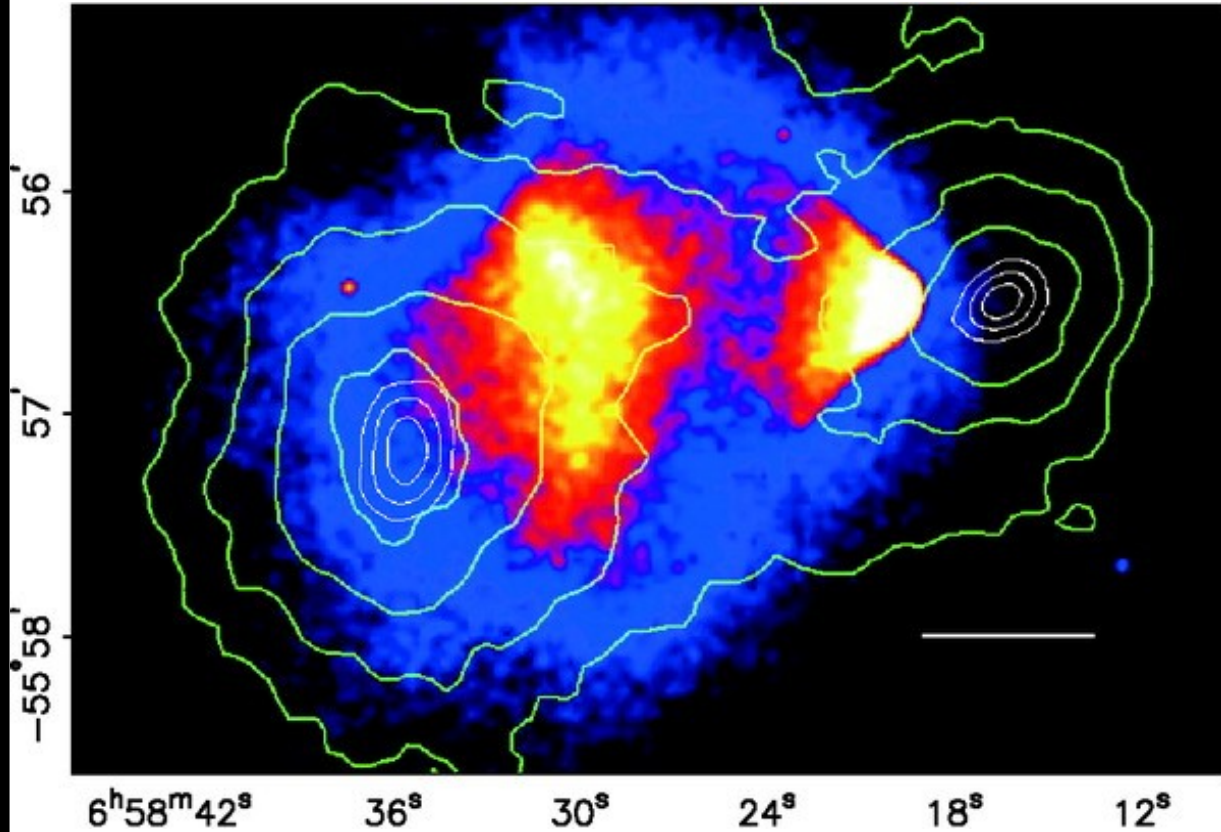
$$\rho_{mm}(r) = \frac{\rho_0}{(1+r/r_s)^4}$$

Converged (physical) total mass:

$$\Rightarrow M_{mm,tot} = \int_0^\infty \rho(r) 4\pi_{mm} r^2 dr = \frac{4\pi}{3} \rho_0 r_s^3$$

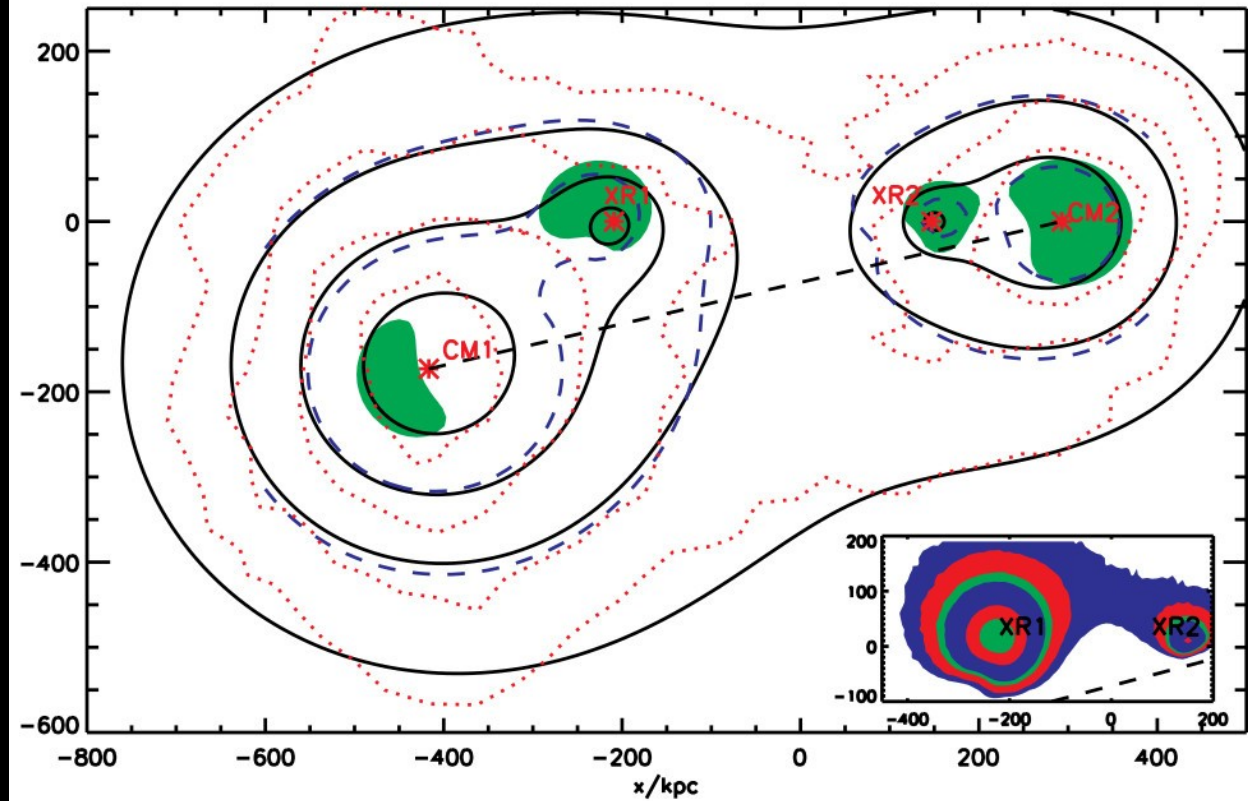
The bullet cluster in MOND: missing mass must be collisionless

OBSERVATIONS (Clowe+2004, ApJ)



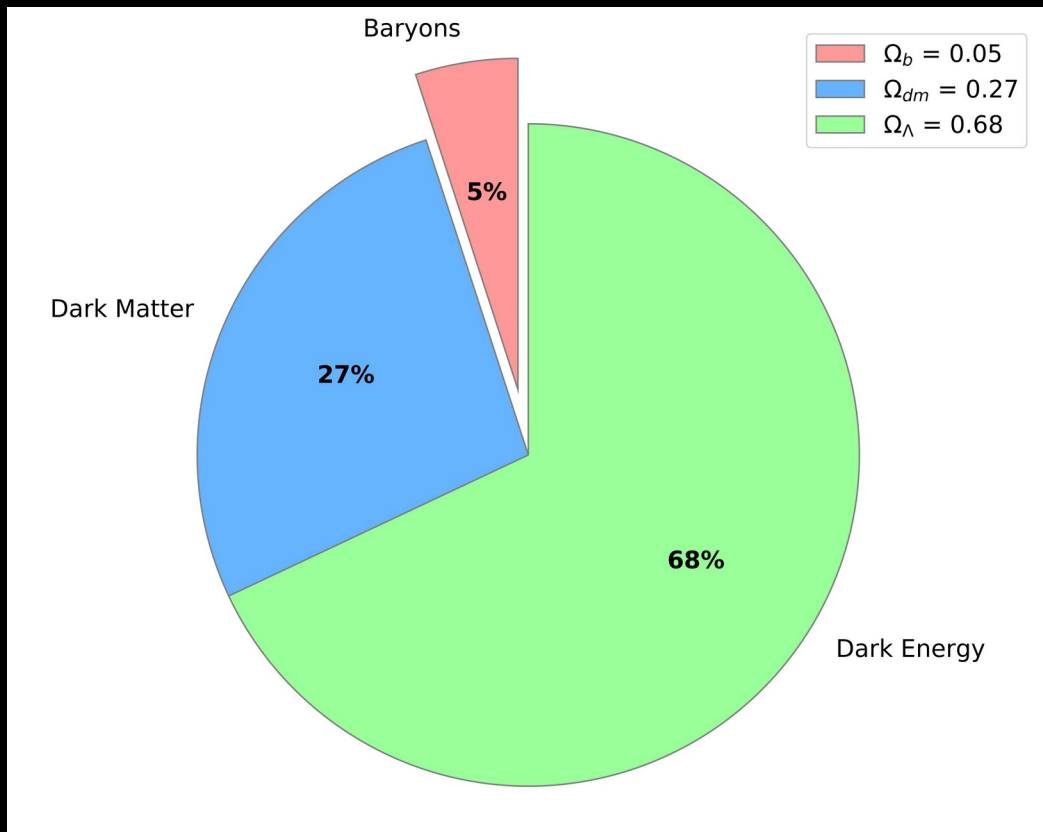
Green: Observed lensing map (total mass)
Blue/Red/Yellow: X-ray emission (hot gas)

MOND (Angus+2006, MNRAS; Angus+2007, ApJ)

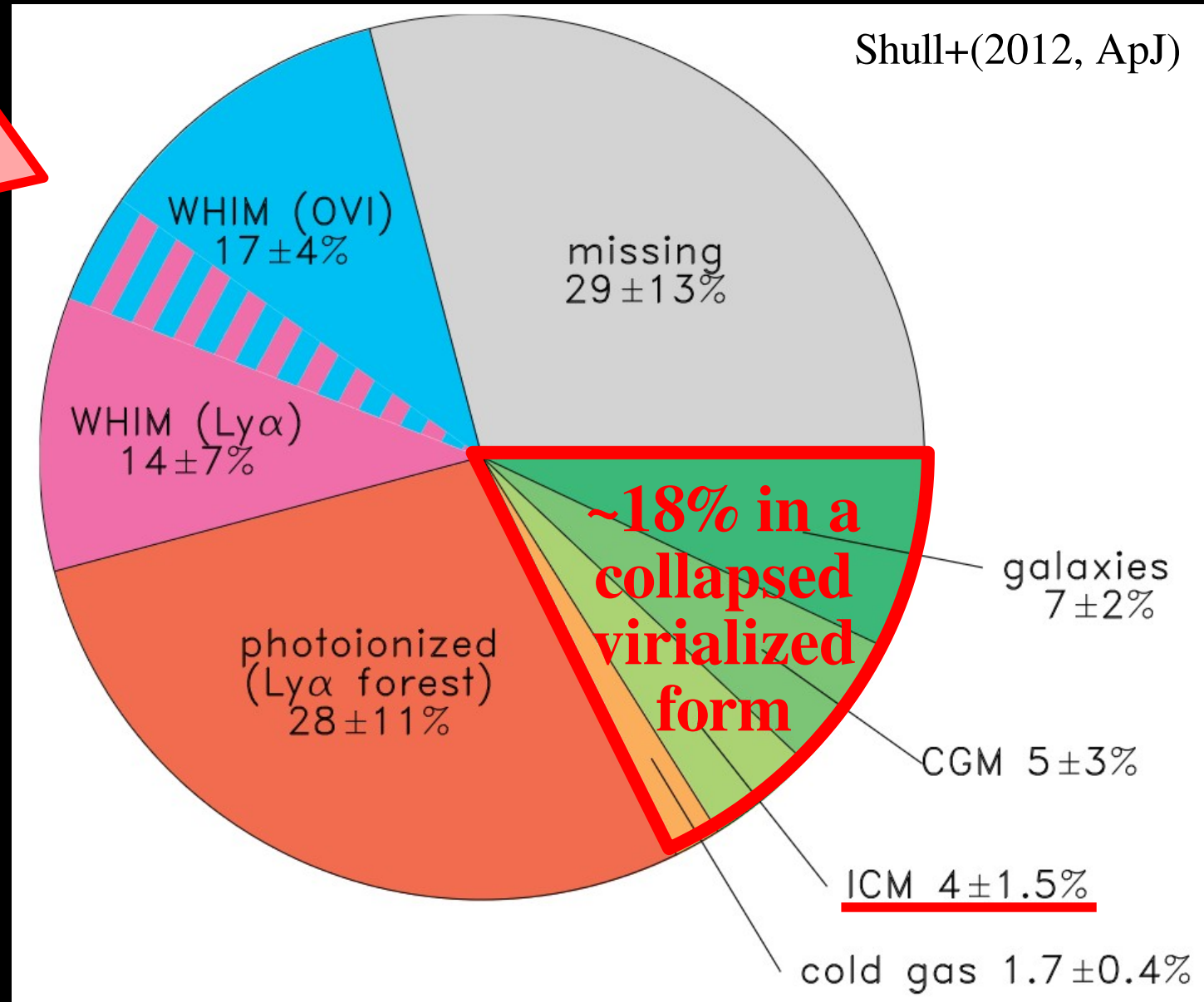
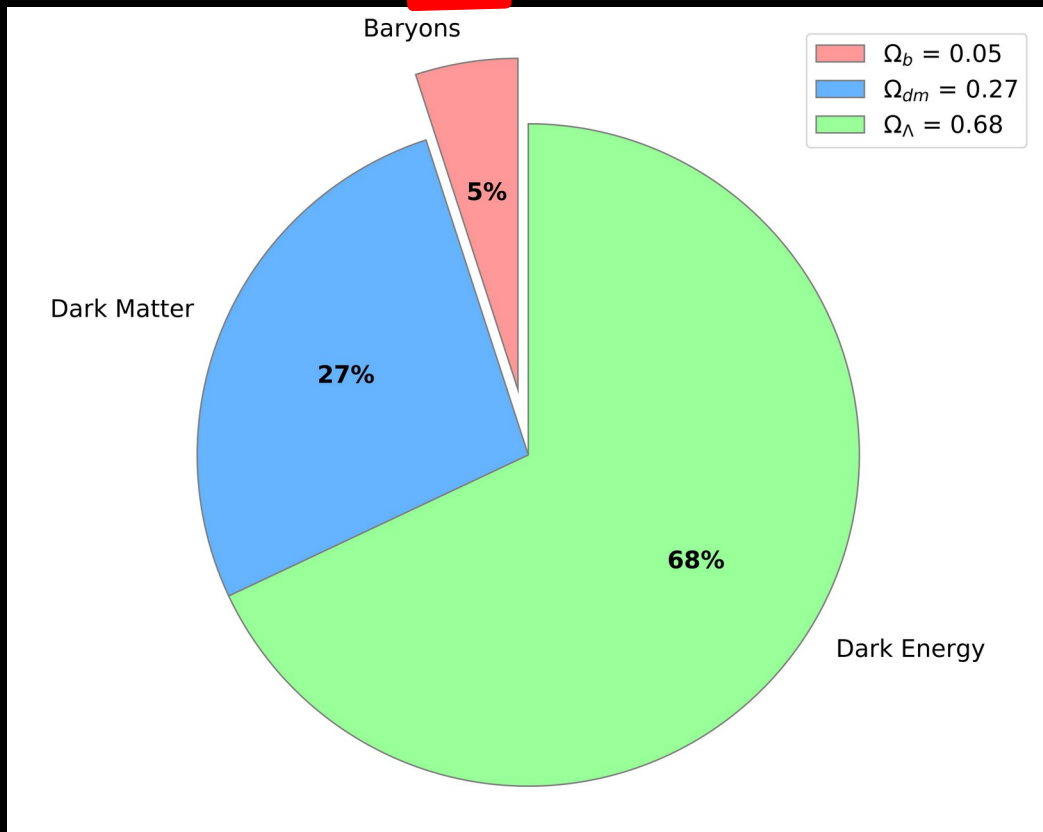


Red: Observed lensing convergence map
Black: TeVeS model with 2eV neutrinos
Blue: total surface densities (baryons + ν)

The missing baryons problem in Cosmology



The missing baryons problem in Cosmology



Talk Outline

1. General MOND paradigm
2. Tests on galaxy scales
- ~~3. Tests on groups/clusters scales~~
4. MOND theories & cosmology

MOND paradigm

```
graph TD; A[MOND paradigm] --> B[Modified Gravity (→ ∇²Φ = 4πGρ)]; A --> C[Modified Inertia (→ F = ma)];
```

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Modified Inertia ($\rightarrow F = ma$)

MOND paradigm

```
graph TD; A[MOND paradigm] --> B["Modified Gravity (→ ∇²Φ = 4πGρ)"]; A --> C["Modified Inertia (→ F = ma)"];
```

Modified Gravity (→ $\nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

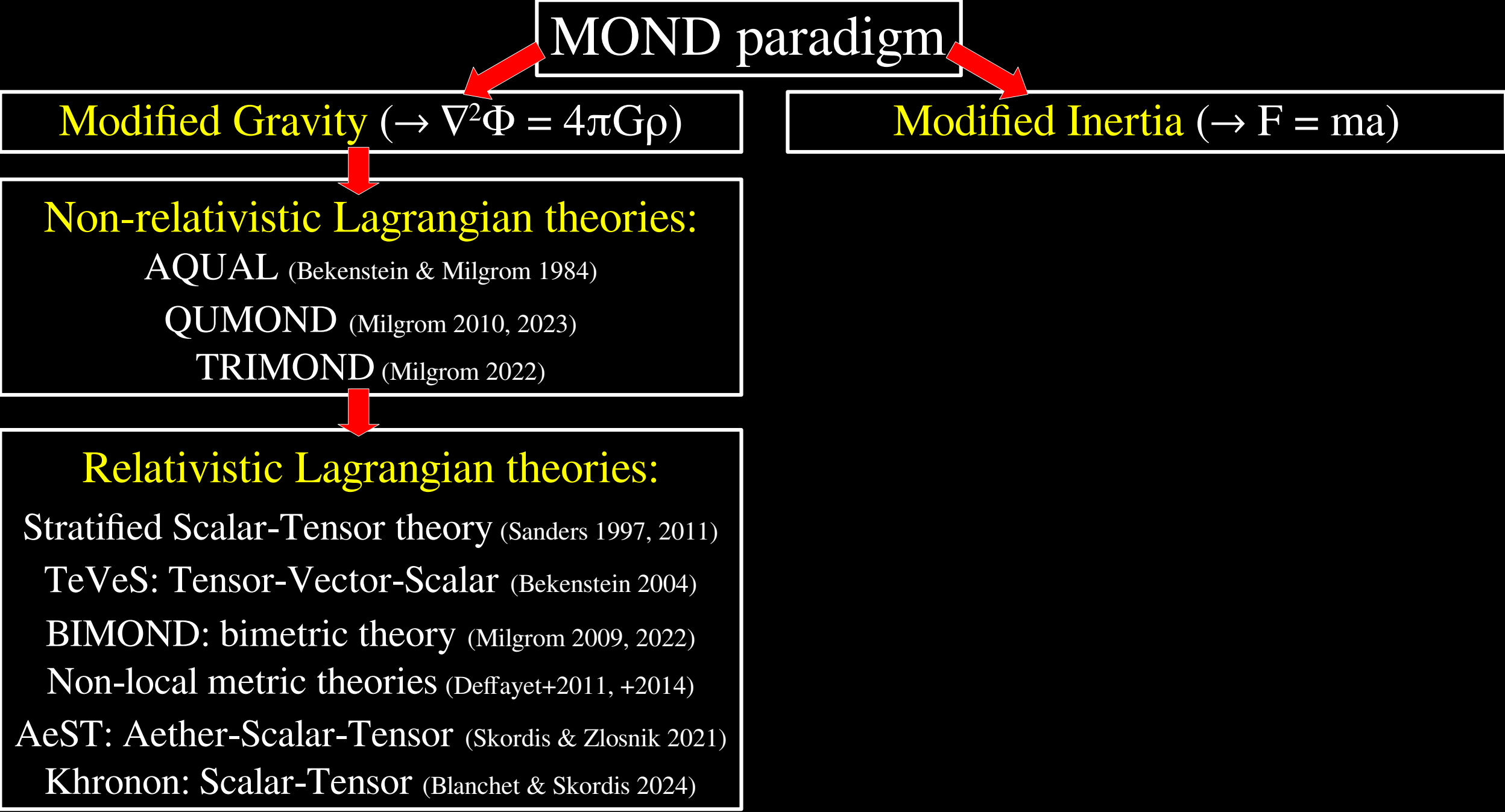
AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Modified Inertia (→ $F = ma$)

MOND paradigm



```
graph TD; A[MOND paradigm] --> B[Modified Gravity (→ ∇²Φ = 4πGρ)]; A --> C[Modified Inertia (→ F = ma)]; B --> D[Non-relativistic Lagrangian theories: AQUAL (Bekenstein & Milgrom 1984), QUMOND (Milgrom 2010, 2023), TRIMOND (Milgrom 2022)]; D --> E[Relativistic Lagrangian theories: Stratified Scalar-Tensor theory (Sanders 1997, 2011), TeVeS: Tensor-Vector-Scalar (Bekenstein 2004), BIMOND: bimetric theory (Milgrom 2009, 2022), Non-local metric theories (Deffayet+2011, +2014), AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021), Khronon: Scalar-Tensor (Blanchet & Skordis 2024)];
```

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

TeVeS: Tensor-Vector-Scalar (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

MOND paradigm

```
graph TD; A[MOND paradigm] --> B[Modified Gravity (→ ∇²Φ = 4πGρ)]; A --> C[Modified Inertia (→ F = ma)]; B --> D[Non-relativistic Lagrangian theories: AQUAL (Bekenstein & Milgrom 1984), QUMOND (Milgrom 2010, 2023), TRIMOND (Milgrom 2022)]; D --> E[Relativistic Lagrangian theories: Stratified Scalar-Tensor theory (Sanders 1997, 2011), TeVeS: Tensor-Vector-Scalar (Bekenstein 2004), BIMOND: bimetric theory (Milgrom 2009, 2022), Non-local metric theories (Deffayet+2011, +2014), AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021), Khronon: Scalar-Tensor (Blanchet & Skordis 2024)];
```

Modified Gravity ($\rightarrow \nabla^2\Phi = 4\pi G\rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

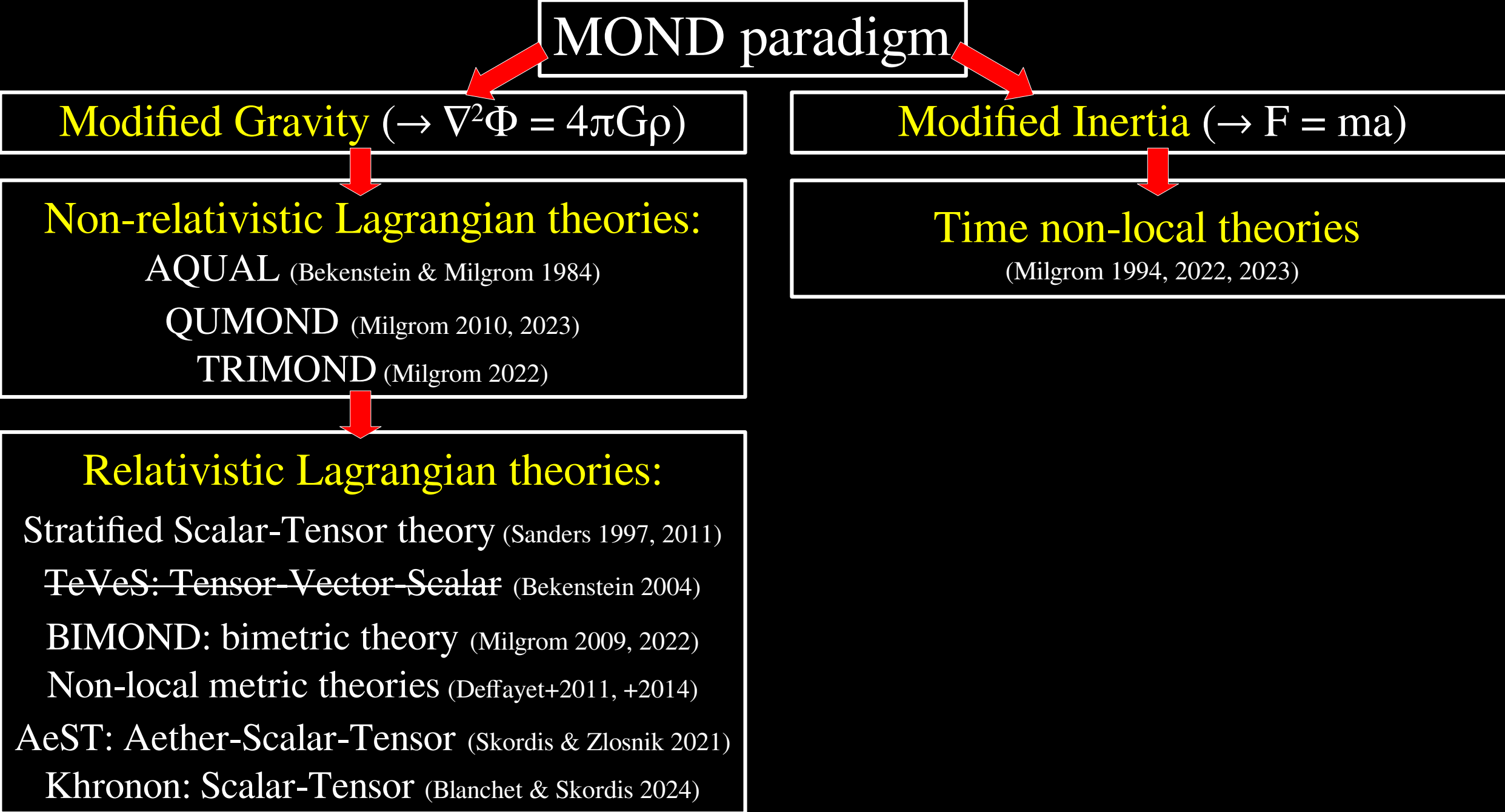
Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

MOND paradigm



```
graph TD; A[MOND paradigm] --> B[Modified Gravity (→ ∇²Φ = 4πGρ)]; A --> C[Modified Inertia (→ F = ma)]; B --> D["Non-relativistic Lagrangian theories:  
AQUAL (Bekenstein & Milgrom 1984)  
QUMOND (Milgrom 2010, 2023)  
TRIMOND (Milgrom 2022)"]; D --> E["Relativistic Lagrangian theories:  
Stratified Scalar-Tensor theory (Sanders 1997, 2011)  
TeVes: Tensor-Vector-Scalar (Bekenstein 2004)  
BIMOND: bimetric theory (Milgrom 2009, 2022)  
Non-local metric theories (Deffayet+2011, +2014)  
AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)  
Khronon: Scalar-Tensor (Blanchet & Skordis 2024)"]; C --> F[Time non-local theories  
(Milgrom 1994, 2022, 2023)];
```

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVes: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c \Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c \Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

MOND-like DM models:

Dipolar DM (Blanchet+2008, 2009, 2015, 2017)

Superfluid DM (Berezhiani & Khoury 2015)

Baryon-interacting DM (Famaey+2018, +2020)

MOND paradigm

Modified Gravity ($\rightarrow \nabla^2 \Phi = 4\pi G \rho$)

Non-relativistic Lagrangian theories:

AQUAL (Bekenstein & Milgrom 1984)

QUMOND (Milgrom 2010, 2023)

TRIMOND (Milgrom 2022)

Relativistic Lagrangian theories:

Stratified Scalar-Tensor theory (Sanders 1997, 2011)

~~TeVeS: Tensor-Vector-Scalar~~ (Bekenstein 2004)

BIMOND: bimetric theory (Milgrom 2009, 2022)

Non-local metric theories (Deffayet+2011, +2014)

AeST: Aether-Scalar-Tensor (Skordis & Zlosnik 2021)

Khronon: Scalar-Tensor (Blanchet & Skordis 2024)

Modified Inertia ($\rightarrow F = ma$)

Time non-local theories

(Milgrom 1994, 2022, 2023)

Heuristic ideas:

Mach's principle for inertia ?

$a_0 \simeq c \Lambda^{1/2} \rightarrow$ quantum vacuum? (Milgrom 1999)

MOND-like DM models:

Dipolar DM (Blanchet+2008, 2009, 2015, 2017)

Superfluid DM (Berezhiani & Khoury 2015)

Baryon-interacting DM (Famaey+2018, +2020)

Let's start from the Newtonian non-relativistic Action

$$S = \int dt L = \int dt (L_{\text{matter}} + L_{\text{gravity}} + L_{\text{coupling}}) = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right)$$

Let's start from the Newtonian non-relativistic Action

$$S = \int dt L = \int dt (L_{\text{matter}} + L_{\text{gravity}} + L_{\text{coupling}}) = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right)$$

Principle of Least Action:

$$\frac{\delta S}{\delta \Phi} = 0 \quad \rightarrow \quad \nabla^2 \Phi = 4\pi G \rho \quad \text{Poisson's equation}$$

$$\frac{\delta S}{\delta \vec{x}} = 0 \quad \rightarrow \quad \vec{a} = -\vec{\nabla} \Phi \quad \text{Newton's 2nd Law}$$

Let's start from the Newtonian non-relativistic Action

$$S = \int dt L = \int dt (L_{\text{matter}} + L_{\text{gravity}} + L_{\text{coupling}}) = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right)$$

Principle of Least Action:

Change this for modified inertia Change this for modified gravity Changing this modify both

$$\frac{\delta S}{\delta \Phi} = 0 \quad \rightarrow \quad \nabla^2 \Phi = 4\pi G \rho \quad \text{Poisson's equation}$$

$$\frac{\delta S}{\delta \vec{x}} = 0 \quad \rightarrow \quad \vec{a} = -\vec{\nabla} \Phi \quad \text{Newton's 2nd Law}$$

AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984)

$$S_N = \int dt L_N = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Lagrangian is quadratic in } \nabla \Phi$$

$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla} \Phi|^2}{a_0^2}\right) \quad \begin{array}{l} F(y) \rightarrow y \text{ for } y = |\nabla \Phi|^2/a_0^2 \gg 1 \\ F(y) \rightarrow y^{3/2} \text{ for } y = |\nabla \Phi|^2/a_0^2 \ll 1 \end{array}$$

AQUAL = Aquadratic Lagrangian (Bekenstein & Milgrom 1984)

$$S_N = \int dt L_N = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Lagrangian is quadratic in } \nabla \Phi$$

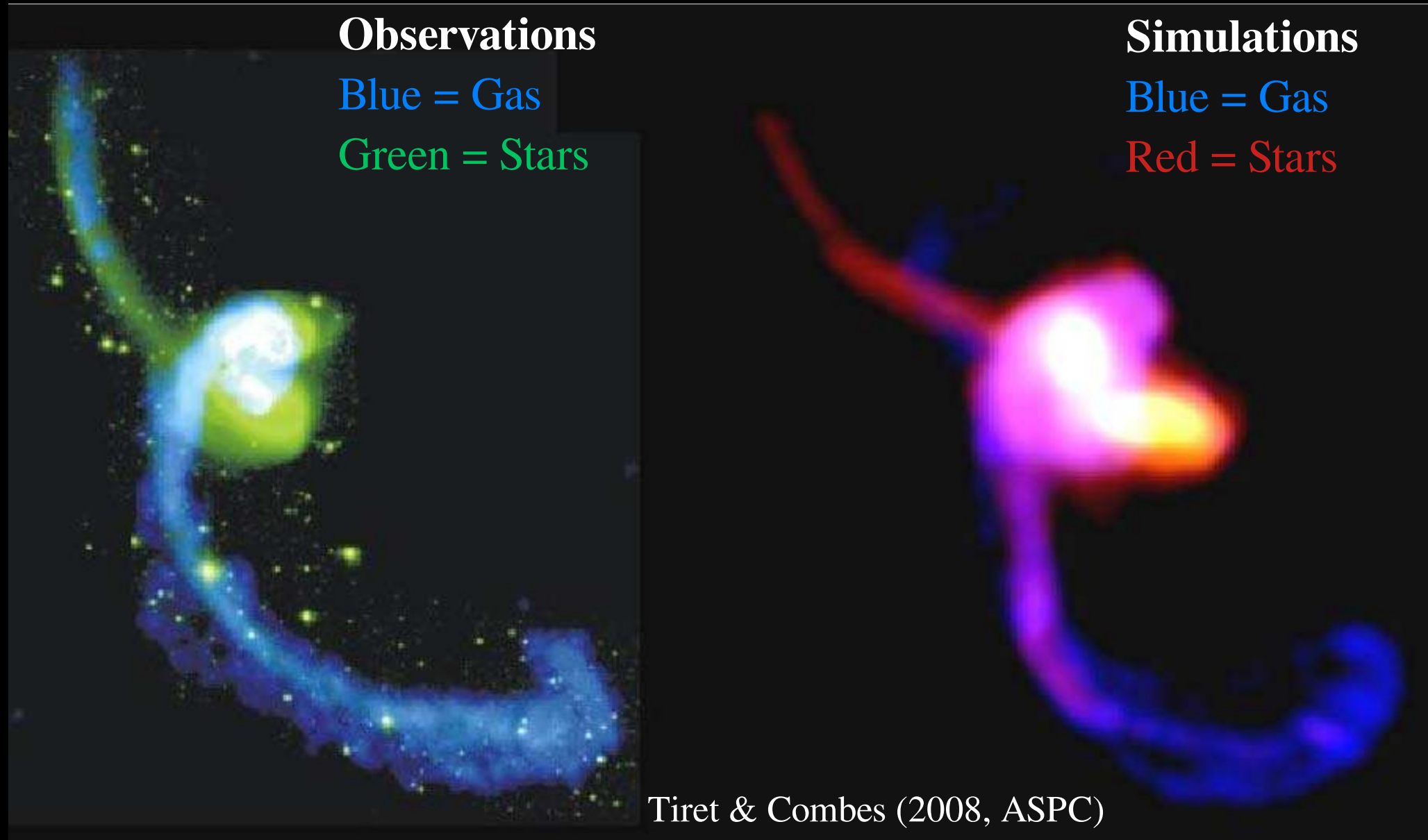
$$\downarrow$$

$$-\frac{a_0^2}{8\pi G} F\left(\frac{|\vec{\nabla} \Phi|^2}{a_0^2}\right) \quad \begin{array}{l} F(y) \rightarrow y \text{ for } y = |\nabla \Phi|^2/a_0^2 \gg 1 \\ F(y) \rightarrow y^{3/2} \text{ for } y = |\nabla \Phi|^2/a_0^2 \ll 1 \end{array}$$

$$\frac{\delta S}{\delta \Phi} = 0 \quad \rightarrow \quad \nabla \cdot \left[\mu \left(\frac{|\vec{\nabla} \Phi|}{a_0} \right) \vec{\nabla} \Phi \right] = 4\pi G \rho \quad \text{Modified Poisson's Equation}$$

$$\mu(x) = \frac{dF(y)}{dy} \quad y = x^2 \quad F(y) \text{ provides the interpolation function } \mu = v^{-1}, \text{ whose full shape is set empirically by the RAR}$$

Application of AQUAL: simulations of interacting galaxies



Lovelock-Grigore Theorem

GR (+ Λ) is the only theory that satisfy these assumptions:

- 1) Geometry is Riemannian
- 2) Action depends only on $g_{\mu\nu}$
- 3) It is diffeomorphism invariant
- 4) It is local
- 5) It leads to 2nd order field equations

Lovelock-Grigore Theorem

GR (+ Λ) is the only theory that satisfy these assumptions:

- 1) Geometry is Riemannian
- ~~2) Action depends only on $g_{\mu\nu}$~~ $\rightarrow$ add new dynamical fields
- 3) It is diffeomorphism invariant
- 4) It is local
- 5) It leads to 2nd order field equations

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

$$n_\mu = -\frac{c}{Q} \nabla_\mu \tau \quad Q = c \sqrt{-g^{\mu\nu} \nabla_\mu \tau \nabla_\nu \tau}$$

Dimensionless unit timelike ($n^\mu n_\mu = -1$) vector, pointing in the future ($n^0 > 0$).

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

$$n_\mu = -\frac{c}{Q} \nabla_\mu \tau \quad Q = c \sqrt{-g^{\mu\nu} \nabla_\mu \tau \nabla_\nu \tau} \quad \text{Dimensionless unit timelike } (n^\mu n_\mu = -1) \text{ vector, pointing in the future } (n^0 > 0).$$

$$A_\mu = c^2 n^\nu \nabla_\nu n_\mu \quad \text{Covariant spacelike acceleration} \quad Y = \frac{A_\mu A^\mu}{c^4} \quad \text{Squared “acceleration” with units of } 1/L^2$$

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (universally coupled to matter fields Ψ)

Scalar $\tau \rightarrow$ Khronon field (units of time; drive a foliation in 3D space-like hypersurfaces)

$$n_\mu = -\frac{c}{Q} \nabla_\mu \tau \quad Q = c \sqrt{-g^{\mu\nu} \nabla_\mu \tau \nabla_\nu \tau} \quad \text{Dimensionless unit timelike } (n^\mu n_\mu = -1) \text{ vector, pointing in the future } (n^0 > 0).$$

$$A_\mu = c^2 n^\nu \nabla_\nu n_\mu \quad \text{Covariant spacelike acceleration} \quad Y = \frac{A_\mu A^\mu}{c^4} \quad \text{Squared “acceleration” with units of } 1/L^2$$

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

Ricci Scalar
(as in GR)

Free function
to get MOND

Free function
to get cosmology

Matter
Action

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

- Cosmology: 3D space is homogeneous & isotropic $\rightarrow Y = 0$ & $Q = d\tau/dt$

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

- Cosmology: 3D space is homogeneous & isotropic $\rightarrow Y = 0$ & $Q = d\tau/dt$

$$K(Q) = \mu_Q^2 (Q - 1)^2 + \dots \quad \Rightarrow \quad \text{It gives dust-like behaviour: } \rho_K \propto a(t)^{-3} \text{ like CDM}$$

$\mu_Q = \text{constant with dimension of inverse length}$

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

- Cosmology: 3D space is homogeneous & isotropic $\rightarrow Y = 0$ & $Q = d\tau/dt$

$$K(Q) = \mu_Q^2 (Q - 1)^2 + \dots \quad \Longrightarrow \quad \text{It gives dust-like behaviour: } \rho_K \propto a(t)^{-3} \text{ like CDM}$$

$\mu_Q = \text{constant with dimension of inverse length}$

- Weak-field stationary limit: no time dependence $\rightarrow A_0 \sim 0; A_i \sim \nabla_i \Phi$

$$\nabla \cdot \left[\left(1 + \frac{dI}{dY} \right) \vec{\nabla} \Phi \right] + \mu_Q^2 \Phi = 4\pi G \rho_m \quad \Longrightarrow \quad \text{AQUAL} + \mu_Q^2 \Phi \quad \mu_Q^{-1} \gg 1 \text{ Mpc}$$

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

- Cosmology: 3D space is homogeneous & isotropic $\rightarrow Y = 0$ & $Q = d\tau/dt$

$$K(Q) = \mu_Q^2 (Q - 1)^2 + \dots \quad \Longrightarrow \quad \text{It gives dust-like behaviour: } \rho_K \propto a(t)^{-3} \text{ like CDM}$$

$\mu_Q = \text{constant with dimension of inverse length}$

- Weak-field stationary limit: no time dependence $\rightarrow A_0 \sim 0; A_i \sim \nabla_i \Phi$

$$\nabla \cdot \left[\underbrace{\left(1 + \frac{dI}{dY} \right)}_{\mu(y)} \vec{\nabla} \Phi \right] + \mu_Q^2 \Phi = 4\pi G \rho_m \quad \Longrightarrow \quad \Lambda_{\text{QUAL}} + \mu_Q^2 \Phi \quad \mu_Q^{-1} \gg 1 \text{ Mpc}$$

$\mu(y) = \mu(|\nabla \Phi|/a_0)$ $\nearrow$ for $y \ll 1$ $\mu(y) = y \Rightarrow I(Y) = \Lambda - Y + \frac{2c^2}{3a_0} Y^{3/2} + O\left(\frac{Y^4}{a_0^2}\right)$

$y = |\nabla \Phi|/a_0$

Khronon = Tensor – Scalar Theory (Blanchet & Skordis 2024)

$$S = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g} [R - 2I(Y) + 2K(Q)] + S_m[\Psi, g]$$

- Cosmology: 3D space is homogeneous & isotropic $\rightarrow Y = 0$ & $Q = d\tau/dt$

$$K(Q) = \mu_Q^2 (Q - 1)^2 + \dots \quad \Longrightarrow \quad \text{It gives dust-like behaviour: } \rho_K \propto a(t)^{-3} \text{ like CDM}$$

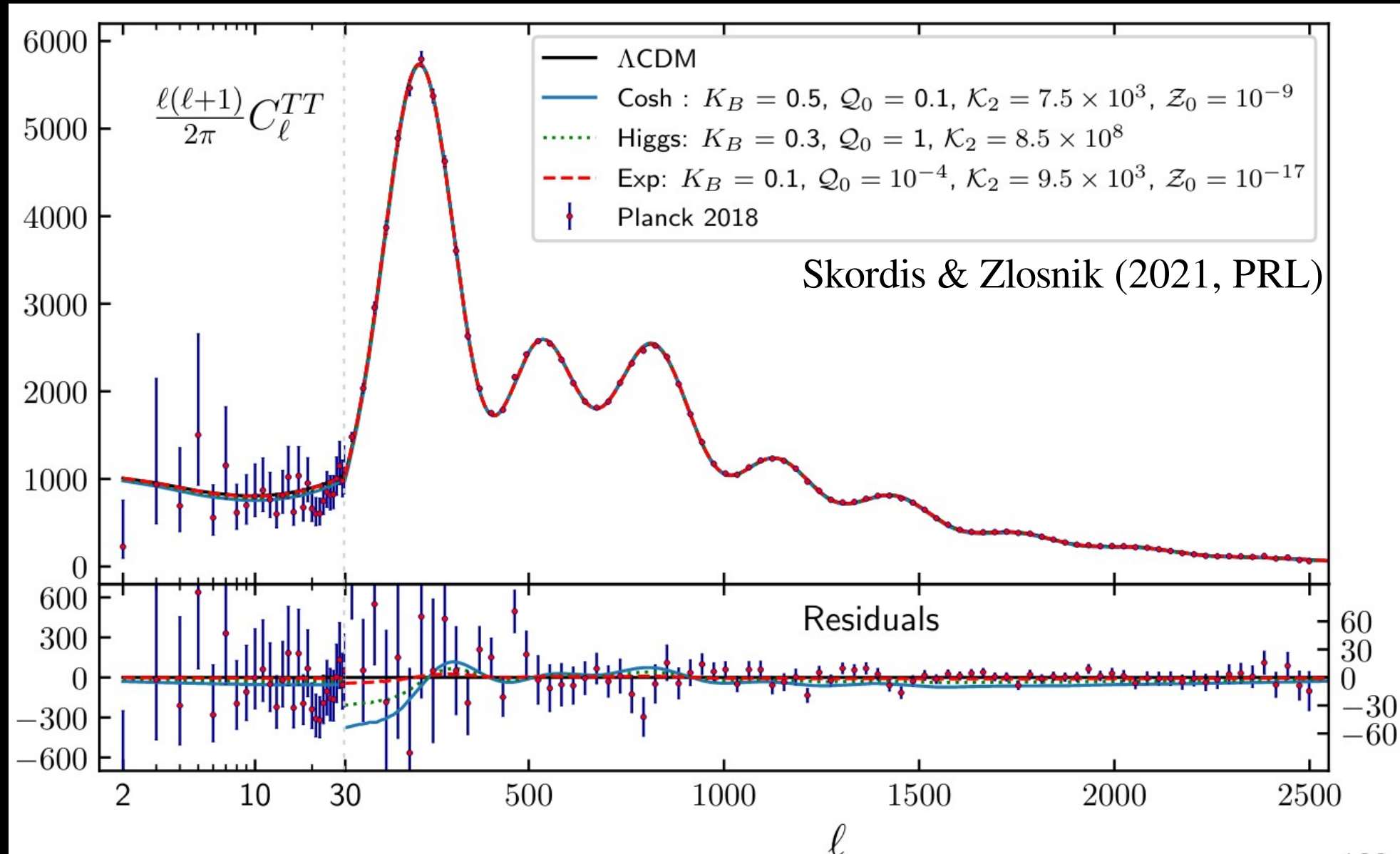
$\mu_Q = \text{constant with dimension of inverse length}$

- Weak-field stationary limit: no time dependence $\rightarrow A_0 \sim 0; A_i \sim \nabla_i \Phi$

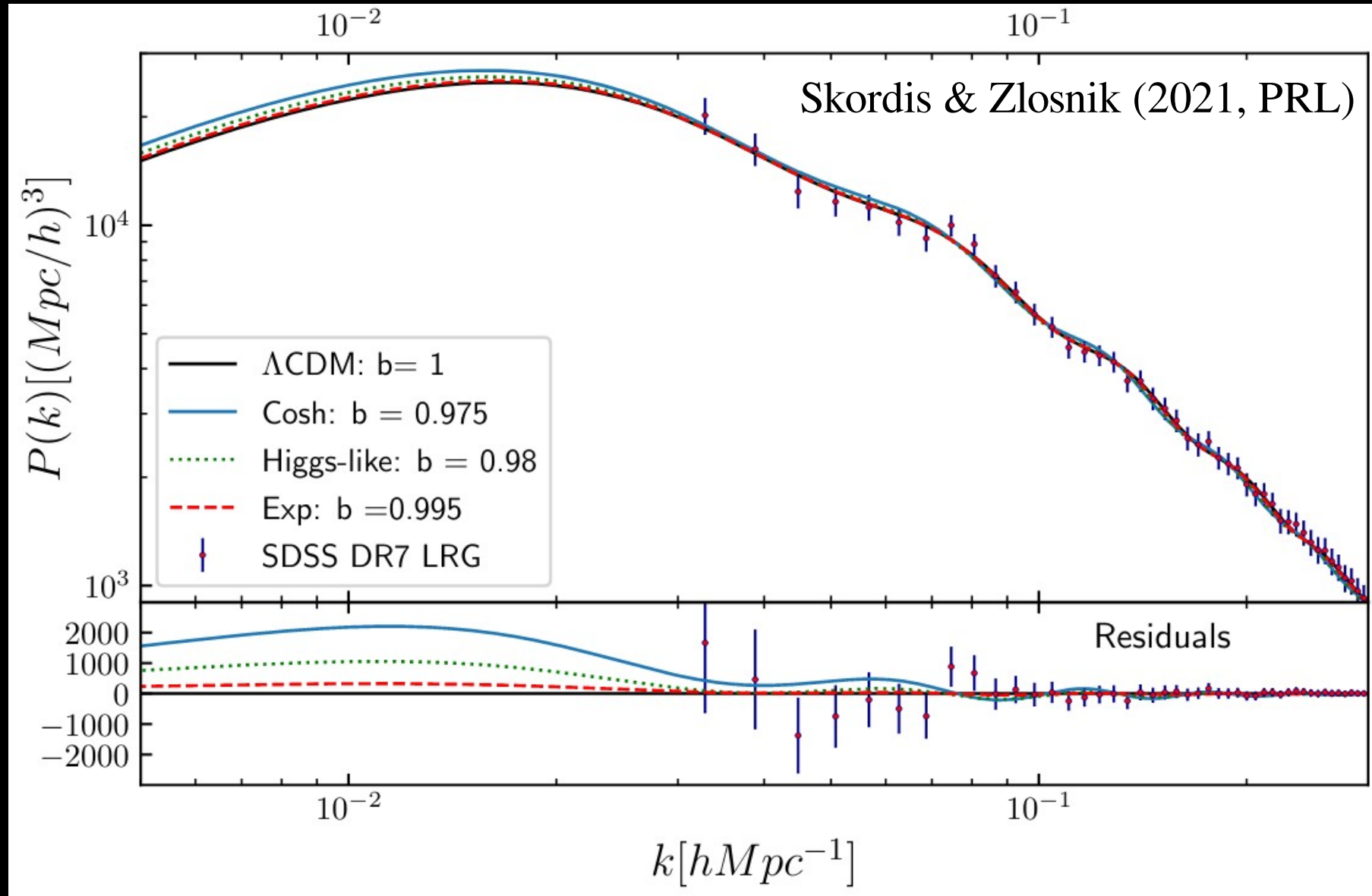
$$\nabla \cdot \left[\underbrace{\left(1 + \frac{dI}{dY} \right)}_{\mu(y)} \vec{\nabla} \Phi \right] + \mu_Q^2 \Phi = 4\pi G \rho_m \quad \Longrightarrow \quad \Lambda_{\text{QUAL}} + \mu_Q^2 \Phi \quad \mu_Q^{-1} \gg 1 \text{ Mpc}$$

$$\begin{aligned} \mu(y) = \mu(|\nabla \Phi|/a_0) & \begin{cases} \text{for } y \ll 1 & \mu(y) = y \Rightarrow I(Y) = \Lambda - Y + \frac{2c^2}{3a_0} Y^{3/2} + O\left(\frac{Y^4}{a_0^2}\right) \\ \text{for } y \gg 1 & \mu(y) = 1 \Rightarrow I(Y) = \Lambda_\infty + O(a_0/y) \end{cases} \\ y = |\nabla \Phi|/a_0 & \end{aligned}$$

CMB Power Spectrum (from AeST or Khronon)



Linear Matter Power Spectrum (from AeST or Khronon)



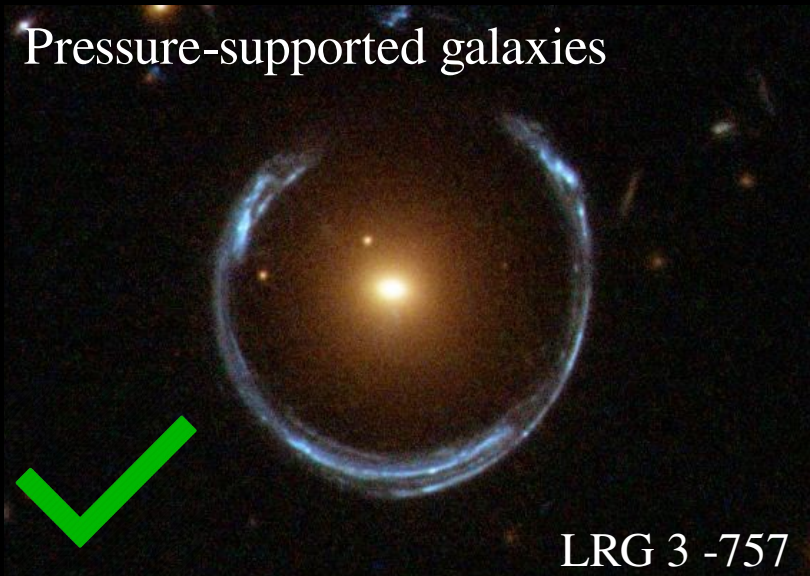
Summary: Status of MOND at Various Scales

Galaxy Scales (~1-100 kpc)

Rotation-supported galaxies

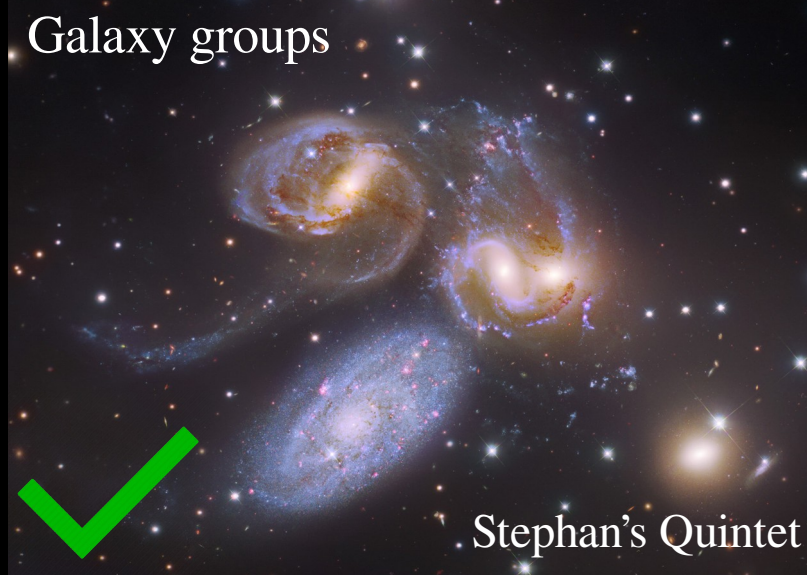


Pressure-supported galaxies

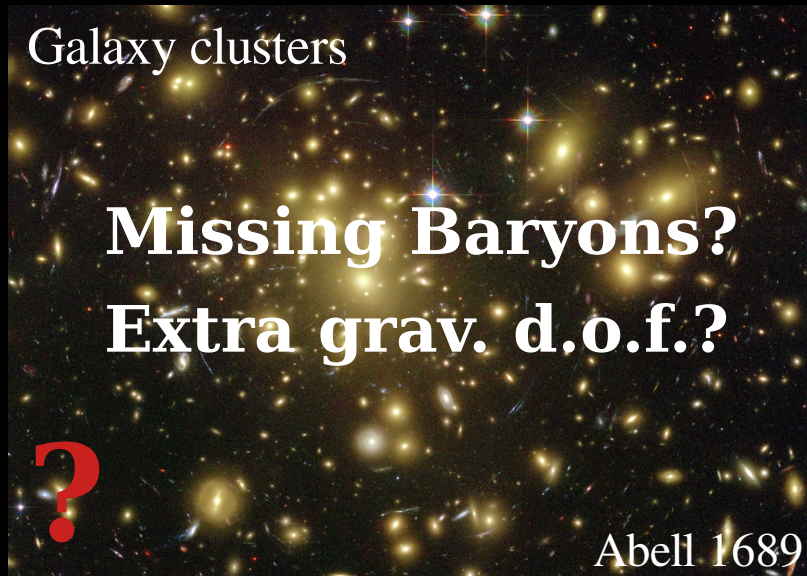


Groups/Clusters Scales (~1-5 Mpc)

Galaxy groups

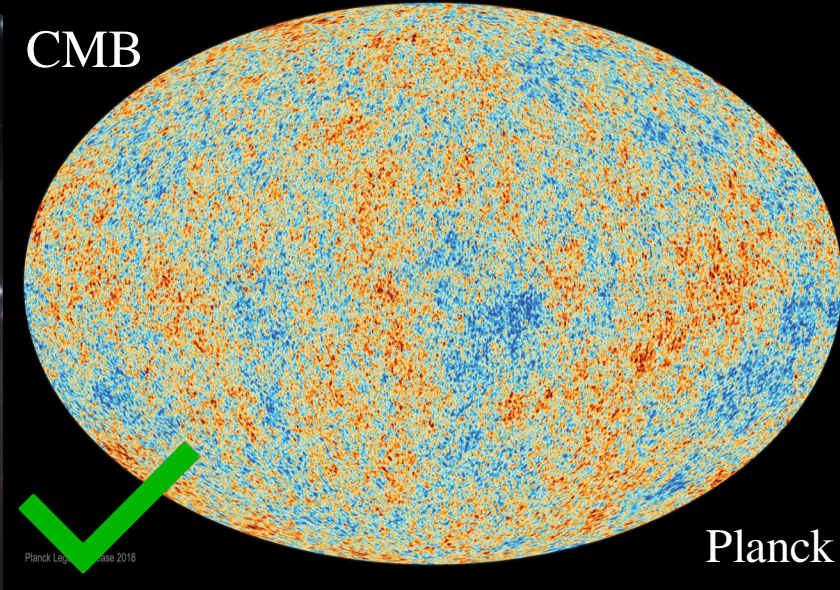


Galaxy clusters

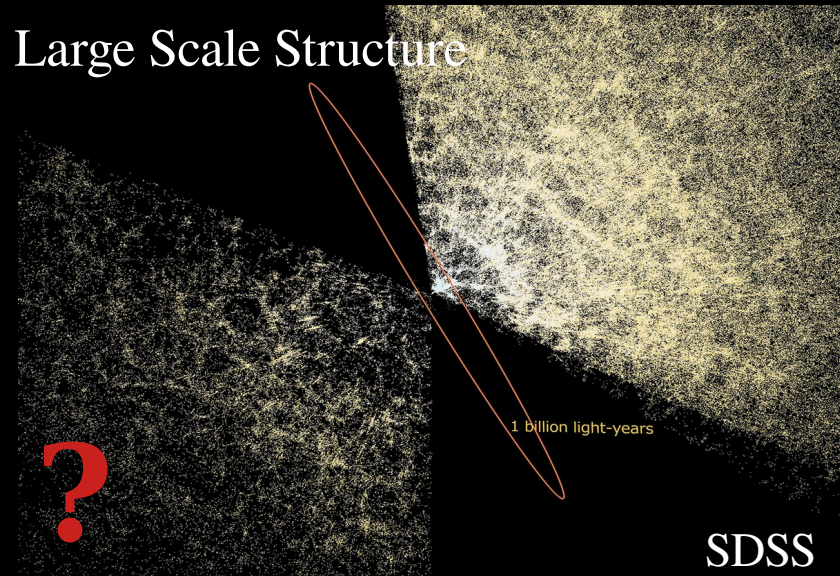


Cosmological Scales (>100 Mpc)

CMB



Large Scale Structure



The MOND white paper: coming soon!



MOND: Alternative Paths in the Dark Matter Problem

1 - 5 September 2025, Leiden, the Netherlands @omega



G
 a_0

Topics

- MOND theories and cosmology
- Tests in rotation-supported galaxies
- Tests in pressure-supported galaxies
- Tests in galaxy groups & galaxy clusters
- Tests on small scales: Solar System, binary stars, star clusters
- Numerical simulations of galaxy formation

Scientific Organizers

- Federico Lelli, INAF – Arcetri Astrophysical Observatory
- Luc Blanchet, Institut d'Astrophysique de Paris
- Erwin de Blok, ASTRON
- Francoise Combes, Observatoire de Paris
- Xavier Hernandez, Universidad Nacional Autonoma de Mexico

The Lorentz Center organizes international workshops for researchers in all scientific disciplines. Its aim is to create an atmosphere that fosters collaborative work, discussions and interactions. For registration see: www.lorentzcenter.nl

More Slides

Khronon Field Equations (Blanchet & Skordis 2024)

$$G^{\mu\nu} + (I - K) g^{\mu\nu} - \frac{2}{c^4} \frac{dI}{dY} A^\mu A^\nu + \left[\frac{2}{c^4} \nabla_\rho \left(\frac{dI}{dY} A^\rho \right) - Q \frac{dK}{dQ} \right] n^\mu n^\nu = \frac{8\pi G}{c^4} T^{\mu\nu}$$

$$T_\tau^{\mu\nu} \stackrel{\text{def}}{=} \frac{c^4}{8\pi G} \left[- (I - K) g^{\mu\nu} + \frac{2}{c^4} \frac{dI}{dY} A^\mu A^\nu - \left[\frac{2}{c^4} \nabla_\rho \left(\frac{dI}{dY} A^\rho \right) - Q \frac{dK}{dQ} \right] n^\mu n^\nu \right]$$

The Khronon stress-energy tensor contains $g_{\mu\nu}$ explicitly and implicitly in Y and Q .

This is very different from adding DM to GR!

AeST = **Aether** - **Scalar** - **Tensor** (Skordis & Zlosnik 2021)

Vector $A^\mu \rightarrow$ **timelike**, first introduced by Sanders (1997) to get gravitational lensing

Scalar $\varphi \rightarrow$ first introduced by Bekenstein & Milgrom (1984) to get NR AQUAL

Tensor $g_{\mu\nu} \rightarrow$ Einstein's metric (as usual)

AeST = **A**ether - **S**calar - **T**ensor (Skordis & Zlosnik 2021)

$Q \equiv A^\mu \nabla_\mu \varphi$ Kinetic term of φ along the A^μ direction (“time”)

$Y \equiv (g^{\mu\nu} + A^\mu A^\nu) \nabla_\mu \varphi \nabla_\nu \varphi$ Kinetic term of φ perpendicular to A^μ (“space”)

$J^\mu \equiv A^\alpha (\nabla_\alpha A^\mu)$ Gradient of A^μ projected on A^μ

AeST = **A**ether - **S**calar - **T**ensor (Skordis & Zlosnik 2021)

$Q \equiv A^\mu \nabla_\mu \varphi$ Kinetic term of φ along the A^μ direction (“time”)

$Y \equiv (g^{\mu\nu} + A^\mu A^\nu) \nabla_\mu \varphi \nabla_\nu \varphi$ Kinetic term of φ perpendicular to A^μ (“space”)

$J^\mu \equiv A^\alpha (\nabla_\alpha A^\mu)$ Gradient of A^μ projected on A^μ

$$\mathcal{L}_{grav} = \frac{\sqrt{-g}}{16\pi\tilde{G}} \left[\underset{\downarrow}{R} - \frac{K_B}{2} \underset{\downarrow}{F^{\mu\nu}} \underset{\downarrow}{F_{\mu\nu}} - \underset{\downarrow}{\lambda} (A^\mu A_\mu + 1) + (2 - K_B) 2 J^\mu \nabla_\mu \varphi - (2 - K_B) F(Y, Q) \right]$$

Ricci scalar
EM-like
 A^μ is unit time-like
Coupling constant
Free function

AeST = **A**ether - **S**calar - **T**ensor (Skordis & Zlosnik 2021)

$Q \equiv A^\mu \nabla_\mu \varphi$ Kinetic term of φ along the A^μ direction (“time”)

$Y \equiv (g^{\mu\nu} + A^\mu A^\nu) \nabla_\mu \varphi \nabla_\nu \varphi$ Kinetic term of φ perpendicular to A^μ (“space”)

- Cosmology (homogeneous & isotropic spacetime):

$F(Y=0, Q) = F_Q = 4\Lambda - 2K_2(Q - Q_0)^2 + \dots \rightarrow$ gives CDM-like behaviour: $\rho_Q \propto a(t)^{-3}$

AeST = **A**ether - **S**calar - **T**ensor (Skordis & Zlosnik 2021)

$Q \equiv \mathbf{A}^\mu \nabla_\mu \varphi$ Kinetic term of φ along the $\mathbf{A}^\mu$ direction (“time”)

$Y \equiv (g^{\mu\nu} + \mathbf{A}^\mu \mathbf{A}^\nu) \nabla_\mu \varphi \nabla_\nu \varphi$ Kinetic term of φ perpendicular to $\mathbf{A}^\mu$ (“space”)

- Cosmology (homogeneous & isotropic spacetime):

$$F(Y=0, Q) = F_Q = 4\Lambda - 2K_2(Q - Q_0)^2 + \dots \rightarrow \text{gives CDM-like behaviour: } \rho_Q \propto a(t)^{-3}$$

- Weak-field quasi-static limit (no time dependence):

$$F(Y, Q=Q_0) = F_Y \begin{cases} \rightarrow (2 - K_B)\lambda_s Y + \dots \rightarrow \text{gives Newton for } Y \rightarrow \infty \\ \rightarrow \frac{(2 - K_B)2\lambda_s}{3(1 + \lambda_s)} \frac{Y^{3/2}}{a_0} + O(Y^2) \rightarrow \text{gives MOND for } Y \rightarrow 0 \end{cases}$$

QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S_N = \int dt L_N = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Single gravitational potential } \Phi$$

$$\frac{-1}{8\pi G} \left[2 \vec{\nabla} \Phi \cdot \vec{\nabla} \Phi_N - a_0^2 Q \left(\frac{|\vec{\nabla} \Phi_N|^2}{a_0^2} \right) \right] \quad \text{Two potentials: } \Phi \text{ and } \Phi_N!$$

QUMOND = Quasi-Linear MOND (Milgrom 2010, MNRAS)

$$S_N = \int dt L_N = \int dt d^3x \left(\rho \frac{V^2}{2} - \frac{|\vec{\nabla} \Phi|^2}{8\pi G} - \rho \Phi \right) \quad \text{Single gravitational potential } \Phi$$

$$\frac{-1}{8\pi G} \left[2 \vec{\nabla} \Phi \cdot \vec{\nabla} \Phi_N - a_0^2 Q \left(\frac{|\vec{\nabla} \Phi_N|^2}{a_0^2} \right) \right] \quad \text{Two potentials: } \Phi \text{ and } \Phi_N!$$

Principle of least Action varying Φ , Φ_N and $\vec{x} \rightarrow$ set of 3 equations

$\nabla^2 \Phi_N = 4\pi G \rho \longrightarrow$ Standard, linear Poisson's equation for Φ_N

$\nabla^2 \Phi = \vec{\nabla} \cdot \left[v \left(|\vec{\nabla} \Phi_N| / a_0 \right) \vec{\nabla} \Phi_N \right] \longrightarrow$ Non-linear step: get Φ from Φ_N $v(\sqrt{x}) = \frac{dQ(x)}{dx}$

$\vec{a} = -\vec{\nabla} \Phi \longrightarrow$ Acceleration/force set by second potential Φ

MOND as Modified Inertia (Milgrom 1994, 1999, 2022)

$$\vec{A}[\vec{x}(t); a_0] = -\vec{\nabla} \Phi_N \quad \bar{A} \text{ is a functional of the full trajectory } \bar{x}(t)$$

For $a \gg a_0$, $A \rightarrow a = d^2x/dt^2$ (Newton's 2nd law)

No full theory yet, but two general results:

(1) Imposing Newtonian and MOND limits + Galilei Invariance $\vec{x}(t) \rightarrow \vec{x}(t) + \vec{v}_0 t$

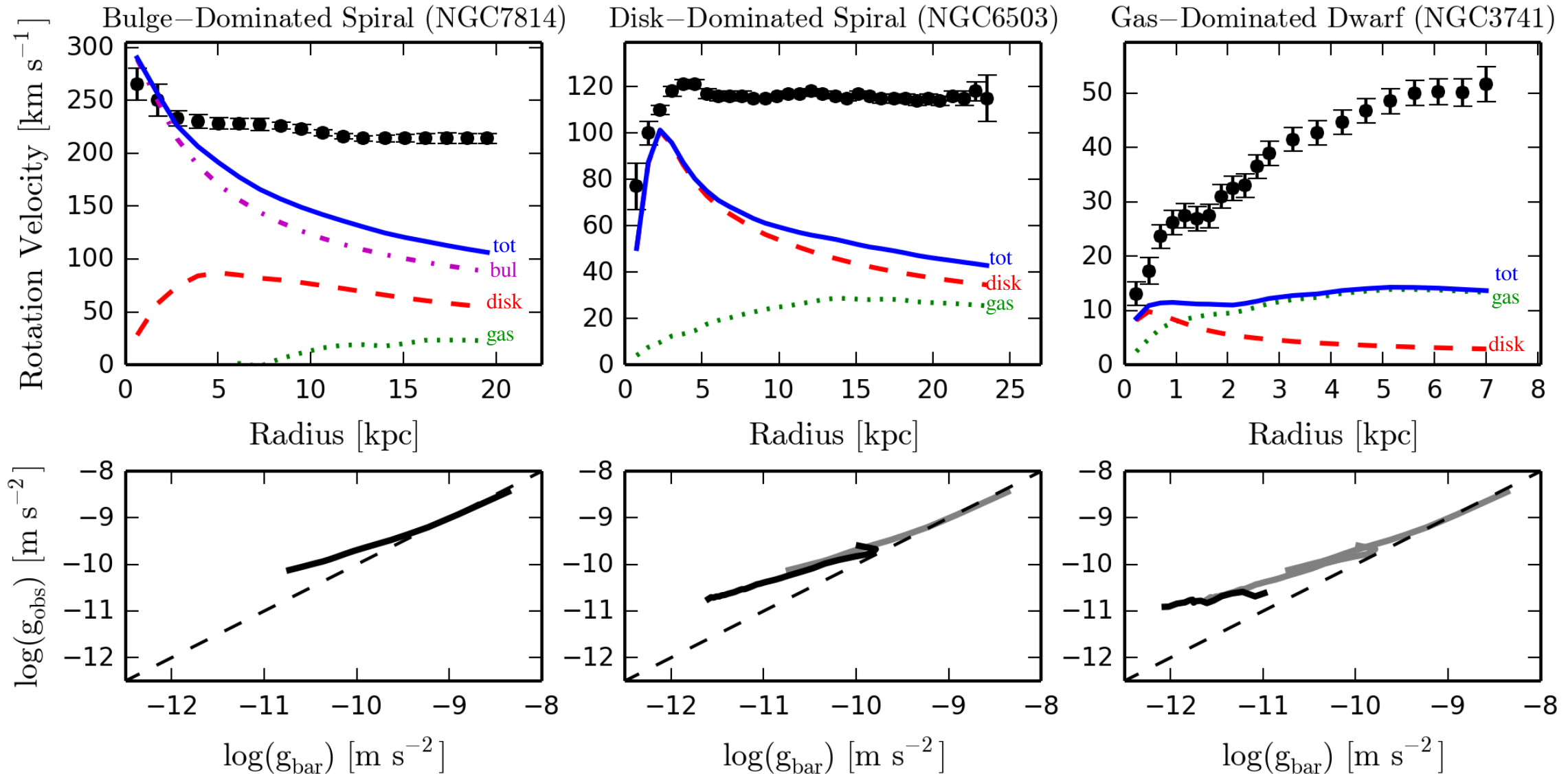
Theory is **time non-local**: $\vec{A}[\vec{x}(t), a_0] \neq F\left(\frac{d^i \vec{x}}{dt^i}; i=1, 2, \dots, N\right)$

Accelerations at $(\bar{x}, t)$ depend on the **full orbital history**!

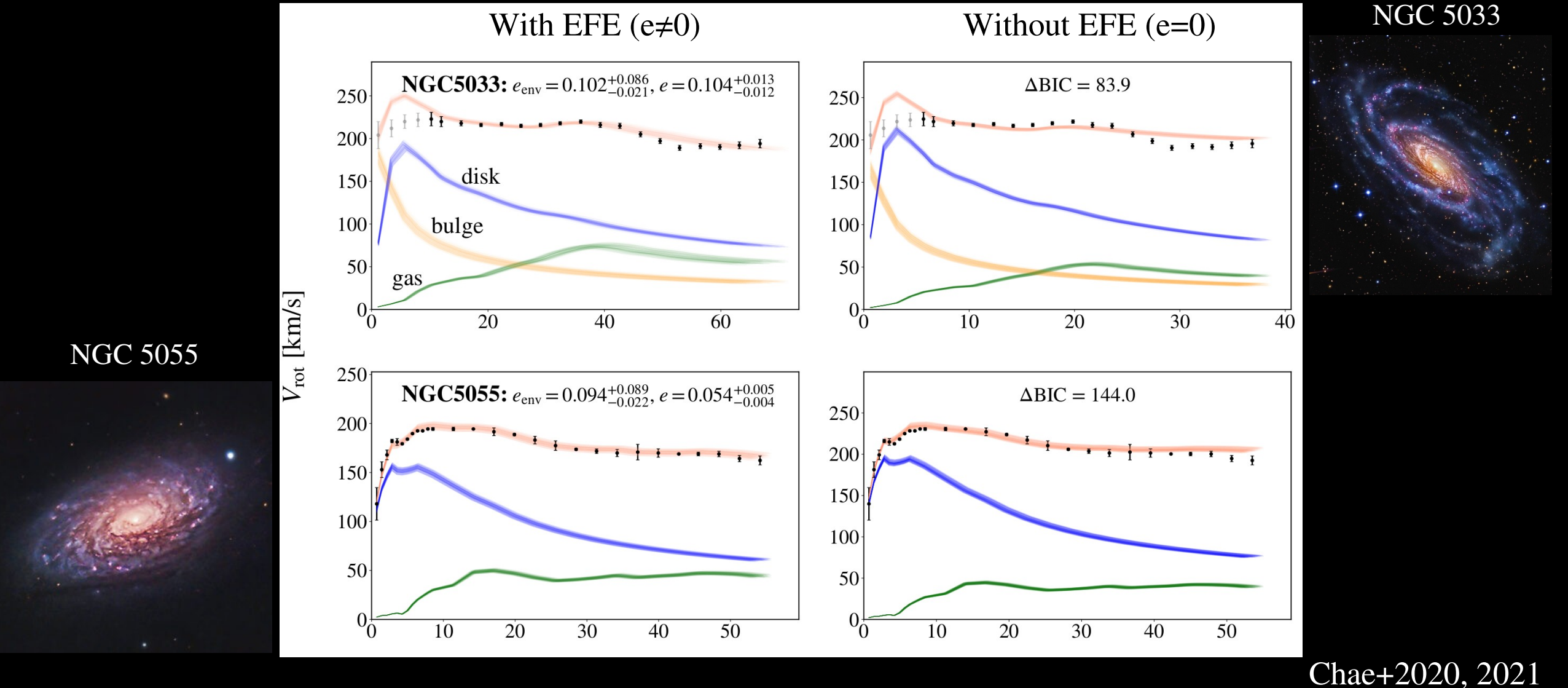
(2) For purely circular orbits: $\vec{a} \mu\left(\frac{a}{a_0}\right) = \vec{g}_N$ holds exactly (RAR for disk galaxies)

The **interpolation function** is a **derived concept** valid for circular orbits!

Galaxies lie on the same RAR *despite* their diversity

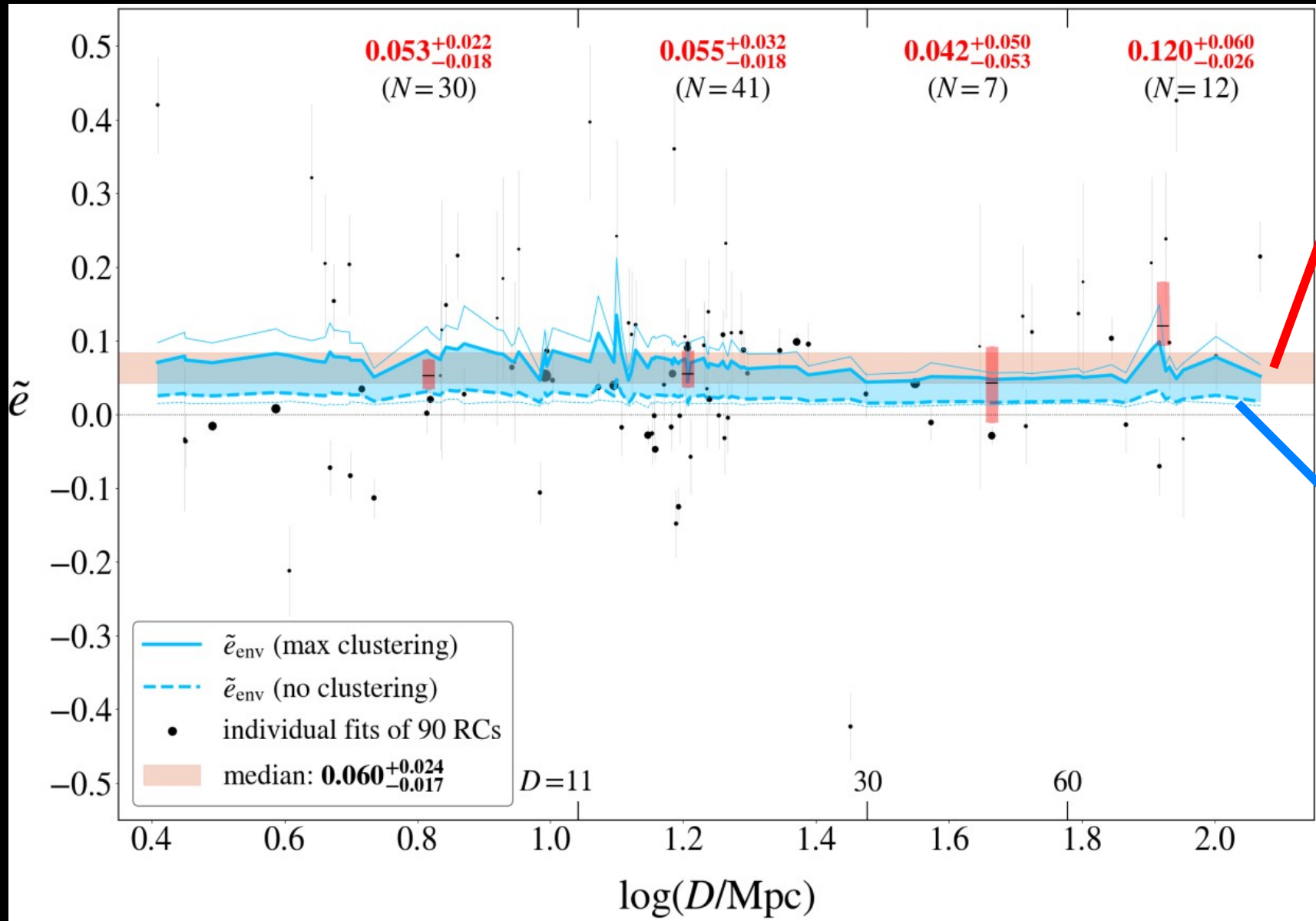


EFE is weak: individual detections only in extreme cases



Statistical approach: $EFE > 0$ at $>4\sigma$ and agrees with LSS

$$\frac{g_{ext}}{a_0} \leftarrow \tilde{e}$$

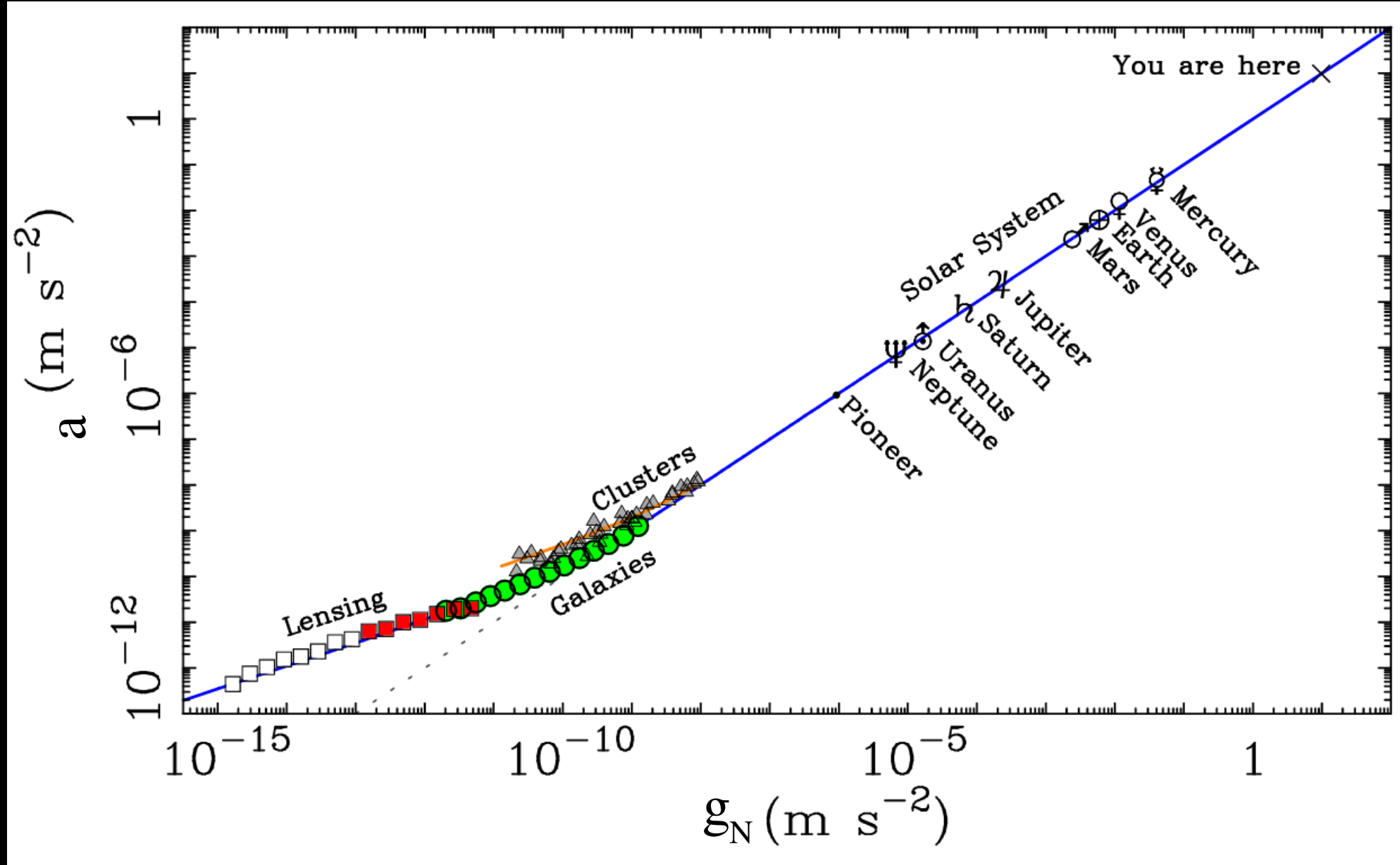


From Rotation
Curve Fits

From Baryon
Large Scale
Structure

Chae+2020, 2021

Galaxy Clusters on the Radial Acceleration Relation



Nature of the missing mass in galaxy clusters?

- **Dark baryons** such as compact clouds of cold gas (Milgrom 2008)

For HI clouds ($T \sim 10^4$ K) $\rightarrow M_{\text{cl}} < 10^5 M_{\odot}$ and $R_{\text{cl}} < 50$ pc

(Kelleher & Lelli 2024)

Below current HI detection limits; SKA may detect them.

- **Sterile neutrinos** of ~ 10 eV (Angus 2008, 2009) but CMB power spectrum may be problematic (Thomas+2016, Kopp+2018, Ilic+2021)

MOND – Cosmology Connection?

Two numerical coincidences (Milgrom 1983a, ApJ; Milgrom 1999, PhLA):

$$a_0 \simeq \frac{H_0 \cdot c}{2\pi} \quad H_0 = \text{Hubble constant} \rightarrow \text{maybe } a_0(t) \sim H(t) ?$$

$$a_0 \simeq \frac{c^2 \sqrt{\Lambda/3}}{2\pi} \quad \Lambda = \text{Cosmological constant} \rightarrow \text{relation to Dark Energy?}$$

IF this numerology has some deeper, fundamental meaning:

Either the state of the Universe at large enters in local dynamics, or the same parameters enters both Cosmology (Λ) and local dynamics (a_0).

Cosmology with modified Newtonian dynamics (MOND)

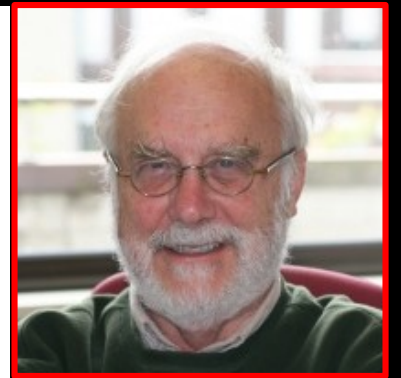
R. H. Sanders

Kapteyn Astronomical Institute, Groningen, The Netherlands

Accepted 1998 January 13. Received 1997 December 22; in original form 1997 October 17

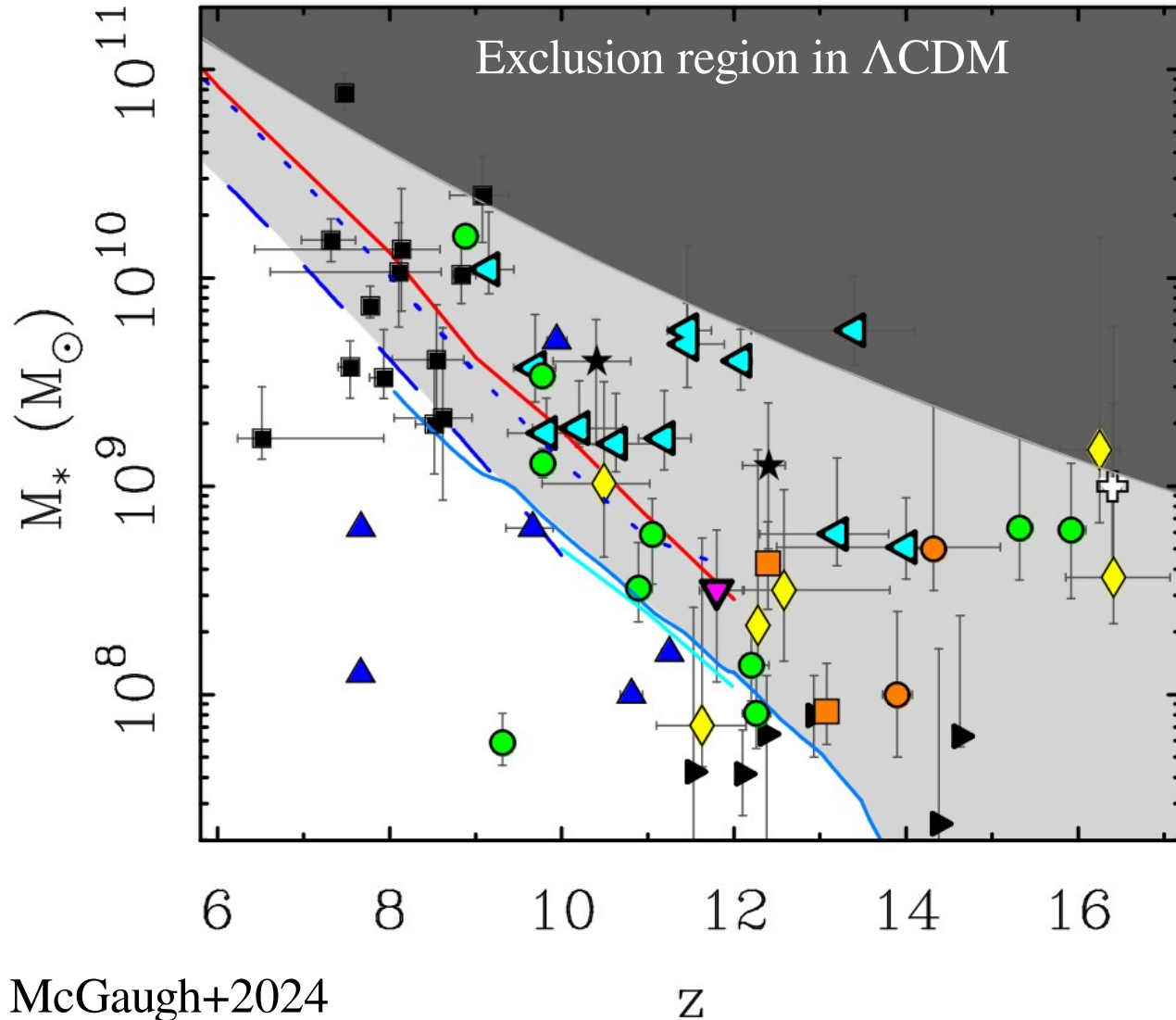
ABSTRACT

It is well known that the application of Newtonian dynamics to an expanding spherical region leads to the correct relativistic expression (the Friedmann equation) for the evolution of the cosmic scalefactor. Here, the cosmological implications of Milgrom's modified Newtonian dynamics (MOND) are considered by means of a similar procedure. Earlier work by Felten demonstrated that in a region dominated by modified dynamics the expansion cannot be uniform (separations cannot be expressed in terms of a scalefactor) and that any such region will eventually recollapse regardless of the initial expansion velocity and mean density. Here I show that, because of the acceleration threshold for the MOND phenomenology, a region dominated by MOND will have a finite size which, in the earlier Universe ($z > 3$), is smaller than the horizon scale. Therefore, uniform expansion and homogeneity on the horizon scale are consistent with MOND-dominated non-uniform expansion and the development of inhomogeneities on smaller scales. In the radiation-dominated era, the amplitude of MOND-induced inhomogeneities is much smaller than that implied by observations of the cosmic background radiation, and the thermal and dynamical history of the Universe is identical to that of the standard big bang model. In particular, the standard results for primordial nucleosynthesis are retained. When matter first dominates the energy density of the Universe, the cosmology diverges from that of the standard model. Objects of galaxy mass are the first virialized objects to form (by $z = 10$), and larger structure develops rapidly. At present, the Universe would be inhomogeneous out to a substantial fraction of the Hubble radius.



Massive galaxies at $z > 10$ is the new normal!

Compilation of JWST observations



Galaxy formation & evolution happens earlier and faster than anticipated in Λ CDM

- Bright, massive galaxies at $z \gtrsim 10$
- Passive galaxies (without star formation) at $z \gtrsim 4$ and inferred formation at $z \gtrsim 10$
- Regularly rotating galaxy disks at $z \gtrsim 5$
- Disk structures (bars, spiral arms) at $z \gtrsim 3$
- Massive galaxy clusters at $z \gtrsim 3$ and protocluster candidates at $z \gtrsim 6$
- $\text{Ly}\alpha$ emission at $z \gtrsim 10$ (early reionization)

See, e.g., McGaugh+2024 for a summary

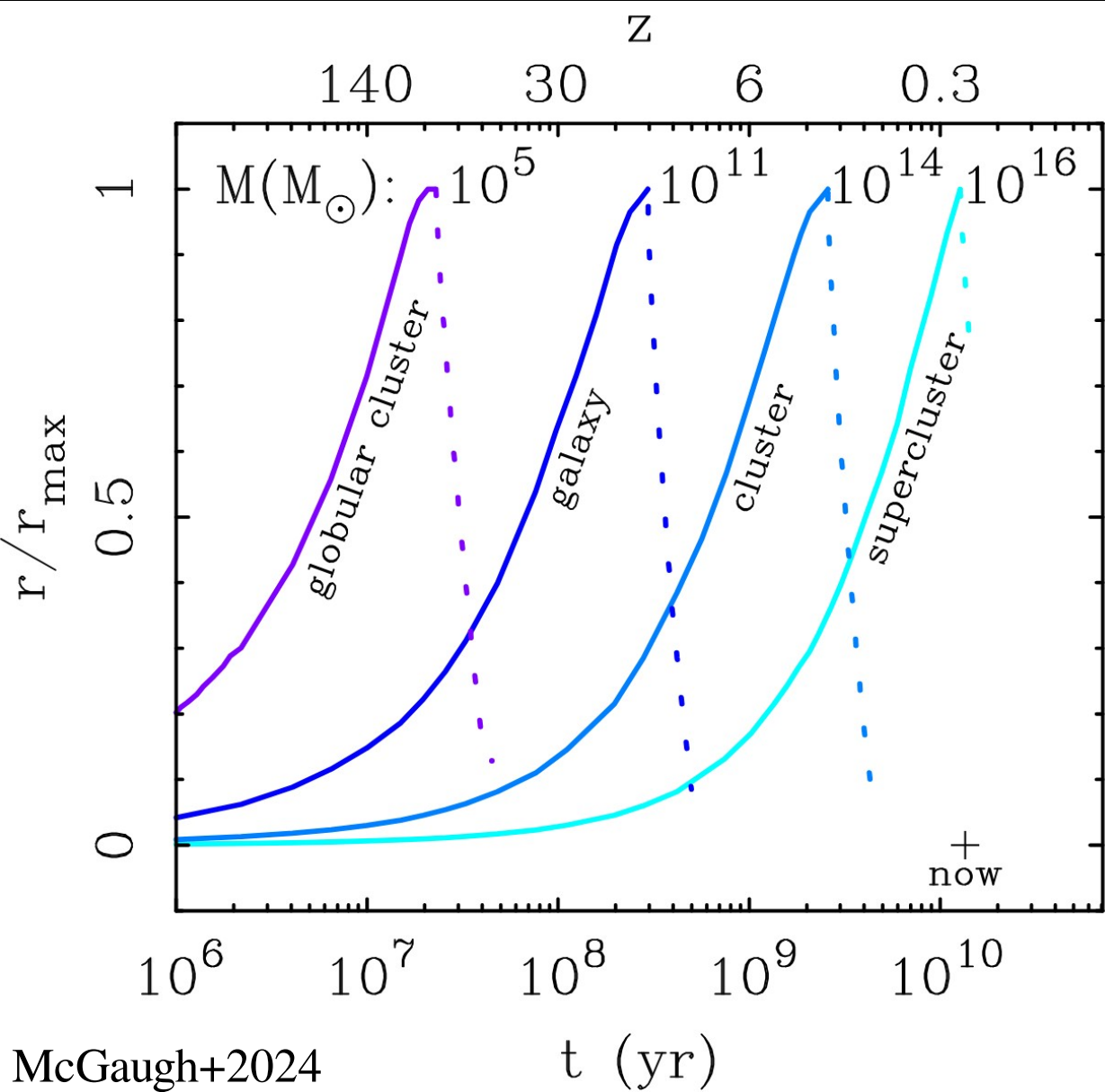
MOND prediction, following Sanders (1998)

Evolution of a MOND spherical region in an expanding Universe (Felten 1984):

$$V^2(t) = V_i^2 - \sqrt{\frac{16\pi}{3} a_0 G \rho_b r_0^3} \ln \left(\frac{r(t)}{r_i} \right)$$

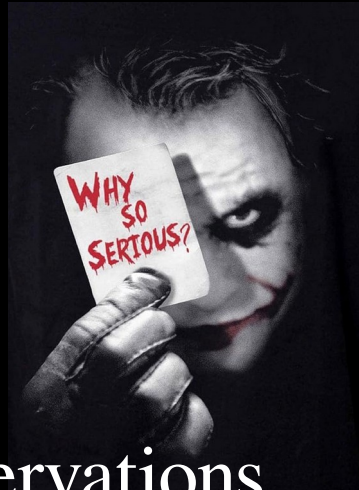
Any spherical region will initially expand, then unavoidably recollapse due to gravity:

- Structures form faster than in Λ CDM
- Late-time Universe is anisotropic (or homogeneity scale is very large)



McGaugh+2024

Why taking MOND seriously?



- MOND made **multiple *a-priori* predictions**, later confirmed by observations. **A-priori predictions are the core of the scientific method.**
- Even if MOND is wrong (or DM is detected), there must be a **baryon – DM coupling in galaxies acting in a MOND way**. It tells us on the nature of DM.
- Over the past ~10 years, **substantial progress in MOND research** in both observations and theory (new tests, new ideas, new relativistic theories).