Cosmological Inflation and Modified Gravity

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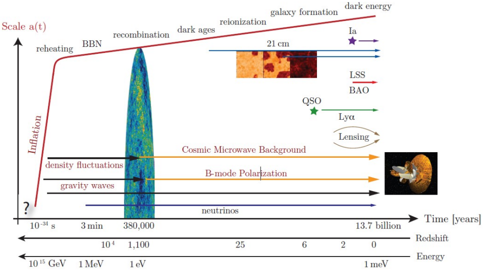
Motivation

- General Relativity is the best theory to describe gravitational interactions on large scales.
- Why modify the description of gravity?
- Attempts to combine GR with quantum field theory: Quantum Gravity?
- The attempts to combine GR with quantum theory yields a nonrenormalizable quantum field theory.
- Consistent theory: String Theory.

Evolution of the Universe

- From 10^{-10} seconds to today the history of the universe is based on well understood and experimentally tested laws of particle physics (SM), nuclear and atomic physics and gravity.
- What about the evolution before 10^{-10} seconds?

	Time	Energy	
Planck Epoch?	$< 10^{-43}$ s	10^{18} GeV	
String Scale?	$\gtrsim 10^{-43}$ s	$\lesssim 10^{18}$ GeV	
Grand Unification?	$\sim 10^{-36}$ s	10^{15} GeV	
Inflation?	$\gtrsim 10^{-34}$ s	$\lesssim 10^{15}$ GeV	
SUSY Breaking?	$< 10^{-10}$ s	> 1 TeV	
Baryogenesis?	$< 10^{-10}$ s	> 1 TeV	
Electroweak Unification	10^{-10} s	1 TeV	
Quark-Hadron Transition	10^{-4} s	10^{2} MeV	
Nucleon Freeze-Out	0.01 s	10 MeV	
Neutrino Decoupling	1 s	1 MeV	
BBN	3 min	0.1 MeV	
			Redshift
Matter-Radiation Equality	10^{4} yrs	1 eV	10^{4}
Recombination	10^{5} yrs	0.1 eV	1,100
Dark Ages	$10^{5} - 10^{8}$ yrs		> 25
Reionization	10^{8} yrs		$25 - 6$
Galaxy Formation	$\sim 6 \times 10^{8}$ yrs		~ 10
Dark Energy	$\sim 10^{9}$ yrs		~ 2
Solar System	8×10^{9} yrs		0.5
Albert Einstein born	14×10^{9} yrs	1 meV	0



Inflation Dynamics

- The simplest models of inflation involve a single scalar field ϕ, the inflaton:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

- From the Eulero-Lagrange equations one obtains the equation of motion in a FLRW universe:

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = 0$$

- The energy momentum tensor is:

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \quad \rightarrow \quad \text{(perfect fluid)} \begin{cases} T_{00} = \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \\ \frac{T_{ij}}{3} = P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \end{cases}$$

If $V(\phi) \gg \dot{\phi}^2$: $P_\phi \simeq -\rho_\phi$.
Therefore, inflation is driven by the potential energy of the inflaton field.

Cosmological Starobinsky (R^2) Inflation

- The original Starobinsky cosmological model demonstrated that gravitational higher-order corrections could drive a successful inflationary phase.
- Extension of the Einstein-Hilbert gravity theory by a term quadratic in the Ricci scalar curvature R:

$$S_{\text{Staro}} = \int d^4x \sqrt{-g}\, \frac{M_p^2}{2} \left[R + \frac{R^2}{6m^2} \right]$$
$$= \int d^4x \sqrt{-g}\, \frac{M_p^2}{2} \left[(1-\chi)\, R - \frac{3}{2} m^2 \chi^2 \right]$$

- Weyl rescaling: $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\phi(x)} g_{\mu\nu}$
- Einstein gravity coupled to a scalar field:

$$S[g_{\mu\nu,\phi}] = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - V(\phi) \right]$$

$$V(\phi) = \frac{3}{4} M_p^2 m^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \phi / M_p \right) \right]^2$$

Slow-roll Conditions/Parameters

- In order to have a period of inflation, i.e. a period of expansion, the condition for inflation $\rho + 3P < 0$ requires that $\dot{\phi}^2 \ll V(\phi)$. This is the so-called *first slow-roll condition*.
- In order to have a suitable phase of inflation, one must require that the kinetic term in the inflaton equation, $\ddot{\phi}$, is negligible compared to the Hubble friction term ($3H\dot{\phi}$) and the potential term. One has a *second slow-roll condition*:

$$\left|\ddot{\phi}\right| \ll \left|3H\dot{\phi}\right| \qquad , \qquad \left|\ddot{\phi}\right| \ll \left|V'(\phi)\right|$$

- It is possible to translate the same conditions into two corresponding dimensionless *slow-roll parameters*:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad , \qquad \eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}$$

The smallness of the first-one ensures the realization of the accelerated phase, while the smallness of the second one ensures that the accelerated phase lasts long enough to sufficiently stretch the universe in a way that is compatible with observations.

Modified Gravity

- Natural modification of gravity: connection and metric independent. The difference between an arbitrary connection and the Levi-Civita connection is the so called Distorsion tensor:

$$C_\mu{}^\rho{}_\sigma \equiv A_\mu{}^\rho{}_\sigma - \Gamma(g)_\mu{}^\rho{}_\sigma$$

- Torsion is defined in terms of the Distorsion $T_{\mu\nu\rho} \equiv C_{\mu\nu\rho} - C_{\rho\nu\mu}$.
- The curvature associated with $A_\mu{}^\rho{}_\sigma$ is:

$$\begin{aligned}\mathcal{R}_{\mu\nu}{}^\rho{}_\sigma &\equiv \partial_\mu A_\nu{}^\rho{}_\sigma - \partial_\nu A_\mu{}^\rho{}_\sigma + A_\mu{}^\rho{}_\lambda A_\nu{}^\lambda{}_\sigma - A_\nu{}^\rho{}_\lambda A_\mu{}^\lambda{}_\sigma \\ &= R_{\mu\nu}{}^\rho{}_\sigma + D_\mu C_\nu{}^\rho{}_\sigma - D_\nu C_\mu{}^\rho{}_\sigma + C_\mu{}^\rho{}_\lambda C_\nu{}^\lambda{}_\sigma - C_\nu{}^\rho{}_\lambda C_\mu{}^\lambda{}_\sigma\end{aligned}$$

- The curvature tensor can be contracted:

$$\mathcal{R} \equiv \mathcal{R}_{\mu\nu}{}^{\mu\nu} \quad , \quad \mathcal{R}' \equiv \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\,\mathcal{R}_{\mu\nu\rho\sigma}$$

$\mathcal{R}' = 0$ for $C_\mu{}^\rho{}_\sigma = 0$, i.e., when the connection is the distorsionless Levi-Civita one.

Einstein-Cartan Gravity

- The connection has torsion but is metric compatible ($\nabla g = 0$).
- We consider a class of models given by an action of the form:

$$S[g_{\mu\nu}, \Phi] = \int d^4x \sqrt{-g}\,(\alpha(\Phi)\mathcal{R} + \beta(\Phi)\mathcal{R}' + \Delta(\Phi, \mathcal{R}, \mathcal{R}') + \Sigma(\Phi, \mathcal{D}\Phi, C))$$

where $\Phi = \{g_{\mu\nu}, \phi, \psi, ...\}$, represents the set of fields that are independent of $C_\mu{}^\rho{}_\sigma$.

Δ is an arbitrary function of the indicated fields and curvatures that brings the non-linear terms, and is crucial to have a dynamical distorsion.

Σ is a quantity that contains the matter fields.

Einstein-Cartan Gravity

- The action S can be equivalently written by introducing one auxiliary scalar field z

$$S = \int d^4x \left[\alpha(\Phi)\mathcal{R} + \left(\beta(\Phi) + \frac{\partial \Delta}{\partial z}\right)\mathcal{R}' + \left(\Delta(z) - z\frac{\partial \Delta}{\partial z}\right)\right]$$

  The field z is called pseudoscalaron and is important since it corresponds to a degree of freedom coming from the distorsion because the field equation of z is $(z - \mathcal{R}')\frac{\partial^2 \Delta}{\partial z^2} = 0$ where

$$\mathcal{R}' = \frac{2}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\left(D_\mu C_{\nu\rho\sigma} + C_{\mu\rho\lambda} C_\nu{}^\lambda{}_\sigma\right)$$

- As an example, we consider a simple class of theories that are exactly solvable and give rise to an interesting set of inflationary models where the inflaton is a pseudoscalar field related to the Holst term.

- They correspond to set the parameters as: no matter fields, $\alpha(\Phi) = M_p^2/2$, $4\gamma\beta(\Phi) = M_p^2$ [Barbero-Immirzi parameter].
- Δ is taken to depend solely by the pseudoscalar curvature , $\Delta(\mathcal{R}') = \xi\,(\mathcal{R}')^p$, with $p > 1$ a real number and ξ a coupling constant.
- On shell the action can be written as the sum of the Einstein-Hilbert action and the lagrangian density of the pseudoscalar field z:

$$S[g_{\mu\nu}, z] = \int d^4x\,\sqrt{-g}\left[\frac{M_P^2}{2}R - K(z)\frac{(\nabla B(z))^2}{2} - V(z)\right]$$

The second term in the action is a kinetic term of z, which is therefore dynamical, while

$$K(z) = \frac{24M_P^2}{1+16B^2(z)} \quad , \quad V(z) = z\frac{\partial\Delta(z)}{\partial z} - \Delta(z) \quad , \quad B(z) = \frac{\beta + \frac{\partial\Delta(z)}{\partial z}}{M_P^2}$$

Einstein-Cartan Gravity

- The pseudoscalar kinetic term in the action can be canonically normalized by a redefinition of the field:

$$\phi(z) = \int^{z} \sqrt{K(\zeta)}\, d\zeta$$

- For the case $\Delta(\mathcal{R}') = \xi(\mathcal{R}')^{p}$, it follows that:

$$V(z) = \xi(p-1)z^{p}$$

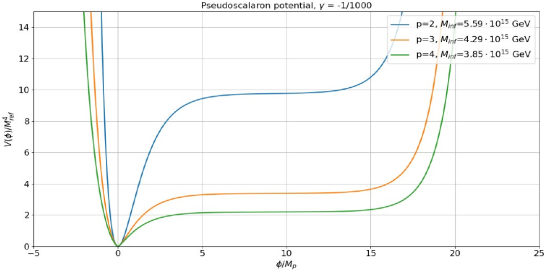
- From $\phi(z)$, inverting with respect to $B(z)$ and then with respect to z, one finds:

$$z^{p-1} = \frac{1}{\xi p}\left[\left(\frac{M_P^2}{4}\right)\sinh\left(\sqrt{\frac{2}{3}}\frac{1}{M_P}\left(\phi(z)-\phi_0\right)\right)-\beta\right]$$

$$\boxed{V(\phi) = \frac{p-1}{p^{p/(p-1)}}\frac{1}{\xi^{1/(p-1)}}\left|\left(\frac{M_P^2}{4}\right)\sinh\left(\sqrt{\frac{2}{3}}\frac{1}{M_P}\left(\phi-\phi_0\right)\right)-\beta\right|^{\frac{p}{p-1}}}$$

Inflation Model

- The sign of the Barbero-Immirzi parameter γ determines the direction of the slow-roll phase.
- The parameter p controls the extension of the inflationary plateau, the asymptotics of the potential for large field values as well as the vacuum geometry. Indeed, as p increases, the plateau region becomes longer and the vacuum geometry becomes more and more cuspy. Smaller values of p favor a potential uphill climbing for smaller values of the scalar field while larger values of p tend to suppress the potential uphill climbing, which begins at larger field values.



Decomposition of Torsion and Contorsion

- Due to the antisymmetry of torsion in the spacetime indices, $T_{\alpha\beta\mu}$ has 24 independent components in $d = 4$. These can be grouped into three irreducible pieces: a vector v_μ (4 components), a pseudovector a_μ (4 components) and the 16-component tensor $\theta_{\alpha\beta\mu}$.
- In terms of its irreducible components, the torsion tensor reads

$$T_{\alpha\beta\mu} = \frac{1}{3}\left(g_{\alpha\beta}v_\mu - g_{\alpha\mu}v_\beta\right) + \frac{1}{6}\epsilon_{\alpha\beta\mu\rho}\, a^\rho + \theta_{\alpha\beta\mu}$$

- From the following combination

$$\nabla_\mu g_{\beta\alpha} + \nabla_\beta g_{\alpha\mu} - \nabla_\alpha g_{\mu\beta} = 0$$

one finds the contorsion tensor

$$\begin{aligned} C_{\alpha\beta\mu} &= \frac{1}{2}\left(T_{\alpha\beta\mu} + T_{\beta\mu\alpha} + T_{\mu\beta\alpha}\right) \\ &= \frac{1}{3}\left(g_{\mu\beta}v_\alpha - g_{\mu\alpha}v_\beta\right) + \frac{1}{12}\epsilon_{\alpha\beta\mu\rho}\, a^\rho + \theta_{\alpha\beta\mu} \end{aligned}$$

Coupling to Spinors

- Let's consider tha Dirac action

$$S_{\mathrm{D}} = \int d^4x \; e \; \frac{i}{2} \left(\overline{\psi}\gamma^\mu D_\mu \psi - \overline{D_\mu \psi}\gamma^\mu \psi \right)$$

- In the presence of torsion, the spin connection can be decomposed into the Leci-Civita part and the distorsion part

$$\omega_{ab\mu} = \mathring{\omega}_{ab\mu}(e) + C_{ab\mu}$$

- The contorsion in curved space is obtained through the vierbein

$$C_{ab\mu} = e_a^\alpha e_b^\beta C_{\alpha\beta\mu}$$

- The contorsion-spinor interaction term in the Dirac case is of the form

$$\mathcal{L}_{\mathrm{Int}} = \frac{i}{2}\frac{1}{4} e_a^\alpha e_b^\beta e_c^\mu C_{\alpha\beta\mu} \overline{\psi} \{\gamma^c, \gamma^{ab}\} \psi = a^\sigma L_\sigma$$

with $L_\sigma = \frac{1}{8}\overline{\psi}\gamma^5\gamma_\sigma\psi$.

Einsten-Cartan gravity + Fermions

- It is possibile to decompose also the curvature and the Holst term

$$\mathcal{R} = \mathring{R} + 2\nabla_\mu v^\mu + \frac{1}{24} a_\mu a^\mu - \frac{2}{3} v_\mu v^\mu - \theta_{\alpha\beta\mu}\,\theta^{\alpha\beta\mu}$$
$$\mathcal{R}' = \nabla_\mu a^\mu - \frac{2}{3} v_\mu a^\mu + 2\epsilon^{\alpha\beta\mu\nu}\theta_{\alpha\sigma\mu}\,\theta^\sigma{}_{\beta\nu}$$

- We can not consider the terms that involve a total derivative $(\nabla_\mu v^\mu)$ and the algebric terms $(\theta\theta)$, and use the following identity

$$\nabla_\mu \left[B(z) a^\mu\right] = \left[\nabla_\mu B(z)\right] a^\mu + B(z)\left(\nabla_\mu a^\mu\right) = 0$$

- Substituting and calculating the variation of the action with respect to the fields a^μ and v^μ it is possibile to find the expressions of the fields wrt the new source term

$$a_\mu = -\frac{24 S_\mu}{1 + 16 B(z)^2} \qquad v_\mu = -\frac{24 B(z) S_\mu}{1 + 16 B(z)^2}$$

where $S_\mu = \nabla_\mu B(z) - \frac{L_\mu}{M_p^2}$.

Local Supersymmetry (Supergravity)

- The Einstein-Cartan model may be extendend to a $\mathcal{N} = 1$ supergravity version.
- The parameter of global supersymmetry transformations are constant spinors ϵ_α. The main idea is that in supergravity supersymmetry is gauged, which means that the spinor parameters become arbitrary functions on a curved spacetime manifold $\epsilon_\alpha(x)$.
- A supergravity theory contains a gravity multiplet, the frame field $e^a_\mu(x)$ describing the graviton, plus a specific number of vector-spinor fields $\psi^i_\mu(x)$, where $i = 1, \ldots, \mathcal{N}$, whose quanta are the gravitinos, the supersymmetric partner of the graviton.
- The basic case of the supergravity theory is $\mathcal{N} = 1$ in $D = 4$, which correspond to the Einstein-Hilbert action plus the gravitino action (Rarita-Schwinger).
- In the first order formalism the equation of motion for the spin connection can be solved to obtain a connection with torsion.

Conclusions and Outlook

- We have considered Einstein-Cartan gravities with dynamical distorsion and higher derivative terms given by powers of the so-called Holst scalar curvature. The resulting effective field theory is equivalent to General Relativity coupled to a pseudoscalar field that can naturally drive a single-field slow-roll inflationary phase fully compatible with the current observational evidences.
- The inflationary paradigm offers an elegant way to solve the old puzzles of the Hot Big Bang Cosmology model.
- We are trying to study the supersymmetric extensions of this class of modified gravities and to embed them in more fundamental theories.
- Further investigations are required, related to the reheating phase and to the coupling to the SM or BSM fields. In particular, thermal and non-thermal leptogenesis are interesting scenarios to deepen using Boltzmann equations.

Thanks for your attention