First-order phase transitions in radiative simmetry breaking models

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Mainly based on:
arXiv:2412.06889 [hep-ph] I.K. Banerjee, F.Rescigno, A.Salvio,
arXiv:2507.21215 [hep-ph] F.Rescigno, A.Salvio

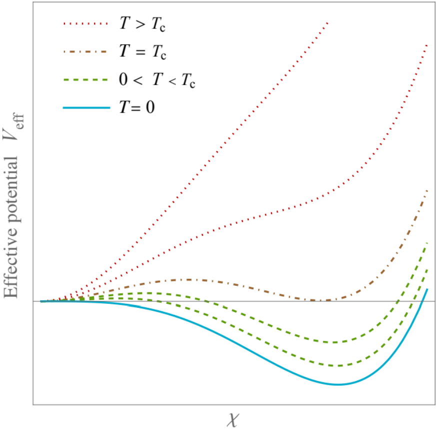
Roadmap

Roadmap

1. Radiative Symmetry Breaking (RSB) and FOPT
2. Late blooming mechanism
3. Reheating after FOPT in Gildner-Weinberg Models: General Properties
4. Radiative Electroweak Symmetry Breaking (EWRSB)
5. Preheating after FOPT

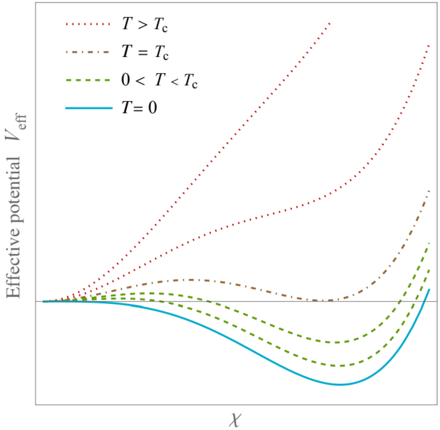
First Order Phase Transitions (FOPT)

- FOPT can occur in the early Universe when the temperature T drops below a critical value T_c
- The finite temperature effective potential $V(\chi, T)$ develops a new stable minima (**true vacuum**), while the old minima becomes metastable (**false vacuum**)
- The false vacuum eventually **decay in the true vacuum**
- The false vacuum decay manifest as the **nucleation of true vacuum bubbles** in a background of false vacuum



Properties of Phase Transition in RSB models

- Phase Transitions in RSB models are **always FOPT**
- FOPTs in RSB feature always a **supercooling phase**, i.e. the temperature drops much below T_c before the bubble nucleation became effective.
  This ensures that thermal effect do **not spoil perturbation theory**
- Supercooling implies a new stage of **inflation**
- FOPT in RSB models are always **strong**, the PT release a large amount of energy



Radiative Symmetry Breaking (RSB)

- We start from the most general no-scale Lagrangian:

$$\mathcal{L}_{\rm ns} = -\frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A + \frac{1}{2}D_\mu\phi_a D^\mu\phi_a + \bar{\psi}_j i\not{D}\psi_j - \frac{1}{2}(Y^a_{ij}\psi_i\psi_j\phi_a + \text{h.c.}) - V_{\rm ns}(\phi),$$

$$D_\mu\phi_a = \partial_\mu\phi_a + i\theta^A_{ab}V^A_\mu\phi_b, \qquad D_\mu\psi_j = \partial_\mu\psi_j + it^A_{jk}V^A_\mu\psi_k,$$

$$V_{\rm ns}(\phi) = \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d.$$

Radiative Symmetry Breaking (RSB)

- Setting $\mu = \tilde{\mu}$ we obtain $\lambda_\chi = 0$, i.e.

$$V_q(\chi) = \frac{\bar{\beta}}{4}\left(\log\frac{\chi}{\chi_0} - \frac{1}{4}\right)\chi^4,$$

$$\bar{\beta} \equiv [\beta_{\lambda_\chi}]_{\mu=\tilde{\mu}}, \qquad \chi_0 \equiv \frac{\tilde{\mu}}{e^{1/4+a_s}}.$$

- When the conditions,

$$\begin{cases} \lambda_\chi(\tilde{\mu}) = 0 & \text{(flat direction)} \\ \beta_{\lambda_\chi}(\tilde{\mu}) > 0 & \text{(minimum condition)} \end{cases}$$

  are fulfilled χ_0 is the zero temperature vacuum-expectation value of χ and is the new absolute minima of $V(\chi)$

- the fluctuation around χ_0 have mass $m_\chi^2 = \bar{\beta}\chi_0^2$

Effective thermal potential

Effective Thermal Potential

$$V_{\text{eff}}(\chi,T) = V_q(\chi) + \frac{T^4}{2\pi^2}\left(\sum_b n_b J_B(m_b^2(\chi)/T^2) - 2\sum_f J_F(m_f^2(\chi)/T^2)\right) + \Lambda_0,$$

$$J_B(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 - e^{-\sqrt{p^2+x}}\right) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x - \frac{\pi}{6}x^{3/2} - \frac{x^2}{32}\log\left(\frac{x}{a_B}\right) + O(x^3),$$

$$J_F(x) \equiv \int_0^\infty dp\, p^2 \log\left(1 + e^{-\sqrt{p^2+x}}\right) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}x - \frac{x^2}{32}\log\left(\frac{x}{a_F}\right) + O(x^3),$$

$$a_B = 16\pi^2 \exp(3/2 - 2\gamma_E), \qquad a_F = \pi^2 \exp(3/2 - 2\gamma_E).$$

Vacuum Decay

Decay rate of the false vacuum per unit of volume (time independent bounce)

$$\Gamma \approx T^4 \mathsf{exp}(-S_3/T),$$

$$S_3 \equiv 4\pi \int_0^\infty dr\, r^2 \left(\frac{1}{2}\chi'^2 + \bar{V}_{\text{eff}}(\chi, T)\right) = -8\pi \int_0^\infty dr\, r^2 \bar{V}_{\text{eff}}(\chi, T),$$

$$\chi'' + \frac{2}{r}\chi' = \frac{d\bar{V}_{\text{eff}}}{d\chi}, \qquad \chi'(0) = 0, \qquad \lim_{r\to\infty} \chi(r) = 0, \qquad \bar{V}_{\text{eff}}(\chi, T) = V_{\text{eff}}(\chi, T) - V_{\text{eff}}(0, T).$$

Supercool Expansion

- Using supercool and improved supercool expansion (JCAP 04 (2023), 051 arXiv:2302.10212 [hep-ph], A. Salvio JCAP 12 (2023), 046 arXiv:2307.04694 [hep-ph], A. Salvio) is possible to calculate several FOTP parameters:
  - **Nucleation temperature**: Defined as the temperature T_n for which

$$\Gamma(T_n) \approx H(T_n)^4 \equiv H_n^4.$$

  - **Inverse duration**: Defined as

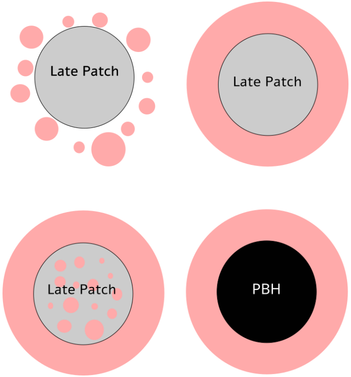
$$\beta \equiv \left[\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right]_{t_n}.$$

  - The **strength parameter** α, that in the case of RSB induced FOPT is always $\alpha \gg 1$

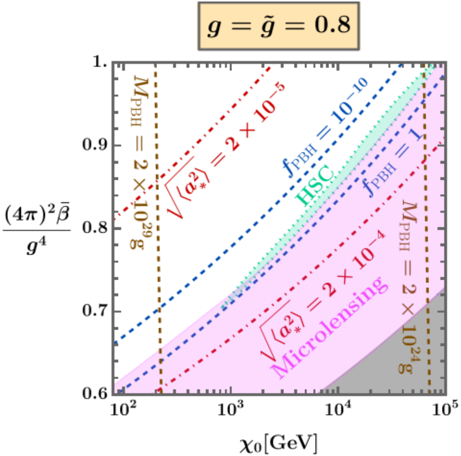
Roadmap

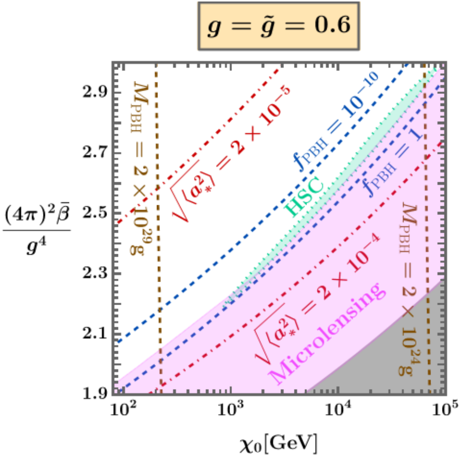
Phys.Rev.D 105 (2022) 2, L021303, J. Liu, L. Bian, R. G. Cai, Z. K. Guo and S. J. Wang

- Vacuum decay is a **probabilistic process**
- There can be some regions that **persist in the false vacuum** for a longer time than the background
- These regions can eventually **collapse into Primordial Black Holes (PBH)** if the mass excess reaches the critical value δ_c

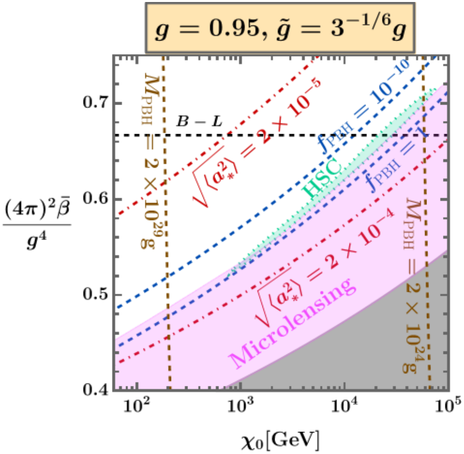


Late Blooming Mechanism





Improved supercool expansion at LO



Improved Supercool at LO

Roadmap

...but after the FOPT?

- Supercooled FOPT are accompanied by a short period of **inflation**.
- After this short period of inflation the Universe must be reheated.
- The field χ starts oscillating around the minimum of the potential in analogy with the inflaton field after the cosmic inflation.
- In analogy with the cosmic inflation the Universe is reheated through:
  - **Perturbative reheating**: decay of the field χ into SM particles. Efficient when the SM is **embedded** in the RSB sector.
  - **Preheating**: particle production through parametric resonance. Important when the SM is coupled to the RSB sector **through a portal**.

Using the definitions

$$\zeta_{Sa} \equiv \frac{4m_a^2}{m_\chi^2} = \frac{4\lambda_a}{\bar{\beta}}, \qquad \zeta_{Fi} \equiv \frac{4m_i^2}{m_\chi^2} = \frac{4y_i^2}{\bar{\beta}}, \qquad \zeta_{VN} \equiv \frac{4m_N^2}{m_\chi^2} = \frac{4g_N^2}{\bar{\beta}}.$$

we can write the two-body decay rates in a simpler form

General decay rates (two-body, ζ-form)

$$\Gamma^{(2S)}(\bar{\beta}, \zeta_S) = \frac{\bar{\beta}^{3/2}}{32\pi}\zeta_S \chi_0, \qquad \Gamma^{(2F)}(\bar{\beta}, \zeta_F) = \frac{\bar{\beta}^{3/2}}{32\pi}\zeta_F \chi_0, \qquad \Gamma^{(2V)}(\bar{\beta}, \zeta_V) = \frac{\bar{\beta}^{3/2}}{32\pi}\zeta_V \chi_0,$$

$$\zeta_S \equiv \frac{1}{4}\sum_a \zeta_{Sa}^2 \sqrt{1-\zeta_{Sa}}\,\Theta\left(1-\zeta_{Sa}\right), \qquad \zeta_F \equiv \sum_i s_i \zeta_{Fi} (1-\zeta_{Fi})^{3/2}\,\Theta\left(1-\zeta_{Fi}\right),$$

$$\zeta_V \equiv \sum_N \left(1 - \zeta_{VN} + \frac{3}{4}\zeta_{VN}^2\right)\sqrt{1-\zeta_{VN}}\,\Theta\left(1-\zeta_{VN}\right).$$

General decay rates (three body)

$$\Gamma^{(3S)} = \sum_{a,b,c} \frac{S\chi_0}{64\pi^5\sqrt{\bar{\beta}}} (\lambda'_{abc})^2 \omega_3 \left(\bar{\beta}, \sqrt{\lambda_a}, \sqrt{\lambda_b}, \sqrt{\lambda_c}\right) \Theta\left(\sqrt{\bar{\beta}} - \sqrt{\lambda_a} - \sqrt{\lambda_b} - \sqrt{\lambda_c}\right),$$

$$\Gamma^{(SV)} = \sum_{N,c} \frac{(G_c^N)^4\chi_0}{32\pi^5\sqrt{\bar{\beta}}} \left(2\omega_3\left(\bar{\beta}, g_N, g_N, \sqrt{\lambda_c}\right) + \frac{1}{g_N^4}\omega_3^{(4)}\left(\bar{\beta}, g_N, \sqrt{\lambda_c}\right)\right) \times \\ \times \Theta\left(\sqrt{\bar{\beta}} - \sqrt{\lambda_c} - 2g_N\right),$$

where

$$\lambda'_{abc} \equiv \lambda_{mnl}\mathcal{O}^T_{ma}\mathcal{O}^T_{nb}\mathcal{O}^T_{lc}, \qquad (G_c^N)^4 \equiv (\bar{T}_a^N \bar{\theta}'^N_{an}\mathcal{O}^T_{nc})^2.$$

and ω_3 and $\omega_3^{(4)}$ are adimensionalized phase space integrals.

Reheating temperature $T_{\rm rh}$ and instantaneous reheating

- We can express the **instantaneous reheating** condition

$$T_{\rm rh} \approx T_{\rm eq},$$

in a general form for GW models (including only two-body decays)

$$\zeta_{\rm tot}^2 \gtrsim \frac{512\pi^3}{3\bar{\beta}^2}\frac{\chi_0^2}{\bar{M}_P^2},$$

where we defined

$$\Gamma_{\rm tot}(\bar{\beta},\zeta_S,\zeta_F,\zeta_V) \equiv \Gamma^{(2S)}(\bar{\beta},\zeta_S) + \Gamma^{(2F)}(\bar{\beta},\zeta_F) + \Gamma^{(2V)}(\bar{\beta},\zeta_V) = \frac{\bar{\beta}^{3/2}}{32\pi}\zeta_{\rm tot}\chi_0,$$

$$\zeta_{\rm tot} = \zeta_S + \zeta_F + \zeta_V .$$

Roadmap

Radiative Electroweak Symmetry Breaking (EWRSB): General Properties

- We start from the conformal SM, that is we replace

$$\mathcal{L}_h = \mu^2 |\mathcal{H}|^2 \longrightarrow \mathcal{L}_{\phi h} = \frac{1}{2}\lambda_{ab}\phi_a\phi_b|\mathcal{H}|^2,$$

where

$$\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix}\eta_1 + i\eta_2 \\ h + i\eta_3\end{pmatrix},$$

- we add a real scalar ϕ_h and assume a $\mathbb{Z}_2$ symmetry such that $\phi_h \to -\phi_h$. The scalar potential is then

$$\begin{aligned} V\left(h, \phi_h\right) &= \frac{1}{4}\lambda_h h^4 + \frac{1}{4}\lambda_\phi \phi_h^4 - \frac{1}{4}\lambda_{\phi h} h^2 \phi_h^2 \\ &= \frac{1}{4}\left(\sqrt{\lambda_h} h^2 - \sqrt{\lambda_\phi}\phi_h^2\right)^2 + \frac{1}{2}\sqrt{\lambda_h}\sqrt{\lambda_\phi} h^2\phi_h^2 - \frac{1}{4}\lambda_{\phi h} h^2 \phi_h^2, \end{aligned}$$

Radiative Electroweak Symmetry Breaking (EWRSB): General Properties

- At the renormalization scale $\tilde{\mu}$ the classical potential is zero along the flat direction, and couplings and fields satisfy

$$\sqrt{\lambda_h} h^2 = \sqrt{\lambda_\phi} \phi_h^2 \text{ (on the flat direction)}, \qquad \lambda_{\phi h} = 2\sqrt{\lambda_h}\sqrt{\lambda_\phi}.$$

- We can rotate the fields in a way that the flat-direction field χ manifestly appear in the Lagrangian

$$\begin{cases} \phi_h = \chi \cos\alpha - H \sin\alpha, \\ h = \chi \sin\alpha + H \cos\alpha, \end{cases}$$

- where the mixing angle α is defined here by

$$\tan\alpha \equiv \sqrt{\frac{\lambda_{\phi h}}{2\lambda_h}},$$

Radiative Electroweak Symmetry Breaking (EWRSB): General Properties

- Including the 1-loop contribution of the potential along the flat direction, and expanding the flat-direction field as $\chi = \chi_0 + \delta\chi$, we get

$$v = \chi_0 \sin\alpha = \chi_0 \sqrt{\frac{\lambda_{\phi h}}{\lambda_{\phi h} + 2\lambda_h}},$$

that relates the RSB scale to the Higgs VEV,such that $h = v + \delta\chi \sin\alpha + H\cos\alpha$.

- the effective 1-loop potential reads

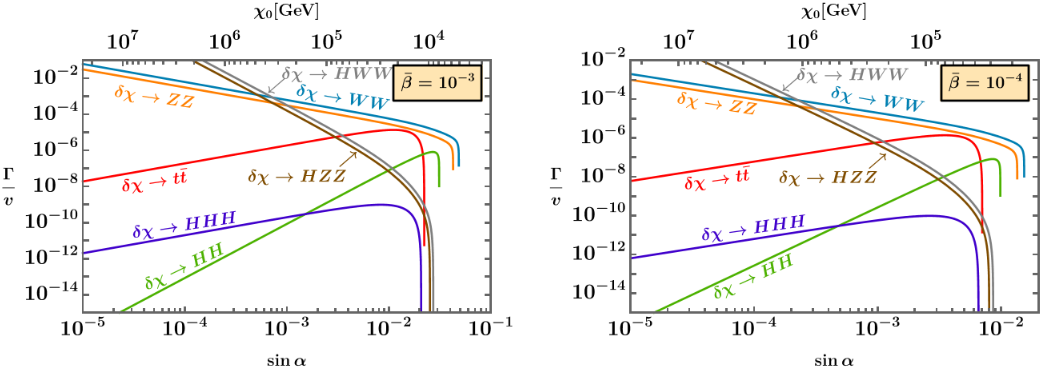
$$\begin{aligned} V_{1-\mathrm{loop}}(H,\delta\chi) =& \frac{\lambda_{\phi h}\chi_0^2}{2} H^2 + \frac{4\lambda_h^2 - \lambda_{\phi h}^2}{2\sqrt{2}\lambda_h} \sqrt{\frac{\lambda_{\phi h}\lambda_h}{(\lambda_{\phi h} + 2\lambda_h)^2}}\,\chi_0 H^3 + \frac{(\lambda_{\phi h} - 2\lambda_h)^2}{16\lambda_h} H^4 \\ &+ \lambda_{\phi h}\chi_0\delta\chi H^2 + \frac{\lambda_{\phi h}}{2}\delta\chi^2 H^2 + \frac{4\lambda_h^2 - \lambda_{\phi h}^2}{2\sqrt{2}\lambda_h} \sqrt{\frac{\lambda_{\phi h}\lambda_h}{(\lambda_{\phi h} + 2\lambda_h)^2}}\,\delta\chi H^3 \\ &+ \frac{\bar{\beta}}{2}\chi_0^2\delta\chi^2 + \delta\chi \text{ self-interactions}, \qquad M_h = \sqrt{\lambda_{\phi h}}\chi_0. \end{aligned}$$

- The EW gauge bosons and SM fermions develop a coupling with $\delta\chi$. For example, for the W bosons the interaction Lagrangian with h is given by

$$
\begin{aligned}
\mathcal{L}_{hWW} =& \frac{1}{4} g_2^2 h^2 W_\mu^+ W^{-\mu} \\
=& \frac{1}{4} g_2^2 \chi_0^2 \sin^2\alpha\, W_\mu^+ W^{-\mu} + \frac{1}{4} g_2^2 H^2 W_\mu^+ W^{-\mu} \cos^2\alpha + \frac{1}{2} g_2^2 \chi_0 \cos\alpha\, \sin\alpha\, H W_\mu^+ W^{-\mu} \\
& + \frac{1}{2} g_2^2 \chi_0 \sin^2\alpha\, \delta\chi W_\mu^+ W^{-\mu} + \frac{1}{4} g_2^2 \sin^2\alpha\, \delta\chi^2 W_\mu^+ W^{-\mu} \\
& + \frac{1}{2} g_2^2 \cos\alpha\, \sin\alpha\, \delta\chi H W_\mu^+ W^{-\mu},
\end{aligned}
$$

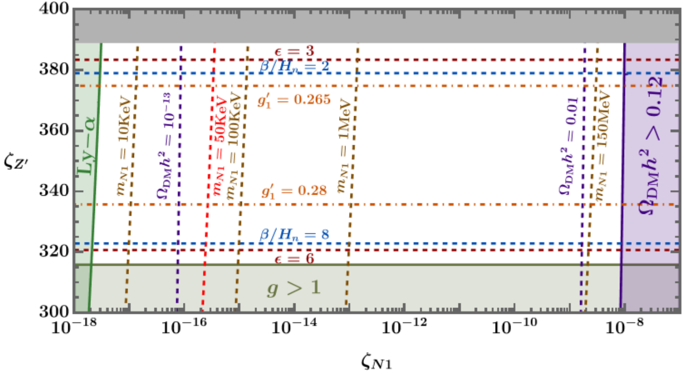
where g_1 and g_2 are the gauge constants of the $SU(2)$ and $U(1)$ SM gauge-group factors.

- The mixing angle α should be small to respect the experimental constraints, so in order to generate the SM VEV v, $\chi_0 \gg v$.
- In this limit we can approximate $\bar{\beta} \approx \left[\mu \frac{d\lambda_\phi}{d\mu}\right]_{\mu=\tilde{\mu}}$.



$$\Omega_{\rm DM}h^2 \approx 0.110 \times \frac{\chi_0}{0.40\,\text{eV}} \frac{\sqrt{\bar\beta}\zeta_{N1}}{4} \frac{\zeta_{N1}(1-\zeta_{N1})^{3/2}}{3},$$

$\chi_0 = 10^5\text{GeV}$



Roadmap

Preheatng: the dark photon model

- There are cases when the reheating is not very efficient: this is typically the case when the **RSB is a dark sector** (feeble interaction with SM).
- At some stage χ undergoes small oscillations around χ_0

$$\chi(t) = \chi_0 + \Phi \sin(m_\chi t),$$

- The dark sector is described by the lagrangian

$$\mathcal{L}_{\rm DP} = -\frac{1}{4}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} - \frac{\eta}{2}\mathcal{A}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{2}(D_\mu\phi)_a(D^\mu\phi)_a - V_{\rm ns}(\phi),$$

equipped with a $U(1)_D$ gauge symmetry.

- The dark sector is coupled with the standard model via the interaction

$$\mathcal{L}_{\chi h} = \frac{\lambda_{\chi h}}{2}\chi^2|\mathcal{H}|^2$$

- We allow for the EWSB in the SM sector, and the RSB only occurs in the dark sector.

Preheatng: the dark photon model

- It must be then

$$v \gg \chi_0 \quad \longrightarrow \quad m_d, m_h \gg m_\chi .$$

- Perturbative reheating can only occur through $\delta\chi \to A'A'/hh \to$ 4 fermions that is suppressed by 4-body phase space.

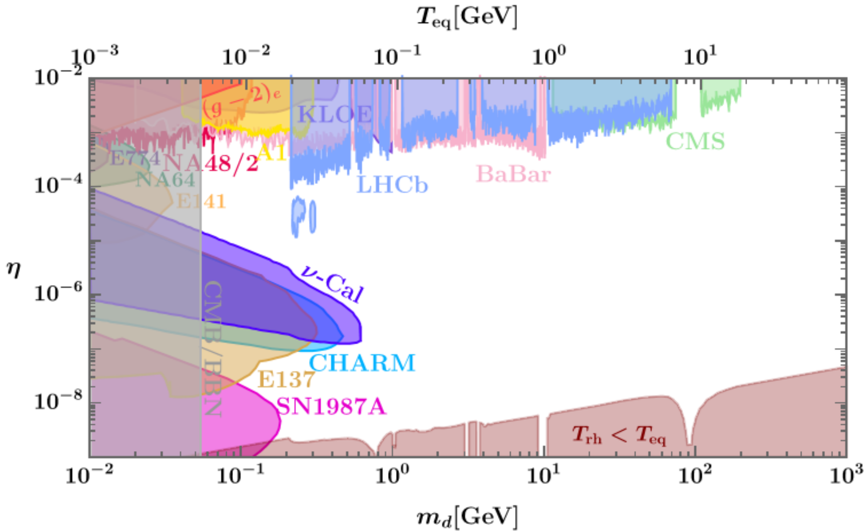
- Resonant production of dark photons can occur

$$\frac{d^2 A'_k}{dz^2} + (a_k - 2q\cos(2z))A'_k = 0,$$

where $a_k \equiv 4(k^2 + e_d^2\chi_0^2)/(\bar{\beta}\chi_0^2)$, $q_i \equiv 4e_d^2\Phi/(\bar{\beta}\chi_0)$, $z \equiv \sqrt{\bar{\beta}}\,\chi_0 t/2 + \pi/4$

- The dark photon that decay into SM and the universe is reheated

Preheatng: the dark photon model



Conclusion and future directions

- Models based on RSB alway feature strong and supercoolded FOPT.
- Strong FOPT produce a GW spectrum that is detectable by future interferometers (LISA).
- Several theoretical tools for the study of FOPT originated by non conformal models have been recently developed.
- An example is the scotogenic extension of the standard model that can feature FOPT.