Hadron phenomenology from Lattice QCD

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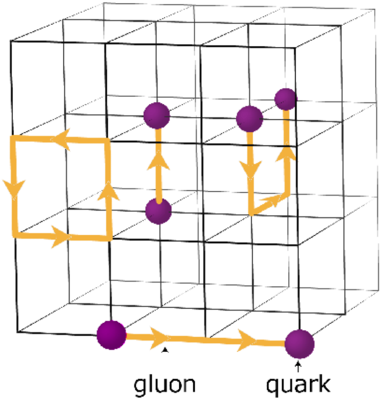
Introduction: Lattice QCD



- QCD action is discretised on a lattice with spacing a which is the UV cut-off of the theory
- Euclidean $4D$ spacetime in a finite volume $L^3 \times T$
- Quark fields live on the sites;
- Gluon fields live on the links;

  parallel transporter: SU(3) matrices:

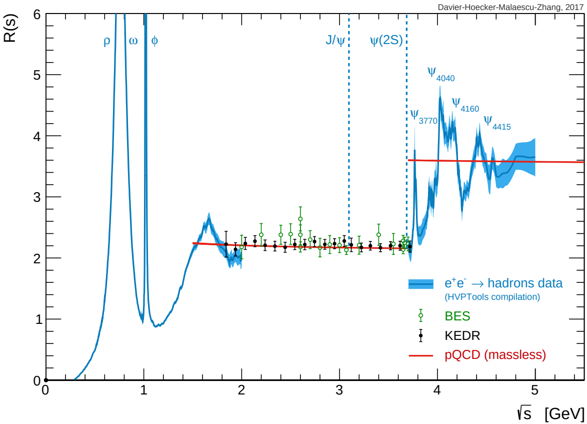
$$U_\mu \sim e^{iagA_\mu}$$

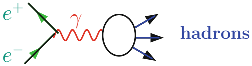
$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[U] e^{-S_E[U]} \mathcal{O}[U]$$

Monte Carlo methods: extract field configurations with probability $P \propto \exp(-S)$ ($\sim$ Boltzmann weight), then compute $\mathcal{O}$ the collected sample and determine $\mathcal{O}$ through the sample mean, $\frac{1}{N}\sum_{i=1}^{N} \mathcal{O}[U_i]$.

R-ratio

The R-ratio plays a fundamental role in particle physics since its introduction





$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

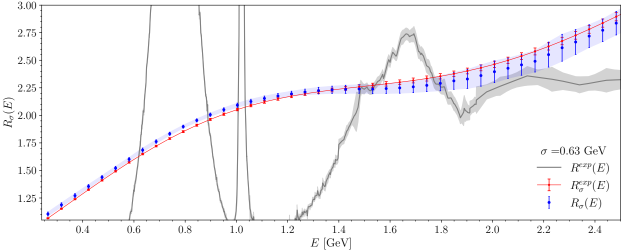
As any other cross-section, it is an energy-dependent probe of the theory and contains an infinite amount of information

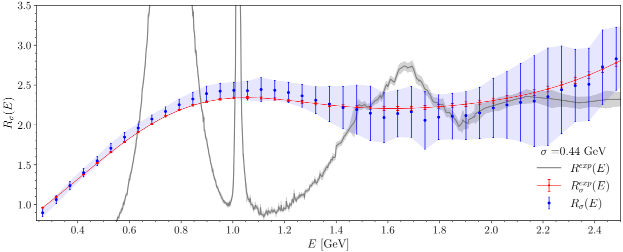
What about computing R-ratio directly on the lattice?

In PRL 130 (2023), the Hansen-Lupo-Tantalo (**HLT**) method is used to evaluate on the lattice:

$$C(t) = -\frac{1}{3}\sum_{i=1}^{3}\int d^3x \mathrm{T}\langle 0|J_i(x)J_i(0)|0\rangle = \frac{1}{12\pi^2}\int_{2m_\pi}^{\infty} d\omega\, e^{-\omega t}\,\omega^2 R(\omega)\ ,$$

$$R_\sigma(E) = \int_0^\infty d\omega\, R(\omega) \underbrace{G_\sigma(E-\omega)}_{\text{G=Gaussian kernels}}\ .$$





- For $\sigma \simeq 0.6$ GeV and around $E \simeq m_\rho$ we observe a $\approx 3\sigma$ deviation w.r.t. $e^+ e^-$ exp. results.
- For $\sigma \simeq 0.4$ GeV the larger errors do not allow us to observe significant deviations from experimental data.

Precision and resolution to be improved with more statistics and by increasing the statistical precision of our lattice correlators.

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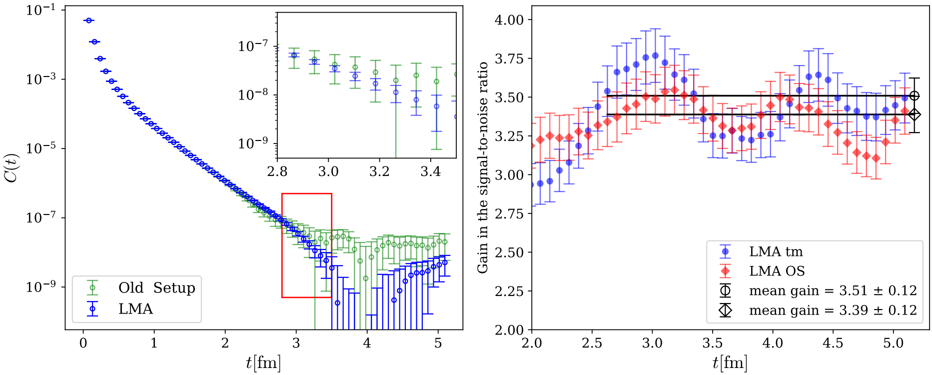

We employ noise-reduction techniques:
by computing a relatively small number of low modes of the Dirac operator exactly, which we refer to as low-mode averaging (LMA).

The smeared R-ratio in isoQCD from first-principles lattice simulations

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Smeared R-ratio in isospin symmetric QCD with Low Mode Averaging

F. Margari*, *S. Bacchio, A. De Santis, A. Evangelista, R. Frezzotti, G. Gagliardi, M. Garofalo, F. Pittler, F. Sanfilippo, C. Schneider, N. Tantalo and for the Extended Twisted Mass Collaboration*

*: *corresponding author*

Full text: pdf

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Abstract

Low Mode Averaging (LMA) is a technique to improve the quality of the signal-to-noise ratio in the long time separation of Euclidean correlation functions. We report on its beneficial impact in computing the vector-vector light connected two-point correlation functions and
derived physical quantities in the mixed action lattice setup adopted by ETM collaboration. We focus on preliminary results of the computation within isospin symmetric QCD (isoQCD) of the R-ratio smeared with Gaussian kernels of widths down to $\sigma \sim 250$ MeV, which is enough to appreciate the ρ resonance around 770 MeV, using the Hansen-Lupo-Tantatlo (HLT) spectral-density reconstruction method.

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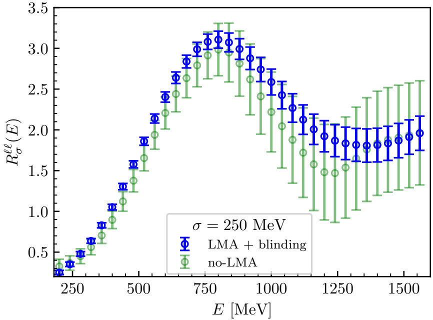


The R-ratio is a phenomenological observable of great relevance, both in itself and in applications such as the dispersive approach to the **muon anomalous magnetic moment**.

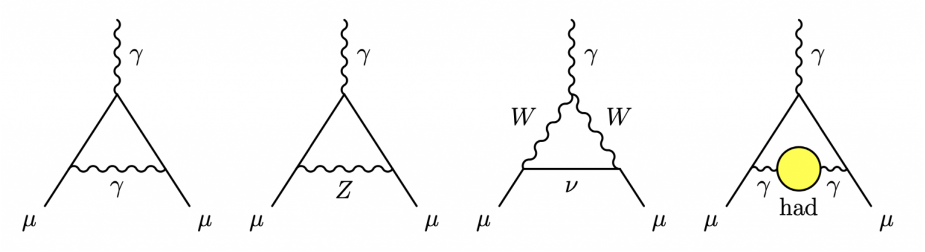
Muon Anomalous Magnetic Moment

The Dirac equation predicts a muon magnetic moment, $\mathbf{M} = g_\mu \frac{e}{2m_\mu}\mathbf{S}$, with gyromagnetic ratio $g_\mu = 2$. Quantum loop effects lead to a small calculable deviation, parameterized by the magnetic anomaly

$$a_\mu \equiv \frac{g_\mu - 2}{2}$$

The SM prediction a_μ^{SM} is generally divided into three parts

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + \boxed{a_\mu^{\text{Had}}}$$

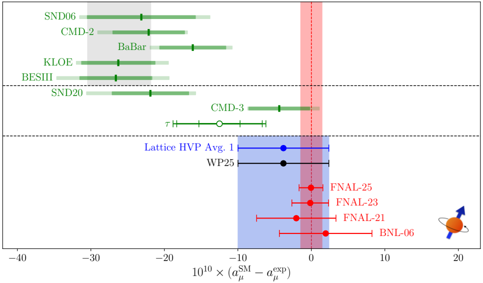


The hadronic contribution a_μ^{Had}

Dispersive approach

$$a_\mu^{\mathrm{HVP-LO}} = \int_{m_\pi}^{\infty} \mathrm{d}E\ R(E) \quad \underbrace{\tilde{K}(E)}_{\text{analytic function}}$$

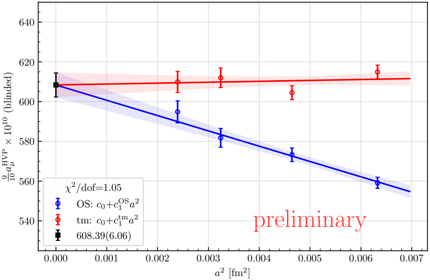
- The idea is to replace $R(E) \to R^{\mathrm{exp}}(E)$ and use previous formula.



Lattice QCD

$$a_\mu^{\mathrm{HVP-LO}} = \int_{0}^{\infty} \mathrm{d}t\ C(t) \quad \underbrace{K(t)}_{\text{analytic function}}$$

- $C(t)$ is the 2-point Euclidean correlation; We use **LMA** technique!
- **ETM Collaboration** effort in computing light-quark contribution to $a_\mu^{\mathrm{HVP-LO}}$ at subpercent accuracy.

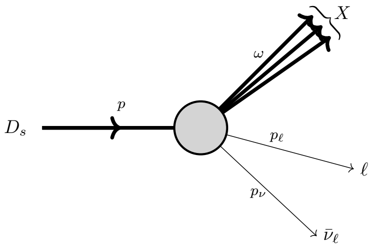


Inclusive semileptonic decays of the D_s meson

$$\Gamma = |V_{cs}|^2 \Gamma_{\bar{c}s} + |V_{cd}|^2 \Gamma_{\bar{c}d} + |V_{us}|^2 \Gamma_{\bar{u}s} \ ,$$

$$\Gamma_{\bar{f}g} = G_F^2 S_{\mathrm{EW}} \int \frac{\mathrm{d}^3 p_\nu}{(2\pi)^3} \, \frac{\mathrm{d}^3 p_\ell}{(2\pi)^3} \, \frac{L_{\mu\nu}(p_\ell, p_\nu) \, \boxed{H_{\bar{f}g}^{\mu\nu}(p, p - p_\ell - p_\nu)}}{4 m_{D_s}^2 e_\ell e_\nu} \ ,$$

$$\boxed{H_{\mu\nu}(p,\omega)} = \frac{(2\pi)^4}{2 m_{D_s}} \langle D_s(p) | J_\mu^\dagger(0) \, \delta^4(\mathbb{P} - \omega) \, J_\nu(0) | D_s(p) \rangle \ ,$$



$$J_{\bar{f}g}^\mu(x) = \bar{\psi}_{\bar{f}}(x) \gamma^\mu (1 - \gamma^5) \psi_g(x) \ ,$$

$$p = m_{D_s}(1, \mathbf{0}) \ , \qquad \omega = m_{D_s}(\omega_0, \boldsymbol{\omega}) \ ,$$

$$p_\ell = m_{D_s}(e_\ell, \boldsymbol{k}_\ell) \ , \qquad p_\nu = m_{D_s}(e_\nu, \boldsymbol{k}_\nu) \ .$$

What about computing Γ directly on the lattice?

Inclusive $D_s \to X\ell\bar{\nu}$

We can relate the lattice correlators to the hadronic tensor:

$$C^{\mu\nu}(t,\boldsymbol{\omega}) = \int_0^{\infty} \mathrm{d}\omega_0 \; e^{-(m_{D_s}\omega_0)t} \; H^{\mu\nu}(\omega_0,\boldsymbol{\omega})$$

The problem of extracting $H^{\mu\nu}(\omega_0,\boldsymbol{\omega})$ from 4-point correlators is equivalent to extracting $\rho(\omega)$ from 2-point correlators

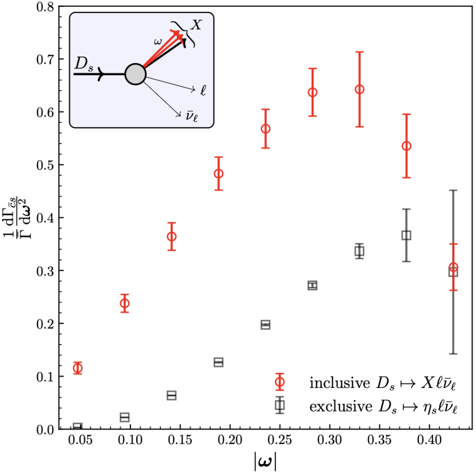
$\Longrightarrow$ **HLT** algorithm!

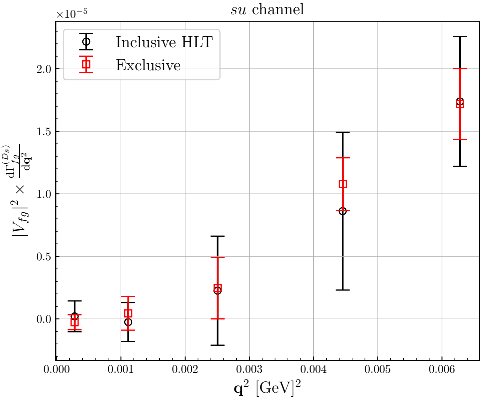
Exclusive $D_s \to P\ell\bar{\nu}$

$$\frac{d\Gamma^{ex}_{fg}}{d\boldsymbol{w}_P^2} = \frac{m_{D_s}^5}{24\pi^3 e_P}|\boldsymbol{w}_P|^3 f_+^2(\boldsymbol{w}_P^2)\;, \quad \langle P|J^{\mu}_{\bar{f}g}(0)|D_s\rangle = (p+p_P)^{\mu} f^{+}_{\bar{f}g} + (p-p_P)^{\mu} f^{-}_{\bar{f}g}\;,$$

Direct computation: The hadronic form factors are obtained by analyzing the asymptotic behavior of lattice correlators at large Euclidean times.

Inclusive vs Exclusive





- Results for $\bar{c}s$ channel: the exclusive contribution does not saturate the decay rate.
- $m_{D_s} < m_D + m_\pi$: the inclusive $\bar{u}s$ channel coincides with the exclusive $D_s \to D\ell\bar{\nu}$
- Perfect agreement with HLT analysis!

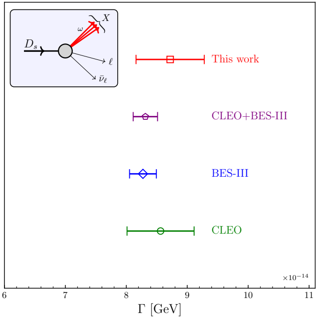
EDITORS' SUGGESTION

Inclusive Semileptonic Decays of the D_s Meson: Lattice QCD Confronts Experiments

Alessandro De Santis, Antonio Evangelista, Roberto Frezzotti, Giuseppe Gagliardi, Paolo Gambino, Marco Garofalo, Christiane Franziska Groß, Bartosz Kostrzewa, Vittorio Lubicz, Francesca Margari, Marco Panero, Francesco Sanfilippo, Silvano Simula, Antonio Smecca, Nazario Tantalo, and Carsten Urbach

Phys. Rev. Lett. **135**, 121901 (2025) - Published 15 September, 2025

Standard model prediction for the semileptonic decay of D_s meson using state-of-the-art lattice QCD calculation agrees well with the experimental determinations.



EDITORS' SUGGESTION

Inclusive semileptonic decays of the D_s meson: A first-principles lattice QCD calculation

Alessandro De Santis, Antonio Evangelista, Roberto Frezzotti, Giuseppe Gagliardi, Paolo Gambino, Marco Garofalo, Christiane Franziska Groß, Bartosz Kostrzewa, Vittorio Lubicz, Francesca Margari, Marco Panero, Francesco Sanfilippo, Silvano Simula, Antonio Smecca, Nazario Tantalo, and Carsten Urbach

Phys. Rev. D **112**, 054503 (2025) - Published 15 September, 2025

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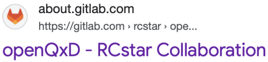
Phenomenological relevance

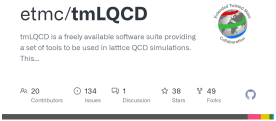
- Lattice results: near-experimental precision and systematic improvability.
- This study opens the way to the inclusive B_s decays rates.

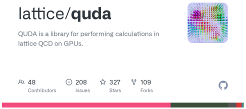
Thanks to several advances in algorithmic methods and the increase of computational resources, **Lattice QCD calculations entered the precision era**, providing accurate and systematically improvable predictions for many observables measured with ever-increasing experimental precision.

To achieve results with a phenomenological relevant precision we need to fully exploit the efficiency of the modern pre-exascale supercomputers.

We interfaced our code to **QUDA** a library for performing calculations in lattice QCD on graphics processing units (GPUs), leveraging NVIDIA's CUDA platform.







Thank you!

Backup slides

HLT method

$$R_\sigma(E) = \int_0^\infty d\omega\, R(\omega)\ \underbrace{G_\sigma(E-\omega)}_{\text{Gaussian kernels}}\ ; \qquad \underbrace{K(\omega;\mathbf{g})}_{\text{Approximated smearing kernels}} = \sum_{\tau=1}^{T/a} g_\tau\, e^{-a\omega\tau}\,.$$

which allows to evaluate $R_\sigma(E)$ from $C(t)$ using

$$\sum_{\tau=1}^{\tau_{\max}} g_\tau(E)\, C(\tau a) = \int_0^\infty d\omega \left(\frac{1}{12\pi^2} \sum_{\tau=1}^{\tau_{\max}} g_\tau(E) e^{-\omega a \tau} \right) R(\omega)\, \omega^2 \simeq R_\sigma(E)$$

Their accuracy is measured by

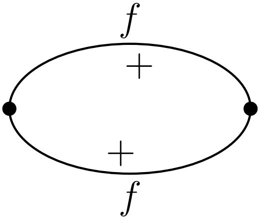
$$A_n[\mathbf{g}] = \int_{E_0}^\infty \mathrm{d}\omega\ w_n(\omega) \left| K(\omega;\mathbf{g}) - \frac{12\pi^2 G_\sigma(E-\omega)}{\omega^2} \right|^2 ,$$

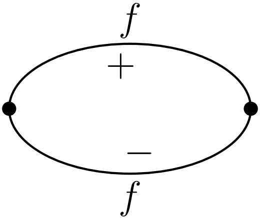
which, for positive weight functions $w_n > 0$, defines a class of weighted L_2-norms in functional space.

The coefficients $\mathbf{g}$ are obtained by minimizing

$$W[\lambda,\mathbf{g}] = (1-\lambda) A_n[\mathbf{g}] + \lambda B[\mathbf{g}], \qquad B[\mathbf{g}] = \sum_{\tau_1,\tau_2=1}^{T/a} g_{\tau_1} g_{\tau_2}\, \mathrm{Cov}_{\tau_1\tau_2}\,.$$

$$J_\mu^{f,\mathrm{OS}} \propto \bar{\psi}_f^+ \gamma_\mu \psi_f^+ \qquad\qquad J_\mu^{f,\mathrm{TM}} \propto \bar{\psi}_f^+ \gamma_\mu \psi_f^-$$





$$R_\sigma^{ff,C,\mathrm{OS}}(a) = A + B^{\mathrm{OS}}\, a^2 \qquad\qquad R_\sigma^{ff,C,\mathrm{TM}}(a) = A + B^{\mathrm{TM}}\, a^2$$