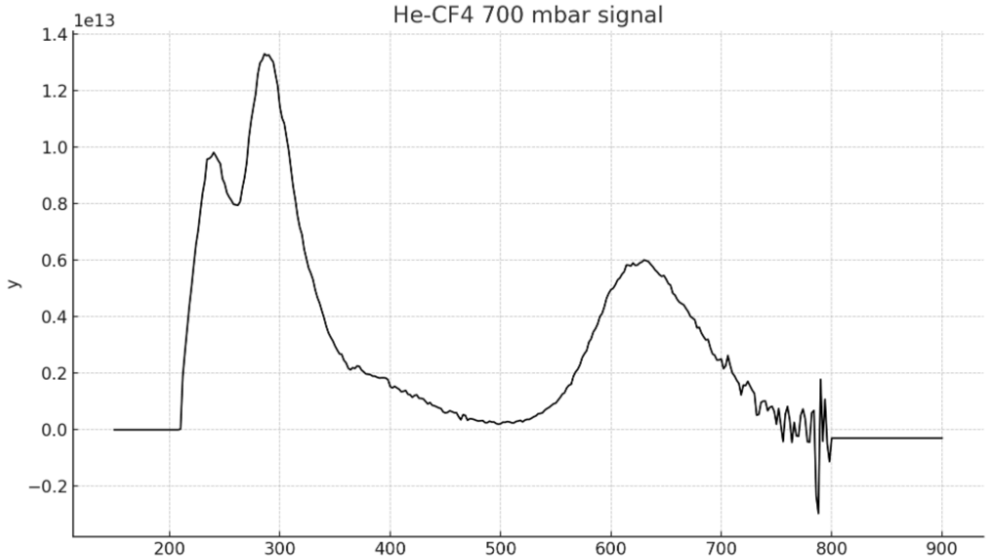
NEGATIVE IONS RESURRECTION

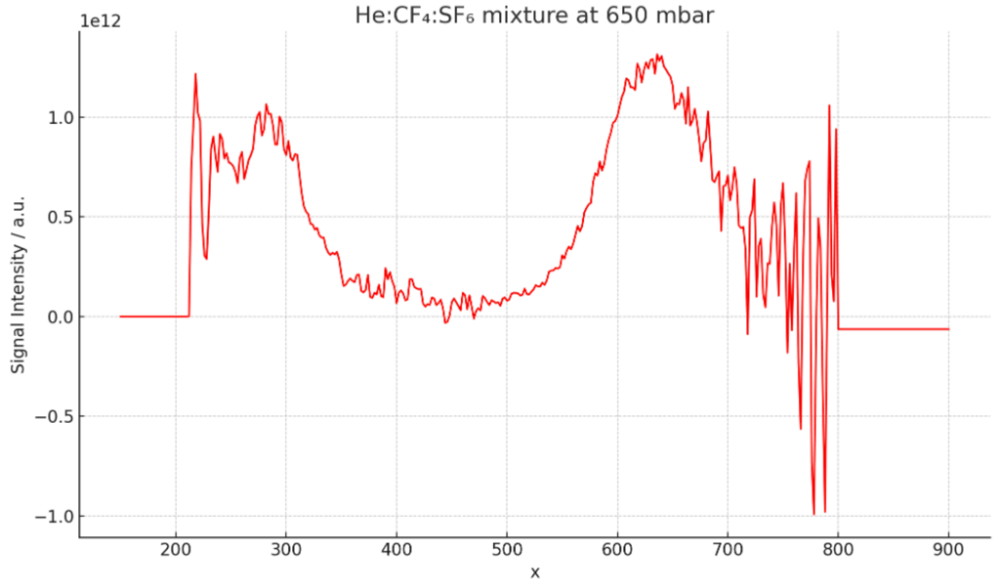
E. Baracchini

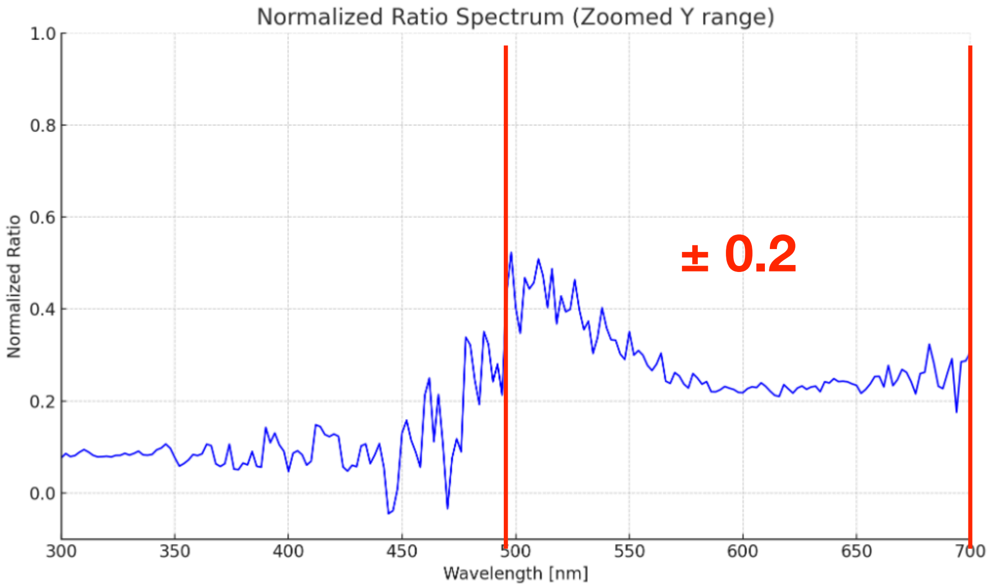
CYGNO Collaboration Meeting 2025 - Clusone

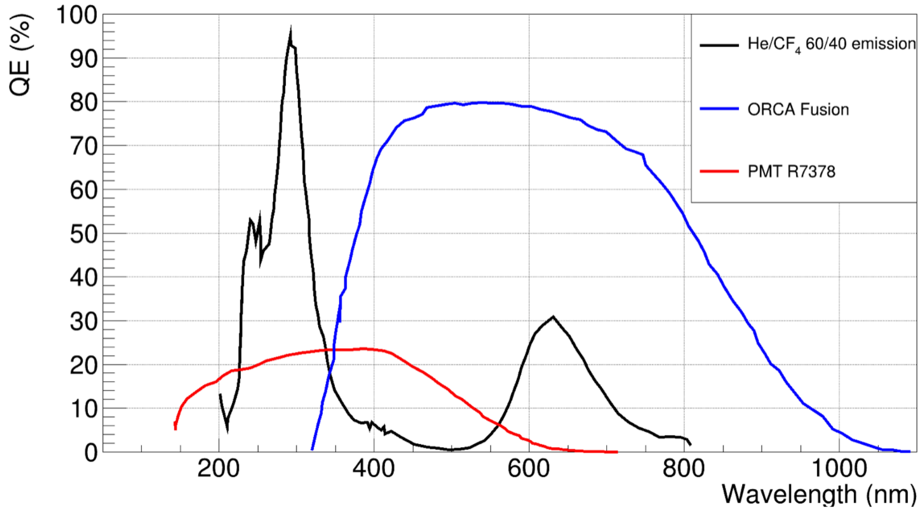
Light Yield Analysis, NID electron stripping and amplification model

CERN spectral measurements

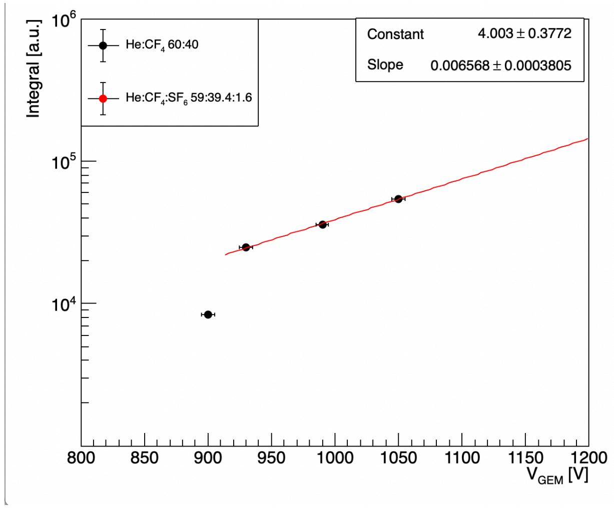


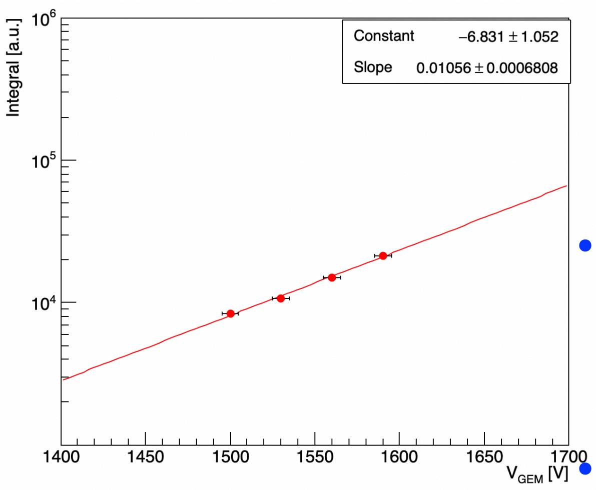


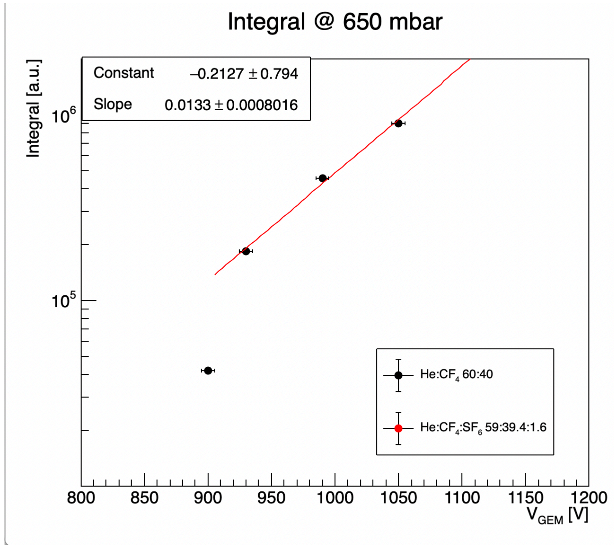


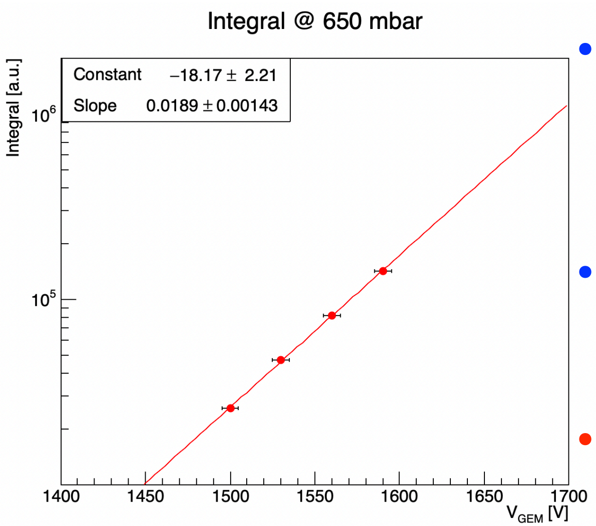


Back on the envelope charge gain evaluation









$$G_{tot} = G_1 \cdot G_2 \cdot G_3 = \frac{(ce^{\alpha V_{\mathrm{GEM}}})^3 \sigma^3}{\sigma^3 + (p/V_{\mathrm{GEM}})(ce^{\alpha V_{\mathrm{GEM}}})^2 (ce^{\alpha V_{\mathrm{GEM}}} - 1)}$$

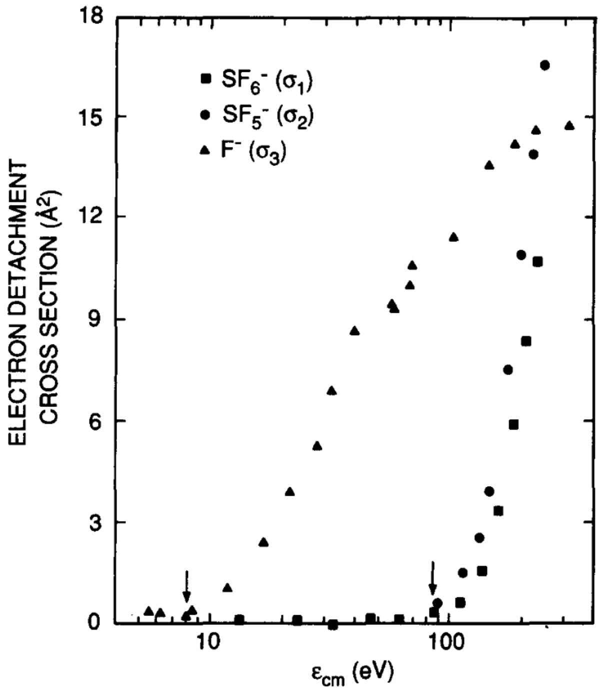
$$\alpha = (2.06 \pm 0.02) \times 10^{-2}\mathrm{V}^{-1},$$
$$c = (1.13 \pm 0.06) \times 10^{-2},$$
$$p = (3.7 \pm 0.2) \times 10^4 \ \mu\mathrm{m}^3\mathrm{V}$$

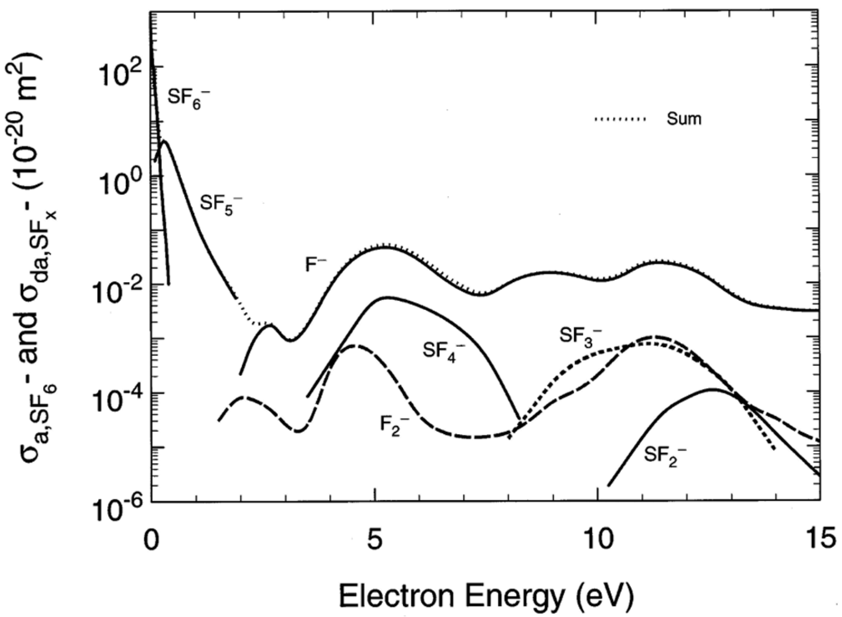
- **With our gain parameterization extract ED gain @ 1050 VGEM and rescale for 900 mbar, we obtain 3.7 x 10^3**
- **If we scale from 1050 VGEM to 930 VGEM with the fitted function, we obtain 750**
- **ALTERNATIVELY, with our gain parameterization extract ED gain @ 930 VGEM and rescale for 900 mbar, we obtain 330**
- **By rescaling for the spectrum ration, we obtain 3.7 x 10^3 /1.7 x 10^3**
- **Bottom line: NID charge gain at 900 mbar and 650 mbar O(10^3)**

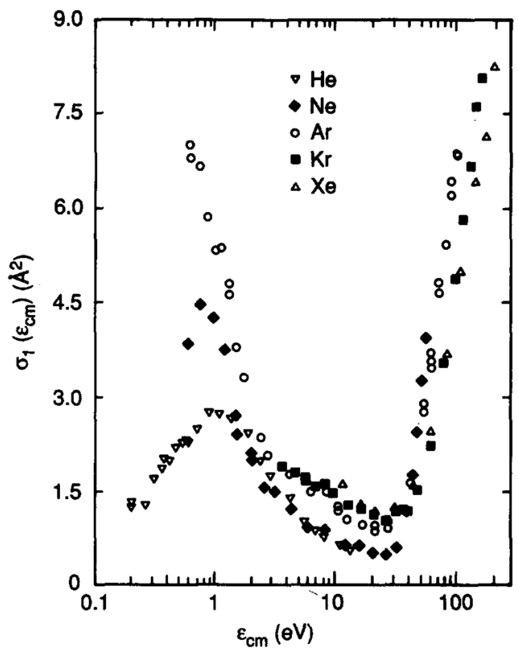
Trying to get insight in the amplification process

Gain: why need such high amplification strength?

1) SF6 electron detachment through collisional processes



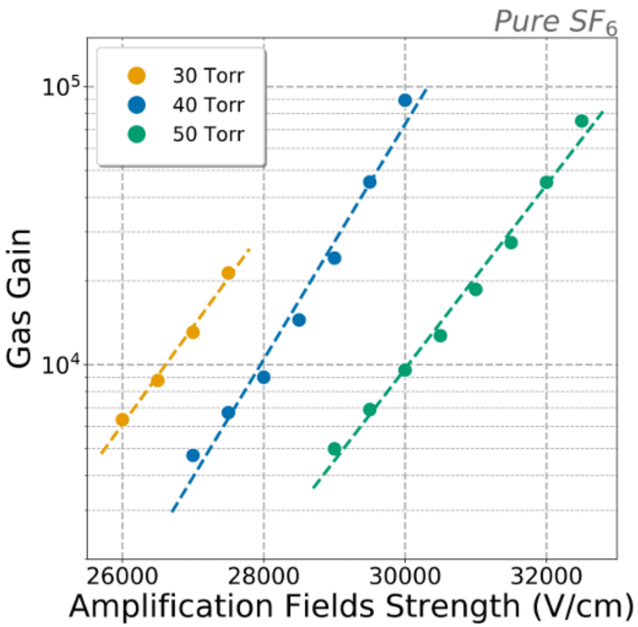




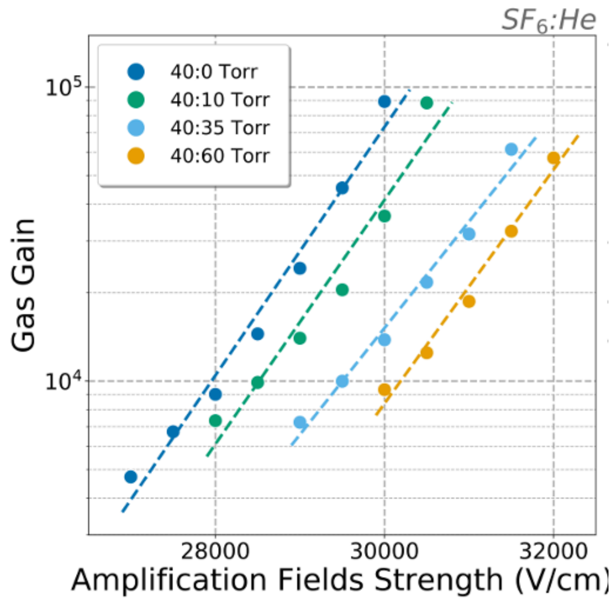
Collisional detachment is highly favoured in presence of noble gases!!!!

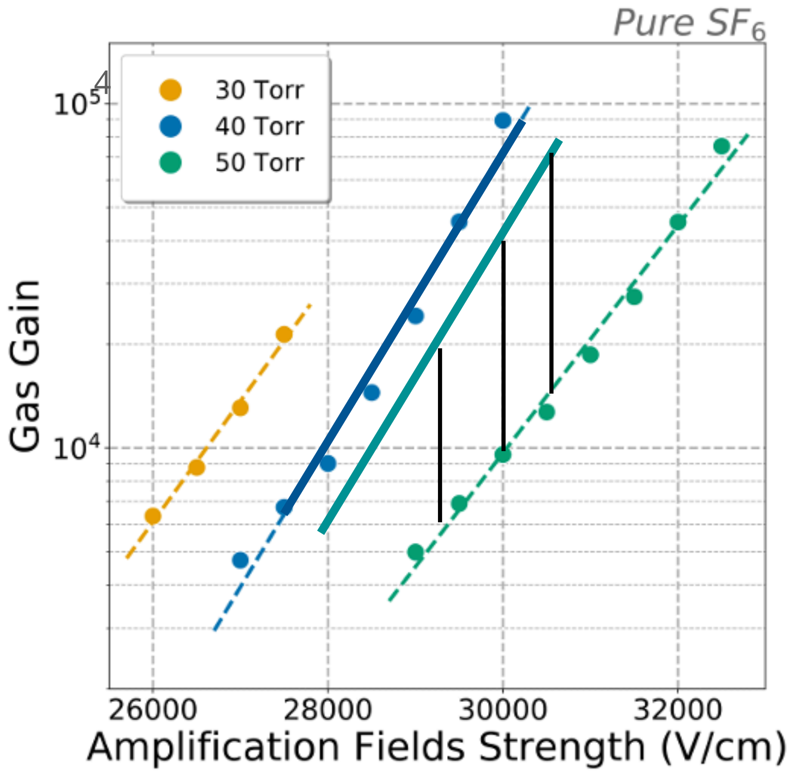
In pure SF_6, detachment strongly suppressed

Is this verified by other published data with our same mixture? YES



$$\frac{\mathrm{d}G}{G} = -\frac{\lambda \ln 2}{\Delta V \, 2\pi\varepsilon_0} \frac{\mathrm{d}\rho}{\rho} .$$

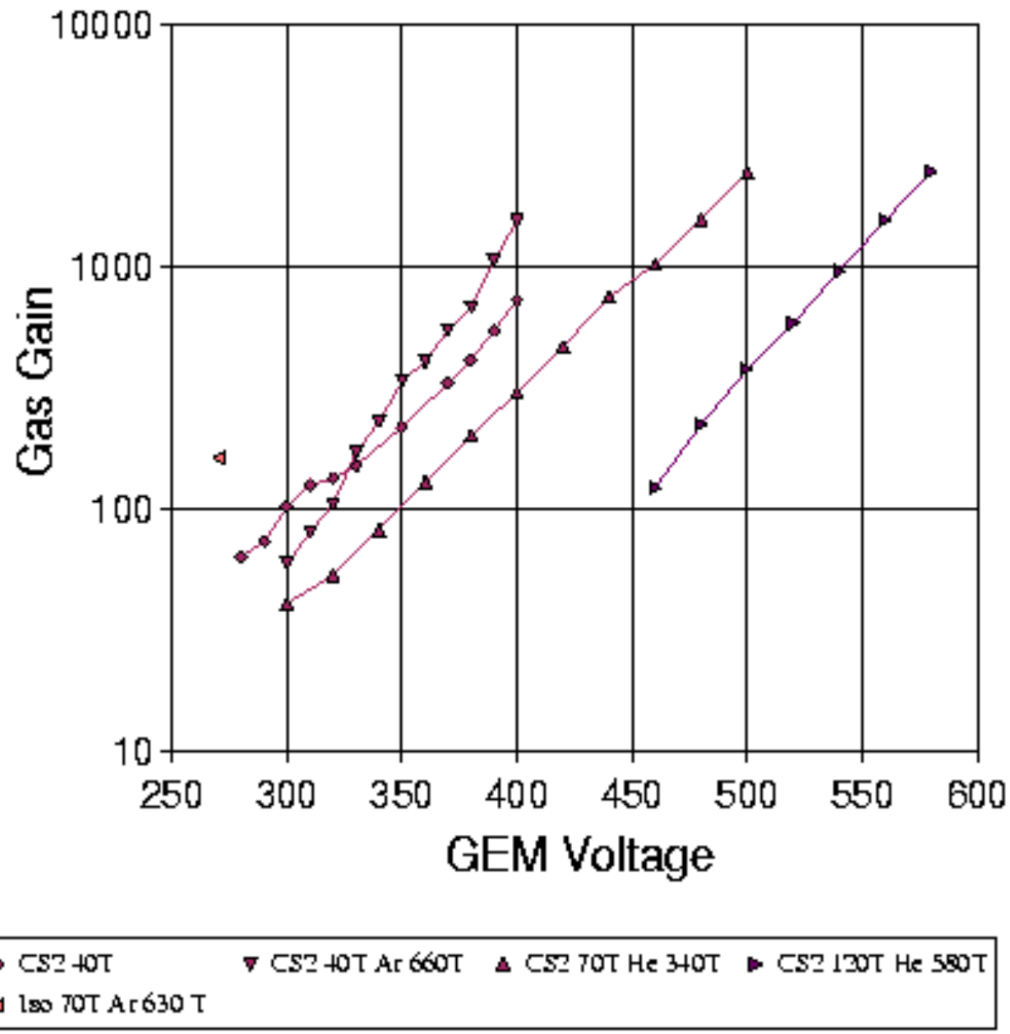




Is this verified by other published data with other mixtures? YES again

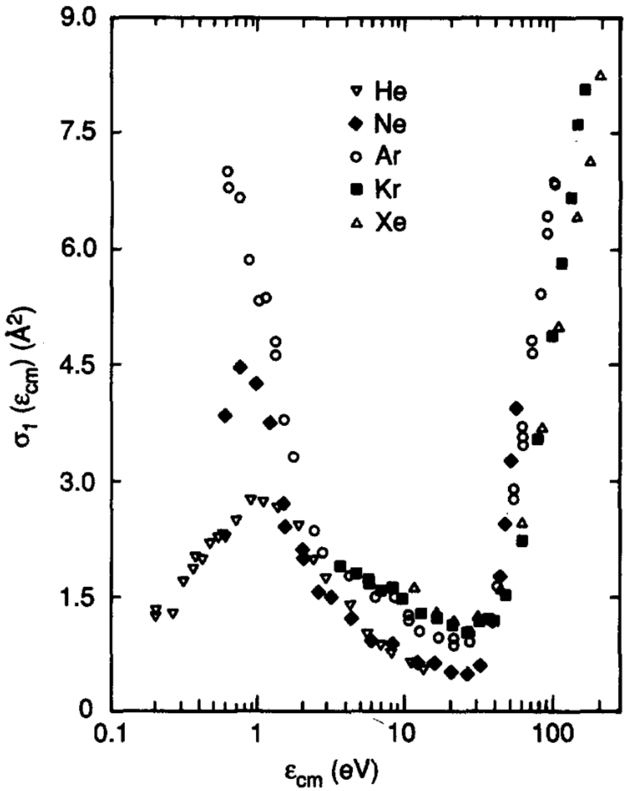
CS_2:Ar 40:660 Torr "scales" better than just because of pressure w.r.t. pure CS_2 at 40 Torr

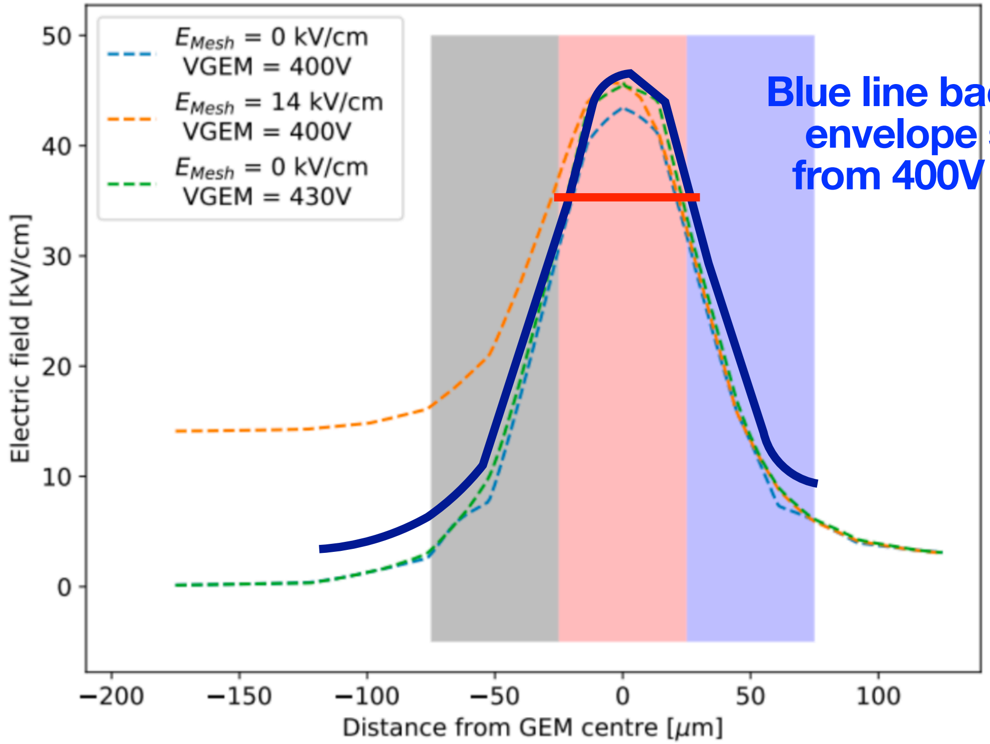
$$\frac{dG}{G} = -\frac{\lambda \ln 2}{\Delta V 2\pi\varepsilon_0}\frac{d\rho}{\rho}.$$



Note: no CS_2 collisional detachment cross section measurements exist

A closer look at our data





Blue line back on the envelope scaling from 400V to 550V

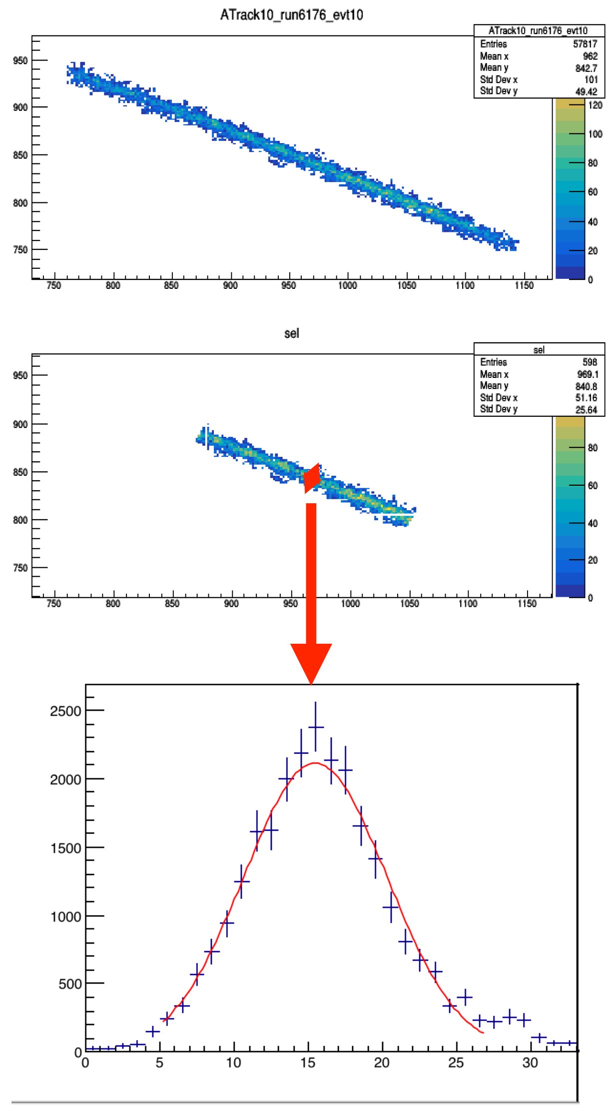
- He and CF_4 can be assumed to be at the thermal energy, i.e. 25 meV
- The energy of the center of mass for V_{GEM} [1500, 1600] in the collision with SF_6^- is
  - ± 0.5 eV with He
  - ± 7 eV with CF_4
- No collisional detachment cross section measurement for SF_6^- on CF_4 exists
- It might be similar to Kr if at first order matter only the mass, but no way to know for sure…
- In any case, the He channel opens "immediately" (i.e. as soon as the anions enter the GEM hole field), while the CF_4 one has an "inner" (w.r.t. GEM hole entrance) onset
  - If similar to Kr, once "open", the CF_4 channel might dominate over the He due to higher cross section

NOTE: since > 500 V_{GEM} is needed to strip electrons, NID gain is intrinsically somehow saturated

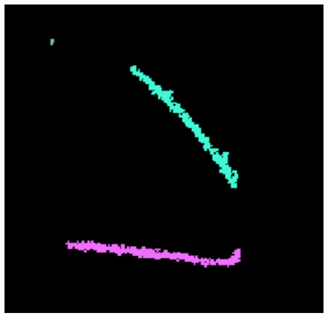
From NID electron stripping and amplification model to transverse profile double gaussian

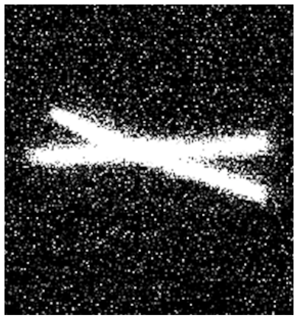
Recap: ED sCMOS images analysis

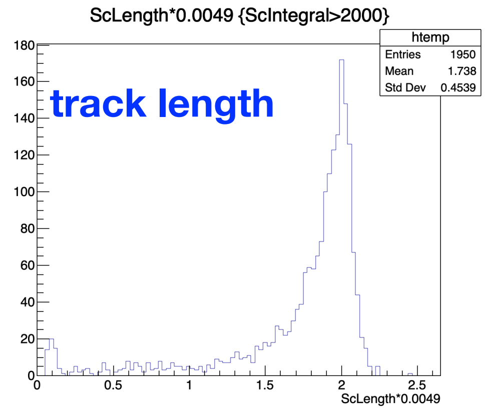
- Alpha tracks selection:
  - tracks reconstructed with iterative DBSCAN algorithm [10]
  - track length > 1.5 cm
  - track slimness < 0.3
  - X and Y track barycenter within ED cosmic ray map cuts (see previous slide)
  - Chi2/nDOF of transverse fit profile < 2 (remove additional multiple tracks)
  - **Selected tracks profile within 100 pixels from the barycenter fitted with a single Gaussian**

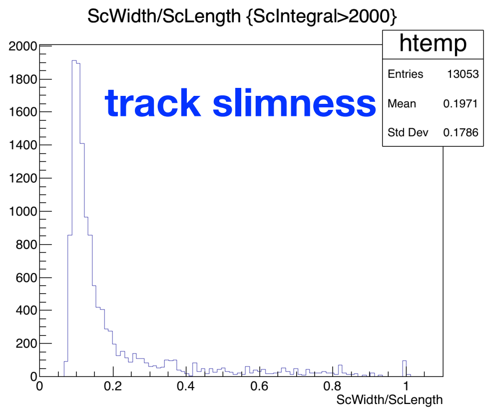


Sigma of track profile and track integral fitted with single Gaussian to estimate diffusion and light yield





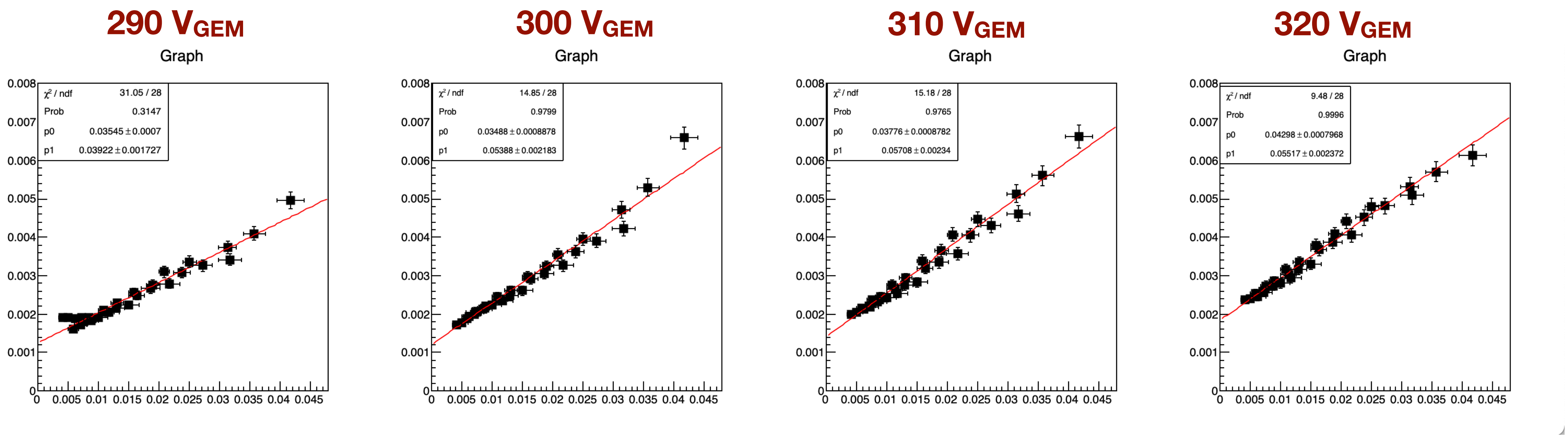




ED analysis to demonstrate methodology robustness

Track transverse profile sigma as a function of L/E for different V_{GEM} (aka different LY)

- **Alphas tracks selection, ED, He:CF_4 60:40 @ 650 mbar**
  - V_{GEM} scan: 290 V, 300 V, 310 V, 320 V on each GEM
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - Total of 120 data sets



$$\sigma^2_{meas} = \sigma^2_0 + \frac{4\epsilon_K}{3e}\frac{L}{E} = \sigma^2_0 + \frac{2kT_{eff}}{e}\frac{L}{E} = p^2_0 + 2p_1\frac{L}{E}$$

$$\epsilon_K = \frac{3}{2}\frac{kT_{eff}}{e}$$

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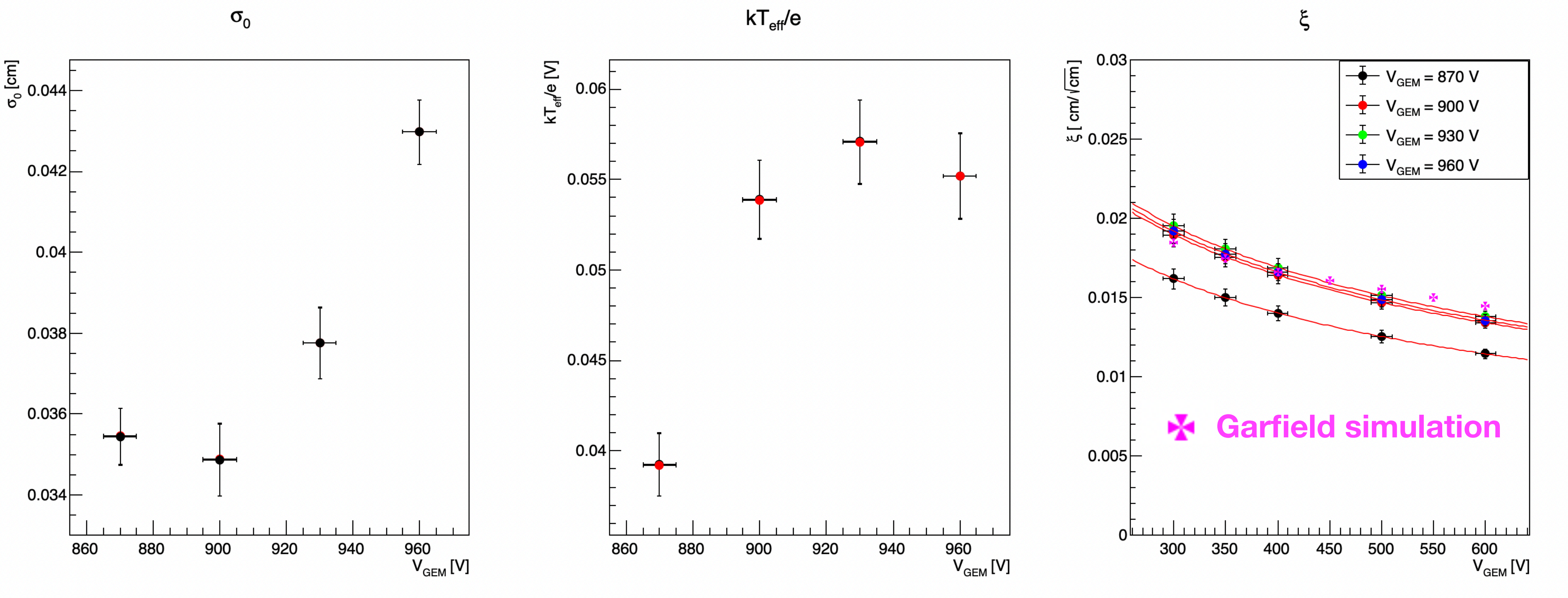
Above 290 V, ED data perfectly consistent among them and with Garfield simulation



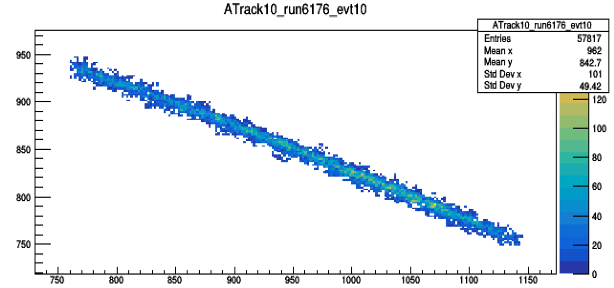
$$\sigma^2_{meas}(V_{GEM}) = \sigma^2_0(V_{GEM}) + \frac{4\epsilon_K(V_{GEM})}{3e}\frac{L}{E} = \sigma^2_0 + \frac{2kT_{eff}(V_{GEM})}{e}\frac{L}{E} = p^2_0 + 2p_1(V_{GEM})\frac{L}{E}$$

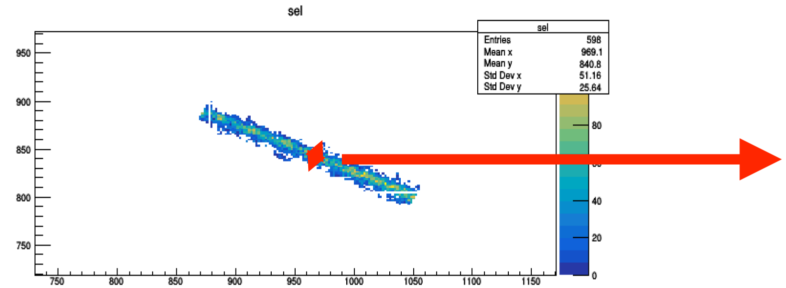
Take away message:

The experimental setup and the analysis methodology is demonstrated to be robust against:

- **Simulation**
- **Data taken with other detectors/setups**
- **LY variation of one order of magnitude**

Recap: NID sCMOS images analysis, aka the infamous double gaussian





- Alpha tracks selection (same as ED):
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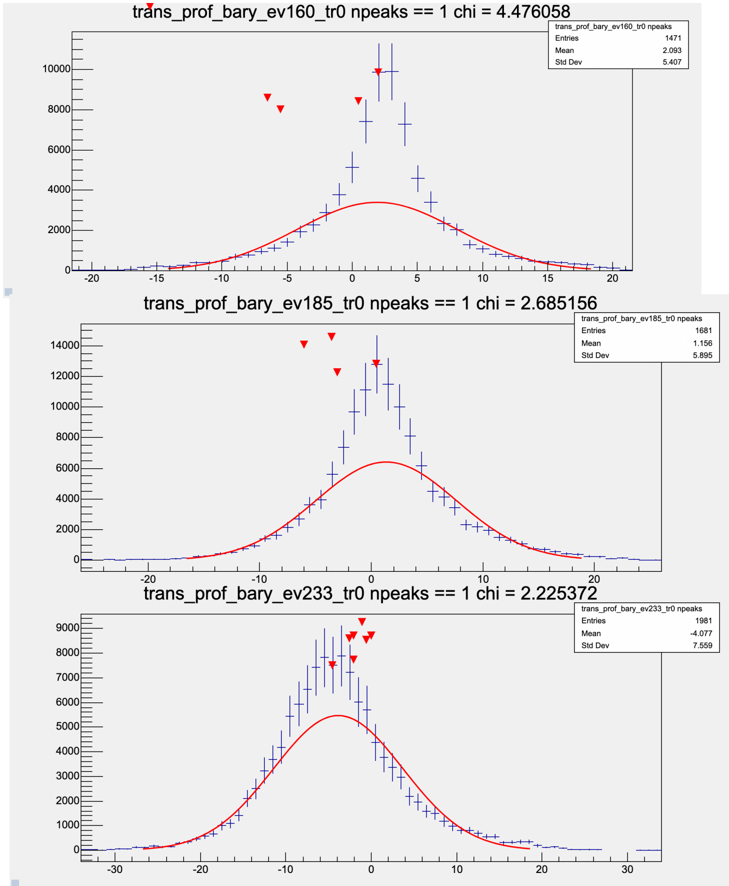

535V 2cm 600V/cm

535V 6cm 400V/cm

535V 12cm 300V/cm

NID transverse profile is NOT a single Gaussian!!!

It is the sum of 2 Gaussian, with proportions and sigmas depending on V_{GEM} (aka gain, aka LY) and L/E!!!

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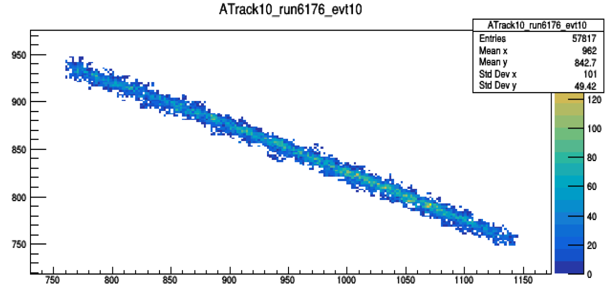

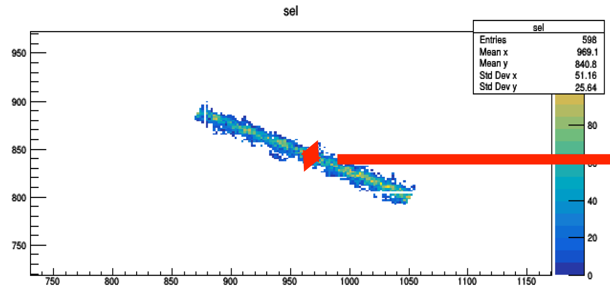

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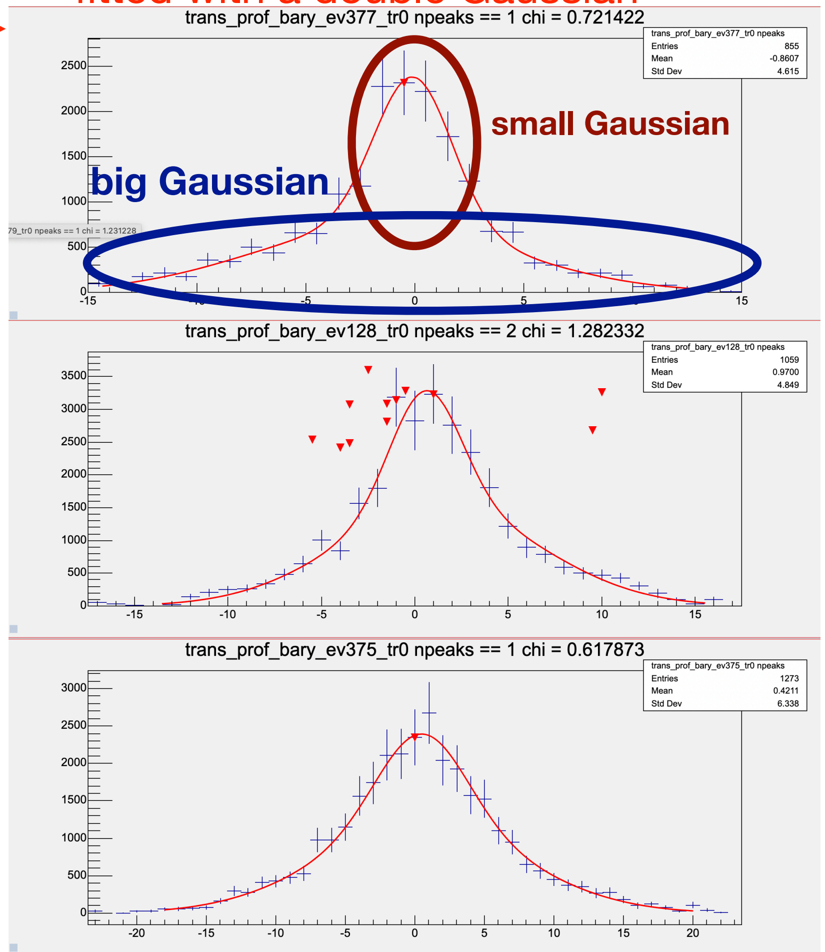

535V 2cm 600V/cm

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535V 12cm 300V/cm

- **Now the measured diffusion has 6 parameters**
  - sigma and mean of the "small" (i.e. less diffused) Gaussian
  - sigma and mean of the "big" (i.e. more diffused) Gaussian
  - Normalisation of the "small" and "big" Gaussian

$$f(y) \sim \mathcal{N}_s\left[\mu_s, \sigma_s\right] + \mathcal{N}_b\left[\mu_b, \sigma_b\right]$$

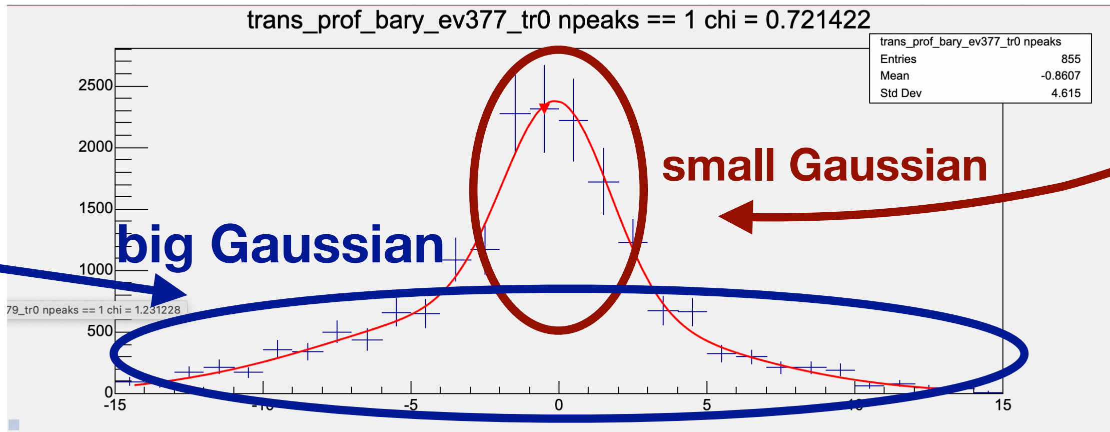
where y is the measured alpha tracks transverse profile

The small Gaussian: where the hell does this come from?

Focusing hypothesis

A focusing regime sets in once a threshold number of negative ions $N_{i^-}^{\mathrm{th}}$ have entered a GEM hole and undergone detachment, producing a sufficient positive ion backflow to attract subsequent i^- into the same hole.

- Below threshold: charge splits among neighbouring holes (no focusing).
- Above threshold: local field distortion leads to collapse into a single hole (focusing).



Ion back flow and GEM gain

Our ansatz: the ion backflow (differently from standard ED case) is seen instantaneously by SF_6^- because of the much slower drift velocity and acts as a "funneling" additional field into the GEM holes

Time-scale argument: why NID is special

- Negative ions drift slowly: typical arrival times at the GEM are in the ms range.
- Positive ions produced in the GEM drift much faster in the transfer field.

Back-of-the-envelope estimate:

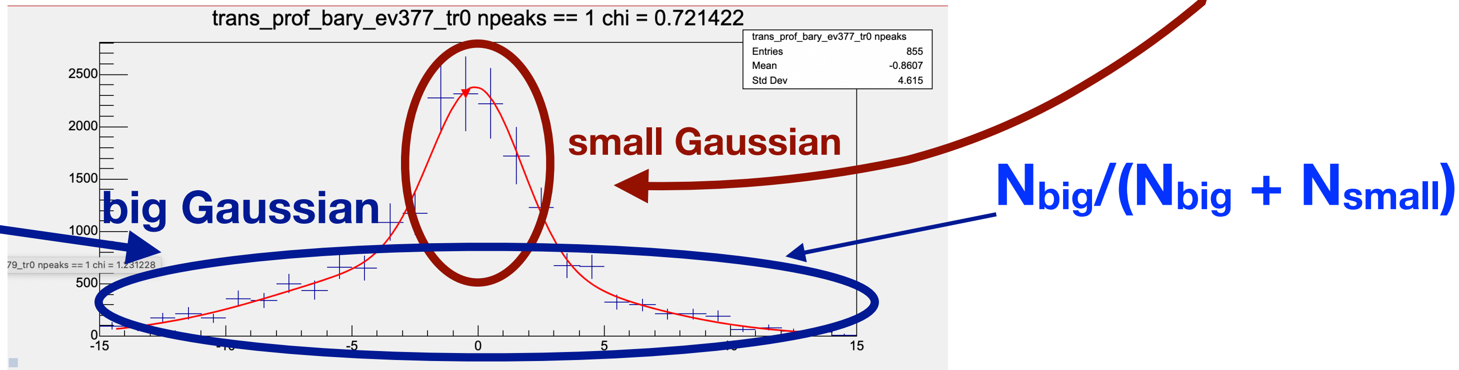
$$t_{i^+}^{\text{clear}} \sim \frac{\Delta z}{\mu_+ E} \sim 10^{-5} - -10^{-4} \text{ s}, \quad (3)$$

while the NID arrival time spread is

$$t_{i^-}^{\text{feed}} \sim \mathcal{O}(10^{-3}) \text{ s}. \quad (4)$$

Conclusion: during NID arrival, ion backflow is continuously regenerated and acts quasi-instantaneously on subsequent ions.

- Below threshold: charge splits among neighbouring holes (no focusing).
- Above threshold: local field distortion leads to collapse into a single hole (focusing).



The number of positive ions produced in the avalanche is proportional to the GEM gain:

$$G_{\mathrm{GEM}}(V_{\mathrm{GEM}}) = a, e^{bV_{\mathrm{GEM}}}. \quad (1)$$

For a given number of negative ions entering a hole:

$$N_{i^+} = G_{\mathrm{GEM}}, N_{i^-}. \quad (2)$$

By purely logical and physical arguments, the number of NID necessary to initiate the focusing effect N^{TH}_{i-}, i.e. the number of NID that end up in the big Gaussian:

- **will be a fraction of the total number of NID**
- **will inversely depend on L/E, i.e. the larger the cloud arrive at the GEM the lower the number of NID in the same hole, hence the larger the fraction of initial NID needed to initiate focusing**
- **will directly depend on the setup effective gain, i.e. for a given L and E, hence given the same number of NID that enters a single hole, the larger the gain the larger the IBF for each single NID, the faster the focusing effect is initiated**

LY as a better proxy for gain given the complex NID amplification mechanism

The measured light integral I is proportional to the total avalanche charge:

$$I \propto N_{\text{aval}} \propto G_{\text{GEM}}, N_{i^-}^{\text{in}}. \tag{5}$$

At fixed drift conditions (L, E):

$$I = I_0(L, E), e^{bV_{\text{GEM}}}, \tag{6}$$

where $I_0(L, E)$ absorbs geometry, diffusion, and optical factors.

Ratio of $N_{big}/(N_{big} + N_{small})$ minimal fitting function

Define $R_{\text{no}}(I)$ as the fraction of charge in the no-focusing component.

Physical constraints:

- $R_{\text{no}}(I = 0) = 1$ (no amplification, no focusing)
- $R_{\text{no}}(I \to \infty) = R_\infty$ (residual geometrical fraction)
- $R_{\text{no}}(I)$ must be monotonic and bounded.

Remove the asymptotic plateau:

$$S(I) = \frac{R_{\mathrm{no}}(I) - R_\infty}{1 - R_\infty}. \tag{7}$$

$S(I)$ represents the survival fraction of the no-focusing regime:

$$S(0) = 1, \qquad S(\infty) = 0. \tag{8}$$

Assume an exponential survival controlled by a cumulative focusing strength $\Lambda(I)$:

$$S(I) = e^{-\Lambda(I)}. \tag{9}$$

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The simplest monotonic choice consistent with dimensional analysis is:

$$\Lambda(I) = \left(\frac{I}{I_0}\right)^n, \tag{10}$$

leading to:

$$\boxed{R_{\text{no}}(I) = R_\infty + (1 - R_\infty)\,\text{exp!}\left[-\left(\frac{I}{I_0}\right)^n\right]} \tag{11}$$

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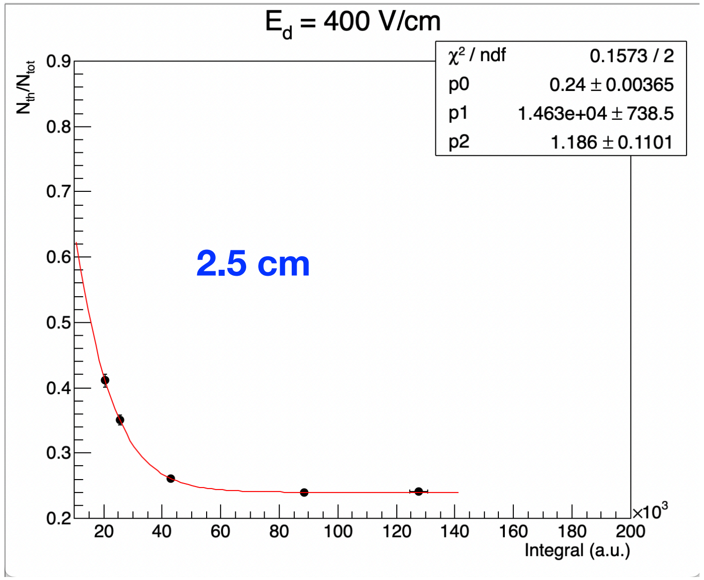
This is the exact function used in the fits.

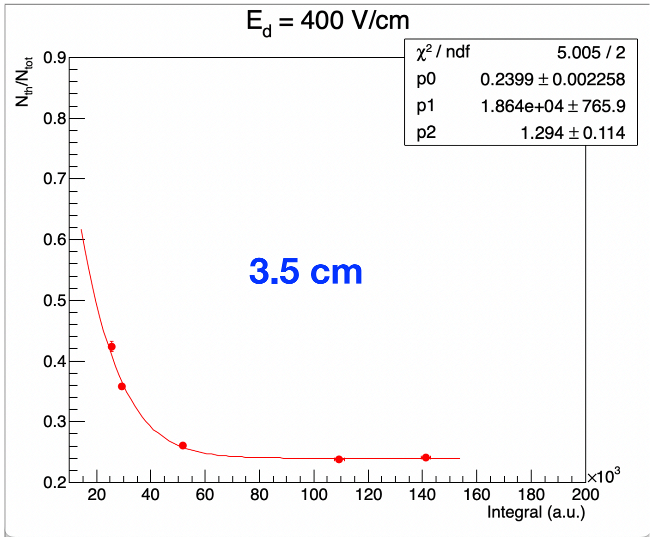
- R_∞: irreducible non-focusing fraction (diffusion + geometry).
- I_0: characteristic intensity required to trigger focusing (backflow threshold).
- n: sharpness of the transition, encoding fluctuations, detachment efficiency, and local field non-linearities.

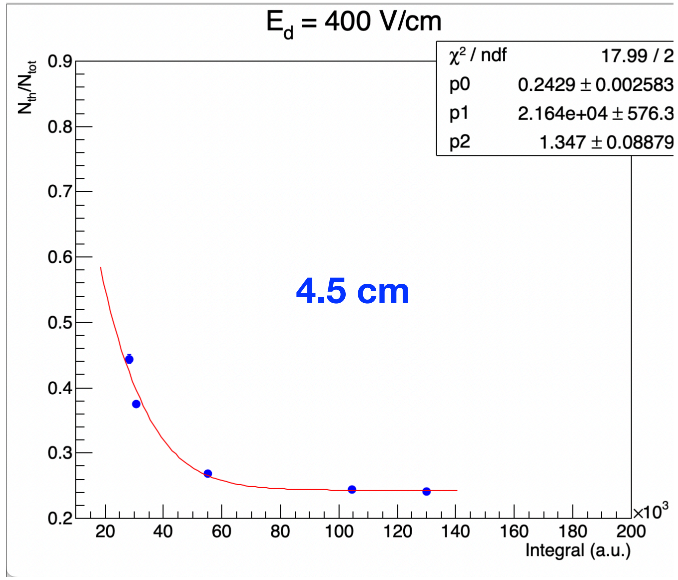
All parameters are expected to depend on transverse diffusion through σ_T.

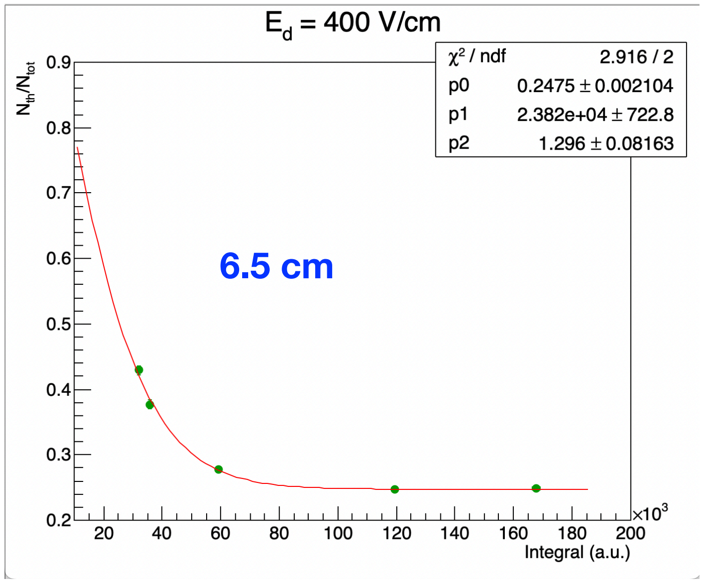
$N_{big}/(N_{big} + N_{small})$ at 400 V/cm fitted with

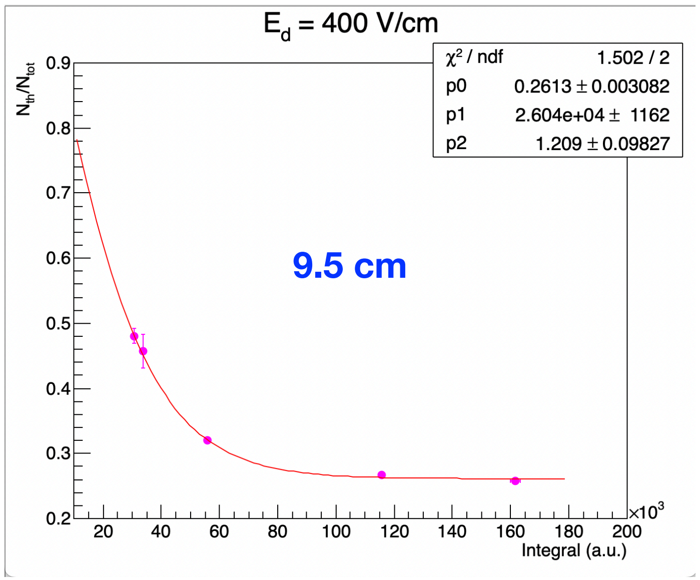
$$R_{no}(I) = R_\infty + (1 - R_\infty)\,\mathrm{exp!}\left[-\left(\frac{I}{I_0}\right)^n\right]$$

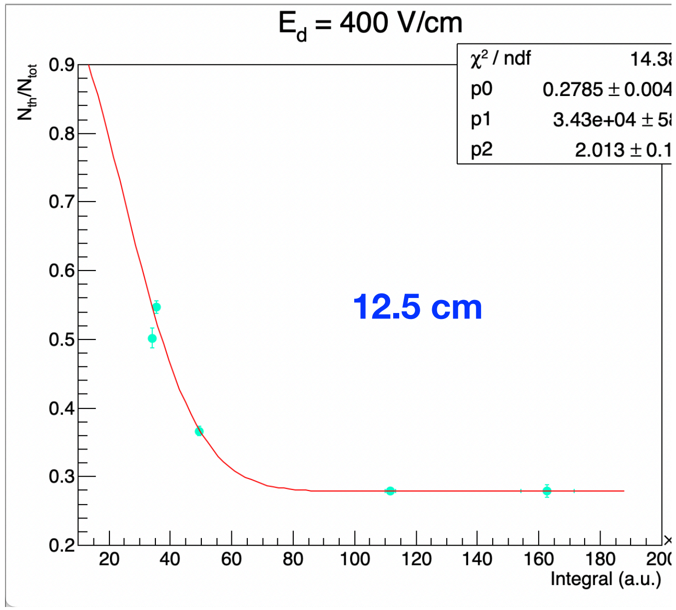












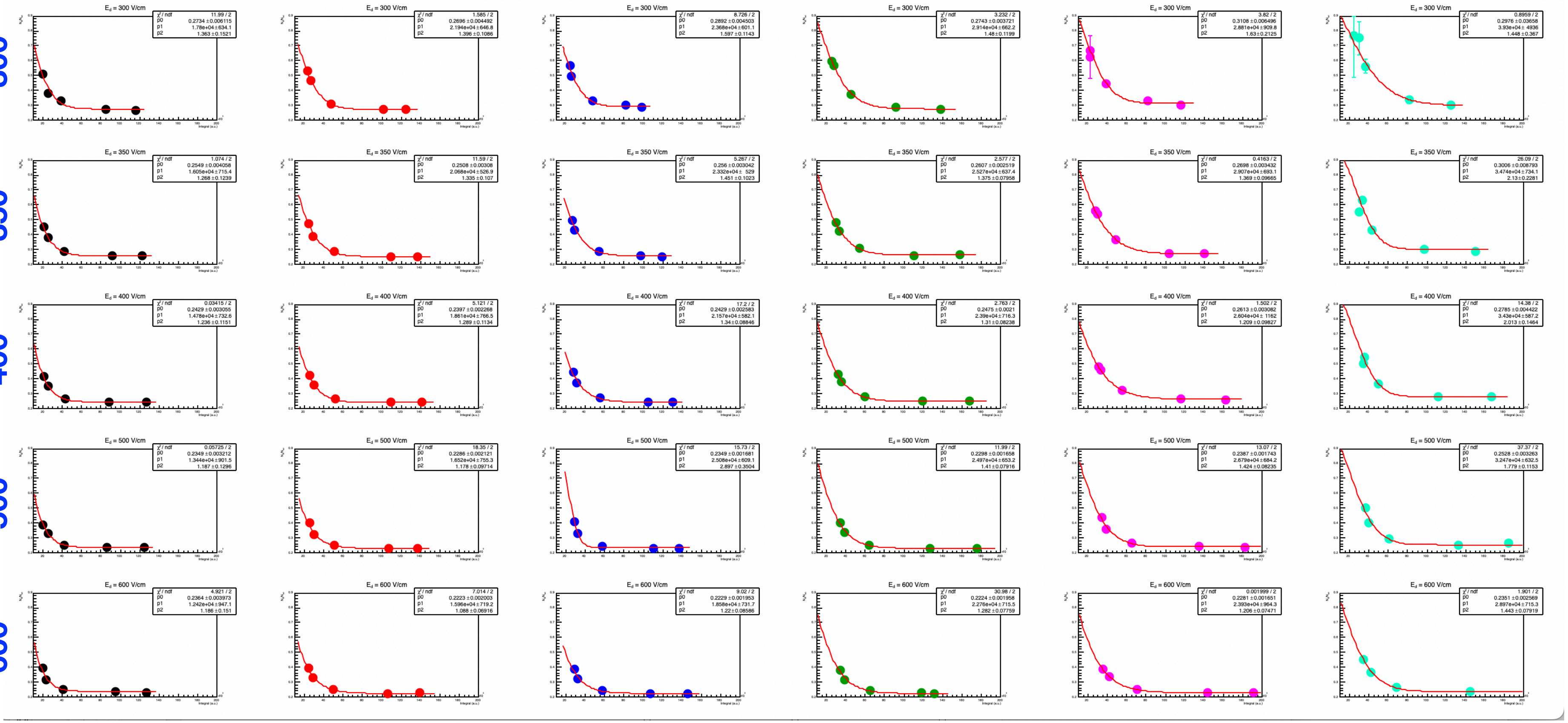
All data together
30 datasets all fitted by same function

Drift Distance

2.5 3.5 4.5 6.5 9.5 12.5

Drift field

300 350 400 500 600



Which dependences to expect?

- R_∞: irreducible non-focusing fraction (diffusion + geometry).
- I_0: characteristic intensity required to trigger focusing (backflow threshold).
- n: sharpness of the transition, encoding fluctuations, detachment efficiency, and local field non-linearities.

All parameters are expected to depend on transverse diffusion through σ_T.

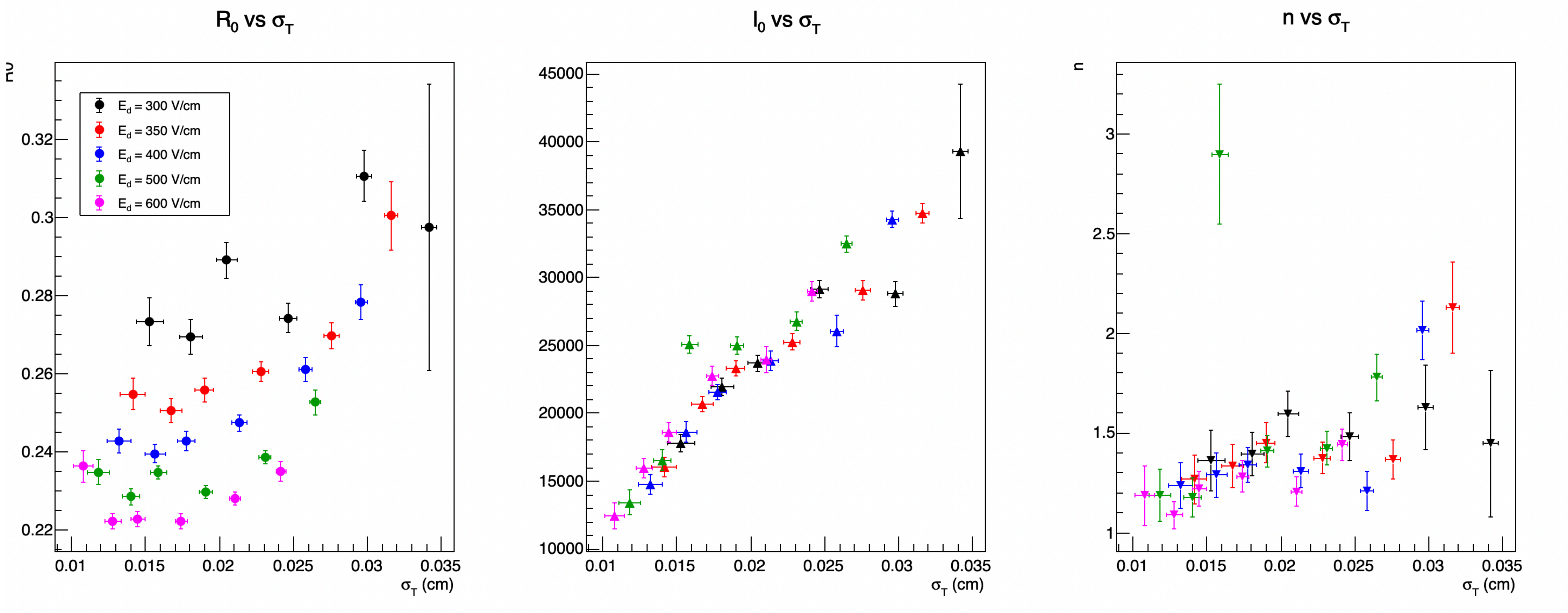
- $\sigma_T^2 = 2\frac{k_B T}{e}\frac{L}{E}$ controls charge density per hole.
- Increasing σ_T lowers local density, increasing I_0 and R_∞.
- The measured evolution of (R_∞, I_0, n) vs σ_T provides a direct probe of focusing physics.

In the thermal (Einstein) regime for NID transport,

$$\sigma_T^2(L, E) = 2D_T(E), \frac{L}{v_d(E)} \simeq 2, \frac{k_B T_{\text{eff}}}{e}, \frac{L}{E}. \tag{12}$$

Therefore, plotting results vs σ_T (instead of L/E) directly tracks the transverse dilution of charge into many holes.

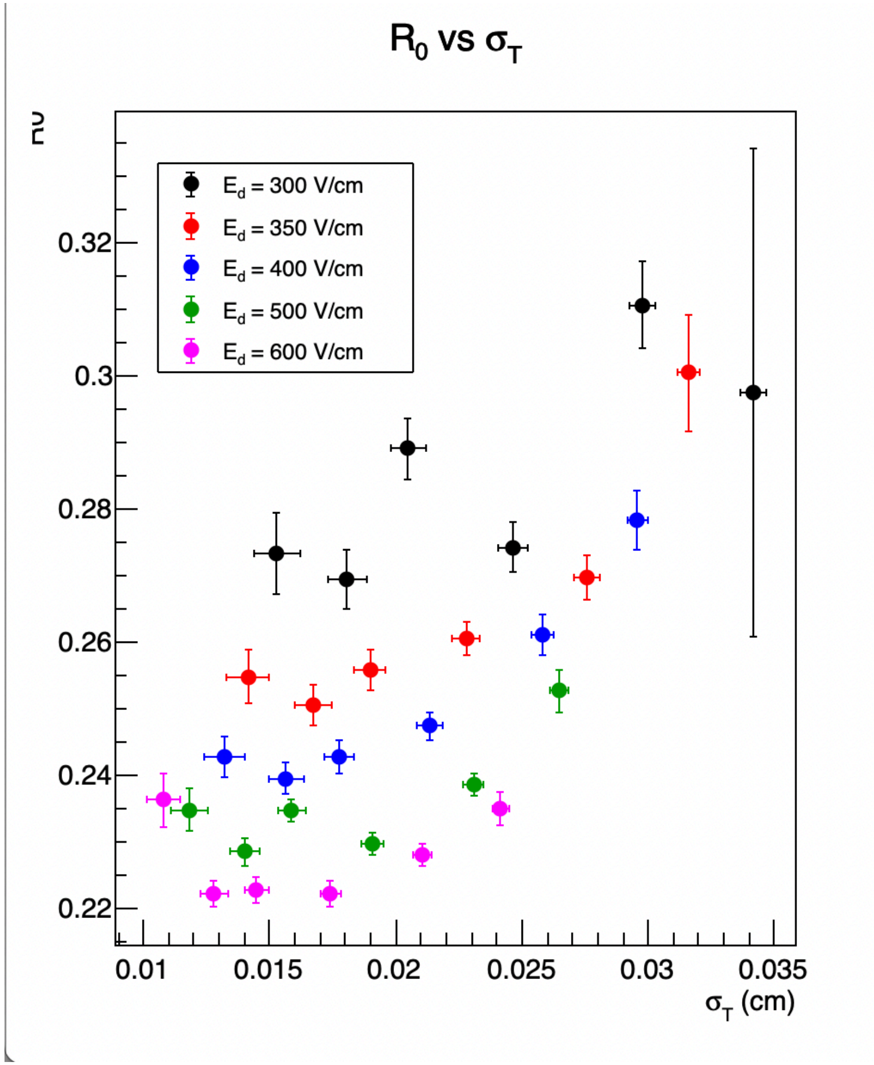
- Larger $\sigma_T \Rightarrow$ lower charge density per hole $\Rightarrow$ focusing becomes harder.
- This must primarily shift the intensity scale I_0, and can also change the residual plateau R_∞.



From the focusing ansatz (threshold in charge density per hole):

- **Intensity scale** $I_0(\sigma_T)$: expected to *increase* with σ_T.
- **Plateau** $R_\infty(\sigma_T)$: can *increase mildly* with σ_T (irreducible splitting into many holes).
- **Sharpness** $n(\sigma_T)$: can vary (transition becomes smoother if local-density fluctuations average out).

Operationally, we test universality by collapsing different (L, E) onto the same curves when re-expressed in σ_T.



$$R_\infty(\sigma_T) = \exp\left(-\frac{r_{\text{foc}}^2}{2\sigma_T^2}\right)$$

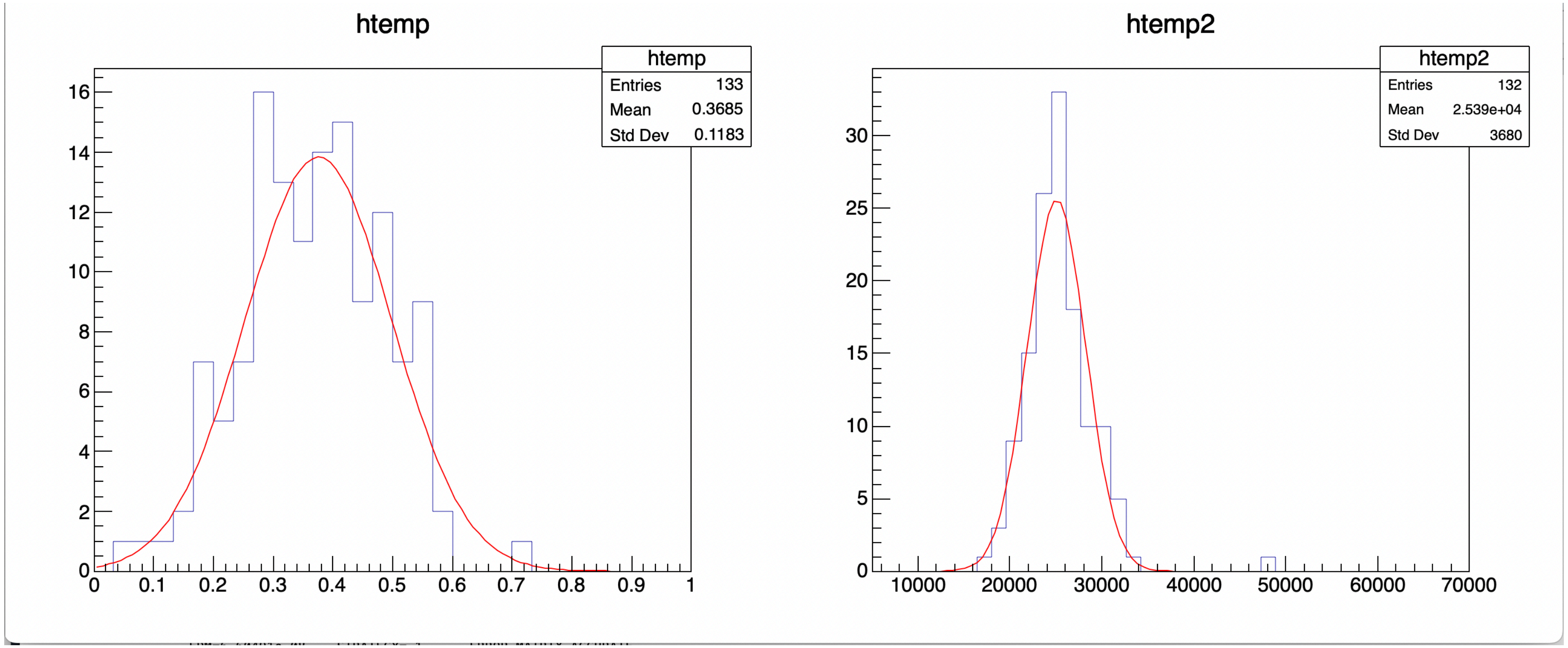
$$\frac{r_{\text{foc}}^2}{2\sigma_T^2} \ll 1$$

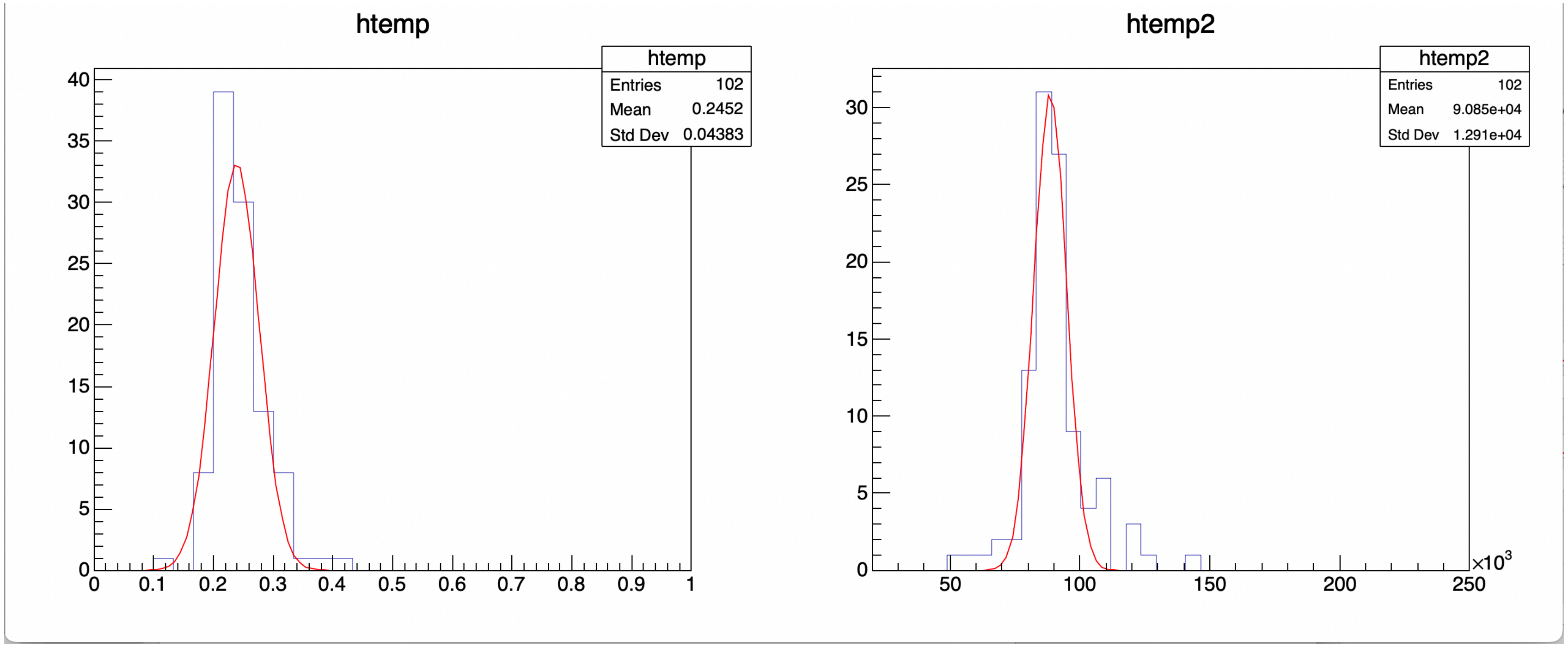
$$R_\infty(\sigma_T) \simeq 1 - \frac{r_{\text{foc}}^2}{2\sigma_T^2} \quad \Longrightarrow \quad R_\infty \approx a + b\,\sigma_T^2$$

The small Gaussian measures the GEM-limited transverse resolutic

- Independent of drift distance.
- Independent of diffusion.
- Fully determined by (pitch, pixel, optics, GEM staggering).
- Provides a direct experimental proof that NID + IBF focusing eliminates diffusion.

This is the "ultimate resolution" regime of NID TPCs.


htemp
htemp
Entries 133
Mean 0.3685
Std Dev 0.1183
htemp2
htemp2
Entries 132
Mean 2.539e+04
Std Dev 3680


htemp
htemp
Entries 102
Mean 0.2452
Std Dev 0.04383
htemp2
htemp2
Entries 102
Mean 9.085e+04
Std Dev 1.291e+04
×10^3

The small Gaussian: where the hell does this come from?

Our ansatz: the ion backflow (differently from standard ED case) is seen instantaneously by SF_6^- because of the much slower drift velocity and acts as a “funneling” additional field into the GEM holes

In practice: after some number of N^-_{th} anions have been stripped of their electrons and these electrons have produced $N^+_{th} = G_{GEM} * N^-_{th}$ backflowing ions through avalanche, all the remaining anions are funneled into the same GEM hole

The threshold N^-_{th} anions behaves with standard transport and hence produce the “Big Gaussian” in the trasverse profile with N_{big} anions, while all the remaining anions funneled into the same GEM hole produce the “Small Gaussian” with N_{small} anions

If the ansatz is correct, the ratio of N_{small}/N_{tot} must have a definite and predictable dependence on V_{GEM} and L/E

If the ansatz is correct, operating with 3 fine pitch GEM with 90 um pitch w.r.t. standard GEM of 140 um should produce a small Gaussian with sigma = 90 um

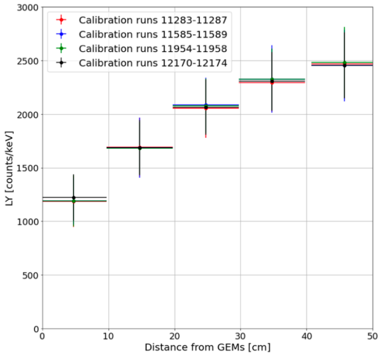
NID diffusion analysis: be careful of saturation!

Track integral vs drift distance as a function of drift field, aka "Saturation plots"

- **Alphas tracks selection, NID, $He:CF_4:SF_6$ 59:39.4:1.6 @ 650 mbar**
  - V_{GEM} scan: 1545 V, 1560 V, 1575 V, 1590 V, 1605 V total V_{GEM} (corresponding to "ladder" 515V, 520V, 525V, 530V and 535V V_{GEM})
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - Total of 150 data sets

- Saturation is when ScIntegral at low distance < ScIntegral at large distance
- NID data 59:39.4:1.6 @ 650 mbar ScIntegral spans from 20k to 150k **(i.e one order of magnitude, same overall values and same span of ED)**
- **NID** data 60:40 @ 650 mbar **are saturating at nearly all field/V_{GEM}**
- **NID data need to be properly selected to avoid additional effects from saturation entering in the diffusion estimation**
- **Note: saturation depends on L/E, aka charge density!**

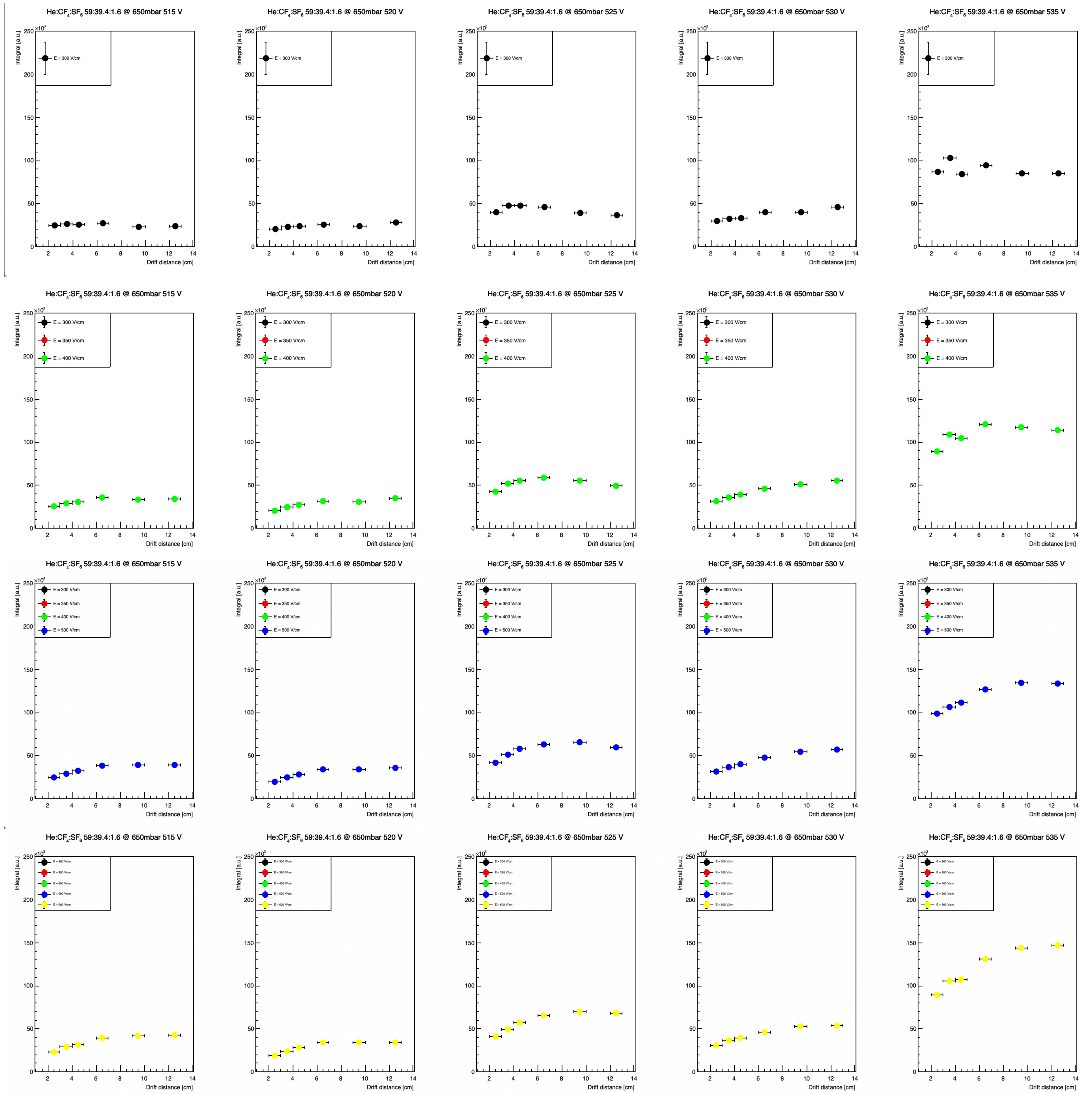
LIME saturated ^{55}Fe integral in Run2



350 V/cm not shown for lack of space

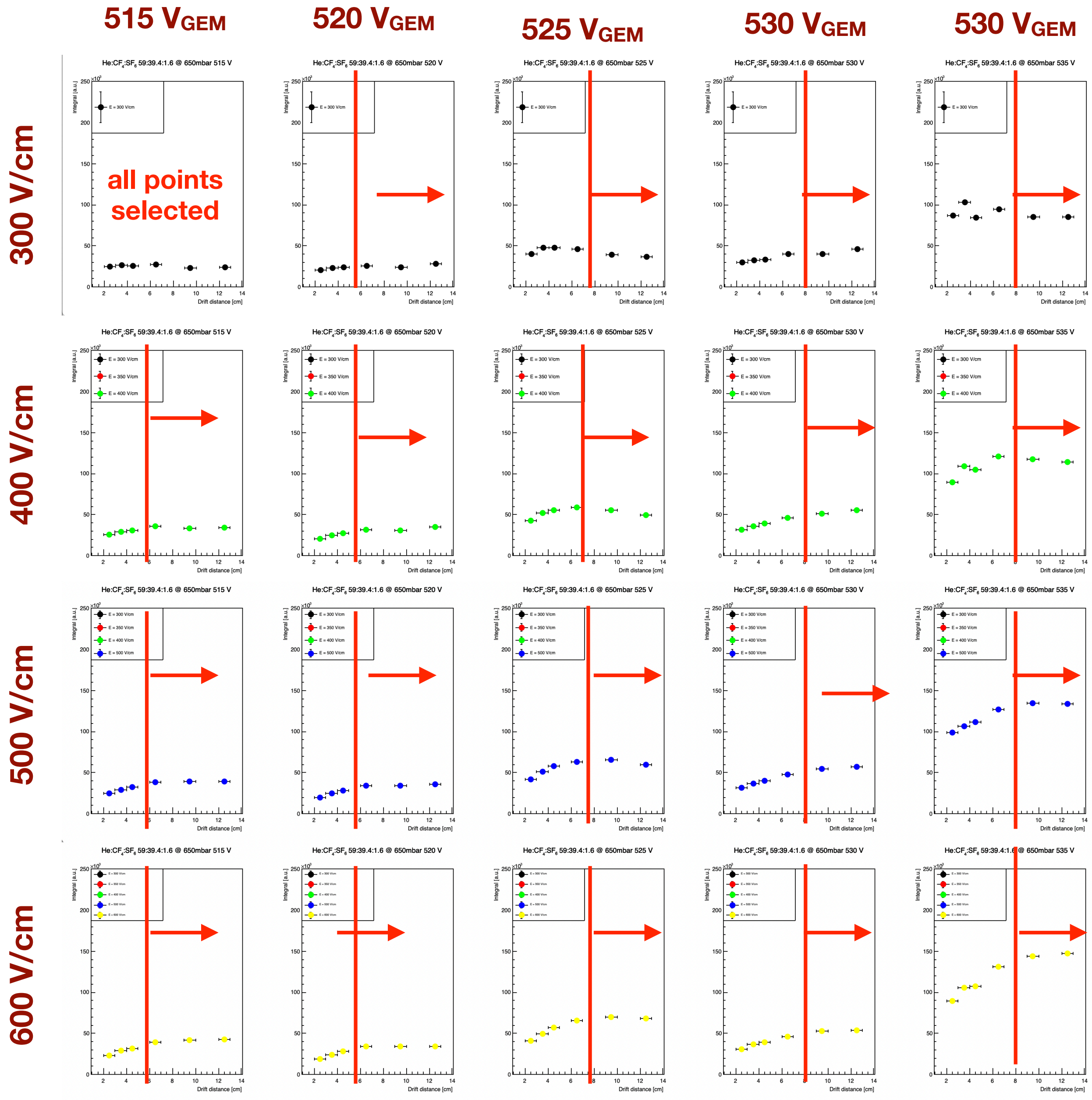
515 V_{GEM} **520 V_{GEM}** **525 V_{GEM}** **530 V_{GEM}** **530 V_{GEM}**

300 V/cm **400 V/cm** **500 V/cm** **600 V/cm**



How to deal with saturation?

Select data sets with constant integral w.r.t. largest distance at a given E_{DRIFT} for each V_{GEM} dataset



350 V/cm not shown for lack of space

Please note: all effects depends on L/E, and actual selection is made on ScIntegral normalised to 12 cm as a function of L/E

*here shown as a function of distance for different E_{DRIFT} for lack of time in redoing the plots ;)

NID small Gaussian diffusion analysis

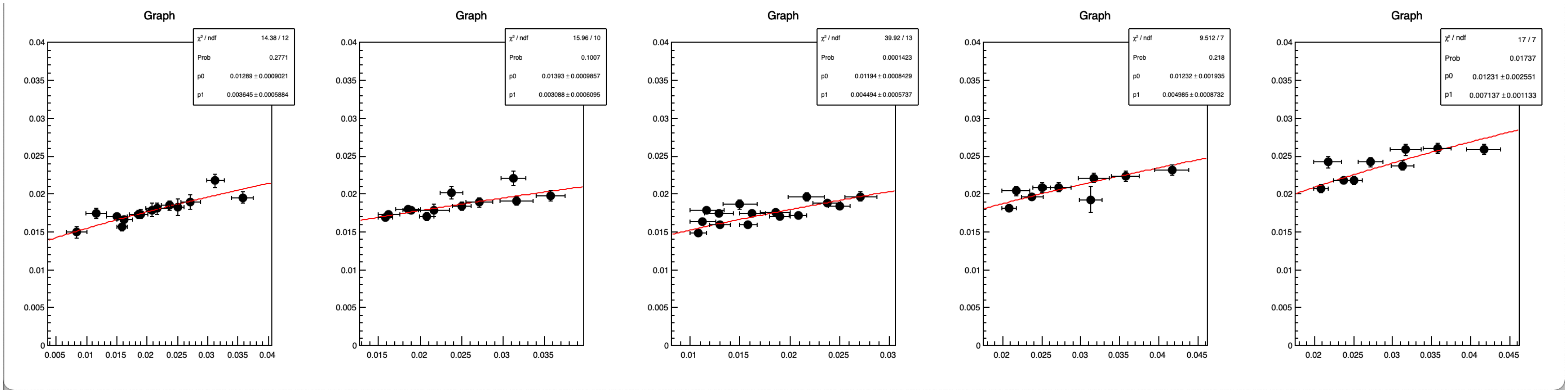
Track transverse profile sigma small as a function of L/E for different V_{GEM} (aka different LY)

- **Alphas tracks selection, NID, $He:CF_4:SF_6$ 59:39.4:1.6 @ 650 mbar**
  - V_{GEM} scan: 1545 V, 1560 V, 1575 V, 1590 V, 1605 V total V_{GEM} (corresponding to “ladder” 515V, 520V, 525V, 530V and 535V V_{GEM})
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - From these sets of data, only “not saturated” data as from previous slide are used in the following analysis

$$f(y) \sim \mathcal{N}_s\left[\mu_s, \sigma_s\right] + \mathcal{N}_b\left[\mu_b, \sigma_b\right]$$

s = small
b = big

515 V_{GEM} **520 V_{GEM}** **525 V_{GEM}** **530 V_{GEM}** **535 V_{GEM}**



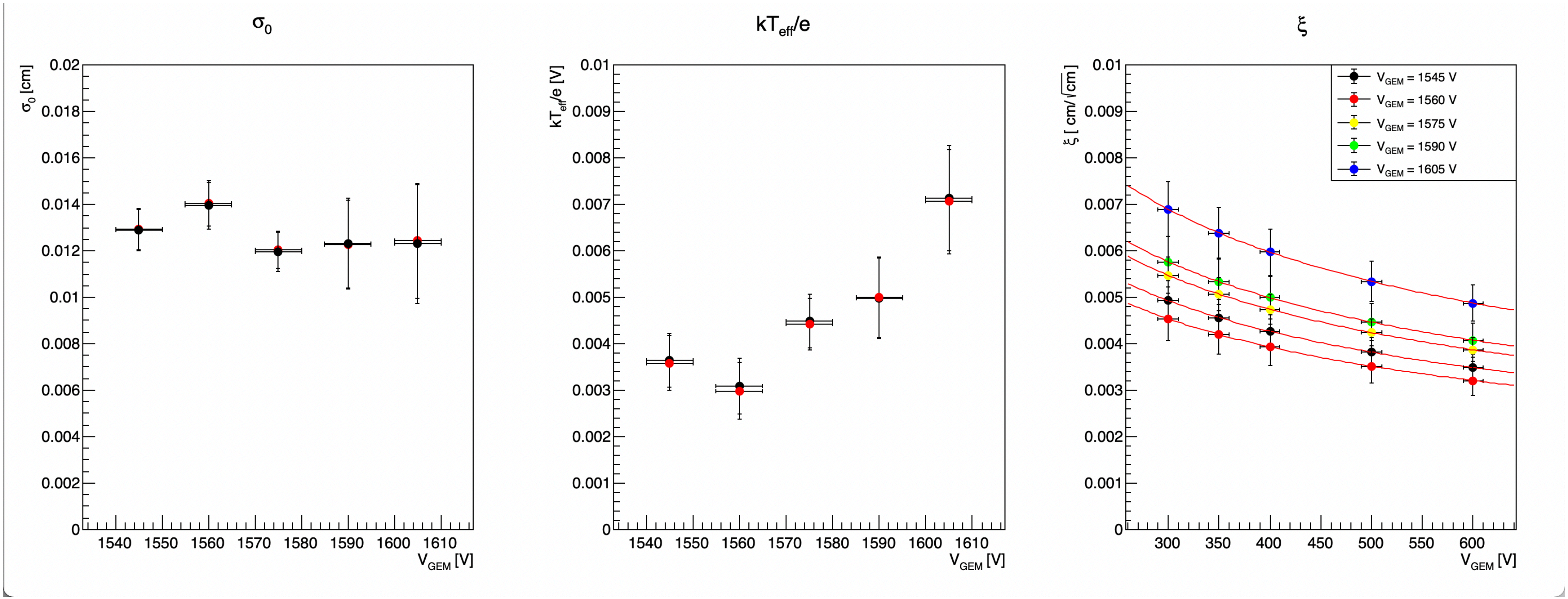
$$\sigma^2_{meas,s} = \sigma^2_{0,s} + \frac{2kT_{eff,s}}{e}\frac{L}{E} = p_0^2 + 2p_1\frac{L}{E}$$

$$p_0 = \sigma_{0,s}$$

$$p_1 = \frac{kT_{eff,s}}{e}$$

NID small Gaussian diffusion analysis

$$\sigma^2_{meas,s}(V_{GEM}) = \sigma^2_{0,s}(V_{GEM}) + \frac{2kT_{eff,s}(V_{GEM})}{e}\frac{L}{E} = p_0^2(V_{GEM}) + 2p_1(V_{GEM})\frac{L}{E}$$



- **sigma$_{0,s}$ is independent of V$_{GEM}$**
- **T$_{eff,s}$ increases with increasing V$_{GEM}$**
- **Small Gaussian effective temperature is ± 5 mV (against 25.7 mV for thermal)**

NID big Gaussian diffusion analysis

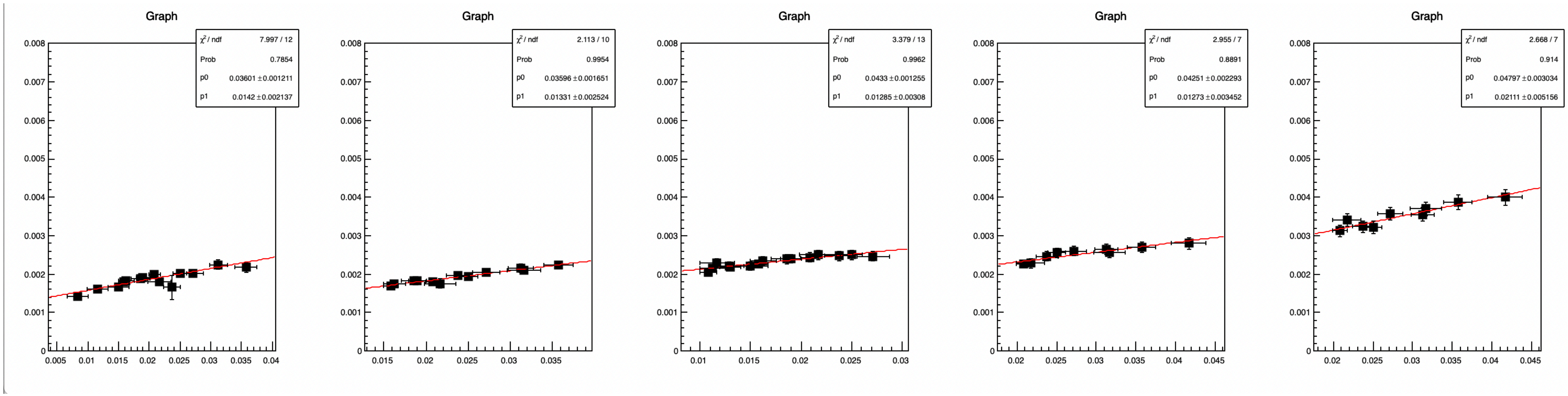
Track transverse profile sigma big as a function of L/E for different V_{GEM} (aka different LY)

- **Alphas tracks selection, NID, $He:CF_4:SF_6$ 59:39.4:1.6 @ 650 mbar**
  - V_{GEM} scan: 1545 V, 1560 V, 1575 V, 1590 V, 1605 V total V_{GEM} (corresponding to "ladder" 515V, 520V, 525V, 530V and 535V V_{GEM})
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - From these sets of data, only "not saturated" data as from previous slide are used in the following analysis

$$f(y) \sim \mathcal{N}_s\left[\mu_s, \sigma_s\right] + \mathcal{N}_b\left[\mu_b, \sigma_b\right]$$

s = small
b = big

515 V_{GEM} **520 V_{GEM}** **525 V_{GEM}** **530 V_{GEM}** **535 V_{GEM}**



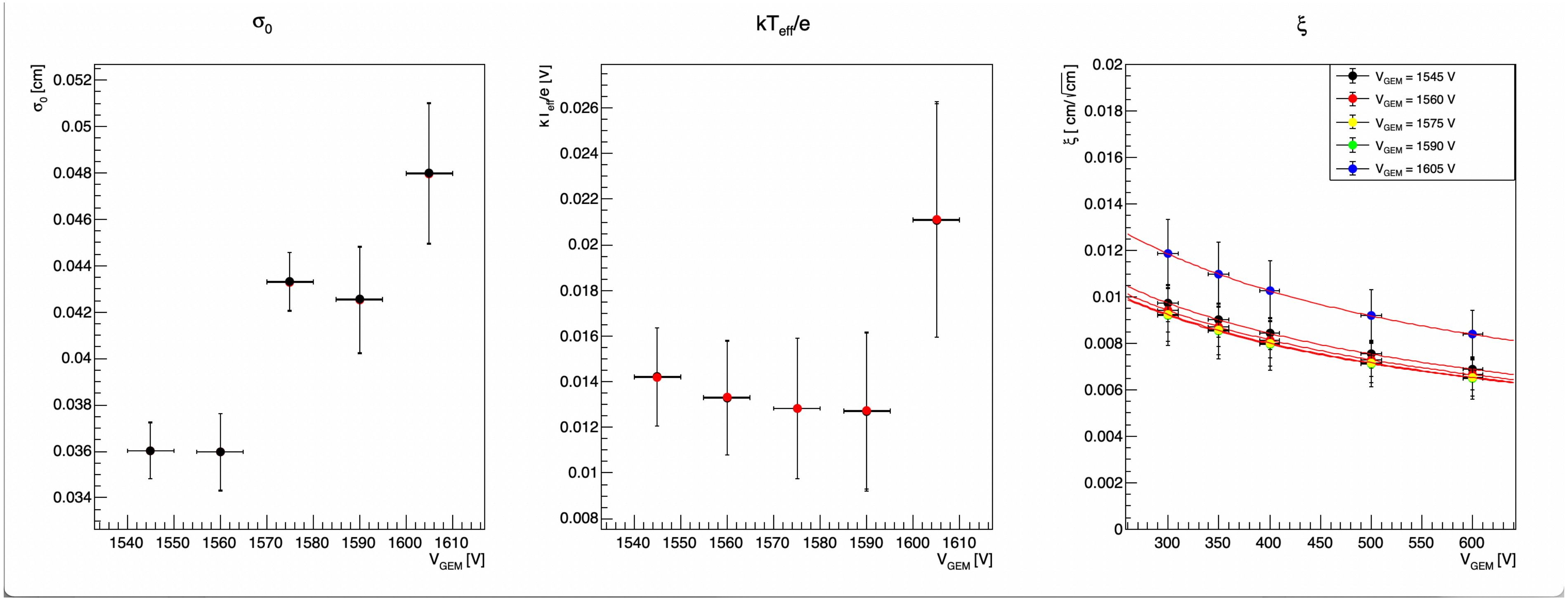
$$\sigma^2_{meas,b} = \sigma^2_{0,b} + \frac{2kT_{eff,b}}{e}\frac{L}{E} = p_3^2 + 2p_4\frac{L}{E}$$

$$p_3 = \sigma_{0,b}$$

$$p_4 = \frac{kT_{eff,b}}{e}$$

NID big Gaussian diffusion analysis

$$\sigma^2_{meas,b}(V_{GEM}) = \sigma^2_{0,b}(V_{GEM}) + \frac{2kT_{eff,b}(V_{GEM})}{e}\frac{L}{E} = p_3^2(V_{GEM}) + 2p_4(V_{GEM})\frac{L}{E}$$



- sigma$_{0,b}$ depends on V$_{GEM}$
- T$_{eff,b}$ is independent V$_{GEM}$ (1605 V data affected by saturation)
- Big Gaussian effective temperature is ± 14 mV (against 25.7 mV for thermal), consistent with Dec 2023 estimate of NID effective temperature

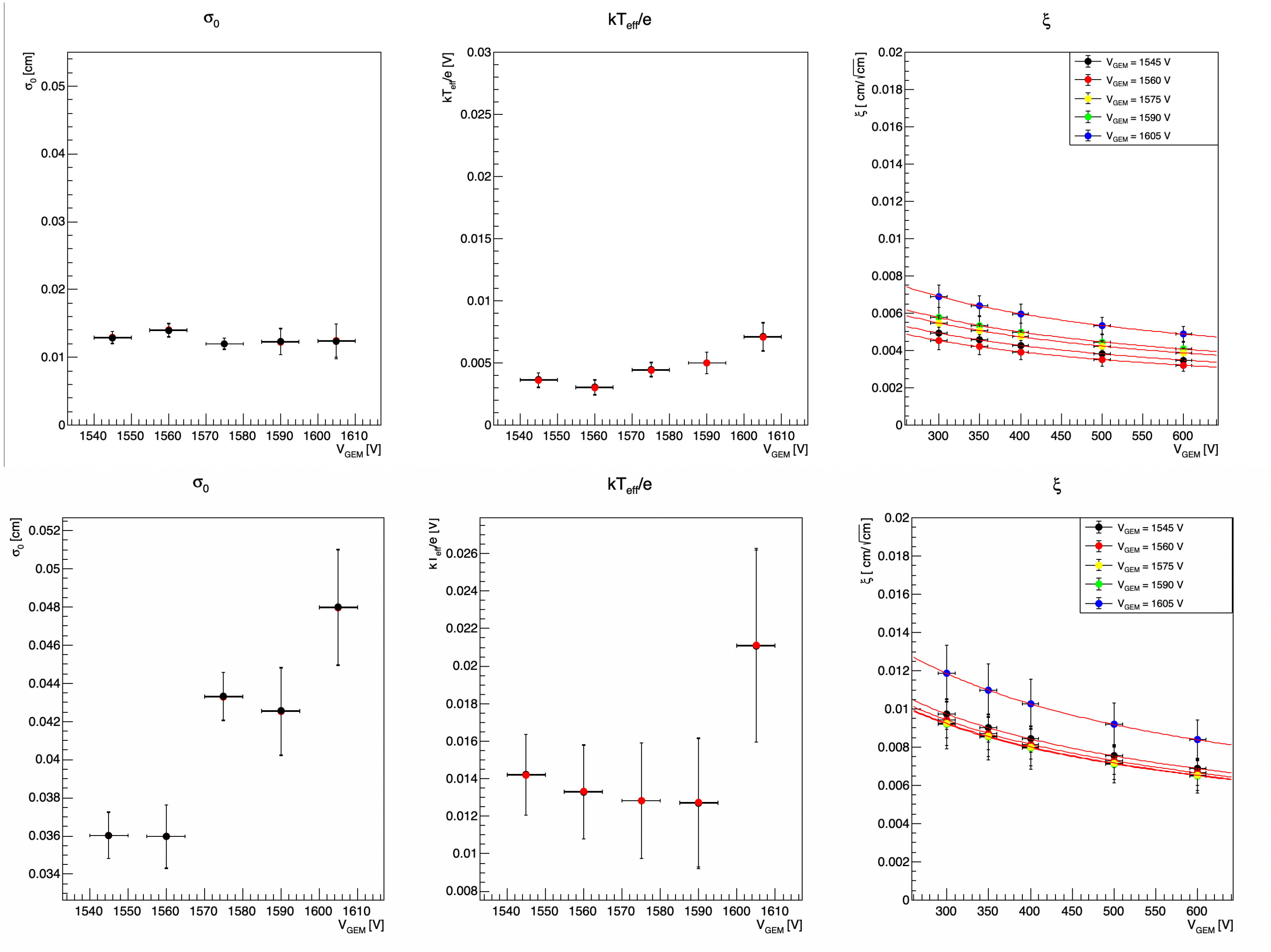
NID diffusion analysis summary

$$\sigma^2_{meas}(V_{GEM}, L/E) = \sigma^2_{0,s} + \frac{2kT_{eff,s}(V_{GEM})}{e}\frac{L}{E} + \sigma^2_{0,b}(V_{GEM}) + \frac{2kT_{eff,b}}{e}\frac{L}{E} = p_0^2 + 2p_1(V_{GEM})\frac{L}{E} + p_3^2(V_{GEM}) + 2p_4\frac{L}{E}$$

small Gaussian　　big Gaussian　　small Gaussian　　big Gaussian

small Gaussian

big Gaussian



*now on same scale

$$n(r,z,t) = \frac{N_0}{(4\pi t)^{3/2} D_T D_L^{1/2}} \exp\left[-\frac{r^2}{4D_T t} - \frac{(z-vt)^2}{4D_L t}\right].$$

At the GEM plane ($z = 0$), after having drifted for a time $t = L/v_d$, the transverse distribution is isotropic Gaussian:

$$\rho_T(x,y;L,E) = \frac{N_0}{2\pi\sigma_T^2} \exp\Big[-\frac{x^2+y^2}{2\sigma_T^2}\Big], \qquad \sigma_T^2(L,E) = 2D_T(E)\,\frac{L}{v_d(E)}. \tag{3}$$

The longitudinal spread around the centroid at $z = 0$ has variance

$$\sigma_L^2(L,E) = 2D_L(E)\,\frac{L}{v_d(E)}. \tag{4}$$

The mean number entering geometrically a *circular* hole of radius ξ centered at $(0,0)$ is the integral of the 2D Gaussian over a disc:

$$N_{\rm in}^{\rm geom}(L,E) = \int_{x^2+y^2\le\xi^2} \rho_T(x,y;L,E)\,dx\,dy = N_0\left[1-\exp\Big(-\frac{\xi^2}{2\sigma_T^2}\Big)\right]. \tag{5}$$

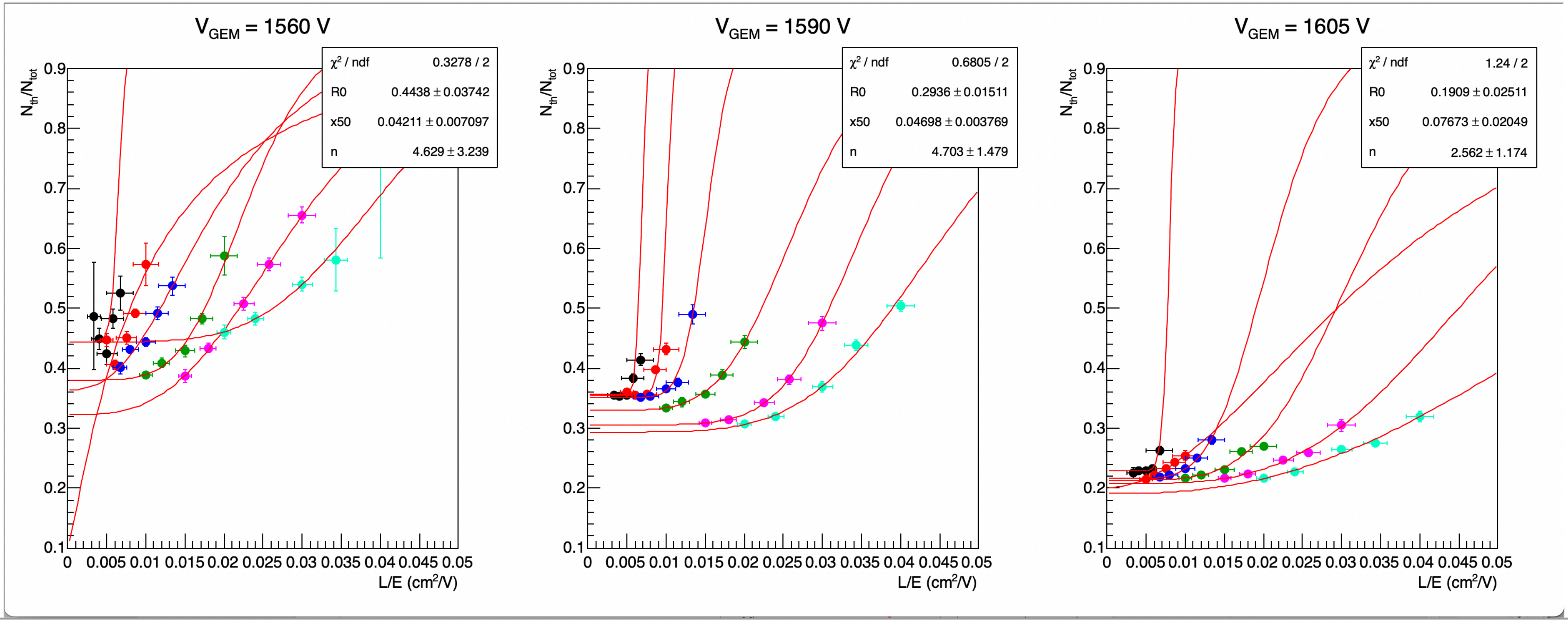
If the centroid is offset by r_0, the exact expression is

$$\frac{N_{\rm in}^{\rm geom}}{N_0} = 1-\exp\Big(-\frac{\xi^2+r_0^2}{2\sigma_T^2}\Big)\, I_0\Big(\frac{\xi\, r_0}{\sigma_T^2}\Big), \tag{7}$$

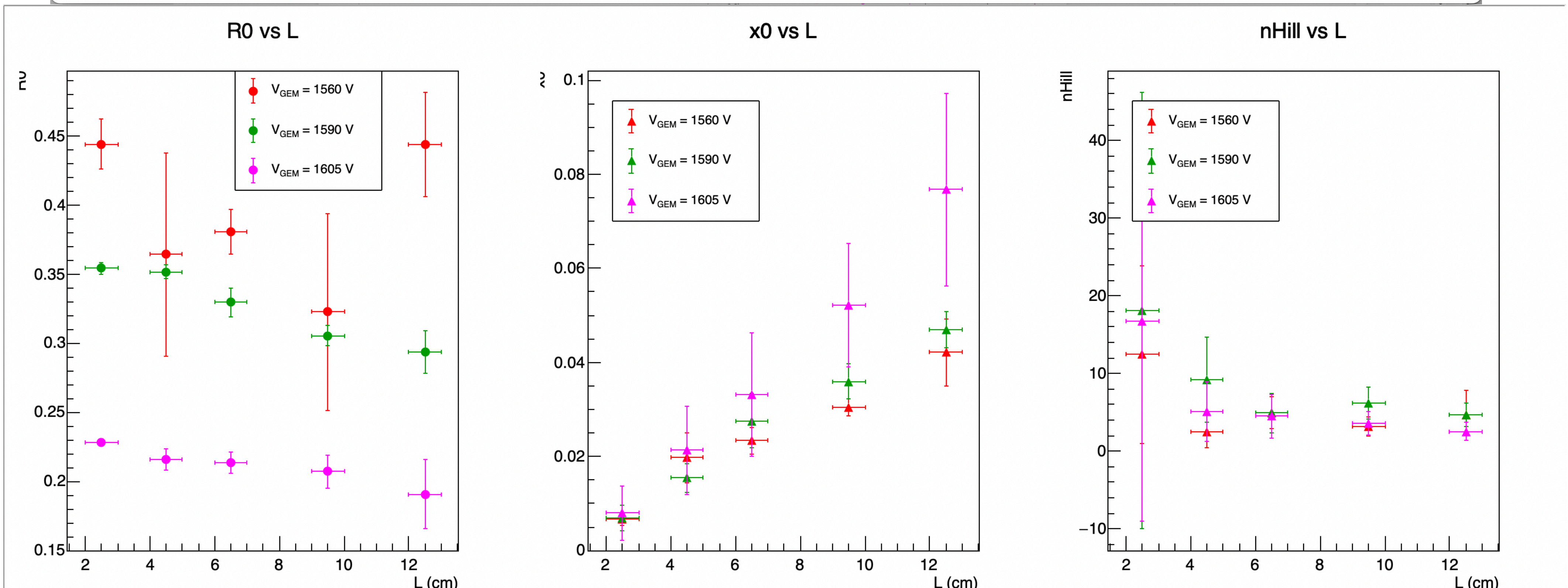
(Sec. 2.1). Restricting the longitudinal integral to a threshold depth z^{th} (interpreted as the condition that N_{i+} has reached N_{i+}^{th}) yields

$$N_{i-}^{th} = \frac{N_{i-}^{\rm geom}}{2}\left[1+\mathrm{erf}\left(\frac{z^{th}-L}{\sqrt{2}\,\sigma_L}\right)\right], \qquad \sigma_L^2 = 2D_L(E/N)\,\frac{L}{v_d(E/N)}. \tag{10}$$

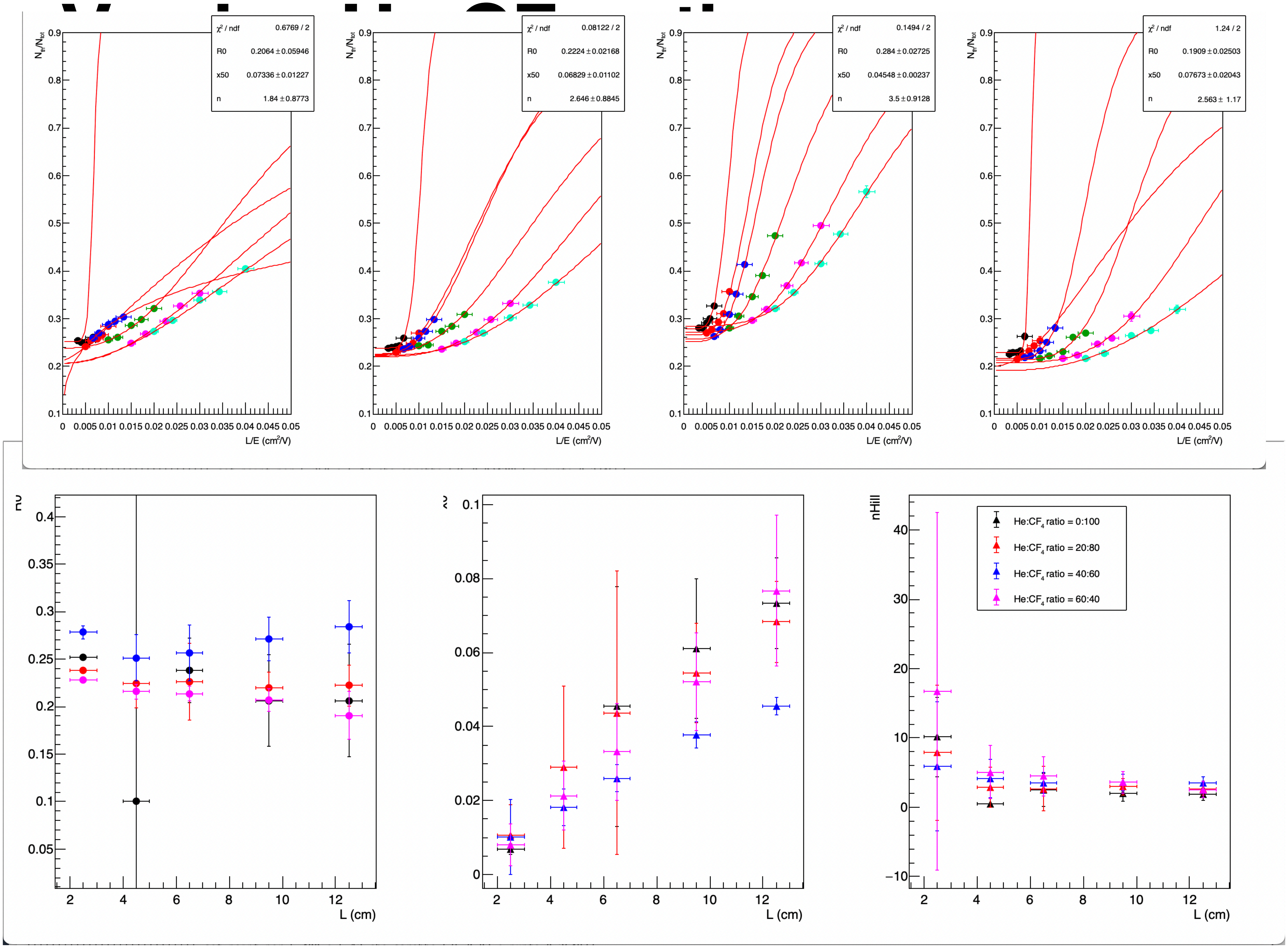
Equation (10) makes explicit that N_{i-}^{th} is a *fraction* of the available geometrical acceptance and is controlled by the longitudinal transport through σ_L—hence by the ratio L/E in the near-thermal regime. In the limits $z^{th} \to +\infty$ (no focusing onset) and $z^{th} \to -\infty$ (no ions meet the condition), one recovers $N_{i-}^{th} \to N_{i-}^{H}$ and $N_{i-}^{th} \to 0$, respectively.

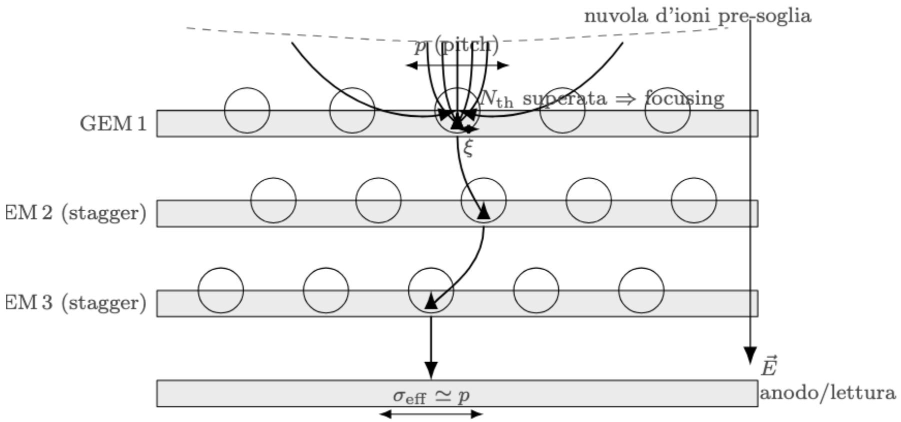


V_GEM = 1560 V
χ² / ndf 0.3278 / 2
R0 0.4438 ± 0.03742
x50 0.04211 ± 0.007097
n 4.629 ± 3.239
V_GEM = 1590 V
χ² / ndf 0.6805 / 2
R0 0.2936 ± 0.01511
x50 0.04698 ± 0.003769
n 4.703 ± 1.479
V_GEM = 1605 V
χ² / ndf 1.24 / 2
R0 0.1909 ± 0.02511
x50 0.07673 ± 0.02049
n 2.562 ± 1.174
N_th/N_tot
L/E (cm²/V)



R0 vs L
x0 vs L
nHill vs L
V_GEM = 1560 V
V_GEM = 1590 V
V_GEM = 1605 V
nHill
L (cm)


χ² / ndf 0.6769 / 2
R0 0.2064 ± 0.05946
x50 0.07336 ± 0.01227
n 1.84 ± 0.8773
χ² / ndf 0.08122 / 2
R0 0.2224 ± 0.02168
x50 0.06829 ± 0.01102
n 2.646 ± 0.8845
χ² / ndf 0.1494 / 2
R0 0.284 ± 0.02725
x50 0.04548 ± 0.00237
n 3.5 ± 0.9128
χ² / ndf 1.24 / 2
R0 0.1909 ± 0.02503
x50 0.07673 ± 0.02043
n 2.563 ± 1.17
L/E (cm²/V)
L (cm)
nHill
He:CF4 ratio = 0:100
He:CF4 ratio = 20:80
He:CF4 ratio = 40:60
He:CF4 ratio = 60:40



Single-event footprint. For a single focusing event, once a hole of GEM 1 has collected $N_{\rm th}$ ions, the remaining ions are funneled into it. The transverse footprint at the exit of that hole is limited by the hole geometry itself, with effective width $\sigma_h \sim \xi/\sqrt{2}$. If no further mechanisms intervened, the ions would remain confined within σ_h all the way to the readout plane.

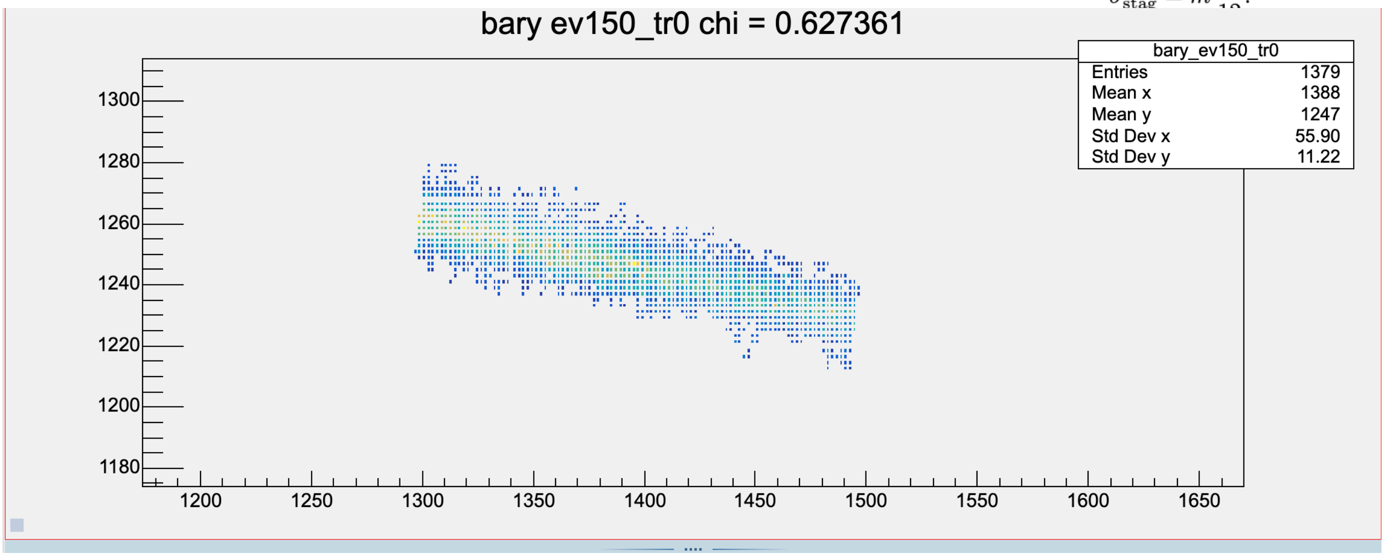
Multi-event distribution and staggering. In practice, however, the three GEM foils are not perfectly aligned: each foil is manufactured with copper–kapton–copper layers, and mounted with small relative displacements. As a result, the ion channel exiting a hole in GEM 1 does not perfectly overlap with a hole of GEM 2, and similarly for GEM 3. The channel is then re-captured by the nearest hole at each stage, effectively introducing a random lateral offset at each interface.

Modeling the offsets. We model each such offset as a random variable uniformly distributed in the interval $[-p/2,\, p/2]$, i.e. we assume that the ion centroid can land anywhere within one pitch relative to the reference hole. The variance of a single uniform distribution on $[-p/2, p/2]$ is

$$\mathrm{Var}[U] = \frac{p^2}{12}. \tag{33}$$

If m such independent offsets contribute, the variance of their sum is

$$\sigma^2_{\rm stag} = m\,\frac{p^2}{12}. \tag{34}$$

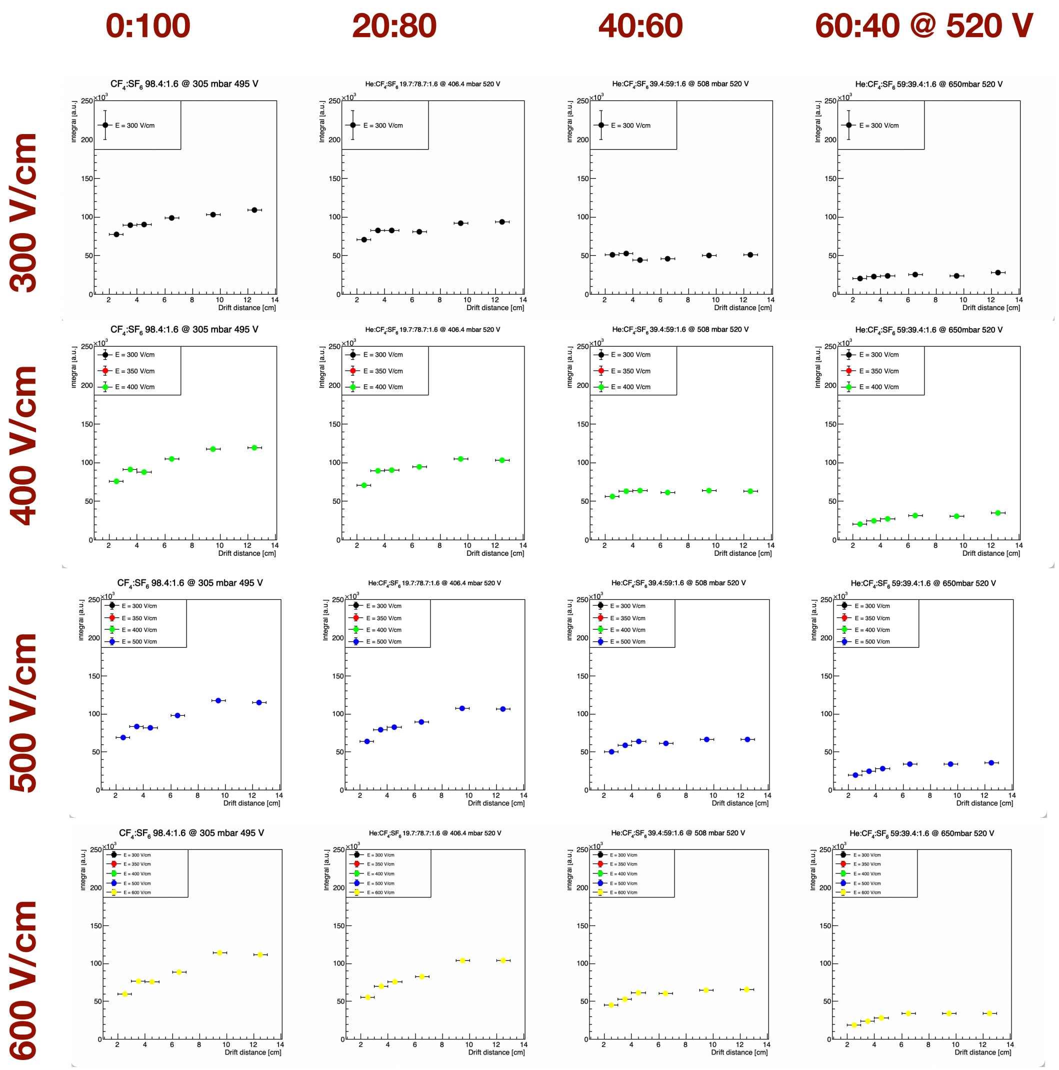


NID “mix” data, i.e. varying He/CF_4 ratio with constant SF_6

- **Alphas tracks selection, NID**
  - He content scan at fixed V_{GEM} for each gas mixture configuration
  - CF_4:SF_6 98.4:1.6, He: CF_4:SF_6 19.7:78.7:1.6, He: CF_4:SF_6 39.4:59:1.6, He: CF_4:SF_6 59:39.4:1.6, aka 0:100, 20:80. 40:60, 60:40
  - Varyng pressure to keep similar LY with stable configuration, hence 0:100 @ 305 mbar, 20:80 @ 406 mbar, 40:60 @ 508 mbar and 60:40 @ 650 mbar
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)

Track integral vs drift distance as a function of drift field, aka “Saturation plots”

- Saturation is when ScIntegral at low distance < ScIntegral at large distance
- NID “mix” ScIntegral spans from 20k to 130k **(i.e one order of magnitude, same overall values and same span of ED)**
- **0:100 ScIntegral > 20:80 ScIntegral > 40:60 ScIntegral > 60:40 ScIntegral**
  - This is by chance! We were just checking “by eye” that LY was of same order!
- **NID** “mix” data **are saturating at nearly all field/ V_{GEM}**
- **NID “mix” data need to be properly selected to avoid additional effects from saturation entering in the diffusion estimation**



NID “mix” small Gaussian diffusion analysis

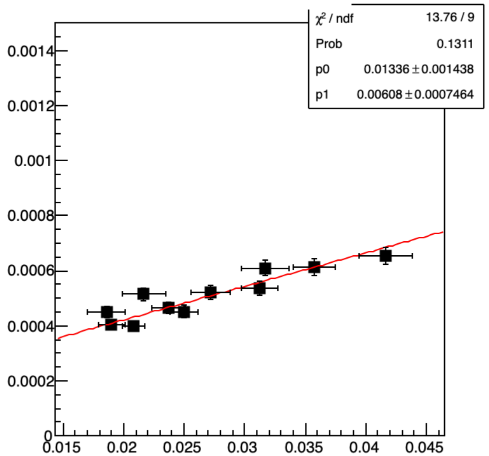
Track transverse profile sigma small as a function of L/E for different V_{GEM} (aka different LY)

- **Alphas tracks selection, NID**
  - He content scan at fixed V_{GEM} for each gas mixture configuration
  - CF_4:SF_6 98.4:1.6, He: CF_4:SF_6 19.7:78.7:1.6, He: CF_4:SF_6 39.4:59:1.6, He: CF_4:SF_6 59:39.4:1.6, aka 0:100, 20:80. 40:60, 60:40
  - Varyng pressure to keep similar LY with stable configuration, hence 0:100 @ 305 mbar, 20:80 @ 406 mbar, 40:60 @ 508 mbar and 60:40 @ 650 mbar
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - From these sets of data, only “not saturated” data are used in the following analysis

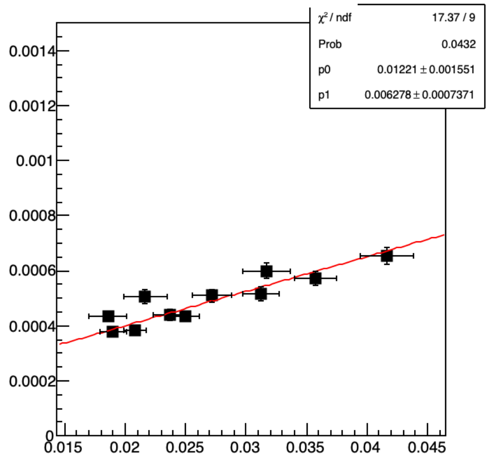
$$f(y) \sim \mathcal{N}_s\left[\mu_s, \sigma_s\right] + \mathcal{N}_b\left[\mu_b, \sigma_b\right]$$

s = small
b = big

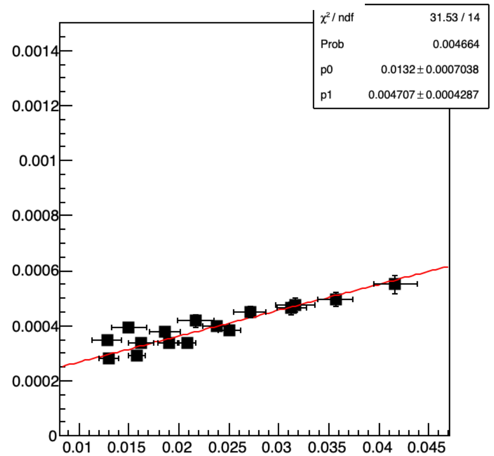
0:100



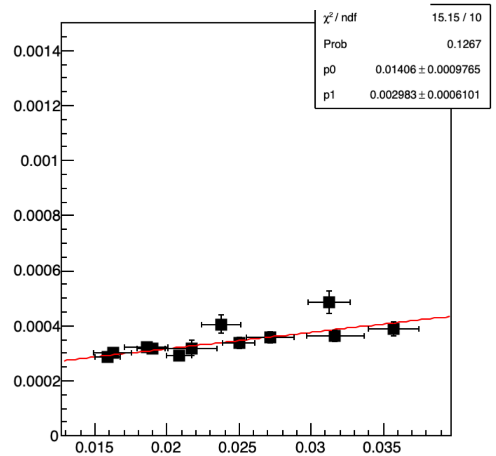
20:80



40:60



60:40 @ 520 V



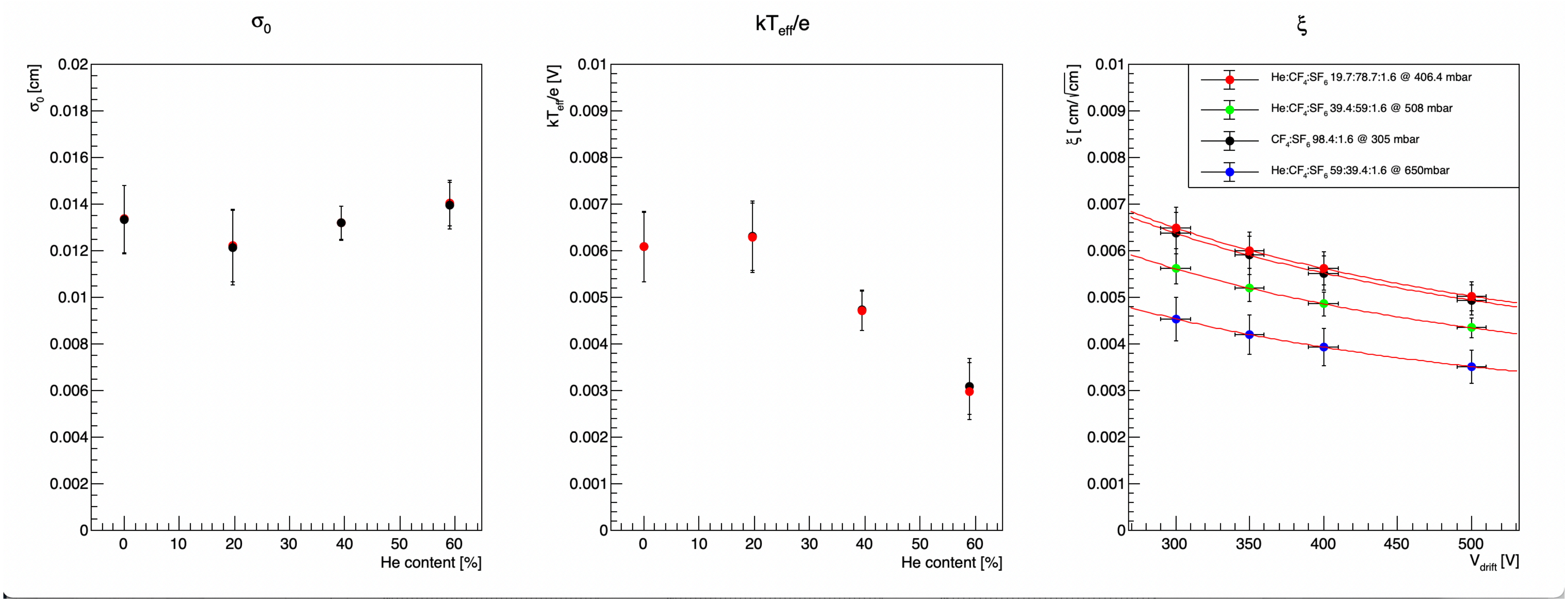
$$\sigma^2_{meas,s} = \sigma^2_{0,s} + \frac{2kT_{eff,s}}{e}\frac{L}{E} = p_0^2 + 2p_1\frac{L}{E}$$

$$p_0 = \sigma_{0,s}$$

$$p_1 = \frac{kT_{eff,s}}{e}$$

NID “mix” small Gaussian diffusion analysis

$$\sigma^2_{meas,s}(V_{GEM}) = \sigma^2_{0,s}(V_{GEM}) + \frac{2kT_{eff,s}(V_{GEM})}{e}\frac{L}{E} = p_0^2(V_{GEM}) + 2p_1(V_{GEM})\frac{L}{E}$$



- sigma$_{0,s}$ is independent of V_{GEM}, consistent with NID V_{GEM} scan analysis at 60:40
- $T_{eff,s}$ decreasing with increasing He content, consistent with the LY behaviour as a function of He content, hence consistent with NID V_{GEM} scan analysis at 60:40
- Small Gaussian effective temperature is < 6 mV (against 25.7 mV for thermal)

NID “mix” big Gaussian diffusion analysis

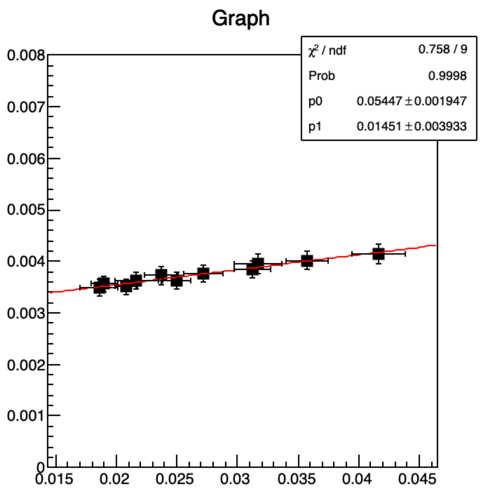
Track transverse profile <u>sigma big</u> as a function of L/E for different V_{GEM} (aka different LY)

- **Alphas tracks selection, NID**
  - He content scan at fixed V_{GEM} for each gas mixture configuration
  - CF_4:SF_6 98.4:1.6, He: CF_4:SF_6 19.7:78.7:1.6, He: CF_4:SF_6 39.4:59:1.6, He: CF_4:SF_6 59:39.4:1.6, aka 0:100, 20:80. 40:60, 60:40
  - Varyng pressure to keep similar LY with stable configuration, hence 0:100 @ 305 mbar, 20:80 @ 406 mbar, 40:60 @ 508 mbar and 60:40 @ 650 mbar
  - For each V_{GEM}, 6 drift distance positions (2, 3, 4, 6, 9, and 12 cm) and 5 E_{DRIFT} values (300, 350, 400, 500 and 600 V/cm)
  - From these sets of data, only “not saturated” data are used in the following analysis

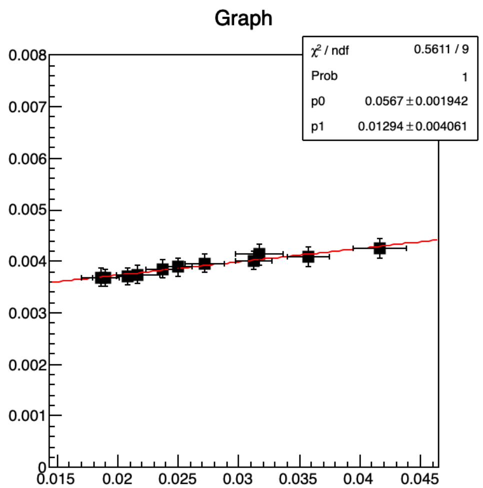
$$f(y) \sim \mathcal{N}_s\left[\mu_s, \sigma_s\right] + \mathcal{N}_b\left[\mu_b, \sigma_b\right]$$

s = small
b = big

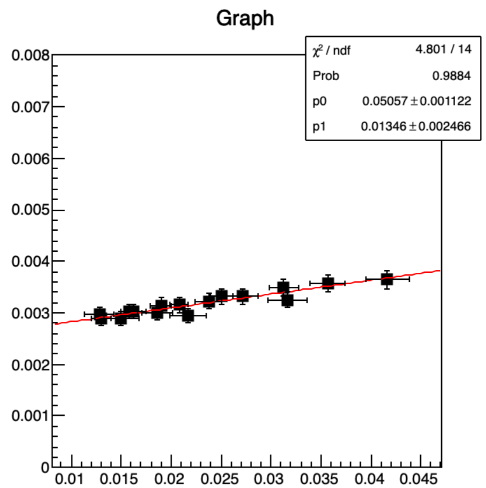
0:100



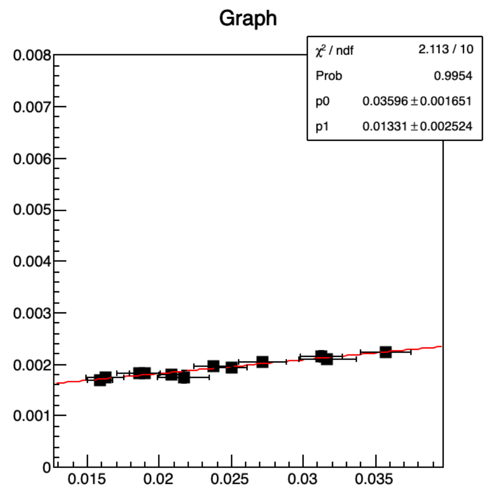
20:80



40:60



60:40 @ 520 V

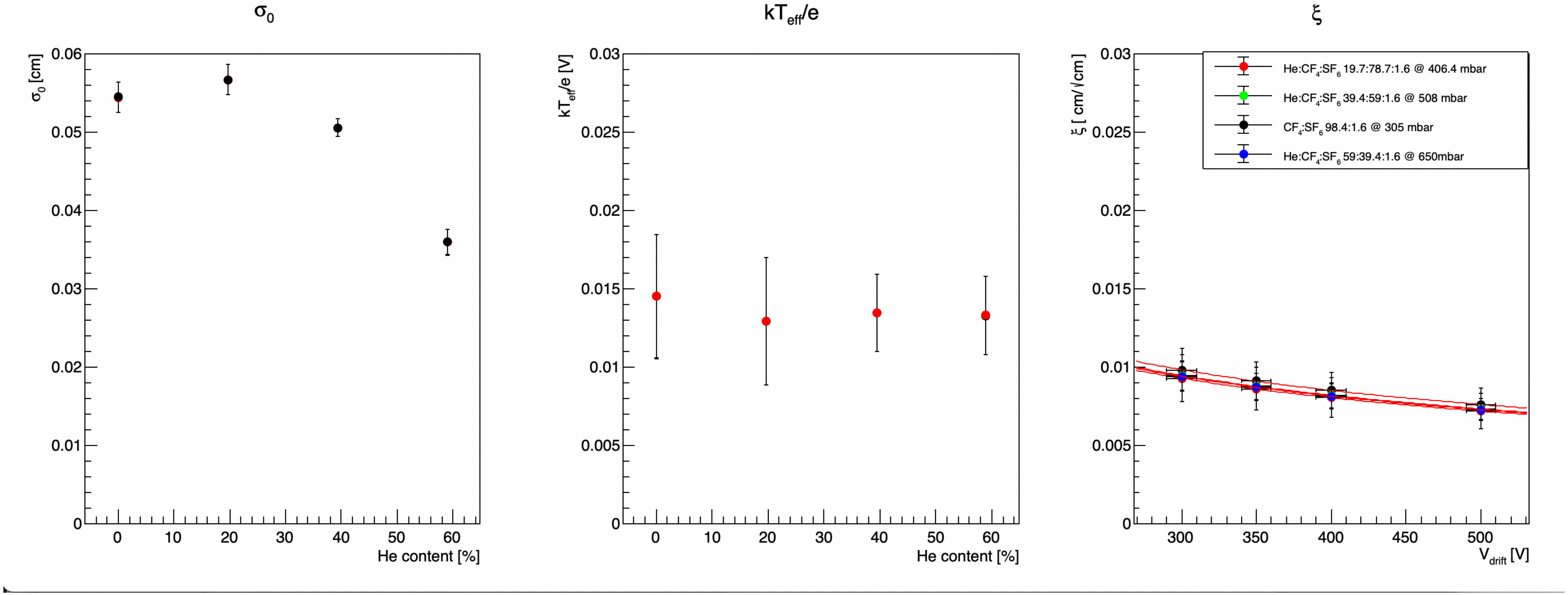


$$\sigma^2_{meas,b} = \sigma^2_{0,b} + \frac{2kT_{eff,b}}{e}\frac{L}{E} = p_3^2 + 2p_4\frac{L}{E}$$

$$p_4 = \frac{kT_{eff,b}}{e}$$

NID “mix” big Gaussian diffusion analysis

$$\sigma^2_{meas,b}(V_{GEM}) = \sigma^2_{0,b}(V_{GEM}) + \frac{2kT_{eff,b}(V_{GEM})}{e}\frac{L}{E} = p_3^2(V_{GEM}) + 2p_4(V_{GEM})\frac{L}{E}$$



- $sigma_{0,b}$ depends on He content, aka LY, consistent with NID V_{GEM} scan analysis at 60:40
- $T_{eff,b}$ is independent V_{GEM} and consistent in values and behaviour as a function of LY with NID V_{GEM} scan analysis at 60:40
- Big Gaussian effective temperature is ± 14 mV (against 25.7 mV for thermal), consistent with NID V_{GEM} scan analysis at 60:40 and with Dec 2023 estimate of NID effective temperature

Summary of the experimental measurements:

ED 60:40

$$\sigma^2_{meas}(V_{GEM}, L/E) = \sigma_0^2(V_{GEM}) + \frac{2kT_{eff}}{e}\frac{L}{E} = p_0^2(V_{GEM}) + 2p_1\frac{L}{E}$$

- **ED data can be fitted with a single Gaussian, as expected from prime principles of electrons transport in gas**
- **Fitted track transverse profile distributions are consistent with our understanding of:**
  - a constant term (aka σ_0) due to the diffusion within the 3 GEMs stack and dependent only on the V_{GEM} applied to the GEMs (consistent with LIME and Lemon measurement of such term)
  - a term dependent only L/E (aka kT_{eff}/e) and NOT on V_{GEM}, due to the diffusion during drift, consistent with prime principle expections, Garfield simulation, and LIME and Lemon measurement of such term

NID

$$\sigma^2_{meas}(V_{GEM}, L/E) = \sigma^2_{0,s} + \frac{2kT_{eff,s}(V_{GEM})}{e}\frac{L}{E} + \sigma^2_{0,b}(V_{GEM}) + \frac{2kT_{eff,b}}{e}\frac{L}{E} = p_0^2 + 2p_1(V_{GEM})\frac{L}{E} + p_3^2(V_{GEM}) + 2p_4\frac{L}{E}$$

small Gaussian | big Gaussian | small Gaussian | big Gaussian

- **NID data (both V_{GEM} scan and "mix" scan with one order of magnitude LY span) are fitted with a double Gaussian**
  - the sum of two Gaussian from a statistical point of view implies the existance of two "population", aka bimodal distribution
- **Fitted track transverse profile distributions of "big Gaussian" are consistent with (ED beahviour):**
  - a constant term (aka $\sigma_{0,b}$) due to the diffusion within the 3 GEMs stack and dependent only on the V_{GEM} applied to the GEMs (similar to ED behaviour)
  - a term dependent on L/E (aka $kT_{eff,b}/e$) due to the diffusion during drift, 60% below the thermal limit
- **Fitted track transverse profile distributions of "small Gaussian" show:**
  - a constant term (aka $\sigma_{0,s}$) independent of the V_{GEM}
  - a term dependent very "slowly" dependent on L/E (aka $kT_{eff,b}/e$) BUT ALSO on V_{GEM}, at about 15% of the thermal limit

Our understanding of the picture:

- Big Gaussian is the one effectively describing the diffusion:
    - the constant term depends on V_{GEM} as for ED
    - the T_{eff} depends on L/E as for ED
    - the measured T_{eff} is about 60% of $T_{thermal}$
    - the measured diffusion coefficient is ± 65 um/sqrt(cm) @ 600 V/cm (against ± 140 um/sqrt(cm) for ED at same E_{DRIFT})

- Small Gaussian is an additional term unrelated to the diffusion along the drift distance (aka systematics) and due to disuniformities of the NID electron stripping and amplification process close to the first GEM
    - It depends on a combination of GEM pitch, V_{GEM} and L/E
    - we have some ideas on the small Gaussian origin that we want to verify before sharing with you

From Dinesh Loomba's paper on SF_6 (ariXiv:1609.05249) Chapter on systematics on the diffusion measurements:

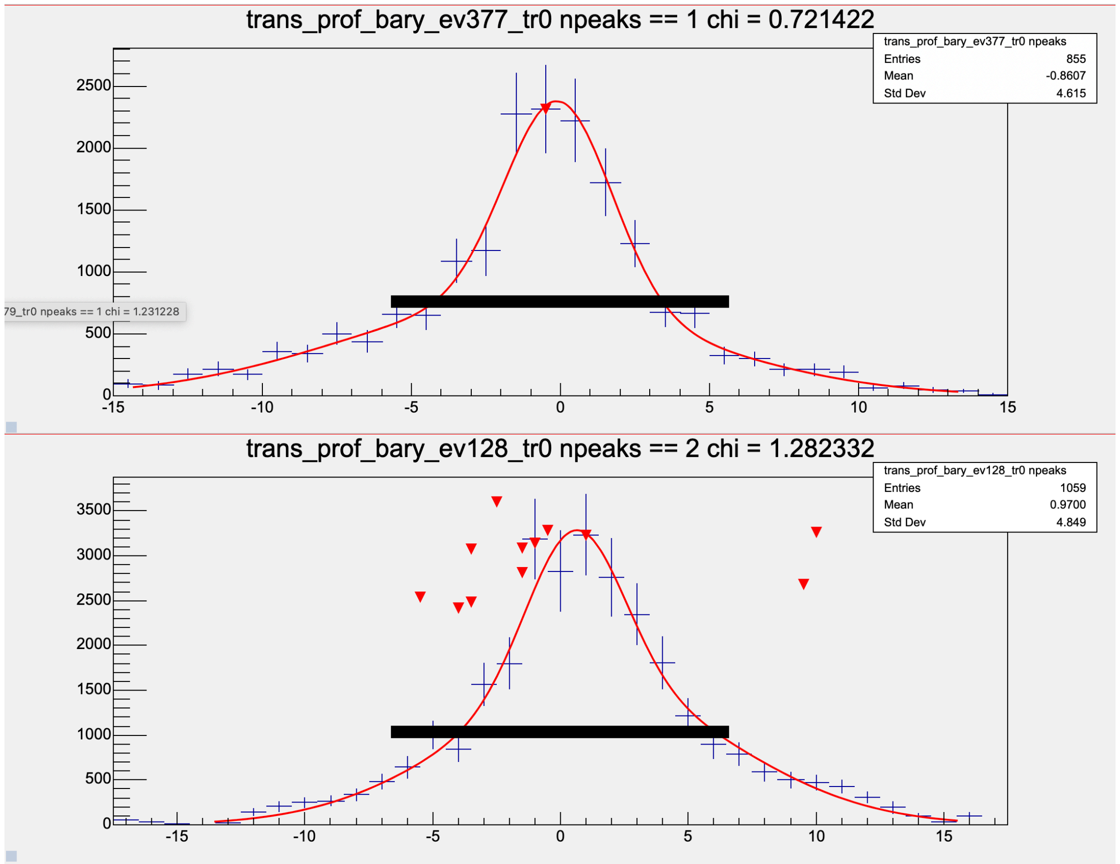
[..] the primary sources of systematic effects that contribute to our estimate of the diffusion width: [..] is the non-uniformity of the electric field near the THGEM. [..] The broadening effect due to the non-uniformity in the drift field close to the THGEM should depend on the THGEM pitch (aka GEM pitch), and the fields in the holes (aka V_{GEM}) and TPC drift region (aka L/E).

…in any case…

…whatever is the origin of the small Gaussian….

…its overall effect is ….

improve directionality!!!!



Backup slides

Center of mass energy full calculation

1. Definizione del problema

- Miscela: **He:CF_4:SF_6** a **650 mbar** (~0.65 atm).
- Consideriamo una singola collisione **SF_6^- + M** (M = He o CF_4).
- Vogliamo l'**energia nel centro di massa**:

$$\varepsilon_{\rm cm} = \frac{\mu}{m_{\rm SF_6^-}} E_{\rm ion},$$

dove:

- $E_{\rm ion}$ = energia cinetica dell'SF_6^- rispetto al laboratorio,
- $\mu = \frac{m_{\rm SF_6^-} m_M}{m_{\rm SF_6^-} + m_M}$ è la massa ridotta.

2. Energia dell'ione in laboratorio

Per uno ione carico $q = -e$ in un campo elettrico uniforme E, l'ener
legata al **mean free path** λ:

$$E_{\rm ion} \sim eE\lambda.$$

Con gas termici a pressione p, la λ vale:

$$\lambda = \frac{1}{n\sigma}, \quad n = \frac{p}{k_B T}.$$

Stime numeriche

- Pressione: $p = 0.65\,\rm atm \approx 6.6\times10^4\,Pa.$
- Temperatura: $T = 300\,\rm K.$
- Numero di molecole per m³:

$$n = \frac{p}{k_B T} \approx \frac{6.6\times10^4}{1.38\times10^{-23}\cdot300} \approx 1.6\times10^{25}\,\rm m^{-3}$$

- Sezione d'urto tipica per collisioni ion–molecola a bassa energia: $\sigma \sim 10^{-19}\,\rm m^2$ (ordine di grandezza).
- Mean free path:

$$\lambda \sim \frac{1}{n\sigma} \approx \frac{1}{1.6\times10^{25}\cdot10^{-19}} \approx 6\times10^{-7}\,\rm m = 0.6\,\mu$$

3. Campo elettrico nella GEM

Spessore GEM $d = 50\,\mu\rm m$.
Differenza di potenziale: $V_{\rm GEM} = 1545\text{–}1620\,\rm V$.
Campo medio nel foro:

$$E_{\rm GEM} \approx \frac{V_{\rm GEM}}{d} = \frac{1545\,\rm V}{50\times10^{-6}\,\rm m} \to \frac{1620}{50\times10^{-6}}.$$

Numeri:

- $E_{\rm GEM}(1545) \approx 3.1\times10^7\,\rm V/m.$
- $E_{\rm GEM}(1620) \approx 3.24\times10^7\,\rm V/m.$

4. Energia dell'SF_6^- per collisione

Usiamo $E_{\rm ion} \sim eE_{\rm GEM}\lambda$:

$$E_{\rm ion} \approx (1.6\times10^{-19})(3.1\times10^7)(6\times10^{-7})$$
$$\approx 3.0\times10^{-18}\,\rm J.$$

Convertiamo in eV:

$$E_{\rm ion} \approx \frac{3.0\times10^{-18}}{1.6\times10^{-19}} \approx 19\,\rm eV.$$

Per 1620 V:

$$E_{\rm ion} \approx 20\,\rm eV.$$

5. Energia nel centro di massa

Ora calcoliamo $\varepsilon_{\rm cm} = \frac{\mu}{m_{\rm SF_6^-}} E_{\rm ion}$.

- Masse atomiche:
  - $m_{\rm SF_6^-} \approx 146\,u.$
  - $m_{\rm He} = 4\,u.$
  - $m_{\rm CF_4} \approx 88\,u.$
  - ($u = 1$ unità di massa atomica.)
- Fattore di riduzione:
  - Per He: $\mu/m_{\rm SF6} = \frac{146\cdot4}{(146+4)\cdot146} \approx 0.027.$
  - Per CF_4: $\mu/m_{\rm SF6} = \frac{146\cdot88}{(146+88)\cdot146} \approx 0.38.$
- Energia di c.m.:
  - He: $\varepsilon_{\rm cm} \approx 0.027\times20\,\rm eV \approx 0.54\,eV.$
  - CF_4: $\varepsilon_{\rm cm} \approx 0.38\times20\,\rm eV \approx 7.6\,eV.$

6. Risultato finale

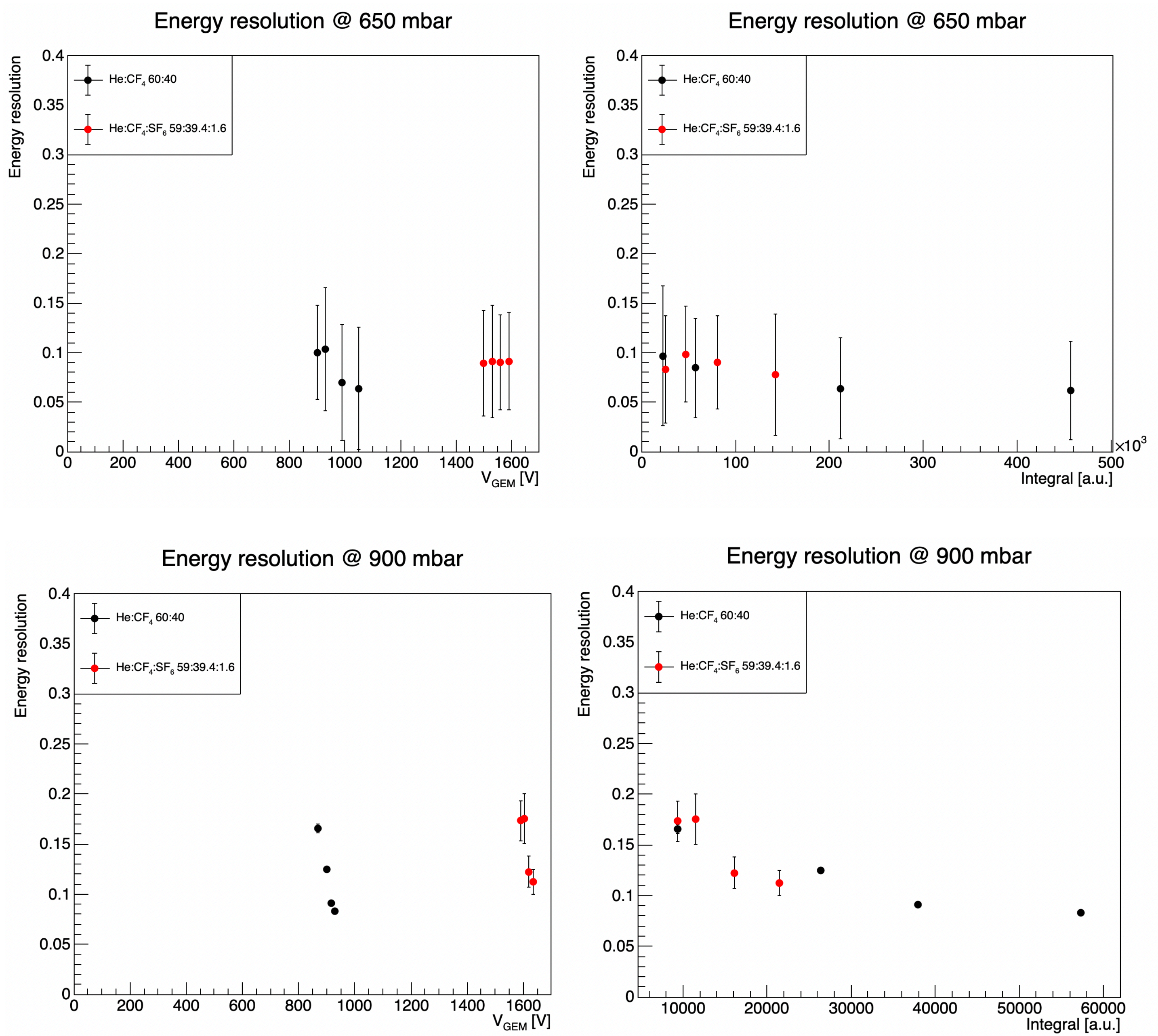
Per $V_{\rm GEM} \in [1545, 1620]$ V (50 µm GEM, 650 mbar, λ ≈ 0.6 µm):

- **He:** $\varepsilon_{\rm cm} \sim 0.5$ eV.
- **CF_4:** $\varepsilon_{\rm cm} \sim 7\text{–}8$ eV.

Quindi nel tuo intervallo operativo:

- **He è attivo nel regime sub-eV** (onset dolce, massimo intorno a 1 eV
- **CF_4 è attivo a energie già più alte (multi-eV)**, con un contributo sig
grande.

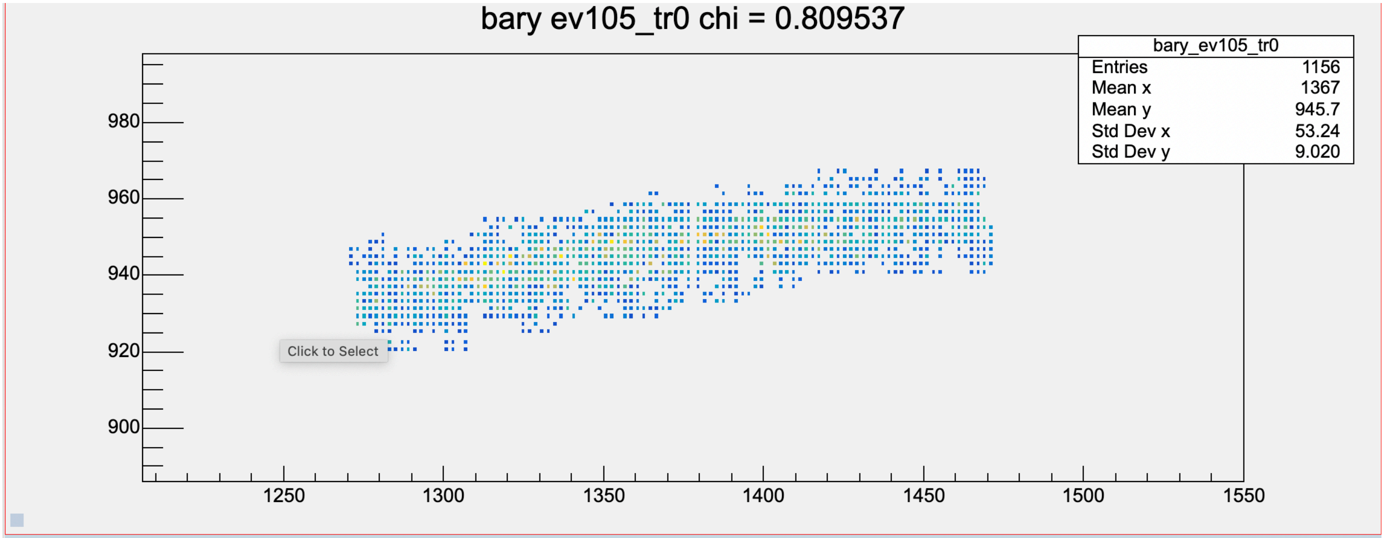
ED & NID energy resolution



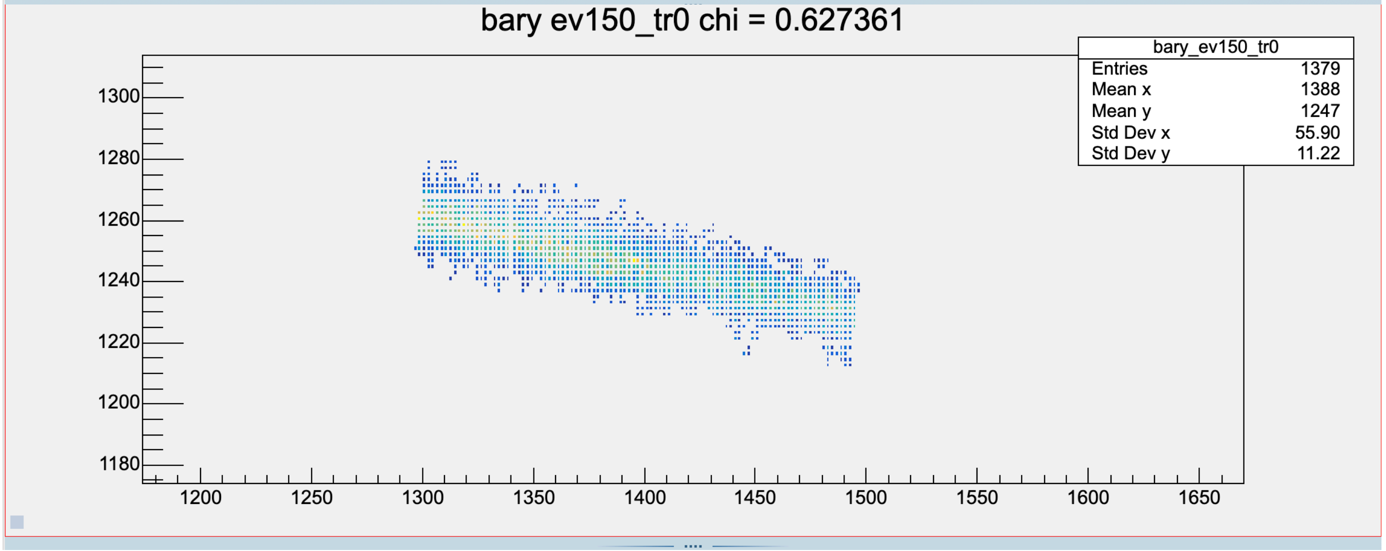
Why 290 V inconsistent with other V_{GEM}?

Because at too low gain we miss the tails!

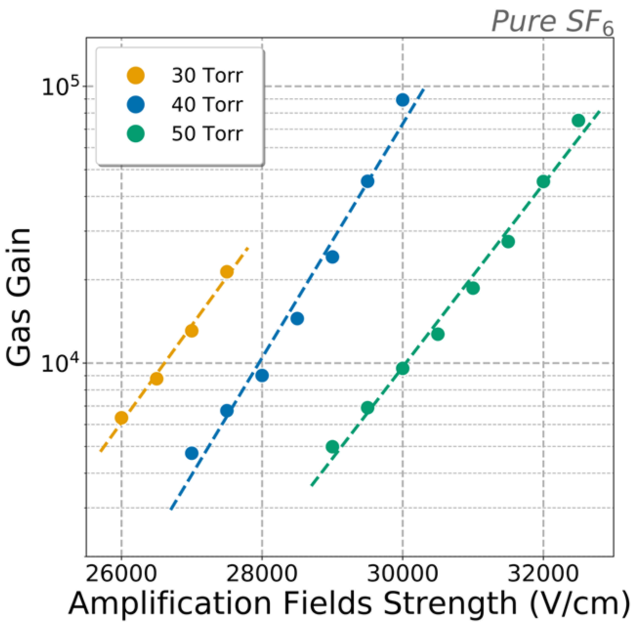
290 V_{GEM}
2 cm 600 V/cm
(aka lowest diffused track, i.e. best case scenario)

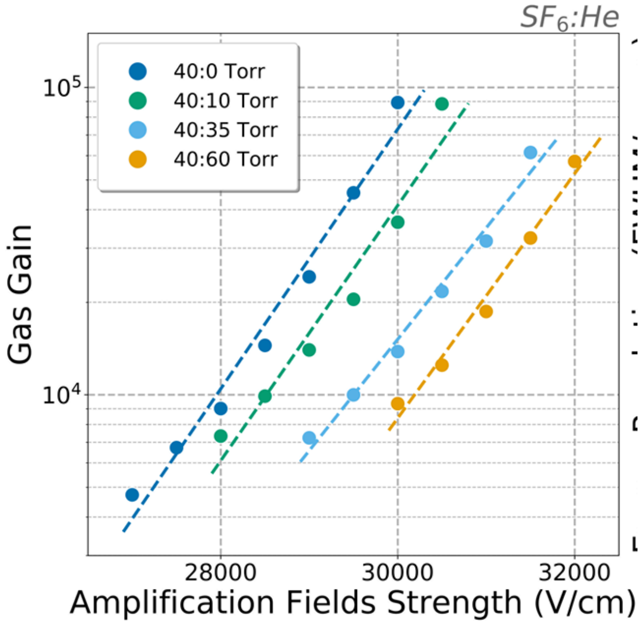


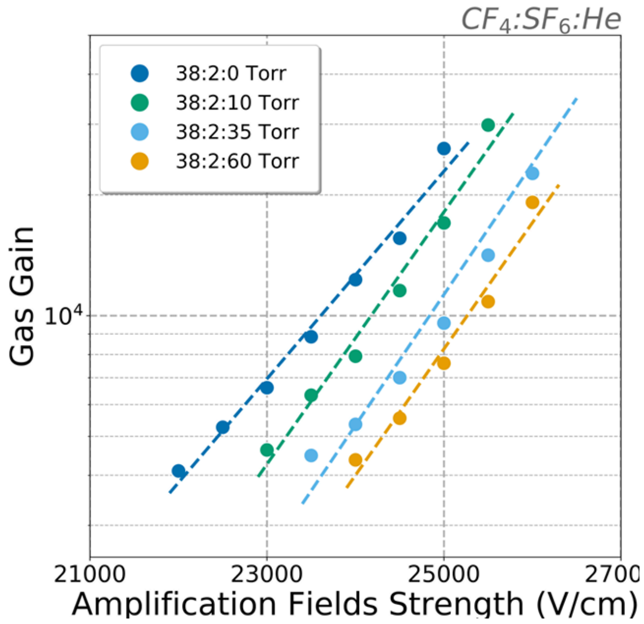
300 V_{GEM}
2 cm 600 V/cm



Comparison with published results: atmospheric pressure absolute value







d (mm)	p (Torr)	ΔV (V)	E_h (kV·cm^{-1})	E_h/p (V·cm^{-1}Torr^{-1})	G_{eff}	$\sigma/\mathcal{E}$ (%)
0.4	30	820	20.50	683	3000	25
1.0	30	1005	10.05	335	3000	45
0.4	40	880	22.00	550	2000	42
0.4	60	1020	25.50	425	-	-