Towards two-loop EW corrections in OpenLoops

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Based on 2312.07927 and 26XX.YYYY

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Introduction

- LHC and future colliders (will) explore the multi TeV energy scale range
  - ✦ Search for New Physics (NP)
  - ✦ Study of Higgs properties: EWSB, Higgs self-couplings, ….
  - ✦ **EW** like **QCD** and **QED**, but BN/KLN not applicable

Ciafaloni *et al.*, hep-ph/0001142

- Higher order corrections are mandatory in theoretical predictions to match the experimental accuracy: **NLO** minimum, but not anymore sufficient

- At this collider energy **EW** radiative corrections exhibit a logarithmic enhancement, known as *Sudakov logarithms/mass-singularities*

- For generic **N**^{n}**LO** **EW** correction, *Sudakov logs* are of the form

$$\alpha^n \log^k \frac{|r_{ij}|}{m^2}, \quad 1 \leq k \leq 2n$$

$$|r_{ij}| = |(p_i + p_j)^2| \sim Q^2$$
$$m^2 \sim m_W^2$$

Introduction

- At this collider energy **EW** radiative corrections exhibit a logarithmic enhancement, known as *Sudakov logarithms/mass-singularities*

- For generic **N^{n}LO** **EW** correction, *Sudakov logs* are of the form

$$\alpha^n \log^k \frac{|r_{ij}|}{m^2} = \alpha^n \left[L + l_{ij}\right]^k , \quad 1 \le k \le 2n$$

$$L \equiv L(Q,m) := \log \frac{Q^2}{m^2}$$

$$l_{ij} \equiv l(r_{ij},Q) := \log \frac{|r_{ij}|}{Q^2}$$

$$|r_{ij}| = |(p_i + p_j)^2|$$

NLO EW

LL: $\delta_{\mathrm{LL}}^{\text{1-loop}} \propto -\alpha L^2 \overset{1\ \mathrm{TeV}}{\sim} -20\%$

NLL: $\delta_{\mathrm{NLL}}^{\text{1-loop}} \propto \alpha\,(\underline{a} + \underline{l_{ij}})\,L \overset{1\ \mathrm{TeV}}{\sim}$ **a.d.** → 10%; **a.i.** → 5%

NNLO EW

LL: $\delta_{\mathrm{LL}}^{\text{2-loop}} \propto \alpha^2 L^4 \overset{1\ \mathrm{TeV}}{\sim} 2-3\%$

NLL: $\delta_{\mathrm{NLL}}^{\text{2-loop}} \propto \alpha^2\,(\underline{b} + \underline{l_{ij}})\,L^3 \overset{1\ \mathrm{TeV}}{\sim}$ **a.d.** → -2%; **a.i.** → 1%

EW Sudakov corrections: *all orders*

NB: also via *SCET*, not discussed here

Chiu *et al.*, 0806.1240
Fuhrer *et al.*, 1011.1505
Denner & Rode, 2402.10503

- Properties of **EW** logarithmic corrections investigated at *all orders* via *IREE*
  - ✦ **LL + NLL** (arbitrary process) — Melles *et al.* hep-ph/0004056 , hep-ph/0012157 , hep-ph/0102097 , hep-ph/0108221
  - ✦ **NNLL** (only NC massless 4-fermion processes) — Kühn *et al.* hep-ph/9912503 , hep-ph/0106298

- *IREE* approach:
  - ✦ Virtual corrections described by generalized RGE in logarithmic approximation (**LA**)
    - Not mass-suppressed Born matrix elements $\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i_n}} \sim E^d \implies$ keep **LL + NLL,** neglect (αE^d) and $\left(\alpha m^n E^{d-n} \log^k\right)$
    - On-shell external momenta $p_i^2 = m_i^2$ and ***all*** r_{ij} much larger than W/Z masses

$$|r_{ij}| = |(p_i + p_j)^2| \approx 2|p_i p_j| \sim Q^2 \gg m^2_{W,Z,H,t}, \qquad i \neq j$$

  - ✦ **EW** splits in two regimes matched at $\mu = M = m_W \implies$ Resummed **EW** corrections factorize in
    - $Q \geq \mu \geq M$: Symmetric **EW** part, computed in $\mathbf{SU(2)_L \otimes U(1)_Y}$, with unique cut-off parameter $M = m_{W,Z,\gamma}$

$$\mathcal{M}^{\mathrm{sew}}_{\mathrm{RG}} \overset{\mathrm{NLL}}{\simeq} \exp\left[\frac{\alpha}{4\pi}\sum_i \sum_{j<i} C^{\mathrm{ew}}_{ij}\left[-L^2 + (a + l_{ij})L\right]\right] \cdot \mathcal{M}_0\,,$$

    - $\mu \leq M$: **EM** part, computed in $\mathbf{U(1)_{em}}$, contain divergences from gap between physical photon mass and $m_\gamma = M$; will *cancel* with real radiation

EW Sudakov corrections: *fixed order*

- Properties of **EW** logarithmic corrections at *fixed order* investigated via explicit diagrammatic computations in **LA**

✦ **One-loop** (arbitrary process ⇒ **DP** algorithm)

Denner & Pozzorini: hep-ph/0010201, hep-ph/0104127
Pozzorini, hep-ph/0201077

✦ **Two-loop**

→ **LL+NLL** (fermionic process)

Denner *et al.*, hep-ph/0608326 & hep-ph/0809.0800

→ **LL+NLL a.d.** (arbitrary process)

Denner *et al.*, hep-ph/0301241

Factorization wrt Born ME at one- and two-loop

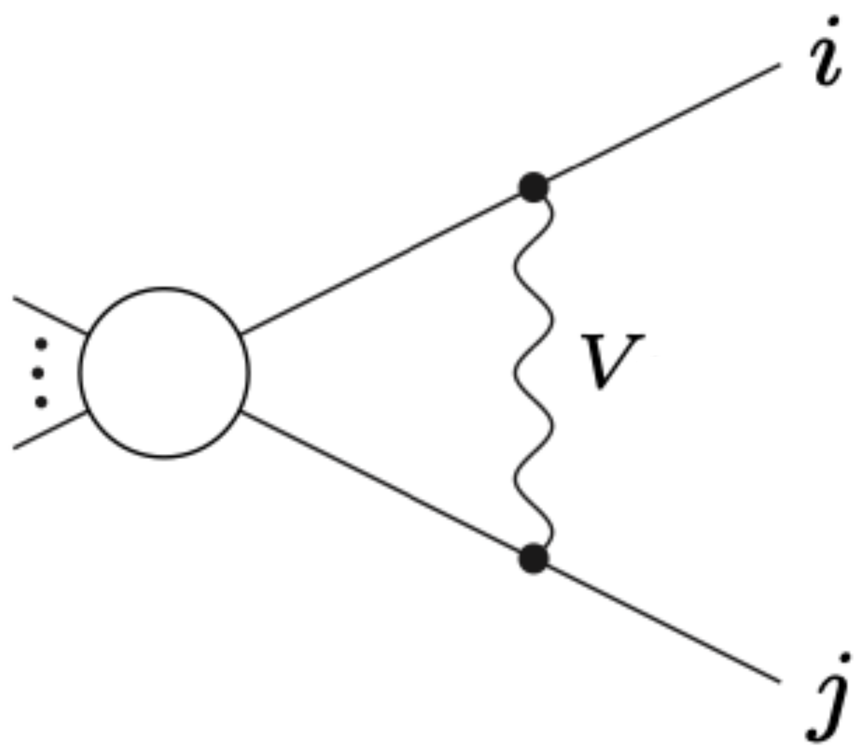
$$\mathcal{M}^{(1)} \sim \left(\frac{\alpha}{4\pi}\right) C_{\rm ew} \left[-L^2 + (a + l_{ij})L\right] \mathcal{M}_0$$

$$\mathcal{M}^{(2)} \sim \left(\frac{\alpha}{4\pi}\right)^2 C^2_{\rm ew} \left[\frac{L^4}{2} + (b + l_{ij})L^3\right] \mathcal{M}_0$$

in agreement with *IREE* and its expansion at FO

- **EW** *Sudakov logarithms* at ***one-loop*** implemented in
  - ✦ ALPGEN (specific process): Chiesa *et al*, 1305.6837; *2013*
  - ✦ Sherpa (general process): Bothmann, Napoletano 2006.14635; *2020*
  - ✦ MadGraph (general process): Pagani, Zaro 2110.03714; *2021*
  - ✦ **OpenLoops** (general process + resonances): Lindert, **L.M.**, 2312.07927; *2023*

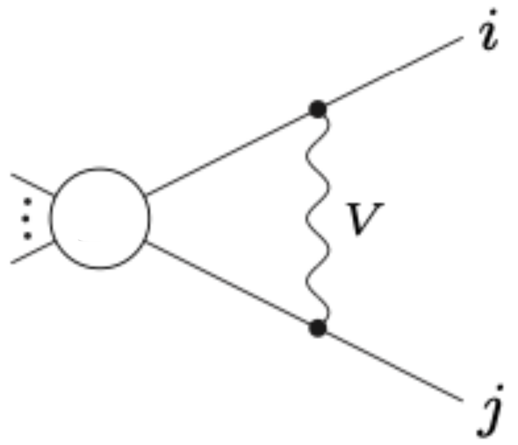
One-loop mass singularities



- One-loop mass singularities originate from diagrams where a vector boson V couples to two different external legs

- Total contribution

$$\mathcal{M}_1 = \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{V=A,Z,W^\pm}\left[\;\cdots\;\right]_{q^\mu\to xp_i^\mu,\,xp_j^\mu}$$



- Diagrams are evaluated in the approximation of V being:

LA: $|r_{ij}| = |(p_i+p_j)^2| \approx 2|p_ip_j| \sim Q^2 \gg m^2_{W,Z,H,t}, \qquad i\neq j$

✦ ***Soft and collinear*** to i or j → **DL:** $\log^2\frac{|r_{ij}|}{m_V^2} = L^2 + 2l_{ij}L + l_{ij}^2$

$q^\mu \to xp_i^\mu, xp_j^\mu, \quad x\to 0$

LL (L^2) **NLL a.d.** ($2l_{ij}L$) **NNLL a.d.** (l_{ij}^2)

l_{ij}^2 : a.d. **NNLL** term formally not part of **LA**, but needed for reliable estimates (Pagani, Zaro 2110.03714; 2021).

In our implementation it can be either excluded (**NLL**) or included (**NLL'**) as a tool to estimate uncertainties beyond **LA**

✦ ***Collinear*** to i or j → **SL:** $\log\frac{s}{m_V^2} = L$ ⟶ **NLL a.i.**

$q^\mu \to xp_i^\mu, xp_j^\mu, \quad x\neq 0$

Denner *et al.* hep-ph/0608326

Implementation in OpenLoops

- Automated tools for full **NLO** **EW** corrections:

OpenLoops (OL), Recola, MadGraph5_aMC@NLO, Sherpa, MoCaNLO, …

Buccioni *et al*, 1907.13071; *2019*

Actis *et al*, 1211.6316; *2012*
Actis *et al*, 1605.01090; *2016*

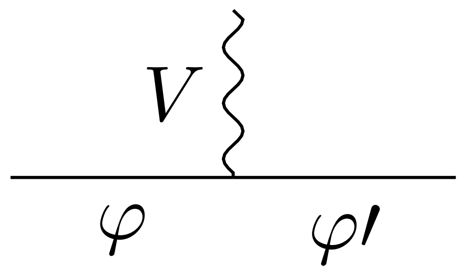
Alwall *et al*, 1405.0301; *2014*

Bothmann *et al*, 1905.09127; *2019*
Bothmann *et al*, 2410.22148; *2024*

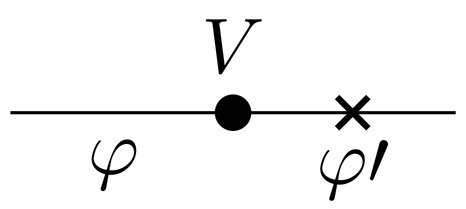
Denner *et al*, 2602.19842; *2026*

- Why implementing *Sudakov logarithms*:
  - ✦ Even if automated, full **NLO** **EW** corrections can be non-trivial in high multiplicity processes
  - ✦ **EW** *Sudakov logs* have nice properties: ***factorization*** w.r.t. Born amplitudes, leading contribution of radiative corrections
  - ✦ Full **NNLO/two-loop** automation still out of reach. *Sudakov approximation* might help to access missing higher order terms and reduce theoretical uncertainty from missing higher order terms, but still not automated… up to now!

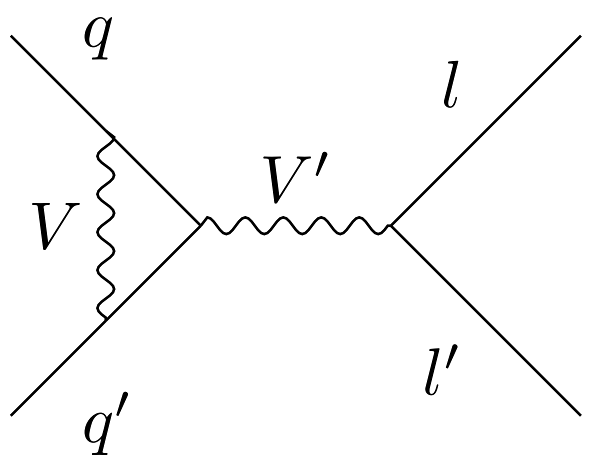
- In **OL**: define *helicity-dependent* two-point vertex rules / *pseudo-CT* $\Longrightarrow$ any ***soft-collinear*** **NLO** topology "converted" into Born one via double *pseudo-CT**



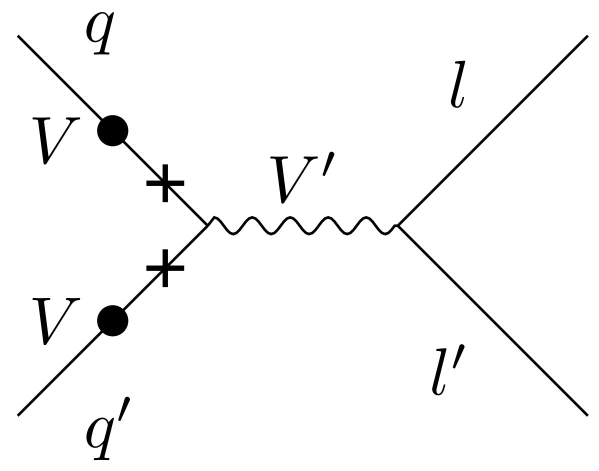
$= ieI^V_{\varphi\varphi'}T \quad \longrightarrow$



$= ieI^V_{\varphi\varphi'} \quad \Longrightarrow$



$\xrightarrow{\text{CT}}$



$$\Rightarrow \mathcal{M} \sim e^2 \sum_{V=A,Z,W^\pm} I^V_q I^V_{q'} K^V_{\text{ew}} \mathcal{M}_0$$

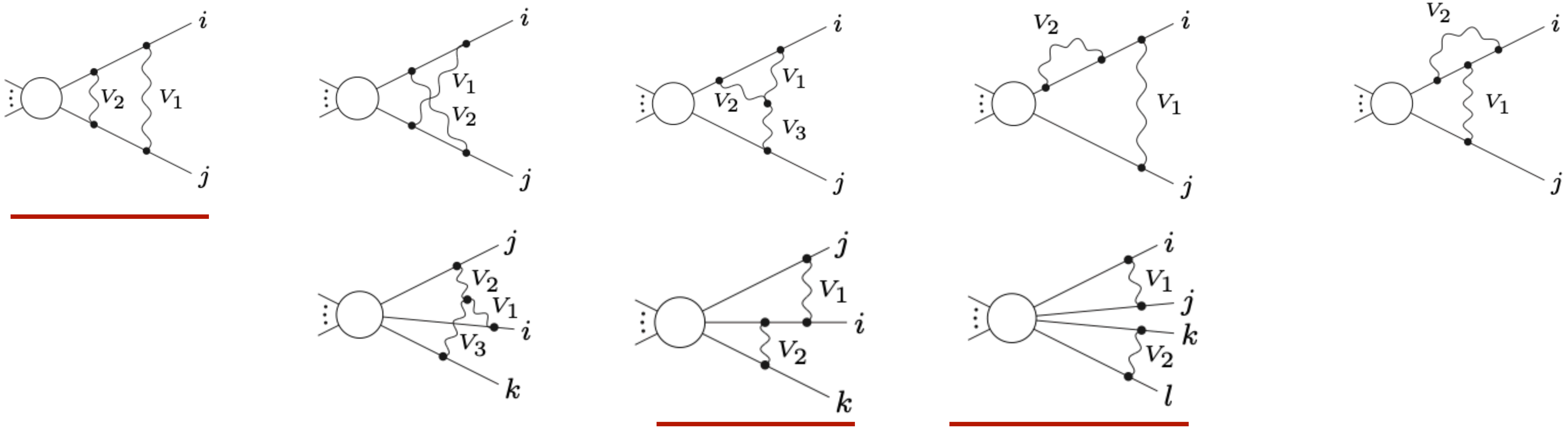
$\downarrow$

K_{ew} contains **LL**, **NLL**, **LL+NLL**,…

*up to 4 CTs are need at two-loop

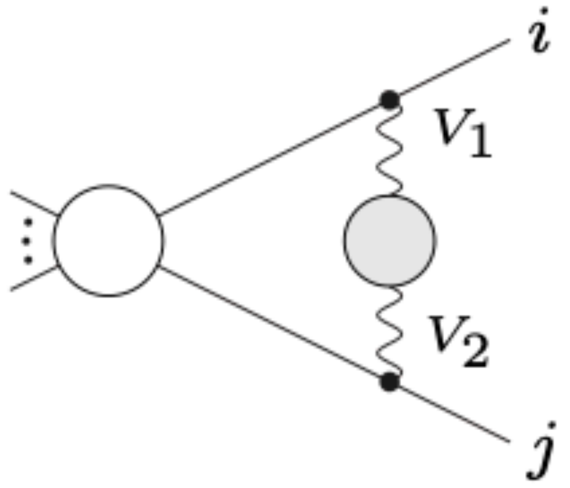
Two-loop mass singularities

- **LL+NLL** mass singularities @ two-loop are obtained from one-loop diagrams yielding **LL** from ***soft and collinear*** regions of V_1, by inserting:
    - ✦ A second ***soft and/or collinear*** gauge boson, coupled to either an external leg or to



NB: **EW** Ward Identities induce cancellations ⇒ only *three* (out of eight) classes of topologies are required

- ✦ A self-energy sub diagram in the propagator of V_1, yielding an additional **NLL**



Denner *et al.*
hep-ph/0608326 & hep-ph/0809.0800

Mass singularities of UV origin

Additional (*angular-independent!*) **NLL** contributions from UV renormalisation

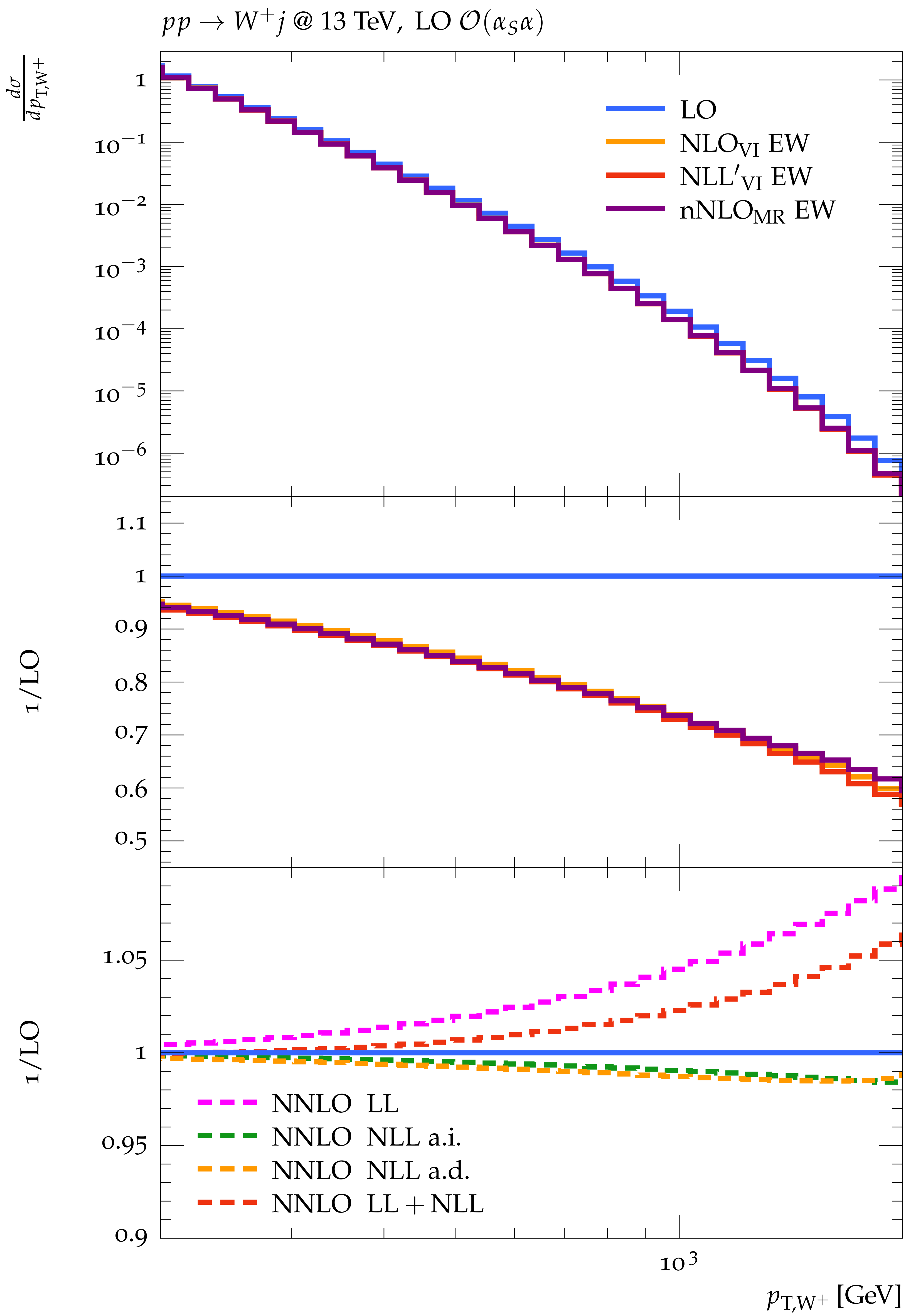
- At one-loop:
  - **PR**: UV renormalisation of **EW** *dimensionless* parameters $\qquad \mathcal{M}_1^{\mathrm{PR}} \sim \delta Z_{g_i}^{(1)} \mathcal{M}_0, \qquad \delta Z_{g_i}^{(1)} \overset{\mathrm{LA}}{\sim} \alpha L$
  - **WF**: yields to the ***factorized*** correction $\qquad \mathcal{M}_1^{\mathrm{WF}} \sim \delta Z_i^{(1)} \mathcal{M}_0, \qquad \delta Z_i^{(1)} \overset{\mathrm{LA}}{\sim} \alpha L$

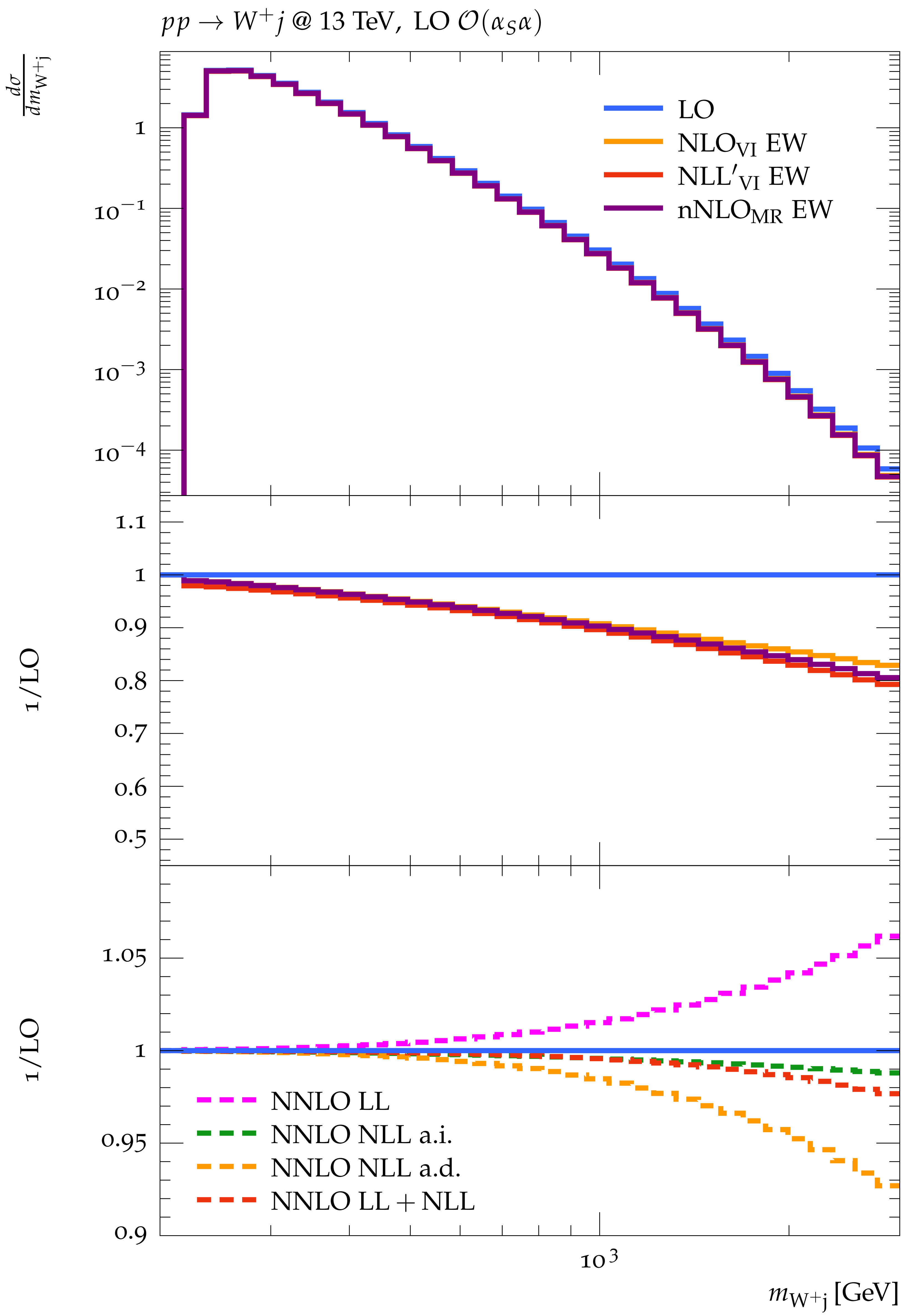
- At two-loop: combination of one-loop **LL** with
  - One-loop (*dimensionless*) **PR:** $\qquad \mathcal{M}_2^{\mathrm{PR}} \sim \alpha L^2 \times \mathcal{M}_1^{\mathrm{PR}}$
  - One-loop **WF** : $\qquad \mathcal{M}_2^{\mathrm{WF}} \sim \alpha L^2 \times \mathcal{M}_1^{\mathrm{WF}}$

- Two-loop UV renormalisation, i.e. $\delta Z_{ii'}^{(2)}, \delta Z_{g_i}^{(2)}$ is at **NNLL** accuracy $\sim \alpha^2 \log^2(s/m_W^2) \Rightarrow$ neglected

Denner *et al.*
hep-ph/0608326 & hep-ph/0809.0800

Two-loop results: hadron level





NLO$_{VI}$:
exact one-loop **EW**

NLL'$_{VI}$:
one-loop **EW** in **LA+ NNLL a.d.**

nNLO$_{MR}$:
exact one-loop **EW** + **NNLO LL + NLL**

NNLO LL + NLL:
two-loop **EW** @ NLL accuracy

In $p_{T,W}$ clear Sudakov behavior, i.e. $r_{ij} \sim Q^2 \; \forall \{ij\}$:

- **NLL'$_{VI}$** in % level agreement with **NLO$_{VI}$**
- **LL > NLL: LL** drives **NNLO** (*positive*)

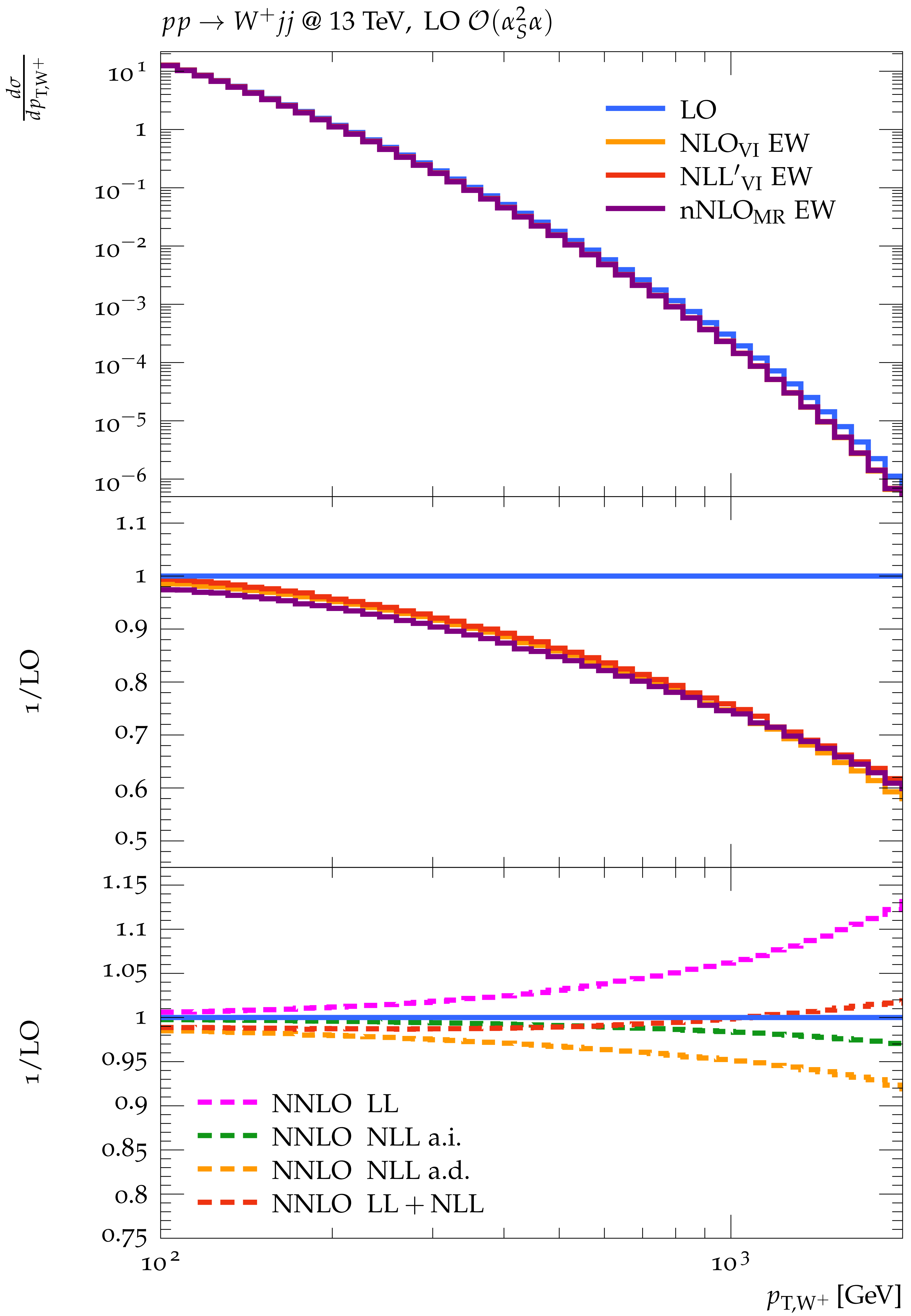
Results consistent with V+j predictions first presented in Lindert *et al.* 1705.04664

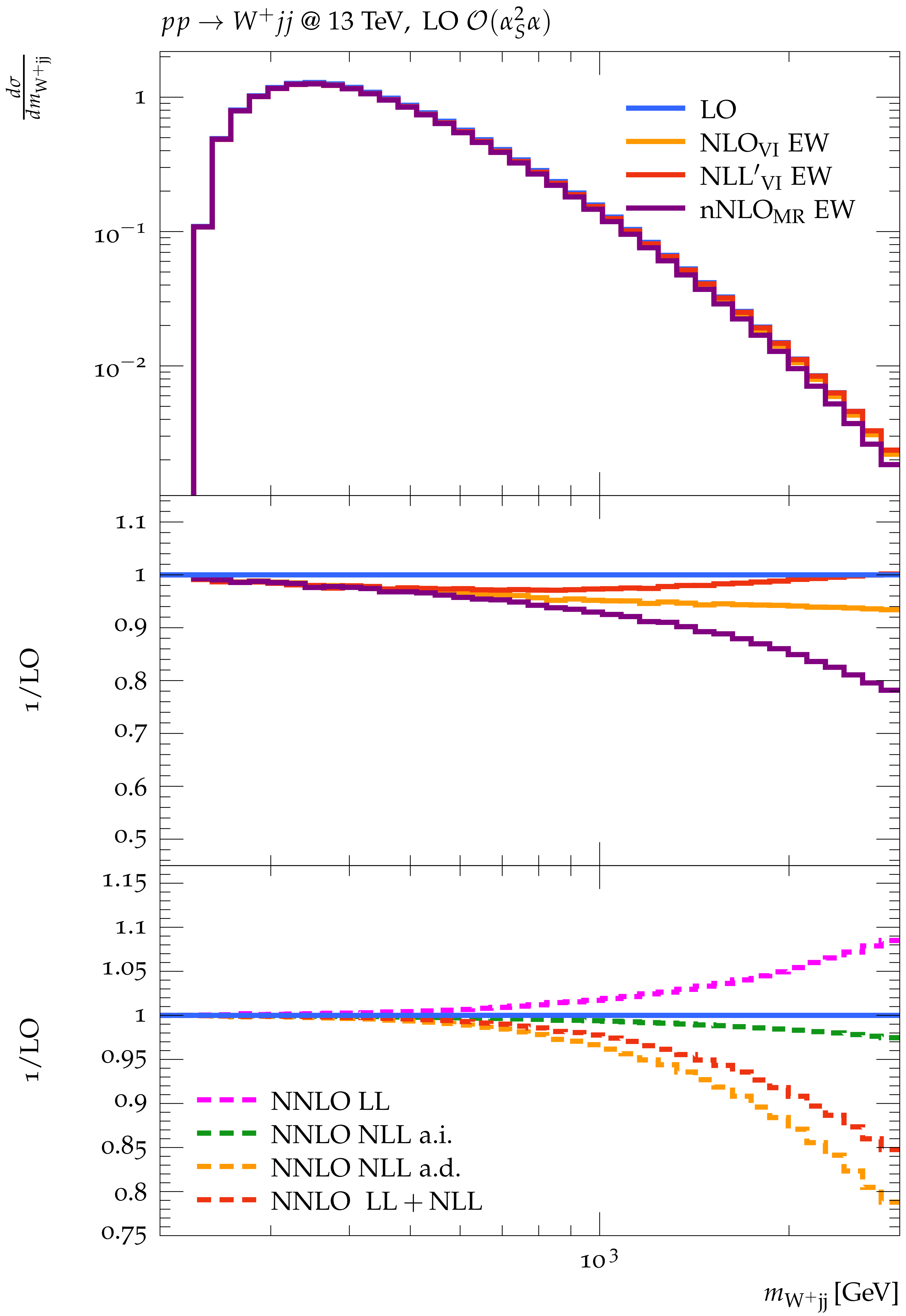
In m_{Wj} **LA** is partially spoilt due to forward configs
$\Longrightarrow$ large ratios of invariants $Q^2 \sim r_{ij} \gg r_{kg}$:

- **NLL'$_{VI}$** not good approximation for **NLO$_{VI}$**
- **LL $\lesssim$ NLL a.d.: NNLO** is *negative*

Lindert, **L.M.**, *In preparation*

Two-loop results: hadron level





NLO_{VI}:
exact one-loop **EW**

NLL'_{VI}:
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$nNLO_{MR}$:
exact one-loop **EW** + **NNLO LL + NLL**

NNLO LL + NLL:
two-loop **EW** @ NLL accuracy

Additional cuts: $|\eta_W| < 3$ & $|y_j| < 4.4$

In $p_{T,W}$ still Sudakov behavior $r_{ij} \sim Q^2\ \forall\{ij\}$:

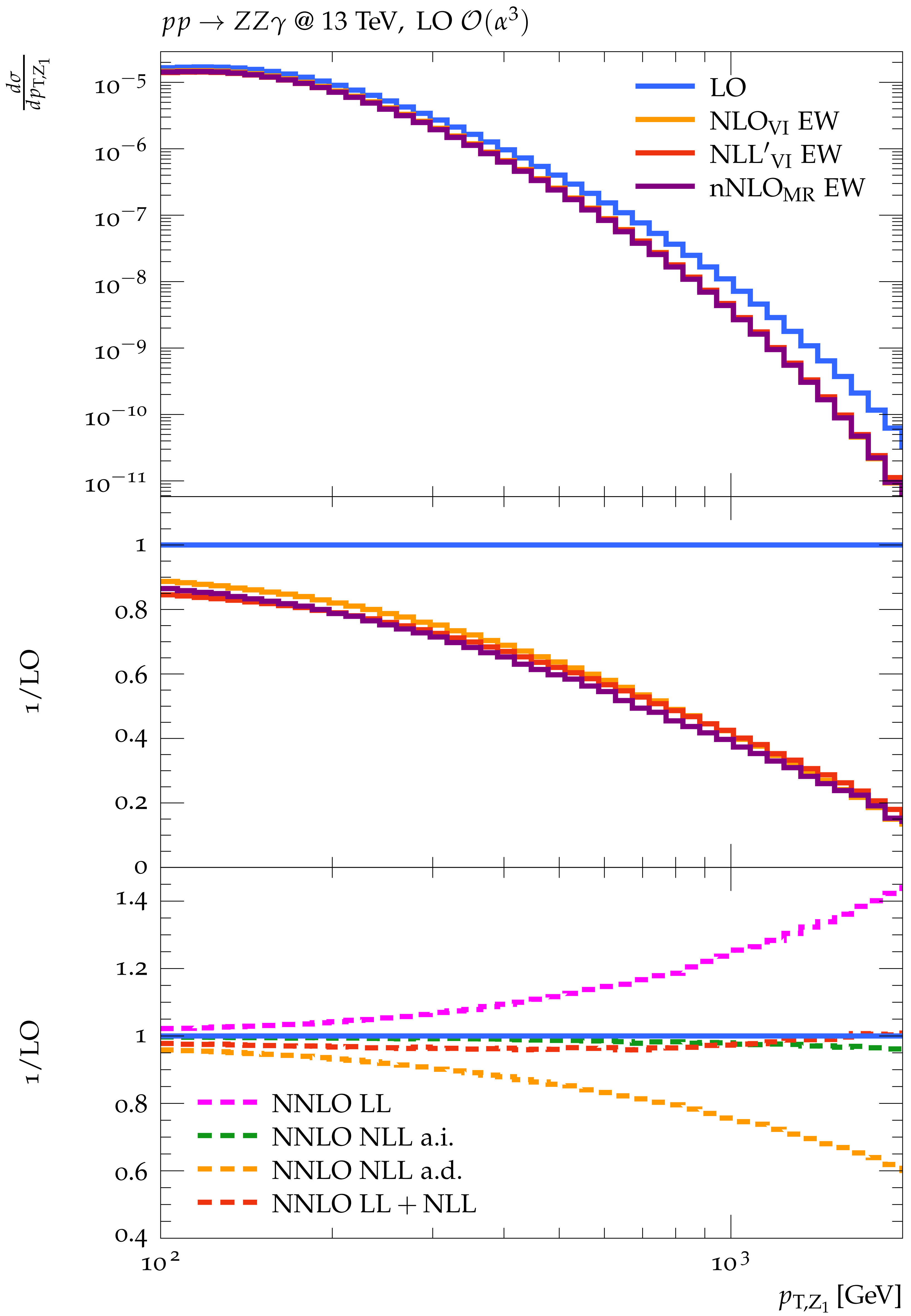
- **NLL'_{VI}** in % level agreement with **NLO_{VI}**
- **LL > NLL: NNLO** *positive*

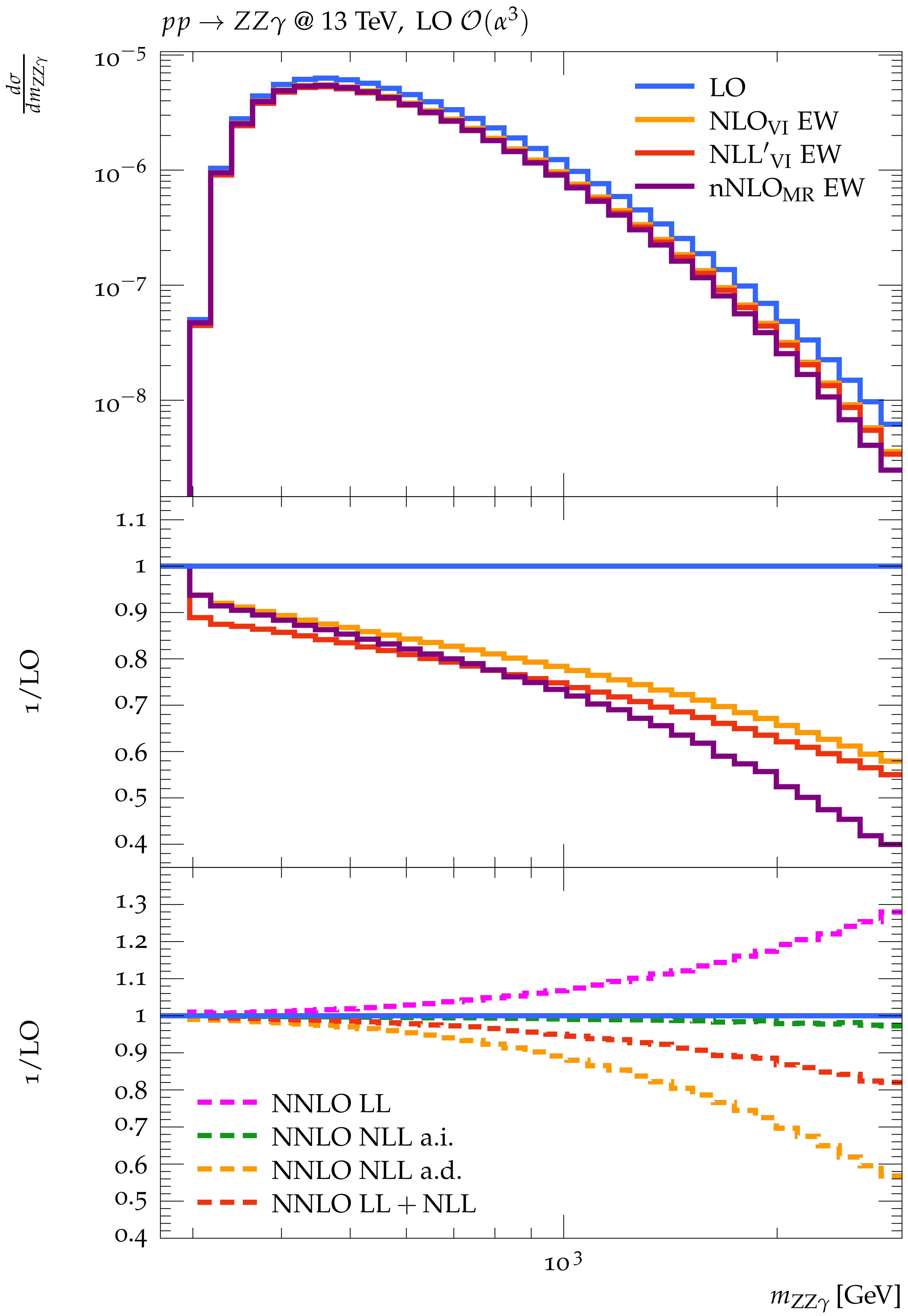
In m_{Wj} **LA** still spoilt despite additional cuts:

- **NLL'_{VI}** not good approximation for **NLO_{VI}**
- **LL < NLL a.d.: NNLO** is *negative*

Lindert, **L.M.**, *In preparation*

Two-loop results: hadron level





NLO$_{VI}$:
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NNLO LL + NLL:
two-loop **EW** @ NLL accuracy

In p_{T,Z_1} **LA** is partially spoilt due to configs with two hard V and one soft; **NNLO** *positive*, $\sim 2\,\%$ in tail

In $m_{ZZ\gamma}$ **LA** spoilt despite cut $|\eta_V| < 3$ for all V

For more on triboson processes see also

- TH: Stefan's, Giovanni's & Paolo's talks
- EXP: Trisha's & Anke's talks

Lindert, **L.M.**, *In preparation*

Conclusion and outlook

- **EW** corrections at high energies dominated by *Sudakov logarithms*: enhancement in tail of distributions by $\mathcal{O}(20-50\%)$ at one-loop and still $\mathcal{O}(5-10\%)$ at two-loop

- ***Factorization*** of *Sudakov logs* allows for implementation via effective CT vertices in **OL** $\Rightarrow$ loop corrections reduced to Born complexity, with % level agreement with exact **NLO**

- Additional aspects of the implementation @ one-loop:
  - Direct employment in Parton Shower Event Generators with **OL** interface
  - Can be used together with differential QED radiation at **NLO** (both MR and DR are available)
  - Support **EW** corrections for resonant processes

- Extension of our algorithm @ two-loop will allow for the *first time* a simple and automated access to **EW** corrections beyond **NLO**, enabling a systematic reduction of theory uncertainties

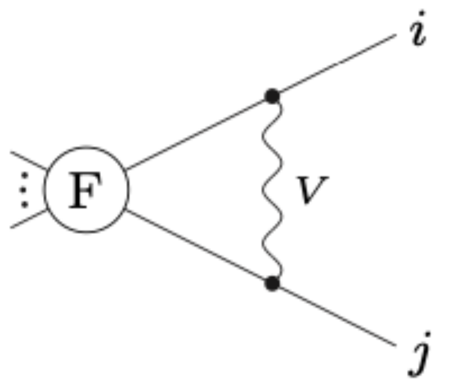
- Outlook:
  - Complete **two-loop** **EW** extension for a generic SM process (massive fermions, scalars)
  - Extend implementation to **mixed** **QCD-EW** corrections (see Federico's talk)

Backup

Alternative approach to one-loop mass singularities

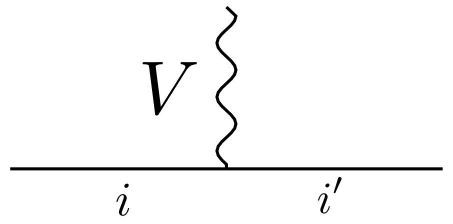
- Employ *Eikonal approximation*[1] for ***Factorizable*** diagrams, i.e.

$$\mathcal{M}_1^{\mathrm{F}} = \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{V=A,Z,W^\pm}\left[\ \cdots\ \right]_{q^\mu \to x p_i^\mu,\, x p_j^\mu,\quad x\to 0}$$



[1]NB: Eikonal approximation implies the substitution $N(q) \to N(0)$ in the numerator of loop integral and the removal of any mass term in it. For external longitudinal gauge bosons the **GBET** must be applied first

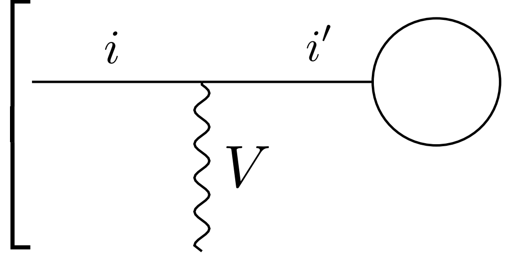
- ***Factorizable*** diagrams will therefore contain **only** ***soft-collinear*** singularities
- Generic **EW** Feynman rules as in Denner 0709.1075; *2007*



$$= ieI^V_{ii'}T$$

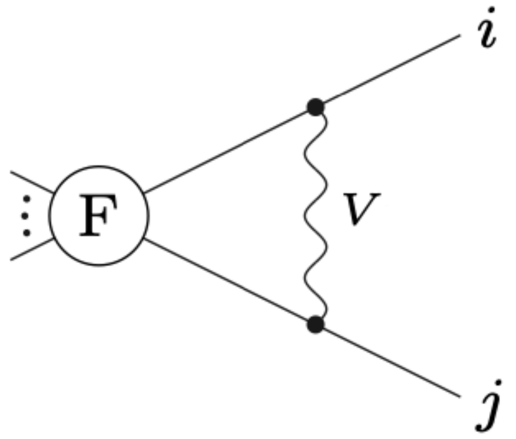
T contains the Lorentz structure of the vertex (e.g. in $Af\bar{f}$: $I^A_{ii'} = -Q_f, \quad T = \gamma^\mu$)

- When contracted with external w.f.s. in computing one-loop amplitudes, it follows



$$\left[\ \cdots\ \right]_{\mathrm{Eik.}} \propto 2ieI^V_{ii'}p_i^\mu \mathcal{M}_0^{\ldots i' \ldots}$$

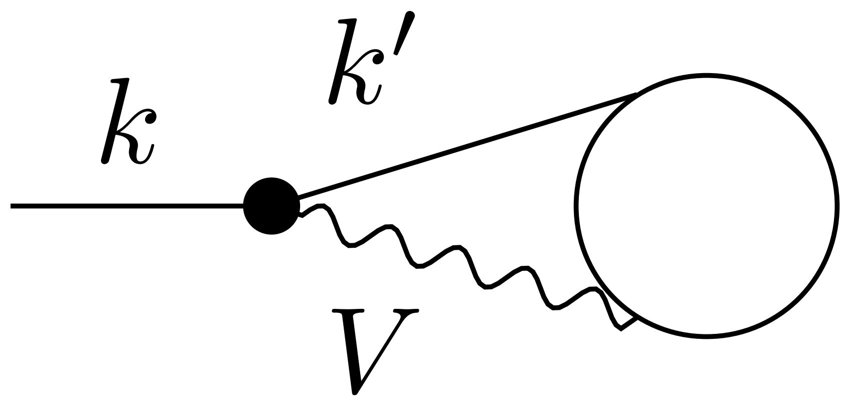
so that



$$\sim \sum_V \sum_{i',j'} \frac{\alpha}{4\pi} I^V_{ii'} I^{\bar{V}}_{jj'}\, r_{ij}\, C_0|_{\mathrm{Eik.}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_i}\cdots\varphi_{i'_j}\cdots\varphi_{i_n}}, \qquad C_0|_{\mathrm{Eik.}} \propto \frac{1}{r_{ij}}\left[\log^2 \frac{r_{ij}}{m_V^2}\right]$$

Denner & Pozzorini, hep-ph/0010201, hep-ph/0104127

Coll sketch



- Pure ***collinear*** singularities are extracted from type (iii) diagrams
- Original integral

$$\sim \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{N(q)}{(q^2 - m_V^2 + i0)[(p_k - q)^2 - m_k^2 + i0]}$$

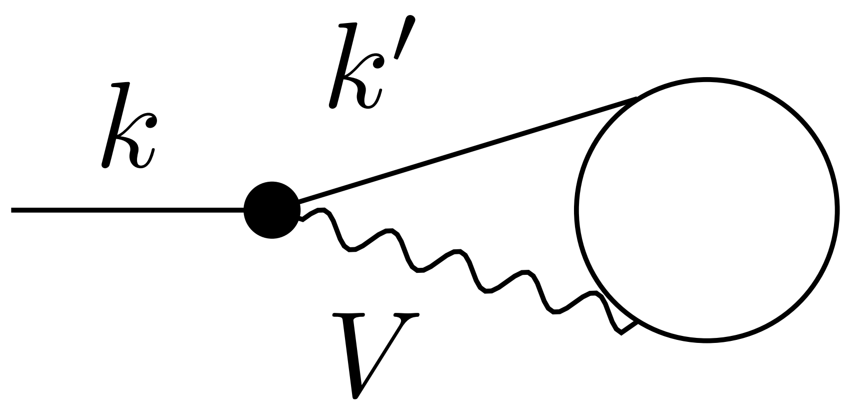
- In Sudakov parametrisation $q^\mu = x p_k^\mu + y l^\mu + q_{\mathrm{T}}^\mu$, after *y* integration the integral reduces to

$$\sim \mu^{4-D} \int_0^1 dx \int \frac{d^{D-2} q_{\mathrm{T}}}{(2\pi)^{D-2}} \frac{N(x, y_i, q_{\mathrm{T}})}{|\vec{q}_{\mathrm{T}}|^2 + \Delta(x)}$$

with $\Delta(x) = (1-x) m_V^2 + x m_{k'}^2 - x(1-x) p_k^2$ regulating the logarithmic singularity

Denner & Pozzorini, hep-ph/0010201, hep-ph/0104127

Coll sketch



- In Sudakov parametrisation

$$\sim \mu^{4-D} \int_0^1 dx \int \frac{d^{D-2}q_{\mathrm{T}}}{(2\pi)^{D-2}} \frac{N(x, y_i, q_{\mathrm{T}})}{|\vec{q}_{\mathrm{T}}|^2 + \Delta(x)}$$

with $\Delta(x) = (1-x)m_V^2 + xm_{k'}^2 - x(1-x)p_k^2$ regulating the logarithmic singularity

- Since we restrict to logarithmic mass-singular contributions, all terms of order

$$|\vec{q}_{\mathrm{T}}|^2,\ p_k^2,\ m_V,\ m_k,\ y_i\,(\propto |\vec{q}_{\mathrm{T}}|^2/p_k l)$$

can be neglected in $N(q)$

- <u>*Collinear approximation*</u>
  - ▲ Substitute $N(x, y, q_{\mathrm{T}}) \to N(x, 0, 0)$, i.e. $q^\mu \to xp_k^\mu$
  - ▲ Neglect all mass terms in $N(x, 0, 0)$

$$\delta^{\mathrm{coll}} \mathcal{M}^{\varphi_{i_1} \cdots \varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_k \sum_{k'} \sum_V \delta_{kk'}^{\mathrm{coll,\ V}} \mathcal{M}_0^{\varphi_{i_1} \cdots \varphi_{i'_k} \cdots \varphi_{i_n}}$$

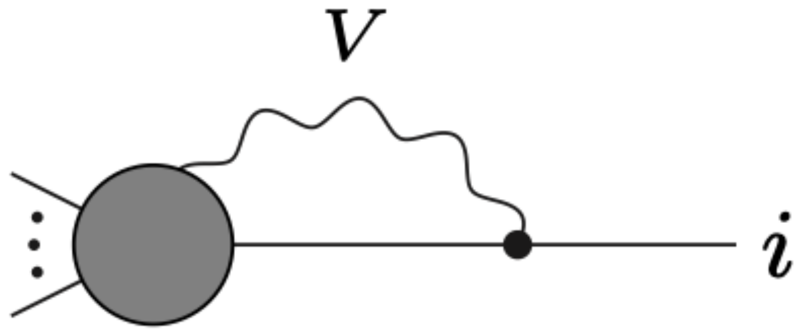
- Final result: $\overset{\mathrm{LA}}{\sim} \log \frac{\mu_{\mathrm{D}}^2}{M^2} \int_0^1 \mathrm{d}x N(x, 0, 0), \quad M^2 = \max(p_k^2, M_{\varphi_c}^2, m_{k'}^2) \Rightarrow$

$$\delta_{kk'}^{\mathrm{coll,\ V}} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right)$$

Note: the scale μ_{D} of DR is always set to s

One-loop mass singularities

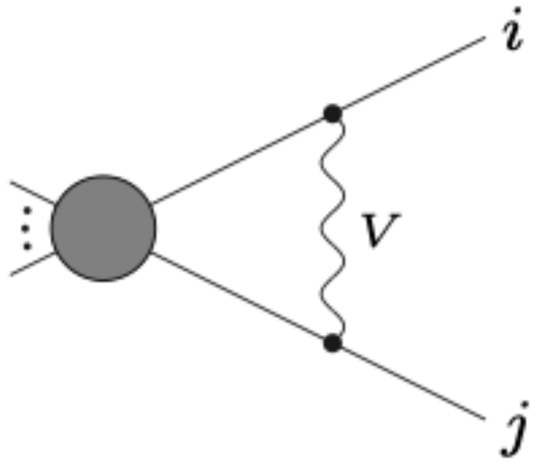
- One-loop mass singularities originate from diagrams where a vector boson V couples to one external line and to any other line of the diagrams, i.e.



- These diagrams can be classified into three groups

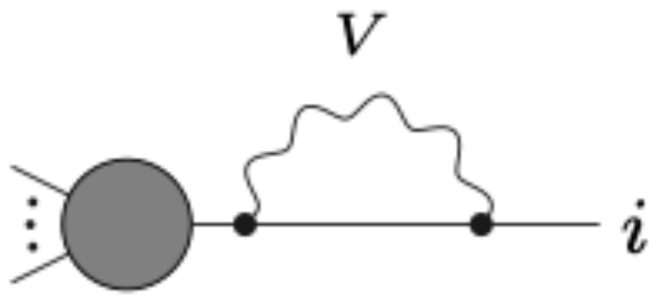
Type (iii) : V coupled to one internal and one external line

Type (i): V coupled to two different external lines



Give rise to *mass singularities* from region of integration where V is ***soft and /or collinear*** to one of the external legs

Type (ii): external leg self-energy



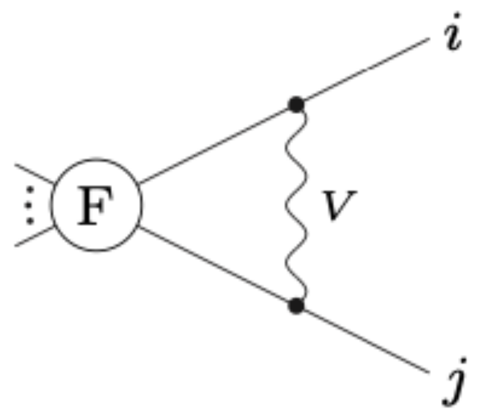
Omitted, as we express S- matrix elements in term of truncated Green functions and on-shell renormalized fields

Denner *et al.* hep-ph/0608326

One-loop mass singularities: *F* vs *NF*

- From type (i) diagrams is possible to extract mass-singularities which factorize w.r.t. corresponding Born subdiagram. These are known as ***Factorizable (F)*** contributions

$$\mathcal{M}_1^{\mathrm{F}} = \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{V=A,Z,W^\pm}\left[\quad\right]_{q^\mu\to xp_i^\mu,\,xp_j^\mu}$$



Diagrams are evaluated in the approximation of V being:

- *soft and collinear* ⟶ **LL + NLL a.d.** $\propto L^2 + 2l_{ij}L + l_{ij}^2$
- *collinear* ⟶ **NLL a.i.** $\propto L$

l_{ij}^2 : a.d. **NNLL** term formally not part of **LA.**
Needed for reliable estimates as first pointed out in Pagani, Zaro 2110.03714; *2021.*
In our implementation it can be either excluded (**NLL**) or included (**NLL'**) as a tool to estimate uncertainties beyond **LA**

Denner *et al.*:hep-ph/0010201, hep-ph/0104127, hep-ph/0608326

One-loop mass singularities: *F* vs *NF*

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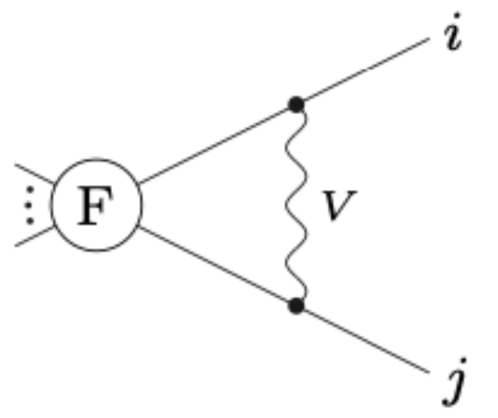

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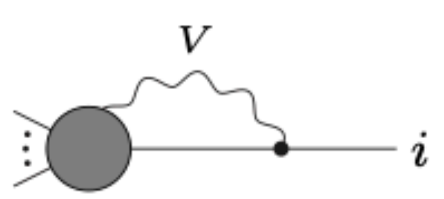
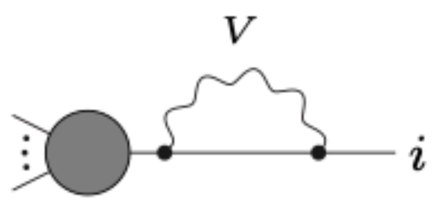
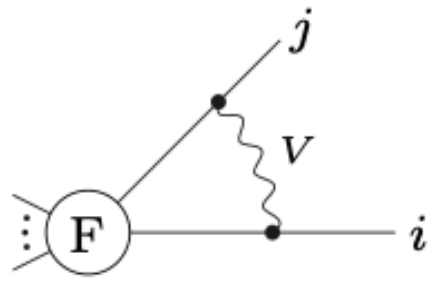
- *soft and collinear* ⟶ **LL + NLL a.d.** $\propto L^2 + 2l_{ij}L + l_{ij}^2$
- *collinear* ⟶ **NLL a.i.** $\propto L$

- The remaining one-loop mass singularities are obtained by subtracting type (i) and (ii) diagrams from the original set. These are referred to as ***Non-Factorizable (NF)***

$$\mathcal{M}_1^{\mathrm{NF}} = \sum_{i=1}^{n}\sum_{V=A,Z,W^{\pm}} \Big[\quad - \quad - \sum_{\substack{j=1\\ j\neq i}}^{n} \quad \Big]_{q^\mu \to x p_i^\mu}$$







- Collinear Ward Identities make all ***NF*** diagrams to vanish $\Longrightarrow$ ***Factorizable*** contributions contain ***all*** one-loop mass singularities of *soft and/or collinear* origin

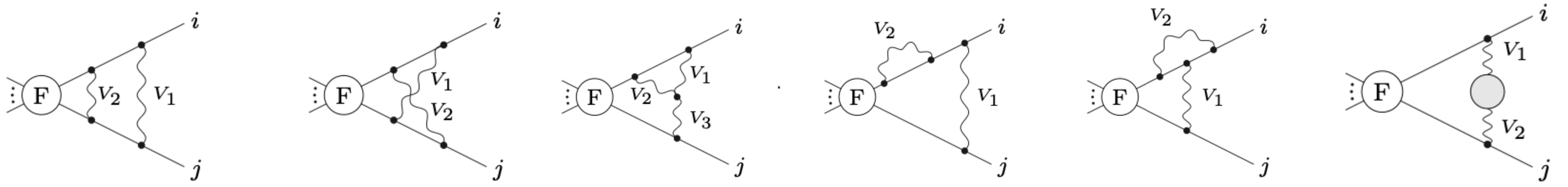
Denner *et al.*: hep-ph/0010201, hep-ph/0104127, hep-ph/0608326

Two-loop mass singularities

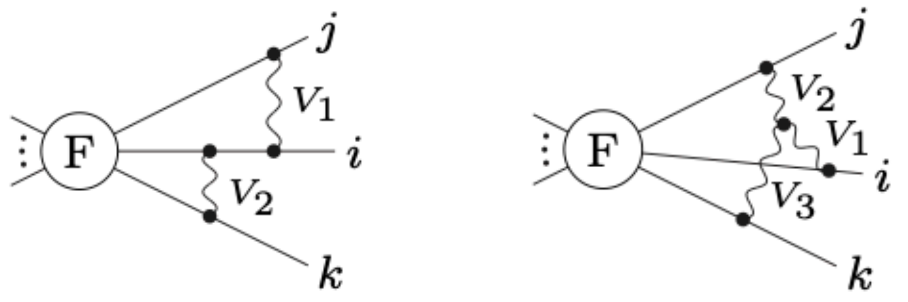
- Also two-loop mass singularities can be split into ***Factorizable*** and ***Non-Factorizable***

    ✦ *F*: 9 topologies sub-split into

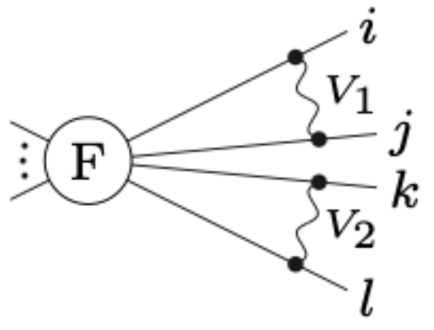
    ✤ Diags related to two external lines



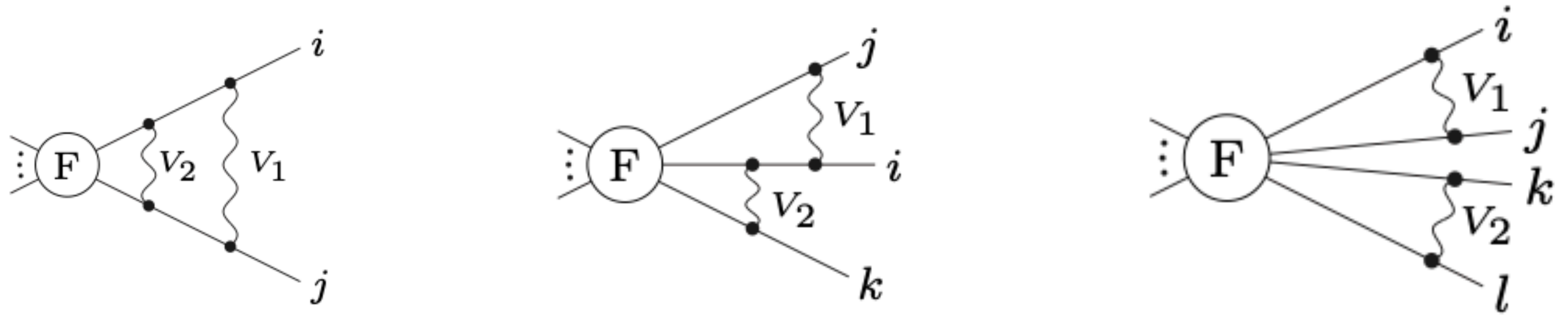
    ✤ Diags related to three external lines



    ✤ Diags related to four external lines



- **EW** Ward Identities induce cancellations ⇒ only three classes of topologies are required



- Two-loop *NF* *vanish* due to collinear Ward Identities: *F* contain ***all*** two-loop **NLL** mass singularities

Denner *et al.*
hep-ph/0608326 & hep-ph/0809.0800

Two-loop mass singularities: final factorization

- Combining one- and two-loop corrections at **NLL** accuracy

$$\mathcal{M} \stackrel{\text{NLL}}{=} F^{\text{sew}} F^{\text{Z}} F^{\text{em}} \mathcal{M}_0$$

$$F^{\text{sew}} \stackrel{\text{NLL}}{=} 1 + \frac{\alpha}{4\pi} F_1^{\text{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 \left[\frac{1}{2}\left(F_1^{\text{sew}}\right)^2 + G_2^{\text{sew}}\right] \stackrel{\text{NLL}}{=} \exp\left[\frac{\alpha}{4\pi} F_1^{\text{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 G_2^{\text{sew}}\right]$$

$$F^{\text{em}} \stackrel{\text{NLL}}{=} 1 + \frac{\alpha}{4\pi} \Delta F_1^{\text{em}} + \left(\frac{\alpha}{4\pi}\right)^2 \left[\frac{1}{2}\left(\Delta F_1^{\text{em}}\right)^2 + \Delta G_2^{\text{em}}\right] \stackrel{\text{NLL}}{=} \exp\left[\frac{\alpha}{4\pi} \Delta F_1^{\text{em}} + \left(\frac{\alpha}{4\pi}\right)^2 \Delta G_2^{\text{em}}\right]$$

$$F^{\text{Z}} \stackrel{\text{NLL}}{=} 1 + \frac{\alpha}{4\pi} \Delta F_1^{Z}$$

- Symmetric **EW** part, where all gauge bosons masses are set to m_W
- **EM** part, resulting from the mass gap between photon and W boson masses. As such, it contains divergences and depends on the scheme used to regularize them
- m_Z dependent part, due to difference between m_Z and m_W. Absent in IREE predictions

with

$$F_1^{\text{sew}} \stackrel{\text{NLL}}{=} -\frac{1}{2} \sum_{\substack{i=1 \\ j=1, j\neq i}}^{n} \sum_{V} I_i^{\bar{V}} I_j^{V} I(\epsilon, m_W; -r_{ij}), \quad I(\epsilon, m_W; -r_{ij}) \stackrel{\text{NLL}}{\supset} -L^2 + (3 - 2l_{ij})L$$

$$\Delta F_1^{\text{em}} \stackrel{\text{NLL}}{=} -\frac{1}{2} \sum_{\substack{i=1 \\ j=1, j\neq i}}^{n} I_i^{A} I_j^{A} \Delta I(\epsilon, 0; -r_{ij}), \quad \Delta I(\epsilon, 0; -r_{ij}) \stackrel{\text{NLL}}{\supset} L^2 - (3 - 2l_{ij})(\epsilon^{-1} + L) - 2\epsilon^{-2}$$

$$\Delta F_1^{\text{Z}} \stackrel{\text{NLL}}{=} -\frac{1}{2} \sum_{\substack{i=1 \\ j=1, j\neq i}}^{n} I_i^{Z} I_j^{Z} \Delta I(\epsilon, m_Z; -r_{ij}), \quad \Delta I(\epsilon, m_Z; -r_{ij}) \stackrel{\text{NLL}}{\supset} 2L \log(m_Z^2/m_W^2)$$

$$G_2^{\text{sew}} \stackrel{\text{NLL}}{=} \frac{1}{2} \sum_{i=1}^{n} \left[\frac{b_1^{(1)}}{c_w^2}\left(\frac{Y_i}{2}\right)^2 + \frac{b_2^{(1)}}{s_w^2} C_i\right] J(\epsilon, m_W; \mu_{\text{R}}^2), \quad J(\epsilon, m_W; \mu_{\text{R}}^2) \stackrel{\text{NLL}}{\supset} \frac{1}{3}L^3 - l_{\mu_{\text{R}}} L^2$$

$$\Delta G_2^{\text{em}} \stackrel{\text{NLL}}{=} \frac{1}{2} \sum_{i=1}^{n} Q_i^2 \left[b_e^{(1)}\left(\Delta J(\epsilon, 0; \mu_{\text{R}}^2) - \Delta J(\epsilon, 0; m_W^2)\right) + b_{\text{QED}}^{(1)} \Delta J(\epsilon, 0; m_W^2)\right],$$

$$\Delta J(\epsilon, 0; \mu_{\text{R}}^2) - \Delta J(\epsilon, 0; m_W^2) \stackrel{\text{NLL}}{\supset} l_{\mu_{\text{R}}}\left(-2\epsilon^{-2} + \epsilon^{-1}(l_{\mu_{\text{R}}} - 2L) + l_{\mu_{\text{R}}} L - \frac{1}{3} l_{\mu_{\text{R}}}^2\right)$$

$$\Delta J(\epsilon, 0; m_W^2) \stackrel{\text{NLL}}{\supset} \frac{3}{2}\epsilon^{-3} + 2L\epsilon^{-2} + L^2\epsilon^{-1}$$

NB:

$$L = \log\left(\frac{Q^2}{m_W^2}\right), \quad l_{ij} = \log\left(\frac{|r_{ij}|}{Q^2}\right)$$

- Divergences in **EM** part cancel when combining virtual and real corrections. The symmetric **EW** part has large but finite logs which do not vanish when real *Z* and *W* are included as a consequence of BN/KLN violation in **EW** theory

Denner *et al.*
hep-ph/0608326 & hep-ph/0809.0800

Implementation in OpenLoops: resonances

- DP algorithm in a nutshell:

  - ✦ At $\sqrt{s} \gg m_W$, **NLO EW** radiative corrections are
  - ✦ These corrections are ***universal***, i.e. are associated to *external* legs only

- Not suitable for processes involving the two-body decay of an unstable particle $X \to ij$ as in the resonant region $s \gg r_{ij} \approx m_X^2$ → LA is violated

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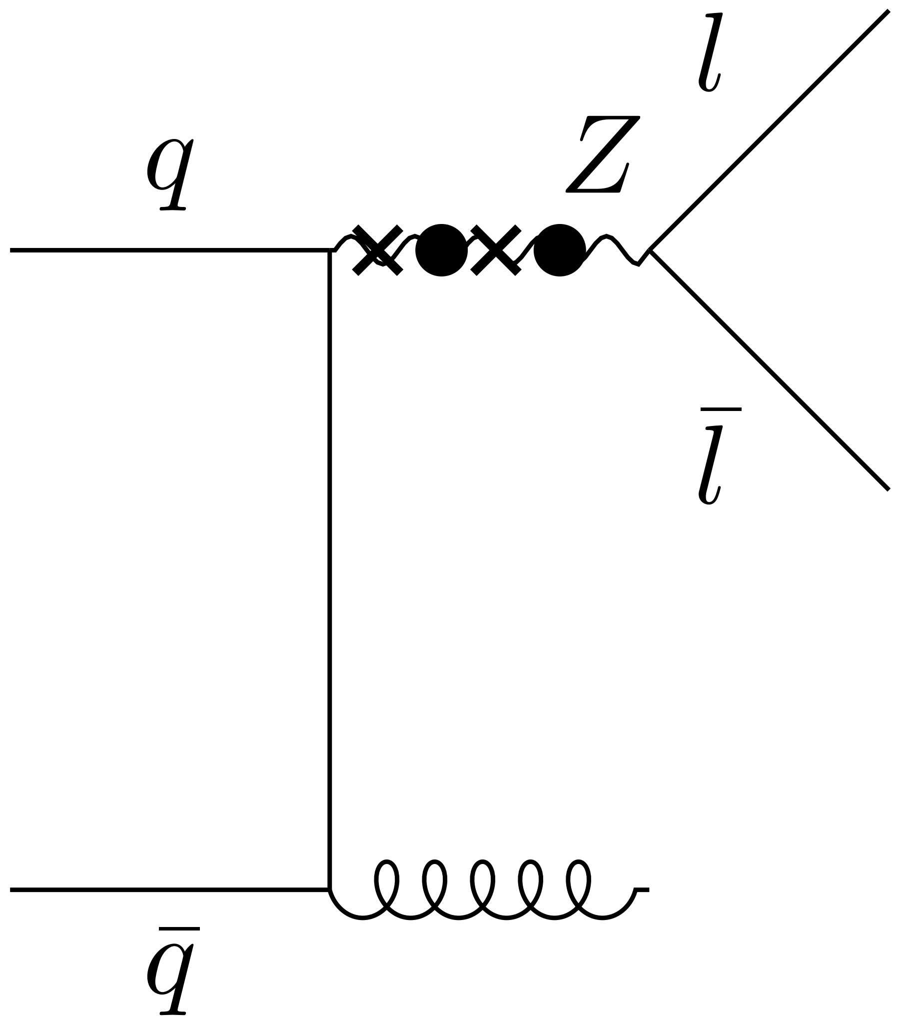
- A possible solution is the strategy adopted within Madspin [1212.3460] and via the HDH handler in Sherpa [1905.09127]:
  - ✦ Employ NWA to generate the hard scattering process including the associated $\mathcal{O}(\alpha)$ EW corrections, then adding the decay
  - ✦ LO off-shell effects and spin correlations can be retained via subsequent Breit-Wigner smearing

Implementation in OpenLoops: resonances

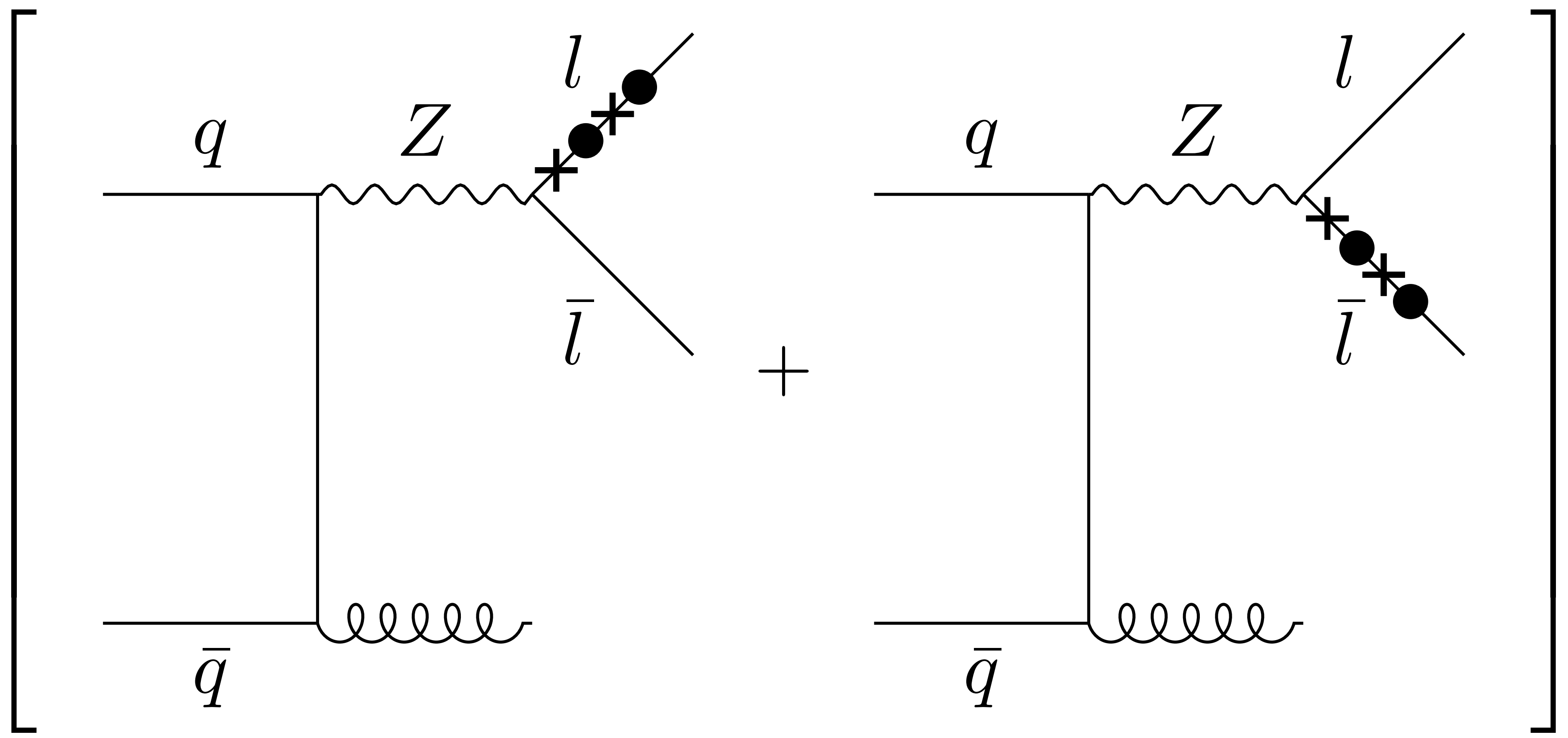
- Our approach: evaluation of *Sudakov logarithms* associated to both X and $\{i,j\}$ with different weights $P_i(k_i)$

NB: also the internal effective two-point counterterm vertices are *helicity- dependent* and project on the helicity of the combined $\{i,j\}$ current of the external states



$\times\, P_Z(k)$

$$P_{X_i}(k_i) = \left| \frac{\mu^2_{X_i} - m^2_{X_i}\Gamma^2_{X_i}}{(k_i^2 - \mu^2_{X_i})^2 + \mu^2_{X_i}} \right| = \begin{cases} 1 & \text{if} \quad k_i^2 \to m^2_{X_i} \\ 0 & \text{if} \quad k_i^2 \to \infty \end{cases}$$



$\times\, P_{\{l,\bar{l}\}}(k) = 1 - P_Z(k)$

Implementation in OpenLoops: projectors

- Explicit expression of the projectors for unstable particles X

$$P_{X_i}(k_i) = \left| \frac{\mu_{X_i}^2 - m_{X_i}^2 w_{\text{rescale}}^2 \Gamma_{X_i}^2}{(k_i^2 - m_{X_i}^2 + i m_{X_i} w_{\text{rescale}} \Gamma_{X_i})^2 + \mu_{X_i}^2} \right| = \begin{cases} 1 & \text{if} \quad k_i^2 \to m_{X_i}^2 \\ 0 & \text{if} \quad k_i^2 \to \infty \end{cases}$$

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- w_{rescale} is a technical parameter which determines the resonance region; it should be chosen of order 10 to capture the entire resonance enhancement of the off-shell amplitude

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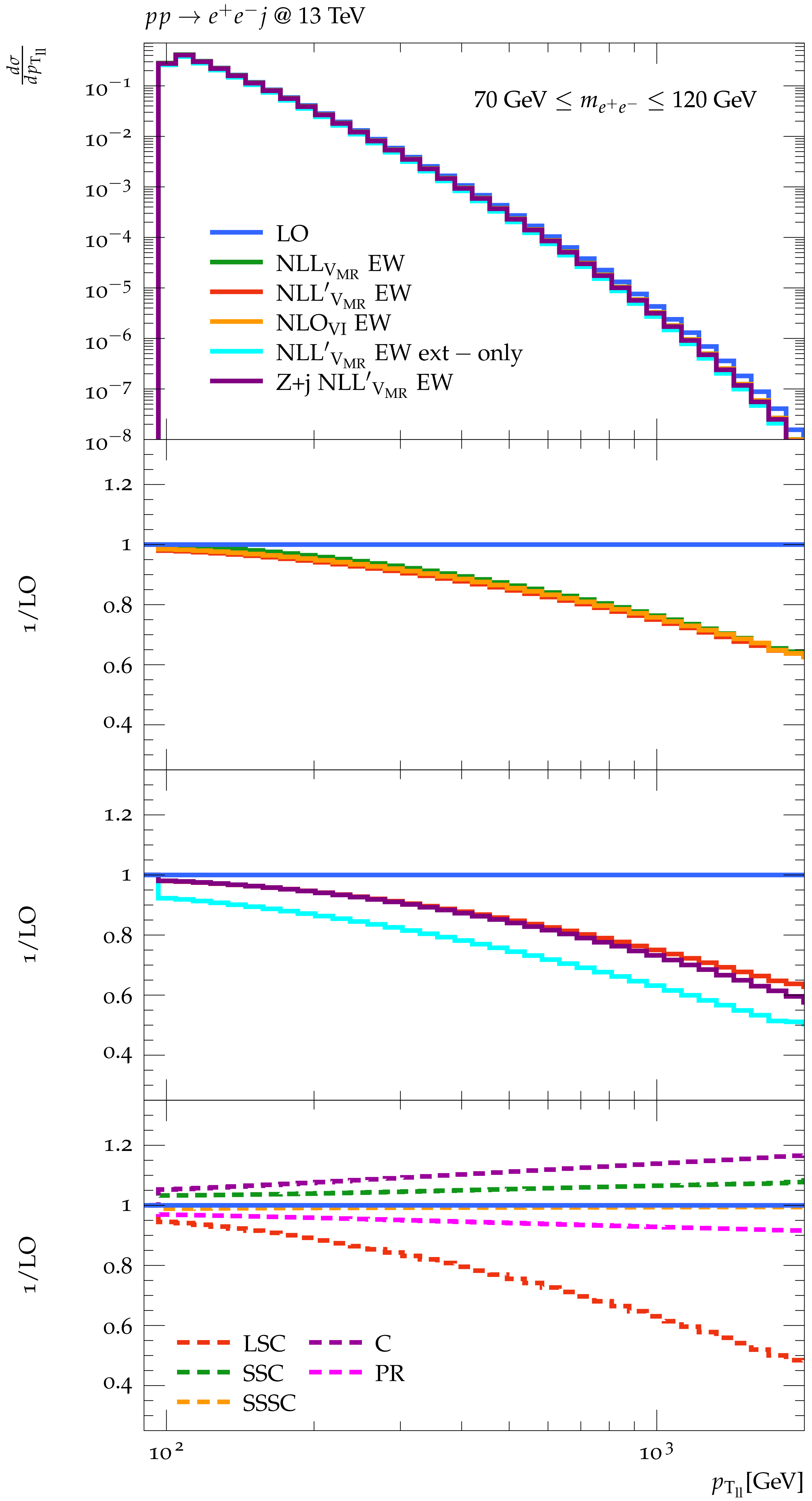
- w_{rescale} is a technical parameter which determines the resonance region; it should be chosen of order 10 to capture the entire resonance enhancement of the off-shell amplitude

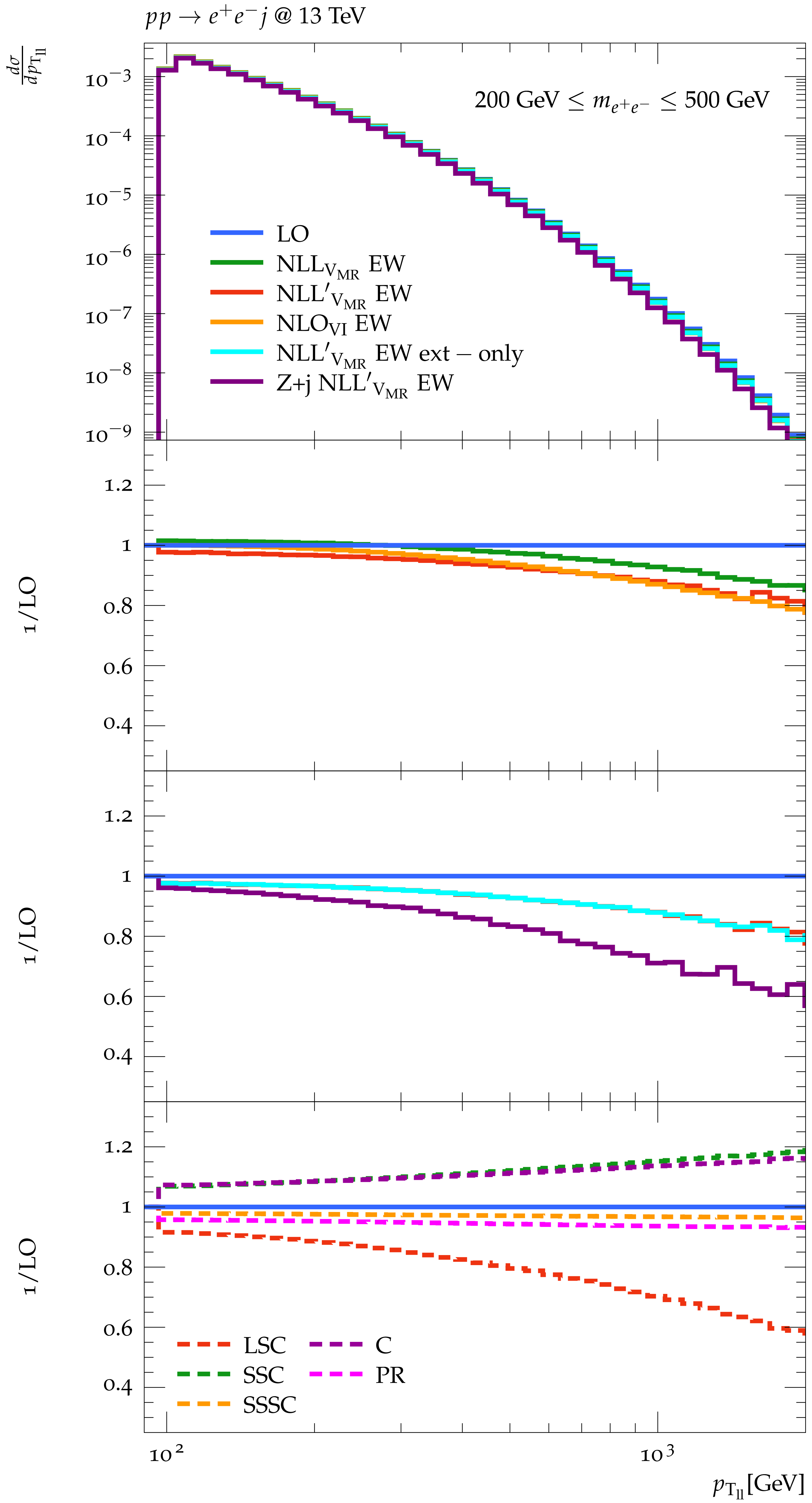
- The direct employment of projectors would violate unitarity but this can be prevented as follows:
  - Evaluation of $P_{X_i}(k_i)$ for a given psp
  - Generation of random number $0 \leq a \leq 1$
  - Choice $P_{X_i} = \begin{cases} 1 & \text{if } P_{X_i} \geq a \\ 0 & \text{if } P_{X_i} \leq a \end{cases}$

NLO EW: Denner *et al*, 1011.6674

NLO QCD+EW: Kallweit *et al*, 1511.08692

Results: $pp \to e^+e^- j$





The standard implementation based on external insertions fails in reproducing the full NLO prediction for the $m_{e^+e^-}$ range capturing the resonance

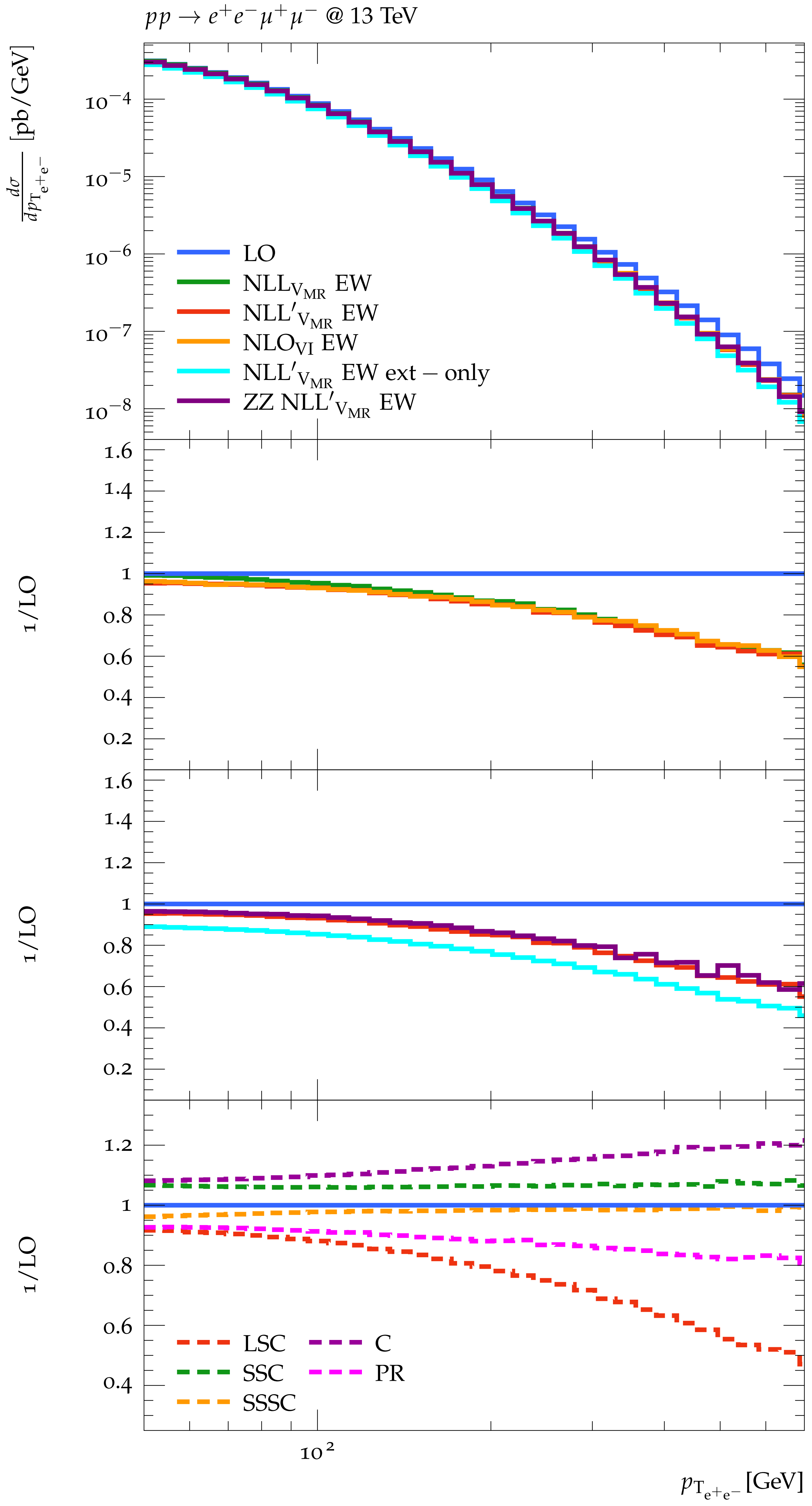
Issue naturally solved with internal insertions controlled by projectors

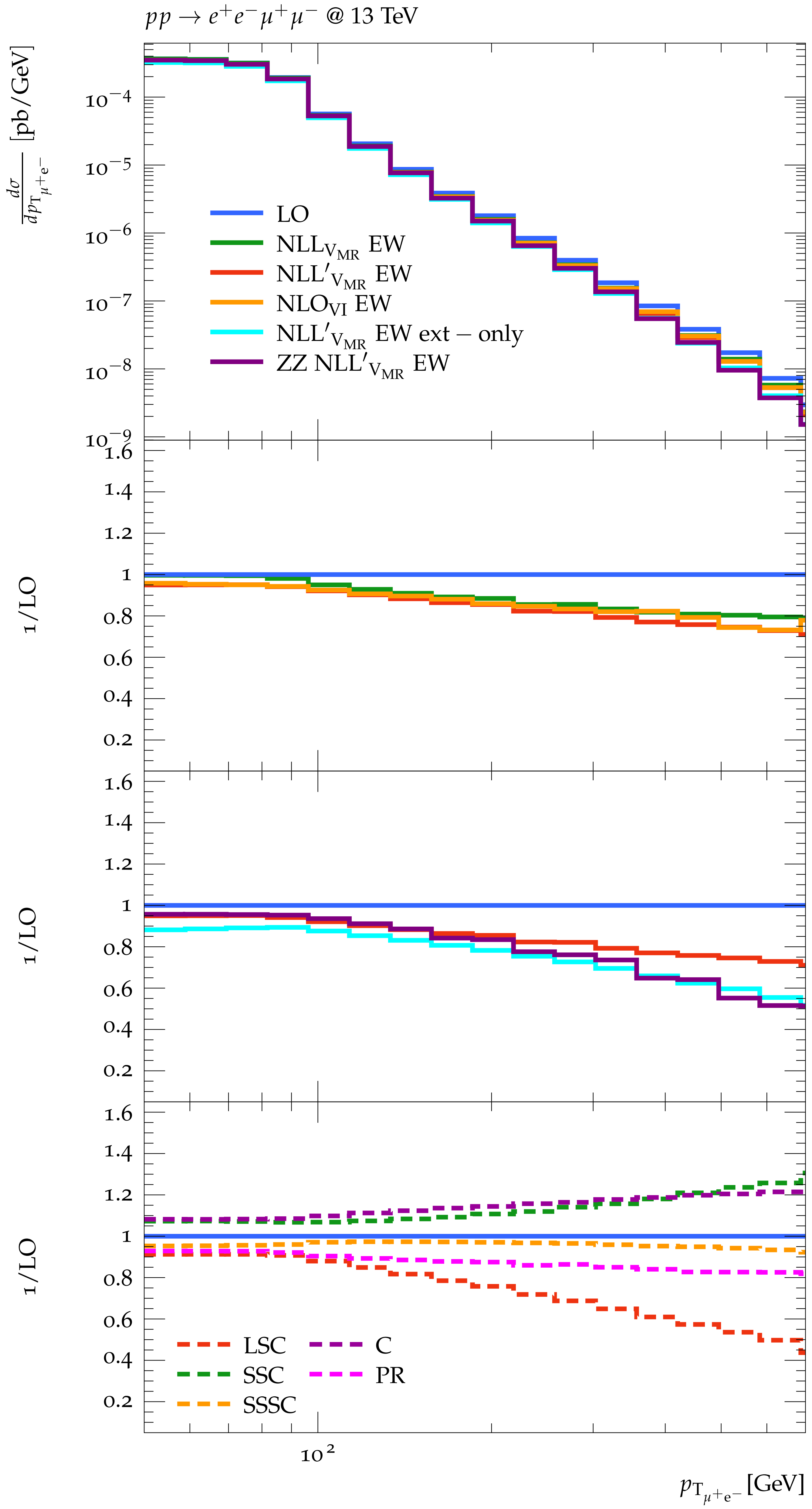
Automatic recover of standard algorithm when far from the resonance

NB: all NLL' and NLL predictions are evaluated in MR with $\lambda = m_W$ for QED contributions. However, consistency of $\mathrm{NLL}'_{\mathrm{V_{MR}}}$ with:

- $Z + \mathrm{j}\ \mathrm{NLL}'_{\mathrm{V_{MR}}}$ in their respective on-shell phase spaces
- $\mathrm{NLL}'_{\mathrm{V_{MR}}}$ ext-only in the off-shell phase space

Results: $pp \to e^+e^-\mu^+\mu^-$



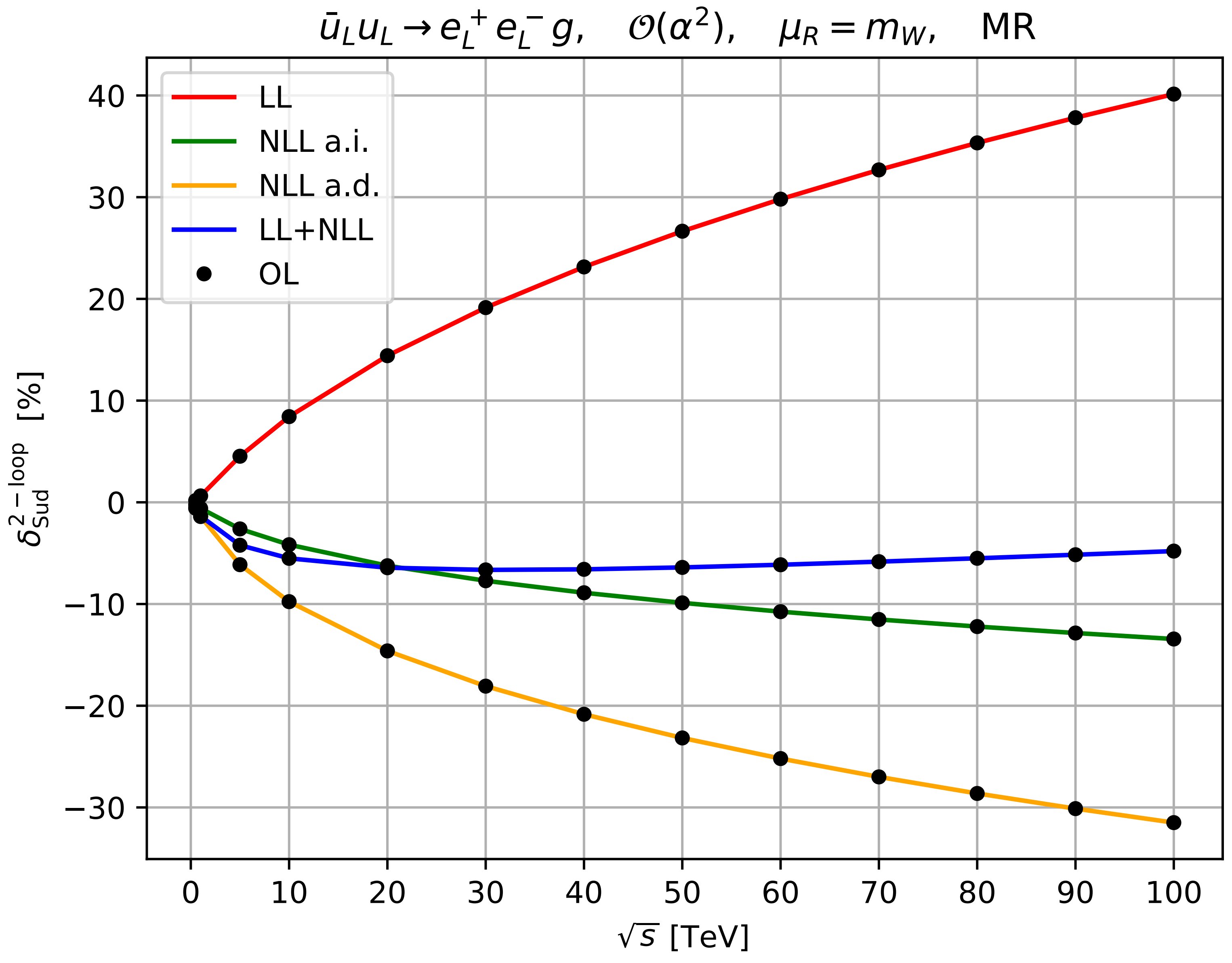


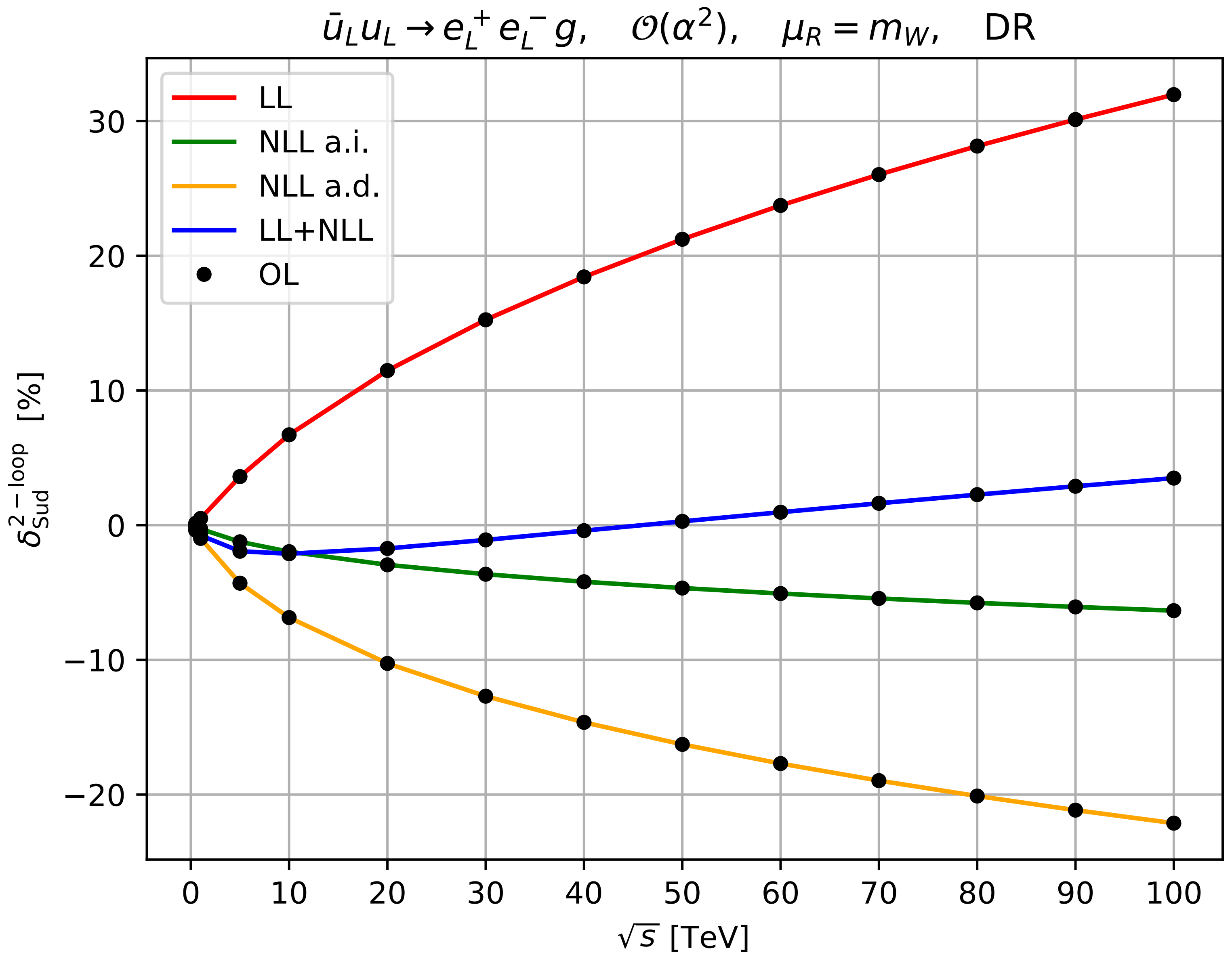
Internal insertions work accurately also for processes with more unstable particles:

- Resonant configuration correctly captured in $p_{T_{e^+e^-}}$
- Overlapping on-shell and off-shell effects properly interpolated in $p_{T_{\mu^+e^-}}$

Additional results

Two-loop results: amplitude level

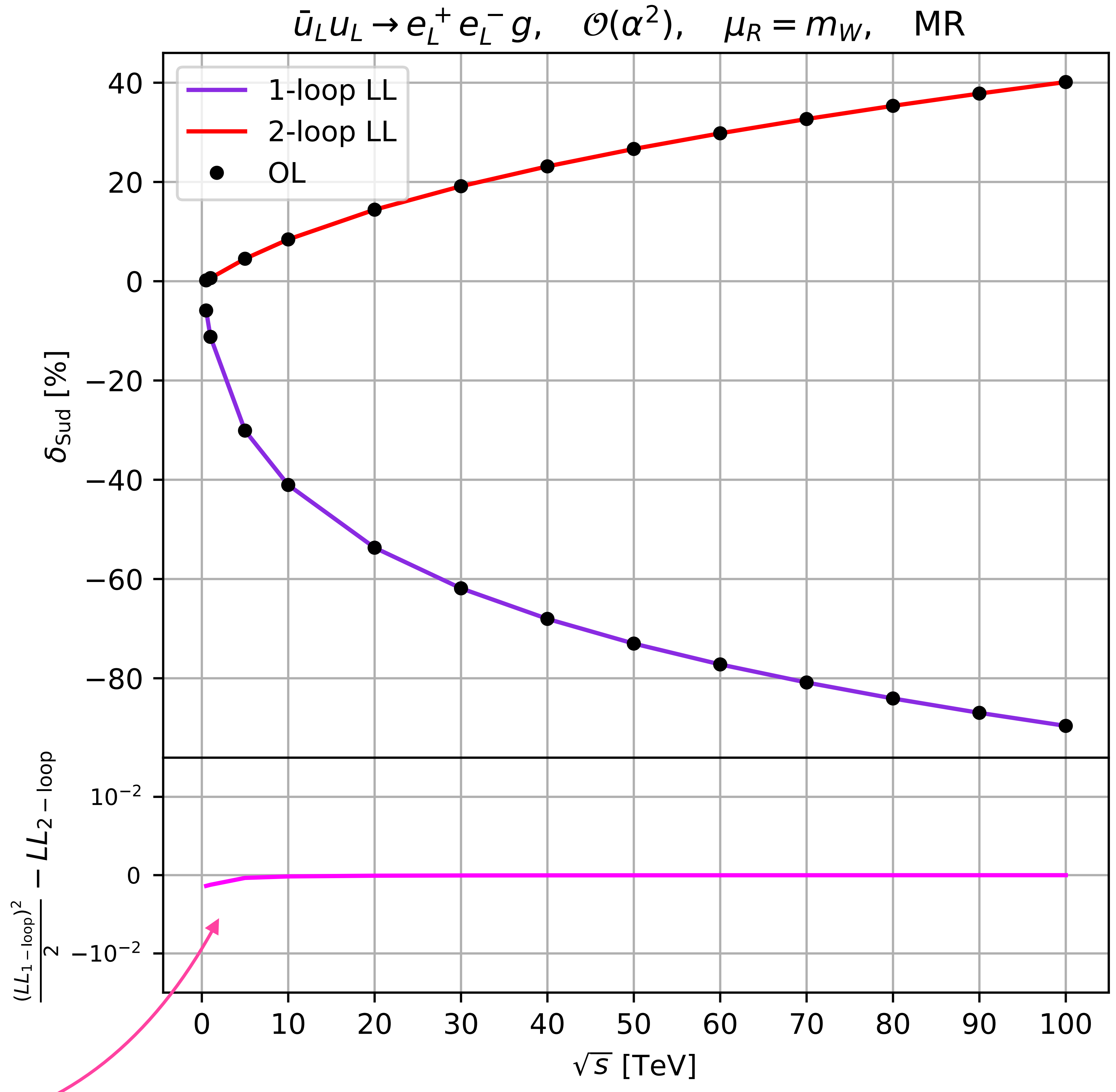




Analytic results firstly presented in
Denner *et al.* hep-ph/0608326

At multi TeV colliders two-loop EW corrections still important!

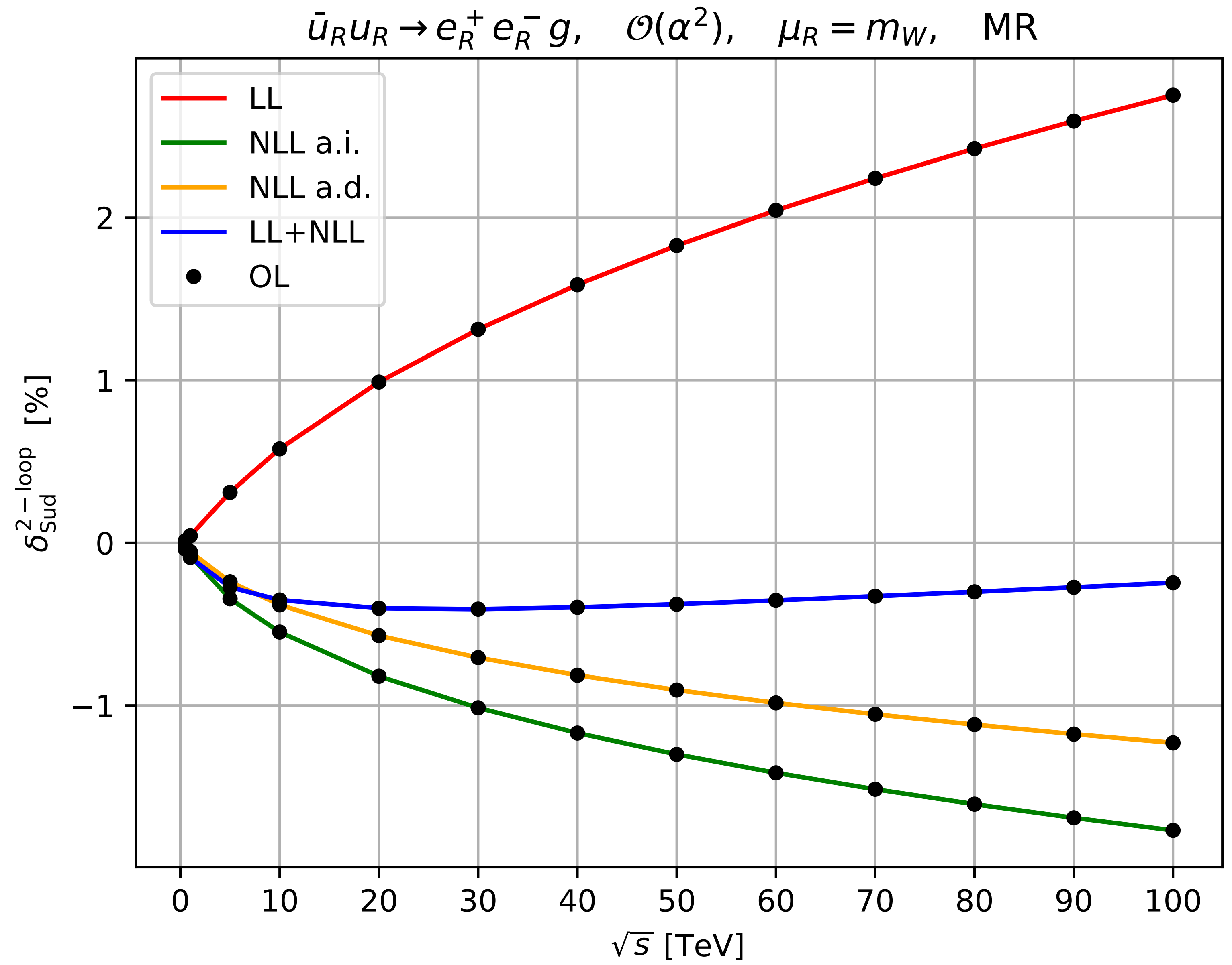
Little deviation at $\sqrt{s} \sim 1-2$ TeV due to difference in W and Z boson's masses in propagator of Born subdiagram

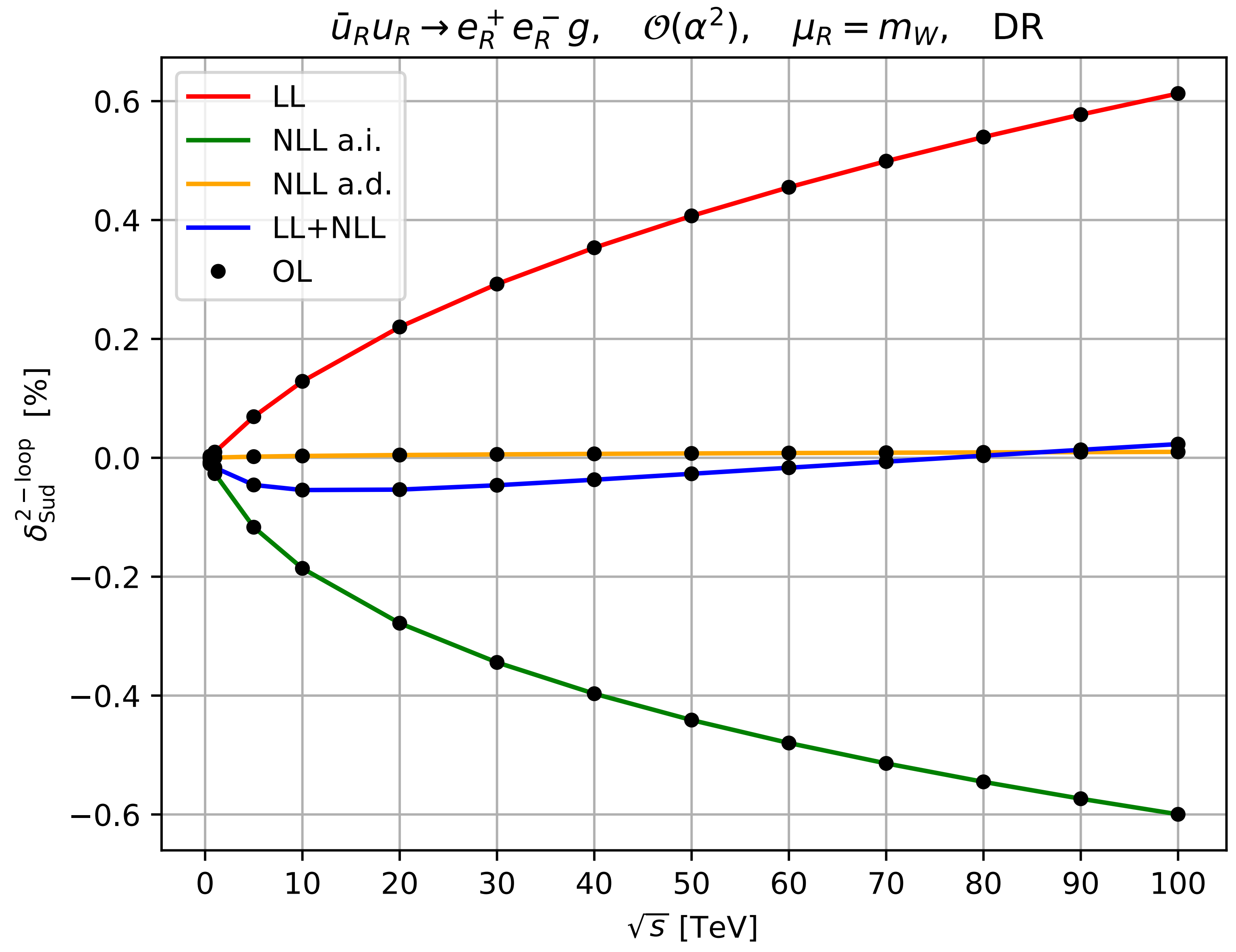


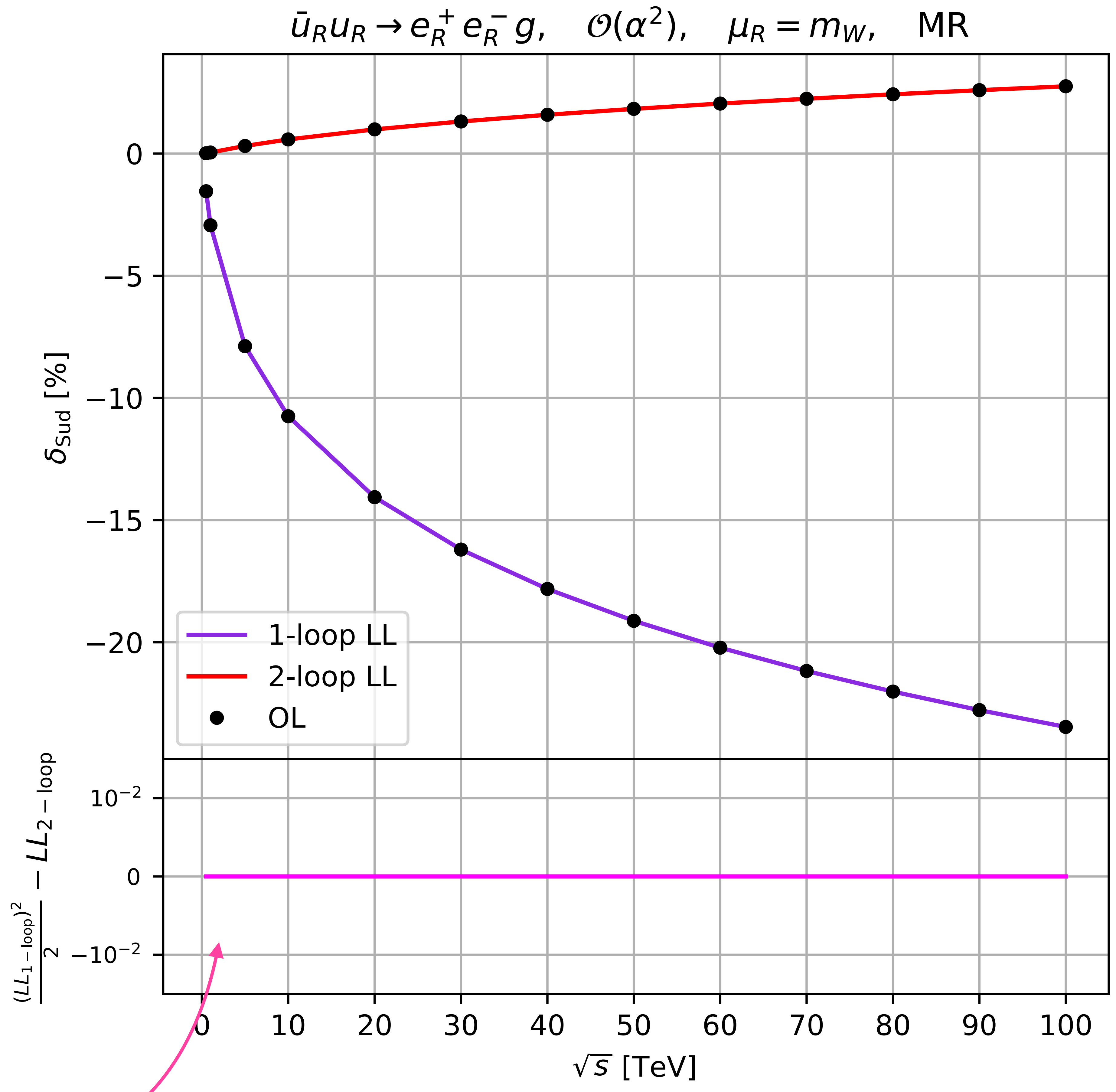
Two-loop LL partially compensate one-loop LL!

Lindert, **L.M.**, *In preparation*

Two-loop results: amplitude level





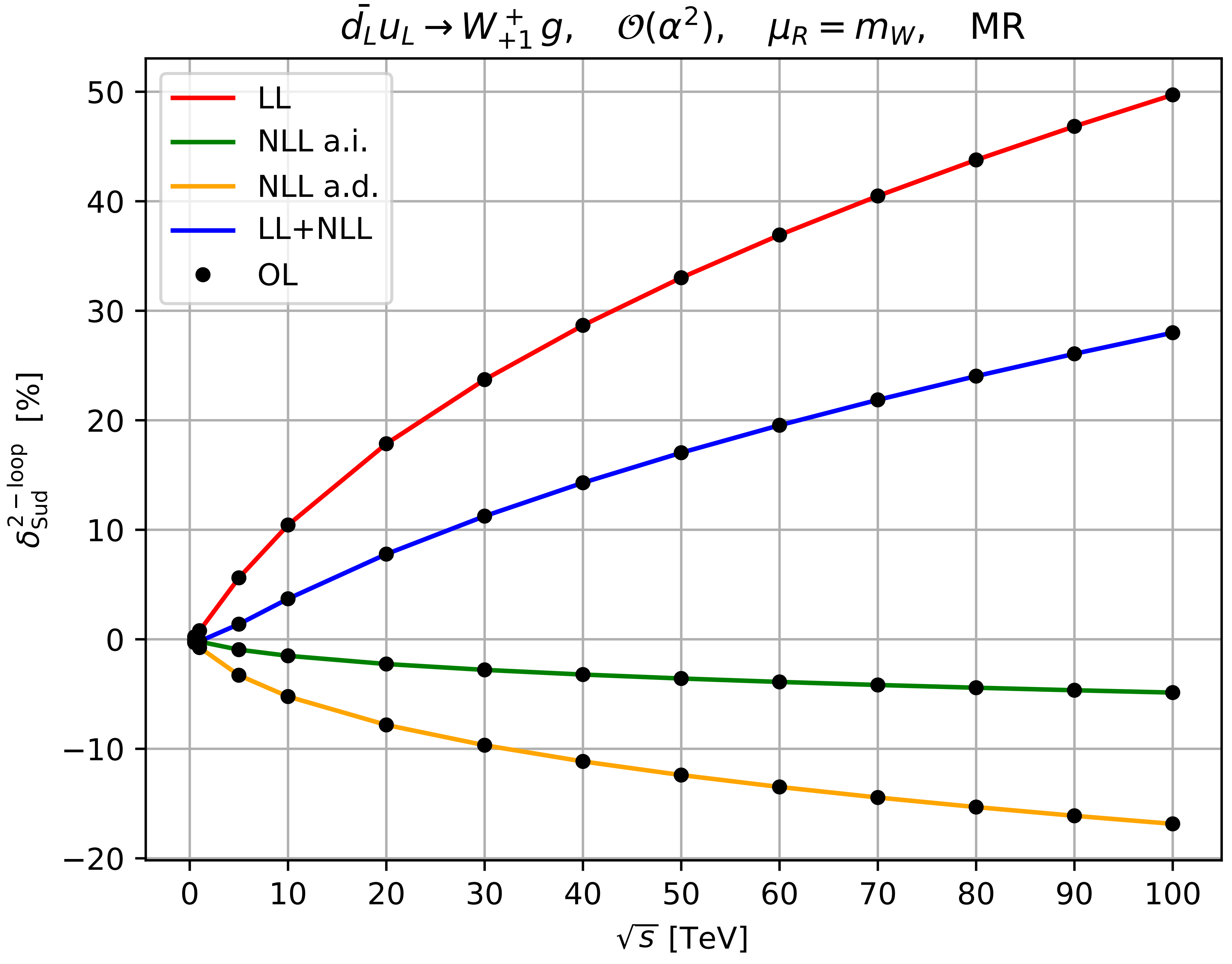


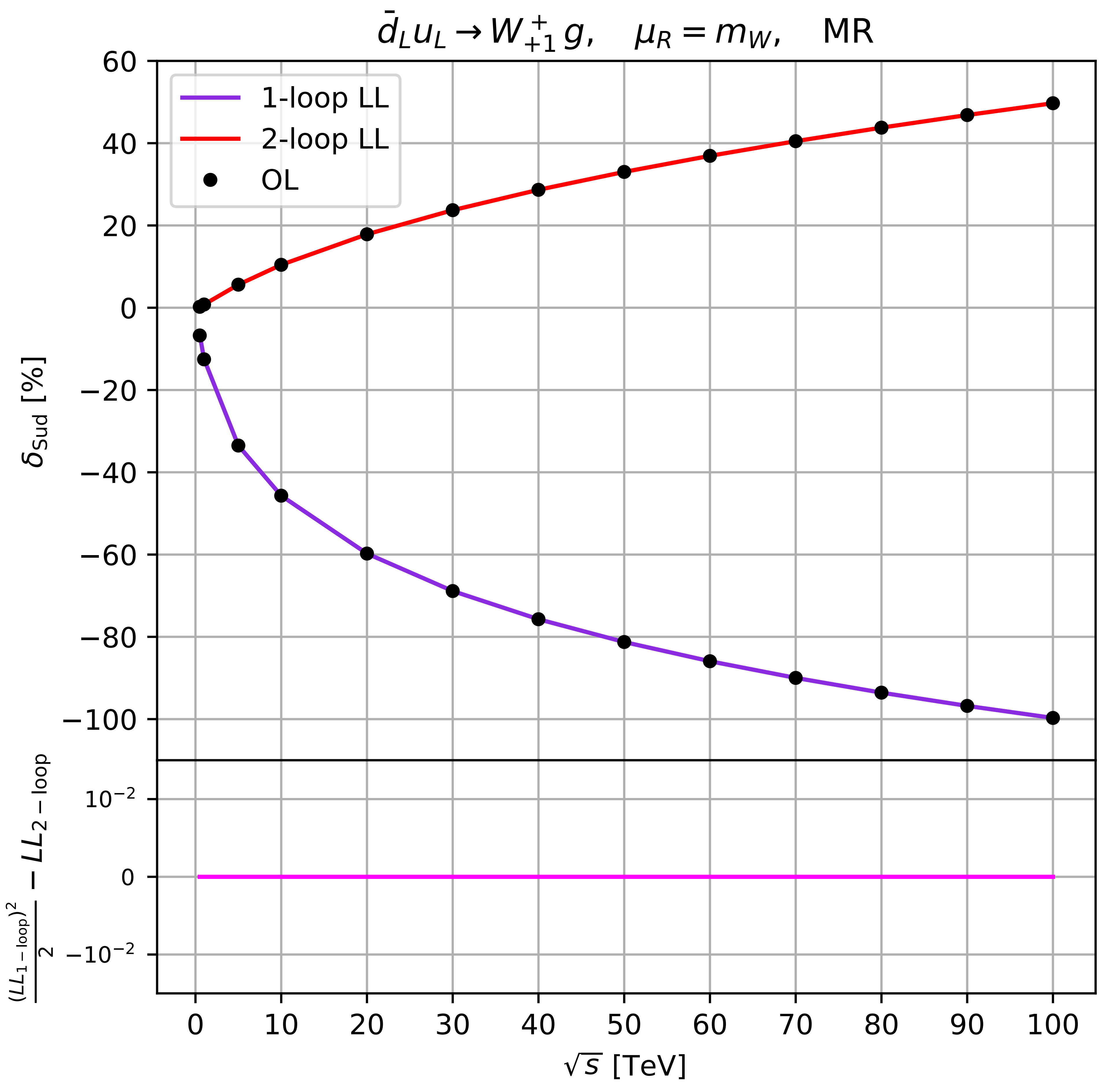
Two-loop LL partially compensate one-loop LL!

No deviation: with all right-handed fields there is no W in Born subdiagram

Lindert, **L.M.**, *In preparation*

Two-loop results: amplitude level





Analytic results firstly presented in:

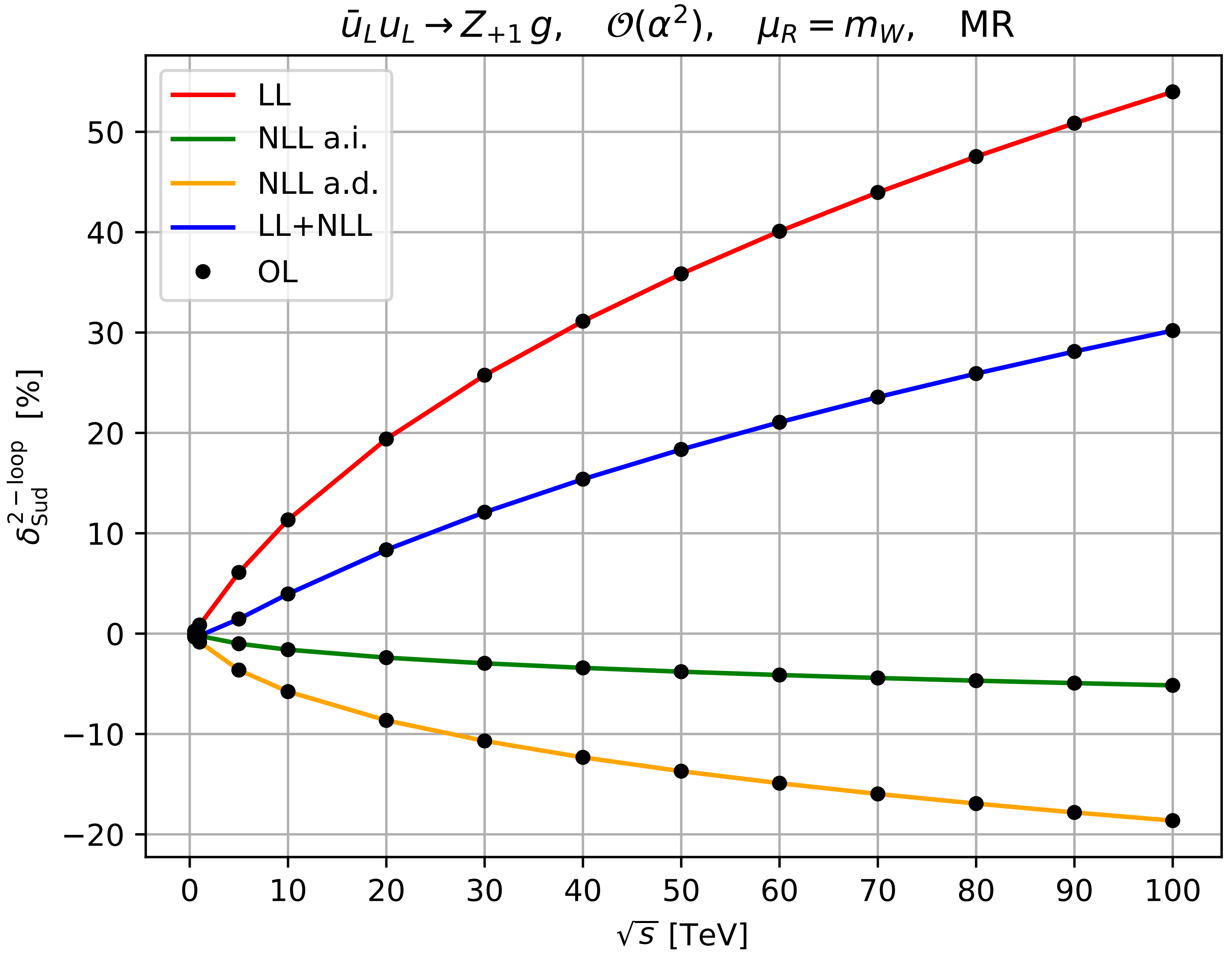
- *Wj*: Kühn *et al.* hep-ph/0703283

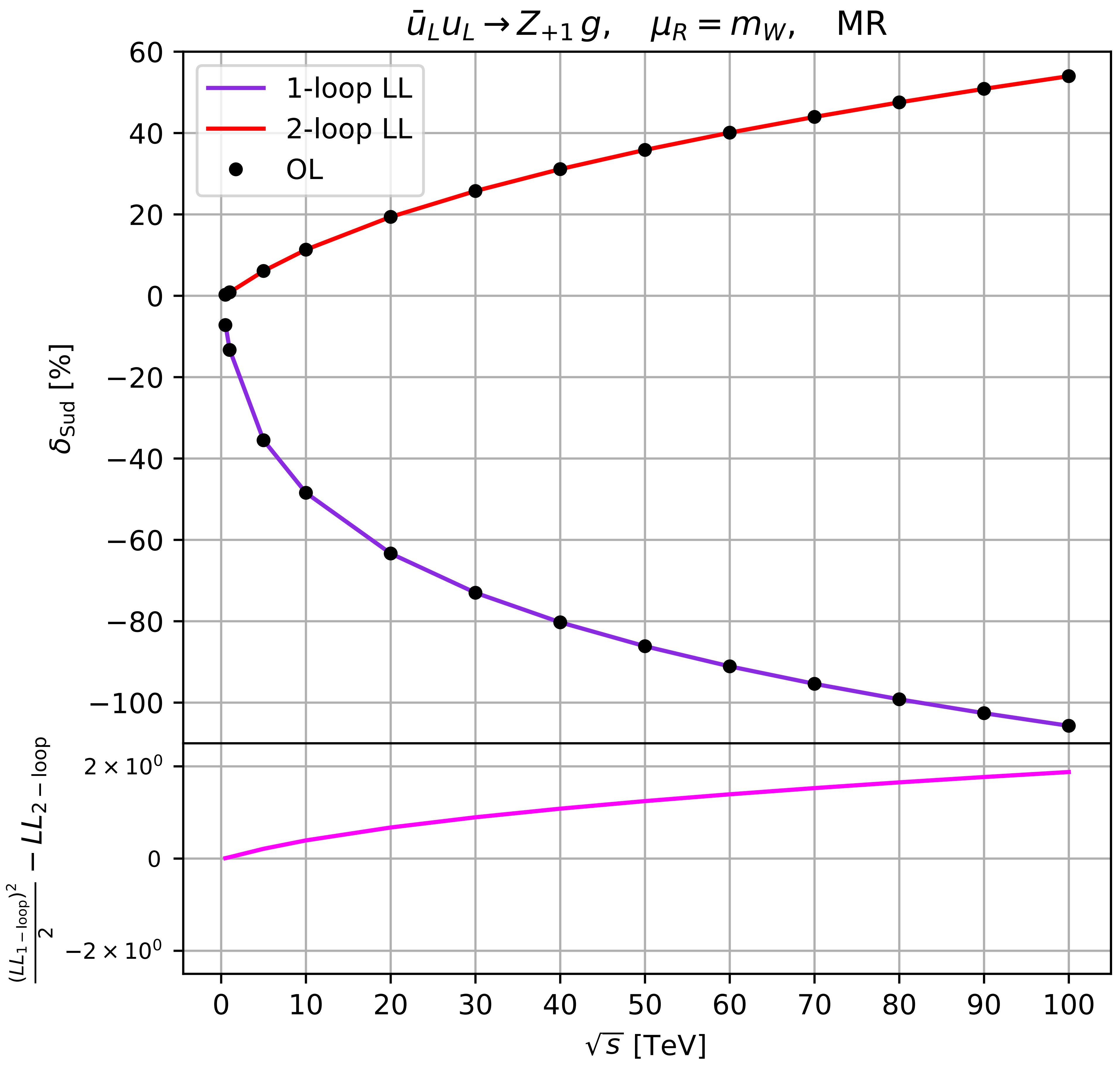
Exponentiation of **LL** still valid but:

- *Wj*: “naive” exponentiation

Lindert, **L.M.**, *In preparation*

Two-loop results: amplitude level





Analytic results firstly presented in:

- *Wj*: Kühn *et al.* hep-ph/0703283
- *Zj*: Kühn *et al.* hep-ph/0408308

Exponentiation of **LL** still valid but:

- *Wj*: “naive” exponentiation
- *Zj*: no “naive” exponentiation due to $A - Z$ mixing in Born sub-amplitude. “Naive” exponentiation is restored in the unbroken phase

Lindert, **L.M.**, *In preparation*

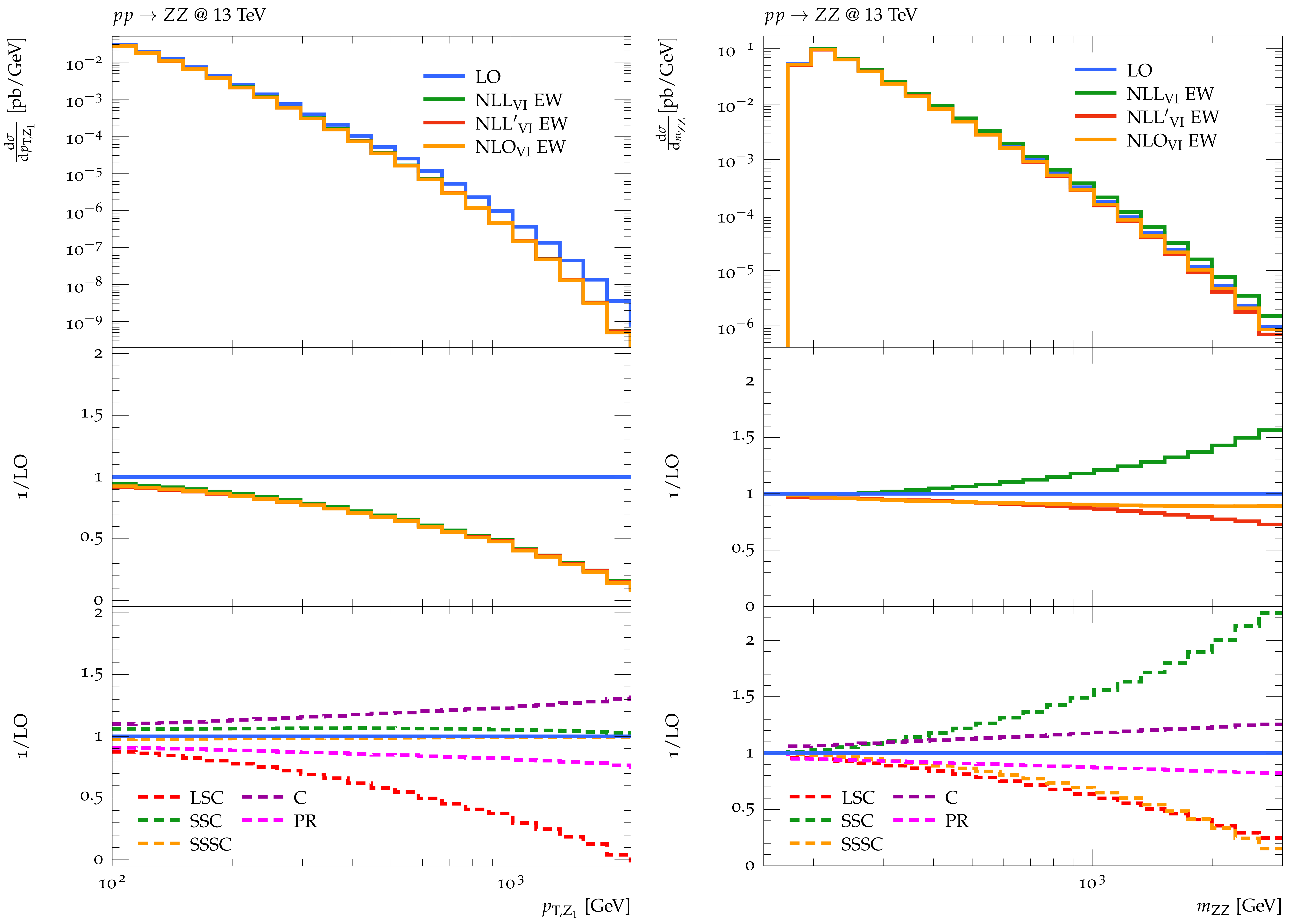
Results: $pp \to ZZ$

Full **NLO EW**: Bierweiler *et al*, 1305.5402

Full **NLO**: Baglio *et al*, 1307.4331

NNLO QCD+NLO EW: Grazzini *et al*, 1912.00068

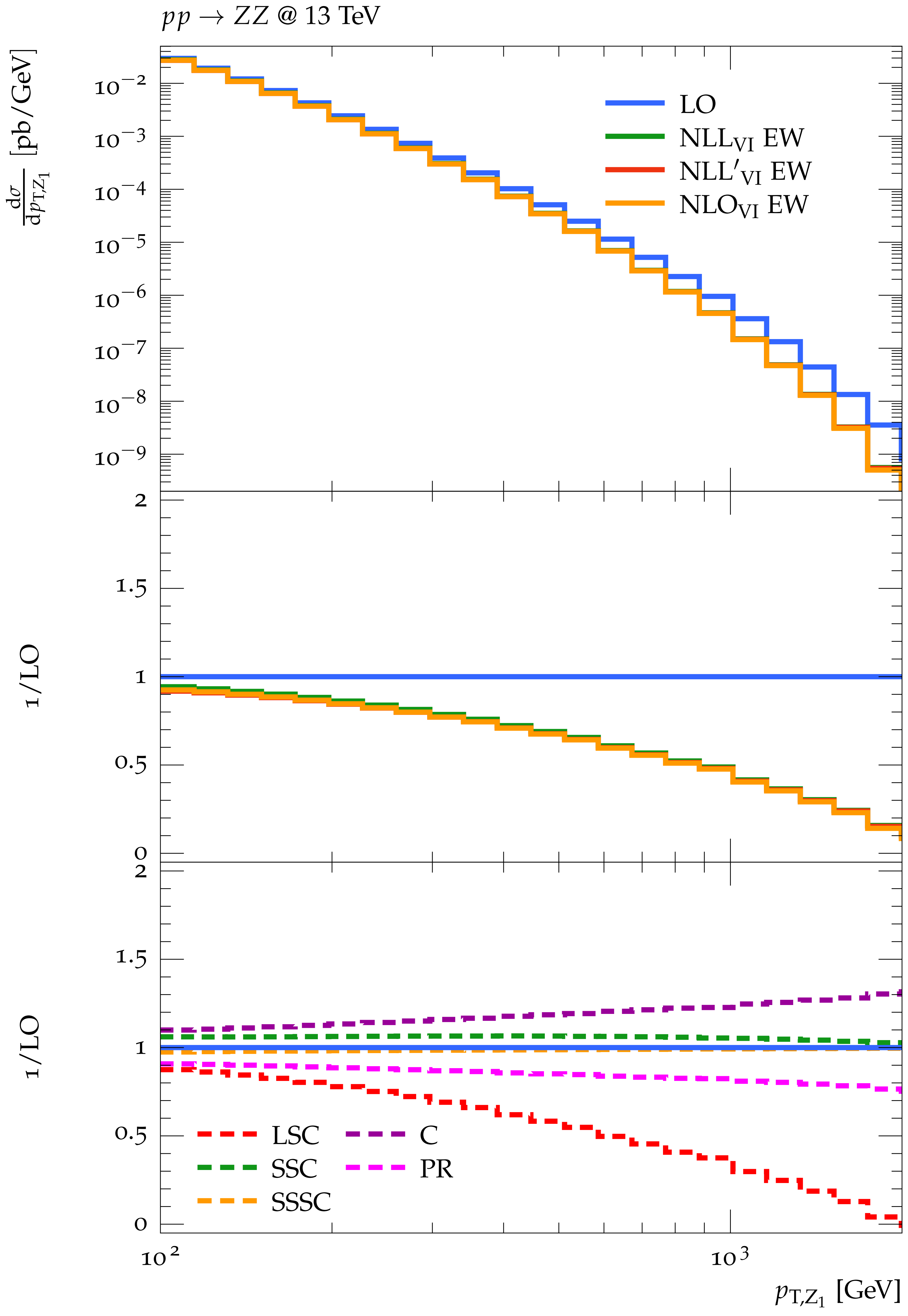
NLO EW vs **NLL EW**: Bothmann *et al*, 2111.13453

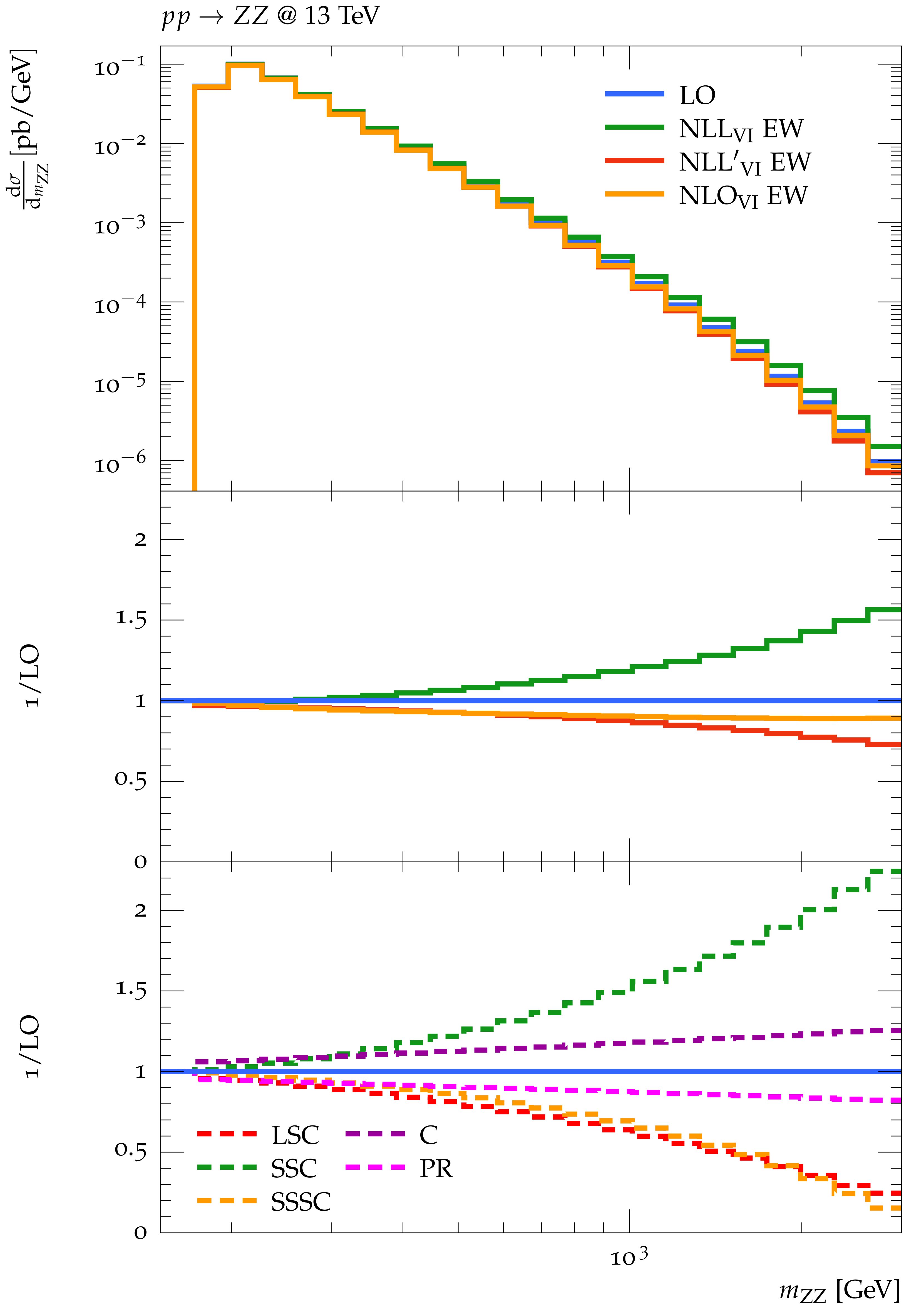


$$\text{NLL}_{\text{VI}}\text{ EW} = (1 + \text{LSC} + \text{SSC} + \text{C} + \text{PR} + \mathbf{I})\text{LO}$$

$$\text{NLL'}_{\text{VI}}\text{ EW} = (1 + \text{LSC} + \text{SSC} + \text{SSSC} + \text{C} + \text{PR} + \mathbf{I})\text{LO}$$

Results: $pp \to ZZ$





SSC and **SSSC** become very sizeable for PS regions where LA condition

$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m^2_{\mathrm{Z,W}} \quad \forall k, l$$

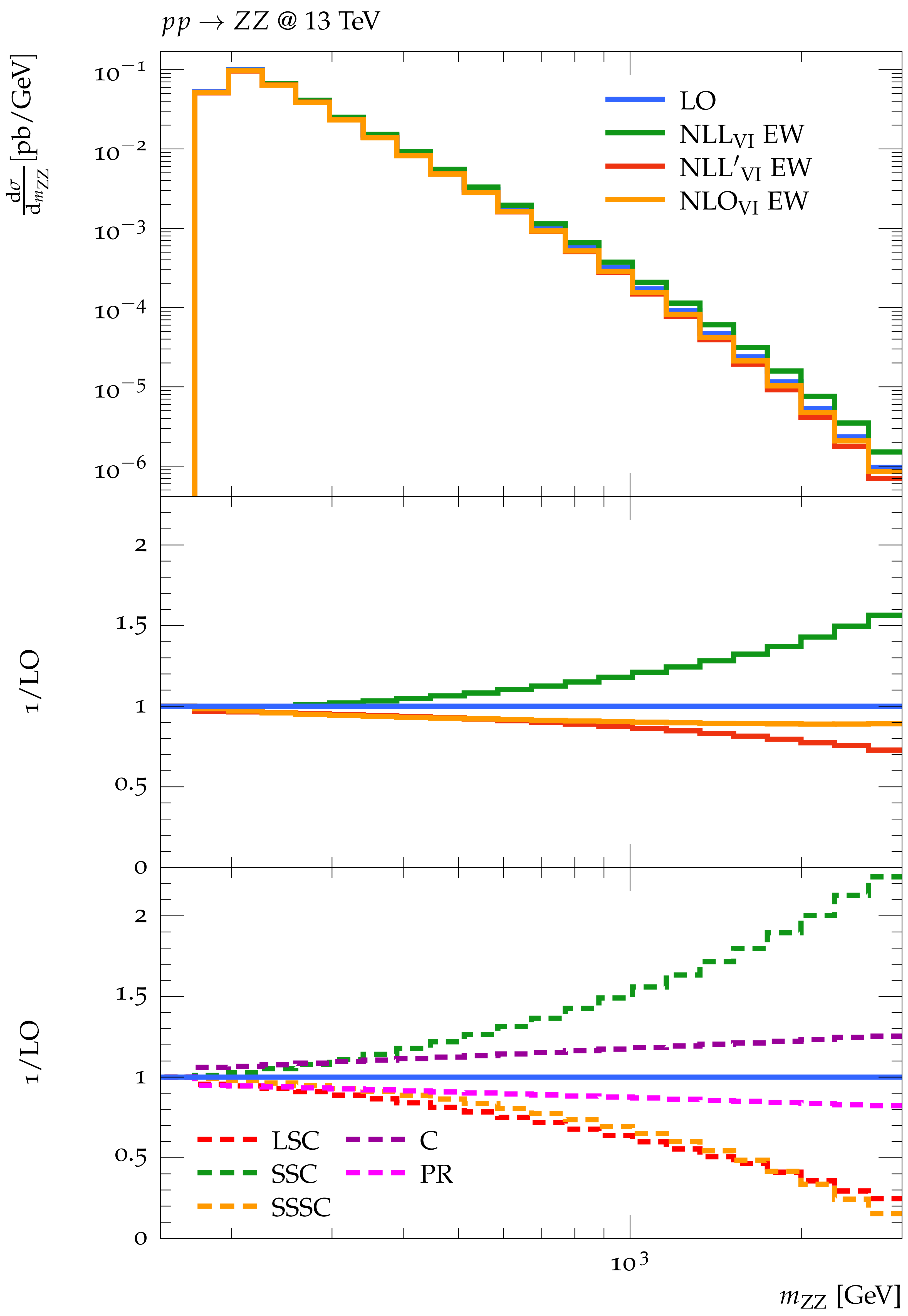
is violated, with hierarchy among invariants

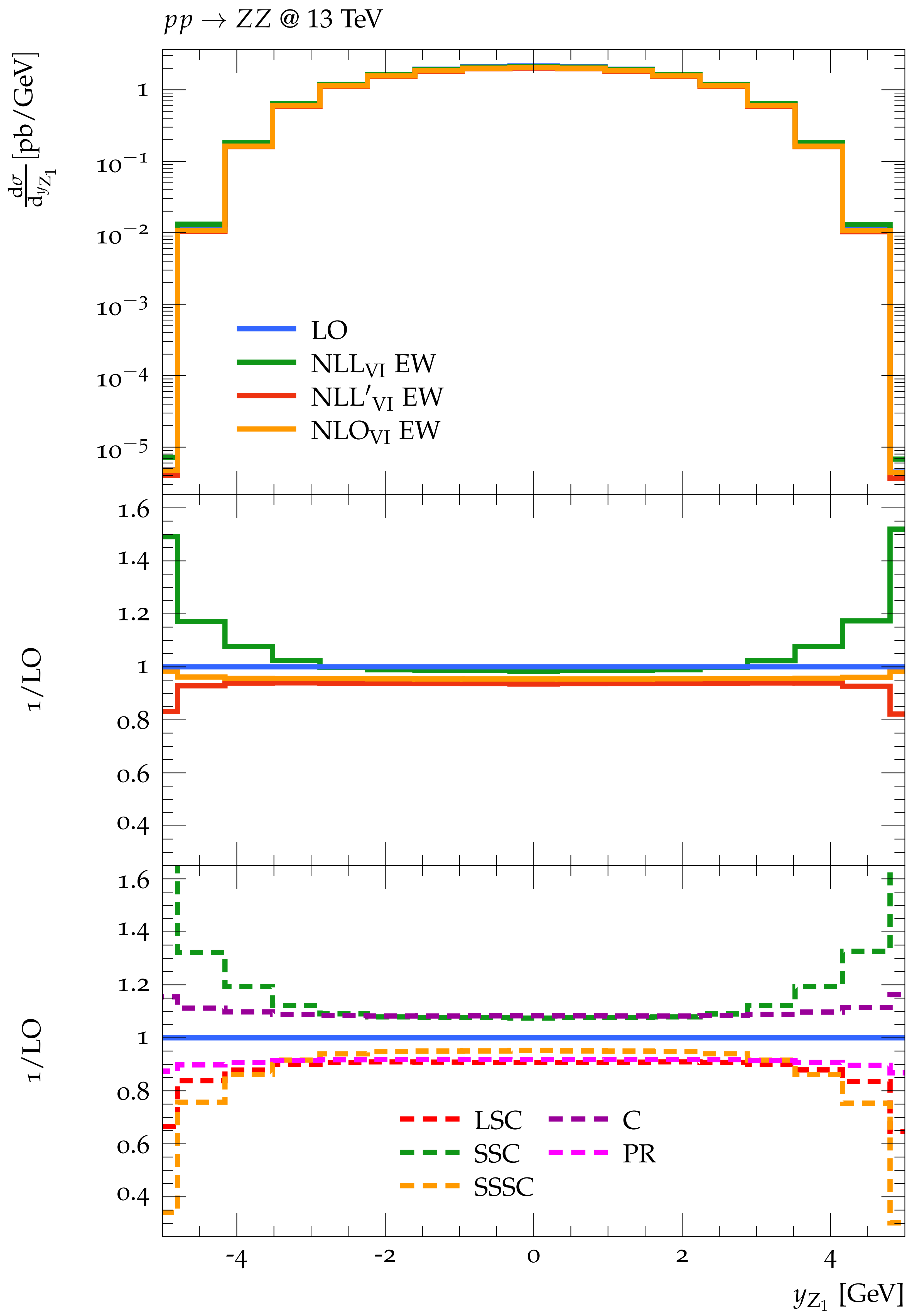
$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg r_{k'l'} \equiv (p_{k'} + p_{l'})^2 \gg m^2_{\mathrm{Z,W}}$$

$$\delta^{\mathrm{SSC,\,V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{m_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

$$\delta^{\mathrm{S\text{-}SSC,\,V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

Results: $pp \to ZZ$

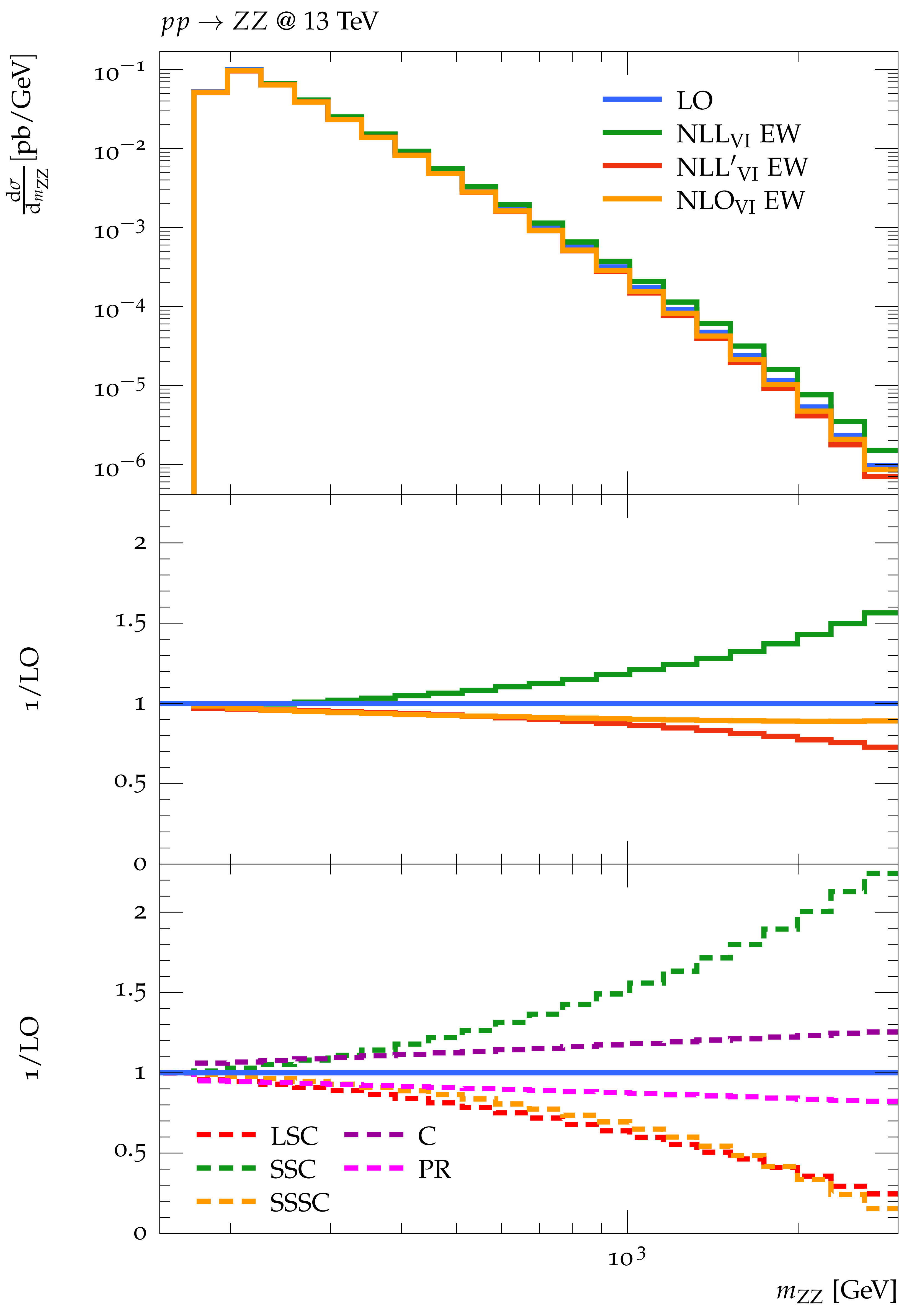


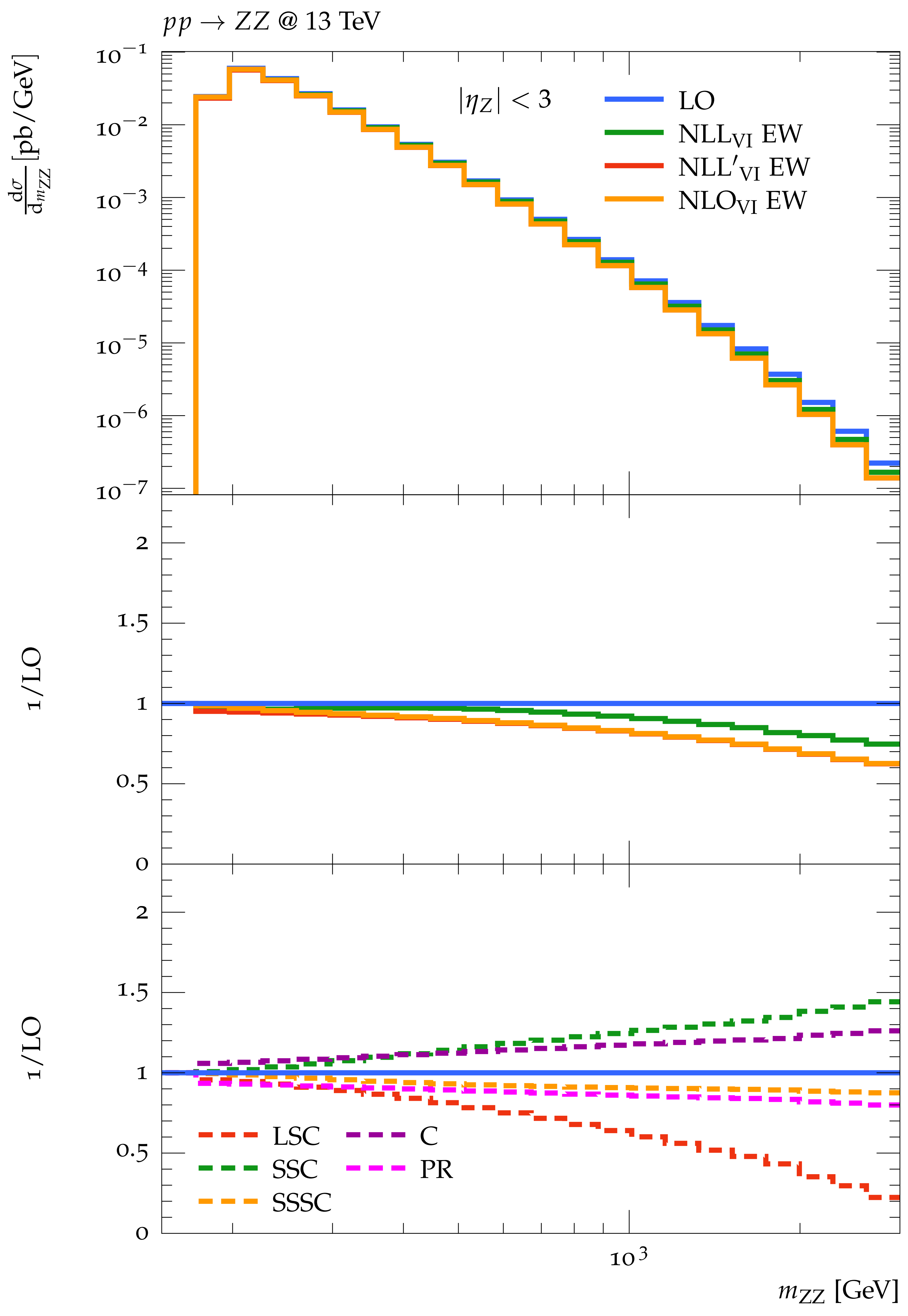


Two considerations from the rapidity distribution:

- The inclusion of SSSC allows for a better *Sudakov* approximation, in particular for $|y_Z| < 3$
- For very forward configurations, i.e. outside the central region $|y_Z| < 3$, SSC and SSSC rapidly grow

Results: $pp \to ZZ$





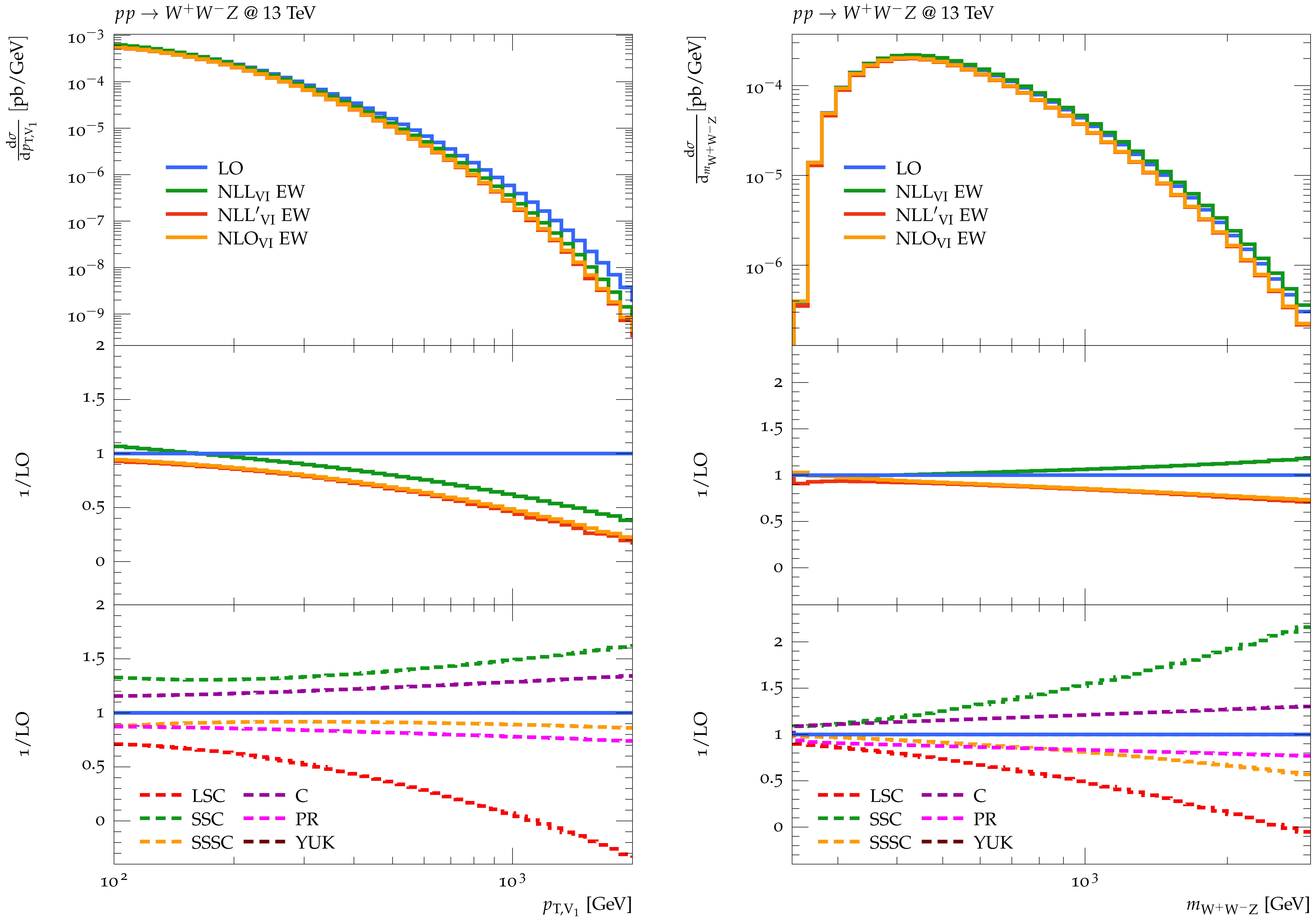
Pseudo-rapidity cut $|\eta_Z| < 3$ avoids pathological forward configurations which violate LA

Inclusion of SSSC leads to more accurate predictions, but there is no full control on it. *Non-universal* SSSC-like terms arising from high-energy expansion of 4-point functions. These angular contributions *cannot* be reliably controlled in LA

Looking at differences between $\mathrm{NLL'}$ and NLL opens two scenarios:

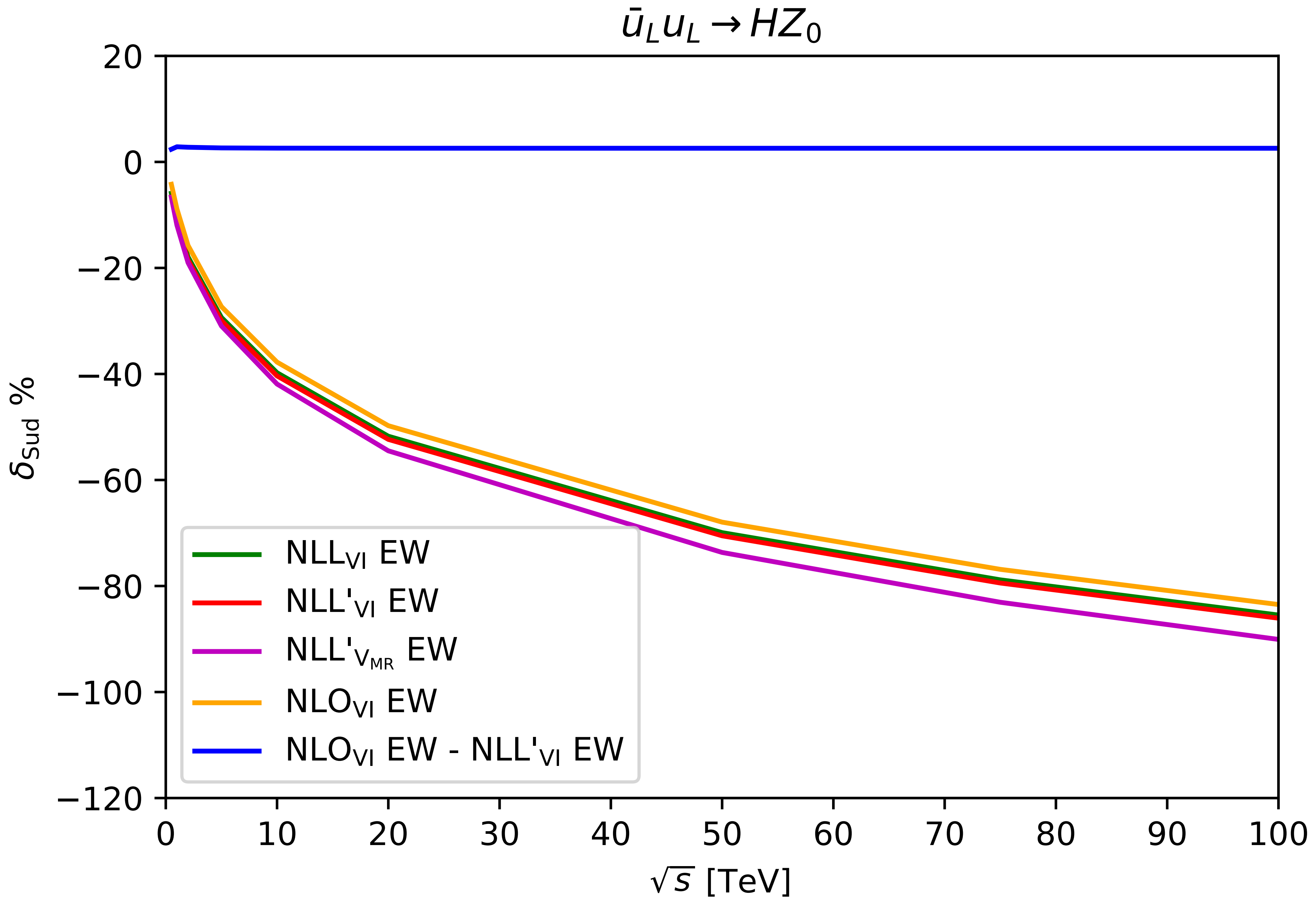
- $\mathrm{NLL'} - \mathrm{NLL} > \mathrm{NLL'} - \mathrm{NLO} \Rightarrow$ SSSC is a reliable estimate of sub-sub-leading angular terms beyond LA
- If NLO is unknown: SSSC might be interpreted as a conservative estimate of uncertainties of LA

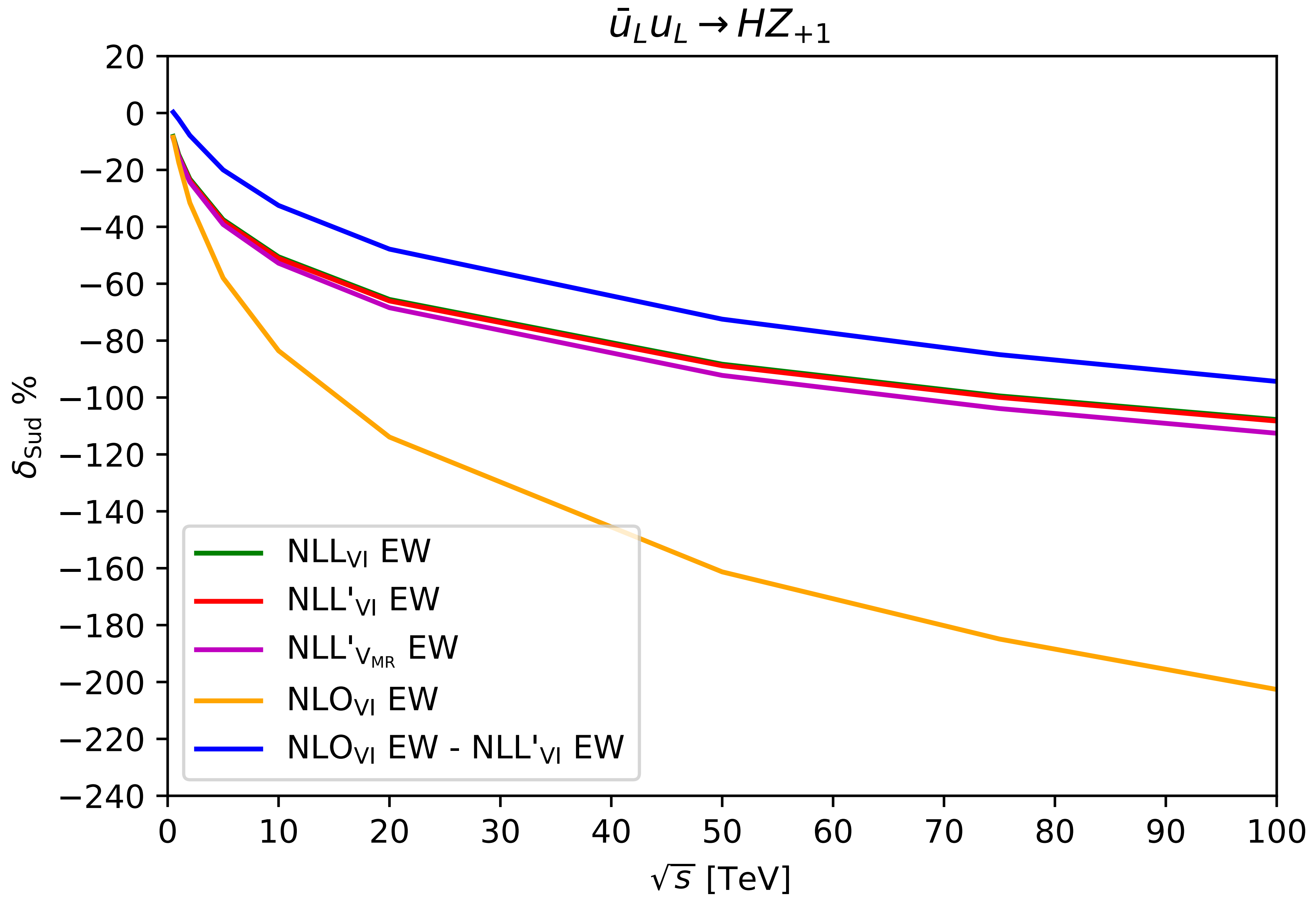
Results: $pp \to W^+W^-Z$



The inclusion of SSSC provides better predictions, but there is no full control on it! (Non-universal) SSSC-like terms arise also from LA of 4-point functions

Amplitude-level validation: $\sqrt{s}$ scan





- In *Sudakov* approximation: keep only double and singular logarithmic corrections

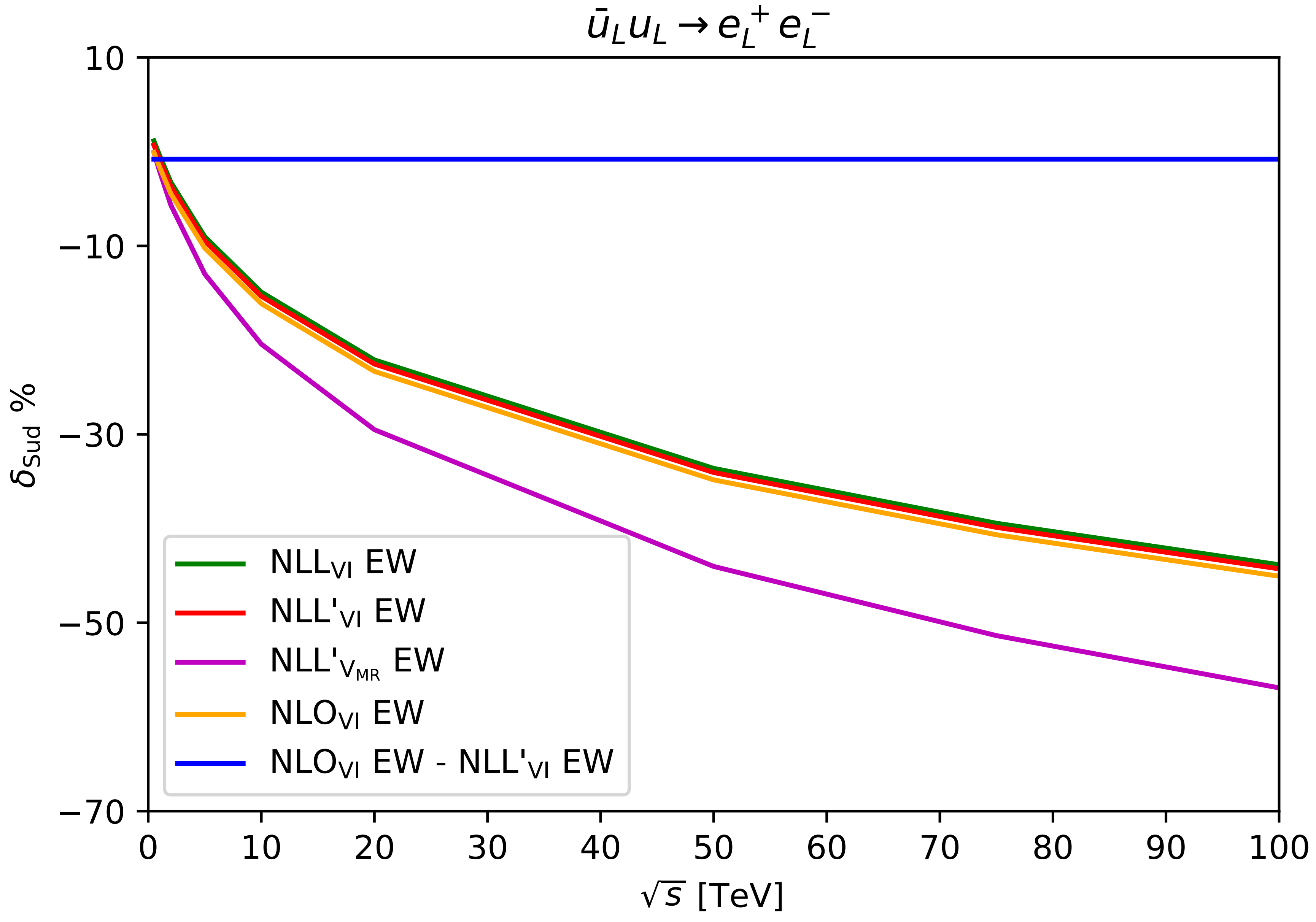
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d L \qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d l$$

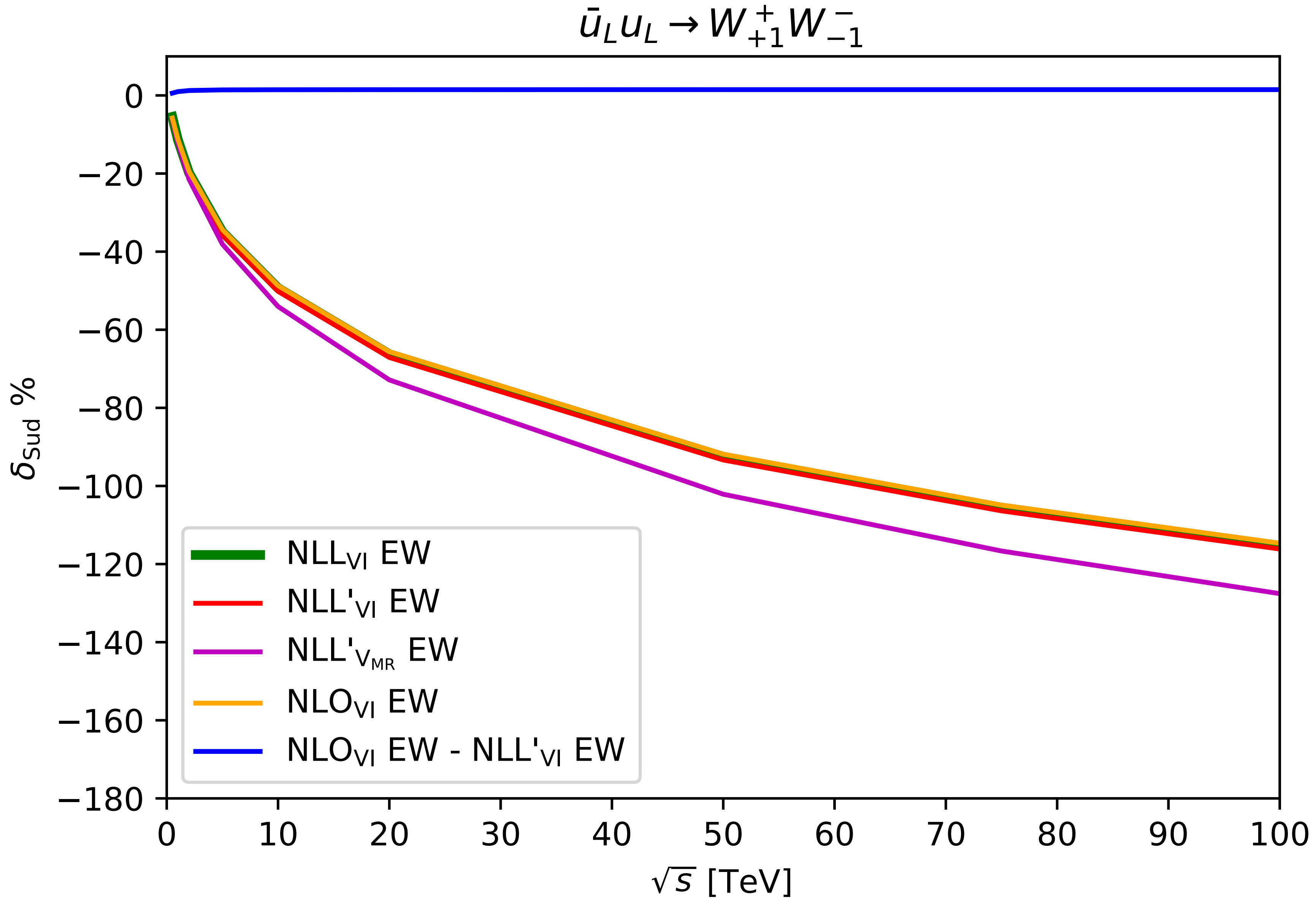
neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ terms

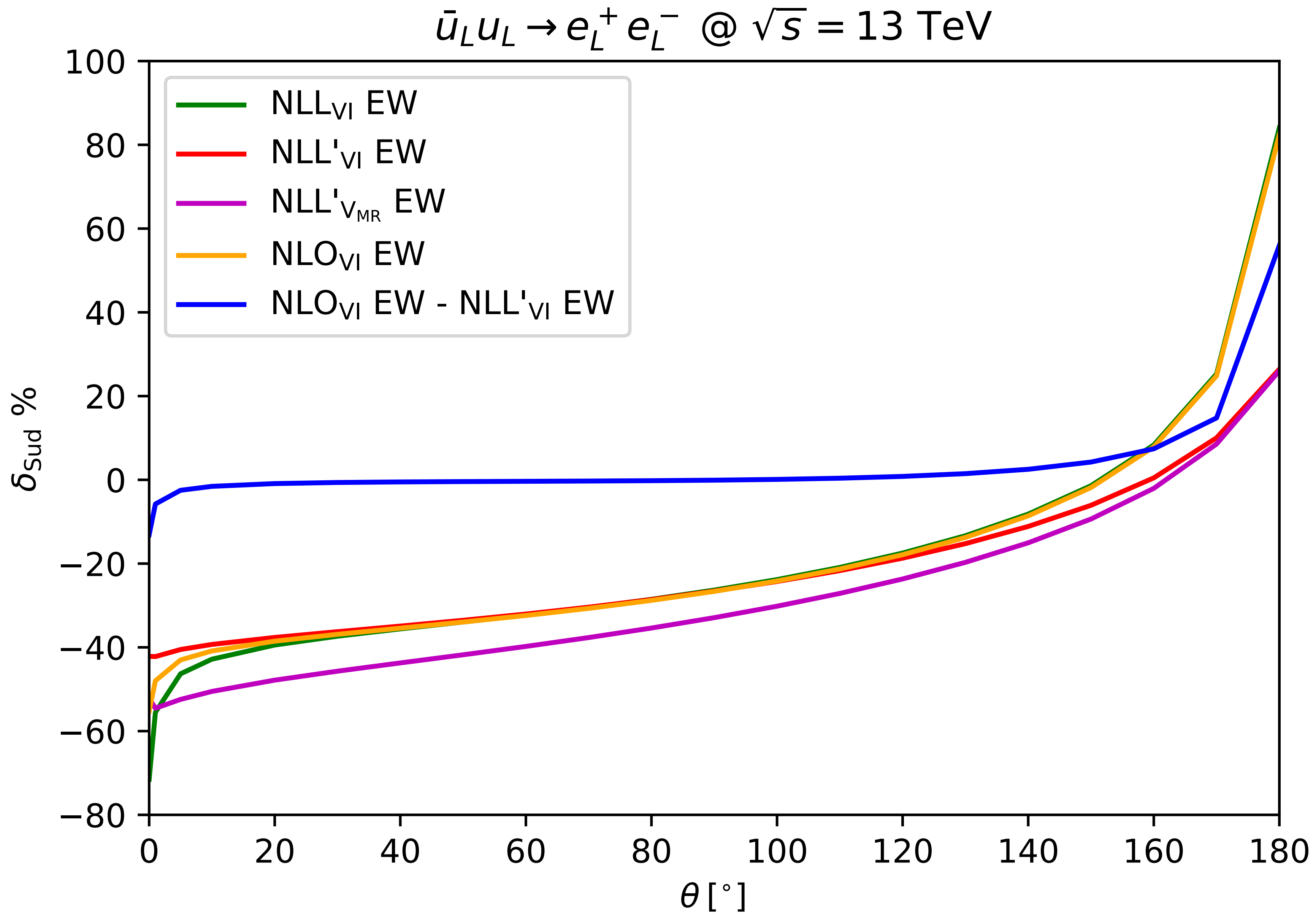
- In the high energy limit and for non mass-suppressed [2] matrix elements we expect $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW} \propto\ const$

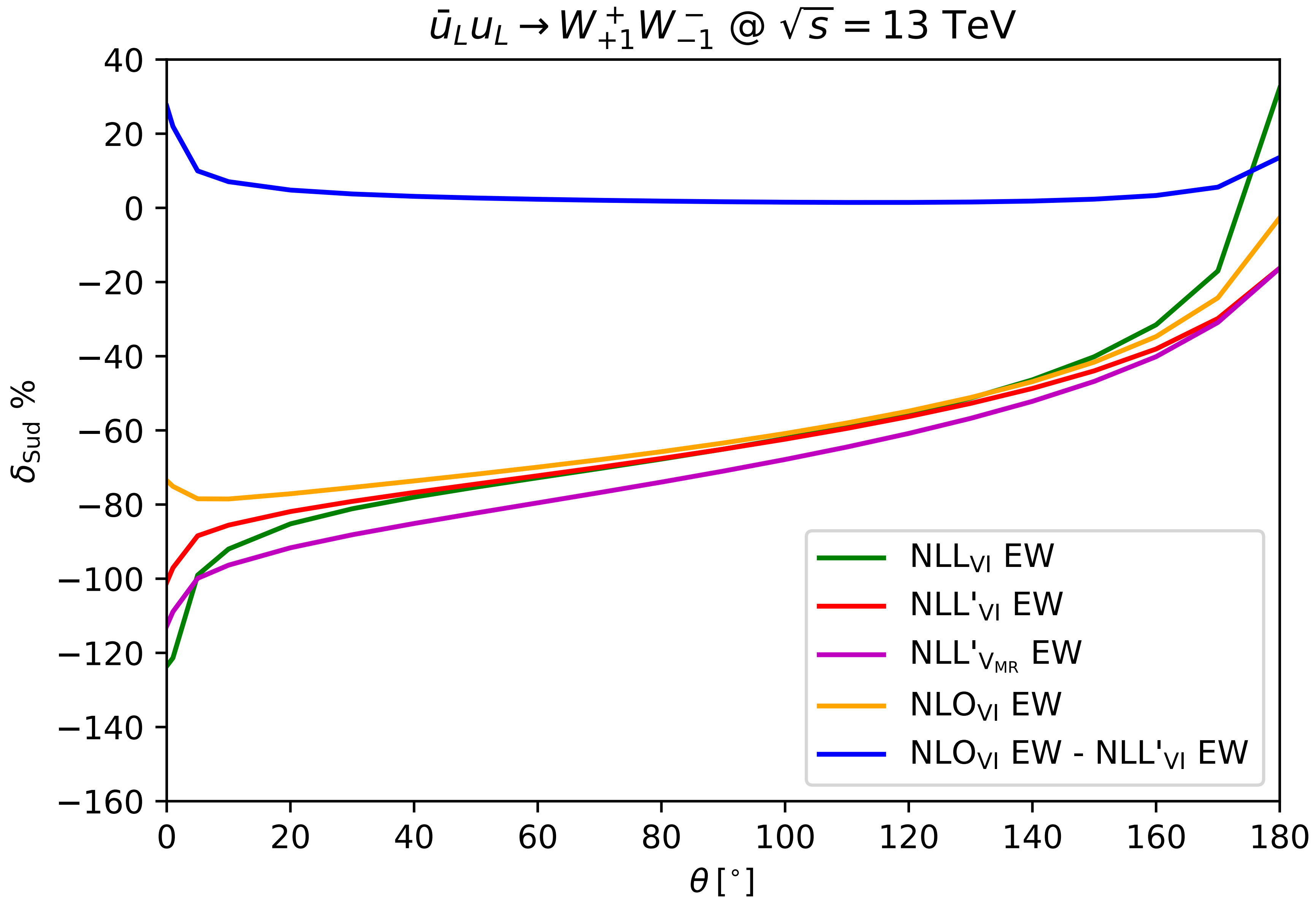
[2]NB: non mass-suppressed configurations scale like $\sim \sqrt{s}^{4-n}$

Amplitude-level validation: $\sqrt{s}$ and θ scans

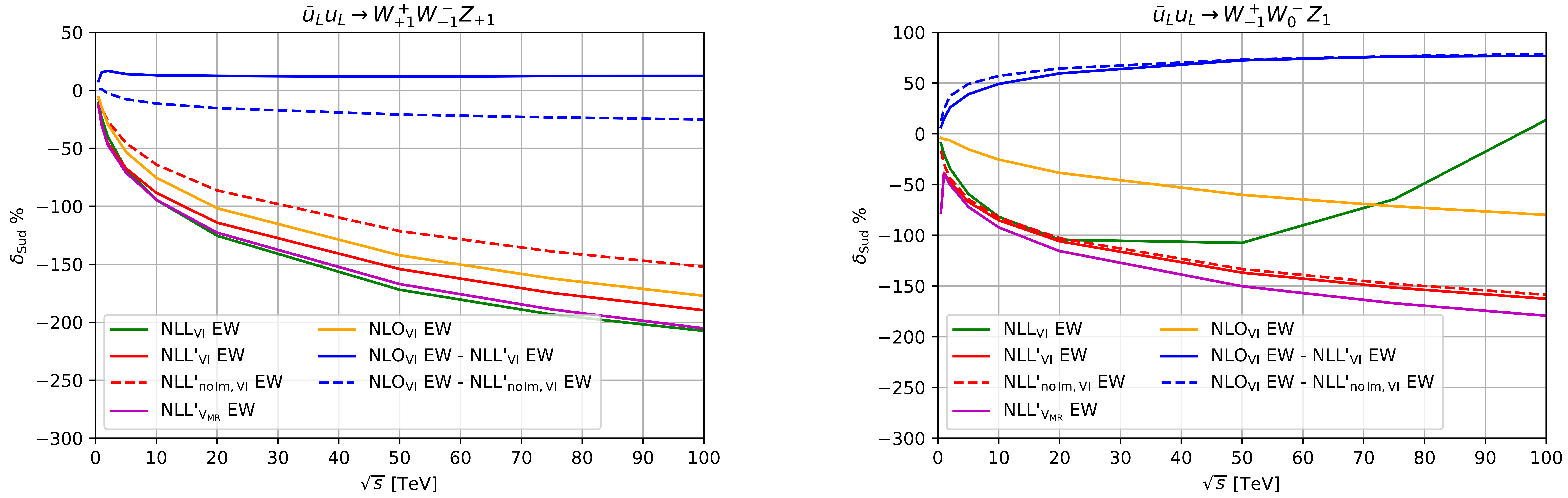








Amplitude-level validation: $\sqrt{s}$ scan



- In the high energy limit and for non mass-suppressed matrix elements we expect $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW} \propto \ const$

- Inclusion of the phase in DL from the LA of C_0, i.e.

$$C_0|_{\rm LA} \propto \left[\log^2\frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl})\log\frac{|r_{kl}|}{M_V^2}\right]$$

is crucial in $2 \to n$ processes with $n \geq 3$: without phase $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW}$ shows a logarithmic dependence. This has been firstly noticed in [Pagani, Zaro 2110.03714; 2021]