Higgs Boson ($H \to VV^*$) Quantum Tomography

Mainly based on: arXiv:2504.03841 (Del Gratta, Fabbri, Lamba, Maltoni, DP) and arXiv:2509.20456 (Del Gratta, Fabbri, Grossi, Maltoni, DP, Pelliccioli, Vicini).

PRIN
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Davide Pagani,

Standard Model at the LHC 2026, Turin (Italy),
09 - 04 - 2026

Outline

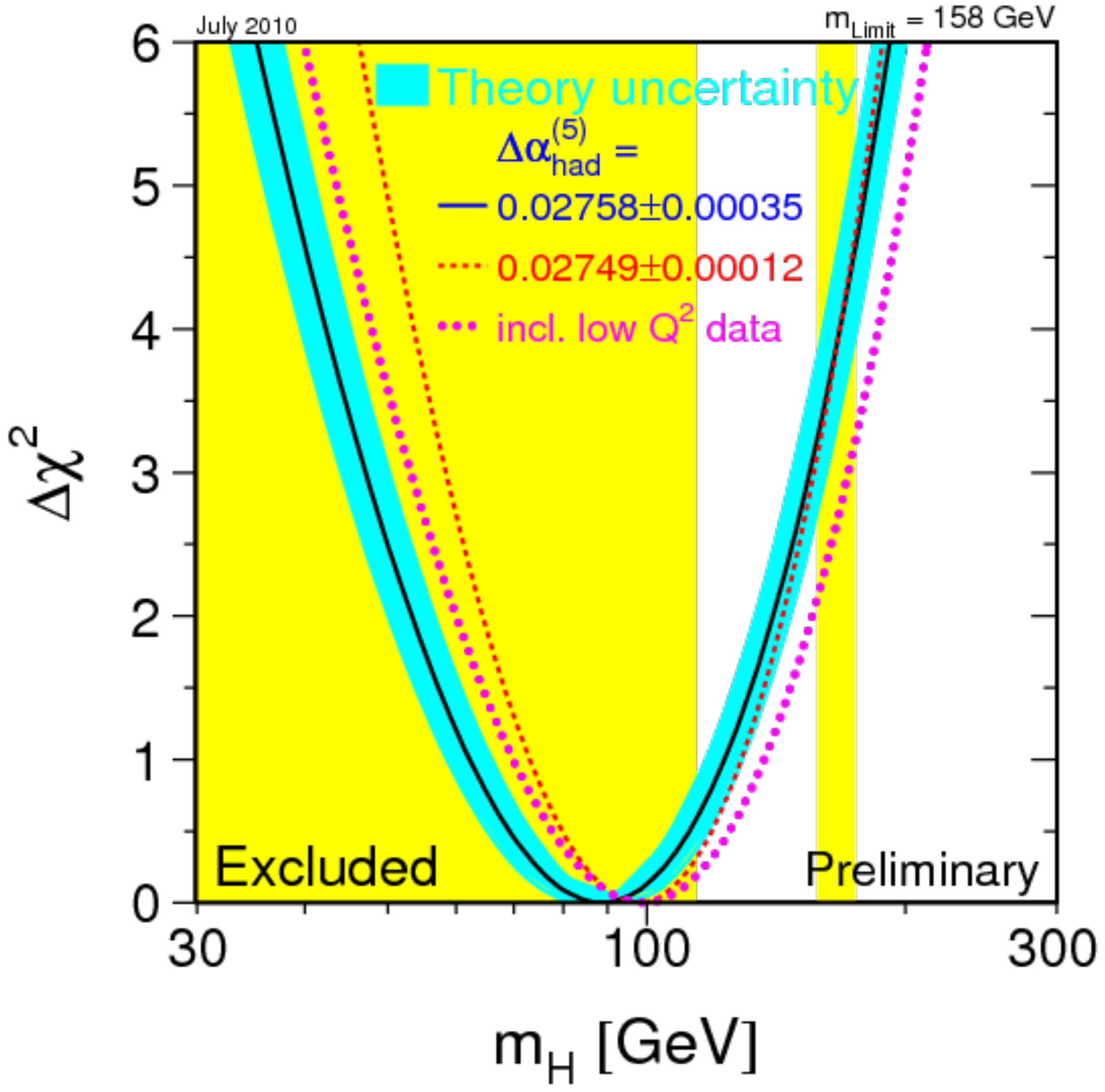
- Motivation: why quantum observables at the LHC? The top-quark pair production as an example.

- Quantum Tomography for $H \to ZZ^* \to 4\ell$, with a focus on the relevance of EW corrections.

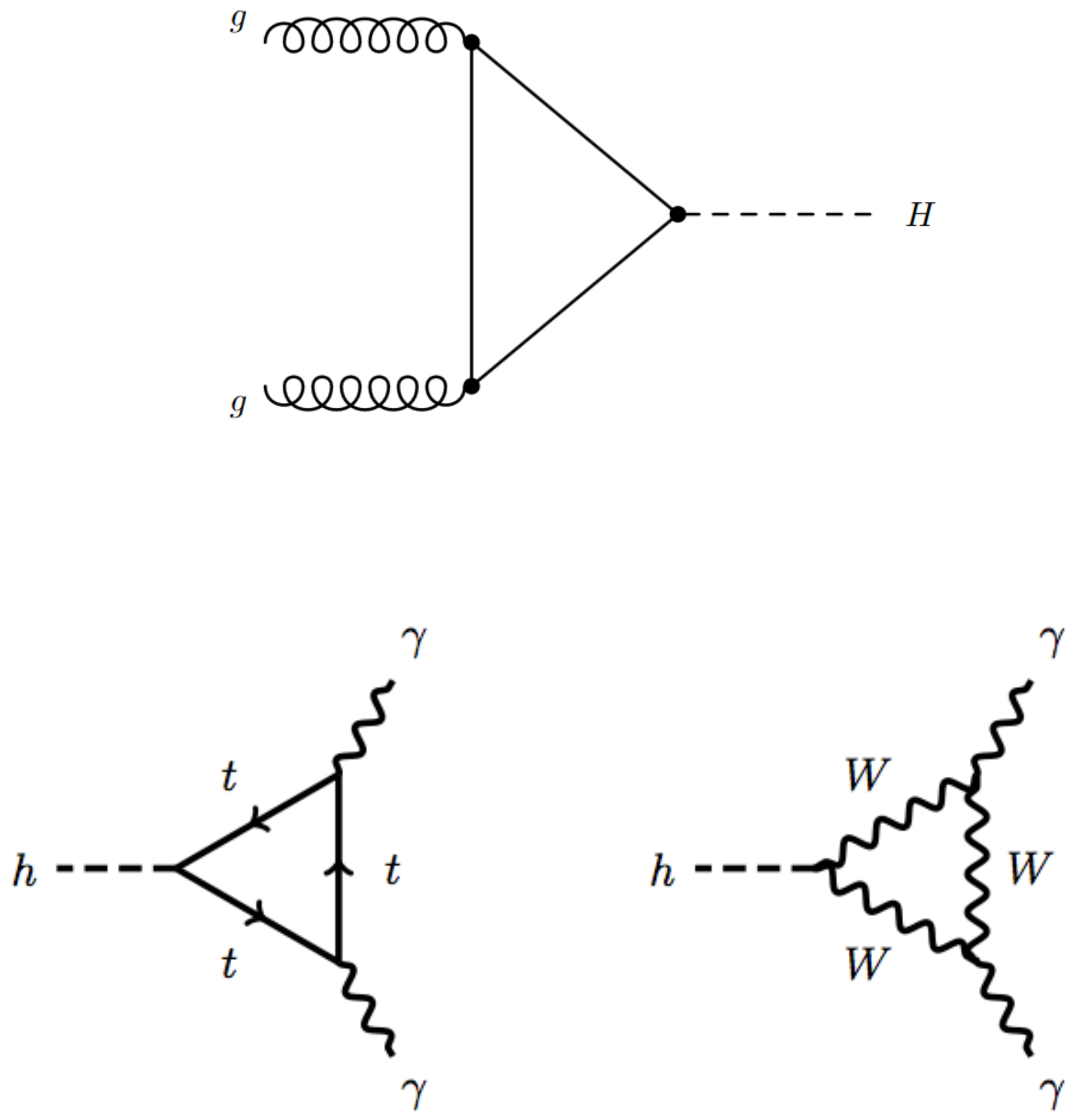
Quantum Mechanics in high-energy physics

For decades, we have been probing, testing, and exploiting Quantum Mechanics and QFT at high energies in collider experiments.

A couple of "(not-so)recent" examples in Higgs physics:



arXiv:1012.2367 [hep-ex]



Accuracies in Exp papers:

Process	Prediction order
ggF H	N^3LO QCD + NLO EW
VBF H	NNLO QCD + NLO EW
VH	NNLO QCD + NLO EW
$gg \to ZH$	NLO + NLL
$t\bar{t}H$	NLO QCD + NLO EW

Quantum Information in high-energy physics

The study of quantum-information observables in high-energy physics is more recent, but it has received significant attention from a growing community in HEP, both theoretical and experimental!

from …

Entanglement and quantum tomography with top quarks at the LHC

Yoav Afik (CERN), Juan Ramón Muñoz de Nova (UCM, Madrid, Dept. Phys.)
Mar 4, 2020

23 pages
Published in: *Eur.Phys.J.Plus* 136 (2021) 9, 907
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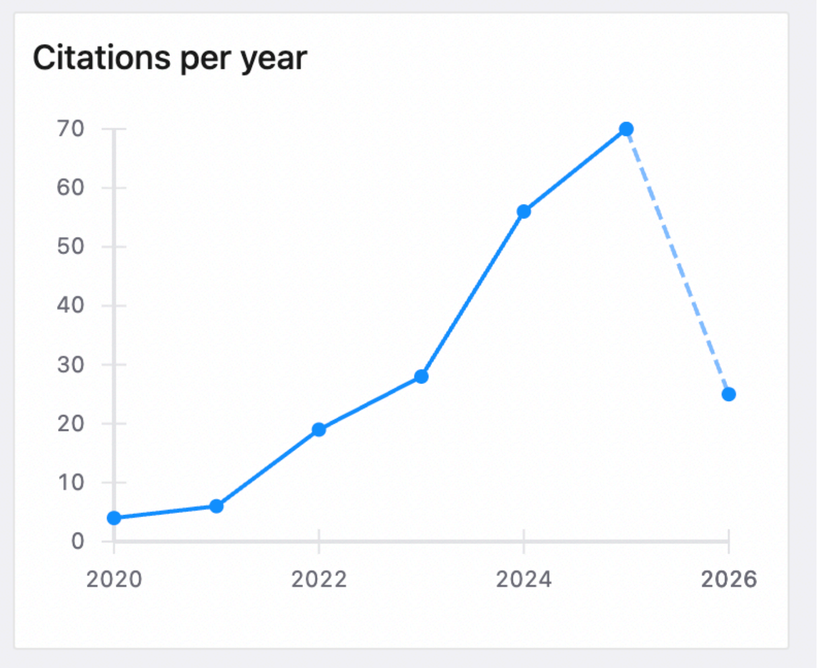
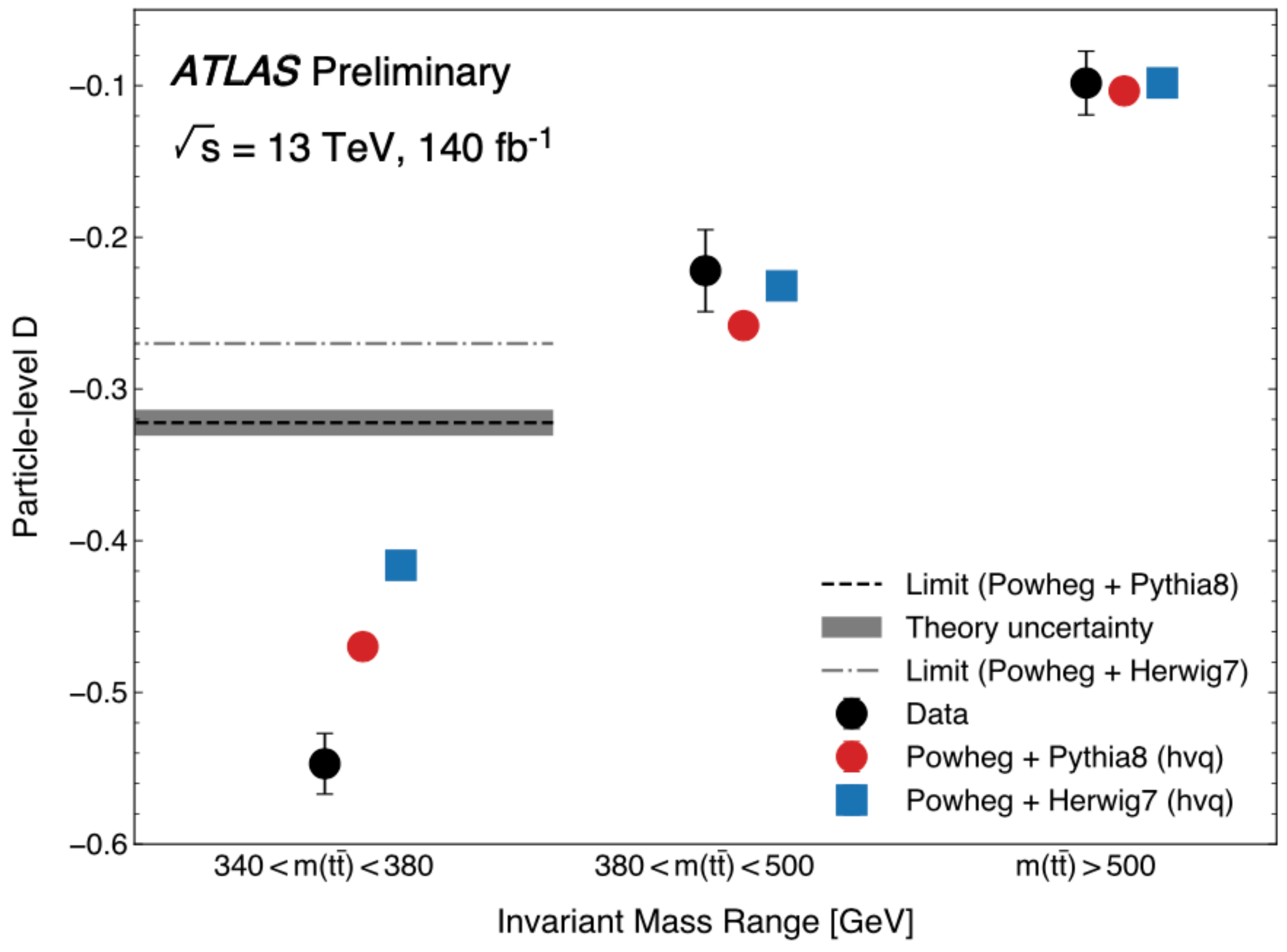


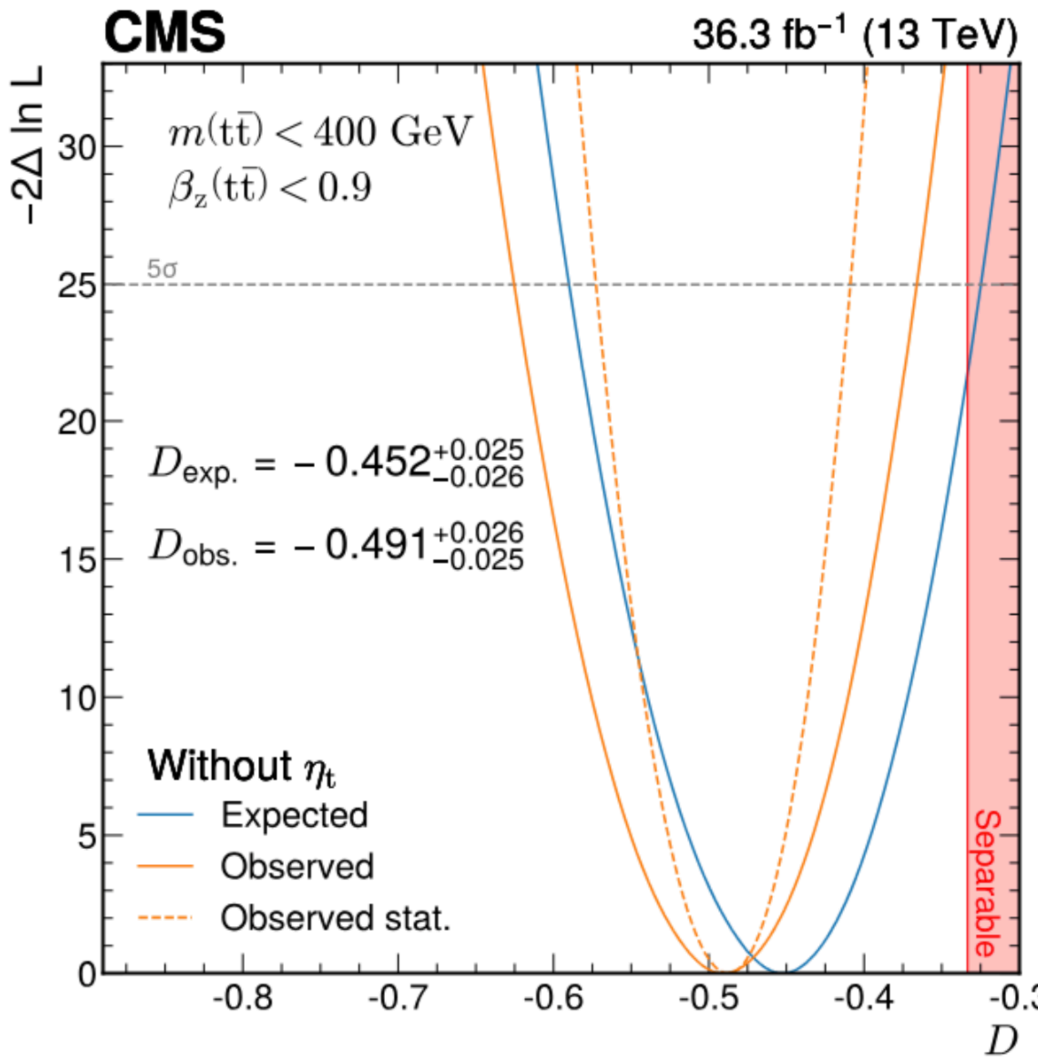
Foundation Paper

… to



ATLAS-CONF-2023-069

TOP-23-001-pas



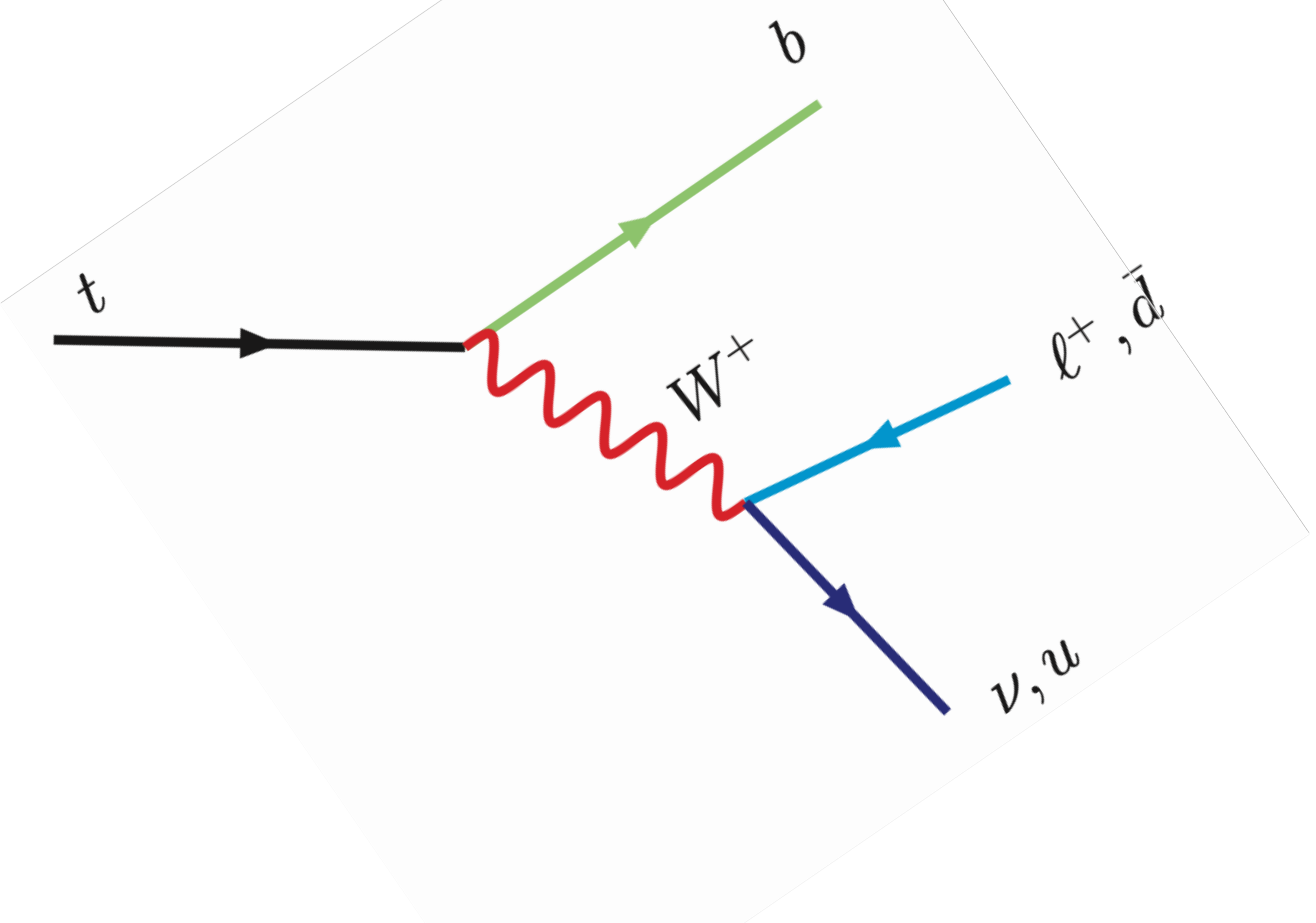
Entanglement observation

Quantum Tomography: what is it and why?

- In $t\bar{t}$ production the spin states of the top and antitop quarks are entangled.
- However, neither ATLAS nor CMS detectors measure the spin of the particles (especially of tops ..).

QUANTUM TOMOGRAPHY:

- We can reconstruct the ρ density matrix of the bipartite qubit system of the $t\bar{t}$ spins via the measurement of distributions of the top-quark decay products. This is QUANTUM TOMOGRAPHY.



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\varphi} = \frac{1+\alpha\cos\varphi}{2}$$

$\alpha_\ell = 1(100\,\%$ correlated)

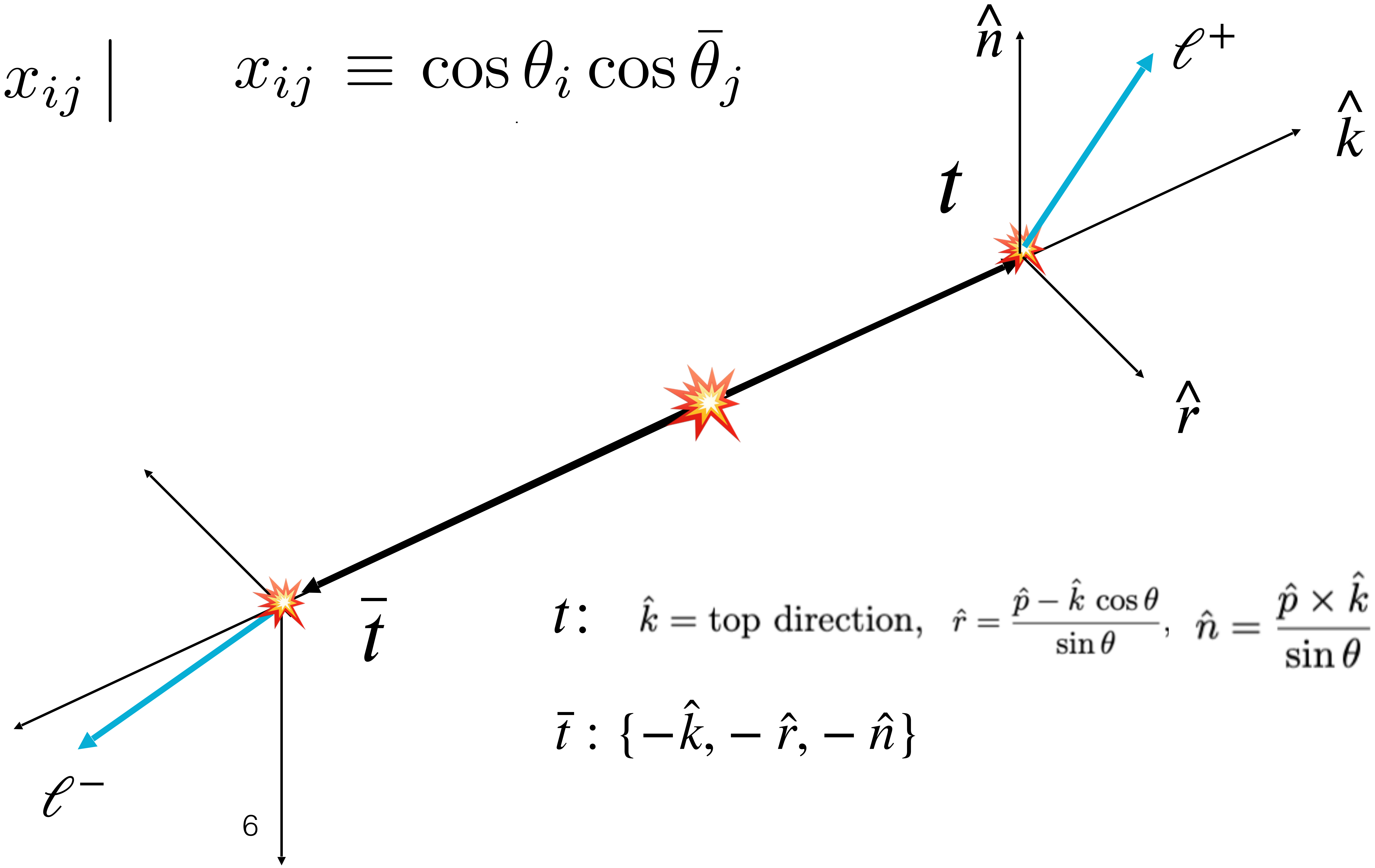
φ is the angle between the spin of the top and the direction of the lepton

Quantum Tomography: how?

$$\rho = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + \sum_{i,j=1}^{3} C_{ij}\, \sigma_i \otimes \sigma_j\right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{dx_{ij}} = \frac{C_{ij}\, x_{ij} - 1}{2} \log\left|x_{ij}\right| \qquad x_{ij} \equiv \cos\theta_i \cos\bar{\theta}_j$$

By fitting doubly differential angular distributions of the leptons from top decays, the spin density matrix can be reconstructed.



t: $\hat{k}$ = top direction, $\hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}$, $\hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$

$\bar{t}$: $\{-\hat{k}, -\hat{r}, -\hat{n}\}$

Simple conditions for Entanglement and BI

$$-C_{kk} - C_{rr} - C_{nn} \equiv -3D^{(1)}$$

$$-C_{kk} + C_{rr} + C_{nn} \equiv -3\,D^{(k)}$$

$$+C_{kk} + C_{rr} - C_{nn} \equiv -3\,D^{(n)}$$

$$+C_{kk} - C_{rr} + C_{nn} \equiv -3\,D^{(r)}$$

$D^{(i)} < -1/3$ $\Rightarrow$ Entanglement

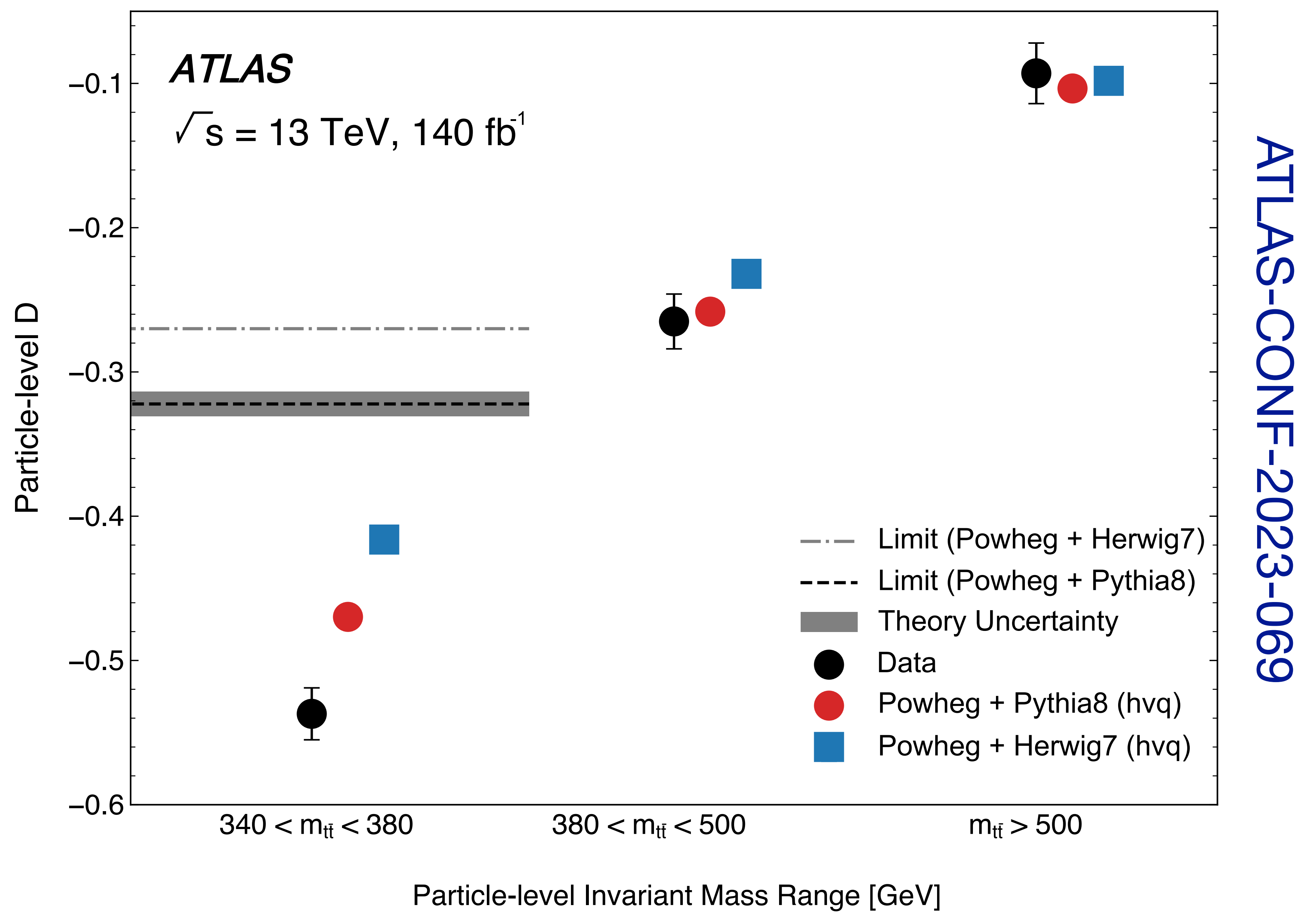
$\sqrt{2}\left| -C_{rr} + C_{nn} \right| \leq 2,$ $\Rightarrow$ Bell violation

D(1) is precisely the quantity that has been measured and that has led to the observation of entanglement!

But that was actually the beginning of another story: Toponium

An example of how the QI-obsevervables enhance sensitivity to new physics.

From Quantum Information to the toponium



$$D = \mathrm{tr}[\mathbf{C}]/3$$

$$D = -3 \cdot \langle \cos \varphi \rangle$$

$D < -1/3$: sufficient condition for entanglement between t and $\bar{t}$ spin-states

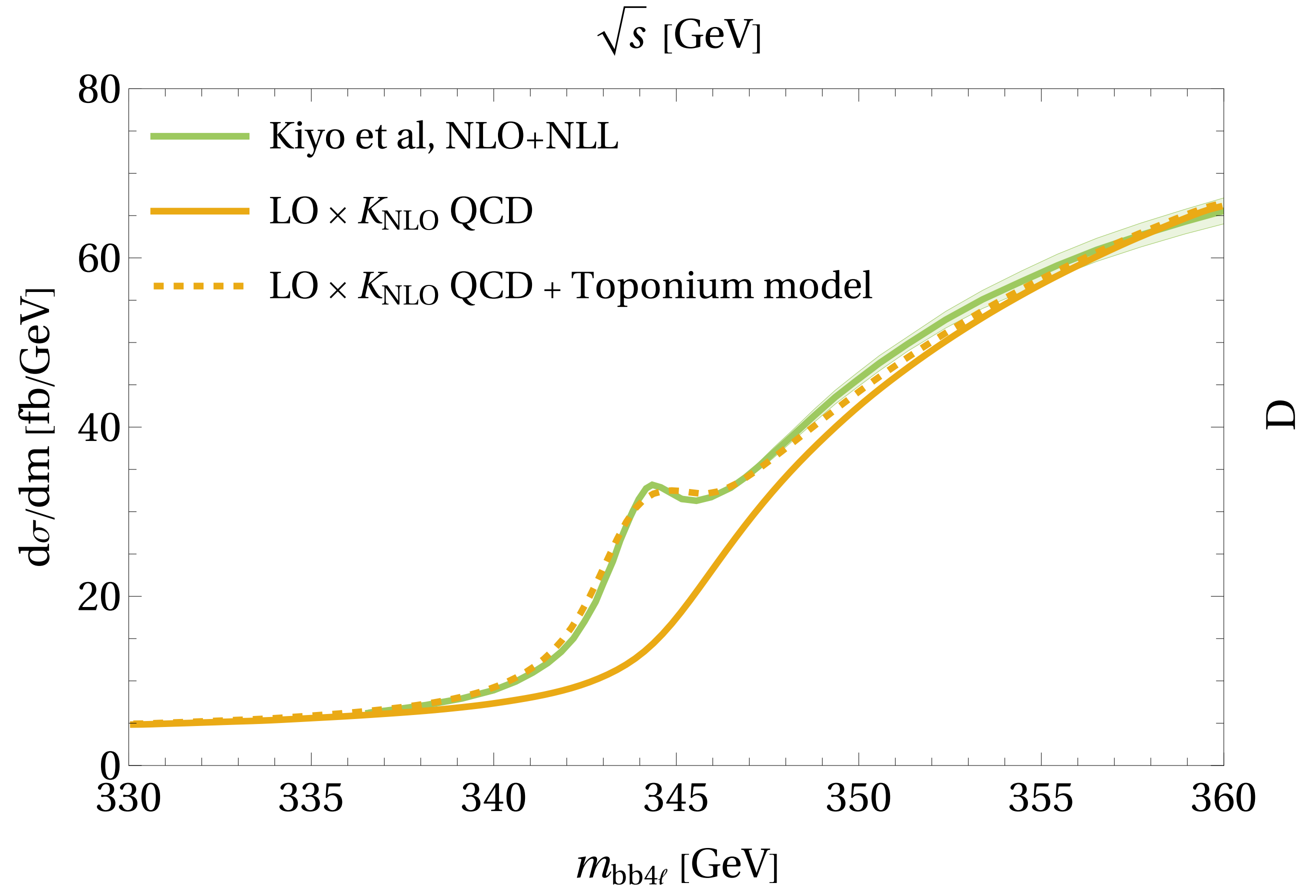
Afik, Muñoz de Nova '20

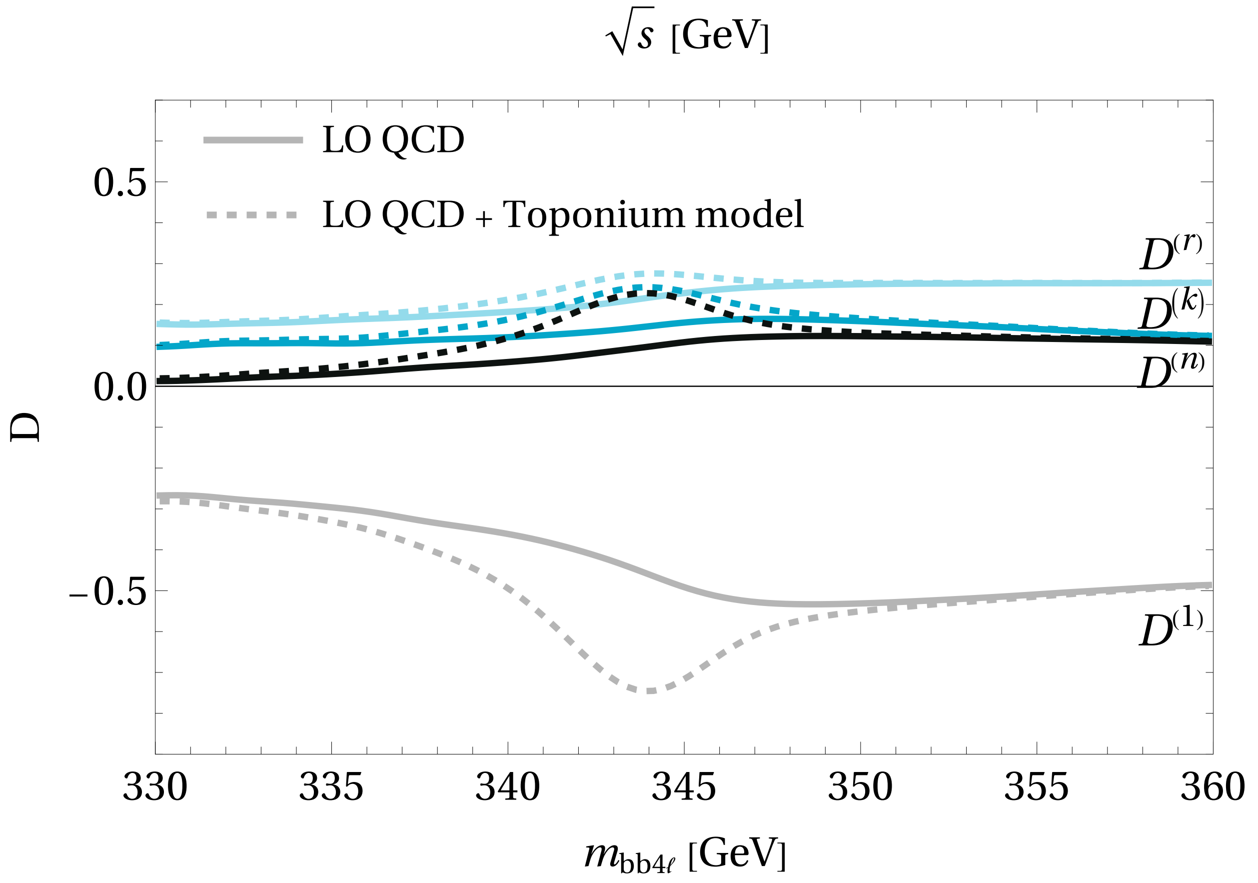
Observation of entanglement, ATLAS '23

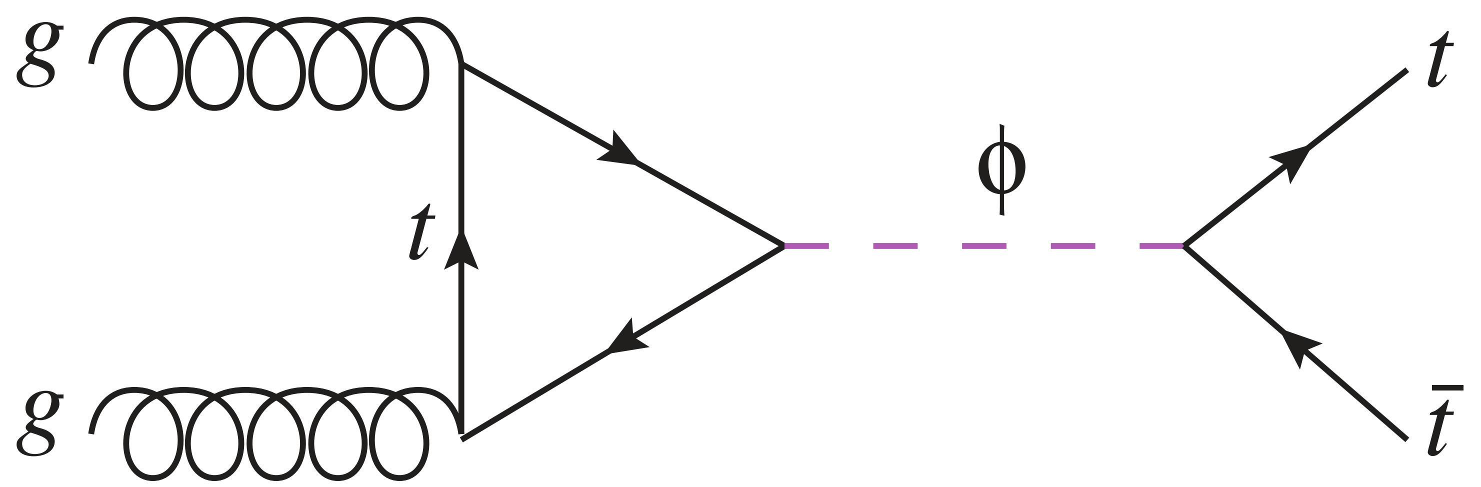
There is still a gap between theory and data.

Is this Toponium?

Toponium observation

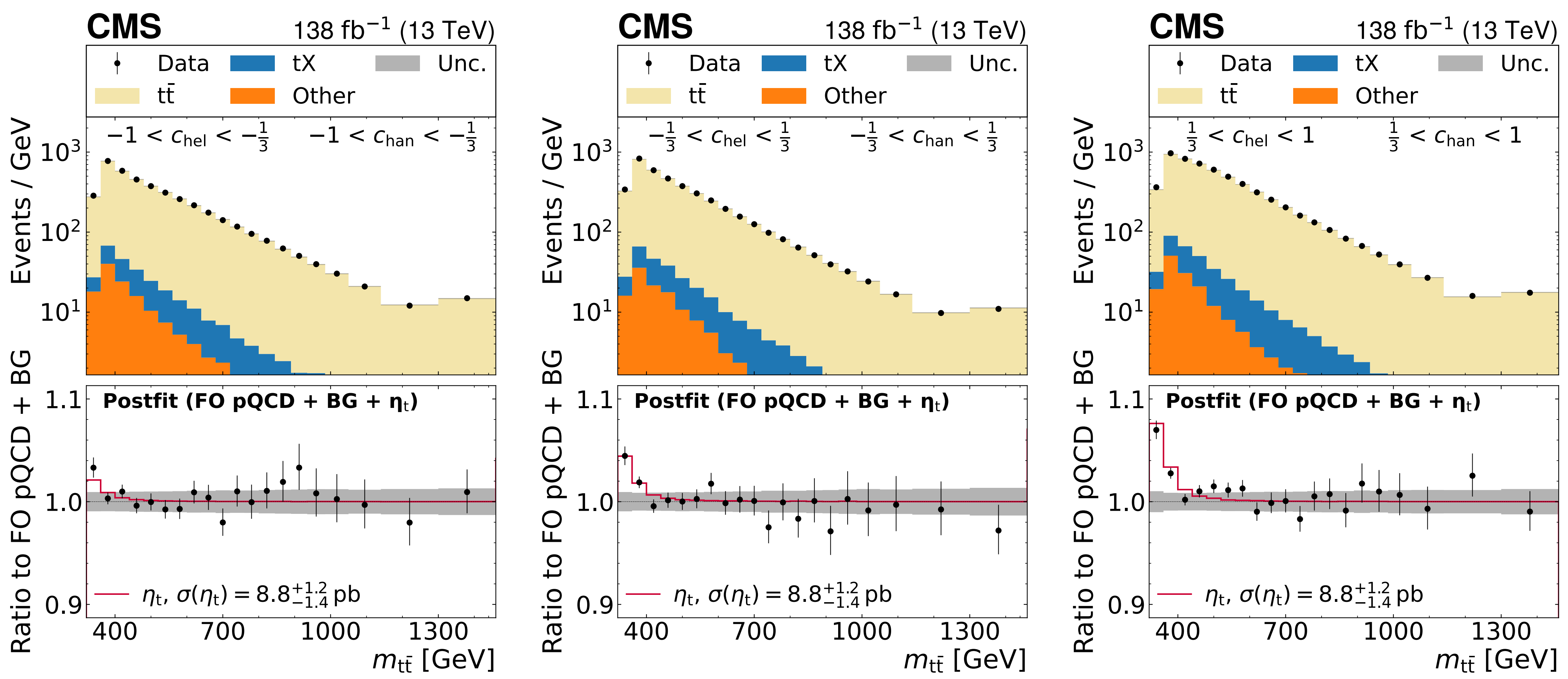






(Maltoni, Severi, Tentori, Vryonidou '24) x 2
Fuks, Hagiwara, Ma, Zheng '24

It is possible to tune a simple colour-singlet pseudoscalar model in order to reproduce the "proper QCD" result for m(bb4l). Doing so, it leads to a reduction of D.



Slicing in m(tt) and in angular variables (c_{hel} and c_{han}) between the two leptons, CMS has seen the effect of toponium. *CMS '25*

This observable would be sensitive also to other pseudoscalar BSM contributions.

The Higgs ($H \to VV^*$)

one scalar decaying into
two massive vector bosons —> 2 qutrits

Studied for the first time in Aguilar-Saavedra, Bernal, Casas, Moreno '22

Several papers appeared on the same topic, exploring different aspects of quantum observables for this process

Spin-density matrix ρ of 2 qutrits

$$\rho = \frac{1}{9}\left[\mathbf{1}_3 \otimes \mathbf{1}_3 + A^a_{L,M}(T_{L,M} \otimes \mathbf{1}_3) + A^b_{L,M}(\mathbf{1}_3 \otimes T_{L,M}) + C_{L_a,M_a,L_b,M_b}(T_{L_a,M_a} \otimes T_{L_b,M_b})\right]$$

$L = 1, 2$ and $-L \leq M \leq L$

$$T_{1,1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_{1,0} = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T_{1,-1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$T_{2,2} = \sqrt{3}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_{2,-2} = \sqrt{3}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T_{2,1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$T_{2,-1} = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \quad T_{2,0} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

If only the Hermiticity of ρ and the condition $\mathrm{Tr}[\rho] = 1$ are required, there are 80 independent parameters.

Several additional conditions can be required, reducing the number of independent parameters.

ρ at LO in $H \to VV^*$ in the SM

$$|\psi\rangle = a_L|0\,0\rangle + a_T\frac{|+-\rangle + |-+\rangle}{\sqrt{2}} \qquad a_L = \frac{-\beta}{\sqrt{2+\beta^2}}\,, \qquad a_T = \frac{\sqrt{2}}{\sqrt{2+\beta^2}}\,, \qquad \beta = 1 + \frac{m_H^2 - (m_a+m_b)^2}{2m_a m_b}$$

where $\beta \equiv p_a \cdot p_b/(m_a m_b)$. The system is maximally entangled at rest $\leftrightarrow$ $\beta = 1$.

β fixed: 1 param

$$\rho_{\rm LO}(\beta) = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & a_L^2 & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \frac{a_T^2}{2} & \cdot & \frac{a_L a_T}{\sqrt{2}} & \cdot & \frac{a_T^2}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

integrated over β: 2 param << 80

$$\rho_{\rm LO} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & y & \cdot & 1-2x & \cdot & y & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$0 \le x \le 1/2$$

Quantum Tomography

- In the $H \to VV^*$ decay, the spin states of the 2 V's are entangled.
- However, neither ATLAS nor CMS detectors measure the spin of the particles (especially of virtual V's ..).

QUANTUM TOMOGRAPHY:

- We can reconstruct the ρ density matrix of the bipartite qutrit system of the VV^* spins via the measurement of distributions of the $V^{(*)}$ decay products.

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{2S_a+1}{4\pi}\frac{2S_b+1}{4\pi}\sum_{\lambda_a,\lambda_a^{'},\lambda_b,\lambda_b^{'}} \rho(\lambda_a,\lambda_a^{'},\lambda_b,\lambda_b^{'})\Gamma_a(\lambda_a,\lambda_a^{'})\Gamma_b(\lambda_b,\lambda_b^{'})$$

$$= \left(\frac{3}{4\pi}\right)^2 \text{Tr}\left[\rho(\Gamma_a \otimes \Gamma_b)^T\right].$$

Γ is the spin decay-density matrix of $Z \to ff$

Quantum Tomography at LO

Γ is the spin decay-density matrix and $Y(\theta_i, \phi_i)$ are spherical harmonics.

$$\Gamma_a = \frac{1}{3}\left[\mathbf{1}_3 + B_1^a (T_{1,M_a} Y_{1,M_a}) + B_2^a (T_{2,M_a} Y_{2,M_a})\right] \qquad B_1^a = \sqrt{2\pi}\alpha_a\,, \qquad B_2^a = \sqrt{\frac{2\pi}{5}}\,.$$

This quantity α, the spin analysing power, will play a crucial role the talk.

I can rewrite

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\big[1 + A^a_{L,M} B^a_L Y_{L,M}(\theta_a,\phi_a) + A^b_{L,M} B^b_L Y_{L,M}(\theta_b,\phi_b) + C_{L_a,M_a,L_b,M_b} B^a_{L_a} B^b_{L_b} Y_{L_a,M_a}(\theta_a,\phi_a) Y_{L_b,M_b}(\theta_b,\phi_b)\big]$$

And finally

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{L,M}(\Omega_j)\, d\Omega_a d\Omega_b = \frac{B^j_L}{4\pi} A^j_{L,M} \quad \text{with} \quad j = a, b\,,$$

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} Y^*_{L_a,M_a}(\Omega_a) Y^*_{L_b,M_b}(\Omega_b)\, d\Omega_a d\Omega_b = \frac{B^a_{L_a} B^b_{L_b}}{(4\pi)^2} C_{L_a,M_a,L_b,M_b}\,,$$

So, at LO approximation, via distributions of fermions in $H \to VV^* \to f\bar{f}f\bar{f}$, it is possible to reconstruct ρ, the spin-density matrix of VV^* system.

$H \to VV^*$ at LO

Assuming SM LO decays of the $\boldsymbol{Z}$ boson, thus knowing all the coefficients $\boldsymbol{B}$ (so also α), one can extract all the $\boldsymbol{A}$ and $\boldsymbol{C}$ coefficients. But, in the SM at LO

$$\rho_{\rm LO} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & y & \cdot & 1-2x & \cdot & y & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & x & \cdot & y & \cdot & x & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$x \equiv \frac{C_{2,2,2,-2}}{3}$$

$$y \equiv \frac{C_{2,1,2,-1}}{3}$$

Only 10 $\boldsymbol{A}$,$\boldsymbol{C}$ coefficients are non-zero, and only two are independent.

$$\begin{gathered} A^a_{2,0} = A^b_{2,0} \neq 0\,, \\ C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0\,, \\ C_{2,-2,2,2} = C_{2,2,2,-2} = -C_{1,0,1,0} = 2 - C_{2,0,2,0} \neq 0\,, \end{gathered}$$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}$$

Quantum-observables at LO

<u>For such a form of ρ ,</u> the Peres-Horodecki criterion tell us that the condition $C_{2,2,2,-2} \neq 0$ or $C_{2,1,2,-1} \neq 0$ are not only necessary but also sufficient for entanglement.

Otherwise, more common criteria are the lower and upper bound on the Concurrence. Lower-bound > 0 implies entanglement, upper-bound = 0 implies no entanglement.

$$(\mathcal{C}(\rho))^2 \geq 2\max(\,\mathrm{Tr}\,\rho^2 - \,\mathrm{Tr}\,\rho_a^2,\,\mathrm{Tr}\,\rho^2 - \,\mathrm{Tr}\,\rho_b^2)\,, \qquad (\mathcal{C}(\rho))^2 \leq 2\min(1 - \,\mathrm{Tr}\,\rho_a^2, 1 - \,\mathrm{Tr}\,\rho_b^2) \qquad \rho_j = \frac{1}{3}(\mathbf{1}_3 + A^j_{L_j,M_j} T_{L_j,M_j})$$

A Bell-type inequality for a two-qutrit system is the CGLMP one. With local theories, the maximum value for $\mathcal{I}_3$ is 2, where $\mathcal{I}_3$ can be evaluated as:

$$\begin{aligned} O_B &= (V^\dagger \otimes U^\dagger) O_{\mathrm{Bell}} (V \otimes U)\,, \\ \mathcal{I}_3 &= \mathrm{Tr}\,[\rho\, O_B]\,, \end{aligned}$$

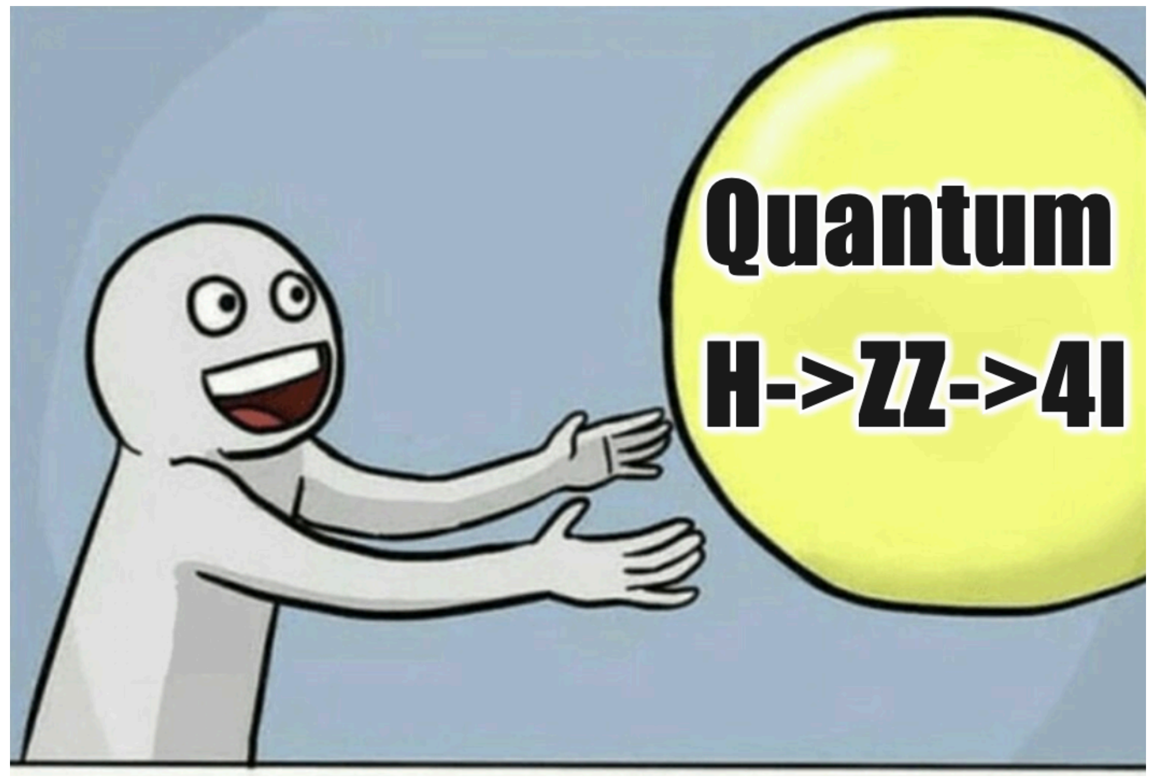
$$O_{\mathrm{Bell}} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \end{pmatrix} .$$

$$\begin{aligned} \mathcal{I}_3 = & \,[P(a_1 = b_1) + P(b_1 = a_2 + 1) + P(a_2 = b_2) + P(b_2 = a_1)] \\ & - [P(a_1 = b_1 - 1) + P(b_1 + a_2) + P(a_2 = b_2 - 1) + P(b_2 = a_1 - 1)] \end{aligned}$$

$H \to ZZ^* \to e^+e^-\mu^+\mu^-$ at LO

$$\rho_{\rm LO} = \begin{pmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 0.195(2) & \cdot & -0.313(3) & \cdot & 0.194(1) & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -0.313(3) & \cdot & 0.612(1) & \cdot & -0.313(3) & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 0.194(1) & \cdot & -0.313(3) & \cdot & 0.195(3) & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix}$$

	inclusive	$m(Z_2) > 10$ GeV	$m(Z_2) > 20$ GeV	$m(Z_2) > 30$ GeV
$C_{2,2,2,-2}$	0.581(5)	0.619(4)	0.713(4)	0.775(3)
$C_{2,1,2,-1}$	-0.938(4)	-0.975(3)	-1.017(3)	-1.014(3)
$\mathcal{I}_3\left(O_B^{O_A,U_{\rm fix}}\right)$	2.601(6)	2.672(4)	2.772(4)	2.794(5)
$\mathcal{I}_3\left(O_B^{(O_A)}\right)$	2.63	2.69	2.77	2.80



Fantastic, there is **entanglement** and the sign of **non-local** effects!
Also, **NLO QCD corrections are not present** for this process and **EW corrections typically affects only at the percent level** ……

$H \to ZZ^* \to e^+e^-\mu^+\mu^-$ at LO

$$\rho_{\rm LO} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.195(2) & \cdot & -0.313(3) & \cdot & 0.194(1) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -0.313(3) & \cdot & 0.612(1) & \cdot & -0.313(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0.194(1) & \cdot & -0.313(3) & \cdot & 0.195(3) & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

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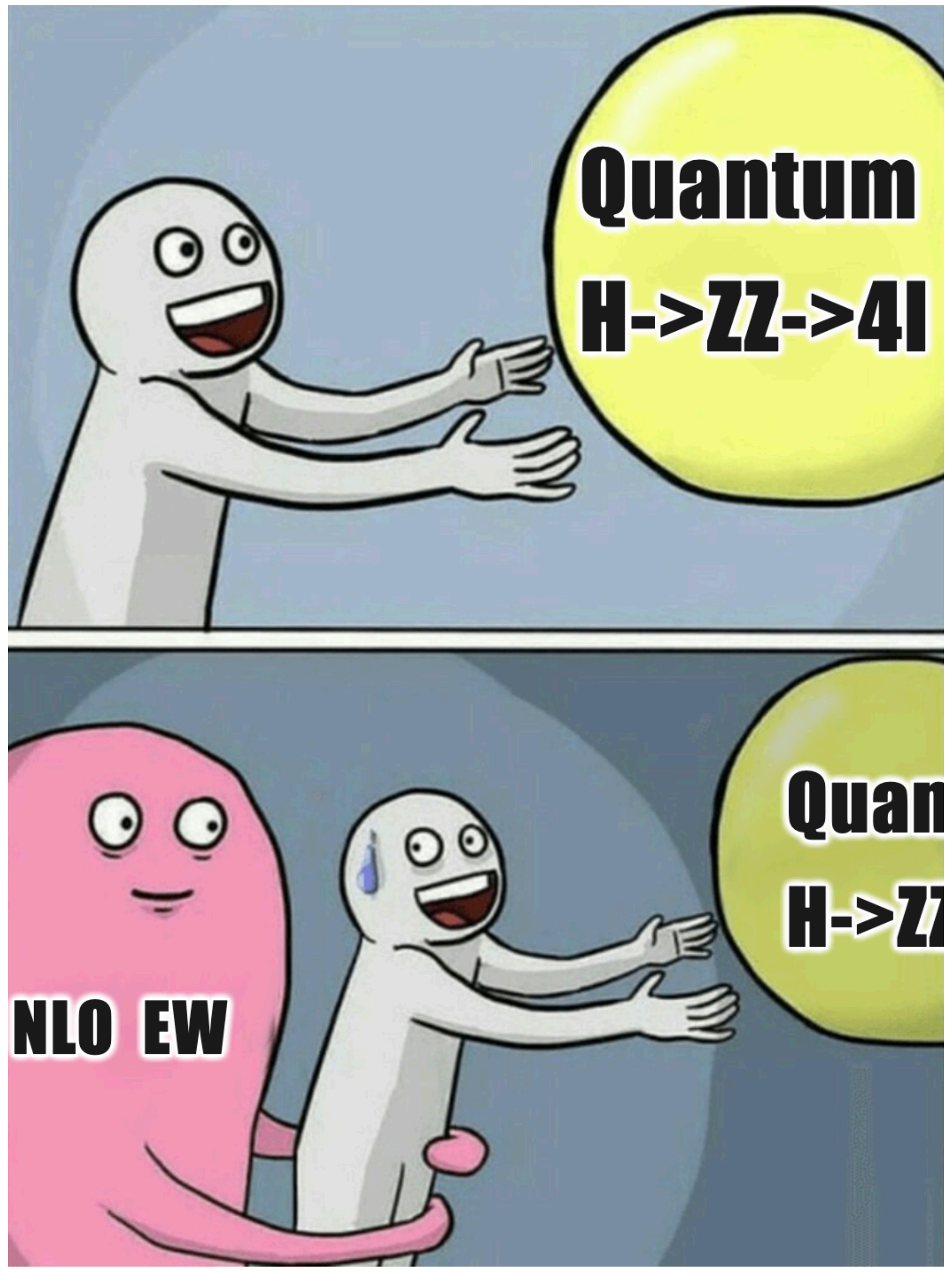
$\mathcal{I}_3\left(O_B^{(O_A)}\right)$	2.63	2.69	2.77	2.80

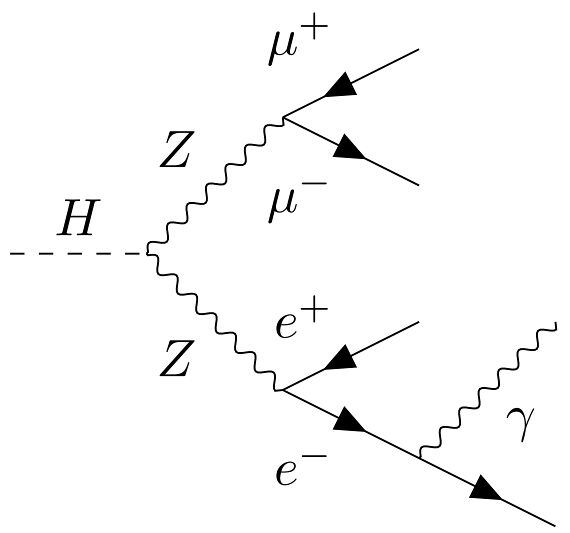
Fantastic, there is **entanglement** and the sign of **non-local** effects!
Also, **NLO QCD corrections are not present** for this process and **EW corrections typically affects only at the percent level** …… **unfortunately it is a bit more complicated**

Del Gratta, Fabbri, Lamba, Maltoni, DP '25
Grossi, Pelliccioli, Vicini '24

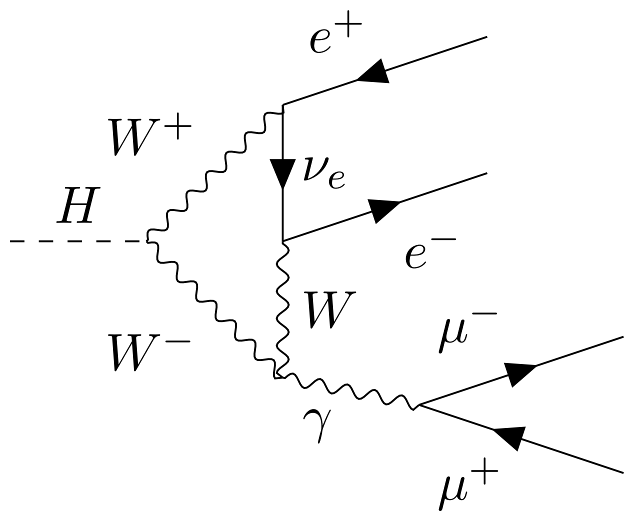


$H \to ZZ^* \to e^+e^-\mu^+\mu^-$ at NLO EW

We can simulate the $H \to ZZ^* \to e^+e^-\mu^+\mu^-$ process at NLO EW and apply QT as before (so assuming LO decay).

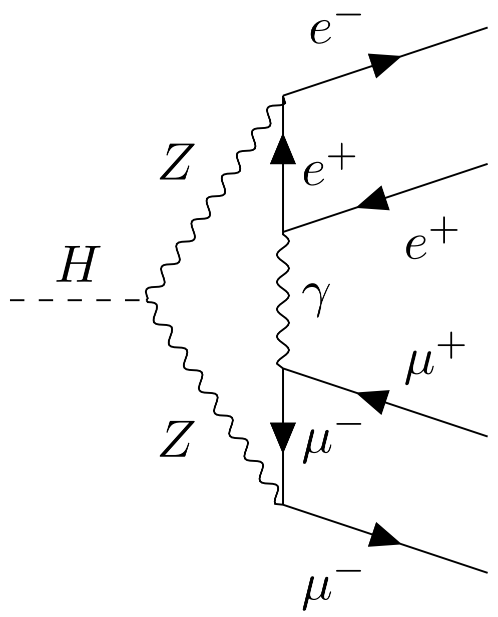


(a) Photon radiation in the final state.

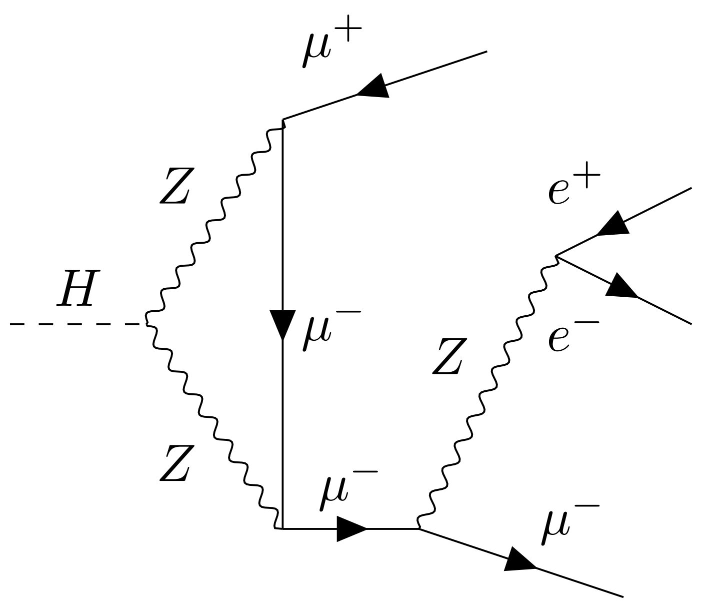


(b) Loop involving W.

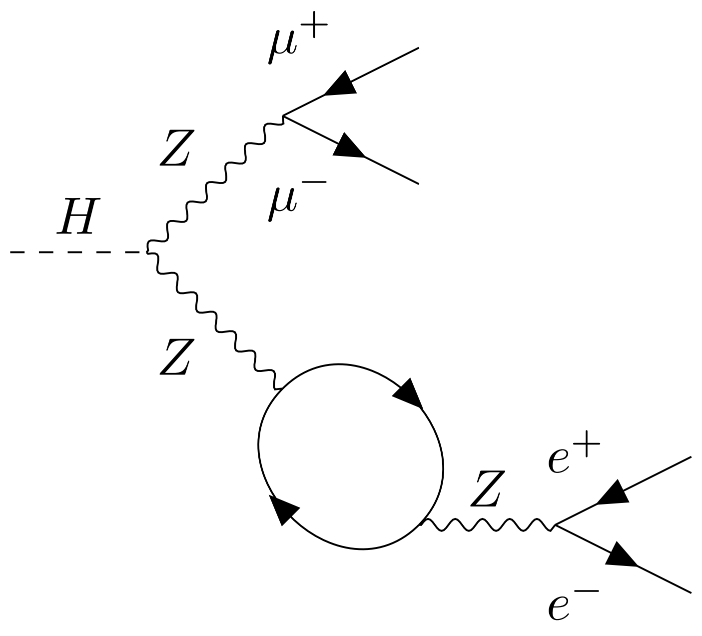
$$\rho_{\rm NLO} = \begin{pmatrix}
0.099(4) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0.004(2) & \cdot & 0.131(4) & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 0.111(4) & \cdot & -0.183(4) & \cdot & 0.189(1) & \cdot & \cdot \\
\cdot & 0.131(4) & \cdot & -0.009(2) & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & -0.183(4) & \cdot & 0.591(1) & \cdot & -0.183(4) & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -0.009(2) & \cdot & 0.131(4) & \cdot \\
\cdot & \cdot & 0.189(1) & \cdot & -0.183(4) & \cdot & 0.110(3) & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 0.131(4) & \cdot & 0.004(2) & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0.099(3)
\end{pmatrix}.$$



(c) Loop with photon-exchange between different-flavour leptons.



(d) Lepton-pair not originating from a Z boson decay.



(e) HVV^* genuine topology.

This matrix has a different texture than the LO one, leading to **negative eigenvalues.** Therefore **it makes no sense.** The situation is as bad as when an NLO calculation leads to negative cross sections.

Del Gratta, Fabbri, Lamba, Maltoni, DP '25

$H \to ZZ^* \to e^+e^-\mu^+\mu^-$ at NLO EW

	LO	NLO	NLO / LO
$A^1_{2,0}$	$-0.592(1)$	$-0.509(2)$	$0.860(2)$
$A^2_{2,0}$	$-0.591(1)$	$-0.565(2)$	$0.956(2)$
$C_{2,1,2,-1}$	$-0.937(2)$	$-0.943(4)$	$1.006(3)$
$-C_{1,1,1,-1}$	$-0.94(1)$	$-0.16(2)$	$0.17(2)$
$A^1_{2,0}/\sqrt{2}+1$	$0.5817(7)$	$0.640(1)$	$1.101(2)$
$C_{2,2,2,-2}$	$0.581(3)$	$0.568(4)$	$0.977(6)$
$-C_{1,0,1,0}$	$0.59(1)$	$0.03(2)$	$0.06(4)$
$C_{2,0,2,0}$	$1.418(3)$	$1.400(5)$	$0.987(3)$
$C_{1,0,1,0}+2$	$1.41(1)$	$1.97(2)$	$1.39(1)$

$$A^a_{2,0} = A^b_{2,0} \neq 0\,,$$
$$C_{1,-1,1,1} = C_{1,1,1,-1} = -C_{2,-1,2,1} = -C_{2,1,2,-1} \neq 0\,,$$
$$C_{2,-2,2,2} = C_{2,2,2,-2} = -C_{1,0,1,0} = 2 - C_{2,0,2,0} \neq 0\,,$$

$$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = C_{2,2,2,-2}$$

Del Gratta, Fabbri, Lamba, Maltoni, DP '25

Giant corrections

LO Relations violated

The culprit: α the spin analysing power

$$\alpha = \frac{c_R^2 - c_L^2}{c_R^2 + c_L^2}$$

f	c_L	c_R	α
ν	$\frac{1}{2}$	0	-1
e	$\frac{-1+2\sin^2\theta_W}{2}$	$\sin^2\theta_W$	-0.219
u	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$	-0.699
d	$-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$	-0.941

Table 1: Spin-analysing power α of the Z decay for different fermions in the SM.

$\frac{\text{NLO}}{\text{LO}} - 1$ [%]	$\ell^+\ell^-\ell'^+\ell'^-$	$u\bar{u}c\bar{c}$	$d\bar{d}s\bar{s}$
$A^1_{2,0}$	-14.0	-5.2	-1.1
$A^2_{2,0}$	-4.4	-1.0	< 1
$C_{2,1,2,-1}$	< 1	< 1	< 1
$C_{1,1,1,-1}$	-83.0	-17.8	-3.1
$C_{2,2,2,-2}$	-2.3	-1.3	< 1
$C_{1,0,1,0}$	-94.0	-14.9	-2.2
$C_{2,0,2,0}$	-1.3	< 1	< 1

For $Z \to \ell^+\ell^-$, there are large cancellations in the numerator of α that are not protected by any symmetry and so they can be spoiled by higher-order corrections, leading to giant effects. **The large effects are in Γ, in particular α, but they are erroneously propagated on ρ via QT at LO.**

We do not see such effects in other channels, including $H \to WW^*$. It is not a problem of tomography, it is not a problem of $H \to VV^*$. It is a problem of $H \to ZZ^* \to 4\ell$.

Del Gratta, Fabbri, Lamba, Maltoni, DP '25

Solution: avoid α dependence

The quantity α affects only coefficients with L=1.

	no cuts	$m(Z_2) > 30$ GeV	85 GeV $< m(Z_1) <$ 95 GeV
LO $\mathcal{C}_{LB}$	0.94	1.18	0.97
LO $\mathcal{C}_{LB}^{L>1}$	0.47	0.59	0.49
NLO $\mathcal{C}_{LB}^{L>1}$	0.49	0.55	0.48

The lower bound on concurrence (with L=1 excluded) is stable under NLO corrections. But it is closer to zero.

	no cuts	$m(Z_2) > 30$ GeV	85 GeV $< m(Z_1) <$ 95 GeV
$\mathcal{I}_3$, LO			
$O_B^{(O_A,U_{\rm fix})}$	2.600 ± 0.003	2.794 ± 0.004	2.639 ± 0.003
$O_B^{(O_A)}$	2.63	2.79	2.65
$O_B^{(O_A,C_{L>1})}$	2.63	2.79	2.65
$\mathcal{I}_3$, NLO			
$O_B^{(O_A,C_{L>1})}$	2.60	2.72	2.64

$\mathcal{I}_3$ is also stable if dependence on L=1 is avoided

Del Gratta, Fabbri, Lamba, Maltoni, DP '25

Solution: avoid α dependence

Measurements of Z-boson pair entanglement in decays of Higgs bosons at the ATLAS experiment

The ATLAS Collaboration

Entanglement is a key property of quantum systems. In this Letter the first measurements of quantum entanglement between spins in pairs of Z bosons are reported, using proton–proton collision data from the Large Hadron Collider (LHC) at center-of-mass energies of 13 TeV and 13.6 TeV, recorded with the ATLAS detector. Measurements of angular observables sensitive to ZZ^* spin-density-matrix elements in the $H \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$ process yield coefficients $C_{2,1,2,-1} = -0.71 \pm 0.45$ and $C_{2,2,2,-2} = 0.08 \pm 0.44$ consistent with their Standard Model predictions. A complementary hypothesis test using the full angular distribution, and relying on several Standard Model assumptions in the decays, provides substantially higher sensitivity to quantum correlations and disfavors the separable-state hypothesis at a significance of 4.7 standard deviations (expected 4.9σ) relative to the entangled Standard Model hypothesis. These results provide strong evidence of quantum entanglement between massive bosons (spin qutrits) at the electroweak scale.

See also CMS PAS HIG-25-011

Coefficients with L=1 avoided, but the new texture of ρ may nevertheless invalidate what is valid at LO.

$$C_{2,2,2,-2} \neq 0 \text{ or } C_{2,1,2,-1} \neq 0$$

ENTANGLEMENT

Please consider this (and maybe cite us ;).

Could we have α, aka η_ℓ, at NLO and correct QT?

YES for on-shell Z, **NO for off-shell Z**. The latter quantity is not gauge invariant and not well defined. In the Higgs decay, at least one Z is off-shell.

If direct ZZ production is considered (no Higgs in the middle), similar large effects are present for A and C coefficients with L=1, but they can be "cured". *Grossi, Pelliccioli, Vicini '24*

CHANGE OF NOTATION: A different name for α is η_ℓ, which we will use in the following.

Level 1 improvement:

Gonçalves, Kaladharan, Krauss, Navarro '25

$$\eta_\ell^{\rm eff} \equiv \eta_\ell^{\rm LO}\Big|_{\sin^2\theta_{\rm w}\to\sin^2\theta_{\rm w}^{\rm eff}} = \frac{1-4\sin^2\theta_{\rm w}^{\rm eff}}{1-4\sin^2\theta_{\rm w}^{\rm eff}+8\sin^4\theta_{\rm w}^{\rm eff}}$$

Level 2 improvement:

Del Gratta, Fabbri, Grossi, Maltoni, DP, Pelliccioli, Vicini '25

Compute $pp \to ZZ \to 4\ell$ in DPA and evaluate η_ℓ with the corresponding set-up: $\eta_\ell^{\rm NLO}$

$$\eta_\ell^{\rm LO} = 0.2131(1) \qquad \eta_\ell^{\rm eff} = 0.1405(8) \qquad \eta_\ell^{\rm NLO} = 0.1313(7)$$

η_ℓ at NLO EW

Calculated directly

Extracted via QT at LO from LO simulation of $pp \to ZZ \to 4\ell$.

$$C_{1010}^{\mathrm{NLO}} \simeq C_{1010}^{\mathrm{LO}} = \frac{8\pi\gamma_{1010}^{\mathrm{LO}}}{(\eta_\ell^{\mathrm{LO}})^2} = 0.977$$

= **Benchmark**

Del Gratta, Fabbri, Grossi, Maltoni, DP, Pellicciolli, Vicini '25

η_ℓ at NLO EW

Calculated directly

Extracted via QT at LO from LO simulation of $pp \to ZZ \to 4\ell$.

$$C^{\rm NLO}_{1010} \simeq C^{\rm LO}_{1010} = \frac{8\pi\gamma^{\rm LO}_{1010}}{(\eta^{\rm LO}_\ell)^2} = 0.977$$ = **Benchmark**

Del Gratta, Fabbri, Grossi, Maltoni, DP, Pelliccioli, Vicini '25

Extracted via QT (different assumptions) from NLO simulation $pp \to ZZ \to 4\ell$ in DPA.

LO QT $$\frac{8\pi\gamma^{\rm NLO}_{1010}}{(\eta^{\rm LO}_\ell)^2} = 0.224$$

NLO QT for both V $$\frac{8\pi\gamma^{\rm NLO}_{1010}}{(\eta^{\rm NLO}_\ell)^2} = 0.593$$

"NLO" QT for both V $$\frac{8\pi\gamma^{\rm NLO}_{1010}}{(\eta^{\rm eff}_\ell)^2} = 0.518$$

NLO QT for expanded $$\frac{8\pi\gamma^{\rm NLO}_{1010}}{(\eta^{\rm LO}_\ell)^2 + 2\,\eta^{\rm LO}_\ell \cdot \delta\eta^{\rm NLO}_\ell} = 0.977$$

"NLO QT" for expanded $$\frac{8\pi\gamma^{\rm NLO}_{1010}}{(\eta^{\rm LO}_\ell)^2 + 2\,\eta^{\rm LO}_\ell \cdot \delta\eta^{\rm eff}_\ell} = 0.710$$

η_ℓ at NLO EW

Calculated directly → C_{1010}^{LO}; Extracted via QT at LO from LO simulation of $pp \to ZZ \to 4\ell$. → $\gamma_{1010}^{\mathrm{LO}}$

$$C_{1010}^{\mathrm{NLO}} \simeq C_{1010}^{\mathrm{LO}} = \frac{8\pi\gamma_{1010}^{\mathrm{LO}}}{(\eta_\ell^{\mathrm{LO}})^2} = 0.977$$ = **Benchmark**

Del Gratta, Fabbri, Grossi, Maltoni, DP, Pellicciolo, Vicini '25

Extracted via QT (different assumptions) from NLO simulation $pp \to ZZ \to 4\ell$ in DPA.

LO QT: $\frac{8\pi\gamma_{1010}^{\mathrm{NLO}}}{(\eta_\ell^{\mathrm{LO}})^2} = 0.224$

NLO QT for both V: $\frac{8\pi\gamma_{1010}^{\mathrm{NLO}}}{(\eta_\ell^{\mathrm{NLO}})^2} = 0.593$

"NLO" QT for both V: $\frac{8\pi\gamma_{1010}^{\mathrm{NLO}}}{(\eta_\ell^{\mathrm{eff}})^2} = 0.518$

NLO QT for expanded: $\frac{8\pi\gamma_{1010}^{\mathrm{NLO}}}{(\eta_\ell^{\mathrm{LO}})^2 + 2\,\eta_\ell^{\mathrm{LO}} \cdot \delta\eta_\ell^{\mathrm{NLO}}} = 0.977$

"NLO QT" for expanded: $\frac{8\pi\gamma_{1010}^{\mathrm{NLO}}}{(\eta_\ell^{\mathrm{LO}})^2 + 2\,\eta_\ell^{\mathrm{LO}} \cdot \delta\eta_\ell^{\mathrm{eff}}} = 0.710$

This results should be interpreted as follow:

η_ℓ^{eff} improves but η_ℓ^{NLO} is better.

DPA at NLO has at most one loop for one Z, never for both of them. The results on the left points to the fact that NNLO is large.

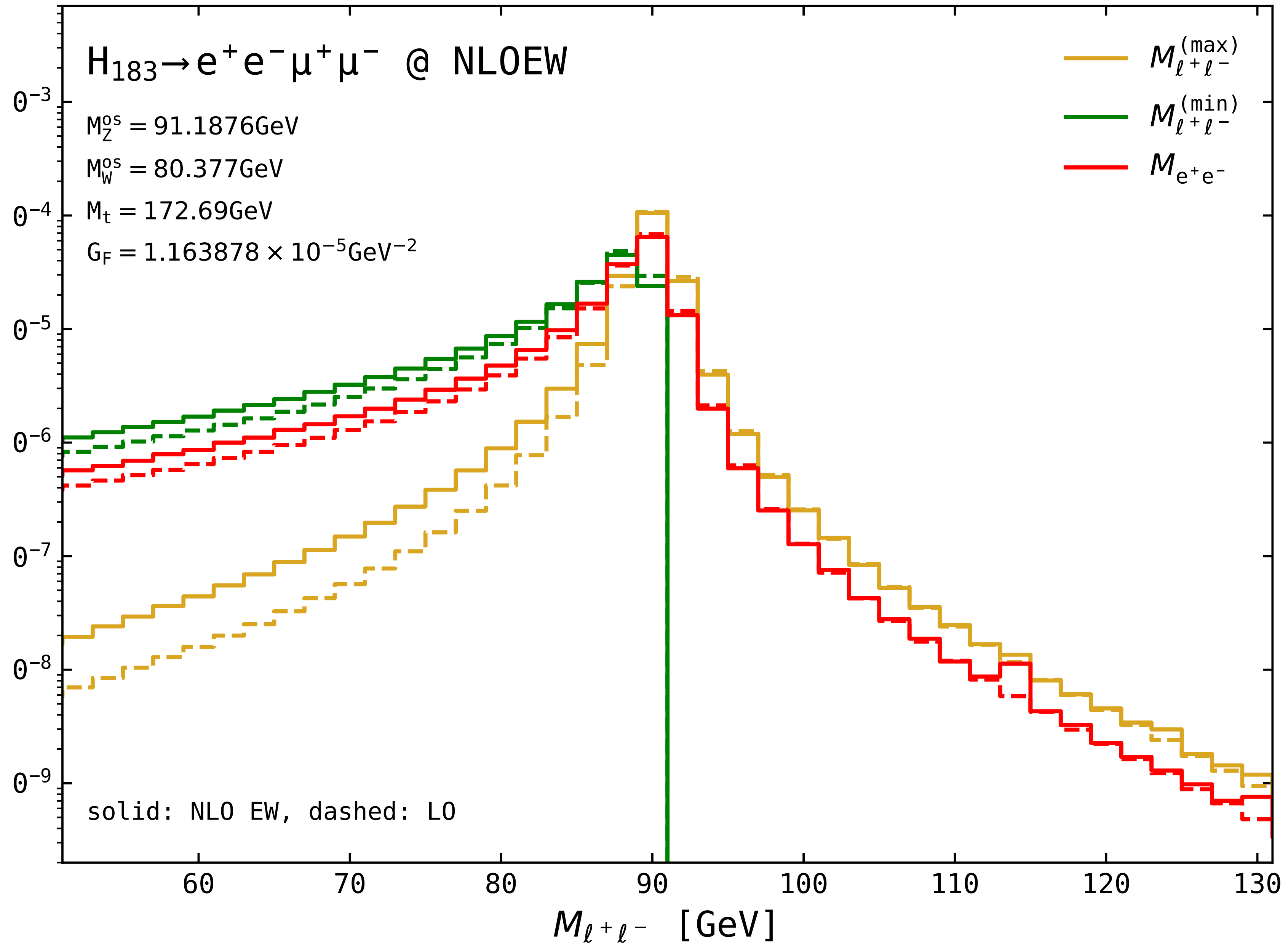
Nature is all-order not NLO so data should be extracted by simply diving by $(\eta_\ell^{\mathrm{NLO}})^2$

and the Higgs?

NOT possible to do this test with an off-shell Z. We considered the case of a heavy SM-like Higgs with $M_H > 2M_Z$, do both Z can be on-shell.

γ_{1010} coefficient $(\times 10^3)$				
	QT		HA	
$M_{\rm H}$	LO	NLO	LO	NLO
183 GeV	$-1.5506(9)$	$-0.44(1)$	$-1.790(1)$	$-0.596(6)$
200 GeV	$-1.3085(5)$	$-0.417(9)$	$-1.364(1)$	$-0.469(5)$
225 GeV	$-0.8601(7)$	$-0.294(7)$	$-0.8776(7)$	$-0.315(3)$
250 GeV	$-0.559(1)$	$-0.200(3)$	$-0.5643(4)$	$-0.208(2)$

$$\gamma_{1010}^{\rm LO} = (\eta_\ell^{\rm LO})^2 (F_\pm^{\rm LO}), \qquad \gamma_{1010}^{\rm NLO} = \gamma_{1010}^{\rm LO} + 2\,\eta_\ell^{\rm LO}\cdot\delta\eta_\ell^{\rm NLO}\cdot F_\pm^{\rm LO} + (\eta_\ell^{\rm LO})^2\cdot\delta F_\pm^{\rm NLO}$$



Del Gratta, Fabbri, Grossi, Maltoni, DP, Pelliccioli, Vicini '25

The lower is M_H, the more the Z boson is off-shell and we are back to the original problem.

Alternative approaches: *Aguilar-Saavedra '25; Aguilar-Saavedra, Giardino '26*

Consider higher-order corrections as a background and subtract them.

BSM $H \to VV^*$ at the tree-level

General Parameterisation

$$\mathcal{V}^{HV_aV_b}_{\mu\nu} = \frac{1}{v}\left[a_1 g_{\mu\nu} M_V^2 + a_2\left(g_{\mu\nu} p_a \cdot p_b - (p_a)_\nu (p_b)_\mu\right) + a_3 \epsilon_{\mu\nu\alpha\beta} p_a^\alpha p_b^\beta\right]$$

a_1 and a_2: CP even

a_3: CP odd

SM tree: only $a_1 \neq 0$ for ZZ and WW

at fixed β

Non-vanishing coefficients also in the SM	Vanishing coefficients in the SM
$A^a_{2,0} = A^b_{2,0}$	$A^a_{1,0} = -A^b_{1,0}$
$\frac{A^a_{2,0}}{\sqrt{2}} + 1 = -C_{1,0,1,0}$	$\frac{A^b_{1,0}}{\sqrt{2}} = C_{2,0,1,0}$
$C_{2,0,2,0} = 2 + C_{1,0,1,0}$	$C_{1,0,2,0} = -C_{2,0,1,0}$
$C_{1,-1,1,1} = -C_{2,-1,2,1}$	$C_{1,-1,2,1} = -C_{2,-1,1,1}$
$C_{1,-1,1,1} = C^*_{1,1,1,-1}$	$C_{1,-1,2,1} = C^*_{1,1,2,-1}$
$C_{2,-1,2,1} = C^*_{2,1,2,-1}$	$C_{2,-1,1,1} = C^*_{2,1,1,-1}$
$C_{2,2,2,-2} = C^*_{2,-2,2,2}$	

$$\rho_{\rm NP}(m_a, m_b) = \tfrac{1}{3} \times$$

$$\begin{pmatrix}
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & \left(1 + \frac{1}{\sqrt{2}} A^2_{2,0} - \sqrt{\frac{3}{2}} A^2_{1,0}\right) & \cdot & \left(-C_{2,1,1,-1} + C_{2,1,2,-1}\right) & \cdot & C_{2,2,2,-2} & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & \left(-C_{2,-1,1,1} + C_{2,-1,2,1}\right) & \cdot & \left(1 - \sqrt{2} A^2_{2,0}\right) & \cdot & \left(C_{2,1,1,-1} + C_{2,1,2,-1}\right) & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & C_{2,-2,2,2} & \cdot & \left(C_{2,-1,1,1} + C_{2,-1,2,1}\right) & \cdot & \left(1 + \frac{1}{\sqrt{2}} A^2_{2,0} + \sqrt{\frac{3}{2}} A^2_{1,0}\right) & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot \\
\cdot\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\cdot
\end{pmatrix}$$

BSM $H \to VV^*$ at the tree-level

Higgs characterisation model

$$\mathcal{L}_{X_0VV} = \Bigg[c_\phi \kappa_{\rm SM} \left[\frac{1}{2} g_{HZZ} Z^\mu Z_\mu + g_{HWW} W^+_\mu W^{-\mu} \right]$$

Artoisenet et al. '13

$$- \frac{1}{4} \left[c_\phi \kappa_{H\gamma\gamma} g_{H\gamma\gamma} A_{\mu\nu} A^{\mu\nu} + s_\phi \kappa_{A\gamma\gamma} g_{A\gamma\gamma} A_{\mu\nu} \widetilde{A}^{\mu\nu} \right]$$

$$- \frac{1}{2} \left[c_\phi \kappa_{HZ\gamma} g_{HZ\gamma} Z_{\mu\nu} A^{\mu\nu} + s_\phi \kappa_{AZ\gamma} g_{AZ\gamma} Z_{\mu\nu} \widetilde{A}^{\mu\nu} \right]$$

$$- \frac{1}{4} \frac{1}{\Lambda} \left[c_\phi \kappa_{HZZ} Z_{\mu\nu} Z^{\mu\nu} + s_\phi \kappa_{AZZ} Z_{\mu\nu} \widetilde{Z}^{\mu\nu} \right]$$

$$- \frac{1}{2} \frac{1}{\Lambda} \left[c_\phi \kappa_{HWW} W^+_{\mu\nu} W^{-\mu\nu} + s_\phi \kappa_{AWW} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} \right]$$

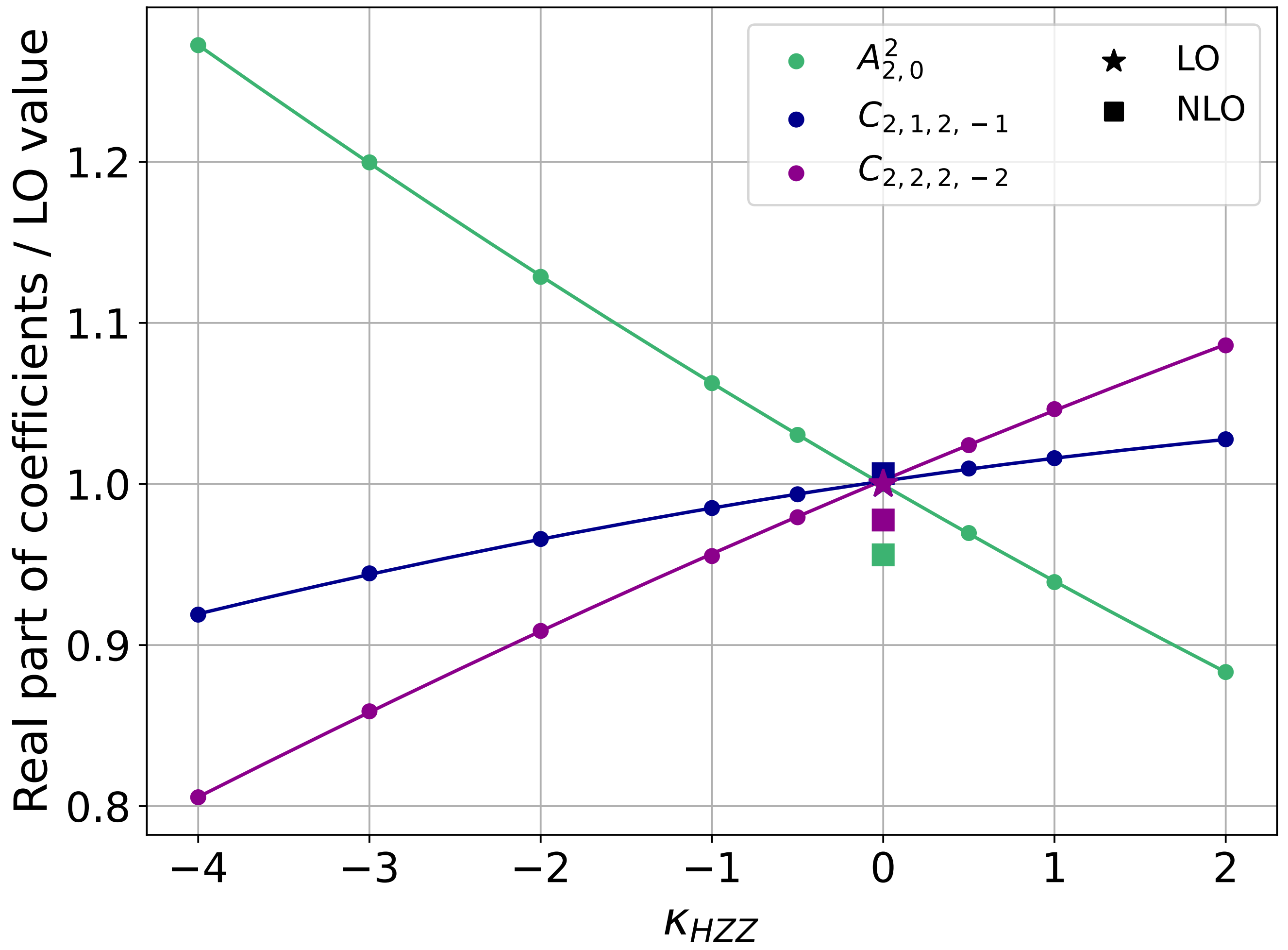
$$- \frac{1}{\Lambda} c_\phi \left[\kappa_{H\partial\gamma} Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\partial Z} Z_\nu \partial_\mu Z^{\mu\nu} + \left(\kappa_{H\partial W} W^+_\nu \partial_\mu W^{-\mu\nu} + \text{h.c.} \right) \right] \Bigg] X_0$$

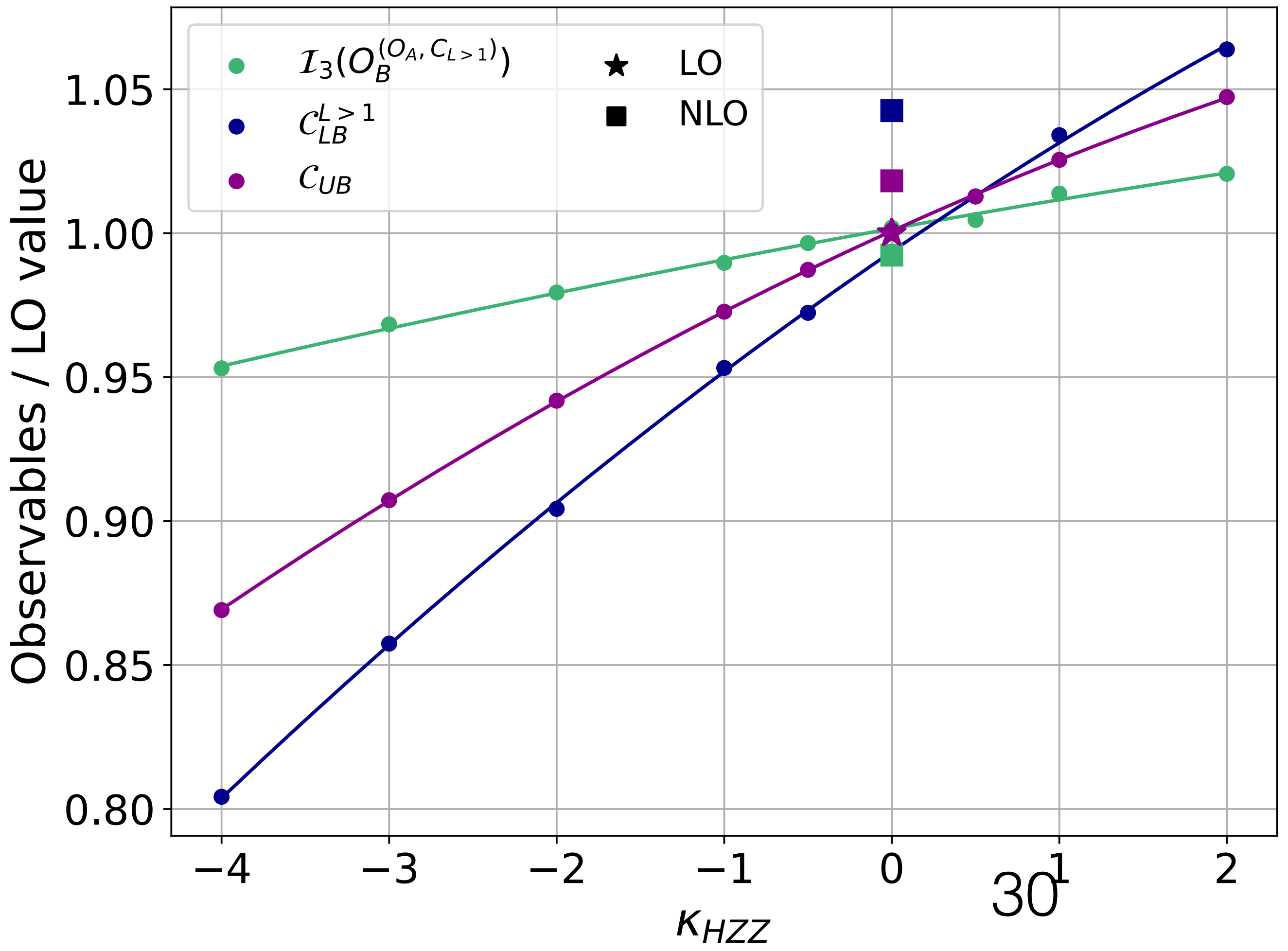
ϕ parametrises CP violation

Relations to general parameterisation

VV		VV	
ZZ	$a_1 = c_\phi \kappa_{\rm SM} + \frac{v}{\Lambda} \frac{(m_a^2 + m_b^2)}{2M_Z^2} c_\phi \kappa_{H\partial Z}$ $a_2 = \frac{1}{2} \frac{v}{\Lambda} c_\phi \kappa_{HZZ}$ $a_3 = \frac{1}{2} \frac{v}{\Lambda} s_\phi \kappa_{AZZ}$	$Z\gamma$	$a_1 = \frac{v}{\Lambda} \frac{m_b^2}{M_Z^2} c_\phi \kappa_{H\partial\gamma}$ $a_2 = v c_\phi \kappa_{HZ\gamma} g_{HZ\gamma}$ $a_3 = v s_\phi \kappa_{AZ\gamma} g_{AZ\gamma}$
WW	$a_1 = 2c_\phi \kappa_{\rm SM} + \frac{v}{\Lambda} \frac{(\kappa_{H\partial W} m_a^2 + \kappa^*_{H\partial W} m_b^2)}{M_Z^2} c_\phi$ $a_2 = \frac{v}{\Lambda} c_\phi \kappa_{HWW}$ $a_3 = \frac{v}{\Lambda} s_\phi \kappa_{AWW}$	$\gamma\gamma$	$a_1 = 0$ $a_2 = \frac{1}{2} v c_\phi \kappa_{H\gamma\gamma} g_{H\gamma\gamma}$ $a_3 = \frac{1}{2} v s_\phi \kappa_{A\gamma\gamma} g_{A\gamma\gamma}$

$$V^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu \text{ and } \widetilde{V}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma}$$

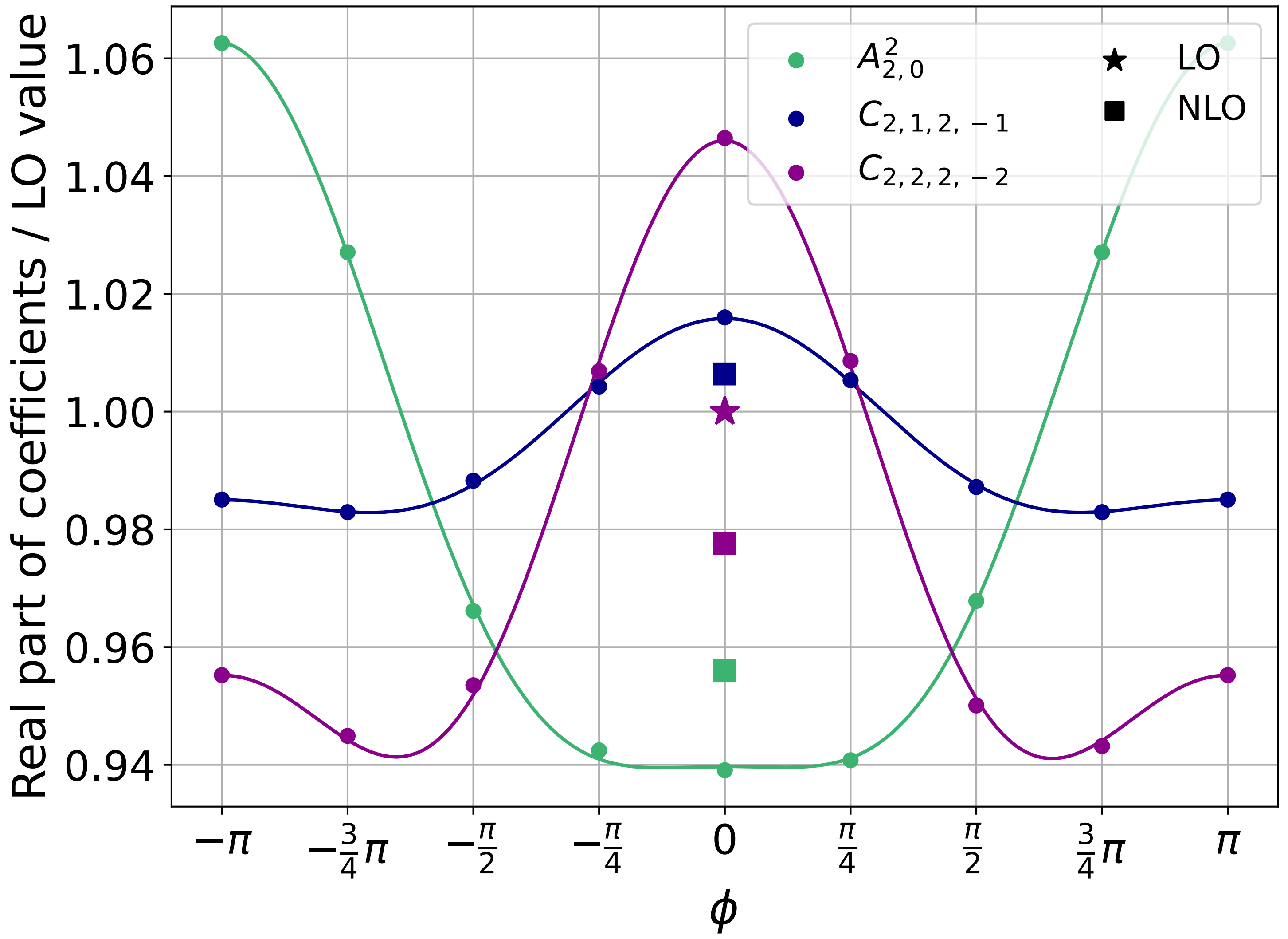


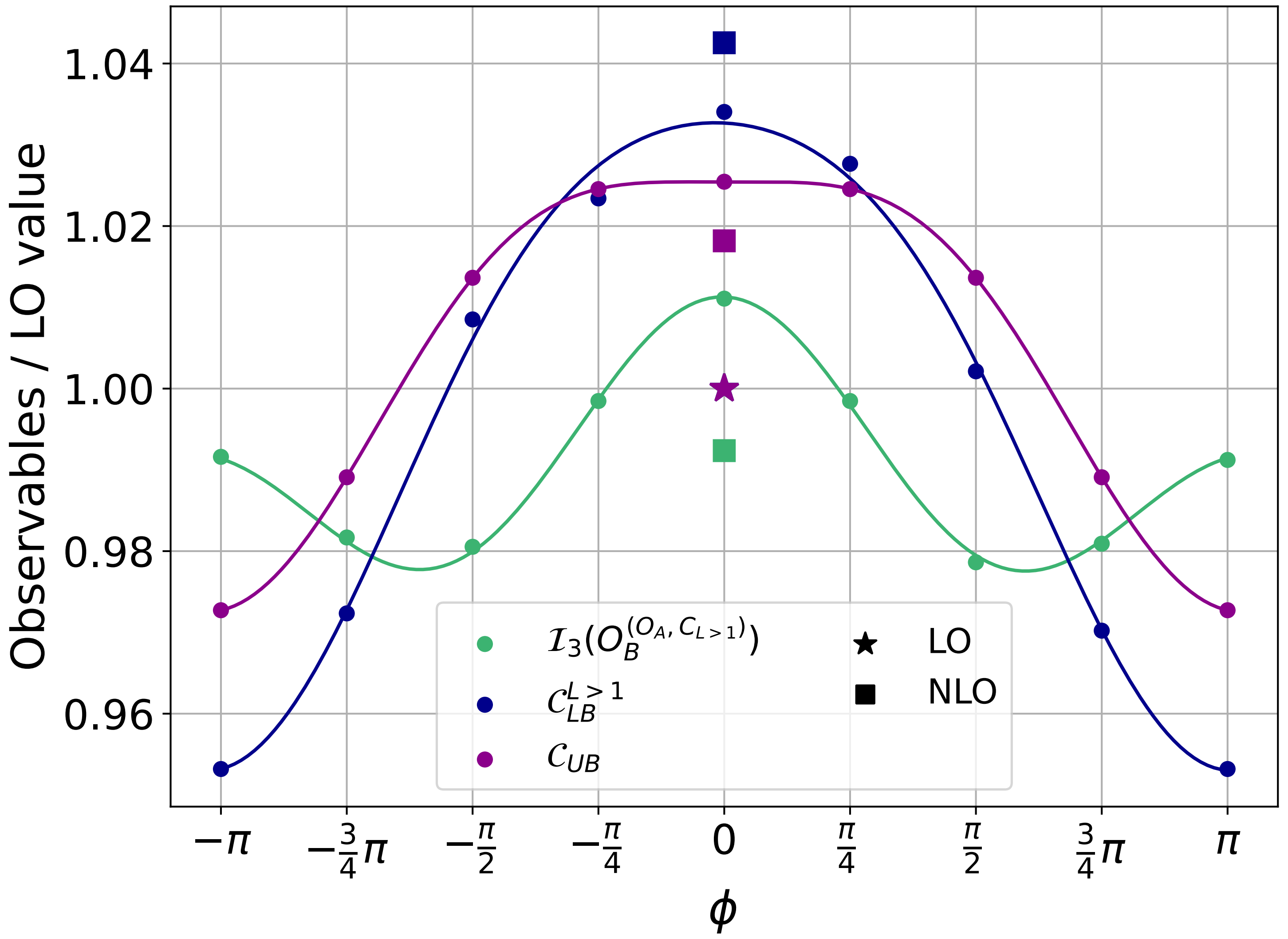


Coefficients and quantum observables are sensitive to BSM effects.

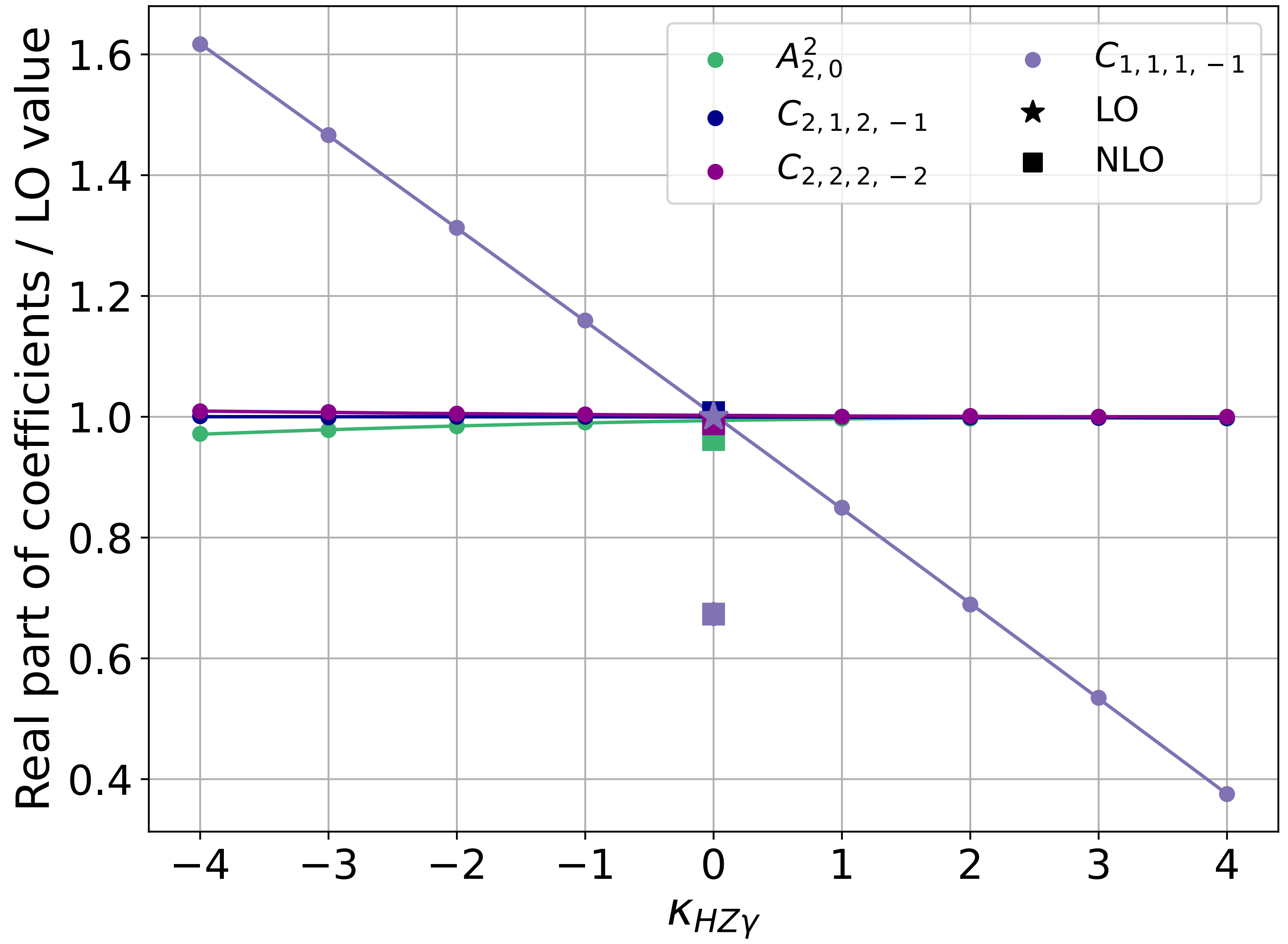
BSM $H \to VV^*$ at the tree-level

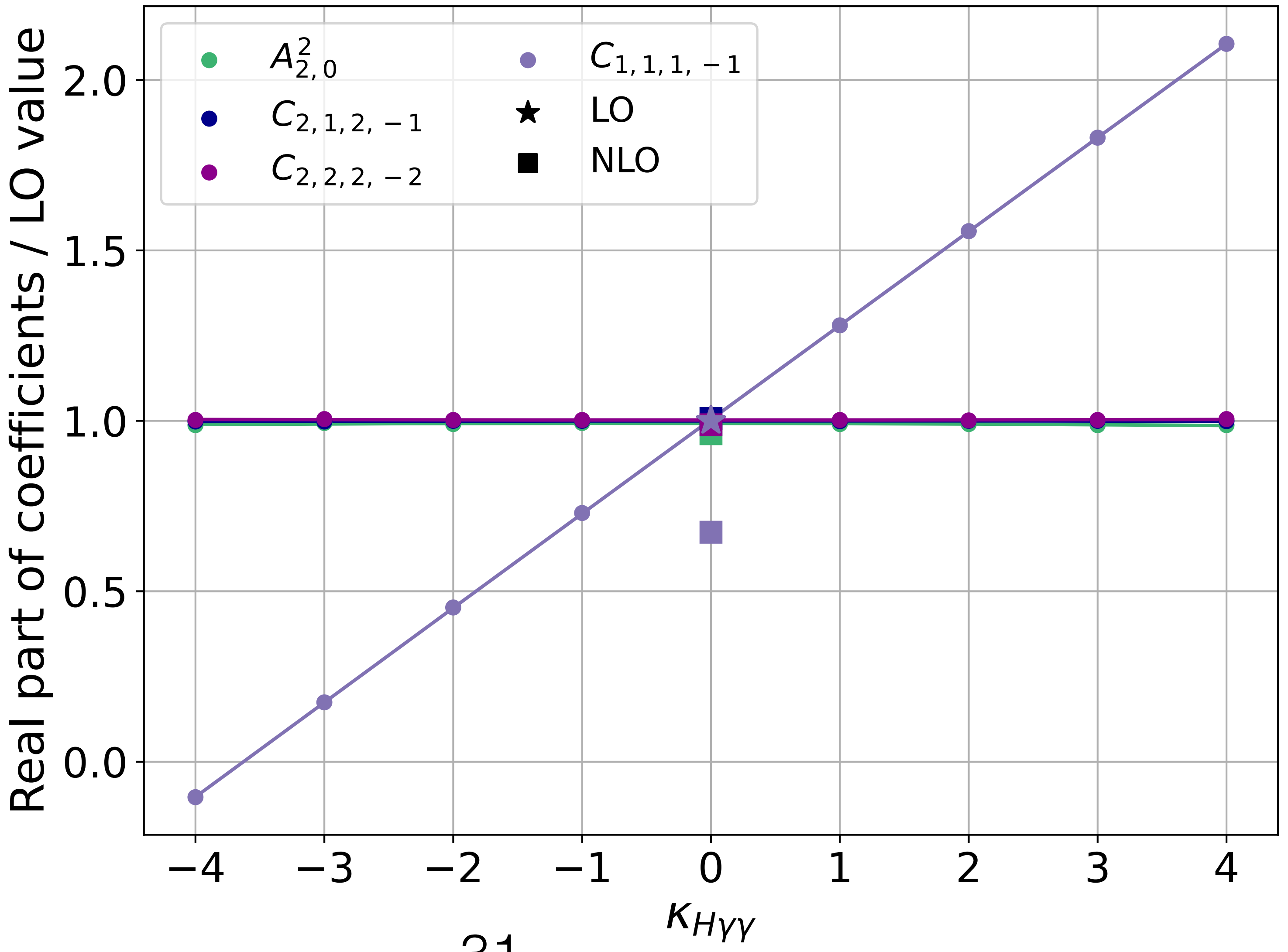
$\Lambda = 1$ TeV $\kappa_{HZZ} = 1$ and $\kappa_{AZZ} = 5$





CP modulation by ϕ





Great sensitivity from $C_{1,1,1,-1}$, but this is not well defined for QI observables. What is the intermediate state, ZZ or $Z\gamma(\gamma\gamma)$?

CONCLUSION

The study of Quantum-Information observables at high energies has started since a few years and it involves the Quantum-Tomography approach.

We have seen for $H \to ZZ^* \to 4\ell$ that once again precise predictions are essential also in this context.

The study of Quantum Observable is per se valuable, but as byproduct it also offers new options for leveraging the sensitivity on New Physics Effects.

Additional Slides

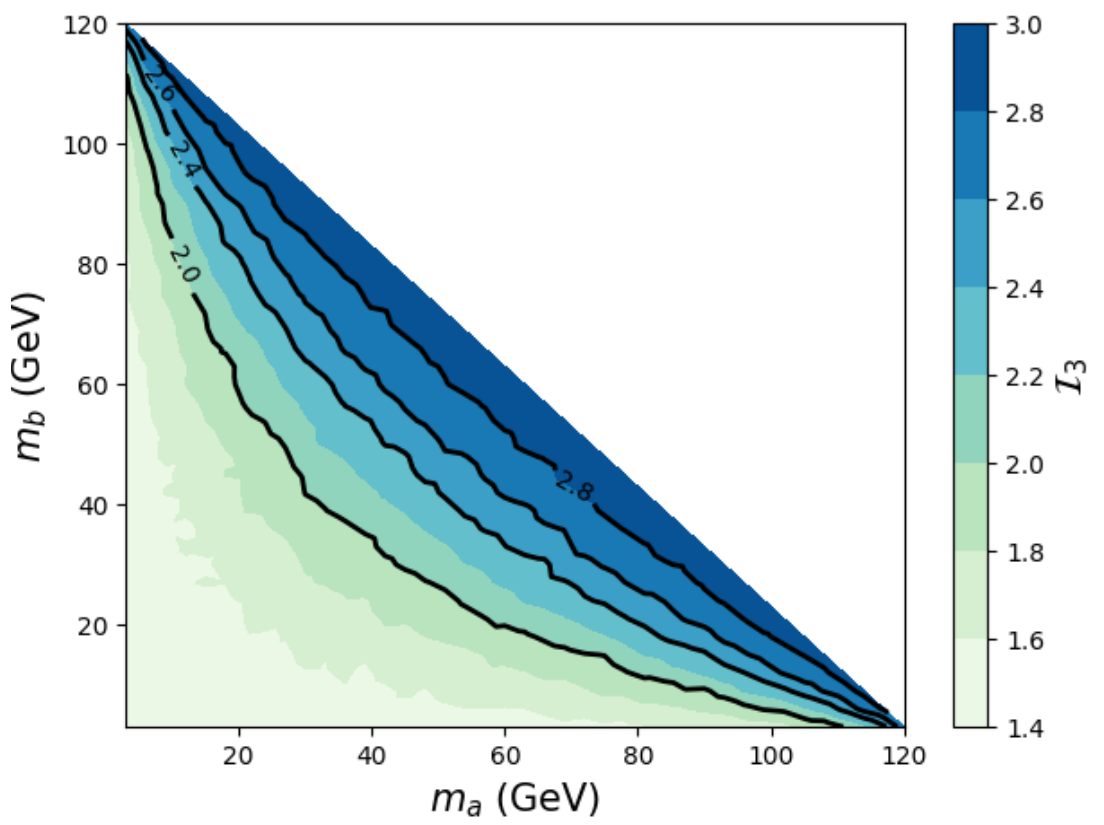
More on $\mathcal{I}_3$

$$O_A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$O_B^{(O_A)} = (U O_A \otimes U)^\dagger O_{\rm Bell} (U O_A \otimes U)$$

Valid only at LO

$$\mathcal{I}_3 \to \ {\rm Tr}\,[\rho\, O_B^{(O_A, U_{\rm fix})}] \equiv \ {\rm Tr}\,[\rho\, (U_{\rm fix} O_A \otimes U_{\rm fix})^\dagger O_{\rm Bell} (U_{\rm fix} O_A \otimes U_{\rm fix})] = \frac{1}{36}(18 + 16\sqrt{3} - \sqrt{2}(9 - 8\sqrt{3}) A^1_{2,0} - 8(3 + 2\sqrt{3}) C_{2,1,2,-1} + 6\, C_{2,2,2,-2})\ .$$



NLO: minimise dependence on L=1

$$\mathcal{I}_3 = A \cdot C_{1,0,1,0} + B \cdot C_{1,-1,1,1} + D\,,$$