SM@LHC, Turin, 8th April 2026

Recent Developments in Precision Calculations for Higgs Boson Pair Production

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Durham University

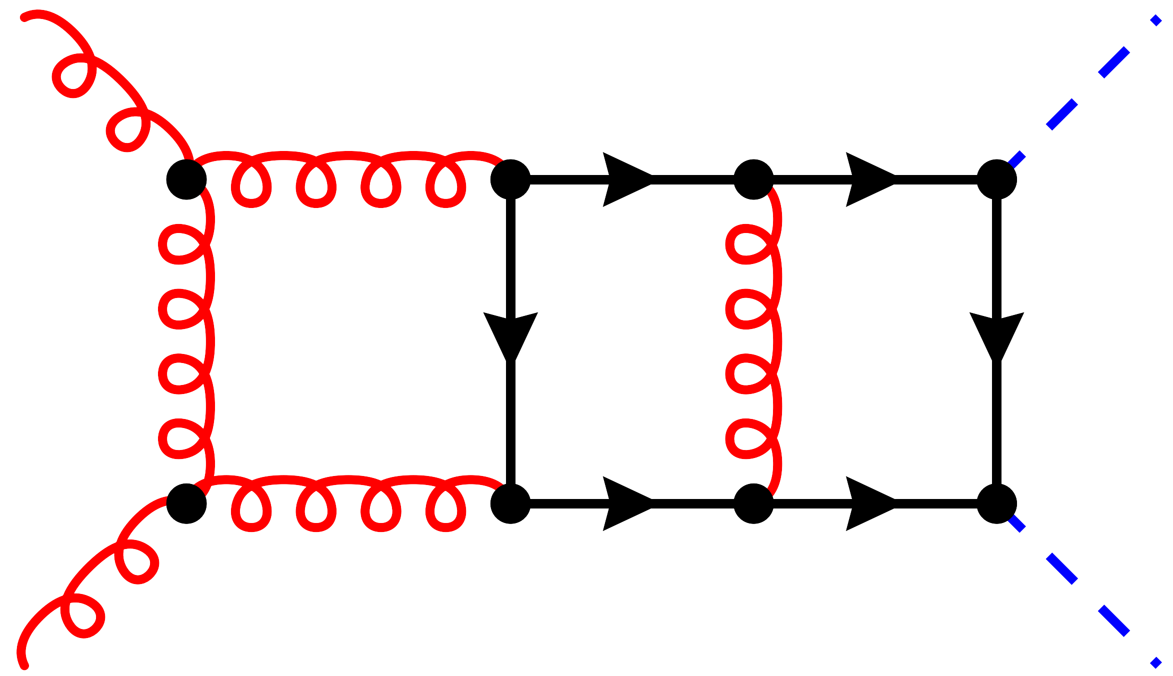
THE ROYAL SOCIETY

Photo Credit: Nasa

Outline

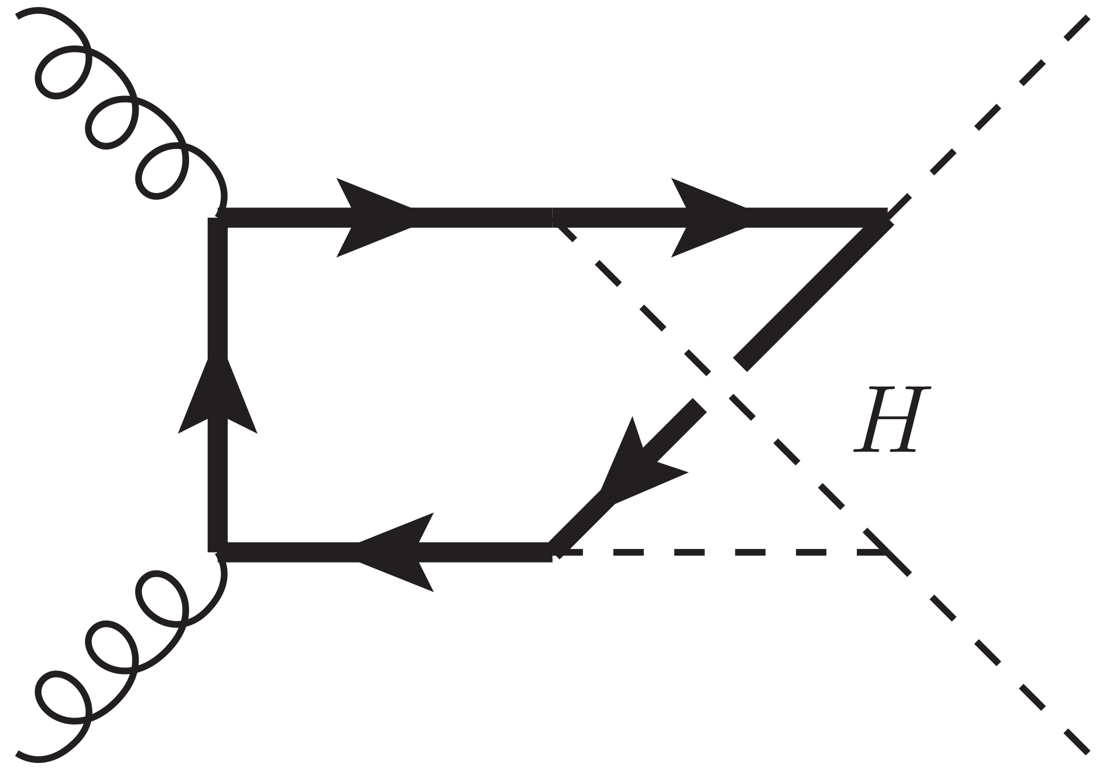
Higgs Boson Pair Production

The Need for Theory Predictions



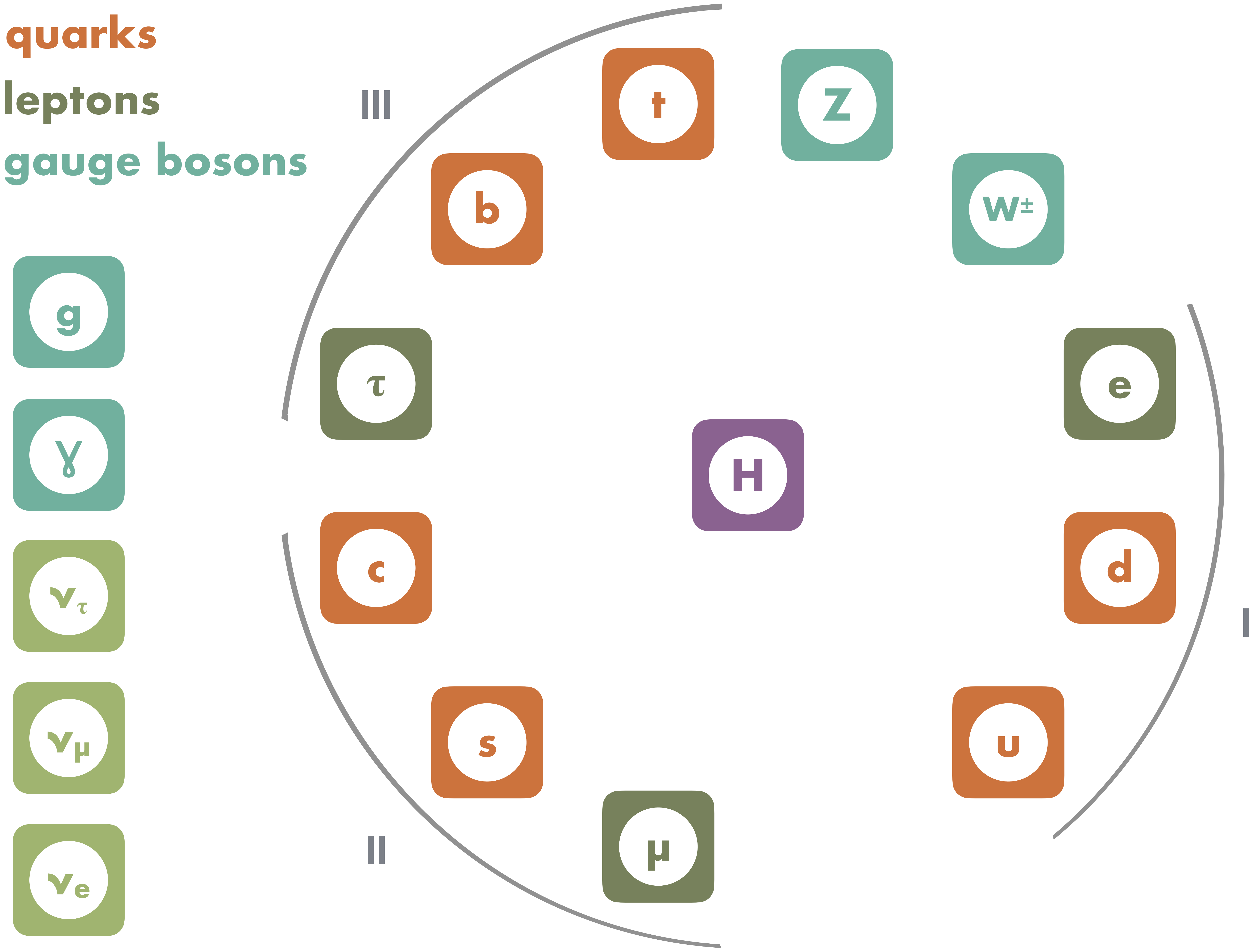
Higgs Pair Production State of the Art

1. Status Summary
2. QCD Corrections
3. Electroweak Corrections
4. Mass Scheme Uncertainty



Outlook

The Need for Theory Predictions


quarks
leptons
gauge bosons
g
γ
ντ
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t
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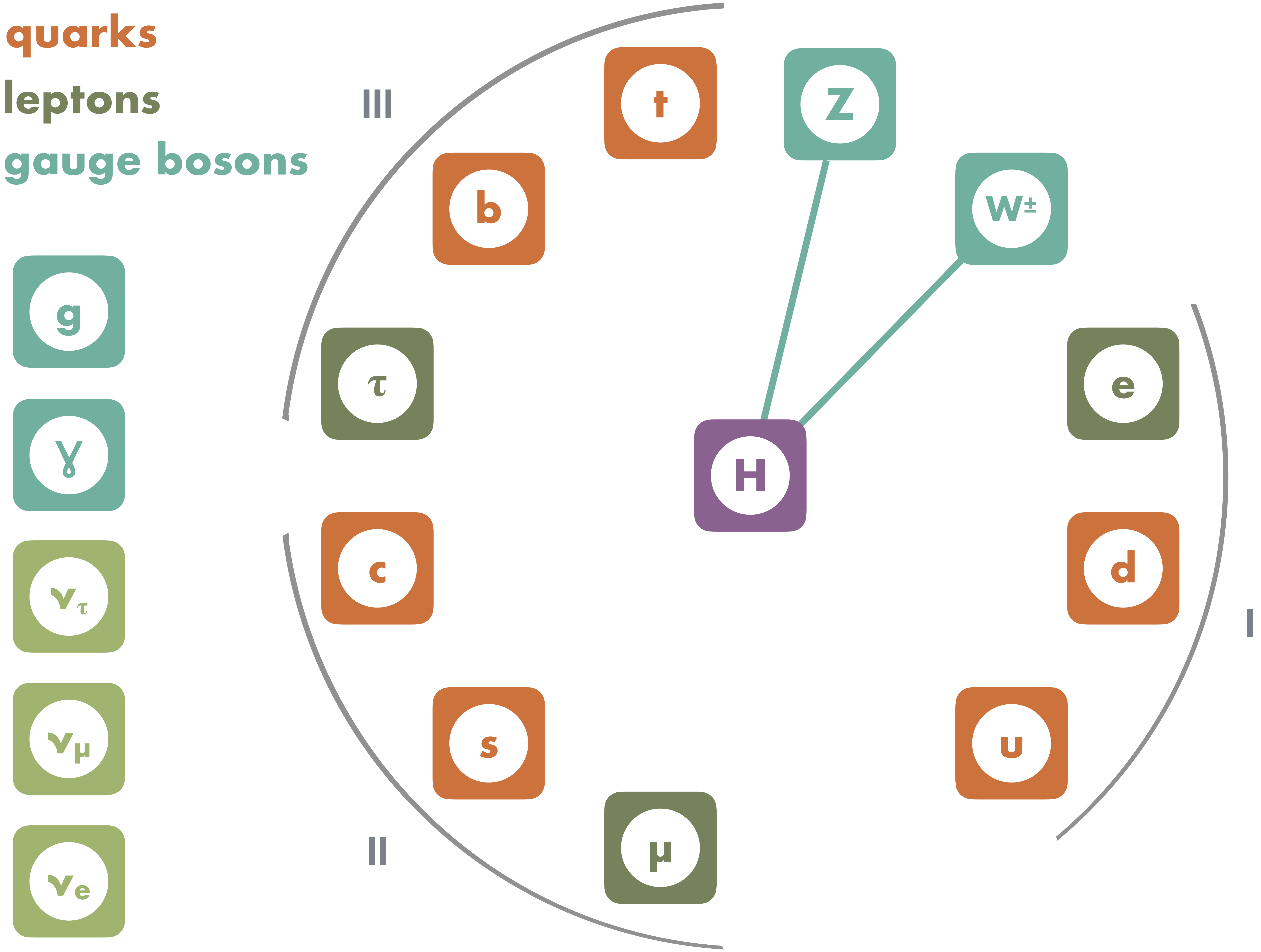
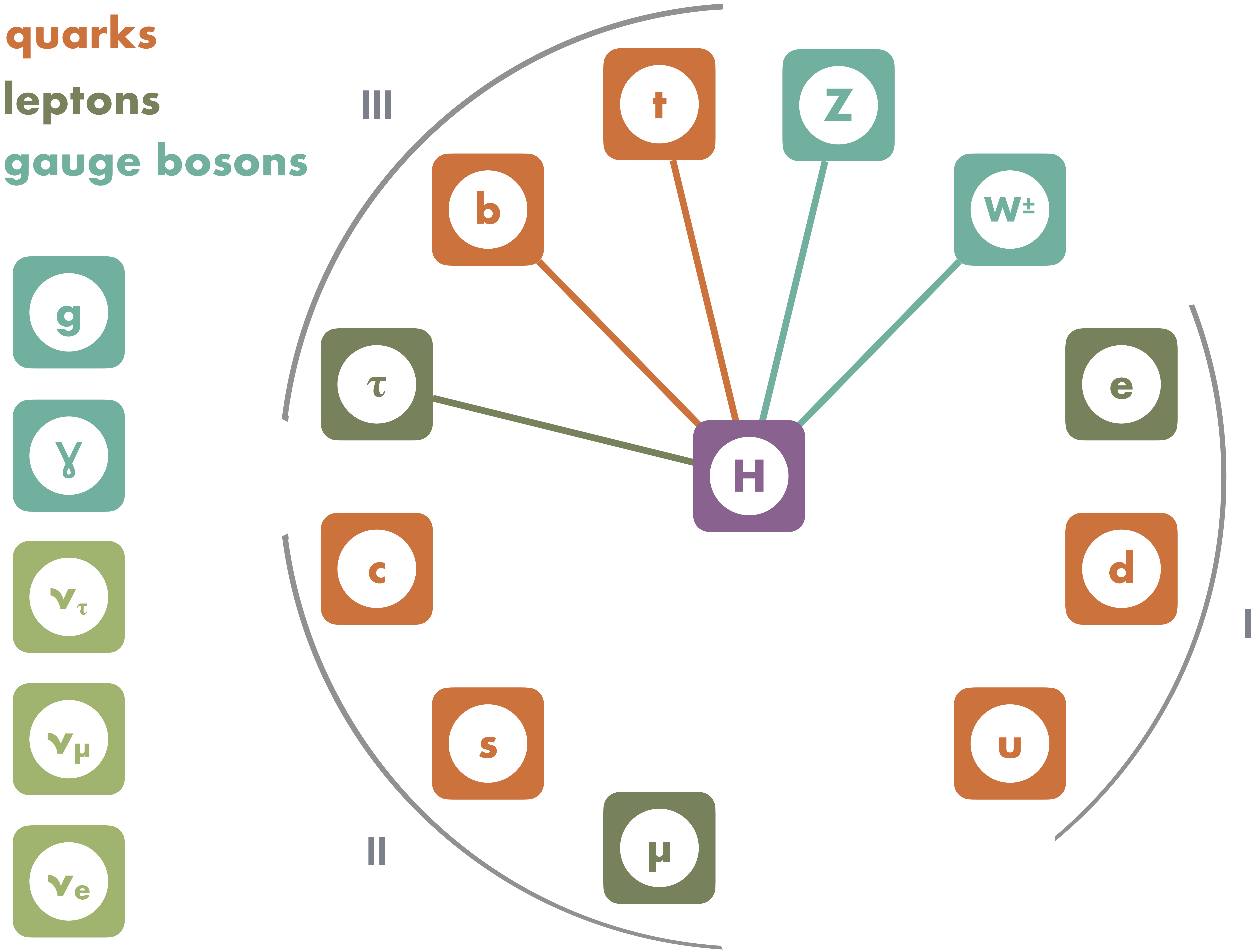


$$\mathscr{L} =$$

$$+ |D_\mu \phi|^2$$ **Gauge Interactions**

~5-10% @ Run 2 LHC



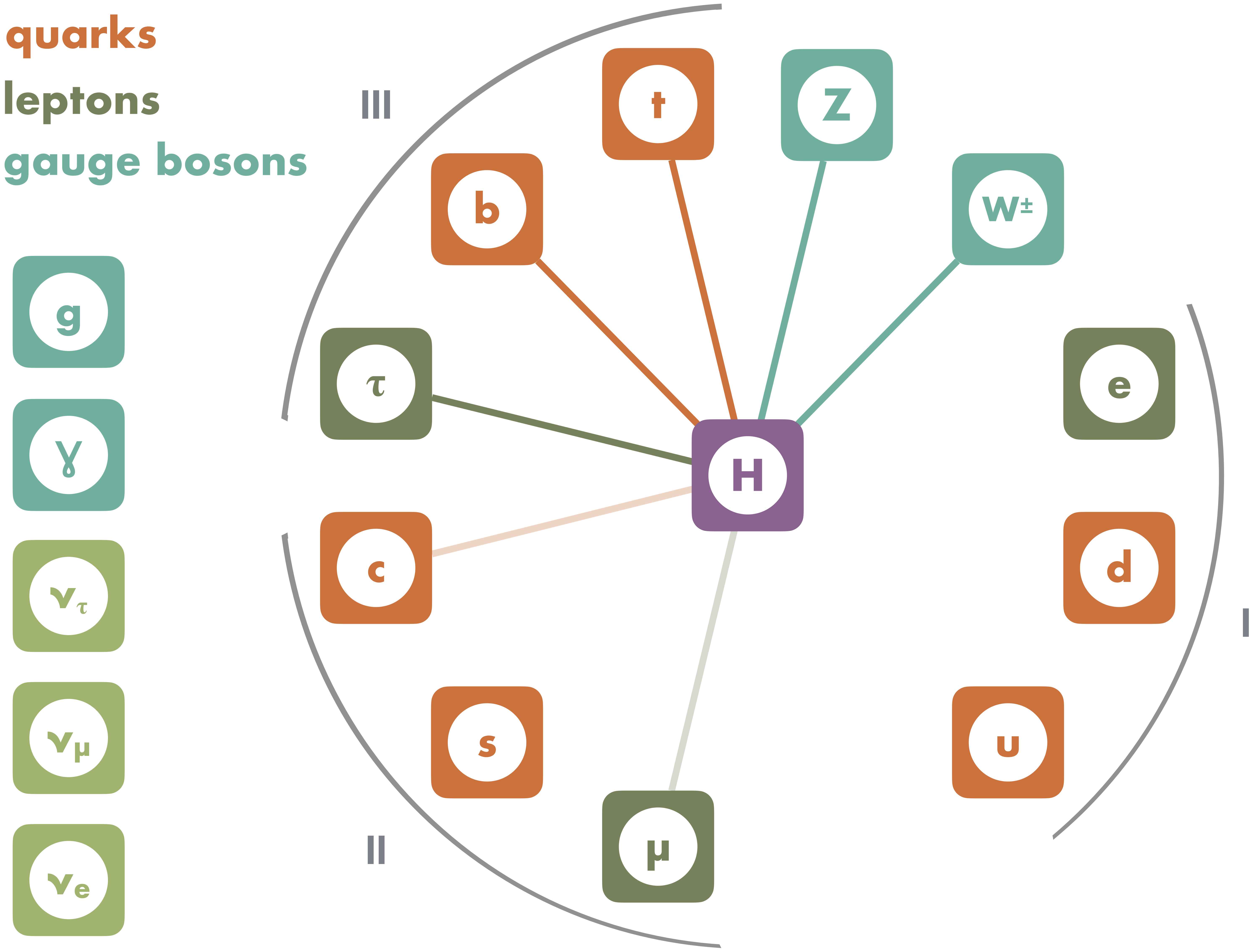
$$\mathscr{L} =$$

$$+\,|D_\mu \phi|^2$$ **Gauge Interactions**

$$+\,\psi_i y_{ij} \psi_j \phi$$ **Yukawa Interactions**

fermion masses ↔ flavour

III ~10-20% @ Run 2 LHC

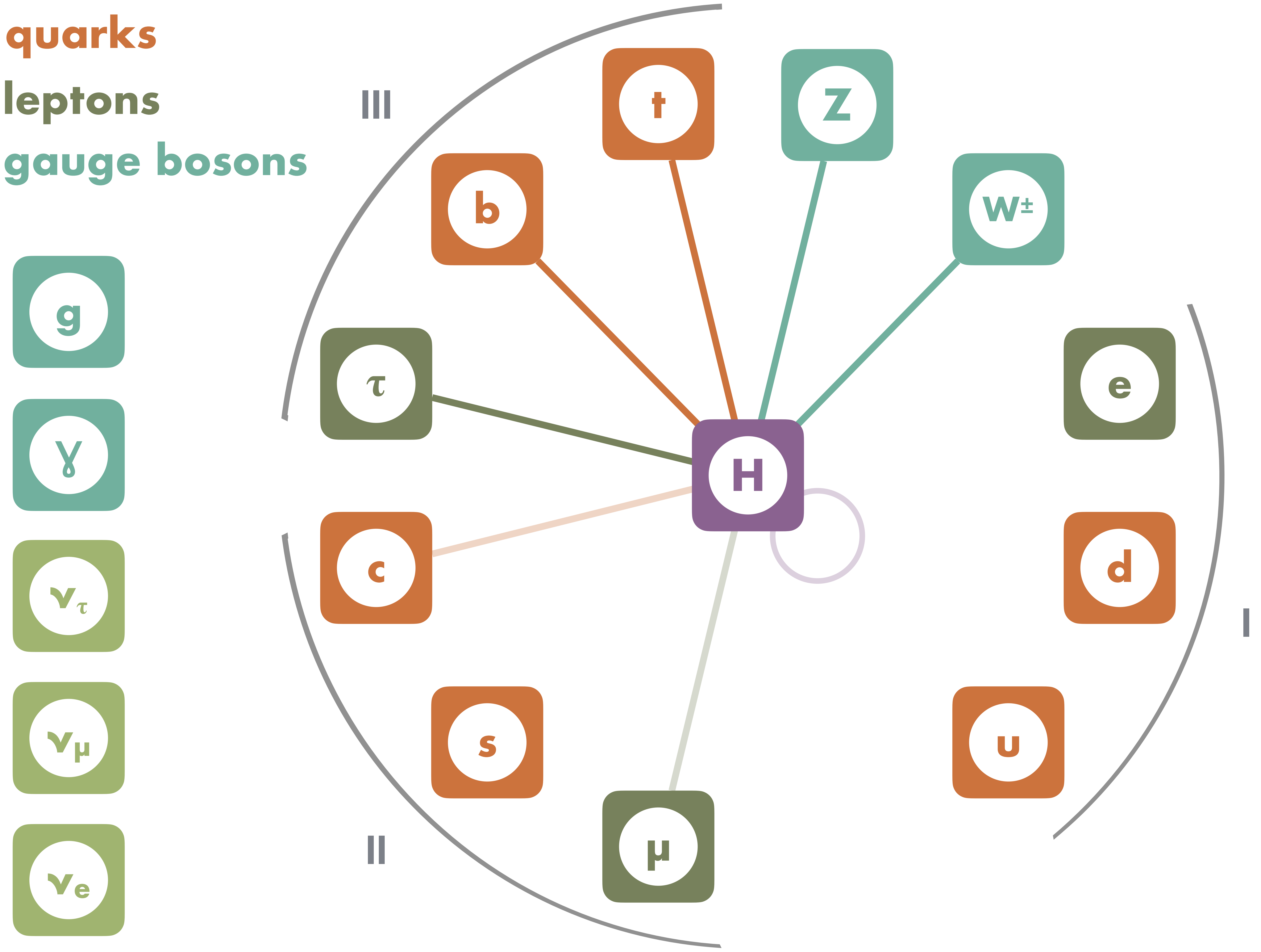


$\mathscr{L} =$

$+ |D_\mu \phi|^2$ **Gauge Interactions**

$+ \psi_i y_{ij} \psi_j \phi$ **Yukawa Interactions**
fermion masses ↔ flavour

III ~10-20% @ Run 2 LHC
II ~60% / first hints



$\mathscr{L} =$

$+|D_\mu\phi|^2$ **Gauge Interactions**

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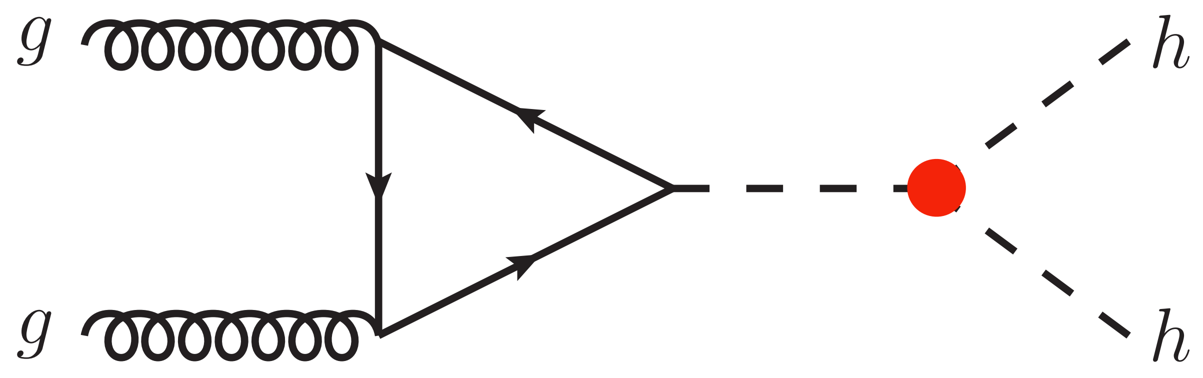
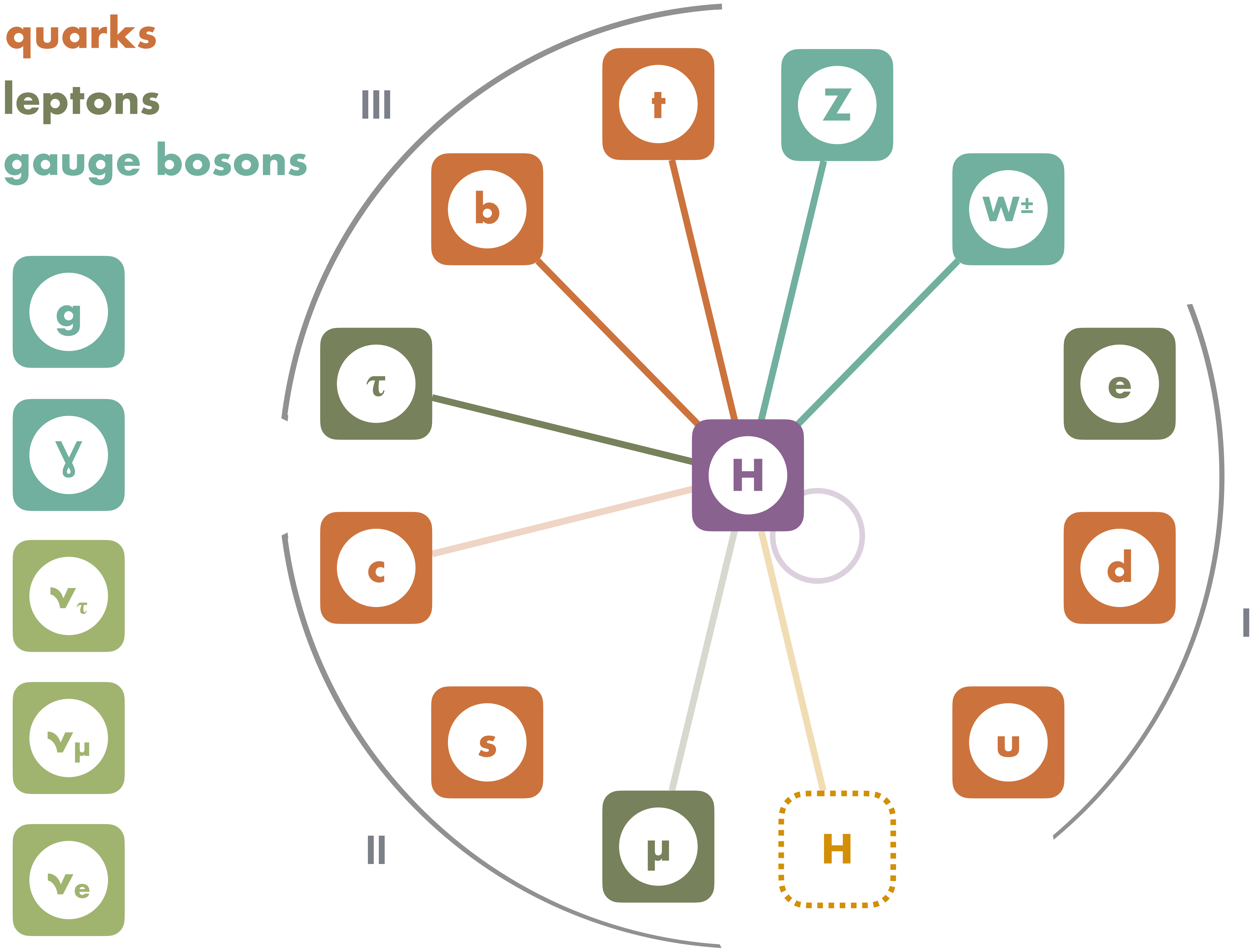
fermion masses ↔ flavour

$-V(\phi)$ **Higgs Potential**
self-coupling ↔ vacuum stability



Direct measurement of the Higgs self-coupling would be a fundamental discovery



$$\mathscr{L} =$$

$+ |D_\mu \phi|^2$ **Gauge Interactions**

$+ \psi_i y_{ij} \psi_j \phi$ **Yukawa Interactions**
fermion masses ↔ flavour

$- V(\phi)$ **Higgs Potential**
self-coupling ↔ vacuum stability

$+ \ldots$ **Surprises**
BSM

Higgs Boson Pair Production

$$\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

EW symmetry breaking

$$\mu^2 = \lambda v^2$$
$$m_H^2 = 2\lambda v^2$$

$$V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4,$$

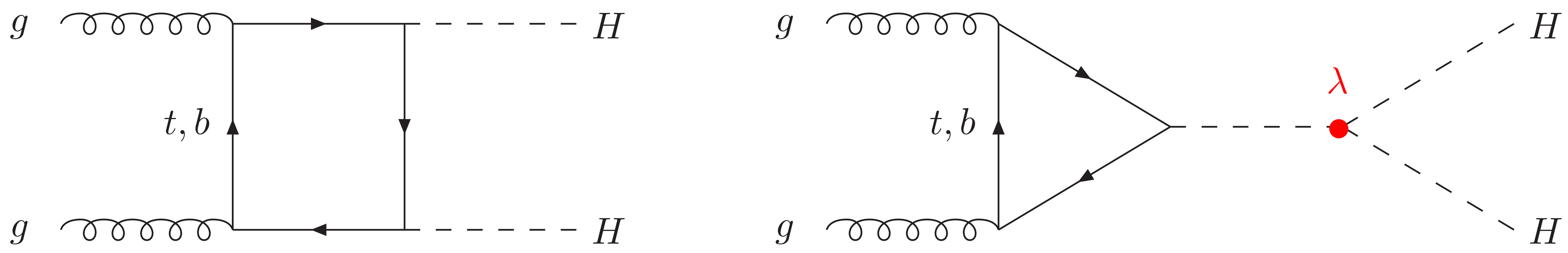
Standard model - self-couplings determined by m_H, v

Experiment - Need measurements to test this

HH Production channels similar to H

Important difference:

$$\sigma(pp \to HH) \sim \frac{\sigma(pp \to H)}{1000}$$



1. Status Summary

Experimental Progress

Experimental Limits (Run 2) @ 95% CL

ATLAS: $-1.2(-1.6) < \kappa_3 < 7.2(7.2)$ $\sigma \leq 2.9\ (2.4) \times \sigma_{SM}$

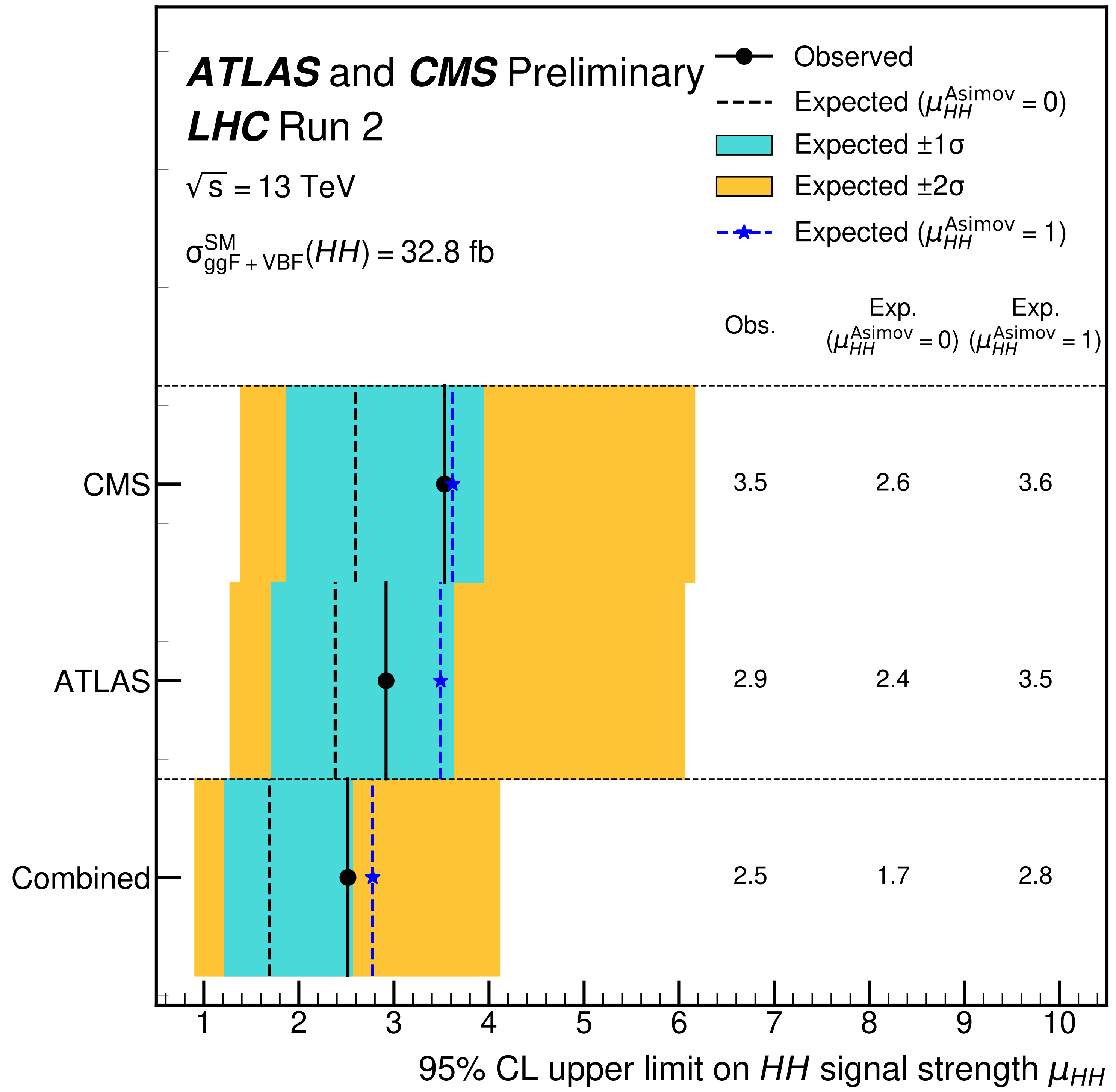
ATLAS CERN-EP-2024-160

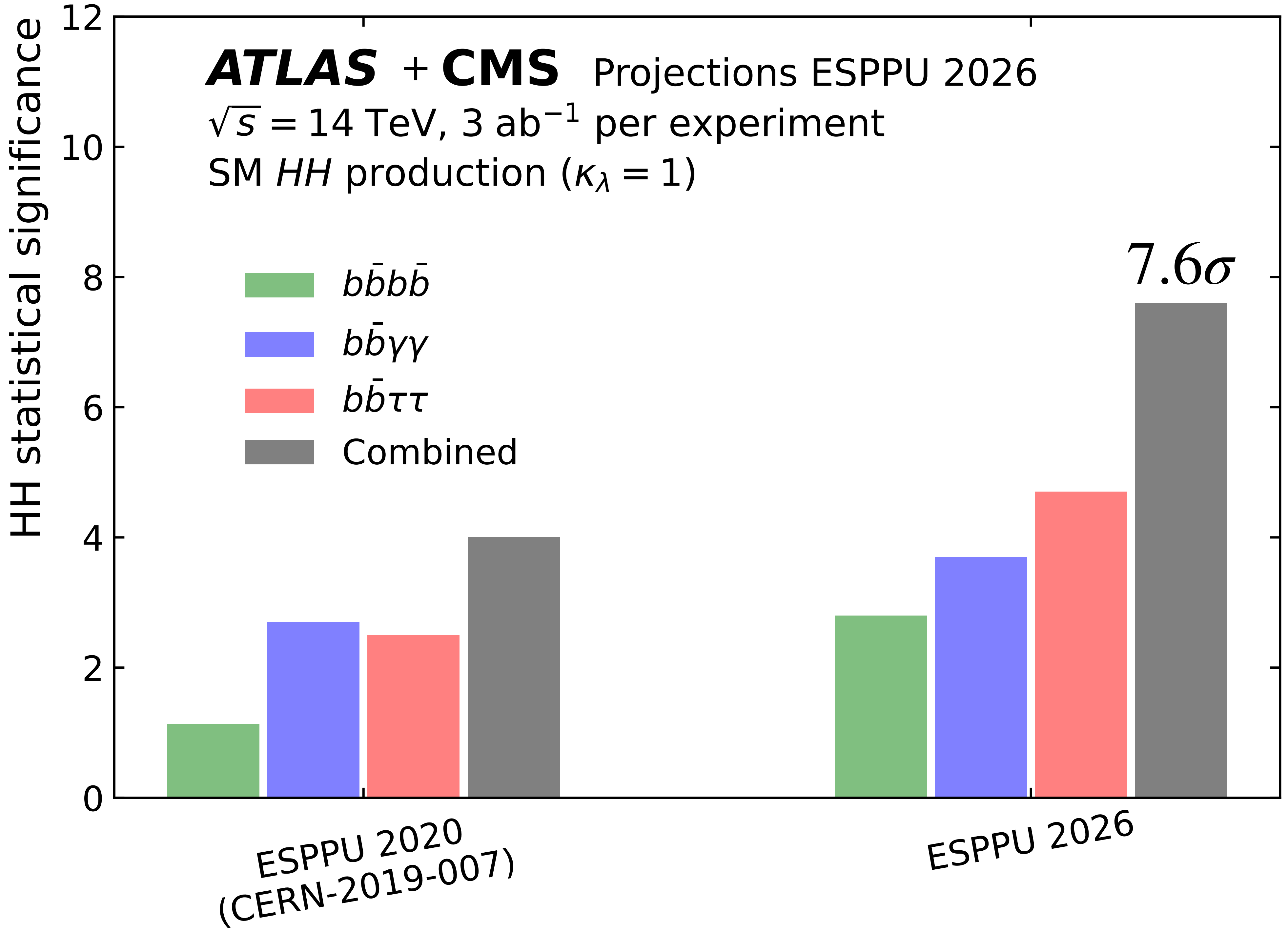
CMS: $-1.39(-1.02) < \kappa_3 < 7.02(7.19)$ $\sigma \leq 3.5\ (2.5) \times \sigma_{SM}$

CMS-PAS-HIG-20-011

A+C: $-0.71(-1.3) < \kappa_3 < 6.1(6.7)$ $\sigma \leq 2.5\ (1.7) \times \sigma_{SM}$

ATLAS-CONF-2025-012





ATL-PHYS-PUB-2025-018 / CMS-HIG-25-002

Projections for HL-LHC (European Strategy 2026)

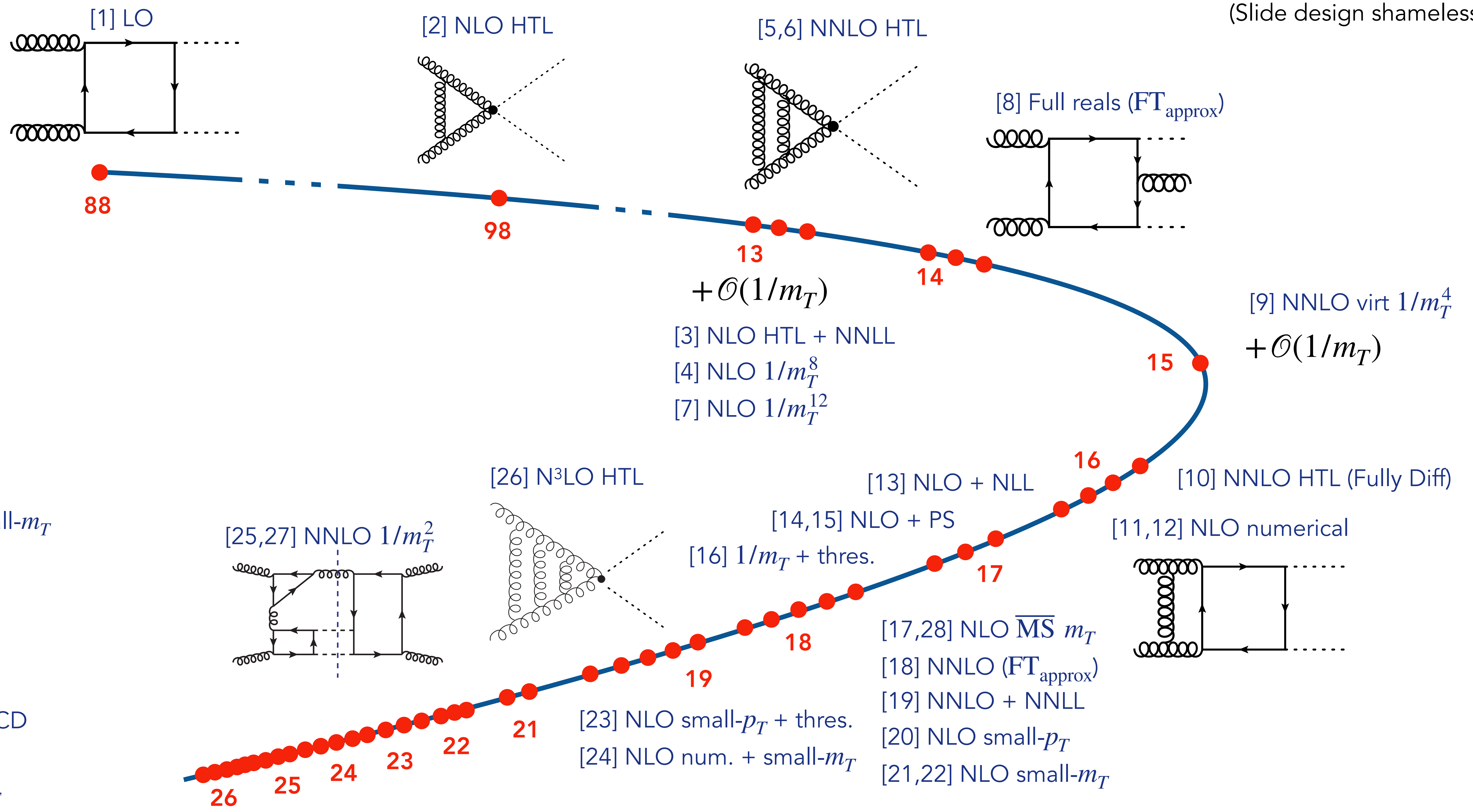
Expected measurement of κ_3 with uncertainty $< 30\,\%$

TH systematics approximately halved in projections

Theory Progress

(Slide design shamelessly stolen from G. Salam)

[29] NLO small-p_T + small-m_T
[30] EW: leading Yukawa vs $\lambda_3^{\rm eff}$
[31] EW: leading Yukawa
[32] N^{3}LO + N^{3}LL + NLO m_T
[33] NLO small-t + small-m_T
[34] NNLO: n_f contrib.
[35] EW: $1/m_T$
[36] NLO comb expansions
[37] NLO EW corrections
[38] NLO + $b\bar{b}\gamma\gamma$ decays
[39] NNLO: reducible
[40] EW: Yukawa/self-coupling
[41] EW: self-coupling
[42] EW: Factorizable
[43] EW: Yukawa/self-coupling small-m_T
[44] NNLO + PS with m_T effects
[45] NNLO: Large-N_c small-p_T
[46] EW: Light quark induced
[47] NLO comb expansions
[48] EW: Top-Yukawa + light-quark
[49] EW: Quark-induced + NLO QCD
[50] N^{3}LO fully diff + NLO m_T
[51] NLO EW corrections, small-m_T



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Mühlleitner, Schlenk, Spira 22; [31] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [32] Ajjath, Shao 22; [33] Davies, Mishima, Schönwald, Steinhauser 23; [34] Davies, Schönwald, Steinhauser 23; [35] Davies, Schönwald, Steinhauser, Zhang 23; [36] Bagnaschi, Degrassi, Gröber 23; [37] Bi, Huang, Huang, Ma Yu 23 [38] Li, Si, Wang, Zhang, Zhao 24; [39] Davies, Schönwald, Steinhauser, Vitti 24; [40] Heinrich, SPJ, Kerner, Stone, Vestner 24 [41] Li, Si, Wang, Zhang, Zhao 24; [42] Davies, Schönwald, Steinhauser, Zhang 24; [43] Davies, Schönwald, Steinhauser, Zhang 25; [44] Alioli, Marinelli, Napoletano; [45] Davies, Schönwald, Steinhauser 25; [46] Bonetti, Rendler, Bobadilla 25; [47] Davies, Schönwald, Stremmer 25; [48] Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira 25; [49] Bonetti, Heinrich, Rendler, Torres Bobadilla 26; [50] Chen, Dai, Tao Li, Li, Shao, Wang 26; [51] Davies, Schönwald, Steinhauser, Zhang 26;

YR4: Total Cross Section & Scale Uncertainty @ 14 TeV

Review of the status of HH shortly after the Yellow Report 4 (~2016-2018)

	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	σ_{N3LO} (fb)
Basic HTL	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$	$37.52^{+5.2\%}_{-7.6\%}$	$38.65^{+0.65\%}_{-2.7\%}$
B-i/proj HTL	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$40.44^{+1.9\%}_{-4.7\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$36.69^{+2.1\%}_{-4.9\%}$	–
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–	–
NLO-i. HTL	–	$32.88^{+13.5\%}_{-12.5\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$39.56^{+0.64\%}_{-2.7\%}$

$\sqrt{s} = 14$ TeV

PDF4LHC15_nlo/nnlo

$m_H = 125$ GeV $\quad m_T = 173$ GeV

$\mu_R = \mu_F = \frac{m_{HH}}{2}$

$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\mathrm{point})$

Chen, Li, Shao, Wang 19, 19; Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18;
de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; Maltoni, Vryonidou, Zaro 14;
Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; Dawson, Dittmaier, Spira 98; Glover, van der Bij 88

If we trust the NLO + N^{m}LO HTL combinations

Scale: $+2.1\,\%\,/-4.9\,\%$ PDF+α_s: $\pm 2.2\,\%$ m_T approx: $\pm 2.7\,\%$ m_T scheme: $+4.0\,\%\,/-18.0\,\%$

LHC HWG HH Twiki

R5: Total Cross Section & Scale Uncertainty @ 14 TeV

Efforts to update the recommendation are almost concluded

Recommendation

$$\sigma(\sqrt{s}) = \sigma_2(\sqrt{s}) \times K_3(\sqrt{s}) \times K_{\rm EW}(\sqrt{s}) \qquad \sigma_2 = \text{NNLO FT}_{\rm approx} \qquad K_3 = \left(\frac{\text{N}^3\text{LO} + \text{N}^3\text{LL HTL}}{\text{NNLO HTL}}\right) \sim 1.03 \qquad K_{\rm EW} = \frac{\sigma_{\rm NLO\ EW}}{\sigma_{\rm LO}} \sim 0.96$$

Uncertainties

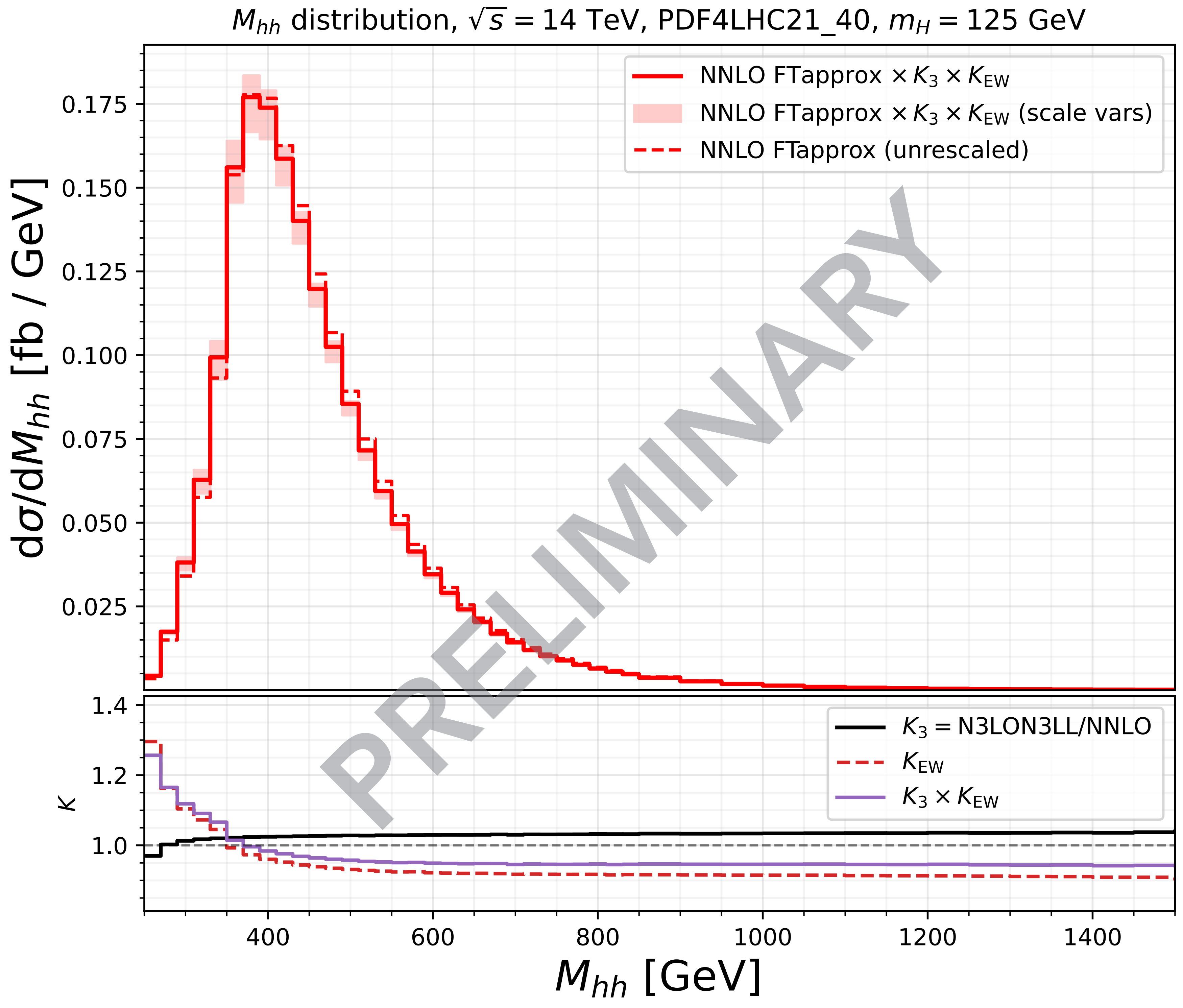
$$\delta_{\rm scale}(\sqrt{s}) = \delta_{\sigma_2\ {\rm scale}}(\sqrt{s})$$

$$\delta_{\rm PDF}(\sqrt{s}) = \sqrt{\left(\delta_{\sigma_2\ {\rm PDF}}(\sqrt{s})\right)^2 + (\delta_{\rm MHOU})^2} \leftarrow \left|\frac{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF} - \sigma^{\rm N^3LO}_{\rm NNLO-PDF}}{\sigma^{\rm N^3LO}_{\rm N^3LO-PDF}}\right| \sim 2.18\ \%$$

$\delta_{m_t} = \pm\ ?$ ← @ NLO $\sim^{+4.0\%}_{-18.0\%}$

Numerics & $r_{\rm cut} \to 0$

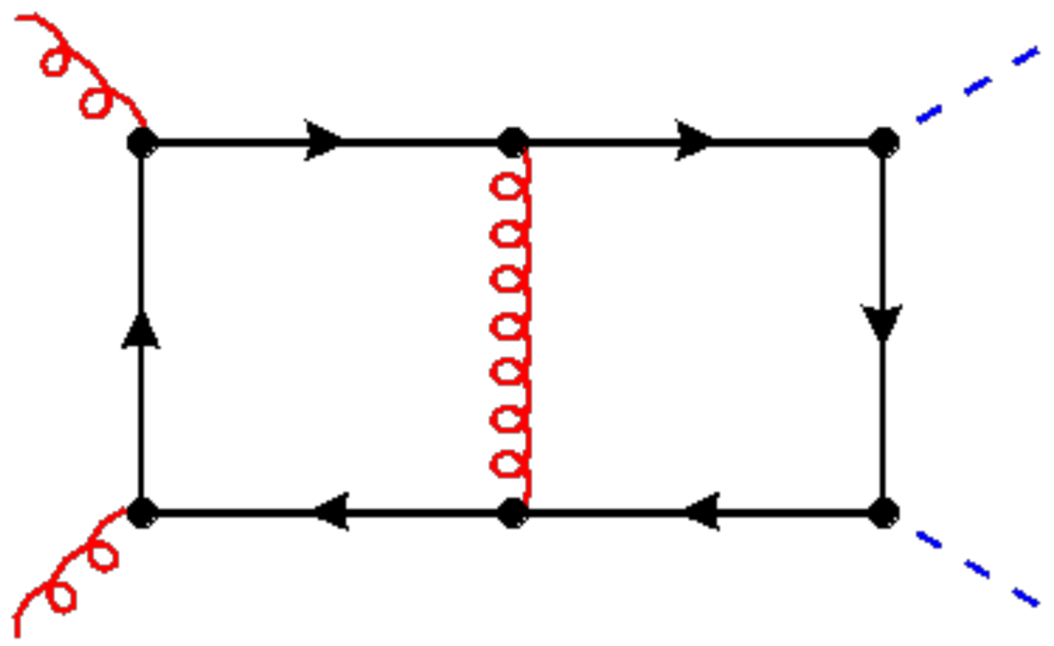
$\sqrt{s}$ [TeV]	$\sigma_{\rm rec.}$ [fb]	± QCD scale unc. [%]	± THU [%]	$\pm\alpha_s$ unc. [%]	± PDF unc. [%]
13	30.40	$^{+2.11}_{-4.98}$	0.14	1.51	2.89
13.6	33.62	$^{+2.07}_{-4.85}$	0.19	1.49	2.90
14	35.81	$^{+2.01}_{-4.78}$	0.21	1.47	2.89



Report 5 contribution on EW corrections in SMEFT/HEFT complete!

LHCHWG-2025-015

2. QCD Corrections



NLO QCD Corrections: ggxy

Davies, Mishima, Schönwald, Steinhauser 23
Davies, Schönwald, Steinhauser, Stremmer 25

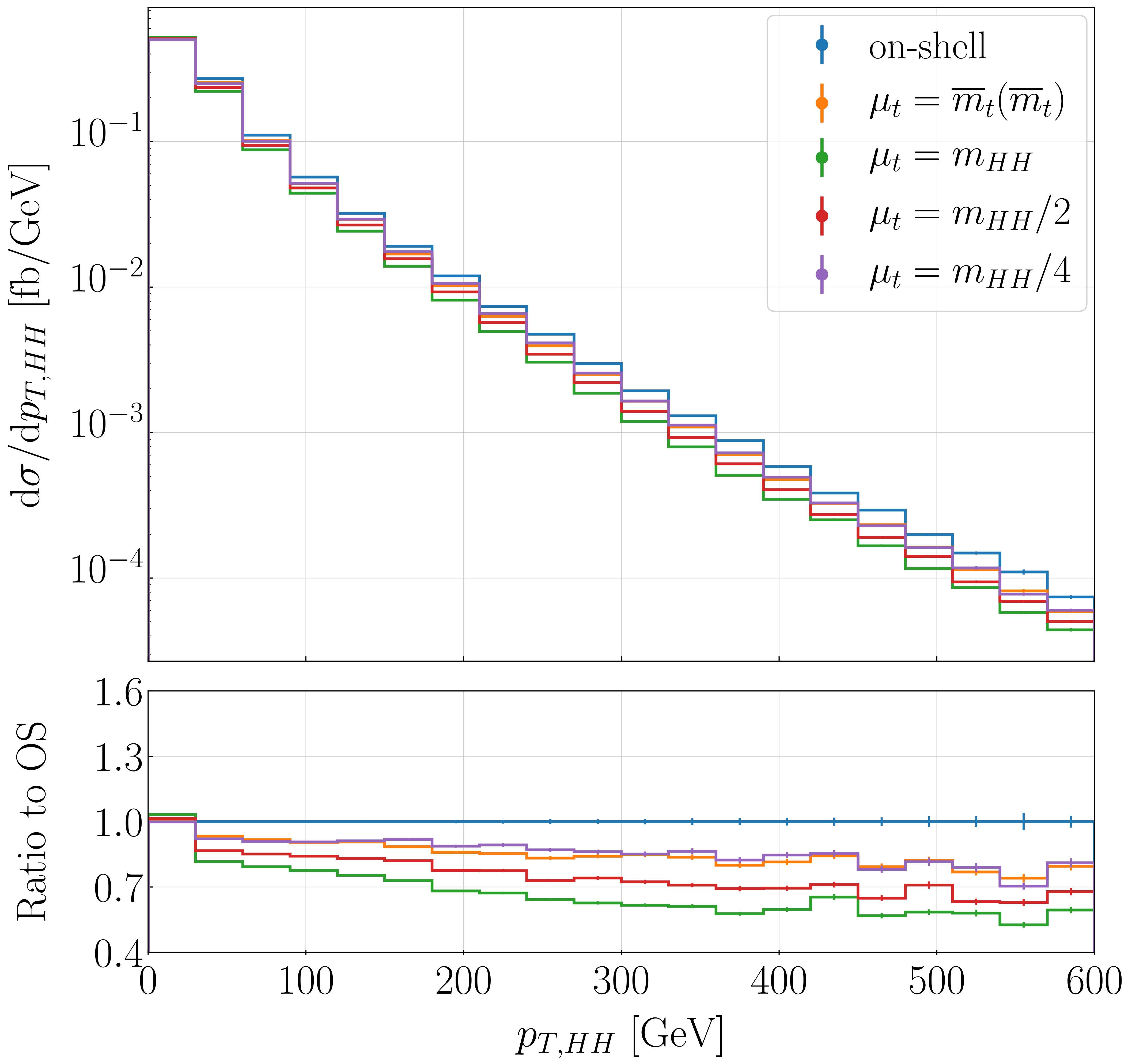
Fast C++ library with NLO QCD results for $gg \to HH$

↪ Combines expansions to cover phase-space: forward limit (to m_H^4 and t^5) + high-energy (to m_H^4 and m_t^{120})

↪ Easily vary m_H, m_t and κ_3 for mass-scheme uncertainties/ BSM scans

↪ Linked to Powheg (ggxy_ggHH), around 4-5x faster than existing code (ggHH) Heinrich, SPJ, Kerner, Luisoni, Vryonidou, (Scyboz) 17, 19;

↪ Validated against fully numeric calculations, yields excellent agreement



Parton Showers: NNLO + PS

Alioli, Billis, Broggio, Gavardi, Kallweit et al 22
Alioli, Marinelli, Napoletano 25

Several NLO+PS results available for some time

↪ MadGraph & POWHEG

↪ Sherpa

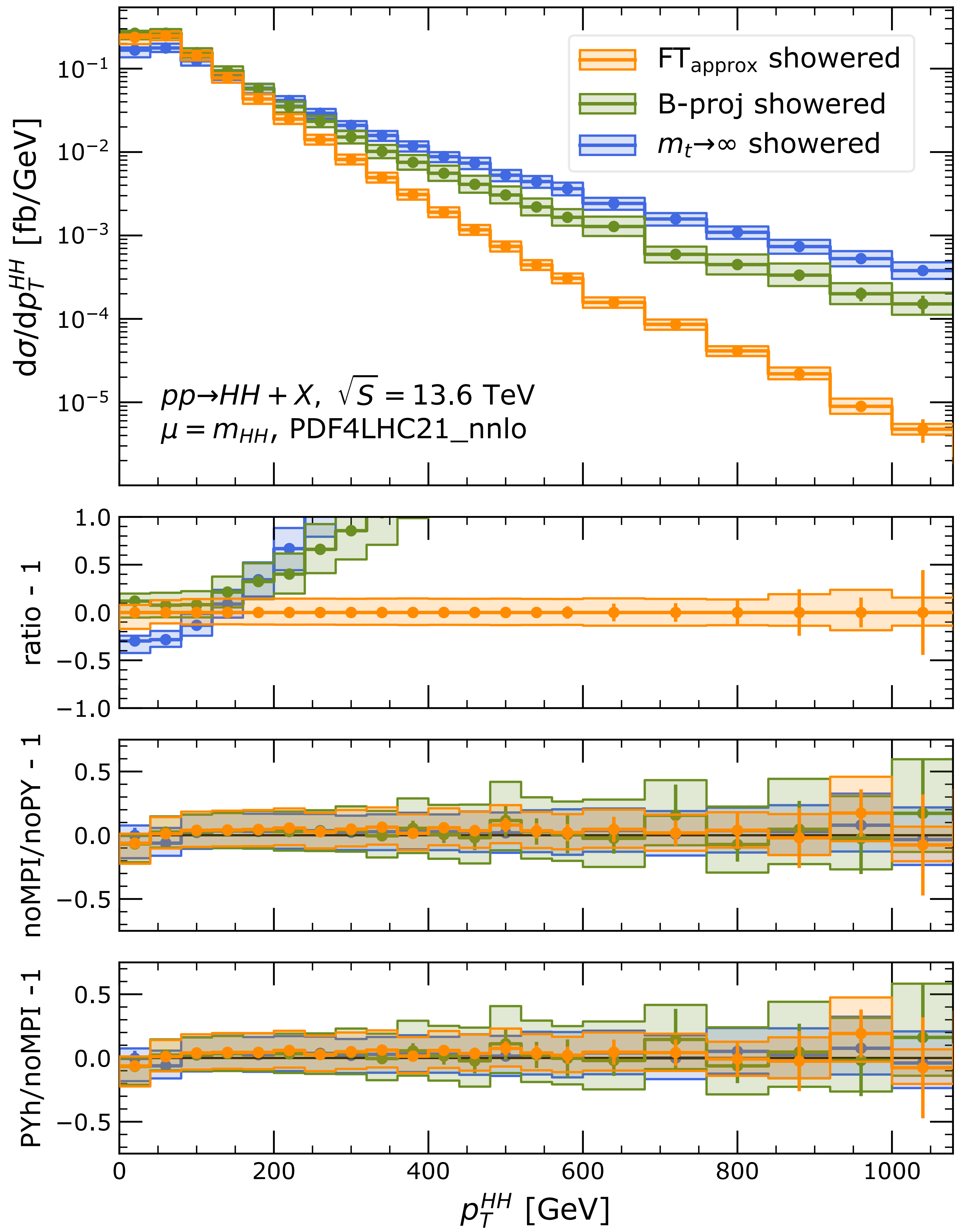
Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; SPJ, Kuttimalai 17;
+κ_3, y_t variations: Heinrich, SPJ, Kerner, Luisoni, Scyboz 19;
Bagnaschi, Degrassi, Gröber 23;
+HEFT: Heinrich, SPJ, Kerner, Scyboz 20
+SMEFT: Heinrich, Lang, Scyboz 22; Heinrich, Lang 23, 24;

Can suffer from very large shower scale uncertainties, especially for LO accurate distributions such as p_T^{HH}

NNLO+PS results including top-mass corrections

↪ Reproduces $\mathrm{FT_{approx}}$ result of MATRIX, adds PYTHIA shower

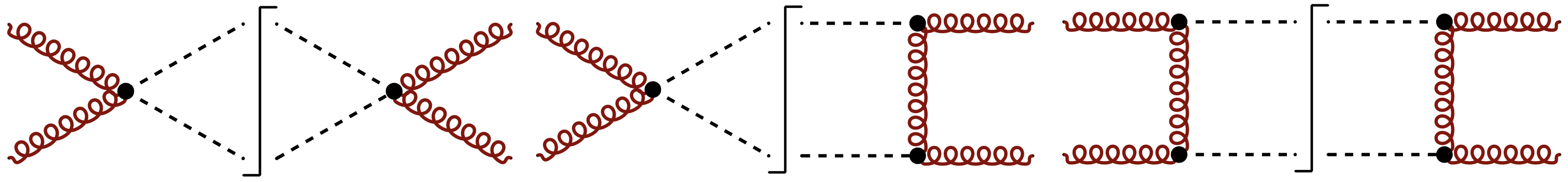
↪ Uses GENEVA framework, adds resummation of τ_0 to NNLL'



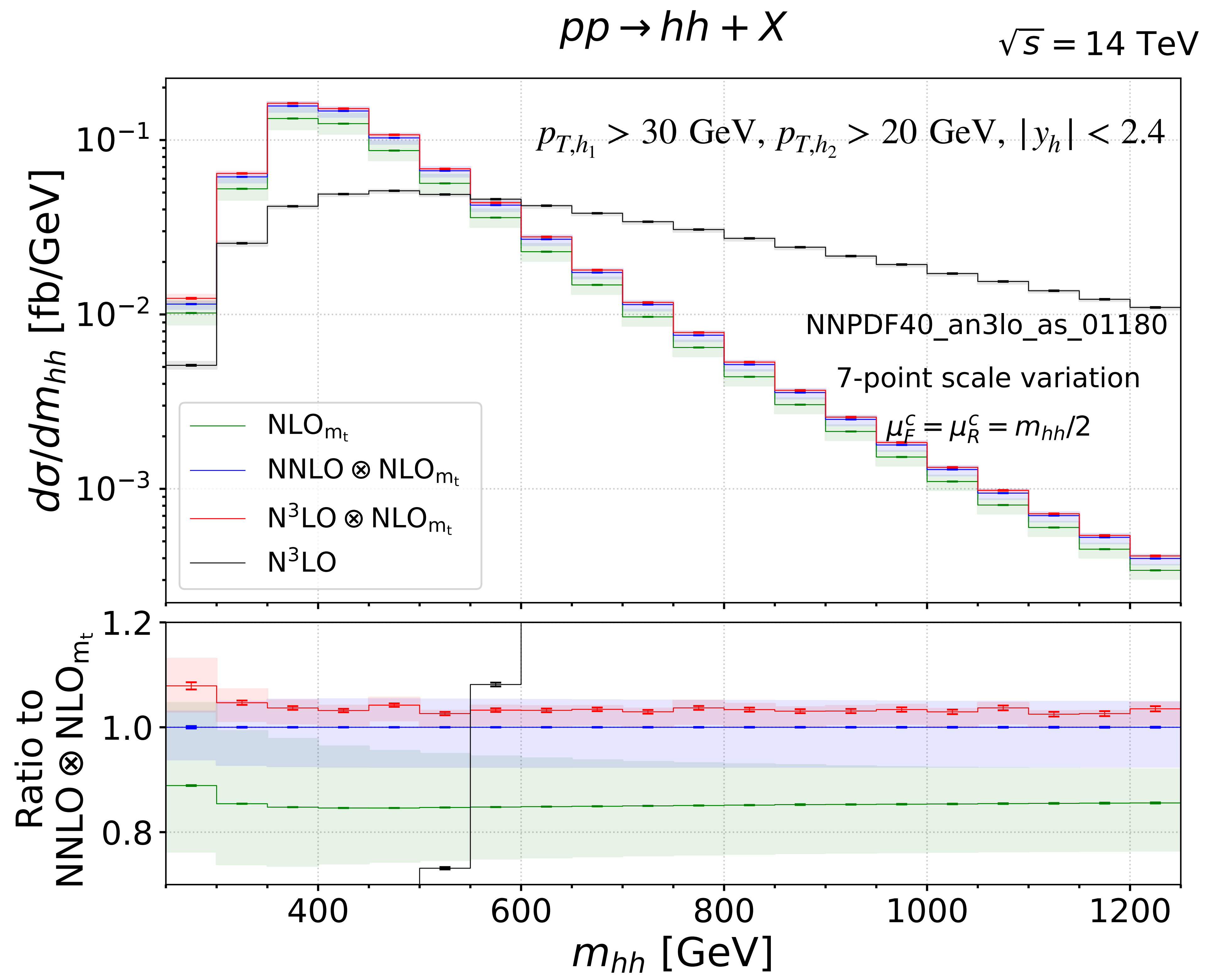
N^3LO HTL + NLO QCD: Fully Differential

Chen, Dai, Tao Li, Li, Shao, Wang 26

Fully differential predictions at N^3LO HTL + NLO QCD, can apply experimental fiducial cuts and produce any distribution



(a) (b) (c)



Consider various scheme for including NLO m_t effects

$$\mathrm{N}^k\mathrm{LO} \oplus \mathrm{NLO}_{m_t} \quad : \quad d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}\oplus\mathrm{NLO}_{m_t}} = d\sigma_{hh}^{\mathrm{NLO}_{m_t}} + d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}} - d\sigma_{hh}^{\mathrm{NLO}} ,$$

$$\mathrm{N}^k\mathrm{LO}_{\mathrm{B-i}} \oplus \mathrm{NLO}_{m_t} \quad : \quad d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}} = d\sigma_{hh}^{\mathrm{NLO}_{m_t}} + \left(d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}} - d\sigma_{hh}^{\mathrm{NLO}}\right) \frac{d\sigma_{hh}^{\mathrm{LO}_{m_t}}}{d\sigma_{hh}^{\mathrm{LO}}} ,$$

$$\mathrm{N}^k\mathrm{LO} \otimes \mathrm{NLO}_{m_t} \quad : \quad d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}\otimes\mathrm{NLO}_{m_t}} = d\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}} \frac{d\sigma_{hh}^{\mathrm{NLO}_{m_t}}}{d\sigma_{hh}^{\mathrm{NLO}}} ,$$

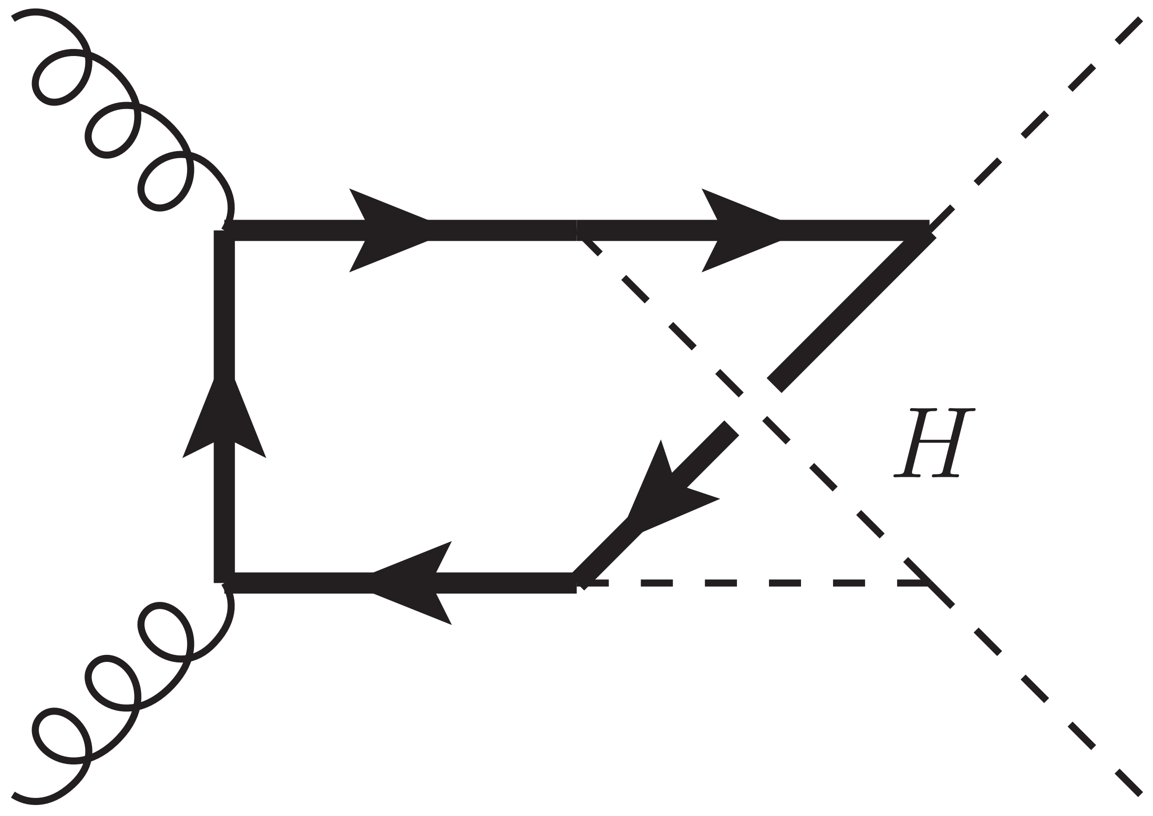
↪ NNLO HTL enhance by 17-20% (depending on comb.)

↪ N3LO HTL enhance by 4% (less dependent on comb.)

Scale uncertainties strongly depend on comb. scheme

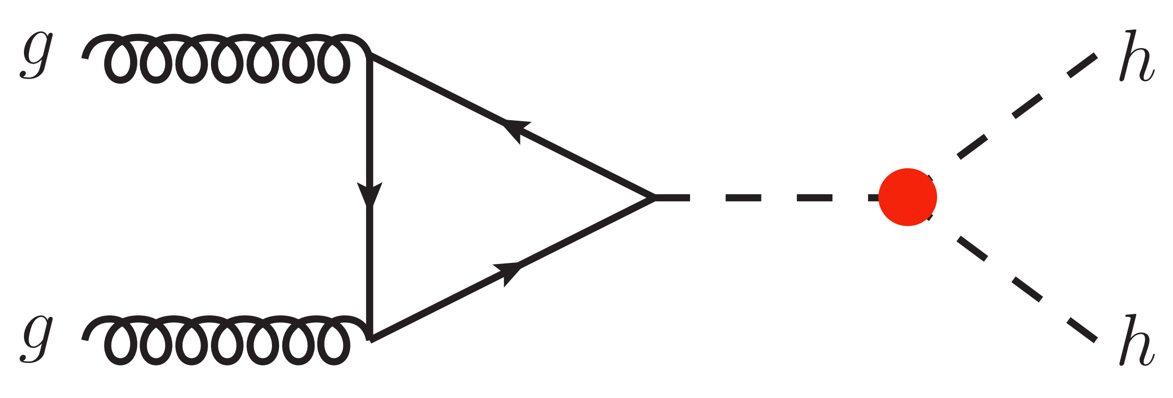
	$\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}\oplus\mathrm{NLO}_{m_t}}$ [fb]	$\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}_{\mathrm{B-i}}\oplus\mathrm{NLO}_{m_t}}$ [fb]	$\sigma_{hh}^{\mathrm{N}^k\mathrm{LO}\otimes\mathrm{NLO}_{m_t}}$ [fb]
$k=1$	$28.44^{+14\%}_{-12\%}$	$28.44^{+14\%}_{-12\%}$	$28.44^{+14\%}_{-12\%}$
$k=2$	$33.30^{+8.1\%}_{-7.2\%}$	$34.03^{+7.6\%}_{-6.8\%}$	$33.40^{+5.2\%}_{-7.3\%}$
$k=3$	$34.56(4)^{+6.8\%}_{-8.0\%}$	$35.47(4)^{+6.6\%}_{-9.2\%}$	$34.68(4)^{+1.4\%}_{-2.9\%}$

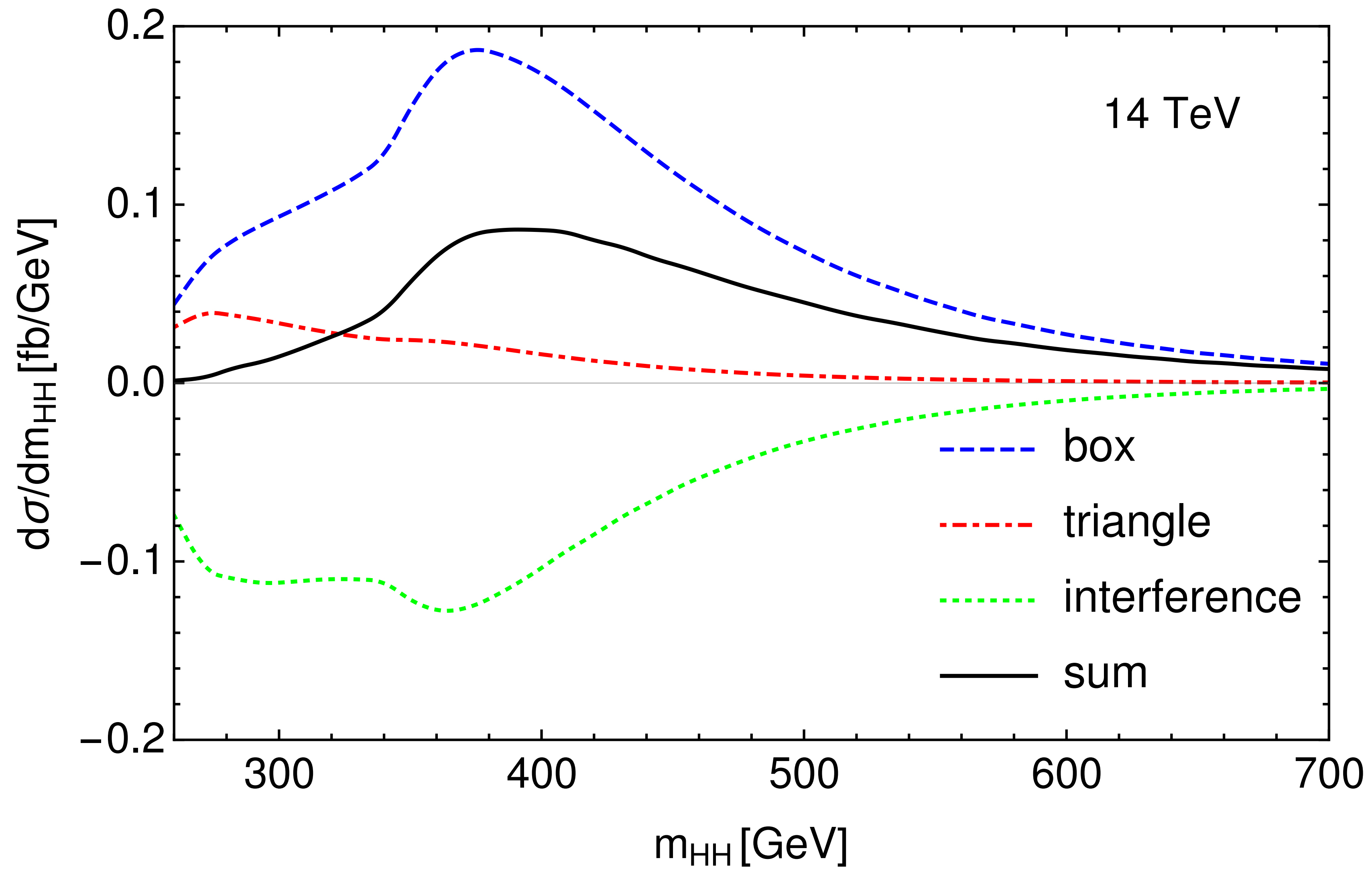
3. Electroweak Corrections



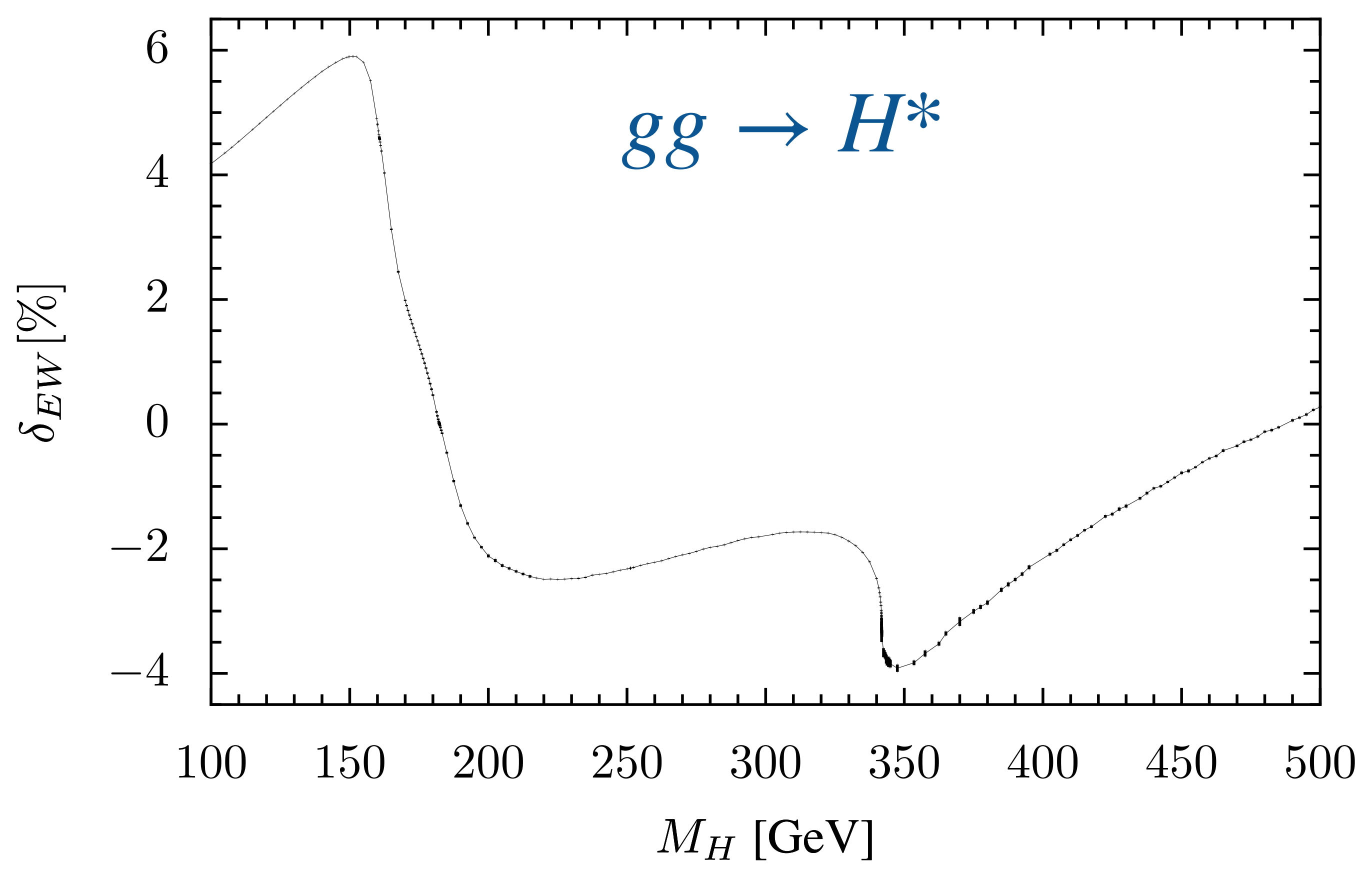
Electroweak Corrections to $gg \to HH$

$pp \to HH$ has a strong interference between the 2 possible LO topologies





Different m_{HH} behaviour of triangle/box heavily exploited by experiments for extracting g_3



Electroweak Corrections to $gg \to H^*$ are $\pm 5\,\%$ and **very dependent on** m_H

Actis, Passarino, Sturm, Uccirati 08

Interesting to explore the impact of EW corrections on HH where they can spoil the delicate cancellations happening at LO

Bestiary

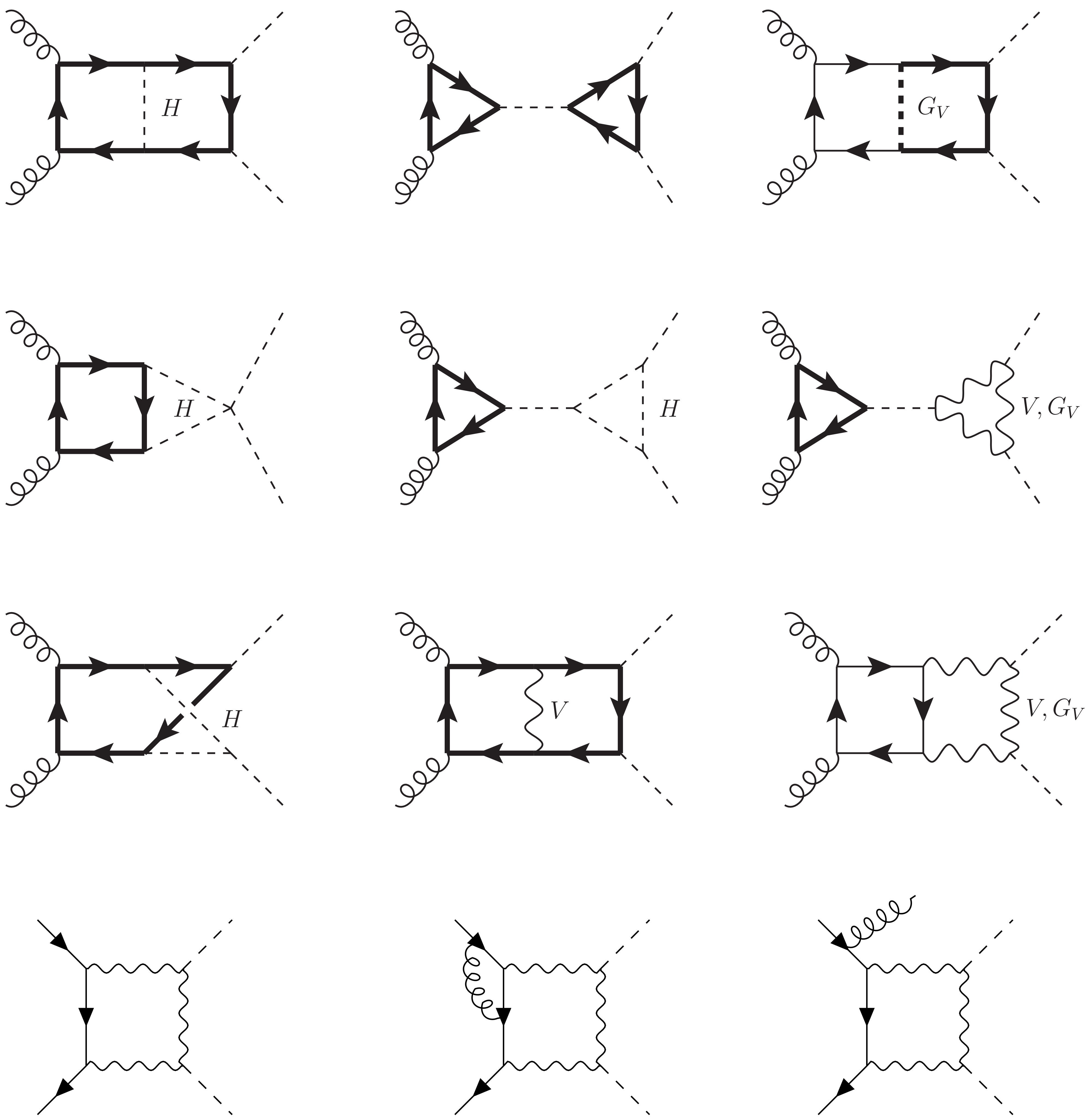
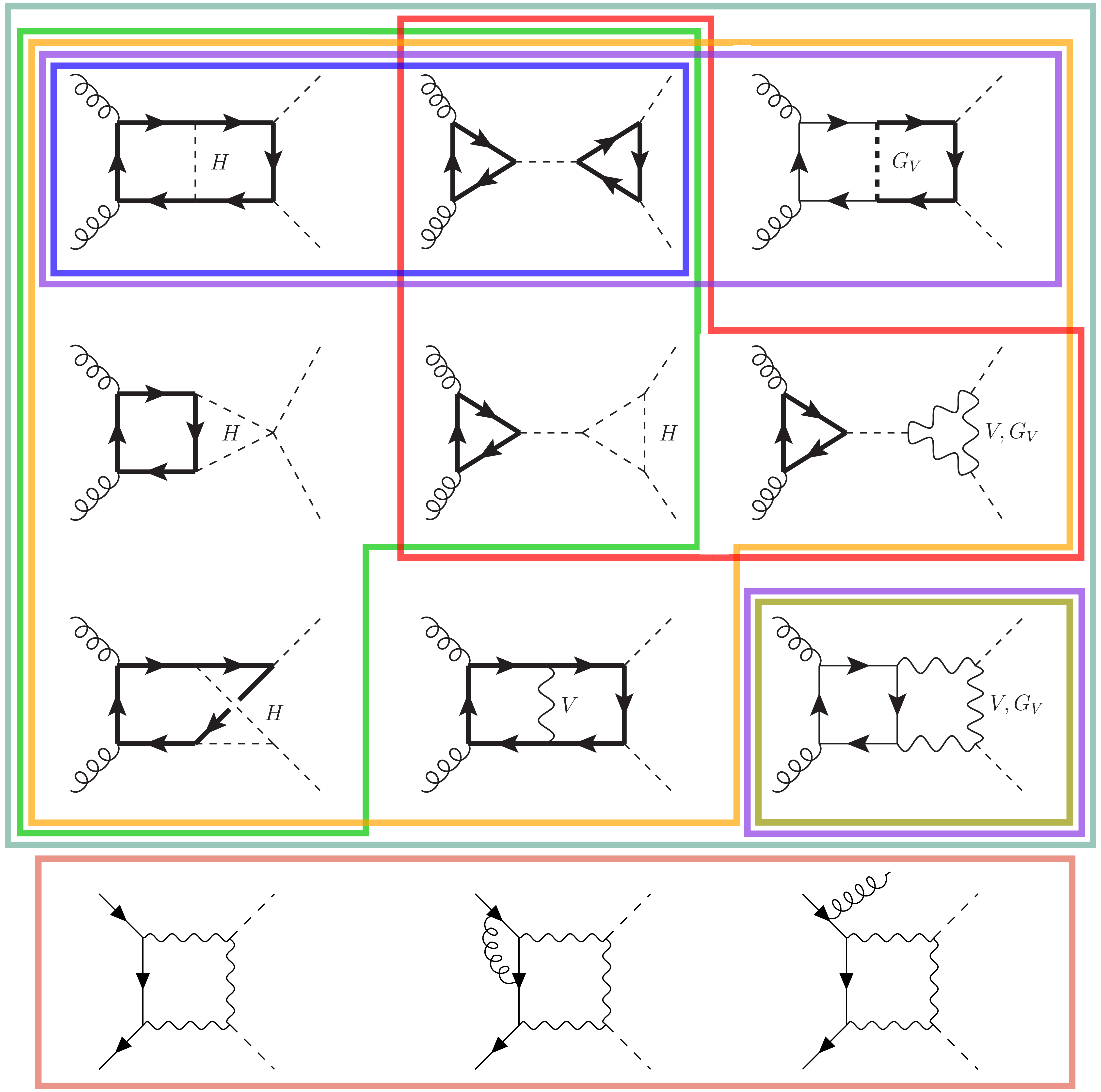


Figure adapted from Marco Bonetti (LHCHWG R5) + Philip Rendler (LHCHWG 25)

Difficult due to the many masses present

$$m_T, m_H, m_W, m_Z$$
$$+s, t$$

Tackled by several groups using an array of techniques!



Small m_T limit, $m_T \to \infty$, full m_T via numerical integration

Davies, Mishima, Schönwald, Steinhauser, Zhang 22; Mühlleitner, Schlenk, Spira 22; Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira 25

Full m_T by numerically solving DEQs, boundary AMFlow

Bi, Huang, Huang, Ma, Yu 23

Large-m_T expansion

Davies, Schönwald, Steinhauser, Zhang 23

Analytic (factorisable contribution)

Mühlleitner, Schlenk, Spira 22; Davies, Schönwald, Steinhauser, Zhang 24

Full m_T via sector decomposition, Small-m_T expansion

Heinrich, SPJ, Kerner, Stone, Vestner 24; Davies, Schönwald, Steinhauser, Zhang 25;

Analytic + series expansion of DEQs, numerical integration

Bonetti, Rendler, Bobadilla 25; Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira 25;

Analytic + series expansion of DEQs

Bonetti, Heinrich, Rendler, Bobadilla 26

Figure adapted from Marco Bonetti (LHCHWG R5) + Philip Rendler (LHCHWG 25)

Gluon-Induced Corrections

Bi, Huang, Huang, Ma, Yu 23

Complete NLO EW corrections to $gg \to HH$

Extremely Challenging Calculation

Fully analytic result beyond reach

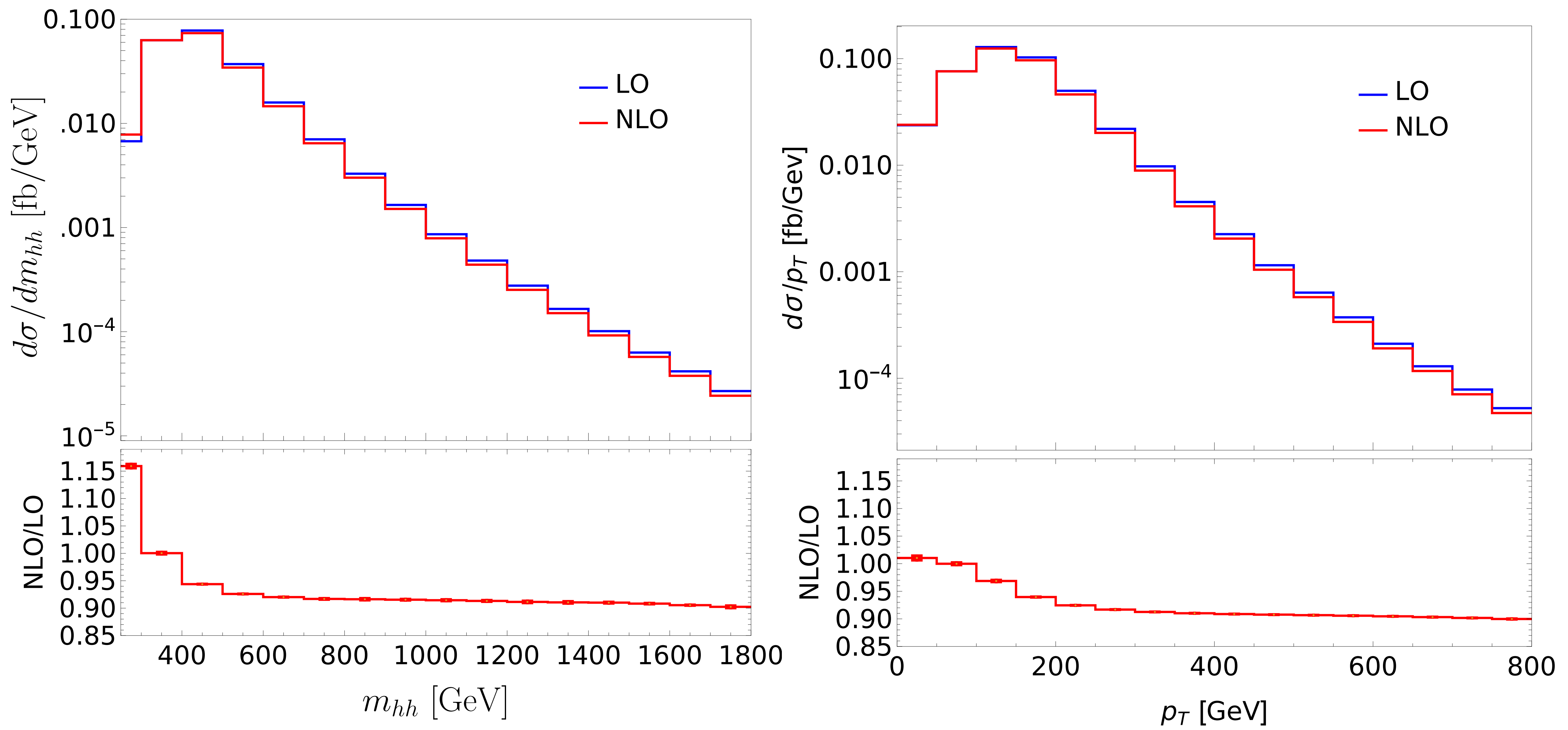
$$\frac{m_H^2}{m_t^2} = \frac{12}{23}, \quad \frac{m_Z^2}{m_t^2} = \frac{23}{83}, \quad \frac{m_W^2}{m_t^2} = \frac{14}{65}, \quad \epsilon = \pm\frac{1}{1000}$$

Renormalise all masses on-shell

Reduce integrals with Blade + FiniteFlow — Guan, Liu, Ma, Wu 25; Peraro 19;

Compute boundary conditions with AMFlow — Liu, Ma, (Wang) (17), 22, 22

Evolve using series solutions of DEQs



Results

K-factor ~ 0.96, very stable with respect to μ_R, μ_F variations

Significant ($\pm 10\,\%$) distortions to m_{HH} and p_T

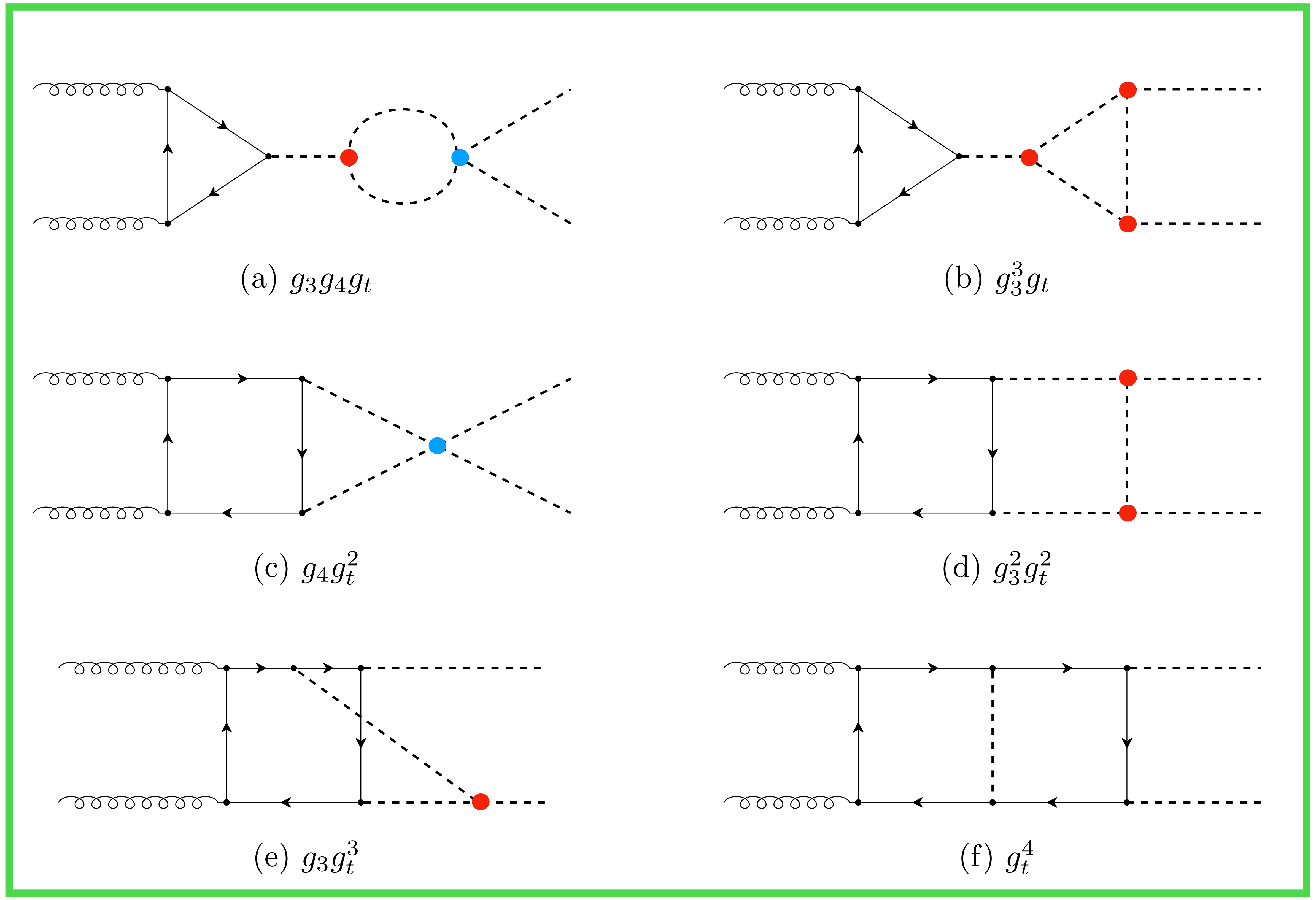
Corrections reduce dependence on EW input scheme

μ	$m_{hh}/2$	$\sqrt{p_T^2 + m_h^2}$	m_h
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)

schemes	G_μ	α_0	α_{m_Z}
LO	19.96	18.84	21.28
NLO	19.12	19.19	19.03
$\mathcal{K}$-factor	0.958	1.019	0.894

Yukawa and Self-Coupling Corrections

Heinrich, SPJ, Kerner, Stone, Vestner 24



Calculation neglects diagrams containing vector bosons and/or Goldstone bosons (fixed to unitary gauge)

Retain **complete control and generality** in all couplings and diagram types (1PI/1PR)

$$g_{t,0} \equiv \frac{m_{T,0}}{v_0}\,, \qquad g_{3,0} \equiv \frac{3m_{H,0}^2}{v_0}\,, \qquad g_{4,0} \equiv \frac{3m_{H,0}^2}{v_0^2}\,.$$

↪ Couplings can be varied for experimental limit setting
↪ Can adapt calculation for EFTs/BSM scenarios

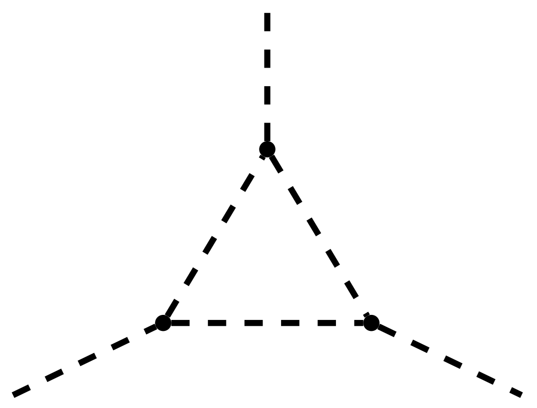
Result

Fully symbolic (s, t, m_T, m_H) reduction to basis of 494 finite d-factorising* master integrals
Up to 11 coupled master integrals within a single sector
Keep exact m_T, m_H dependence, evaluate master integrals numerically using pySecDec Heinrich, SPJ, Kerner, Magerya, Olsson, Schlenk 23

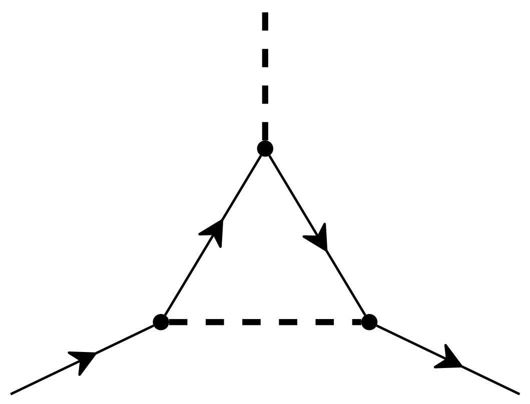
Interpretation

Heinrich, SPJ, Kerner, Stone, Vestner 24

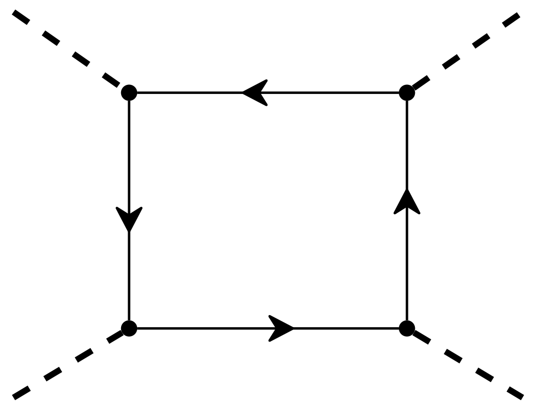
Can fix vev counterterm using any vertex…



$$\delta_v^{g_3}(g_t, g_3, g_4)|_{\mathrm{UV}} = -\frac{1}{32\pi^2 g_3 m_H^4 \epsilon}\left[g_3 g_4 m_H^4 + 8 g_4 g_t m_H^2 m_T^3 N_c - 8 g_3^2 g_t m_T^3 N_c + 2 g_3 g_t^2 m_H^2 \left(m_H^2 + 12 m_T^2\right) N_c - 48 g_t^3 m_H^4 m_T N_c\right]$$



$$\delta_v^{g_t}(g_t, g_3, g_4)|_{\mathrm{UV}} = -\frac{g_3 g_t m_H^2 + 2 g_t^2 m_T \left(m_H^2 - 4 m_T^2\right) N_c}{32\pi^2 m_H^2 m_T \epsilon}$$



$$\delta_v^{g_4}(g_t, g_3, g_4)|_{\mathrm{UV}} = -\frac{2 g_t g_4 N_c (g_t(m_H^4 + 6 m_H^2 m_T^2) - 2 g_3 m_T^3) + g_4^2 m_H^4 - 24 g_t^4 m_H^4 N_c}{32\pi^2 g_4 m_H^4 \epsilon}$$

Get different $1/\epsilon$ in renormalisation constants depending on the vertex!

SM limit →

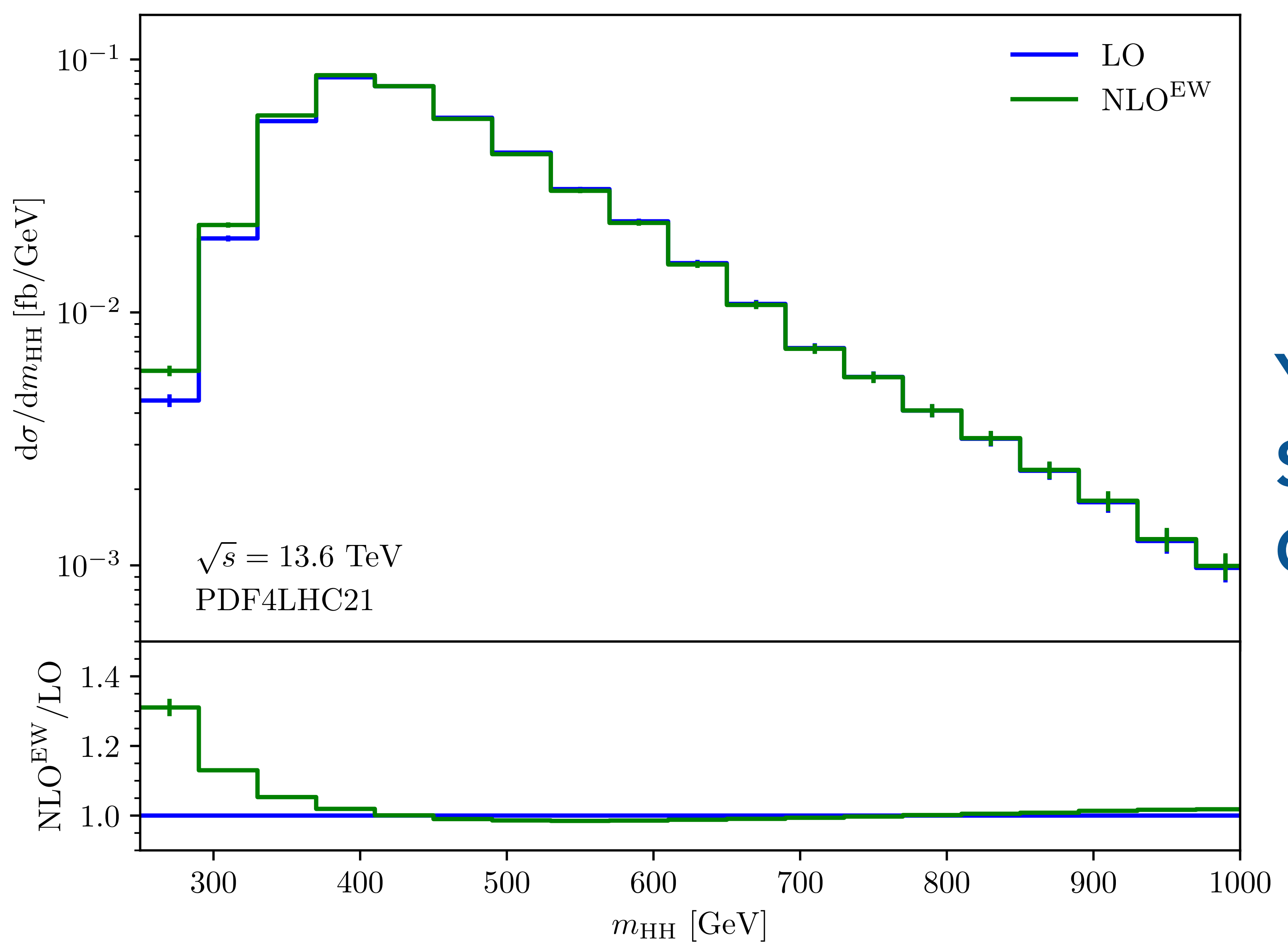
$$\delta_v^{g_t}\left(\frac{m_T}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} = \delta_v^{g_3}\left(\frac{m_T}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} = \delta_v^{g_4}\left(\frac{m_T}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} \stackrel{!}{=} \delta_v|_{\mathrm{UV}}$$

All OK

Ok to talk about κ_3, κ_4 when considering QCD corrections, EW corrections make this much more delicate

Bizoń, Ulrich Haisch, Luca Rottoli 18; Bizoń, Haisch, Rottoli, Gillis, Moser, Windischhofer 24; Tao Li, Si, Wang, Zhang, Zhao 24

NLO Electroweak Corrections to $gg \to HH$



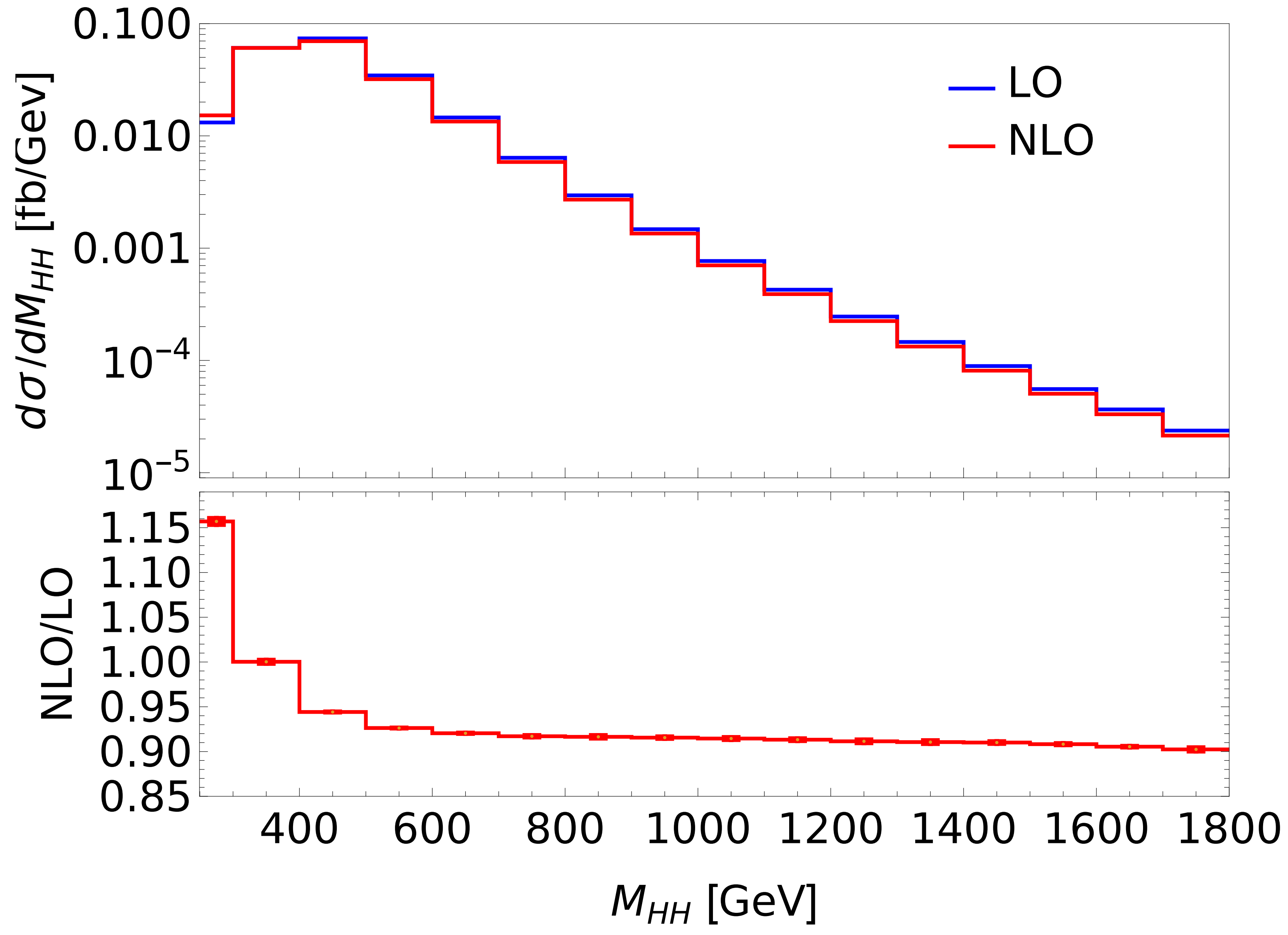
Yukawa/ Self-coupling Corrections

↪ +1% on total cross section

↪ +30% near production threshold*

Can be adapted for EFT analyses

Heinrich, SPJ, Kerner, Stone, Vestner 24



Full Electroweak Corrections

↪ -4% on total cross section

↪ +15% near production threshold*

↪ -10% at high energy (Sudakov-like)

Bi, Huang, Huang, Ma, Yu 23

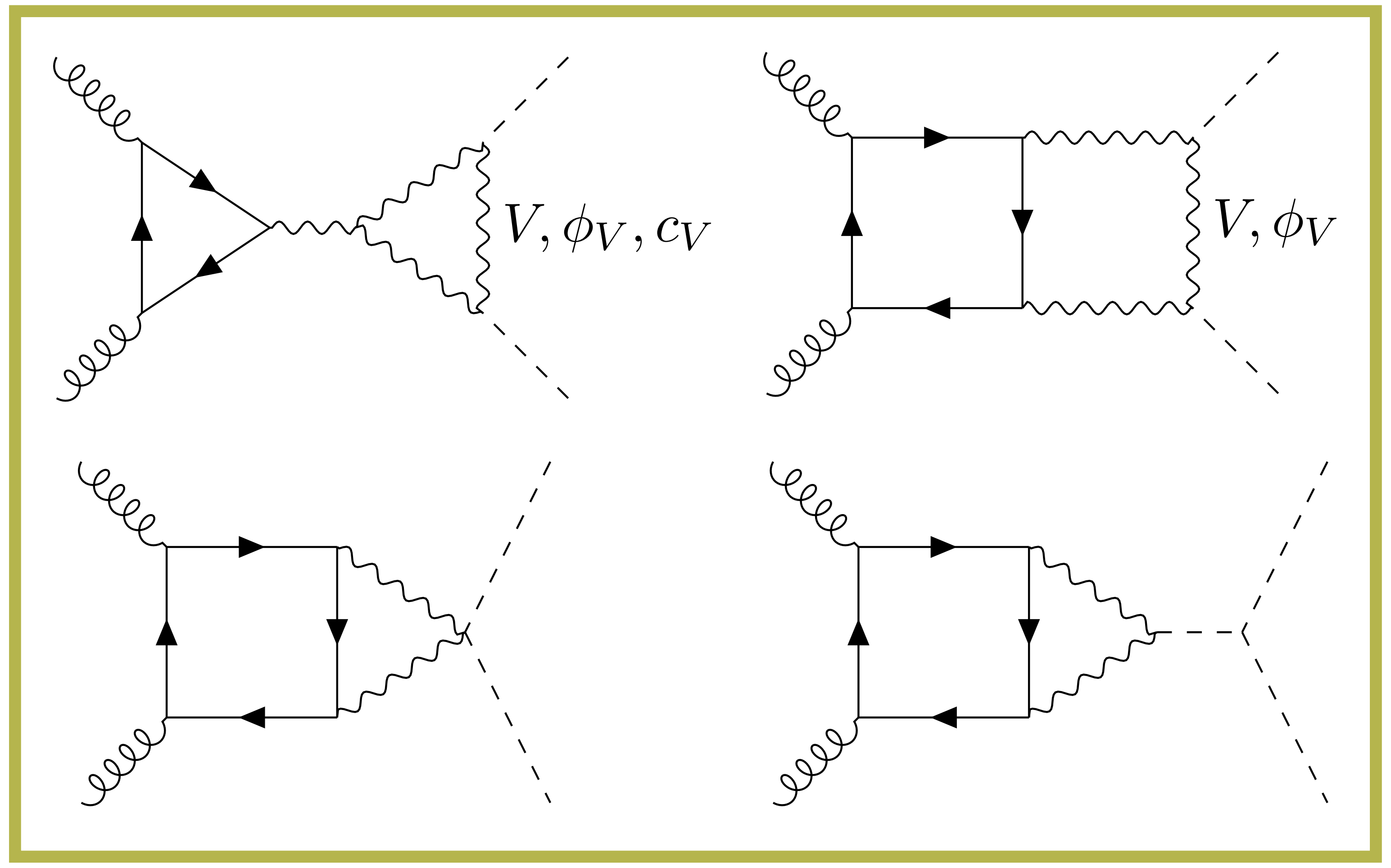
Computing all $gg \to HH$ diagrams (retaining control of all couplings) + combine with $q\bar{q} \to HH$ + QCD corrections

Reduction to 1291 master integrals complete, extensive cross-checks & renormalisation (in progress…)

Bonetti, Heinrich, SPJ, Kerner, Rendler, Stone, Vestner (WIP)

Light-Quark Mediated

Bonetti, Rendler, Torres Bobadilla 25
Bonetti, Heinrich, SPJ, Kerner, Rendler, Stone, Vestner 25



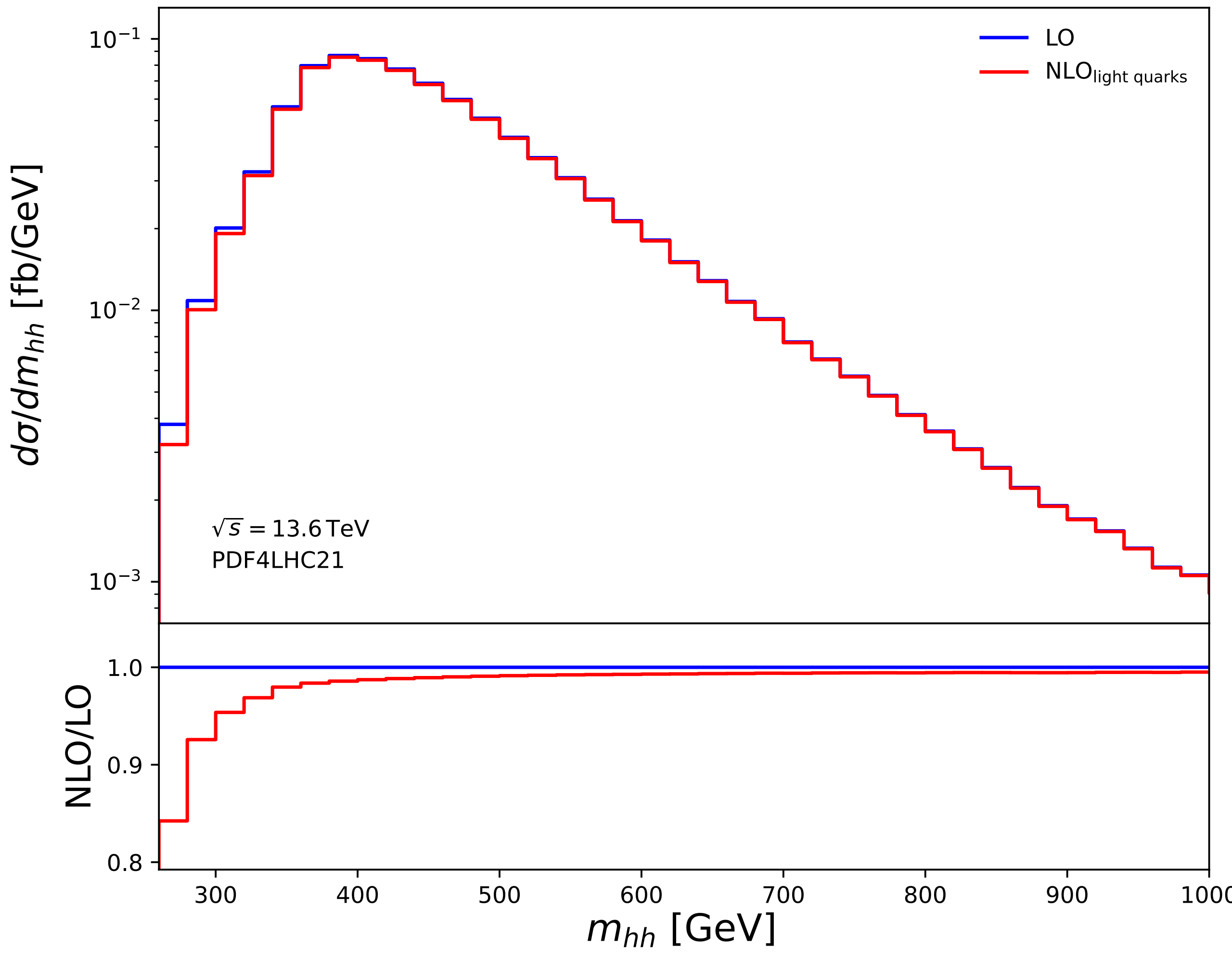
Light-Quark mediated corrections to $gg \to HH$

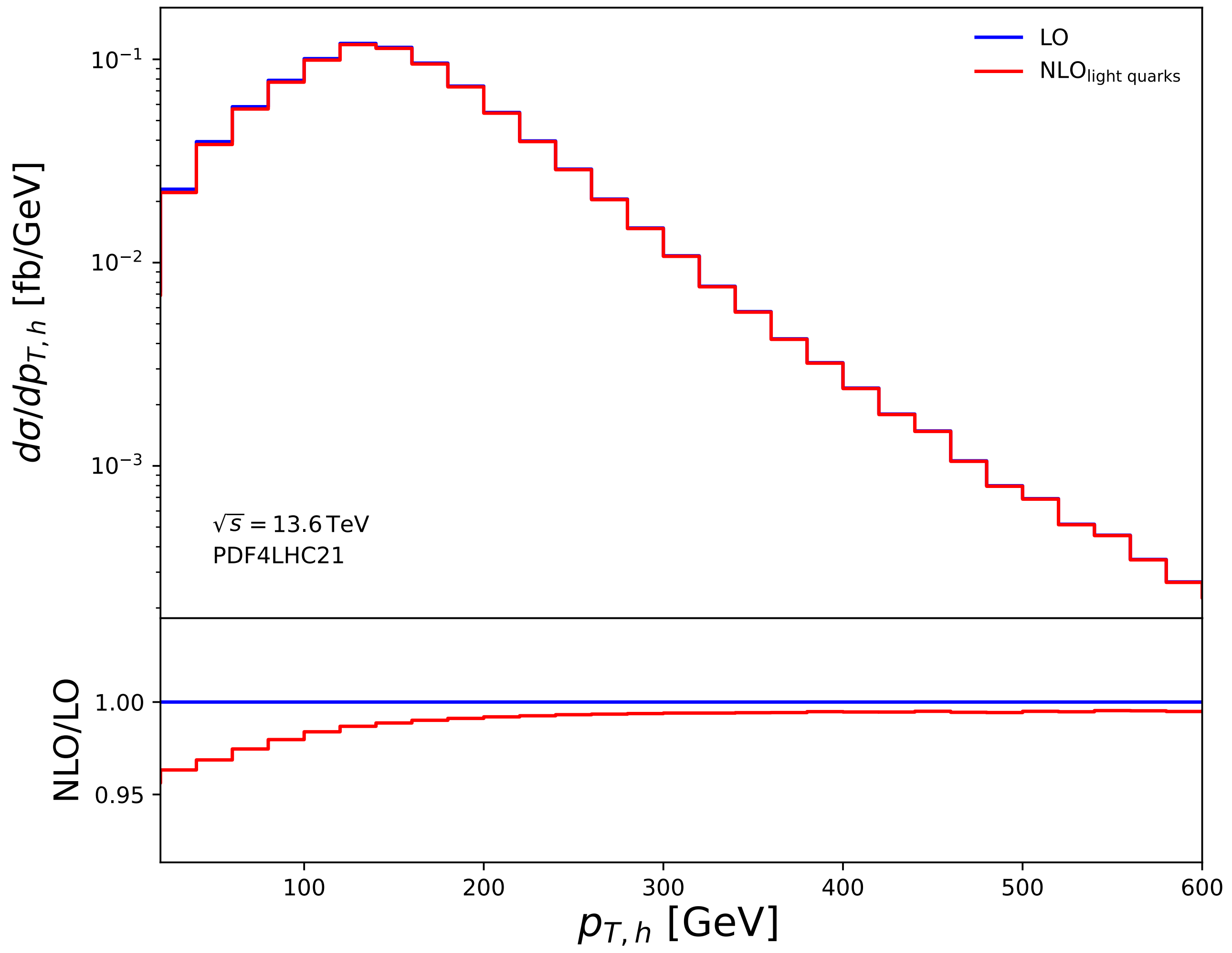
Canonical differential equations obtained (boundary values @ $s, t, u \ll m_V^2$) evolve boundary values using series-solution via DiffExp Moriello 19; Hidding 20;

Grids produced and implemented in Powheg

Small effect at total cross-section level

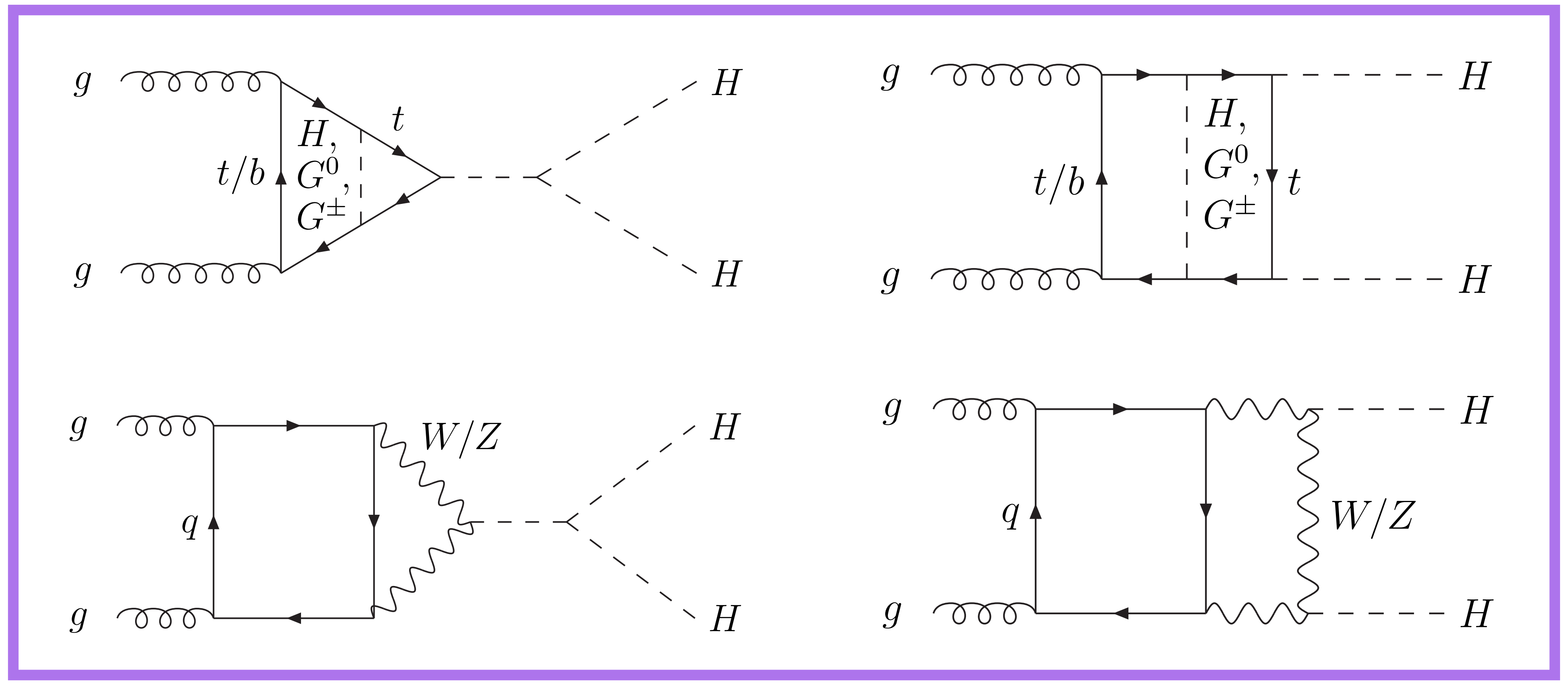
Suppresses m_{HH} and p_T significantly close to production threshold (depends on binning)

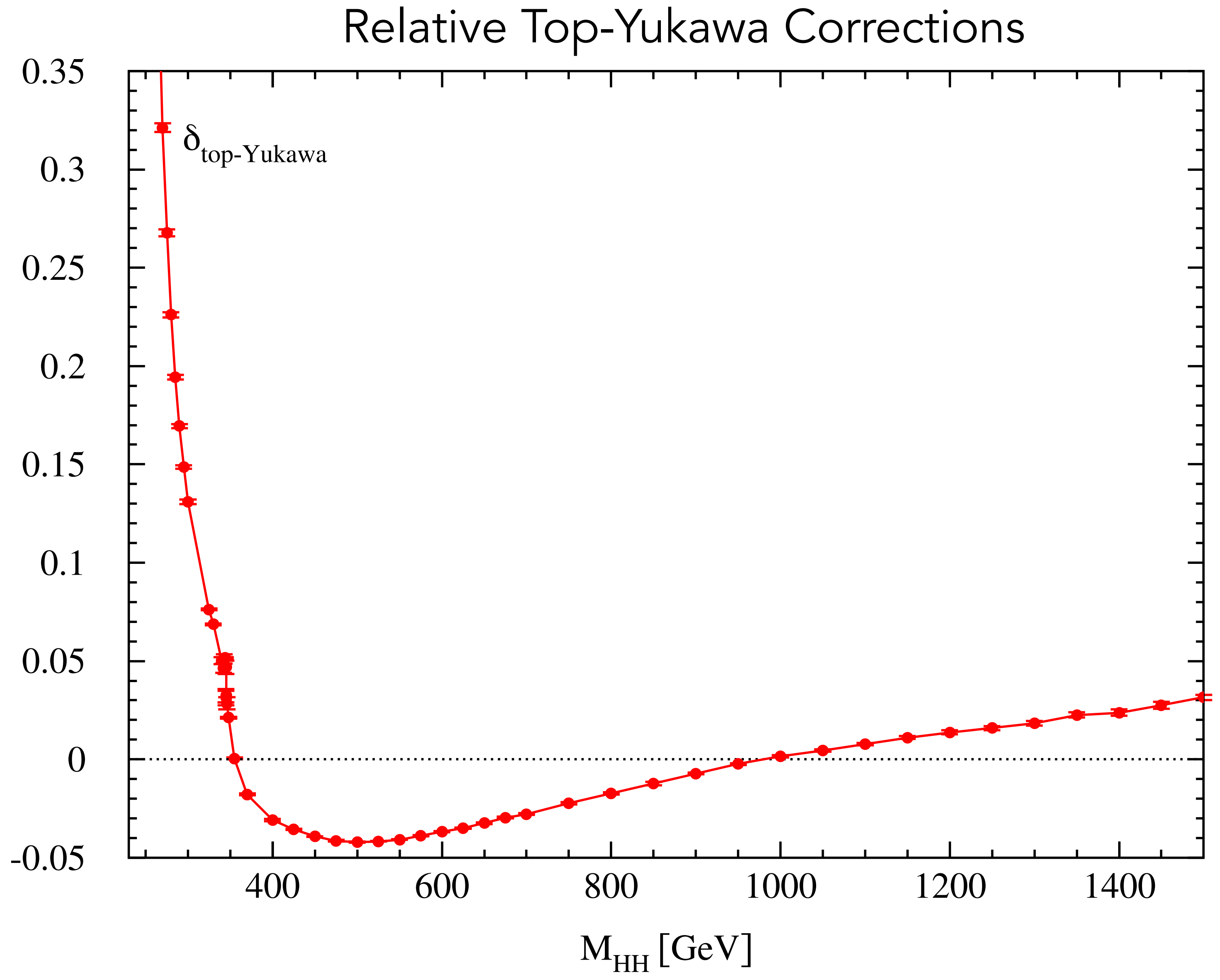




Yukawa and Light-Quark Mediated

Bhattacharya, Campanario, Carlotti, Chang, Mazzitelli, Mühlleitner, Ronca, Spira 25





Light-quark Mediated Corrections

Computed generally using numerical integration

Yukawa Corrections

Calculated in gaugeless limit i.e. neglecting vector bosons but including massless Goldstone bosons

Includes corrections to 1PR ``triangle'' diagrams and 1PI ``box'' diagrams, neglects new $y_t^2\lambda$ topologies arising at NLO EW

Calculation

Numerical integration in Feynman parameter space

Regularise the virtual thresholds by introducing small imaginary parts

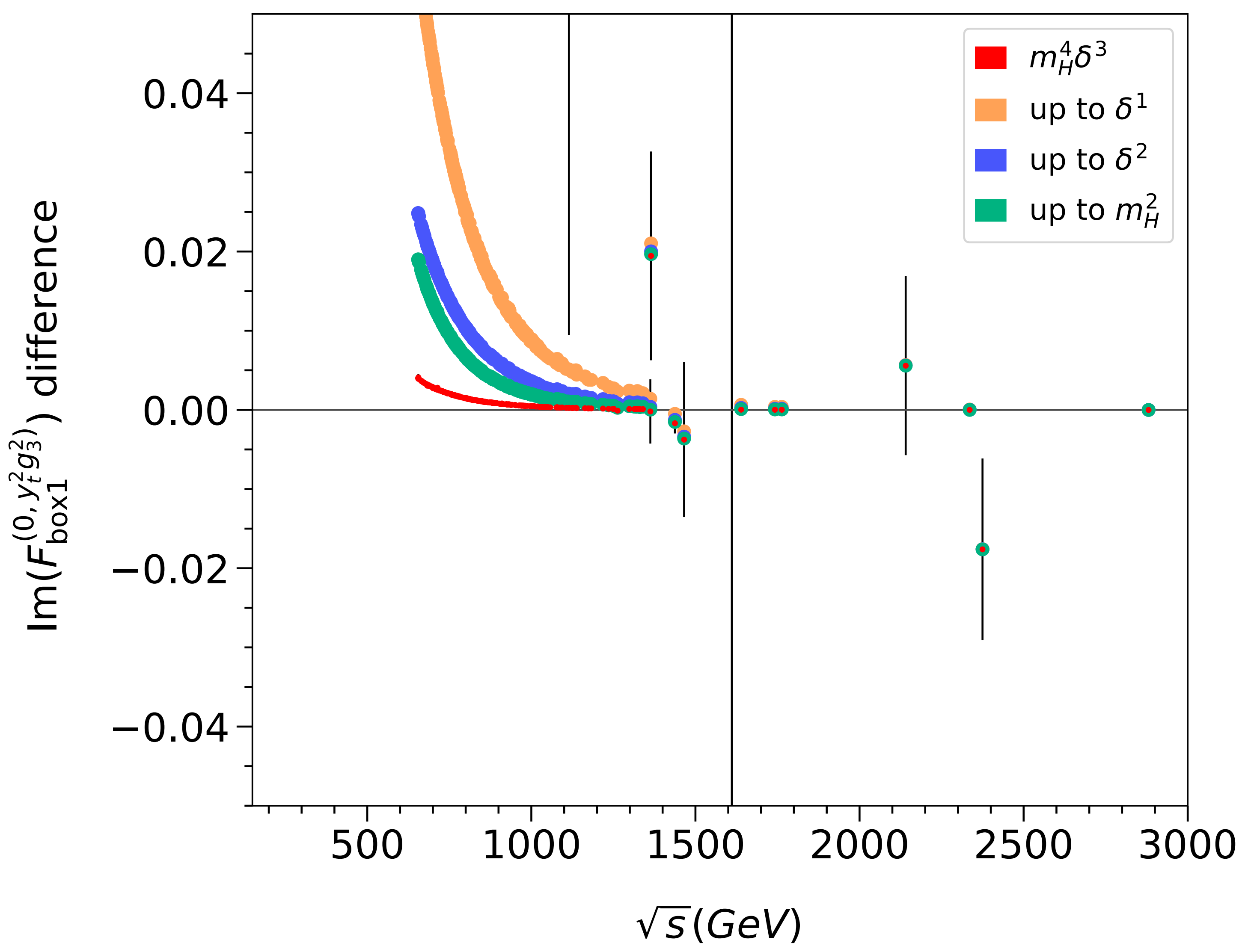
$$m_{t/b}^2 \to m_{t/b}^2(1 - i\delta), \quad m_H^2 \to m_H^2(1 - i\delta)$$

Extrapolate $\delta \to 0$ using Richardson extrapolation

Gluon-Induced Corrections

Davies, Schönwald, Steinhauser, Zhang 25, 26

Complete NLO EW corrections to $gg \to HH$ in the high-energy limit



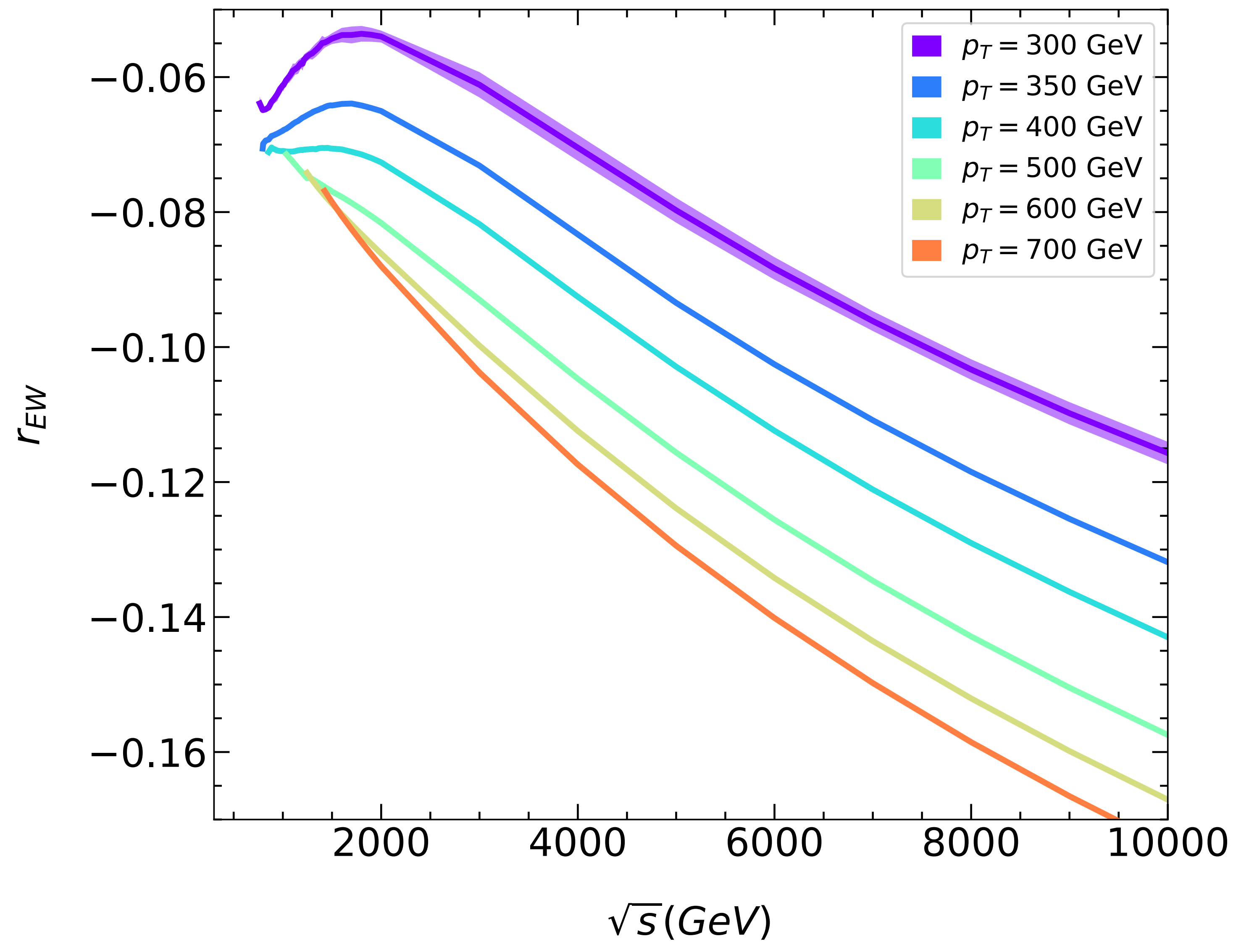
Expansion procedure

↪ Taylor expand in m_H^{ext} and $\delta = 1 - m_X/m_t$ $(m_X = m_W, m_Z, m_H^{\mathrm{int}})$

↪ Asymptotic expansion $m_t \ll s, t$

Provides analytic expressions for studying structure of EW corrections

Can perform detailed comparison/validation of existing results



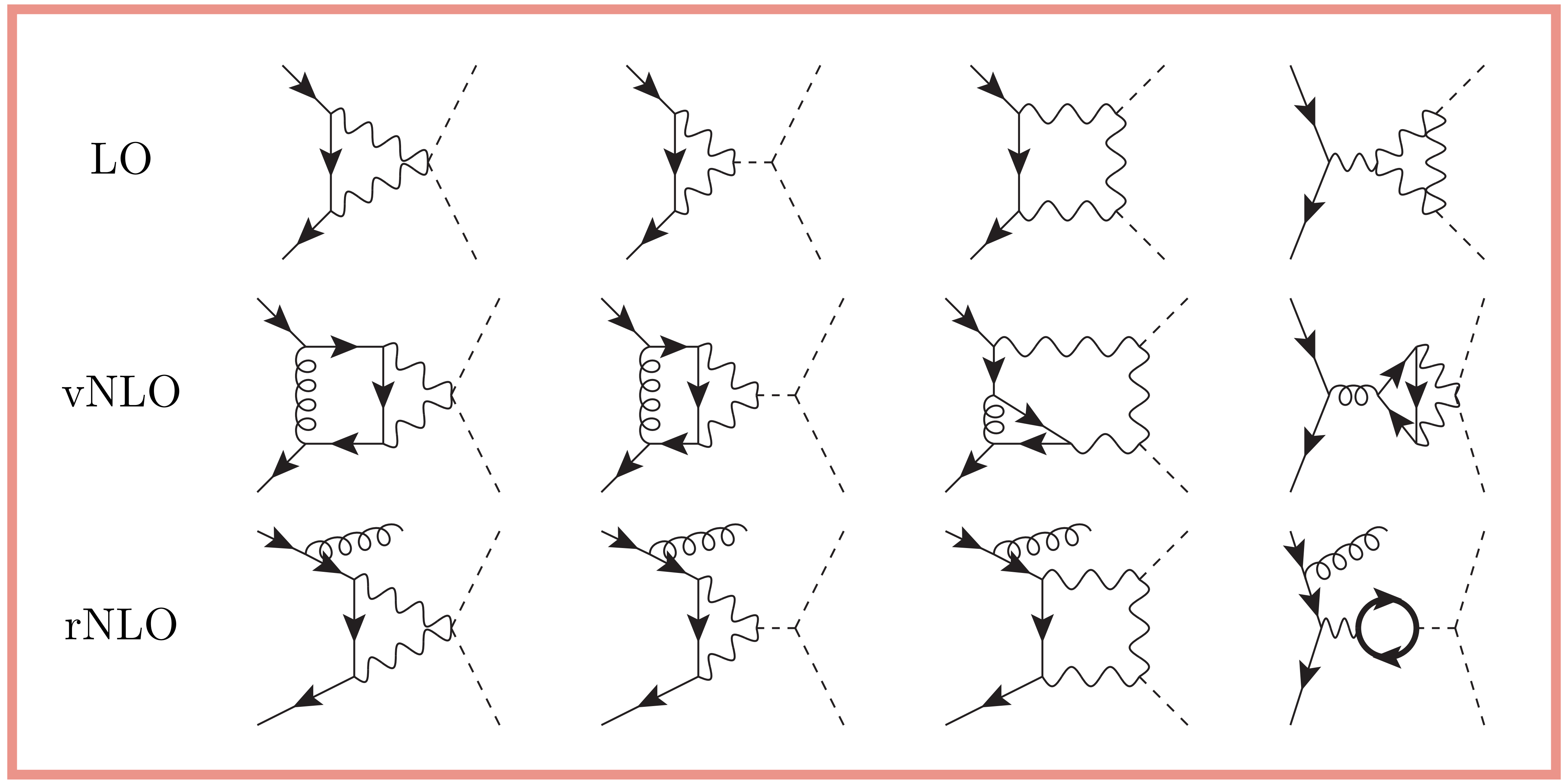
Covers $p_T \gtrsim 350$ GeV with small expansion uncertainty

Reproduce -10%(+) corrections observed at high-energy seen in first calculation of the NLO EW corrections to $gg \to HH$

Results will be made available in ggxy

Light-Quark Induced Corrections

Bonetti, Heinrich, Rendler, Torres Bobadilla 26



Quark-induced contribution from EW topologies + NLO QCD corrections

Canonical differential equations obtained (boundary values @ $s, t, u \ll m_V^2$) evolve boundary values using series-solution via DiffExp Moriello 19; Hidding 20;

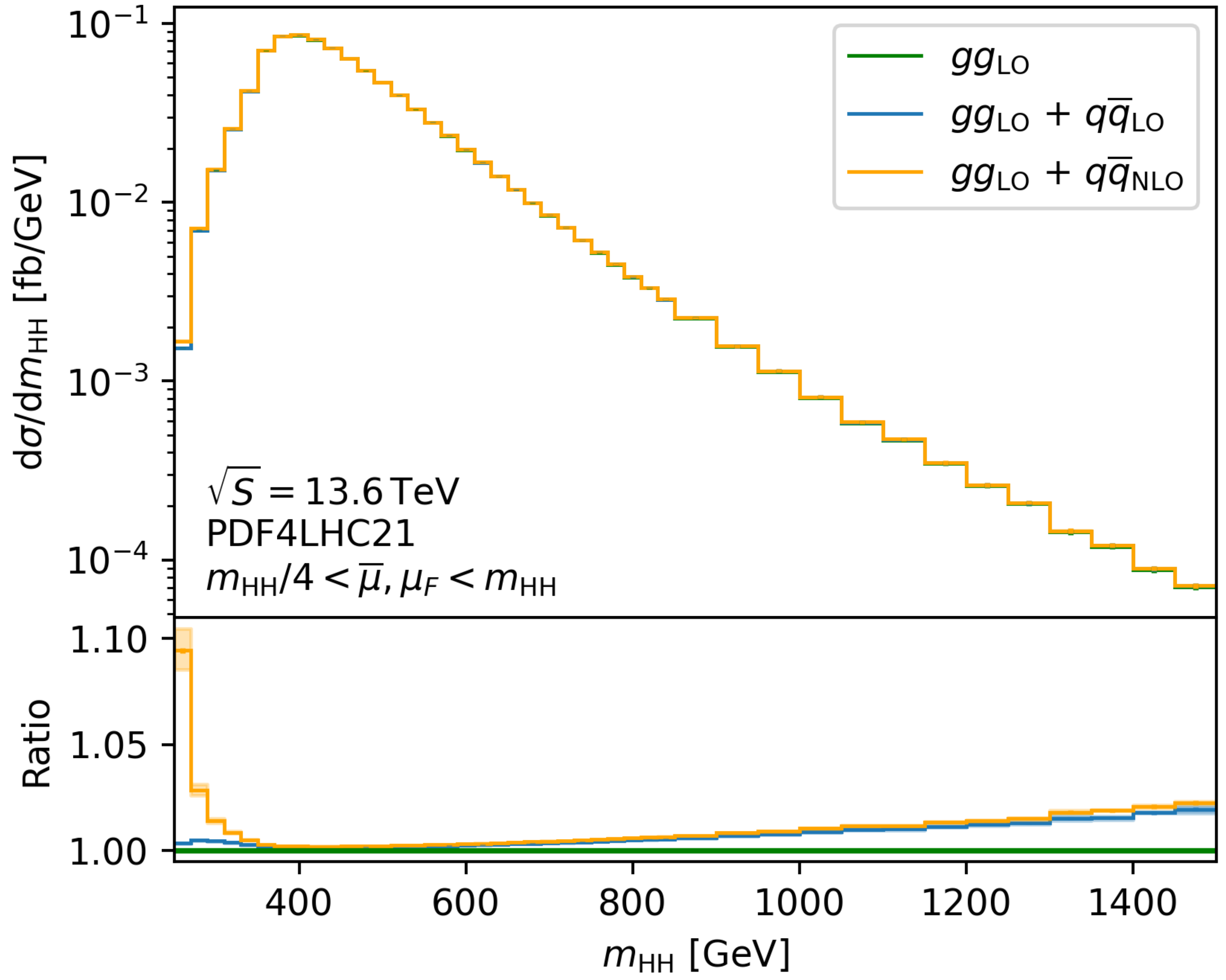
Total cross-section effects are quite small (<1%)

Very large NLO corrections (mainly due to opening gq channel)

Can significantly modify distributions +10% in first m_{HH} bin, p_T at 1-2%

Initial state bottom quarks can lead to further modifications at 1-2% level

$\sqrt{S}$ [TeV]	$q\bar{q}_{\rm LO}$ [fb]	$q\bar{q}_{\rm NLO}$ [fb]	$gg_{\rm LO}$ [fb]	$q\bar{q}_{\rm NLO}/q\bar{q}_{\rm LO}$	$q\bar{q}_{\rm NLO}/gg_{\rm LO}$
13.0	0.039	0.061	16.45	+59%	+0.37%
13.6	0.041	0.066	18.26	+60%	+0.36%
14.0	0.043	0.069	19.52	+60%	+0.35%



4. Mass Scheme Uncertainties

Mass Scheme Uncertainty

Converting the top-quark mass from OS to $\overline{\text{MS}}$ scheme

$$\frac{m(\mu)}{M} = \frac{Z_m^{\text{OS}}}{Z_m^{\overline{\text{MS}}}} \equiv \sum_{n\geq 0} \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n \left(z_m^n(M) + z_m^{n,\log}(\mu)\right)$$

$$z_m(\mu) = 1 + \frac{\alpha_s(\mu)}{2\pi}\left(-2C_F - \frac{3}{2}C_F \ln\frac{\mu^2}{M^2}\right) + \mathcal{O}(\alpha_s^2)$$

4-loop: Marquard, Smirnov, Smirnov, Steinhauser, Wellmann 16

Top quark mass scheme uncertainty

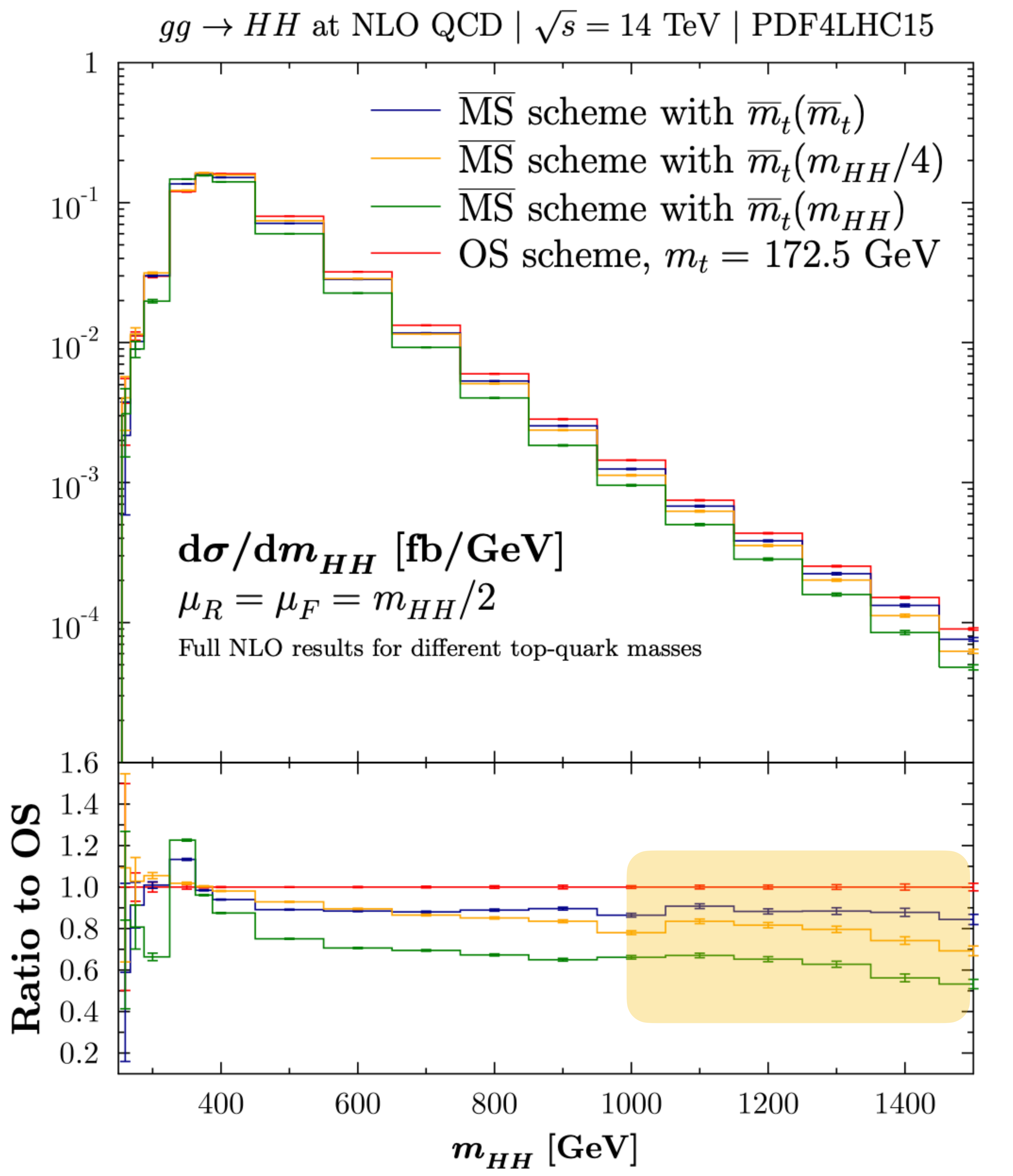
$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=400\text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=600\text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV},$$

Why do we have a large uncertainty comparing OS with $\overline{\text{MS}}$ mass?



Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 18, 20, 20

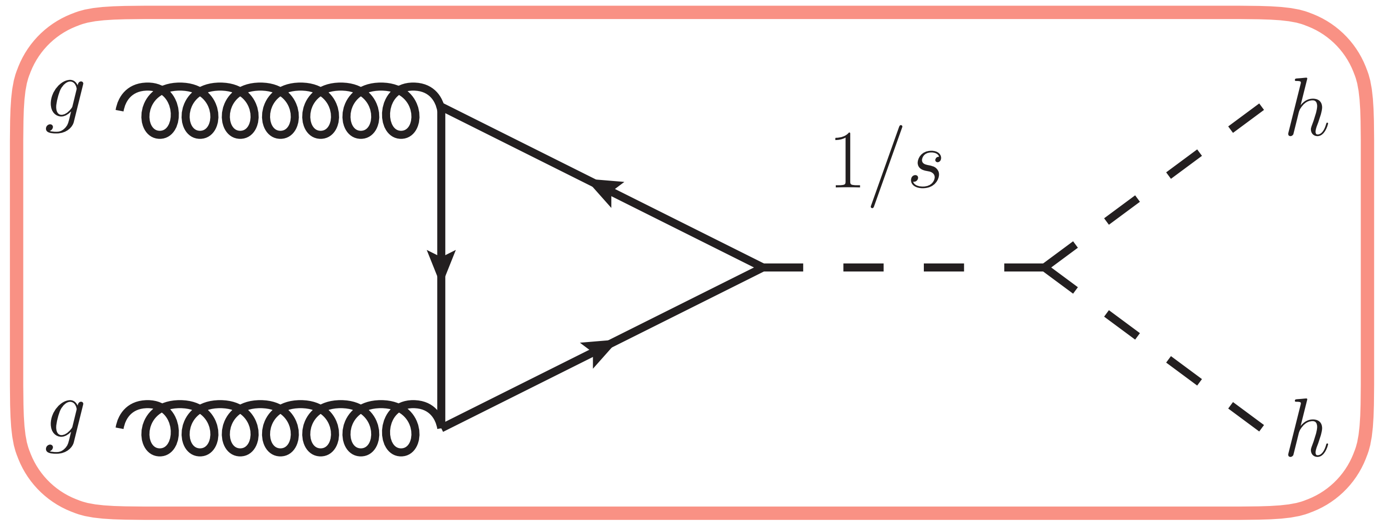
Amplitude Structure

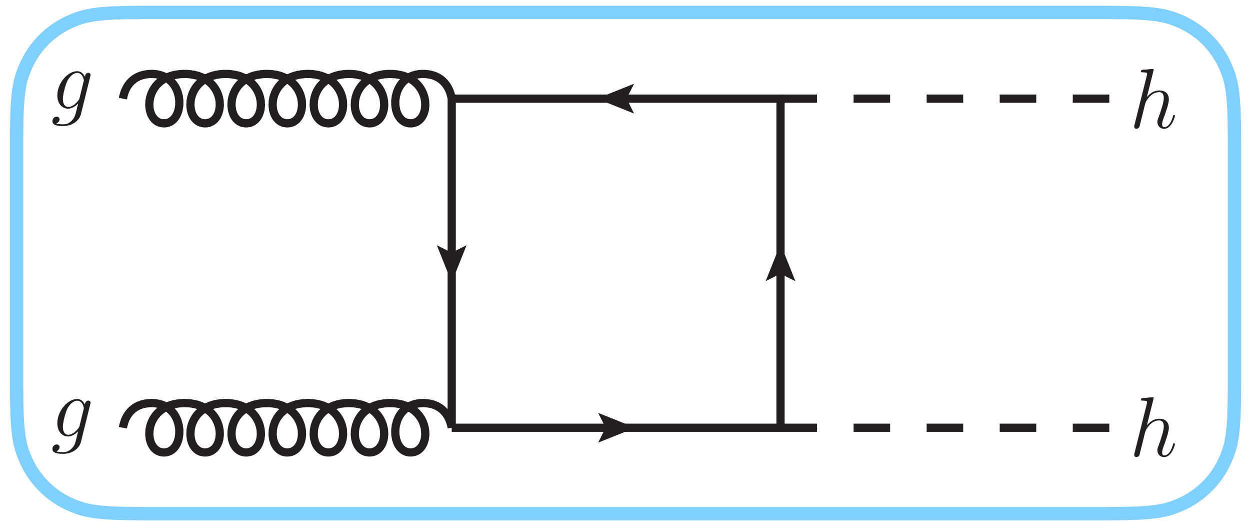
Structure of QCD corrections in the amplitude

$$\mathscr{M} = \varepsilon_{1,\mu}\,\varepsilon_{2,\nu}\,\delta^{AB}\left(A_1 P_1^{\mu\nu} + A_2 P_2^{\mu\nu}\right)$$

$$A_1 = T_F \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{2\pi}s\left[\frac{3m_H^2}{s-m_H^2}A_{1,y_t\lambda} + A_{1,y_t^2}\right]$$

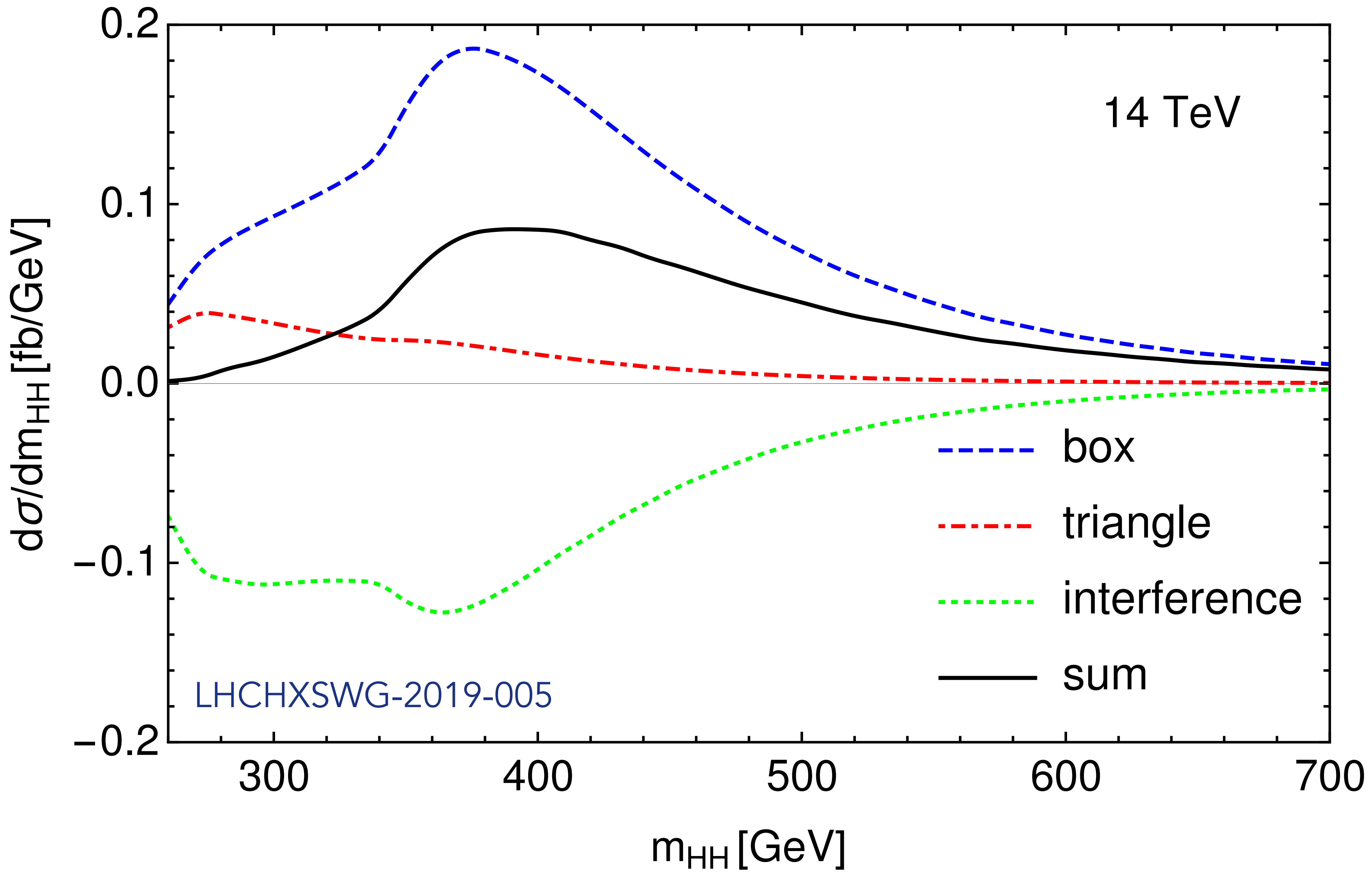
$$A_2 = T_F \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{2\pi}s\left[A_{2,y_t^2}\right]$$





Triangle type amplitudes studied in literature

Liu, Penin 17, 18; Anastasiou, Penin 20; Liu, Modi, Penin 22; Liu, Neubert, Schnubel, Wang 22



We will study the box type amplitudes in the high-energy or small quark mass limit

$$s, |t|, |u| \gg m_t^2 \gg m_H^2$$

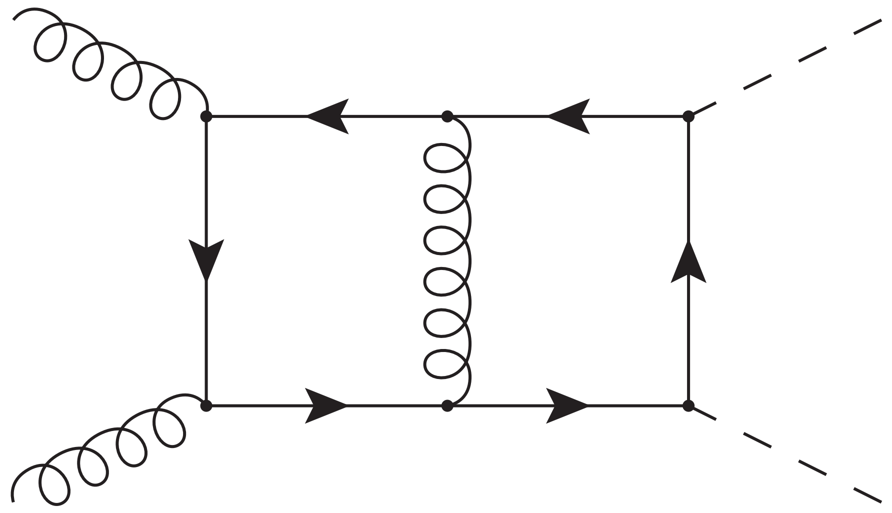
See also: Hu, Liu 25

High-energy limit

Expanding amplitude perturbatively $A_i^{\text{fin}} = \frac{\alpha_s}{2\pi} A_i^{(0),\text{fin}} + \left(\frac{\alpha_s}{2\pi}\right)^2 A_i^{(1),\text{fin}} + \mathcal{O}(\alpha_s^3)$ and around $m_t \sim 0$

$gg \to HH$

Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20



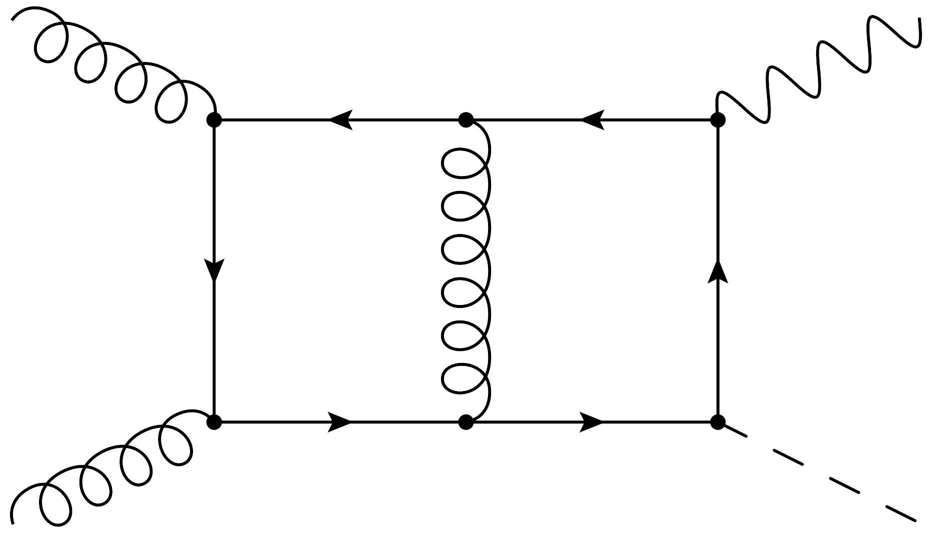
$$A^{(0)}_{i,y_t^2} \sim y_t^2 f_i(s,t) + y_t^2 \mathcal{O}(m_t^2)$$

$$A^{(1)}_{i,y_t^2} \sim 3 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right] + y_t^2 \mathcal{O}(m_t^2)$$

Leading $\log(m_t^2)$ from mass counter term, converting to $\overline{\text{MS}}$ gives $\log\left[\mu_t^2/s\right]$ → scale choice of $\mu_t^2 \sim s$

$gg \to ZH$

Davies, Mishima, Steinhauser 20; Chen, Davies, Heinrich, SPJ, Kerner, Mishima, Schlenk, Steinhauser 22



$$A_i^{(0)} \sim y_t m_t f_i(s,t) \ \log^2\left[\frac{m_t^2}{s}\right]$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{12} A_i^{(0)} \ \log^2\left[\frac{m_t^2}{s}\right]$$

Leading $\log(m_t^2)$ not coming from mass counter term ($C_A - C_F$ structure)

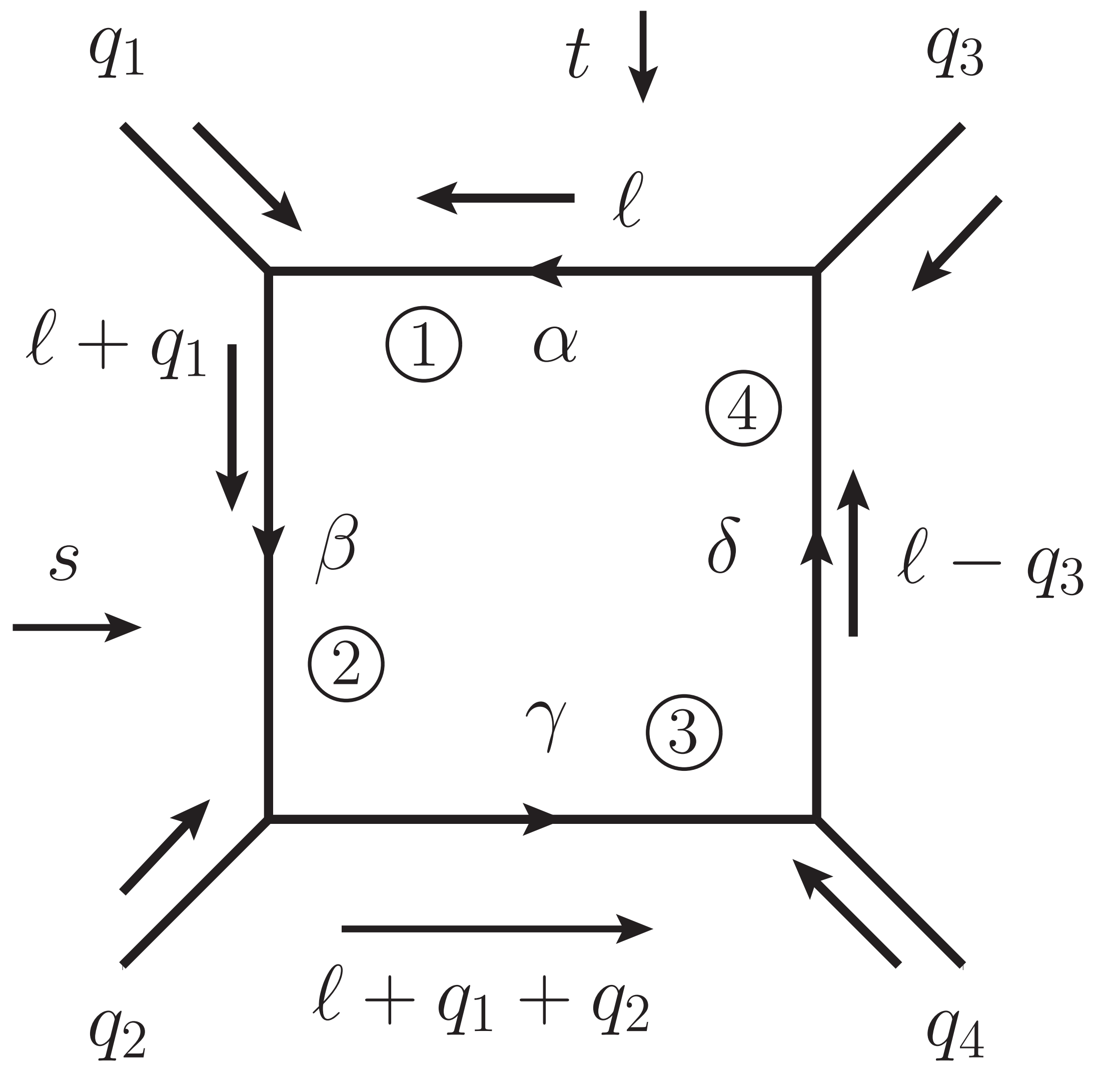
How does this simple structure arise? Does it generalise to all orders?

Expansion of $gg \to HH$ @ 1-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

Limit: $s, |t|, |u| \gg m_t^2 \gg m_H^2, \quad m_H^2 \to 0$ and $\lambda \sim m_t/Q$

Kinematics: $s + t + u = 0, \quad m_H = 0$



Automatically find remaining regions in parameter space

$\mathbf{u}^R$	order	interpretation	routing
(-2, -2, 0, 0)	$4 - 2(\epsilon + \alpha + \beta)$	c_1	ℓ
(0, -2, -2, 0)	$4 - 2(\epsilon + \beta + \gamma)$	c_2	$\ell - q_1$
(-2, 0, 0, -2)	$4 - 2(\epsilon + \alpha + \delta)$	c_3	$\ell + q_3$
(0, 0, -2, -2)	$4 - 2(\epsilon + \gamma + \delta)$	c_4	$\ell - q_1 - q_2$
(0, 0, 0, 0)	0	h	n/a

Using a set of possible loop momenta modes can **systematically search** for momentum routing to give a momentum space interpretation

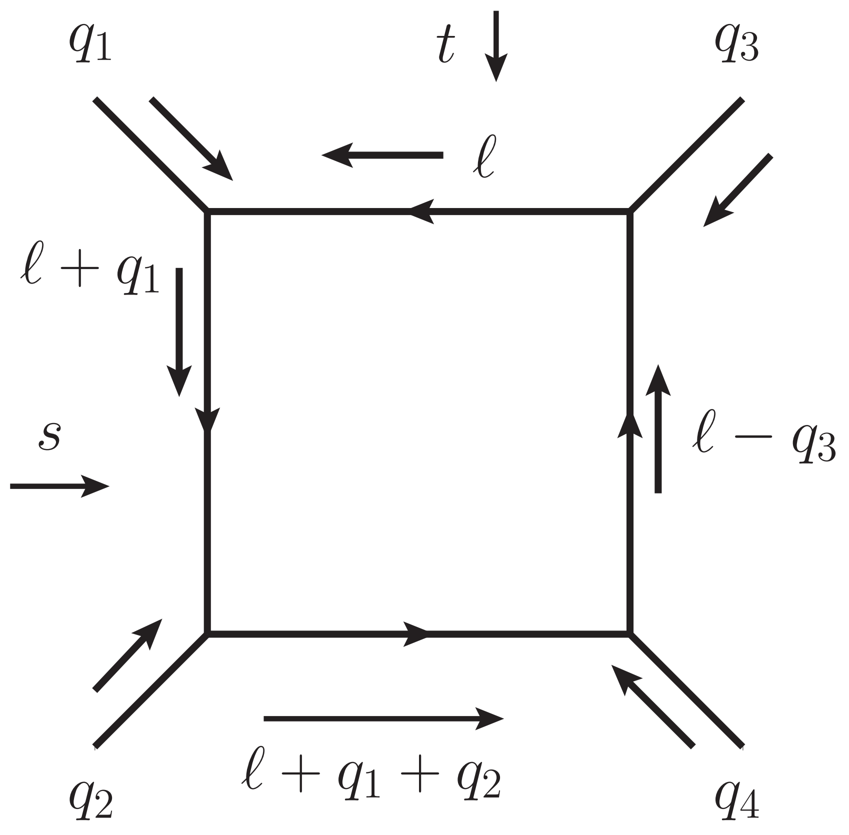
Implemented in pySecDec by Y. Ulrich (TBA)

Amplitude Level Results @ 1-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

Can compute amplitude level results for each region, at the 1-loop level:

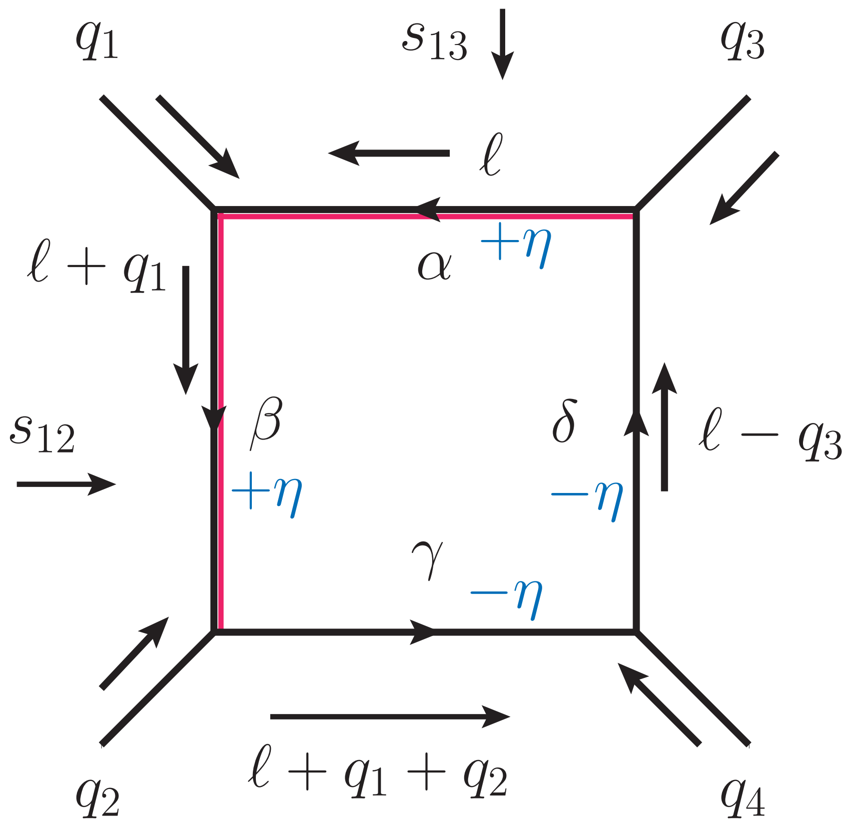
Hard region



Leading Power (LP)

$$
\begin{aligned}
A^{(h)}_{1,y_t^2} = \frac{4y_t^2}{s}\Bigg\{ 2 - 2m_t^2 \Bigg[&-\frac{2}{\epsilon^2 s} - \frac{1}{\epsilon}\left(\frac{s^2+2tu}{stu}\, l_s + \frac{l_t}{u} + \frac{l_u}{t}\right) \\
&+ \frac{-l_s^2 + 2l_t^2 + 2l_u^2}{s} + \frac{l_s\, l_t}{t} + \frac{l_s\, l_u}{u} + \frac{(t-u)^2\, l_t\, l_u}{stu} \\
&- \left(\frac{2}{s} + \frac{1}{t} + \frac{1}{u}\right) l_s - \frac{t\, l_t}{su} - \frac{u\, l_u}{st} + \frac{60 + 13\pi^2}{6s} \Bigg] + \mathcal{O}(m_t^4) \Bigg\}
\end{aligned}
$$

Collinear q_1 region



Next-to-Leading Power (NLP)

$$
\begin{aligned}
A^{(c_1)}_{1,y_t^2} = &\frac{-4y_t^2 m_t^2\, \mu^{2\epsilon} e^{\gamma_E \epsilon}\Gamma(2\eta)^2\Gamma(\epsilon+2\eta)}{s(stu)(1-2\eta-\epsilon)\Gamma(4\eta)\Gamma(1+\eta)^2}\left(\frac{1}{m_t^2}\right)^{2\eta+\epsilon} \\
&\times\Big\{(-t)^\eta(-u)^\eta\Big[s^2\left(1+2\epsilon^2-(2+\eta)\epsilon\right) - 2\eta\, tu\Big] \\
&+(-s)^\eta(-t)^\eta\Big[su\left(1+2\epsilon^2-(2+\eta)\epsilon\right) - tu(1-3\eta-(2-5\eta)\epsilon)\Big] \\
&+(-s)^\eta(-u)^\eta\Big[st\left(1+2\epsilon^2-(2+\eta)\epsilon\right) - tu(1-3\eta-(2-5\eta)\epsilon)\Big]\Big\}
\end{aligned}
$$

Generates $\log(m_t^2)$ at NLP

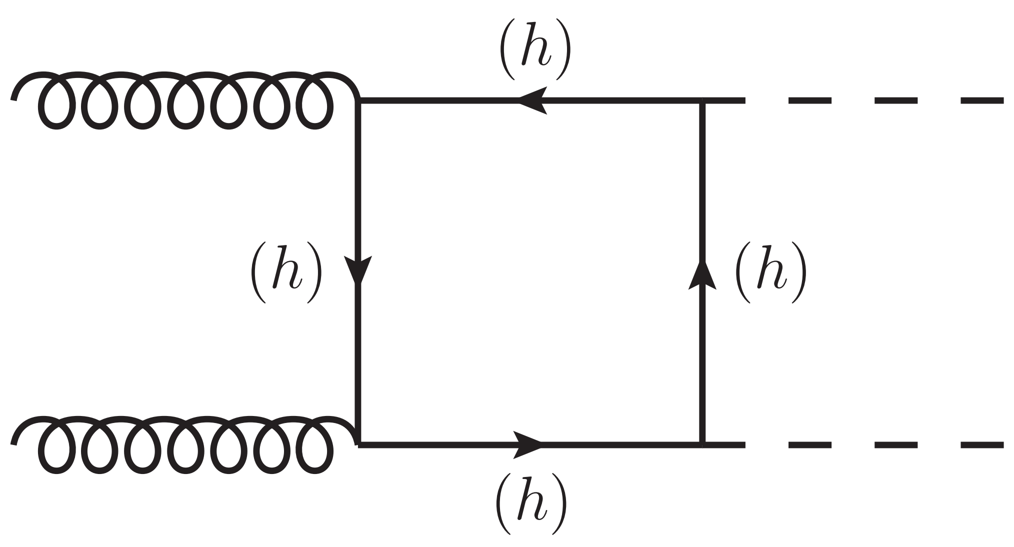
Leading Power Analysis

Jaskiewicz, SPJ, Szafron, Ulrich 25

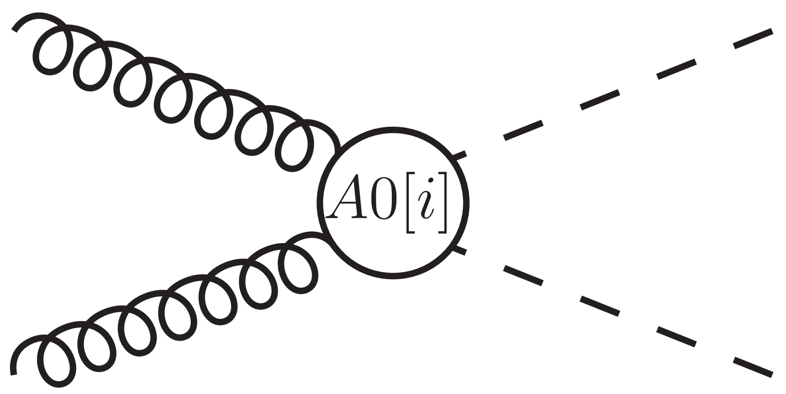
Soft Collinear Effective Theory (SCET) an approximation of QCD based on soft/collinear expansion Bauer, Fleming, Pirjol, Stewart, Beneke, Chapovsky, Diehl, Feldmann, …

$$\psi(x) \to \underbrace{\psi_1(x) + \ldots + \psi_N(x)}_{N \text{ collinear fermion fields}} + q(x)$$

Leading power matching $J_{\mathrm{LP}}^{[i]}(t_1, t_2, t_3, t_4) = y_t^2\, P_i^{\mu\nu}\, \mathscr{A}_{c_1\perp_1\,\mu}(t_1 n_{1+})\, \mathscr{A}_{c_2\perp_2\,\nu}(t_2 n_{2+})\, h_{c_3}(t_3 n_{3+})\, h_{c_4}(t_4 n_{4+})$

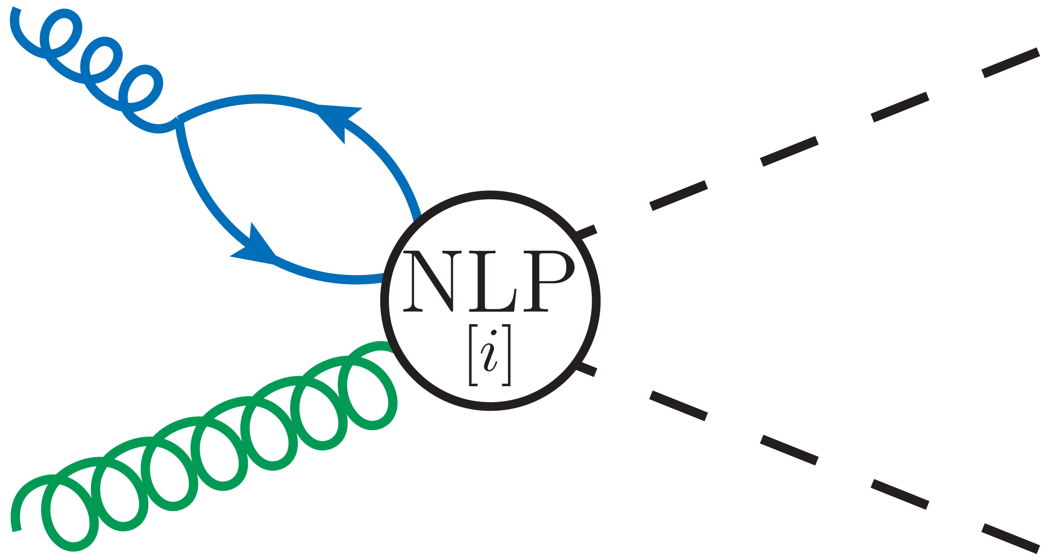


$$+ \ldots = C_{[i]}^{A0}(s,t)$$



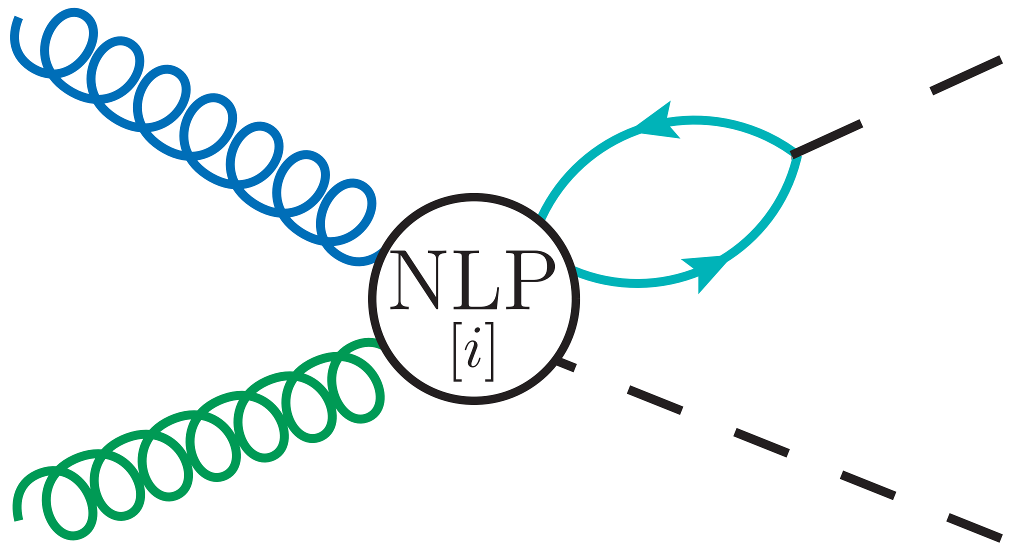
Collinear Regions c_1, c_2

Mixing with the external gluon is forbidden at LP



Collinear Regions c_3, c_4

Mixing with the external Higgs is forbidden at LP



Result holds to all orders in α_s **due to helicity conservation for** $m_t \to 0$

Overview of Structure

Jaskiewicz, SPJ, Szafron, Ulrich 25

The $gg \to HH$ amplitude in the $\overline{\mathrm{MS}}$ scheme has the following leading power structure

$$l_\mu = \log(\mu_t^2/s)$$

$$l_m = \log(\mu_t^2/s), \log(m_t^2/s)$$

$$\mathrm{LO}: \alpha_s y_t^2 (c_0 + m_t n_0),$$

$$\mathrm{NLO}: \alpha_s^2 y_t^2 (a_1 l_\mu + c_1 + m_t n_1),$$

$$\mathrm{NNLO}: \alpha_s^3 y_t^2 (a_2 l_\mu^2 + b_2 l_m + c_2 + m_t n_2),$$

$$\mathrm{N^3LO}: \alpha_s^4 y_t^2 (a_3 l_\mu^3 + b_3 l_m^2 + d_3 l_m + c_3 + m_t n_3),$$

$$\mathrm{N^iLO}: \alpha_s^{i-1} y_t^2 (a_i l_\mu^i + b_i l_m^{i-1} + d_i l_m^{i-2} + \ldots + c_i + m_t n_i)\,.$$

Integrals are known

Henn, Mistlberger, Smirnov, Wasser 20
Caola, von Manteuffel, Tancredi 20;
Bargiela, Caola, von Manteuffel, Tancredi 22;

Leading log structure generated to all orders by RG running

$$m^{\mathrm{LL}}(\mu) = M \exp\left[a_{\gamma_m}^{\mathrm{LL}}(\mu)\right] z_m(M)$$

$$a_{\gamma_m}^{\mathrm{LL}}(\mu) = \frac{3C_F}{2\beta_0} \ln\left(1 - \frac{\alpha_s(\mu)}{2\pi}\beta_0 \ln\left(\frac{\mu^2}{M^2}\right)\right)$$

LP LL
Known from RG running of top-quark mass

LP NLL
RG running + massification

Penin 06; Moch, Mitov 07; Becher, Melnikov 07; Engel et al 19; Wang, Xia, Yang, Ye 23;

LP Constant
Hard region ($m_t = 0$) contribution only, known to NLO

Mass Scheme Uncertainty

Jaskiewicz, SPJ, Szafron, Ulrich 25

For finite **virtual correction** at very high energy

Compare $m_t^{\mathrm{OS}} \leftrightarrow m_t^{\overline{\mathrm{MS}}}$

Very large uncertainty obtained

LO: ~60-70% (blue band)

NLO: ~30-40% (red band)

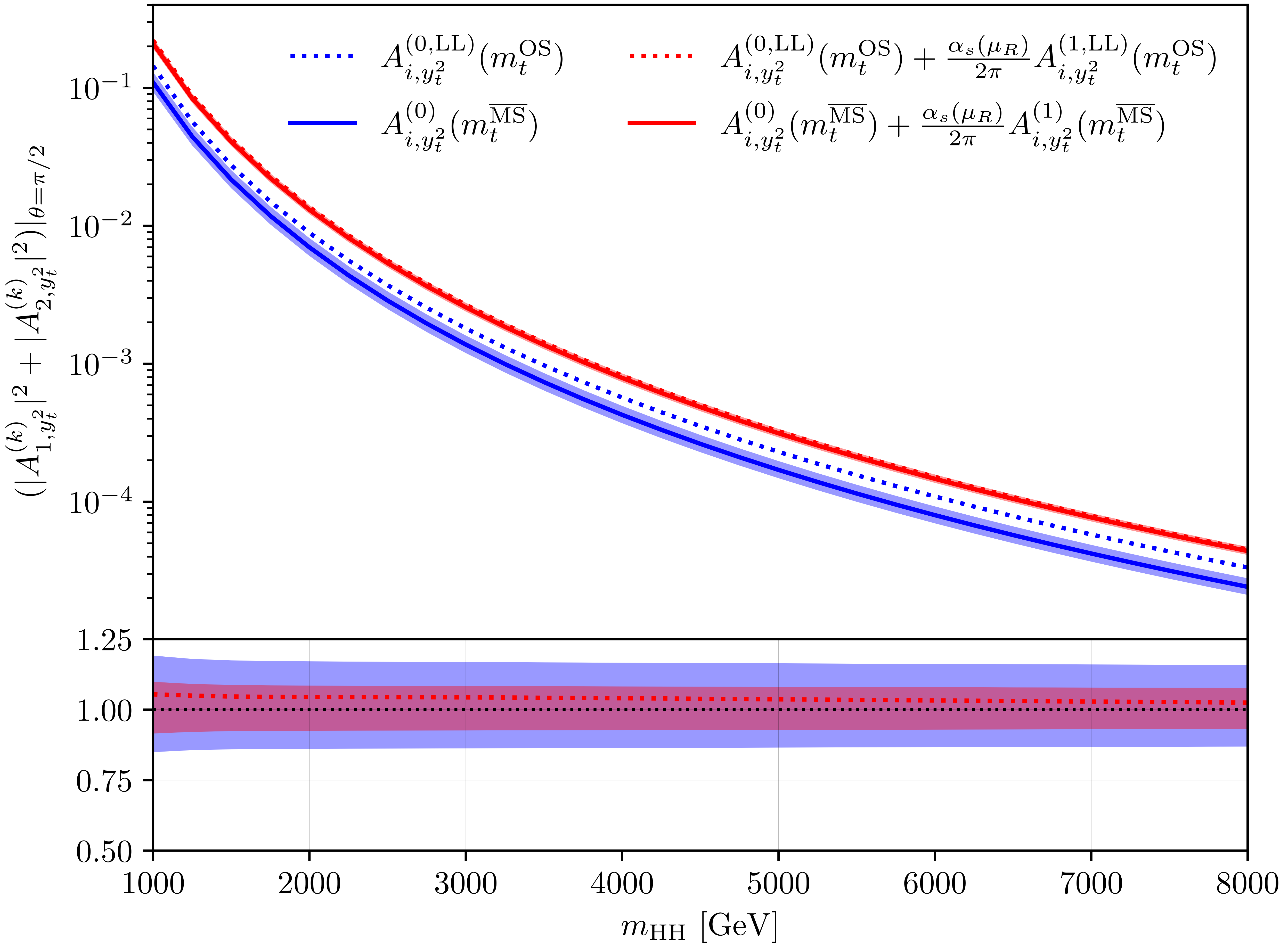
+ **known** LP LL (**green** tower of logs)

LO: ~13-16%

NLO: ~7-8%

Uncertainty significantly reduced

Not yet studied for physical cross-section!



3-loop Massless Limit (c_2)

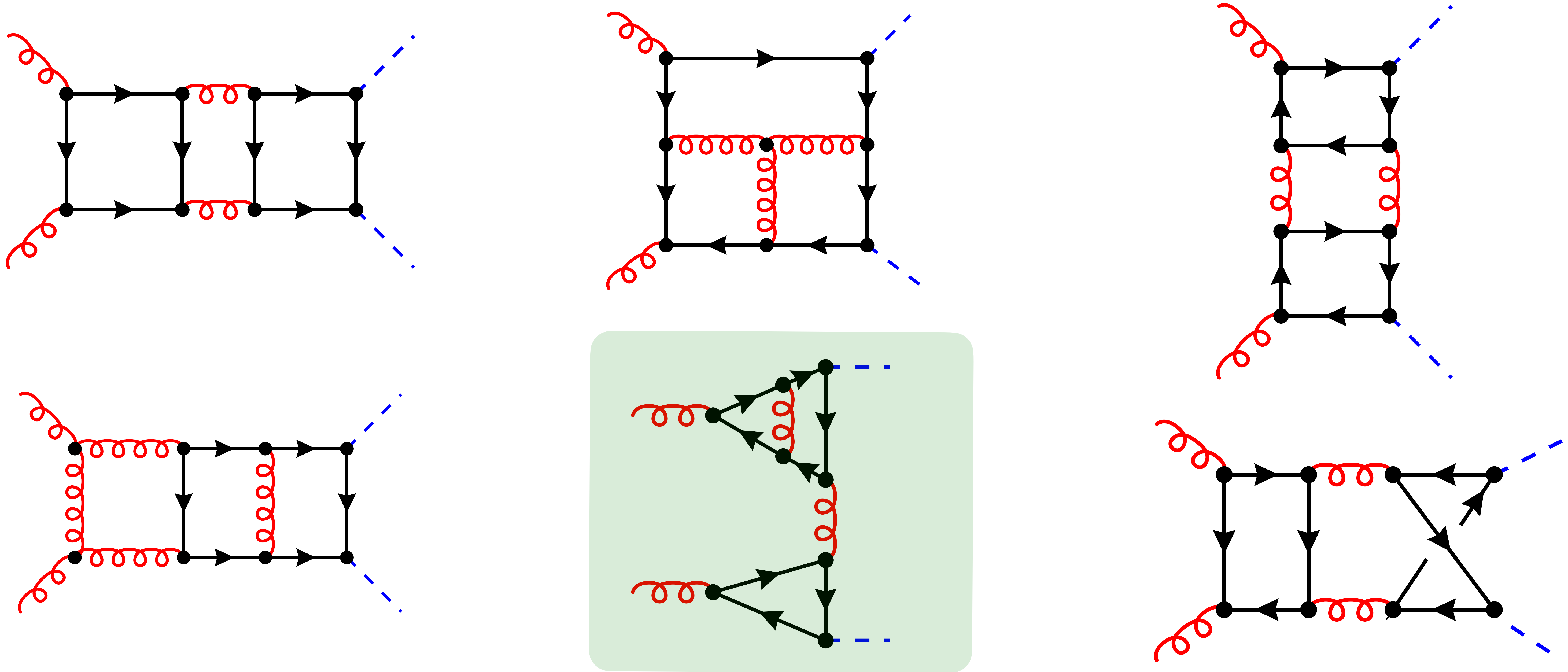
Ajjath, SPJ, Magerya (WIP)

Hard region ($m_H = 0, m_T = 0$) Box (y_t^2) Amplitudes

3728 Feynman diagrams

3 integral families + crossings

Integrals in terms of GPLs



Several pieces known

Reducible contributions with exact m_t $\propto$ (Higgs production with off-shell gluon leg)^2

Davies, Schönwald, Steinhauser, Vitti 24

Large-n_f and large-N_c in the forward limit ($t = 0$, $m_H = 0$)

Davies, Schönwald, Steinhauser 23, 25

Ideally want $gg \to HH$ at NNLO QCD with the full mass dependence

Conclusion

Importance

Theory work needed to match experimental precision

EW corrections can change our interpretation of limits/deviations

Results

Have NLO QCD + N^3LO HTL + NLO EW andNNLO FTapprox + PS

Significant work undertaken to pick apart, understand and validate EW corrections

Largest uncertainty remains the NLO QCD top-quark mass scheme uncertainty

Outlook

Important to have control of complete EW calculation also in EFT context (SMEFT/HEFT)

Full NNLO QCD calculation desperately needed

Thank you for listening!

Backup

Precision Progress

Higgs is just one example, in reality the success of the HL-LHC programme+ requires many accurate predictions

Higgs

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO_{QCD}^{(t,t\times b)}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}^{(HTL)}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.)
$pp \to H+j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO_{QCD}}$
$pp \to H+2j$	$\mathrm{NLO_{HTL} \otimes LO_{QCD}}$ $\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ (incl.) $\mathrm{NNLO_{QCD}^{(VBF^*)}}$ $\mathrm{NLO_{EW}^{(VBF)}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ $\mathrm{NNLO_{QCD}^{(VBF)}}$ $\mathrm{NLO_{QCD}}$
$pp \to H+3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO_{QCD}^{(VBF)}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO_{QCD}^{(VBF^*)}}$
$pp \to VH$	$\mathrm{N^3LO_{QCD}}$ (incl.)$\mathrm{+NLO_{EW}}$ $\mathrm{NLO_{gg\to HZ}^{(t,b)}}$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to VH+j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to HH$	$\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to HH+2j$	$\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ (incl.) $\mathrm{NNLO_{QCD}^{(VBF^*)}}$ $\mathrm{NLO_{EW}^{(VBF)}}$	$\mathrm{NLO_{QCD}}$
$pp \to HHH$	$\mathrm{NNLO_{HTL}}$	$\mathrm{NLO_{QCD}}$
$pp \to H+t\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO_{QCD}}$ (approx.)	$\mathrm{NNLO_{QCD}}$
$pp \to H+t/\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$

Vector Bosons

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD} + N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ + Full $\mathrm{NLO_{QCD}}$ $(gg \to ZZ)$, approx. $\mathrm{NLO_{QCD}}$ $(gg \to WW)$	Full $\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V+j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	hadronic decays
$pp \to V+2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
$pp \to V+b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to W+b\bar{b}$	$\mathrm{NNLO_{QCD}}$	
$pp \to VV'+1j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to VV'+2j$	$\mathrm{NLO_{QCD}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^+W^++2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to W^+W^-+2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to W^+Z+2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to ZZ+2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to VV'V''$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/ decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)
$pp \to WWW$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to W^+W^+(V \to jj)$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to WZ(V \to jj)$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to \gamma\gamma$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma+j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma\gamma+j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $\mathrm{+NLO_{QCD}}$ (gg channel)	
$pp \to \gamma\gamma\gamma$	$\mathrm{NNLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Top

process	known	desired
$pp \to t\bar{t}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$ (w/ decays)	$\mathrm{N^3LO_{QCD}}$
$pp \to t\bar{t}+j$	$\mathrm{NLO_{QCD}}$ (off-shell effects) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+2j$	$\mathrm{NLO_{QCD}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+V'$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+\gamma$	$\mathrm{NLO_{QCD}}$ (off-shell)	
$pp \to t\bar{t}+Z$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to t\bar{t}+W$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to t/\bar{t}$	$\mathrm{NNLO_{QCD}}$*(w decays) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to tZj$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off shell)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays)
$pp \to t\bar{t}t\bar{t}$	$\mathrm{NLO_{QCD}}$ (w decay) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$

Jets

process	known	desired
$pp \to 2\,\mathrm{jets}$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + NLO_{EW}}$
$pp \to 3\,\mathrm{jets}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	

- completed since 2021

- progress since 2021

Huss, Huston, SPJ, Pellen, Röntsch 25 (Les Houches Wishlist)

2-loop Massless Limit (c_1)

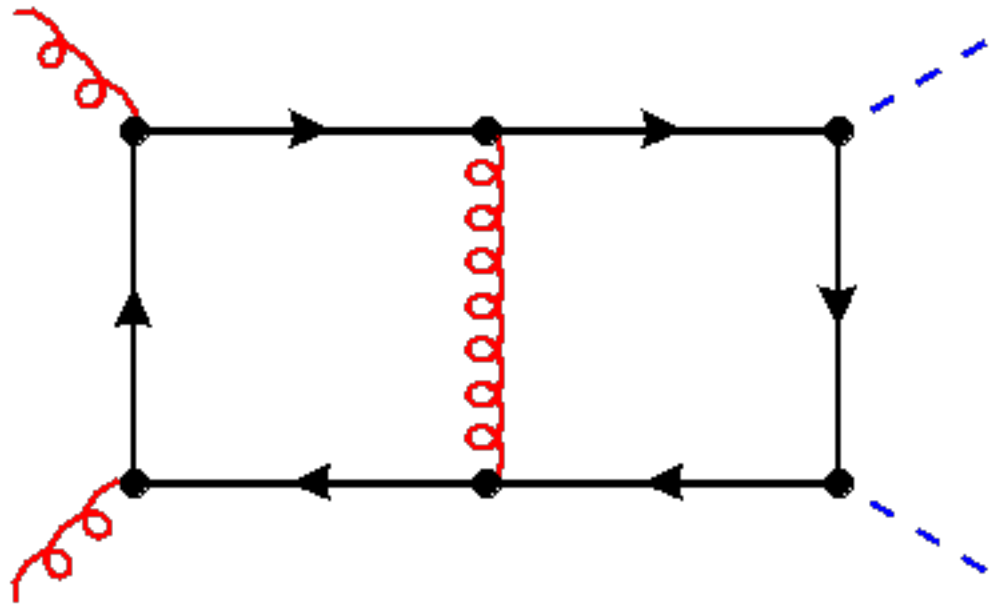
Ajjath, SPJ, Magerya (WIP)

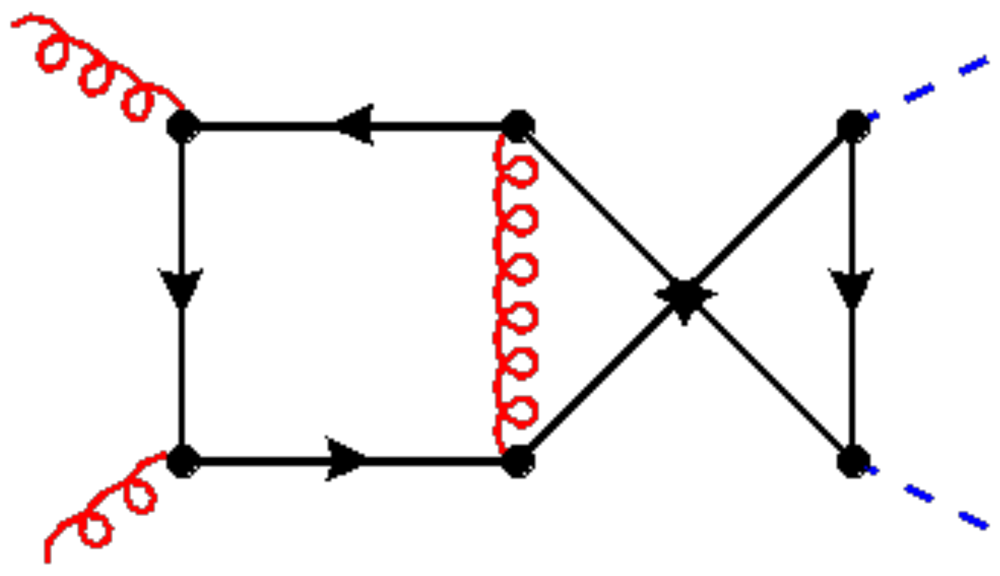
Hard region ($m_H = 0, m_T = 0$) + Box (y_t^2) Amplitudes

138 Feynman diagrams

2 integral families + crossings

Integrals in terms of GPLs





Renormalised amplitudes

$$A_i^{\text{ren}} = \left(\frac{\alpha_s}{2\pi}\right) A_i^{(0),\text{ren}}(m_t^2) + \left(\frac{\alpha_s}{2\pi}\right)^2 A_i^{(1),\text{ren}}(m_t^2) + \mathcal{O}(\alpha_s^3),$$

$$A_i^{(1),\text{ren}}(m_t^2) = S_\epsilon^{-2} \left(\frac{\mu_R^2}{\mu_0^2}\right)^{2\epsilon} A_i^{(1)}(m_t^2) + S_\epsilon^{-1} \left(\frac{\mu_R^2}{\mu_0^2}\right)^{\epsilon} \left[\delta Z_{\alpha_s} A_i^{(0)}(m_t^2) + \delta Z_m \, m_t^2 \frac{\partial A_i^{(0)}(m_t^2)}{\partial m_t^2}\right]$$

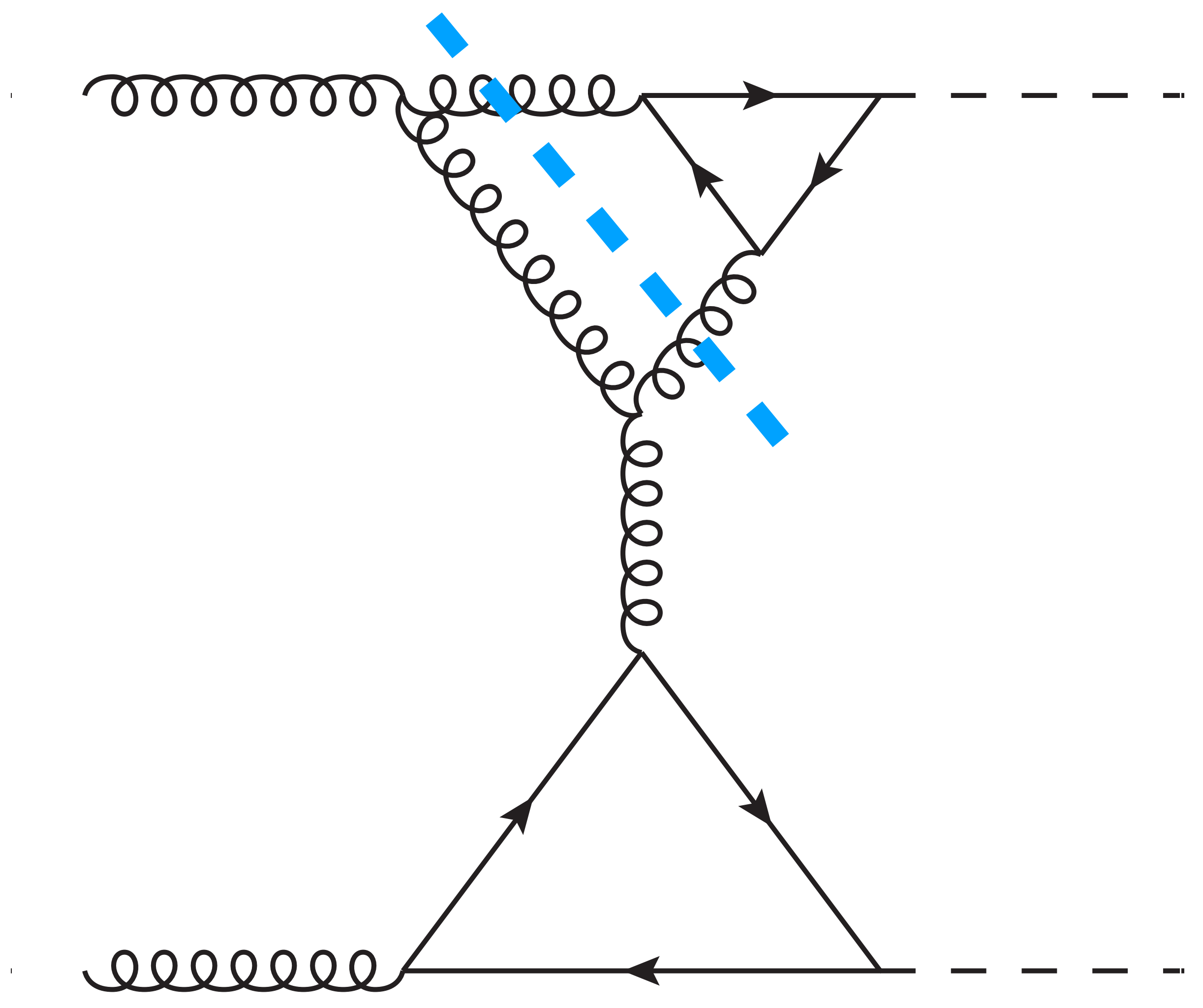
$-24 C_f m_f^2 \log(s)$ $\qquad$ $24 C_f m_f^2 \log\left(m_f^2\right)$

Match small-m_t result at leading power: explicit check that only hard region contributes at LP @ 2-loop ✔

Davies, Mishima, Steinhauser, Wellmann 18;

Delicate Limits

So far we set $m_H = 0$ from the start ($\Longrightarrow$ Taylor exp.) then expanded around $m_T = 0$ ($\Longrightarrow$ asymptotic exp)



Davies, Schönwald, Steinhauser, Vitti 24

The commutativity of these limits is not trivial

Expansion around $m_H \to 0$ can develop $\ln(m_H^2)$ behaviour (even for y_t^2 component!)

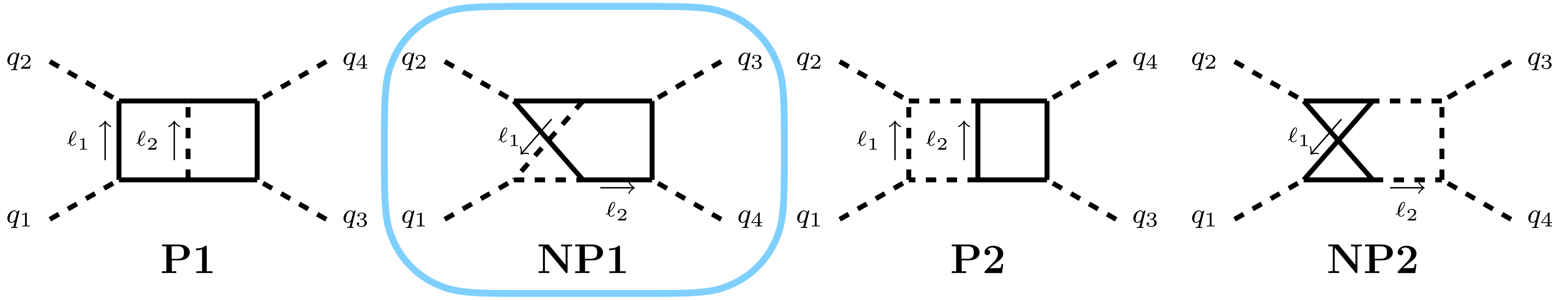
Configurations we identified all involve a single Higgs boson coupled to a closed fermion loop

Suppressed by at least one power of m_t (helicity flip)
Do not affect the $y_t^2 m_t^0$ term but may appear beyond LP

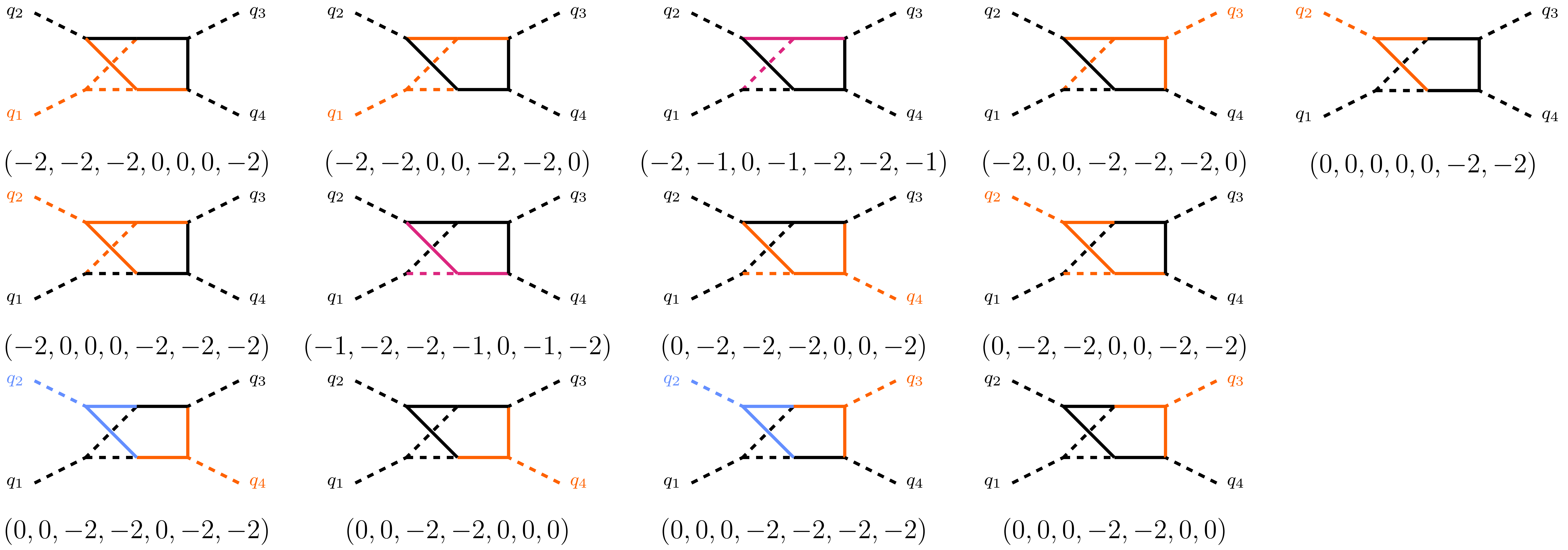
A complete explicit cross-check of this behaviour at fixed order (3-loop/4-loop/...) is desired

High-Energy Expansion of $gg \to HH$ @ 2-loops

Topologies



Regions (Parameter Space)



Amplitude Level Results @ 2-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

Could compute each region at 2-loops (tedious), can instead examine numerator prior to reduction

Region $c_1 c_1$

$$\ell_1^\mu \sim \ell_2^\mu \sim Q(1,\lambda^2,\lambda)$$

$$l_1^2 \sim \lambda^2 Q^2, \quad l_2^2 \sim \lambda^2 Q^2, \quad l_1 \cdot l_2 \sim \lambda^2 Q^2,$$
$$l_1 \cdot q_1 \sim \lambda^2 Q^2, \quad l_2 \cdot q_1 \sim \lambda^2 Q^2,$$
$$l_1 \propto q_1, \quad l_2 \propto q_1 \,.$$

Region ss

$$\ell_1^\mu \sim \ell_2^\mu \sim Q(\lambda,\lambda,\lambda)$$

$$l_1^2 \sim \lambda^2 Q^2, \quad l_2^2 \sim \lambda^2 Q^2, \quad l_1 \cdot l_2 \sim \lambda^2 Q^2,$$
$$l_1 \cdot q_2 \sim \lambda^2 Q^2, \quad l_1 \cdot q_3 \sim \lambda^2 Q^2, \quad l_1 \cdot q_4 \sim \lambda^2 Q^2,$$
$$l_2 \cdot q_2 \sim \lambda^2 Q^2, \quad l_2 \cdot q_3 \sim \lambda^2 Q^2, \quad l_2 \cdot q_4 \sim \lambda^2 Q^2,$$

Inserting into amplitude, projecting form factors and computing traces
Numerator gives a λ^2 suppression for all soft/collinear regions

Consistent with the result of Steinhauser et al. for the $m_t \to 0$ limit

Davies, Mishima, Steinhauser, Wellmann 18;

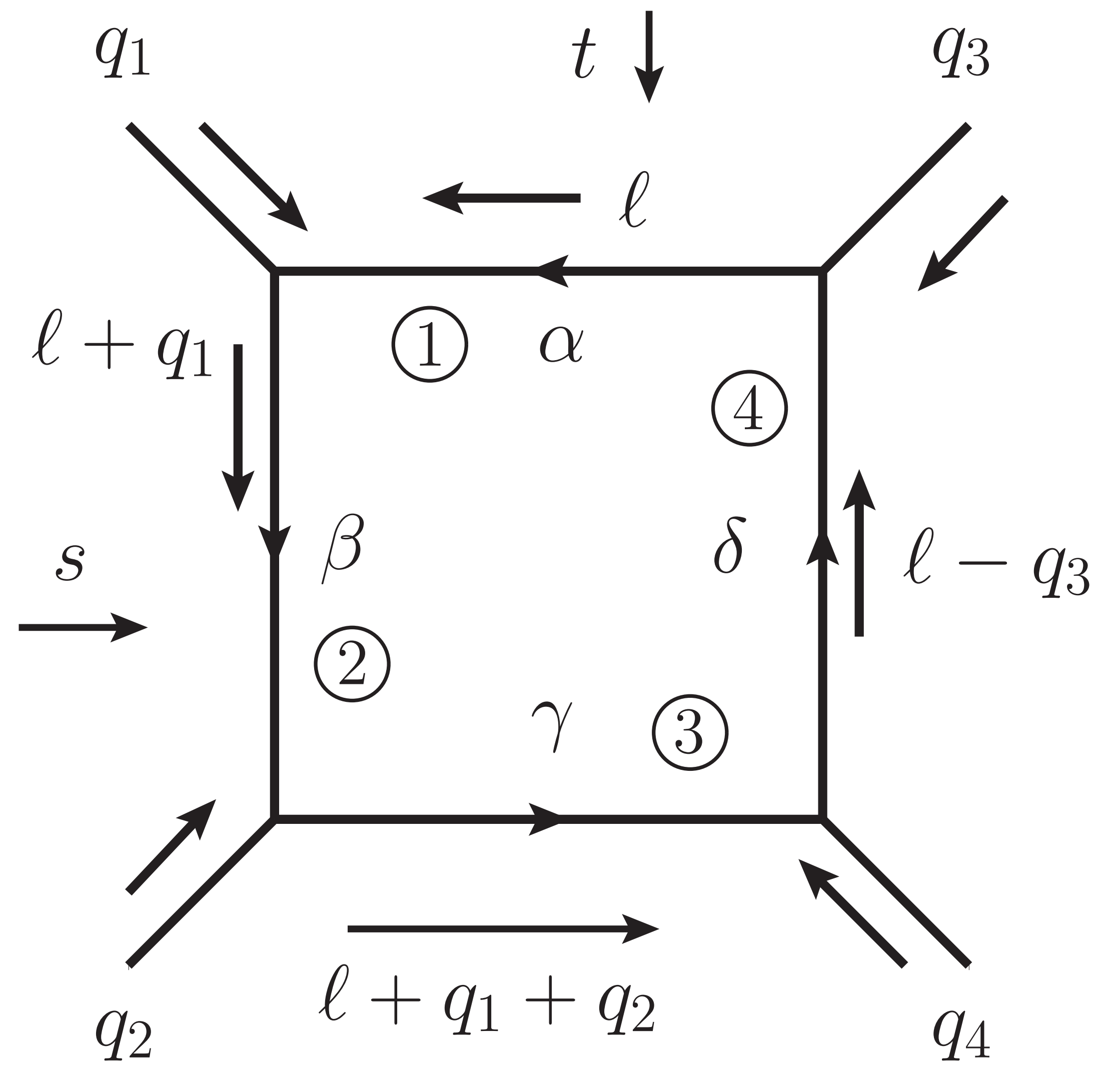
Suggests that regions other than the hard region are **helicity suppressed** by at least $\lambda \sim m_t$

Expansion of $gg \to HH$ @ 1-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

Limit: $s, |t|, |u| \gg m_t^2 \gg m_H^2\,,\quad m_H^2 \to 0$ and $\lambda \sim m_t/Q$

Kinematics: $s + t + u = 0, \quad m_H = 0$



$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{\ell^2 - m_t^2} \frac{1}{(\ell+q_1)^2 - m_t^2} \frac{1}{(\ell+q_1+q_2)^2 - m_t^2} \frac{1}{(\ell-q_3)^2 - m_t^2}$$

↓

Hard region $\mathbf{u}^{(0)} = (0,0,0,0)$

↓

Every propagator scales as λ^0

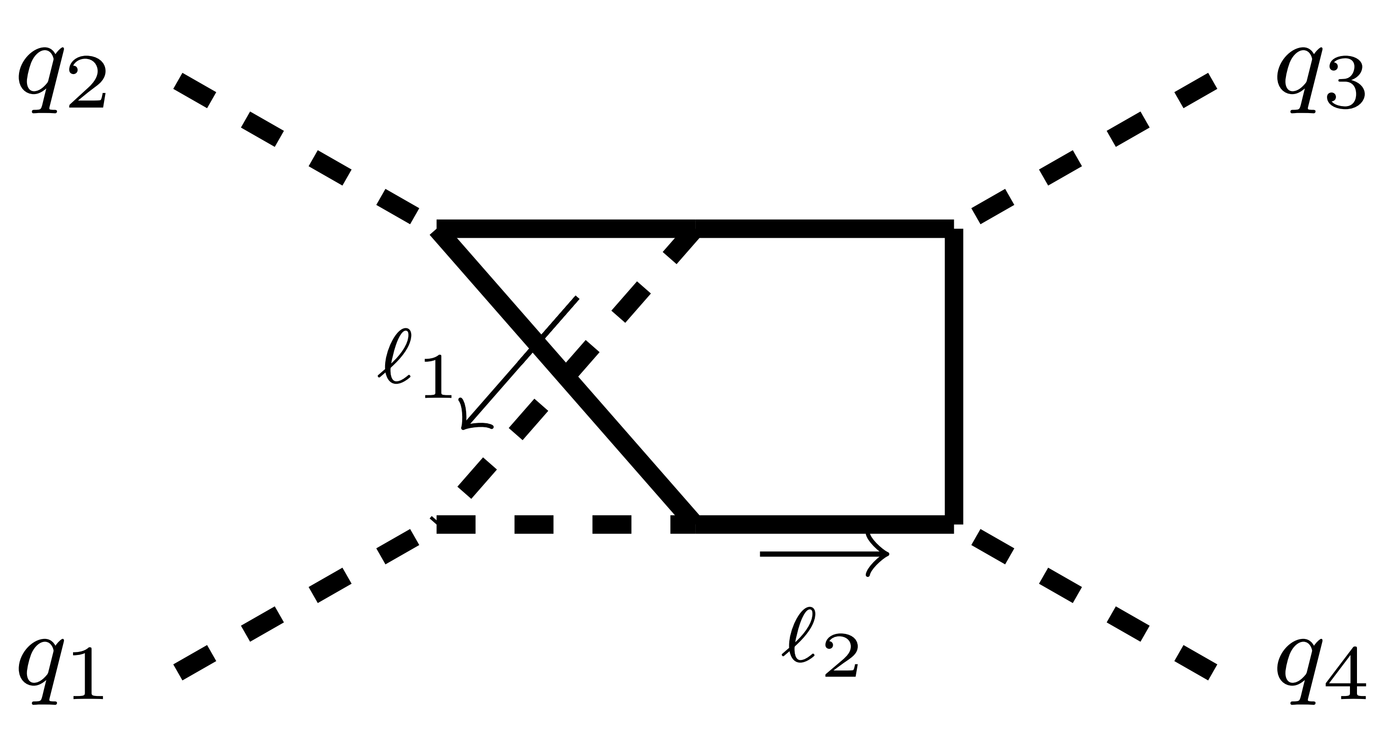
Achieved by **hard scaling** of the loop momenta $\ell^\mu = Q(1,1,1)$

↓

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{\ell^2} \frac{1}{(\ell+q_1)^2} \frac{1}{(\ell+q_1+q_2)^2} \frac{1}{(\ell-q_3)^2} \sim \lambda^0$$

Expansion of $gg \to HH$ @ 2-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25



New features:

1. Soft modes appear $l_S^\mu = Q(\lambda, \lambda, \lambda)$
2. Soft regions are power enhanced at level of scalar integral

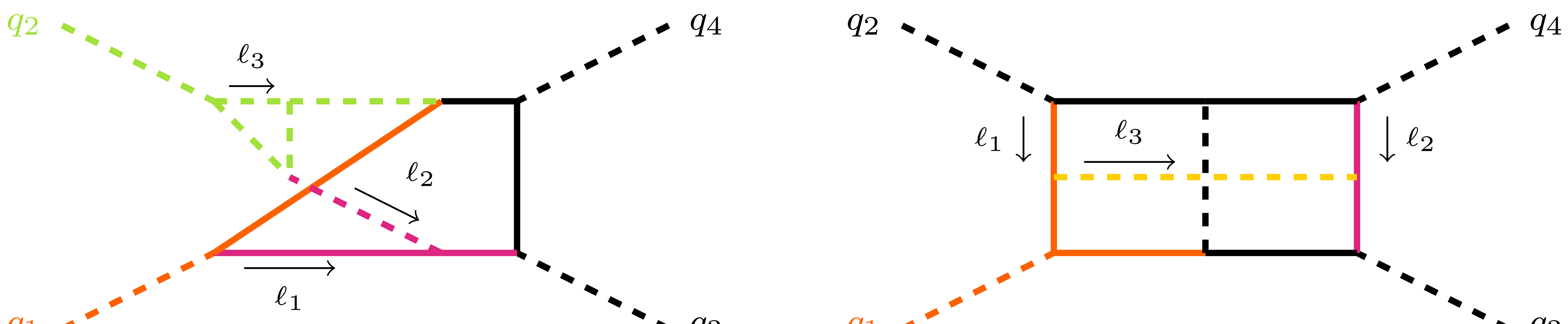
$\mathbf{u}^R$	order	interpretation	routing
(-2, -2, -2, 0, 0, 0, -2)	-4ϵ	c_1c_1	$\ell_1,\ \ell_2$
(-2, -2, 0, 0, -2, -2, 0)	-4ϵ	c_1c_1	$\ell_1,\ \ell_2 - q_3 - q_4$
(-2, -1, 0, -1, -2, -2, -1)	$-1-4\epsilon$	ss	$\ell_1,\ \ell_2 - q_3 - q_4$
(-2, 0, 0, -2, -2, -2, 0)	-4ϵ	c_3c_3	$\ell_1,\ \ell_2 - q_4$
(-2, 0, 0, 0, -2, -2, -2)	-4ϵ	c_2c_2	$\ell_1,\ \ell_2 - q_3 - q_4$
(-1, -2, -2, -1, 0, -1, -2)	$-1-4\epsilon$	ss	$\ell_1 - q_1,\ \ell_2$
(0, -2, -2, -2, 0, 0, -2)	-4ϵ	c_4c_4	$\ell_1 - q_1,\ \ell_2$
(0, -2, -2, 0, 0, -2, -2)	-4ϵ	c_2c_2	$\ell_1 - q_1,\ \ell_2$
(0, 0, -2, -2, 0, -2, -2)	-4ϵ	$c_4\bar{c}_2$	$\ell_1 - \ell_2 + q_3 + q_4,\ \ell_1$
(0, 0, -2, -2, 0, 0, 0)	-2ϵ	c_4h	$\ell_1 - \ell_2 + q_3 + q_4,\ \ell_1$
(0, 0, 0, -2, -2, -2, -2)	-4ϵ	$c_3\bar{c}_2$	$\ell_1 - \ell_2 + q_3,\ \ell_1 - q_4$
(0, 0, 0, -2, -2, 0, 0)	-2ϵ	c_3h	$\ell_1 - \ell_2 + q_3,\ \ell_1 - q_4$
(0, 0, 0, 0, 0, -2, -2)	-2ϵ	hc_2	$\ell_1,\ \ell_1 + \ell_2 - q_3 - q_4$
(0, 0, 0, 0, 0, 0, 0)	0	hh	n/a

Can again find momentum space interpretation

Expansion of $gg \to HH$ @ 3-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

Considering $gg \to HH$ at 3-loops we systematically checked for new loop momenta modes



Indeed find new modes entering

Hard-collinear $l^\mu_{HC_i} = Q(1, \lambda, \lambda^{\frac{1}{2}})$

Soft-collinear $l^\mu_{SC_i} = Q(\lambda, \lambda^2, \lambda^{\frac{3}{2}})$

Ultra-soft $l^\mu_{US} = Q(\lambda^2, \lambda^2, \lambda^2)$

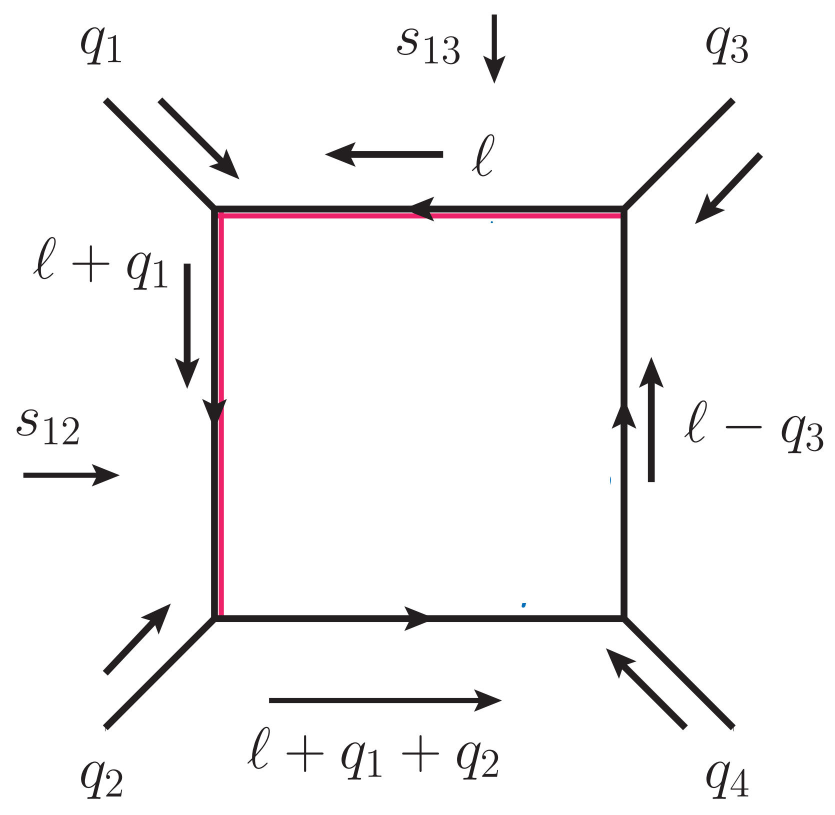
Expect new modes entering at each loop order, consistent with results in the literature

Ma 23

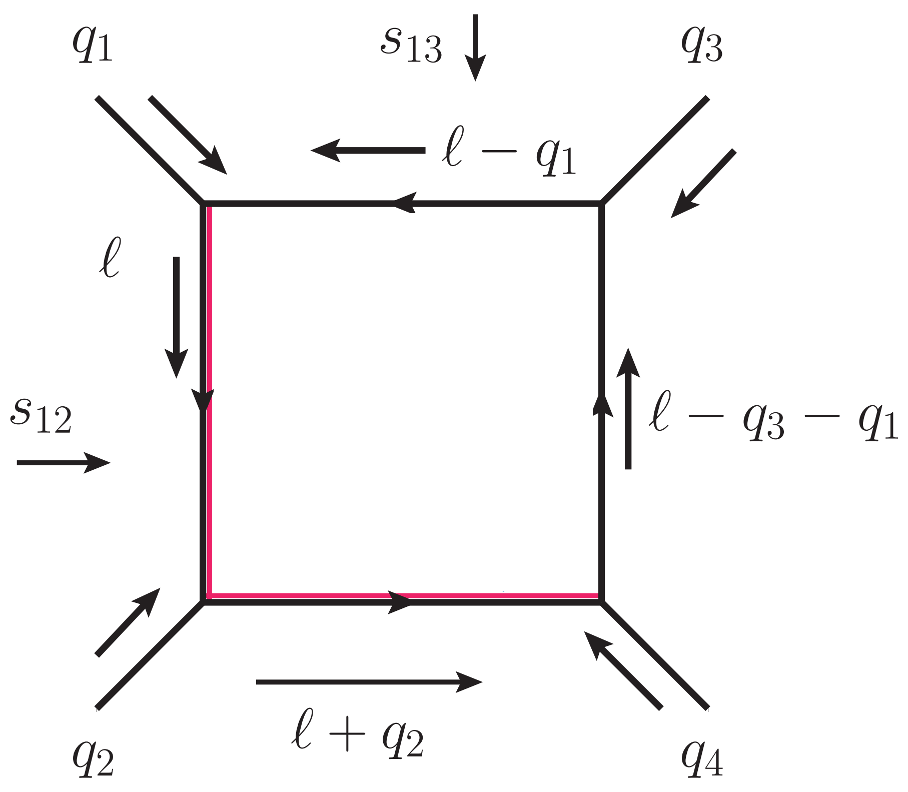
Expansion of $gg \to HH$ @ 1-loop

Jaskiewicz, SPJ, Szafron, Ulrich 25

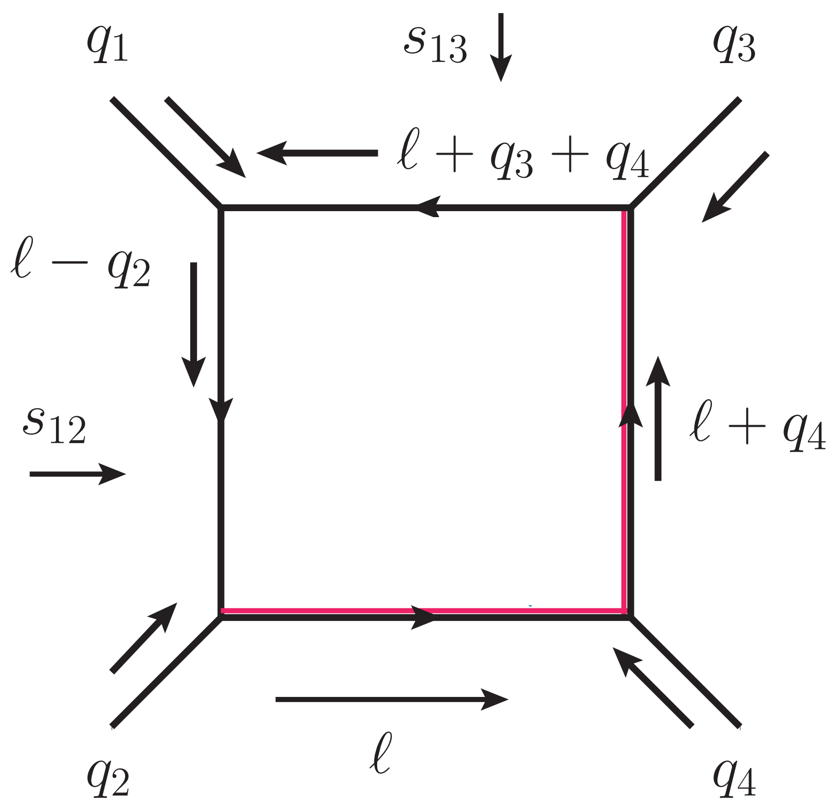
Collinear Regions



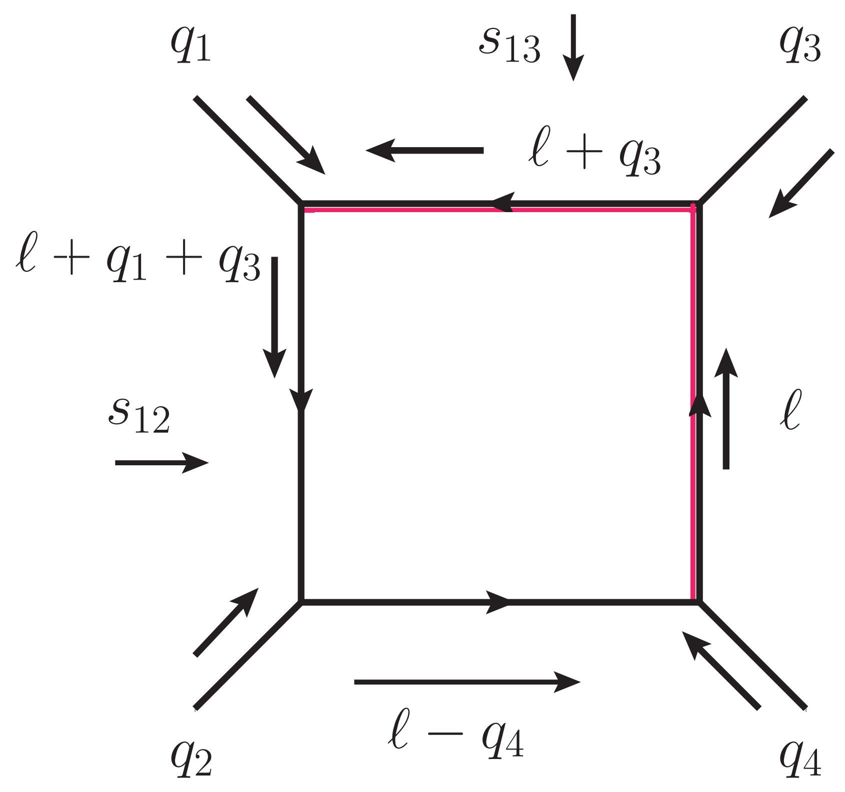
$\mathbf{u}^{(1)} = (-2,-2,0,0)$



$\mathbf{u}^{(2)} = (0,-2,-2,0)$



$\mathbf{u}^{(4)} = (0,0,-2,-2)$



$\mathbf{u}^{(3)} = (-2,0,0,-2)$

Collinear q_1 region $\quad \ell^\mu = Q(1, \lambda^2, \lambda)$

$$\int \underbrace{\frac{d^d\ell}{(2\pi)^d}}_{\lambda^4} \underbrace{\frac{1}{\ell^2 - m_t^2}}_{\frac{1}{\lambda^2}} \underbrace{\frac{1}{(\ell + q_1)^2 - m_t^2}}_{\frac{1}{\lambda^2}} \frac{1}{2(\ell + q_1)\cdot q_2} \frac{1}{-2\ell \cdot q_3}$$

Collinear regions are also leading power at the level of scalar integrals!

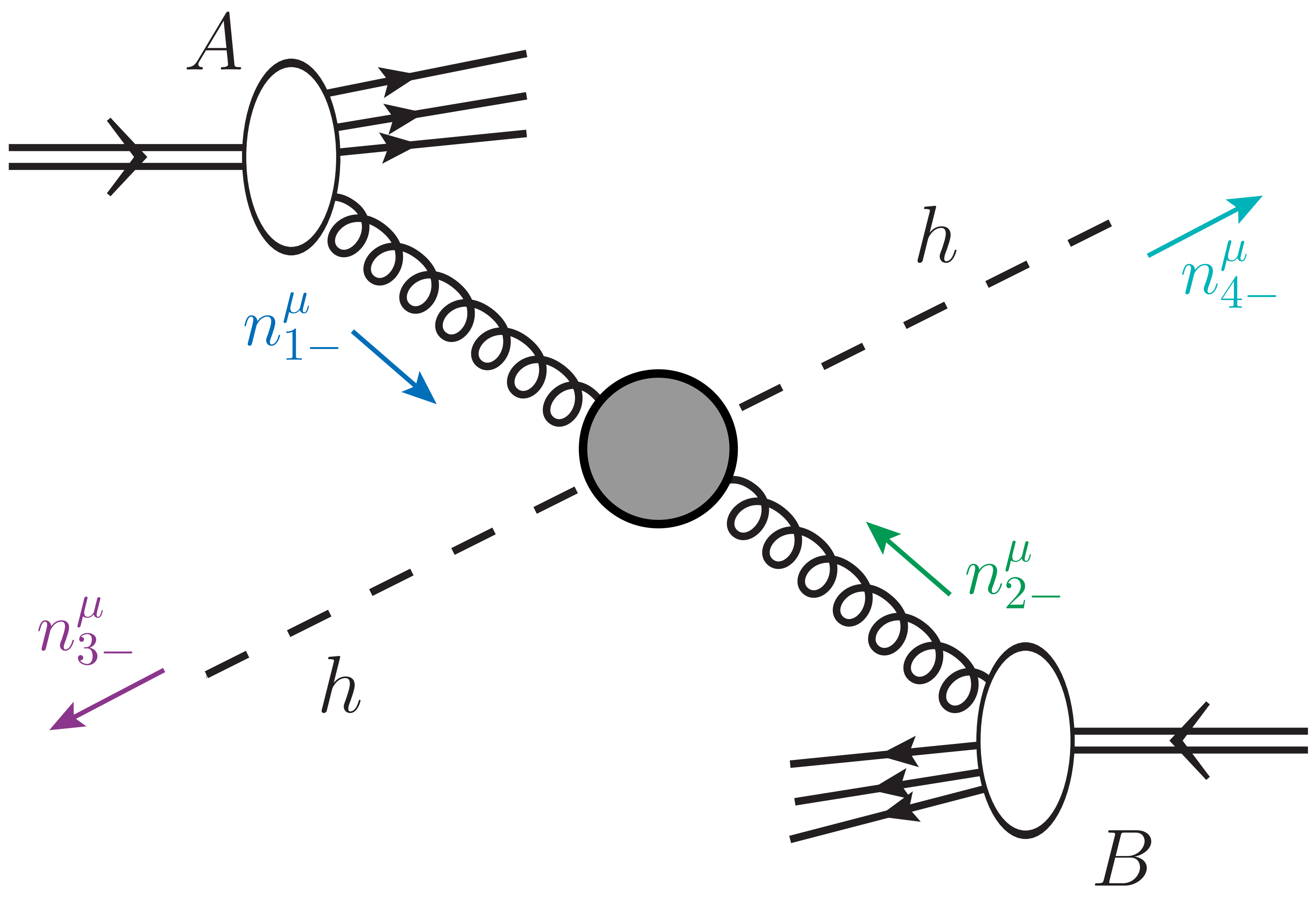
Effective field theory Analysis

Soft Collinear Effective Theory (SCET) an approximation of QCD based on soft/collinear expansion

Bauer, Fleming, Pirjol, Stewart, Beneke, Chapovsky, Diehl, Feldmann, …

$$\psi(x) \rightarrow \underbrace{\psi_1(x) + \ldots + \psi_N(x)}_{N \text{ collinear fermion fields}} + q(x)$$

$$\mathscr{L}_{\text{SCET}} = \sum_{i=1}^{N} \mathscr{L}_{c_i} + \mathscr{L}_{\text{soft}}$$



Lagrangians belong to a specific collinear direction
Can be expanded in powers of the small parameter

$$\mathscr{L}_{c_i} = \underbrace{\mathscr{L}^{(0)}_{c_i}}_{\text{LP}} + \underbrace{\mathscr{L}^{(1)}_{c_i}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathscr{L}^{(2)}_{c_i}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Generic N-jet operator has the form:

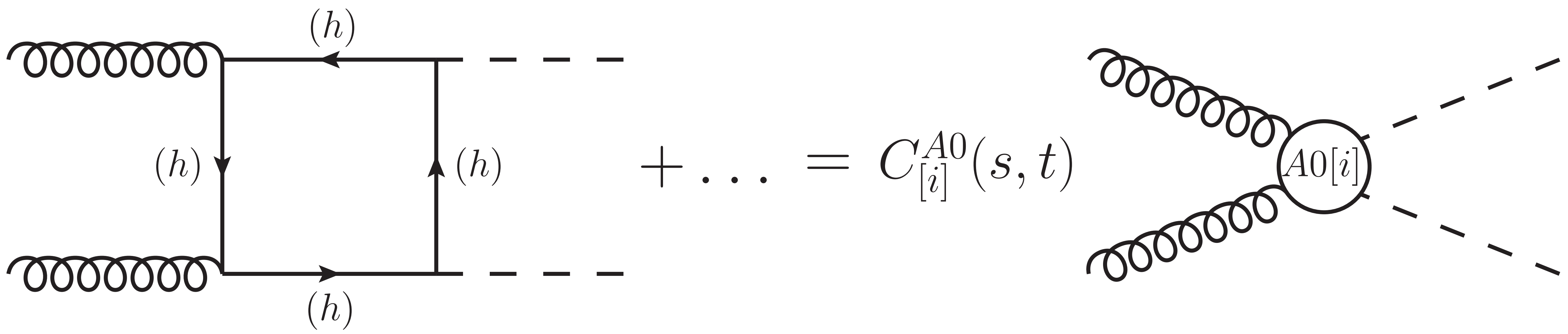
$$J = \int \left[\prod_{ik} dt_{i_k}\right] C(\{t_{i_k}\}) \prod_{i=1}^{N} J_{c_i}(t_{i_1}, t_{i_2} \ldots)$$

Beneke, Garny, Szafron, Wang, 17, 17, 18, 19

Leading Power Analysis

Jaskiewicz, SPJ, Szafron, Ulrich 25

Leading power matching $J^{[i]}_{\mathrm{LP}}(t_1,t_2,t_3,t_4) = y_t^2\, P_i^{\mu\nu}\, \mathcal{A}_{c_1\perp_1\,\mu}(t_1 n_{1+})\, \mathcal{A}_{c_2\perp_2\,\nu}(t_2 n_{2+})\, h_{c_3}(t_3 n_{3+})\, h_{c_4}(t_4 n_{4+})$



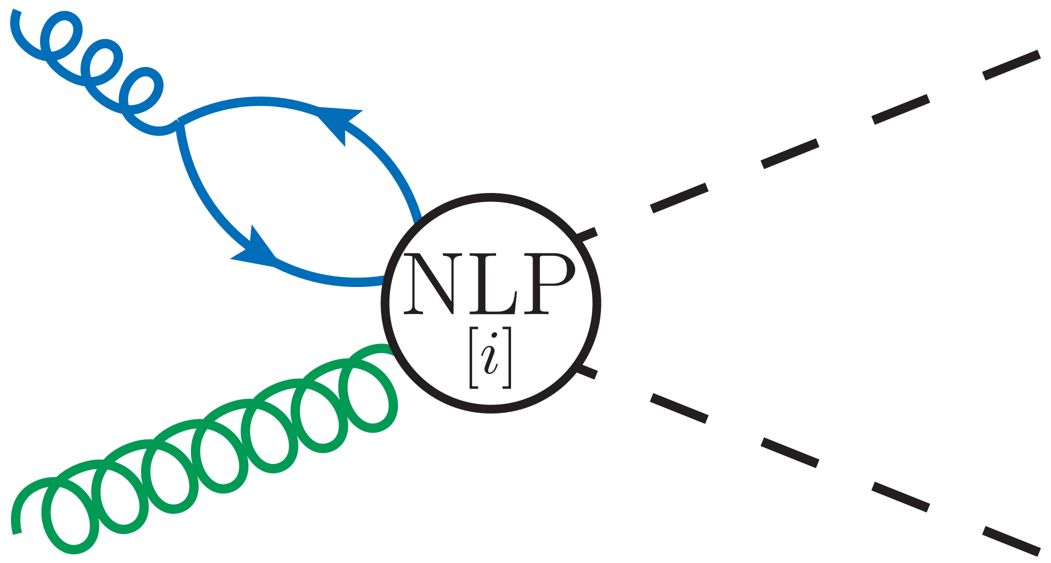
Collinear Regions c_1, c_2 $\mathcal{M}^{\mathrm{QCD}}_{\mathrm{LP}} \propto \left(\bar{v}_{c_1}(\bar{r}q_1)\frac{\not{n}_{1+}}{2}u_{c_1}(rq_1)n_{3-\nu}\varepsilon^{\nu}_{\perp_1}(q_2) + \bar{v}_{c_1}(\bar{r}q_1)\frac{\not{n}_{1+}}{2}\gamma_5 u_{c_1}(rq_1)n^{\mu}_{3-}i\epsilon^{\perp_1}_{\mu\nu}\varepsilon^{\nu}_{\perp_1}(q_2) \right)$

Relevant operator structures are scalar/pseudoscalar

$$J_{S_i}(\{t_{i_1},t_{i_2}\}) = \bar{\chi}_{c_i}(t_{i_2}n_{i+})\frac{\not{n}_{i+}}{2}\,\chi_{c_i}(t_{i_1}n_{i+}),$$

$$J_{P_i}(\{t_{i_1},t_{i_2}\}) = \bar{\chi}_{c_i}(t_{i_2}n_{i+})\frac{\not{n}_{i+}}{2}\,\gamma_5\,\chi_{c_i}(t_{i_1}n_{i+}),$$

Mixing with the external gluon is forbidden at LP



Leading Power Analysis

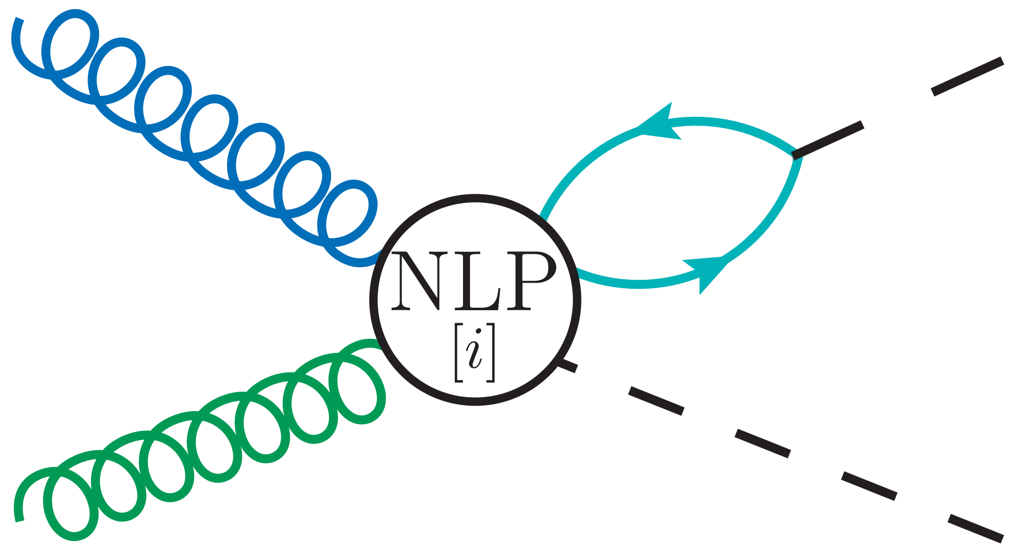
Jaskiewicz, SPJ, Szafron, Ulrich 25

Collinear Regions c_3, c_4

$$\mathcal{M}^{\rm QCD}_{\rm LP} \sim i g_s^2 \mathbf{T}^B\, \mathbf{T}^A \left[\frac{g_W y_t}{2}\right] \bar{v}_{c_3}(q') \frac{1}{\bar{r}(n_{3+}q_3)} \left[\frac{2 n_{3-\mu}}{(n_{1+}q_1) n_{3-} \cdot n_{1-}} \frac{\not{n}_{3+}}{2} \gamma_{\nu\perp_3} u_{c_3}(q) + \frac{1}{r(n_{3+}q_3)} n_{3+\nu} \frac{\not{n}_{3+}}{2} \gamma_{\mu\perp_3} u_{c_3}(q) \right] \varepsilon^{\nu}_{\perp_2}(q_2)\ \varepsilon^{\mu}_{\perp_1}(q_1)$$

Situation reversed, structures appearing at LP are vector-like

Mixing with the external Higgs is forbidden at LP

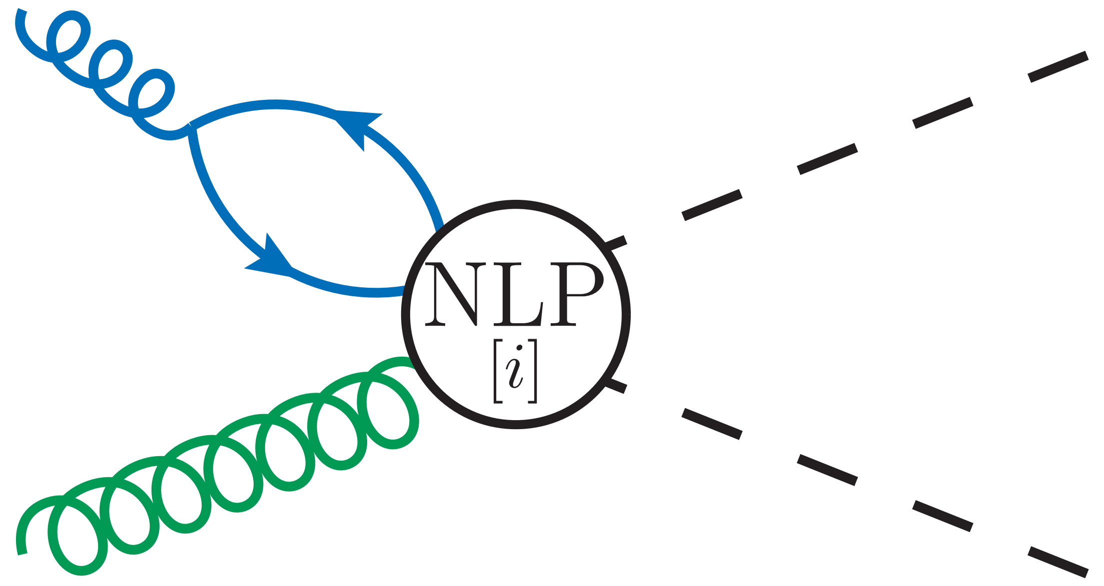


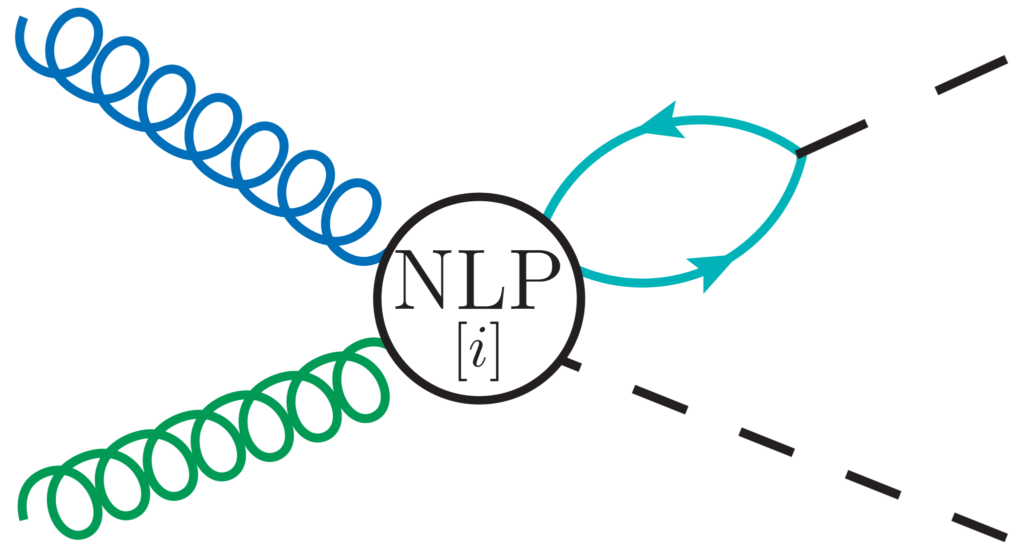
Result holds to all orders in α_s due to helicity conservation for $m_t \to 0$

Next-to-Leading Power

Structure of the amplitude allows mixing with external gluon/Higgs

Expect contributions from collinear/soft regions





Leading Power Expansion

Consider the LO and NLO finite virtual corrections

$$A_{i,j}^{\mathrm{fin}} = \frac{\alpha_s}{2\pi} A_{i,j}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 A_{i,j}^{(1)} + \mathcal{O}(\alpha_s^3)$$

Use ``SCET'' IR scheme for virtuals Becher, Neubert 09, 13;

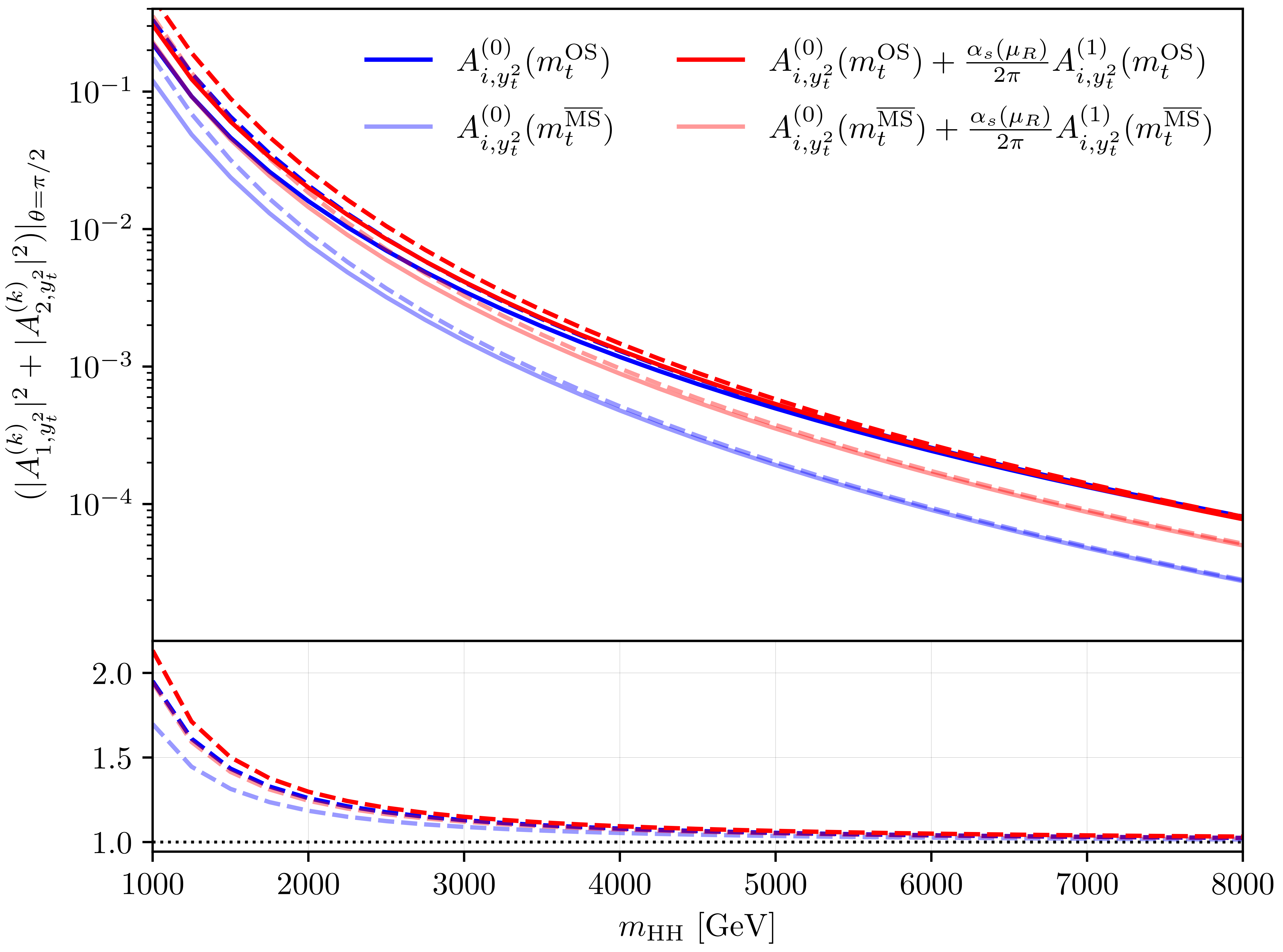
Neglecting real contributions

Solid Lines: Full TH result

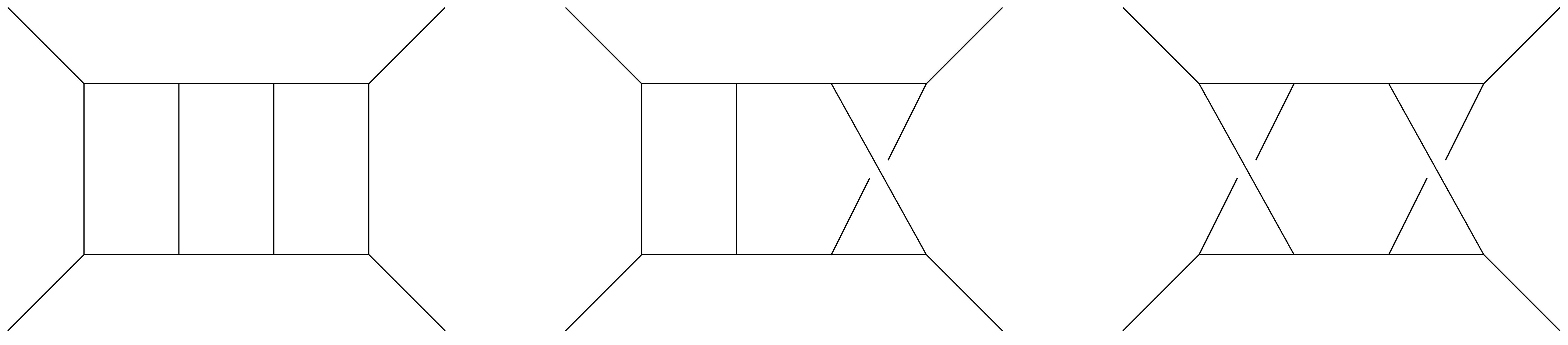
Davies, Mishima, Steinhauser, Wellmann 18;

Dashed Lines: Leading power expansion

Leading power is a good approximation for $\sqrt{s} \gtrsim 1\ \mathrm{TeV}$, let us focus on the **very high energy** behaviour of the **amplitude**



3-loop Massless Limit (c_2)



PL	NPL1	NPL2
k_1^2	k_1^2	k_1^2
k_2^2	k_2^2	k_2^2
k_3^2	k_3^2	k_3^2
$(k_1-p_1)^2$	$(k_1-p_1)^2$	$(k_1-p_1)^2$
$(k_2-p_1)^2$	$(k_2-p_1)^2$	$(k_2-p_1)^2$
$(k_3-p_1)^2$	$(k_3-p_1)^2$	$(k_3-p_1)^2$
$(k_1-p_1-p_2)^2$	$(k_1-p_1-p_2)^2$	$(k_1-p_1-p_2)^2$
$(k_2-p_1-p_2)^2$	$(k_2-p_1-p_2)^2$	$(k_3-p_1-p_2)^2$
$(k_3-p_1-p_2)^2$	$(k_3-p_1-p_2)^2$	$(k_1-k_2)^2$
$(k_1-p_1-p_2-p_3)^2$	$(k_1-p_1-p_2-p_3)^2$	$(k_2-k_3)^2$
$(k_2-p_1-p_2-p_3)^2$	$(k_2-p_1-p_2-p_3)^2$	$(k_1-k_2-p_3)^2$
$(k_3-p_1-p_2-p_3)^2$	$(k_3-p_1-p_2-p_3)^2$	$(k_2-k_3+p_1+p_2+p_3)^2$
$(k_1-k_2)^2$	$(k_1-k_2)^2$	$(k_2+p_3)^2$
$(k_1-k_3)^2$	$(k_2-k_3)^2$	$(k_1-k_3)^2$
$(k_2-k_3)^2$	$(k_1-k_2+k_3)^2$	$(k_2-p_1-p_2)^2$

Bargiela, Caola, von Manteuffel, Tancredi 22;

Integral Families

15 propagators

6 independent crossings per family

486 master integrals → expressible in HPLs (or GPLs)

Henn, Mistlberger, Smirnov, Wasser 20

Amplitude $gg \to HH$

$\mathcal{O}(10^6)$ integrals

Up to 5 inverse propagators (s≤5)

3-loop Massless Limit (c_2)

Ajjath, SPJ, Magerya (WIP)

Reduce entire amplitude as a single job - impossibly slow and exhausts memory (>1 TB RAM)

Many symmetry/crossing/family relations clutter up system

Workflow

1) Separate amplitude into family, crossing, top-level sector

2) **Kira**

Generate IBPs for each top-level sector, crossing, family separately

Ensure IBPs never mix crossings/families

Maierhofer, Usovitsch, Uwer 17; Klappert, Lange, Maierhofer, Usovitsch 20; Lange, Usovitsch, Wu 23; 25

3) **Ratracer**

Record solution of system in a finite field (trace)

Rapidly replay solution for different probes (reconstruction)

Allows complete control of expressions in system and specifying precisely what to solve for

Firefly: Klappert, Lange 19; Klappert, Klein, Lange 20

Ratracer: Magerya 22

```
PL[1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 0]*(-1)
masterCan@1*(2240/(9*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@2*(-320/(3*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@3*(-3584/(3*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@4*(1024/(3*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@5*(160/(3*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@7*(1280/(9*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@8*(-1280/(9*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@9*(640/(9*(-4 + d)^6*s12^4*(-1 - s13/s12)))
masterCan@10*(-1120/(3*(-4 + d)^6*s12^4*(-1 - s13/s12)))
...
```