Dipartimento di Fisica "E. Fermi"
Università di Pisa

Istituto Nazionale di Fisica Nucleare

(Alcuni) Risultati e prospettive sulla fisica dei buchi neri

Walter Del Pozzo
University of Pisa

Gravitational waves

- Einstein 1916-18: quadrupolar radiation in linear theory

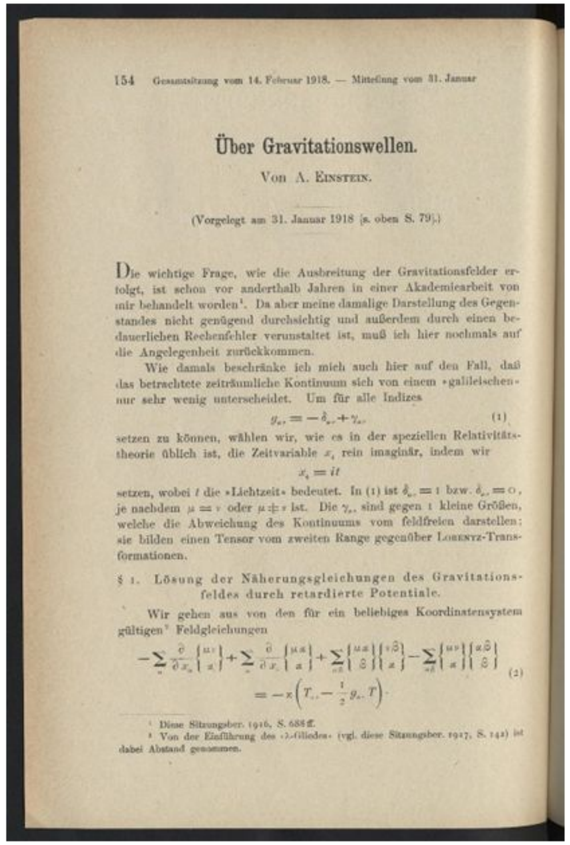
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Generated by time varying quadrupoles (at LO)

$$h \simeq \frac{1}{r}\frac{G}{c^4}\ddot{Q}$$

- Travel at the speed of light

$$v_{GW} = c$$



154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_\sigma \frac{\partial}{\partial x_\sigma}\left\{\begin{matrix}\mu\nu\\ \sigma\end{matrix}\right\} + \sum_{\alpha\beta}\frac{\partial}{\partial x_\nu}\left\{\begin{matrix}\mu\alpha\\ \alpha\end{matrix}\right\} + \sum_{\alpha\beta}\left\{\begin{matrix}\mu\alpha\\ \beta\end{matrix}\right\}\left\{\begin{matrix}\nu\beta\\ \alpha\end{matrix}\right\} - \sum_{\alpha\beta}\left\{\begin{matrix}\mu\nu\\ \alpha\end{matrix}\right\}\left\{\begin{matrix}\alpha\beta\\ \beta\end{matrix}\right\} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.
[2] Von der Einführung des »Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

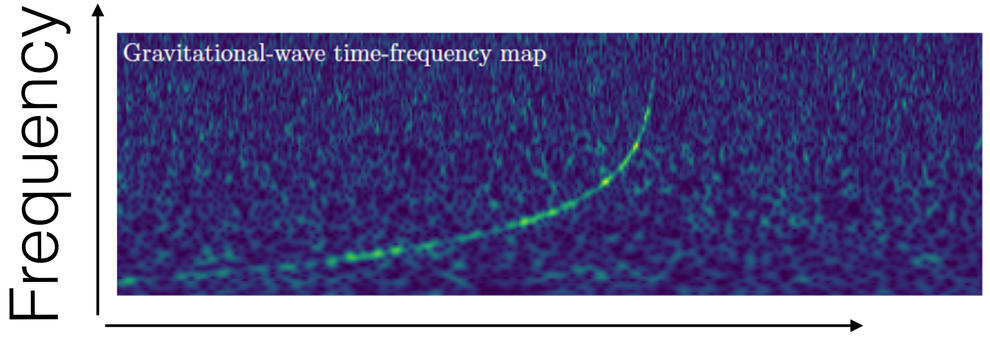
Binary systems

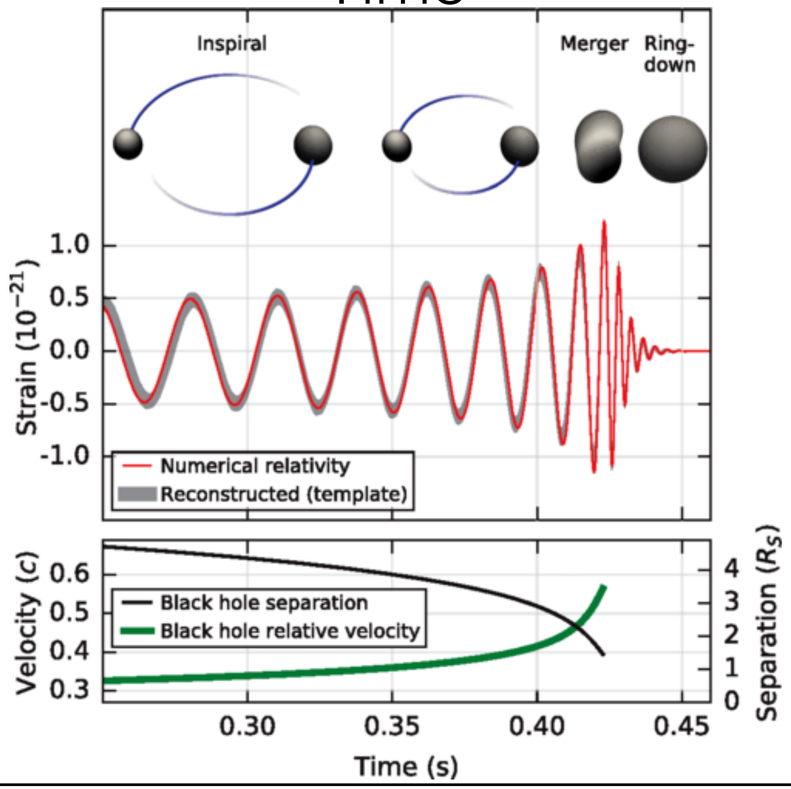
- Binary systems emit GW because of their time varying quadrupole moment
- Radiation emitted at twice the orbital frequency
- Quadrupolar emission pattern
- Evolution of the frequency with time including radiation back-reaction

$$E = -\frac{Gm_1m_2}{2R},\ F = \frac{32}{5}\frac{G^4}{c^5}\frac{m_1^2m_2^2(m_1+m_2)}{R^5}$$

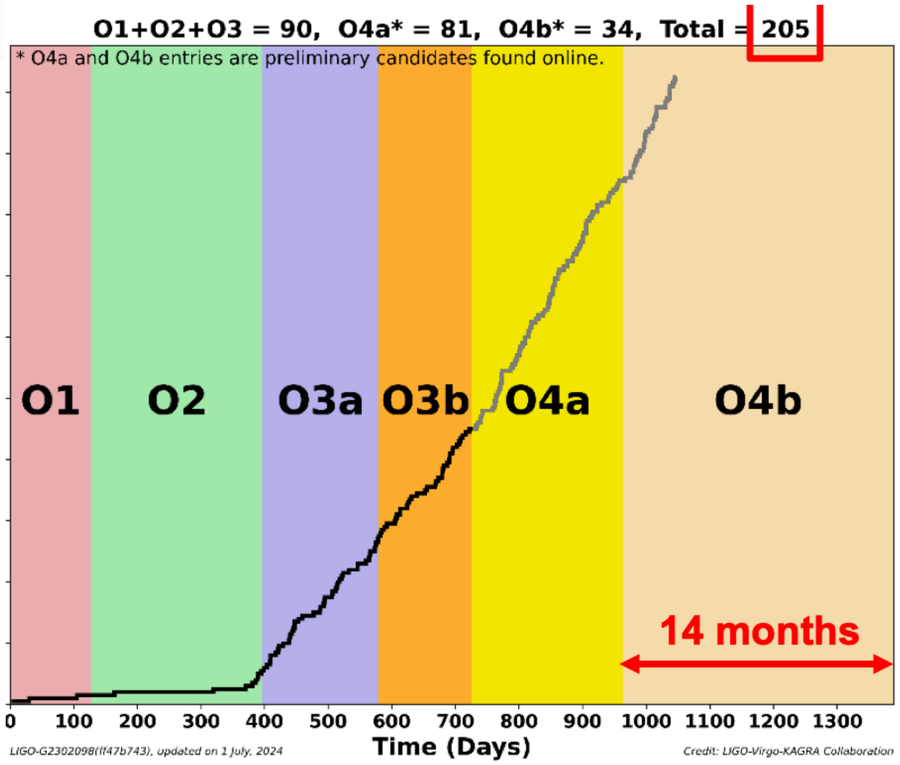
$$\frac{dE}{dt} = -F$$

- From Kepler's law $\omega^2 = G(m_1+m_2)R^{-3}$ is an equation for the frequency evolution

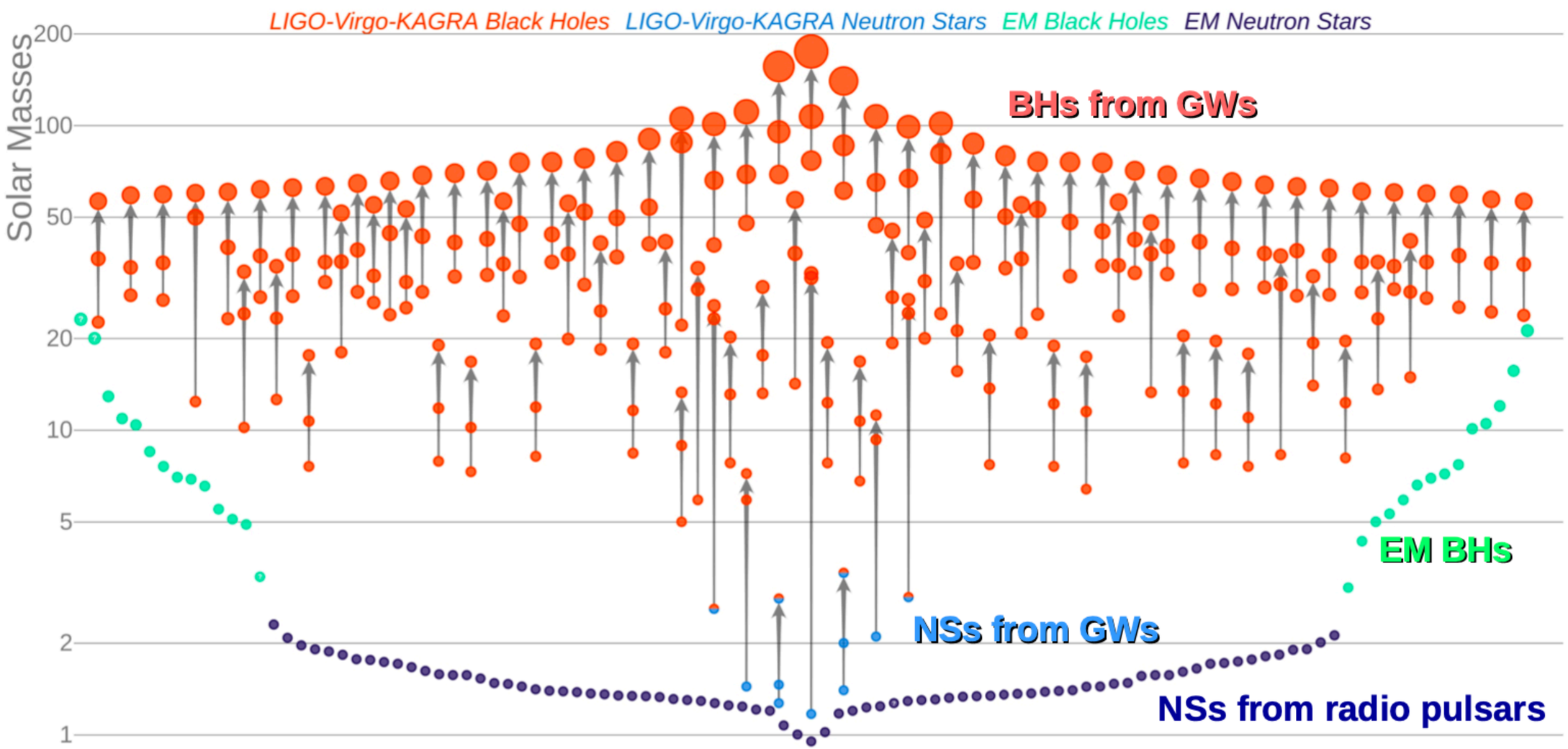




Where are we now - ground-based



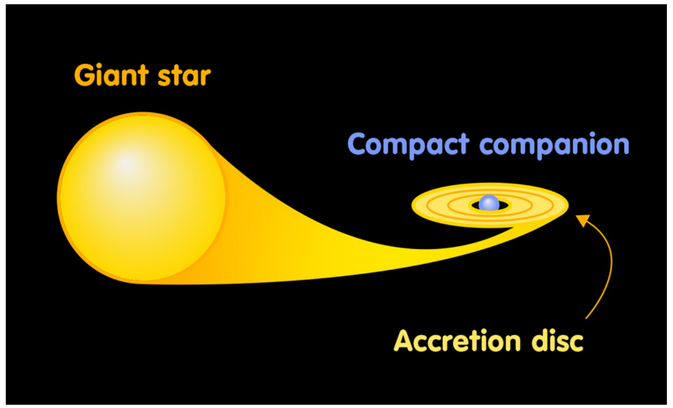
The GW Universe



credit: LIGO-Virgo-KAGRA | A. Geller | Northwestern

Where do BHs come from?

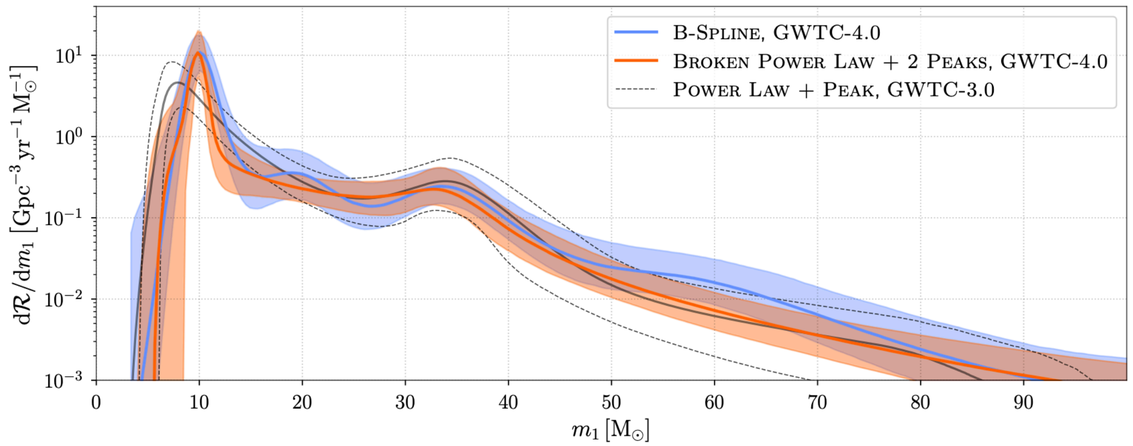
- Isolated/Field
  - common envelope
  - stable mass transfer
- Dynamical/Cluster
  - globular cluster
  - young star clusters
  - AGN disk

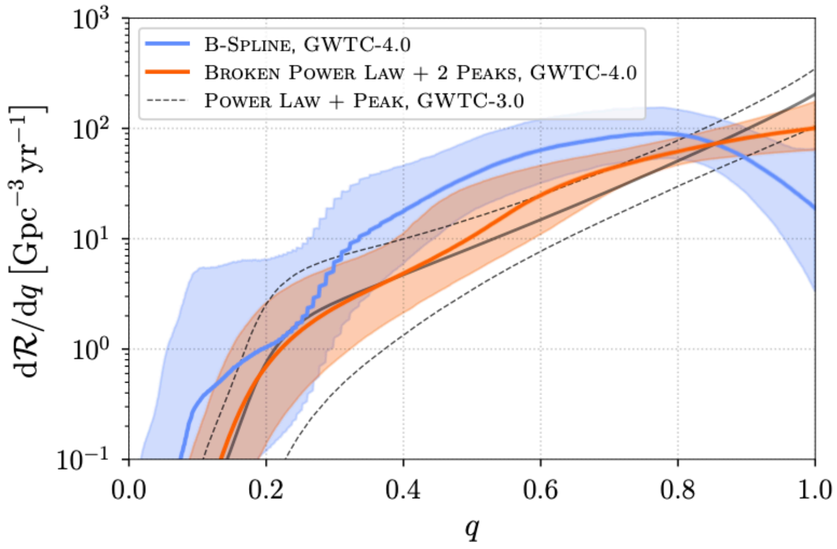


- Primordial black holes
- Triples/quadruples
- Hierarchical

Mass distribution

- LVK phenomenological modeling
- Primary mass modelled as a broken power-law plus peak (Fishbach & Holz 2017, Talbot & Thrane 2018)
- Feature at ~ 10 and 35 $M_\odot$
- Preference for nearly equal mass

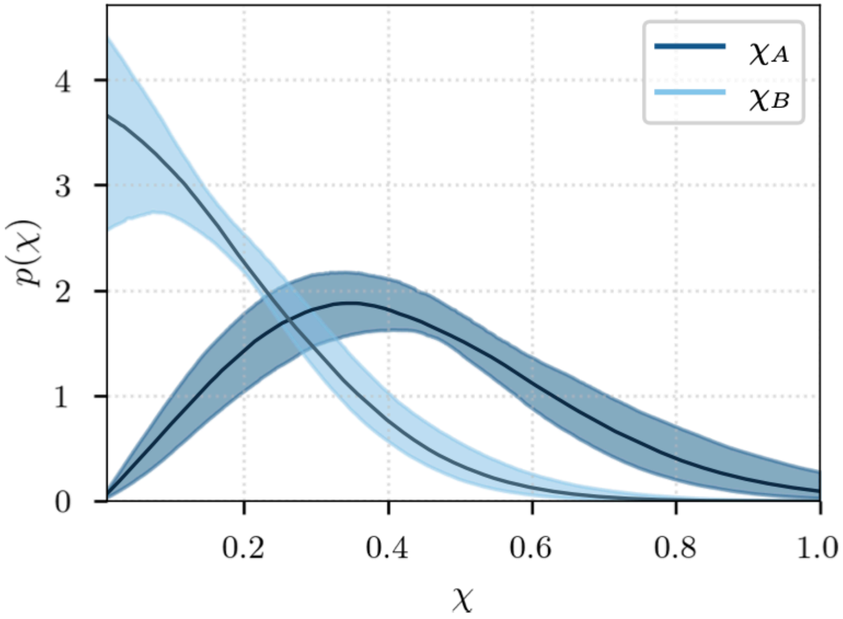


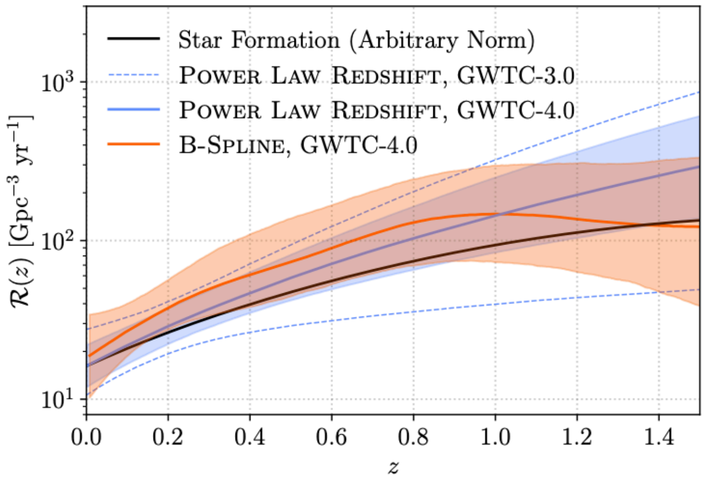


LVK, arXiv:2508.18083

Spin and redshift

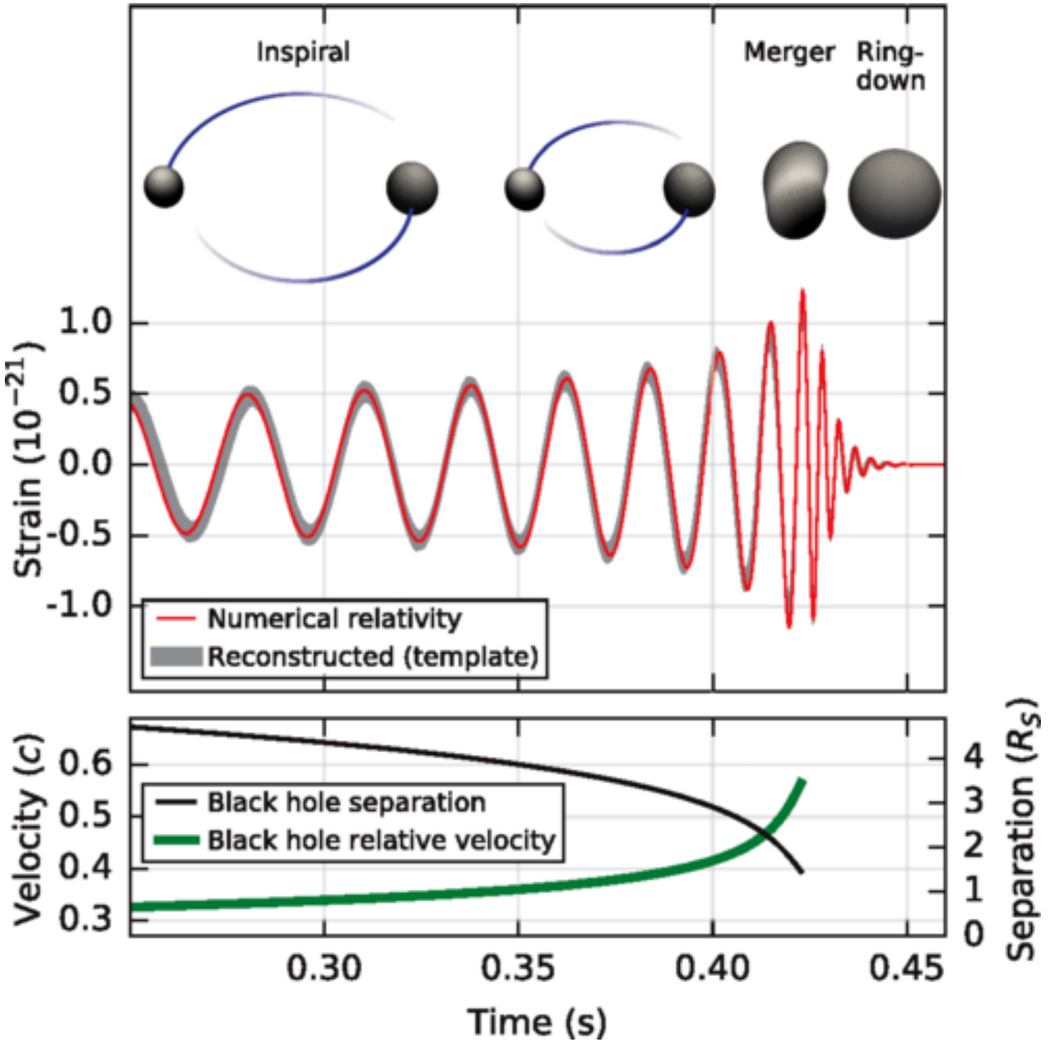
- Primary spin ~ 0.4
- Redshift distribution rules out uniform-in-comoving-volume distribution
  - Evidence for astrophysical evolution
  - SFR?





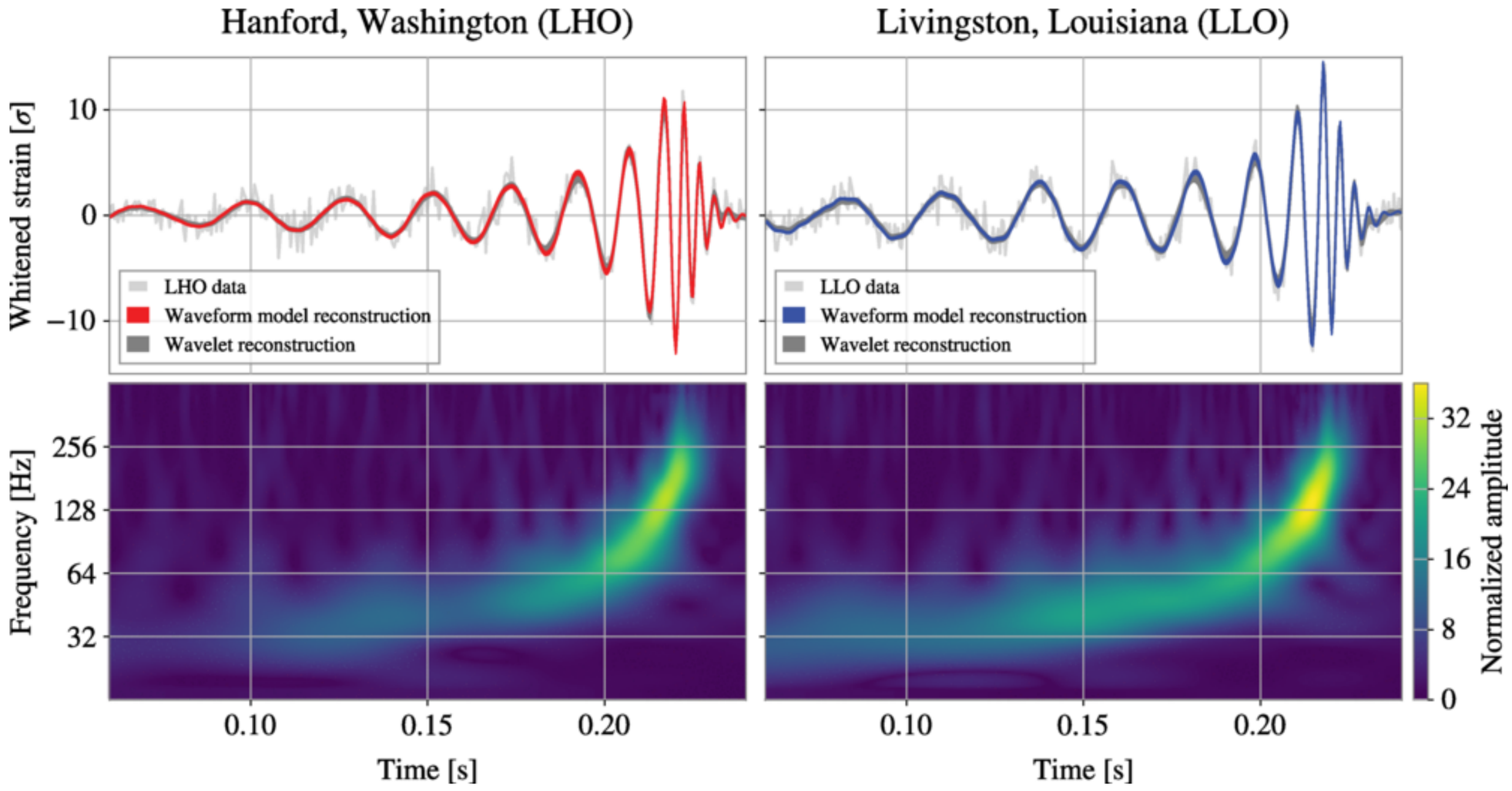
Fundamental physics

- In GR, gravitational waves (GW) are wave solutions to Einstein's equations generated from accelerating masses, propagating at the speed of light
- Shape of GW signal carries information about
  - binary dynamics and component nature
  - non-linear dynamics of space-time
  - final object nature



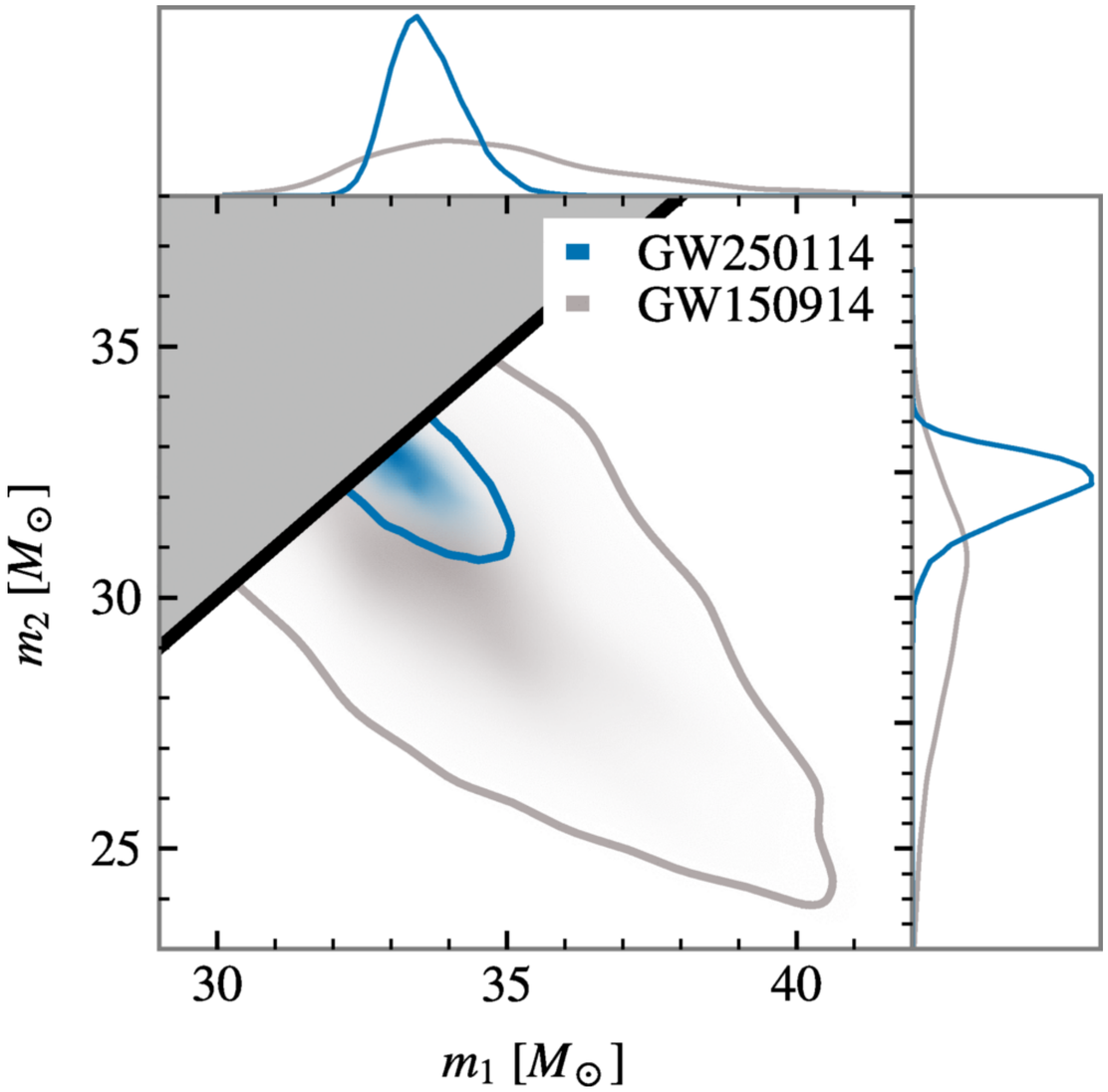
LVC, arXiv:1602.03837

GW250114


Hanford, Washington (LHO)
Livingston, Louisiana (LLO)
Whitened strain [σ]
10
0
−10
LHO data
Waveform model reconstruction
Wavelet reconstruction
LLO data
Waveform model reconstruction
Wavelet reconstruction
Frequency [Hz]
256
128
64
32
0.10
0.15
0.20
Time [s]
Normalized amplitude
32
24
16
8
0

GW250114

- SNR ~ 80
  - Accurate measurement of signal properties
  - Fundamental physics test



Laws of black hole dynamics

- 0th: The horizon has constant surface gravity for a stationary black hole
- 1st: For perturbations of stationary black holes, the change of energy is related to change of area, angular momentum, and electric charge
- 2nd: The horizon area is, assuming the weak energy condition, a non-decreasing function of time
- 3rd: It is not possible to form a black hole with vanishing surface gravity

Bardeen, J. M.; Carter, B.; Hawking, S. W. (1973)

Laws of black hole dynamics

- 0th: The horizon has constant surface gravity for a stationary black hole
- 1st: For perturbations of stationary black holes, the change of energy is related to change of area, angular momentum, and electric charge
- 2nd: The horizon area is, assuming the weak energy condition, a non-decreasing function of time
- 3rd: It is not possible to form a black hole with vanishing surface gravity

Bardeen, J. M.; Carter, B.; Hawking, S. W. (1973)

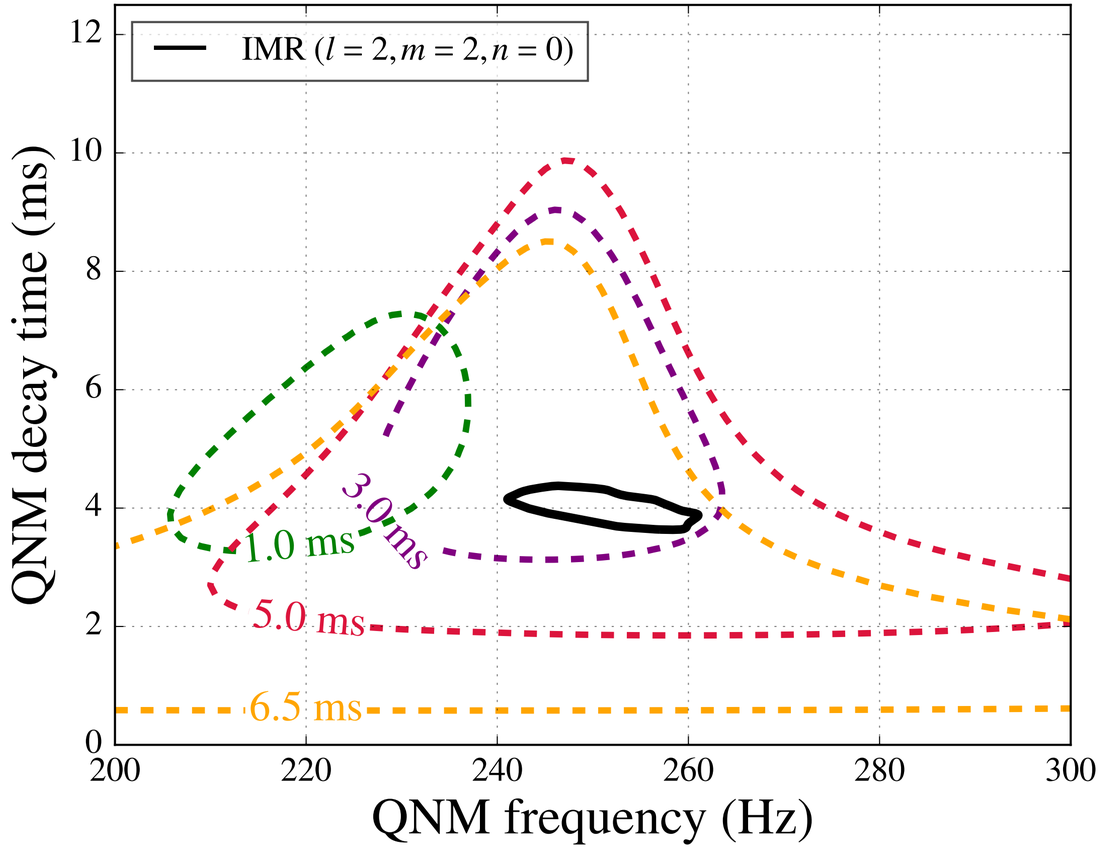
The nature of the final object

- BH perturbation theory predicts that BHs “vibrate”:

$$h(t) = \sum_{nlm} A_{nlm} e^{-\frac{t-t_0}{\tau_{nlm}}} \cos(\omega_{nlm}(t-t_0) + \varphi_{nlm})$$

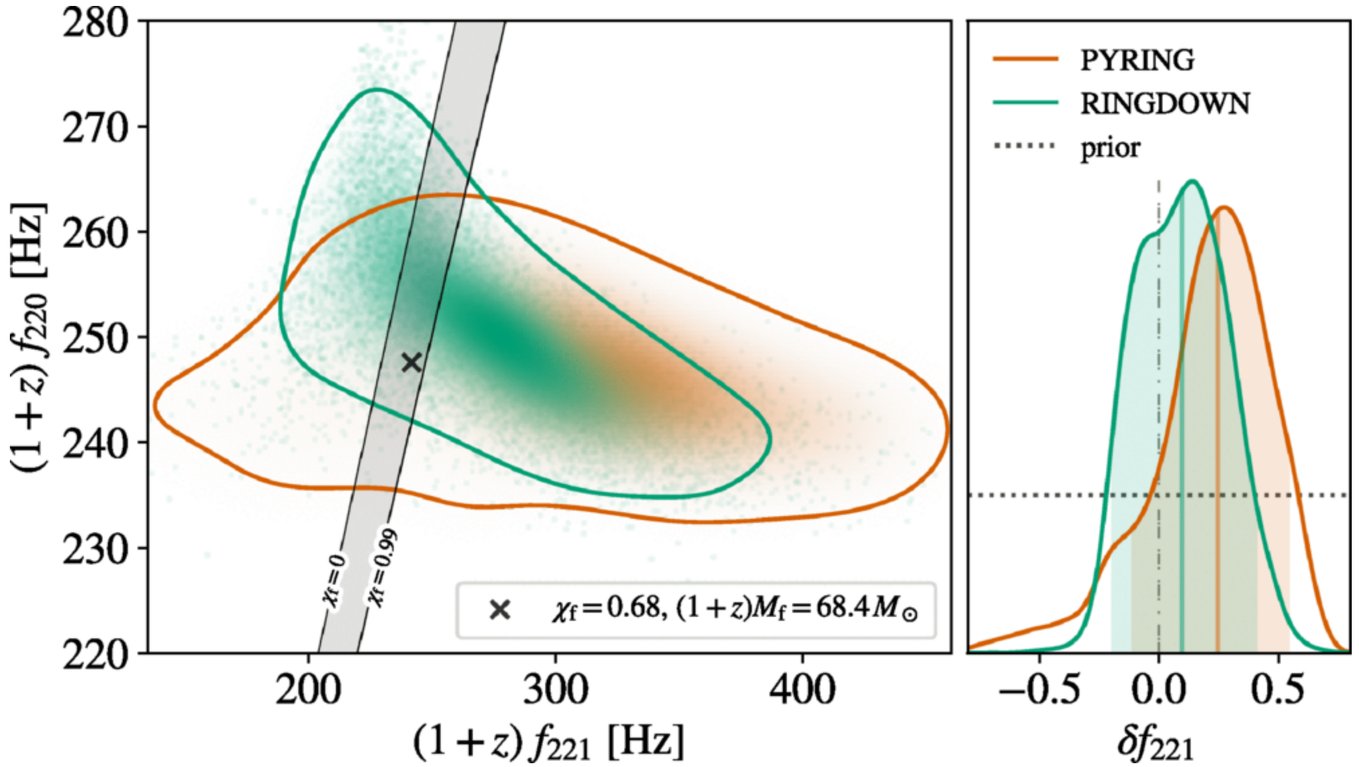
- Central frequencies ω_{nlm} and decay times τ_{nlm} are functions of BH mass and spin only (manifestation of the BH uniqueness hypothesis, Berti et al, arXiv:0512160)

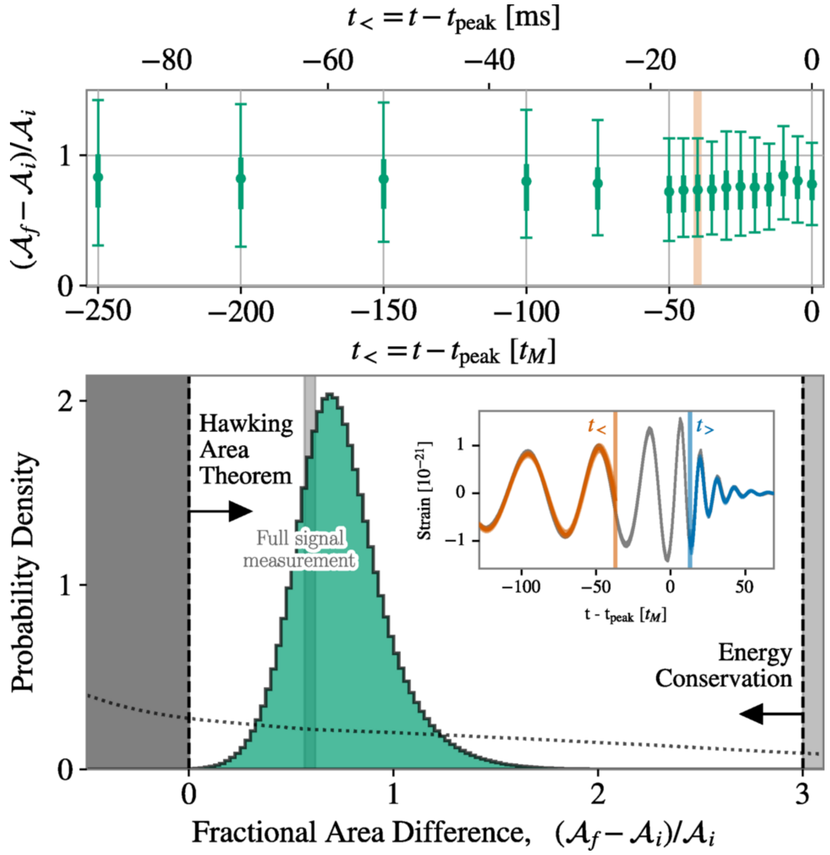
- First observation: GW150914



LVC,arXiv:1602.03841

GW250114 Ringdown and 2nd law



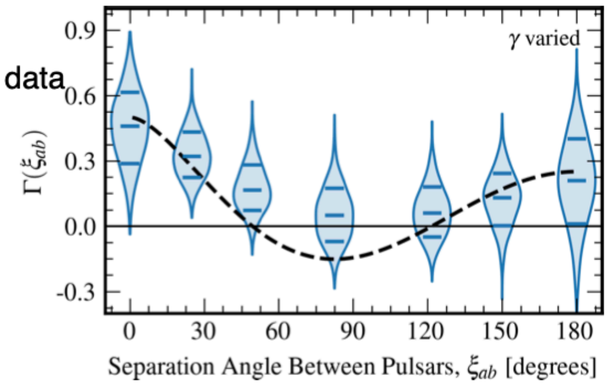


LVK, Phys. Rev. Lett. 135, 2025

Pulsar timing arrays

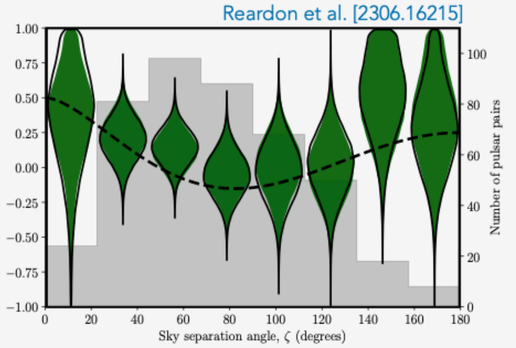
NANOGRav:
68 pulsars, 16 yrs of data
$\sim 3-4\sigma$ significance

Agazie et al. [2306.16213]



PPTA:
32 pulsars, 18 yrs of data
$\sim 2\sigma$ significance

Reardon et al. [2306.16215]

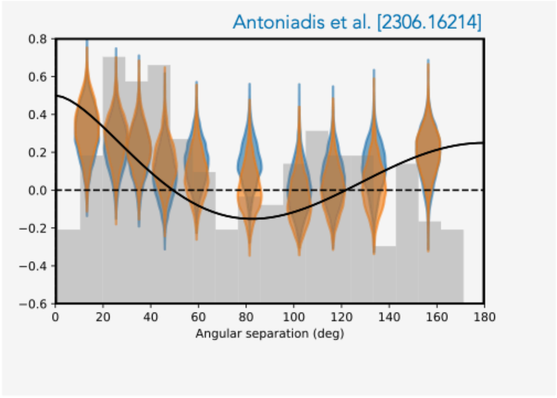


First Detection of a SGWB

EPTA+InPTA:
25 pulsars, 24 yrs of data

$\sim 3\sigma$ significance

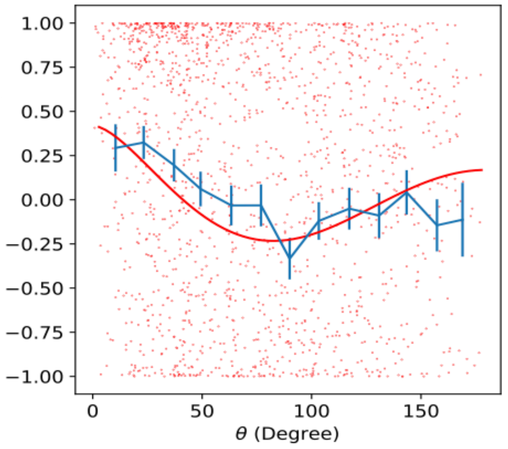
Antoniadis et al. [2306.16214]



CPTA:
57 pulsars, 3 yrs of data

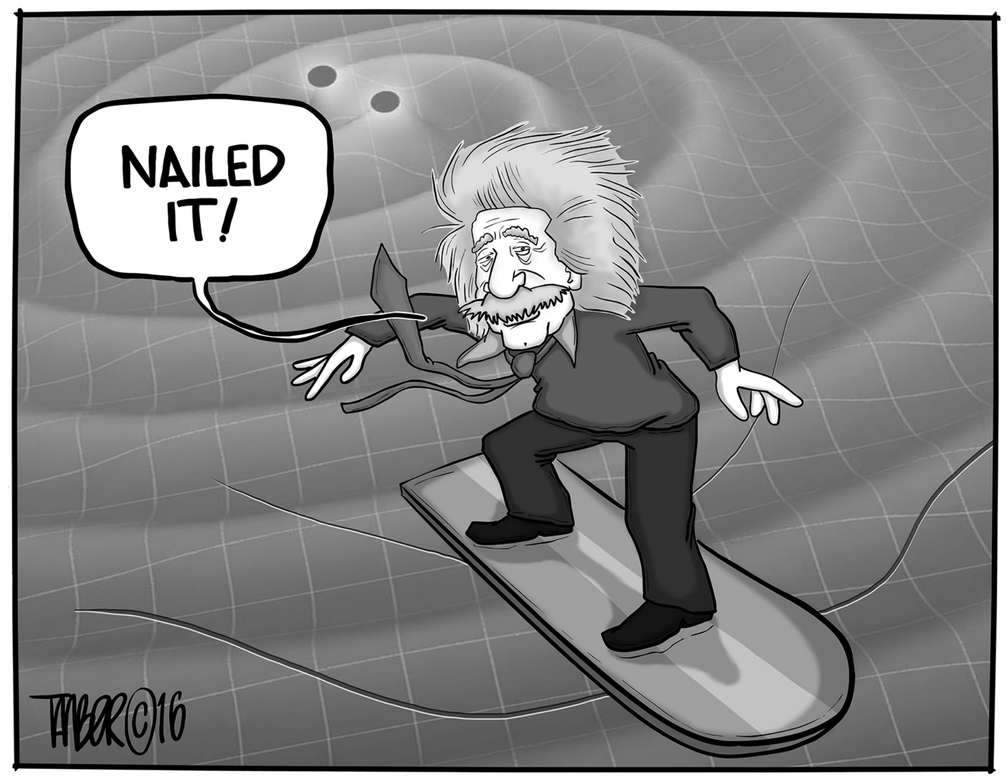
$\sim 4.6\sigma$ significance

Xu et al. [2306.16216]

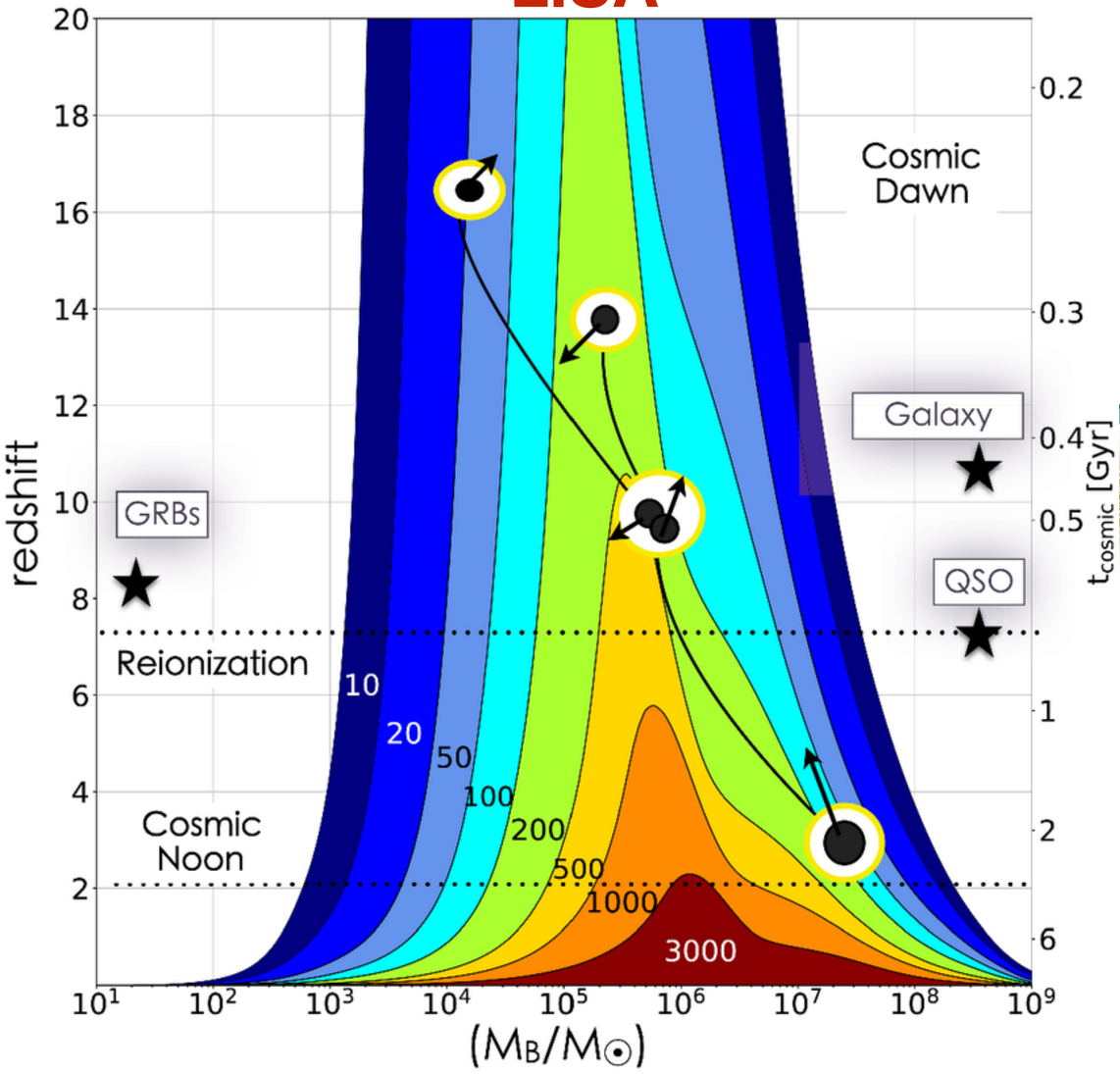


Conclusions

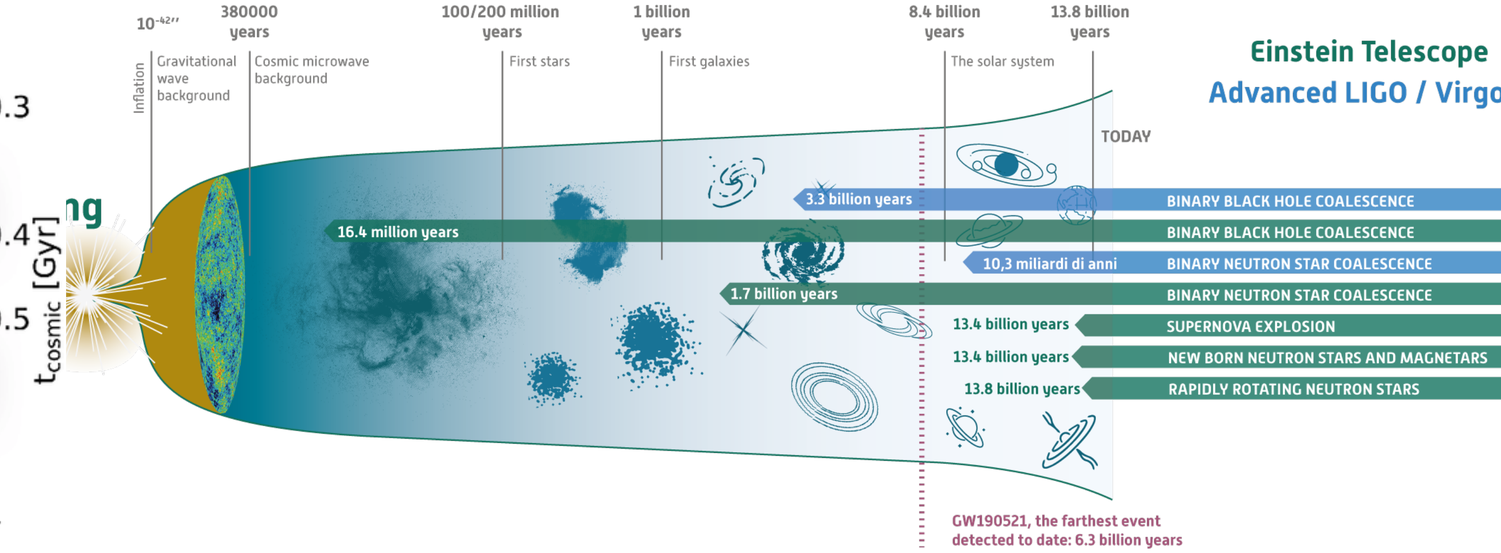
- Astrophysical distributions are initial tool to test stellar evolution
- To date (and in our limited sensitivity):
  - **BBHs and GW behave just like GR predicts**



LISA


redshift
t_{cosmic} [Gyr]
$(M_B/M_\odot)$
Cosmic Dawn
Galaxy
QSO
GRBs
Reionization
Cosmic Noon
10
20
50
100
200
500
1000
3000

ET


Inflation
Gravitational wave background
Cosmic microwave background
First stars
First galaxies
The solar system
TODAY
380000 years
100/200 million years
1 billion years
8.4 billion years
13.8 billion years
Einstein Telescope
Advanced LIGO / Virgo
3.3 billion years
BINARY BLACK HOLE COALESCENCE
16.4 million years
BINARY BLACK HOLE COALESCENCE
10,3 miliardi di anni
BINARY NEUTRON STAR COALESCENCE
1.7 billion years
BINARY NEUTRON STAR COALESCENCE
13.4 billion years
SUPERNOVA EXPLOSION
13.4 billion years
NEW BORN NEUTRON STARS AND MAGNETARS
13.8 billion years
RAPIDLY ROTATING NEUTRON STARS
GW190521, the farthest event detected to date: 6.3 billion years