Onde gravitazionali dallo spazio: LISA e Cosmologia

Angelo Ricciardone

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24 September 2025

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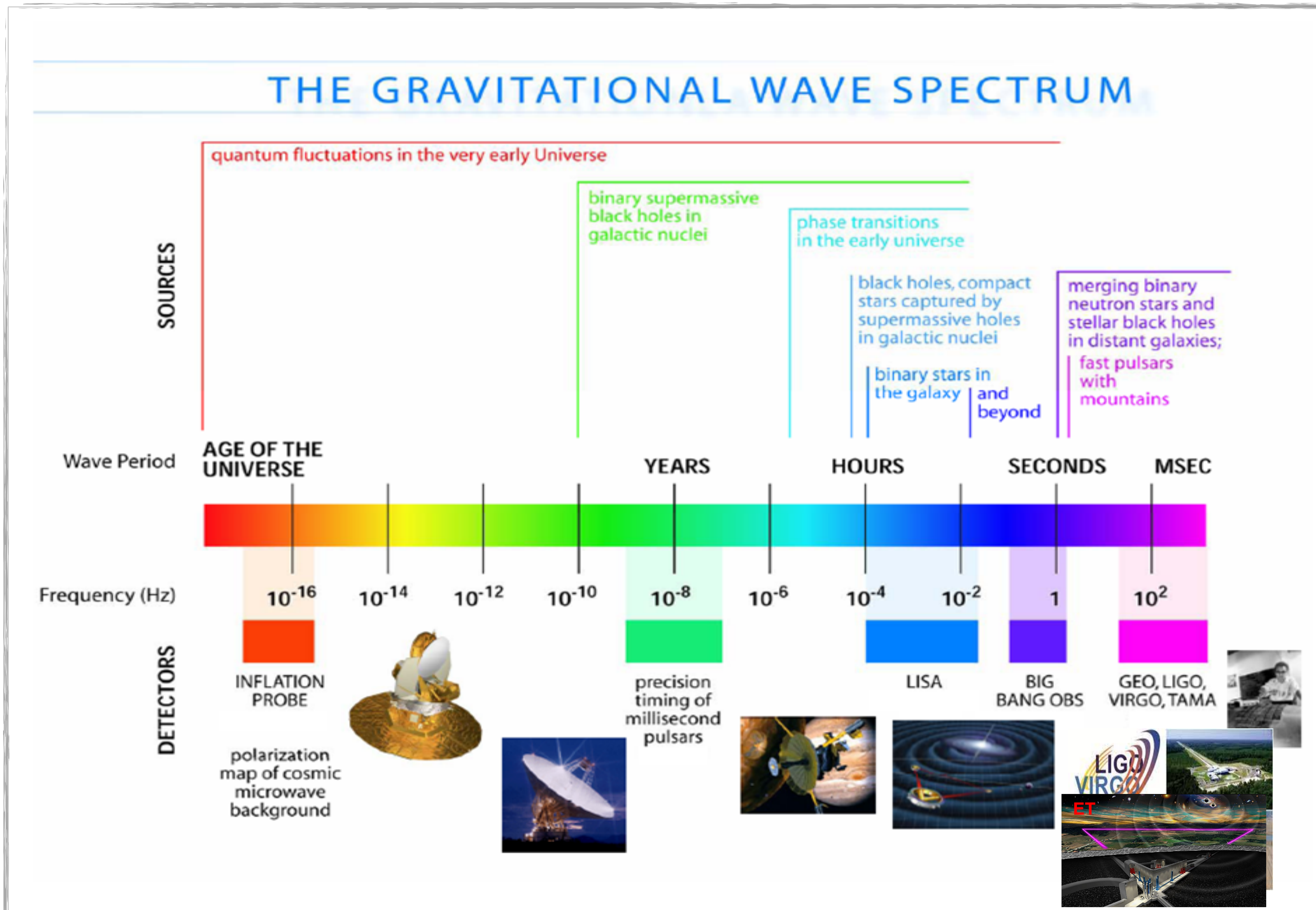
Dieci anni di Onde Gravitazionali - Pisa

The Gravitational Wave Spectrum



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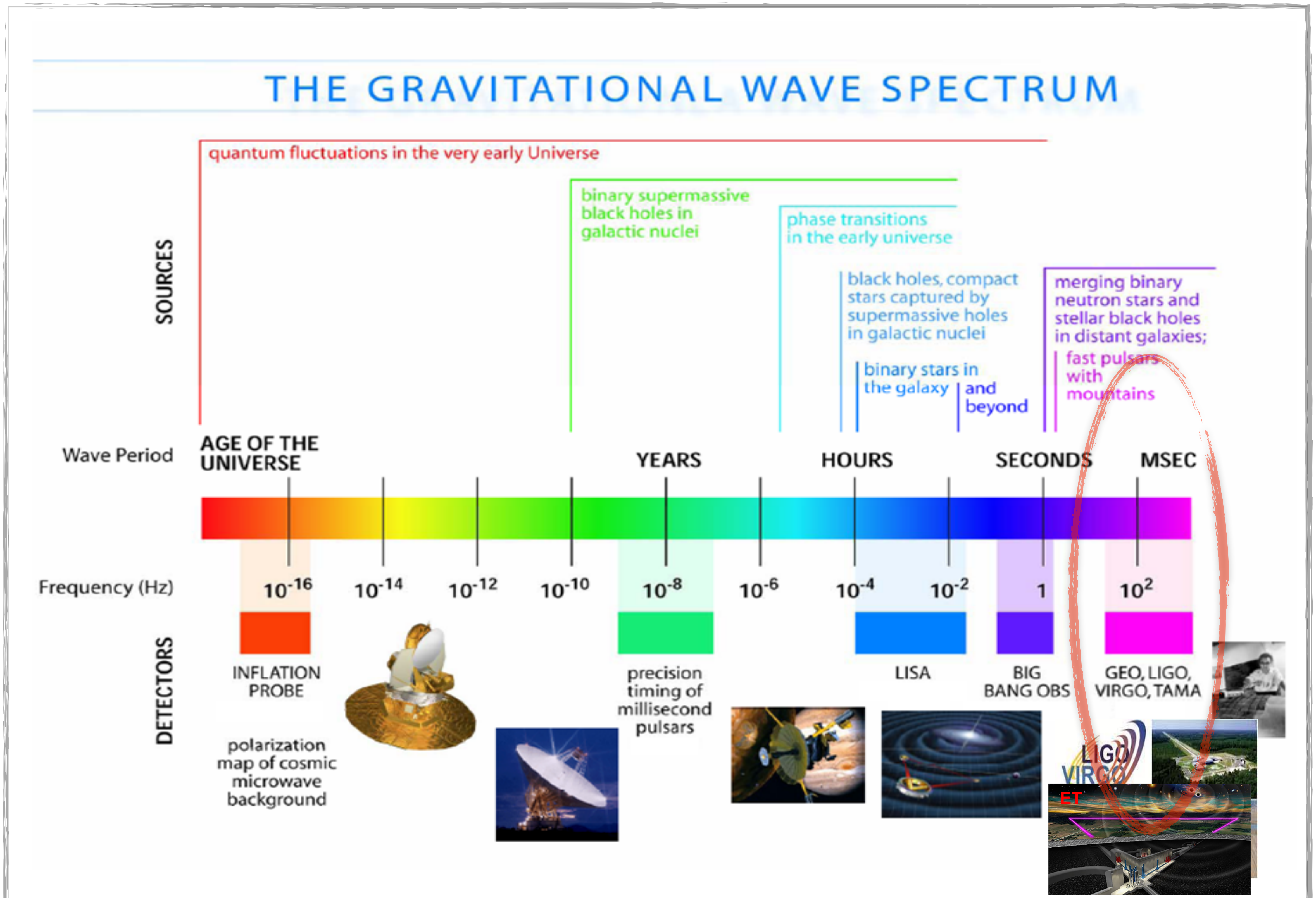

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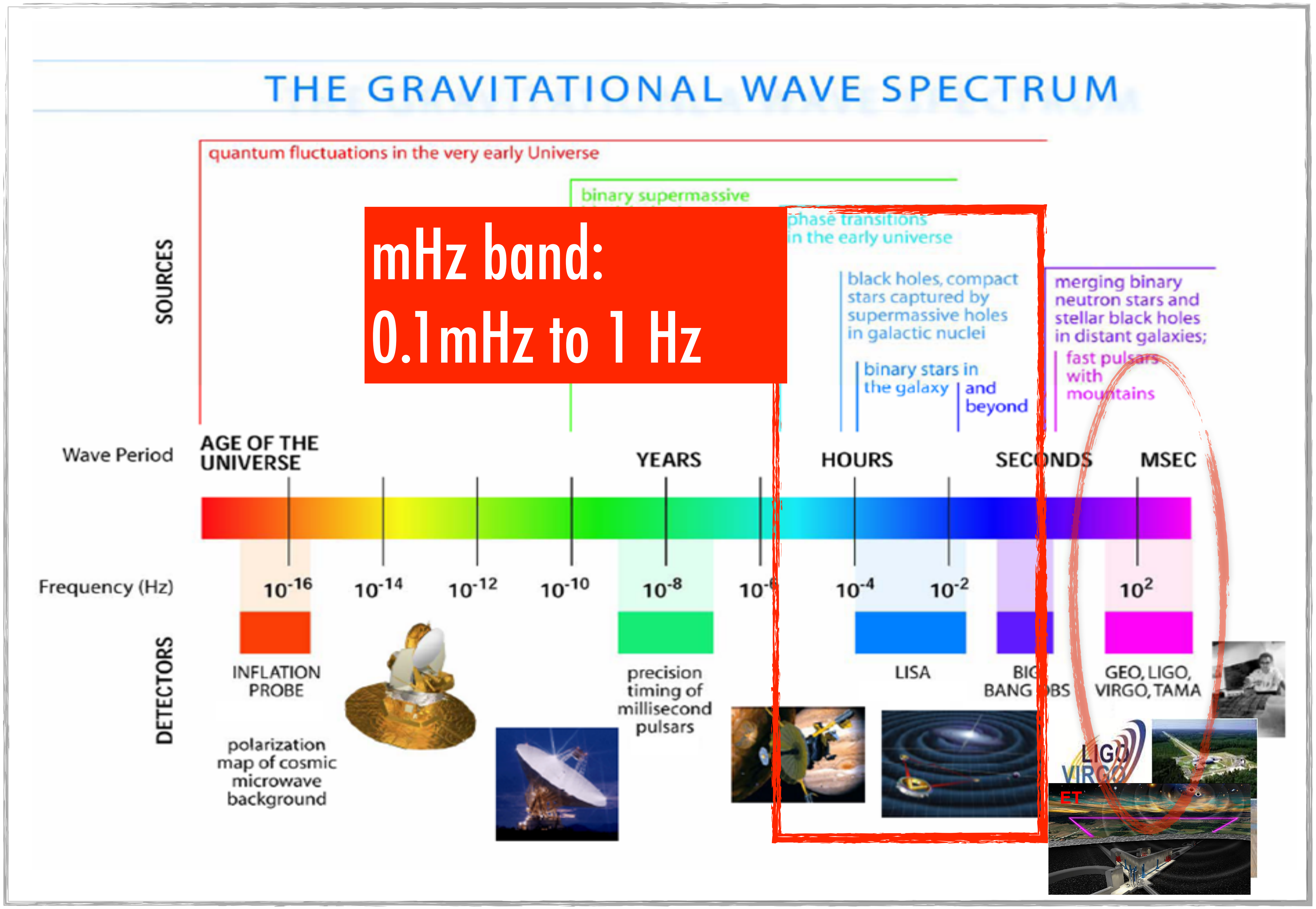
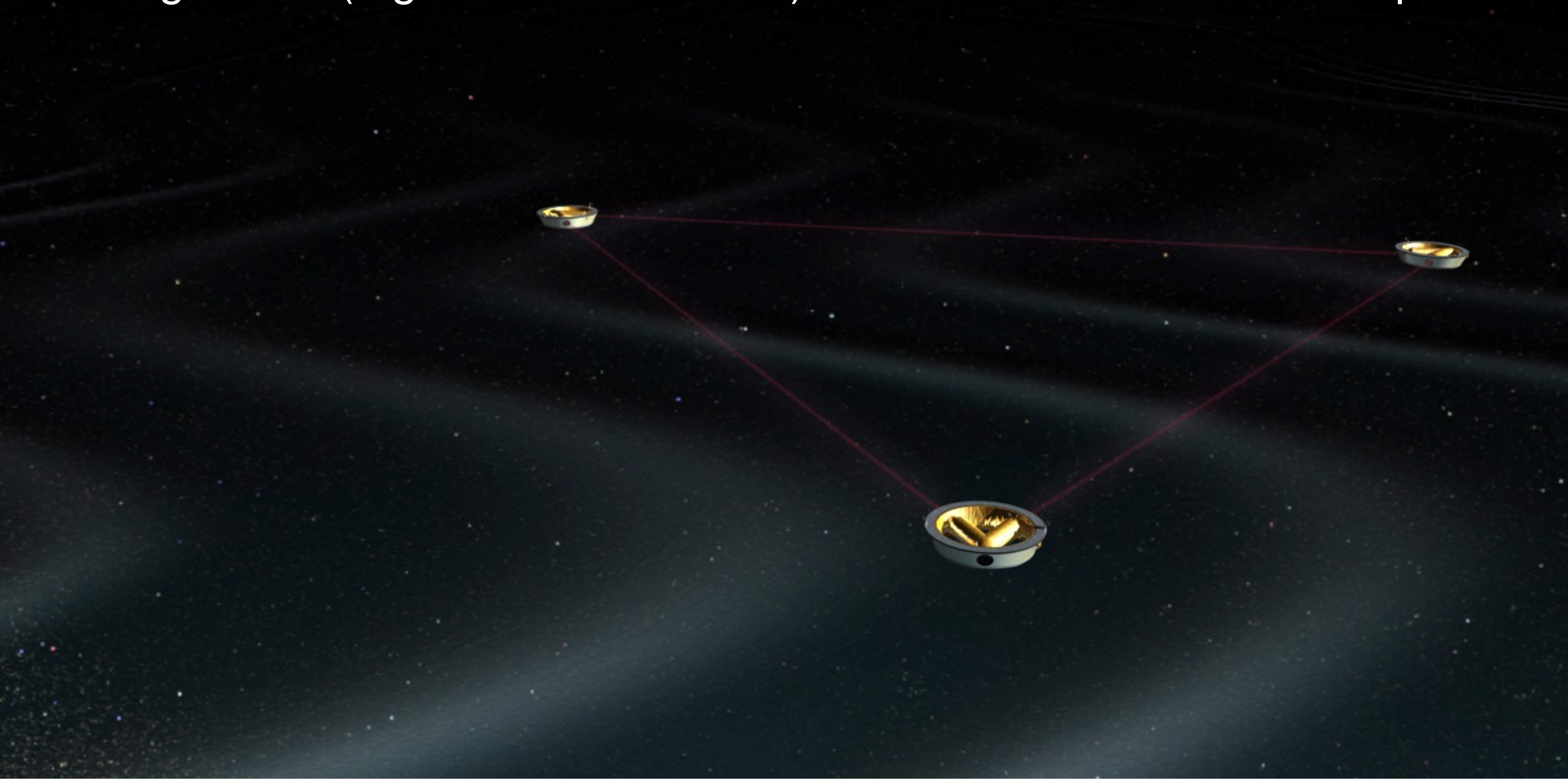


How to go below the Seismic Noise?

Constellation of 3 spacecraft on heliocentric orbits in an equilateral configuration (a giant interferometer) with 2.5 million-km arms in space



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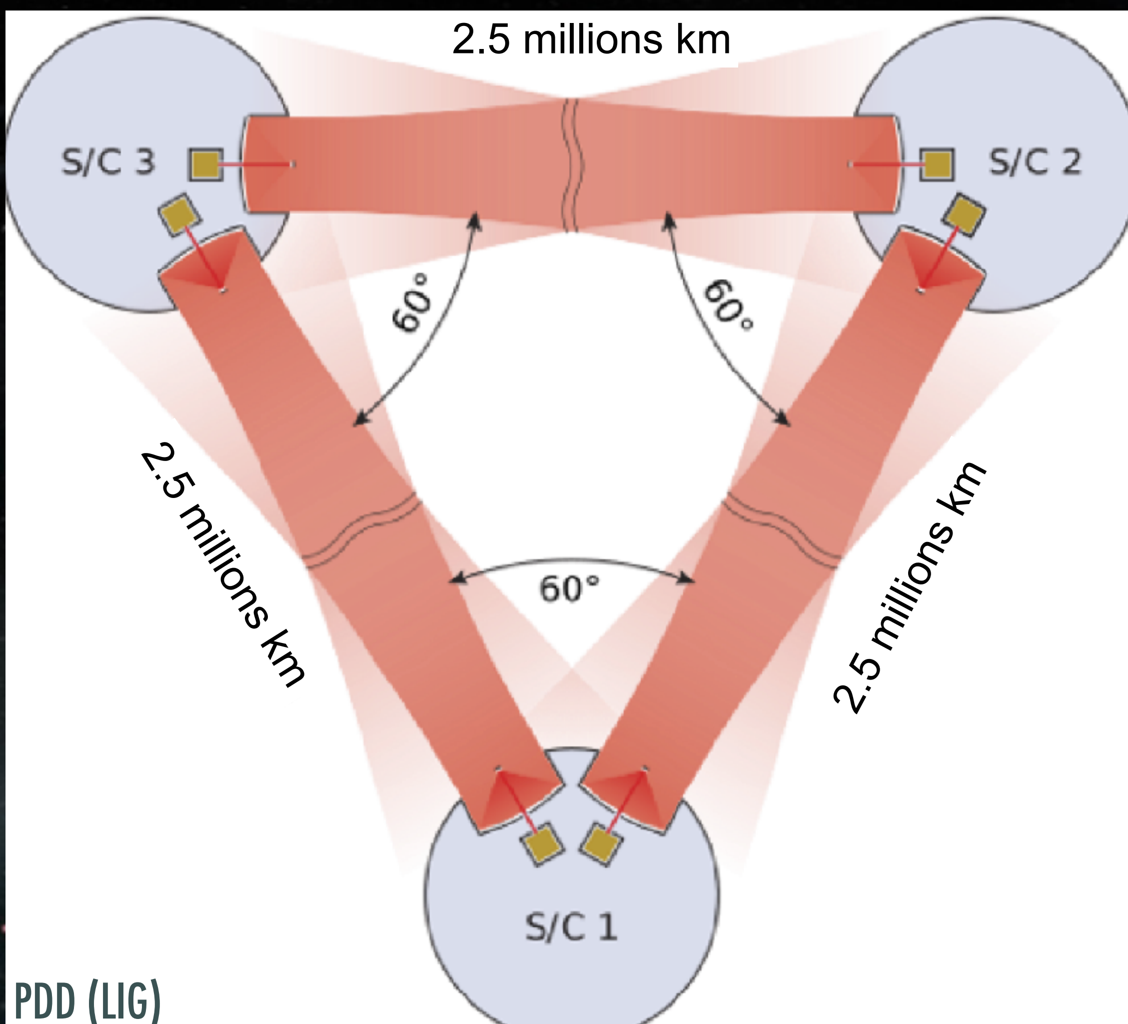

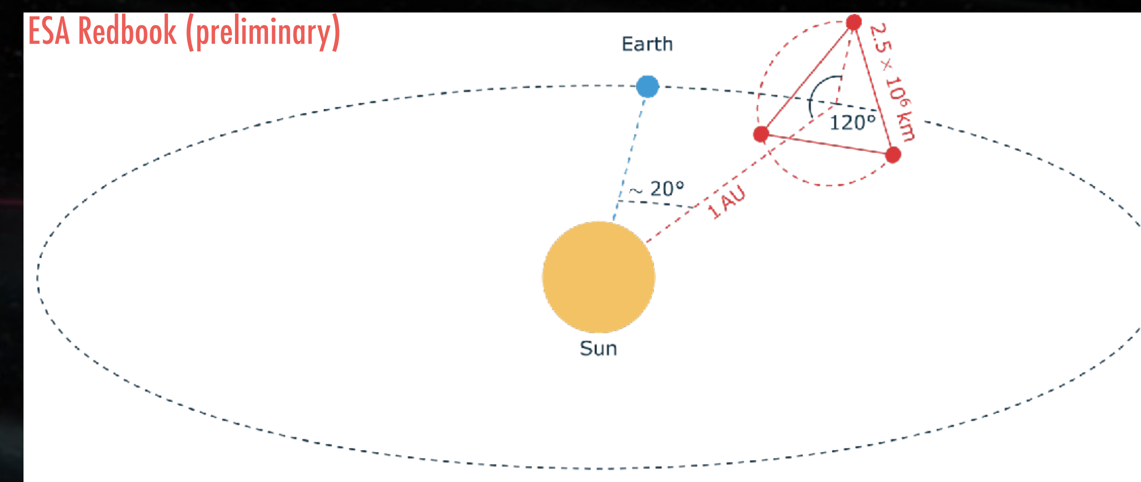

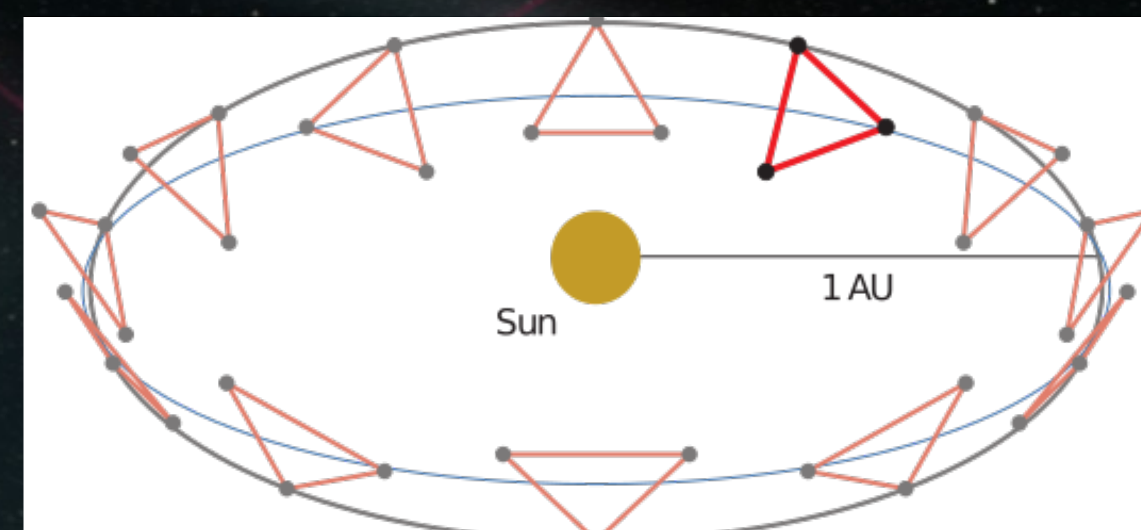

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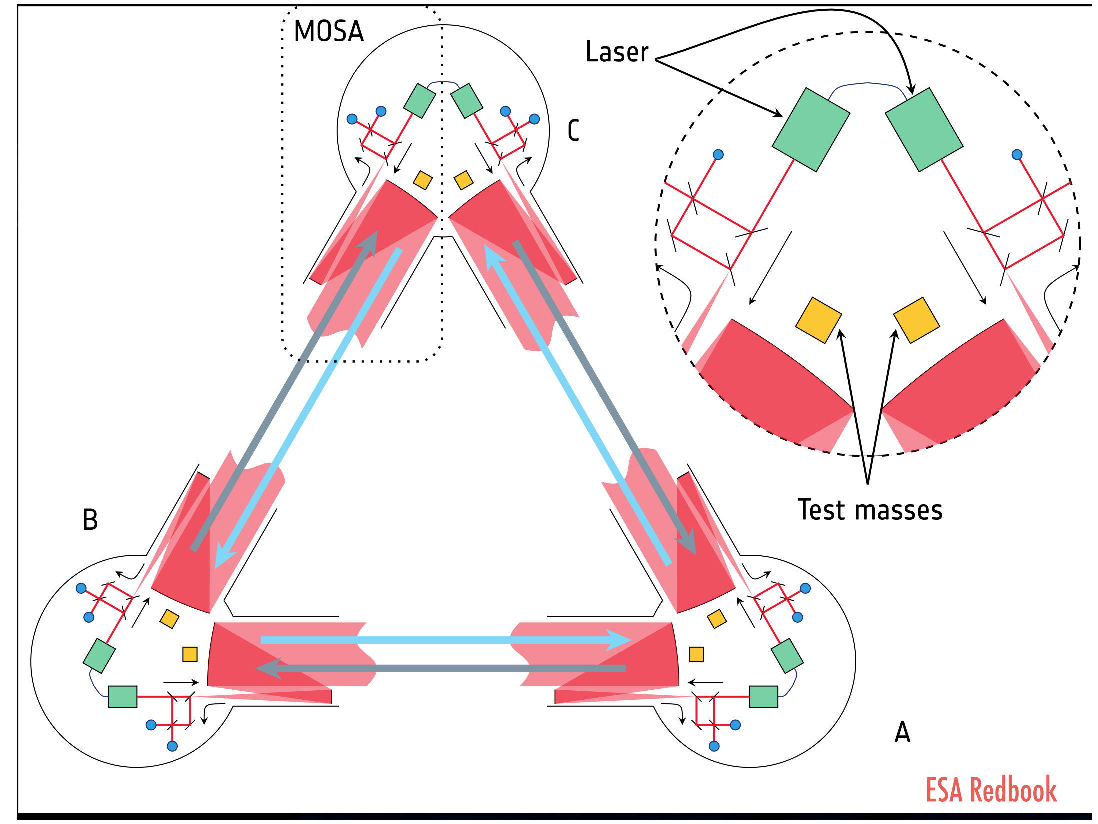
Constellation of 3 spacecraft on heliocentric orbits in an equilateral configuration (a giant interferometer) with 2.5 million-km arms in space

Goal: measure relative distance changes of 10^{-21} on 2.5 million km arms -> picometer displacement of masses

Spacecraft design

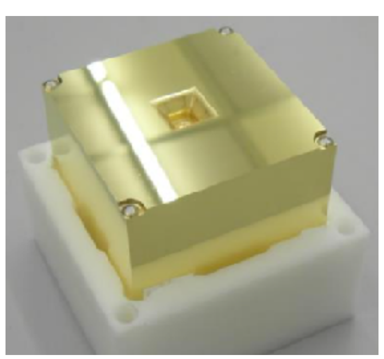
- Measurement points must be **shielded from fluctuating non- gravitational influences**

- **The spacecraft protects test-masses (TMs) from external forces (drag-free system) and always adjusts itself on it using micro-thrusters**



- Readout: **- interferometric (sensitive axis of main laser/GW)**

  - capacitive sensing (in the remaining 5 dofs)



46 mm cubic Au-Pt test mass

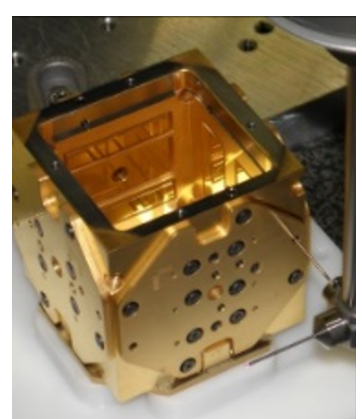
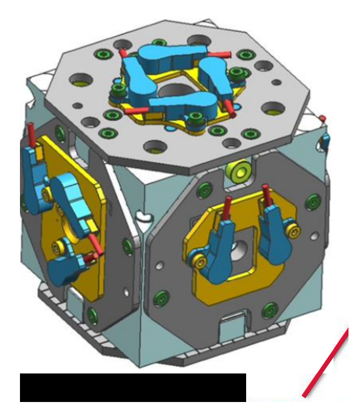


Photo of the test mass electrode housing



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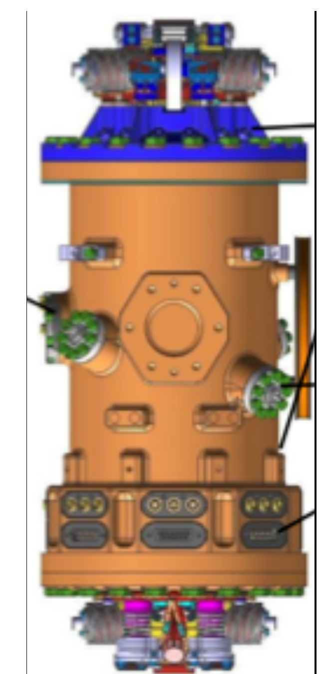


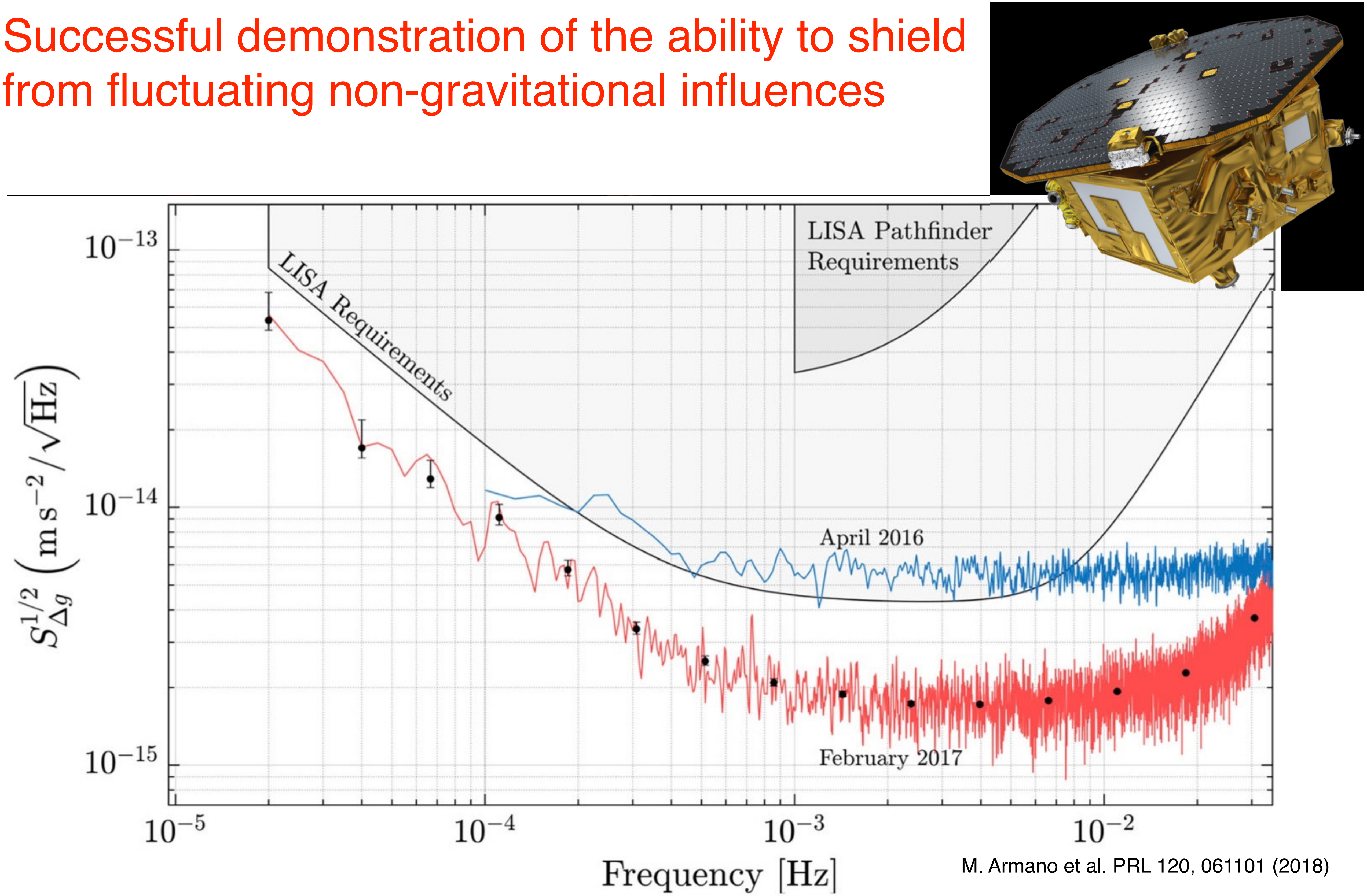
Moving Optical Sub-Assemblies (MOSAs)

structural units that contain elements for optical measurement and define the inertial reference

Gravitational Reference System (GRS)

LISAPathfinder final main results

Successful demonstration of the ability to shield from fluctuating non-gravitational influences



M. Armano et al. PRL 120, 061101 (2018)

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- **Laser emission**: each spacecraft sends a laser beam to the others.

- The received beam is too weak to reflect directly, so it is **phase-locked** and **amplified locally**.

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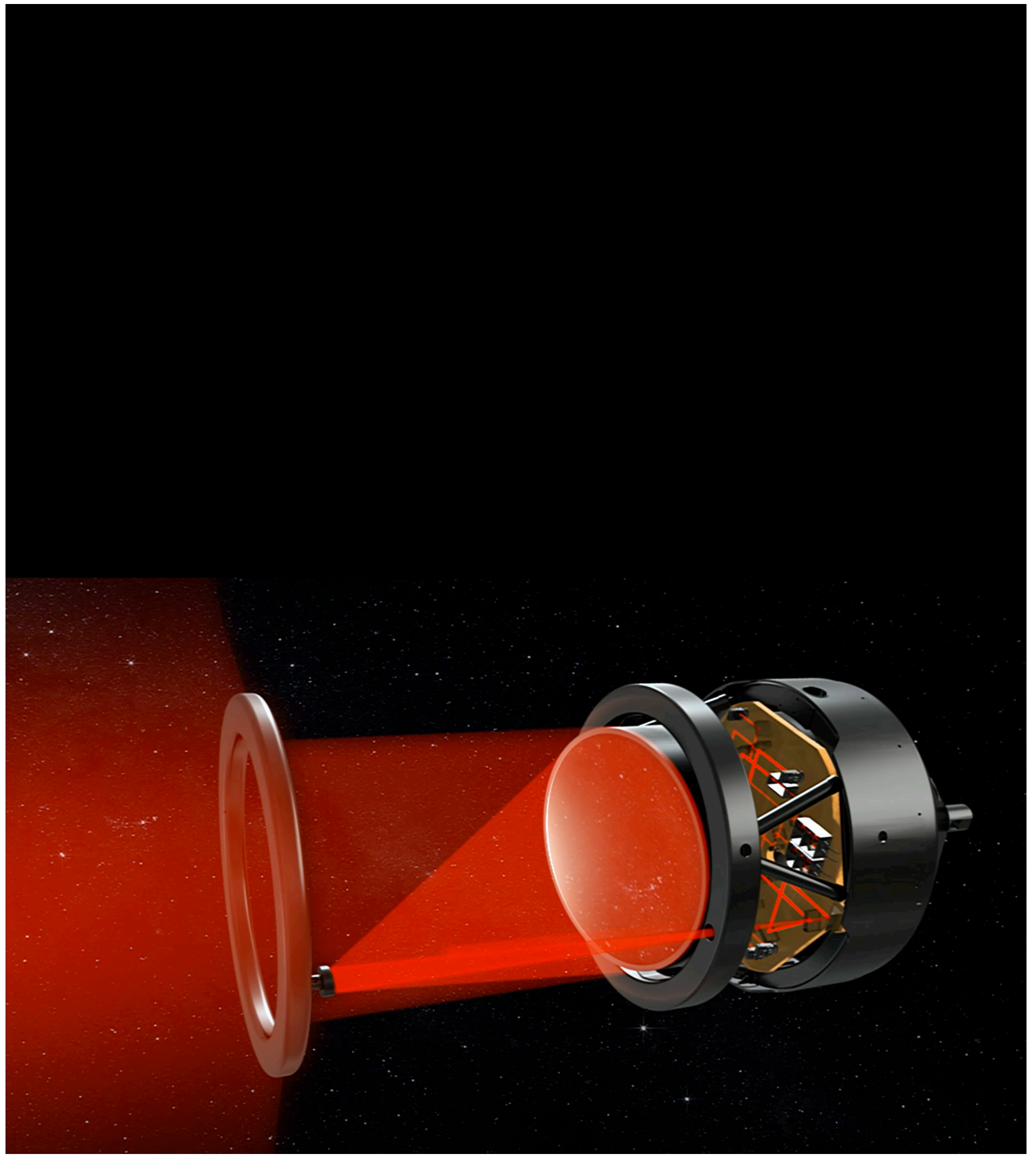
- **Gravitational waves**: passing waves stretch and compress the arm lengths, **producing differential phase shifts.**



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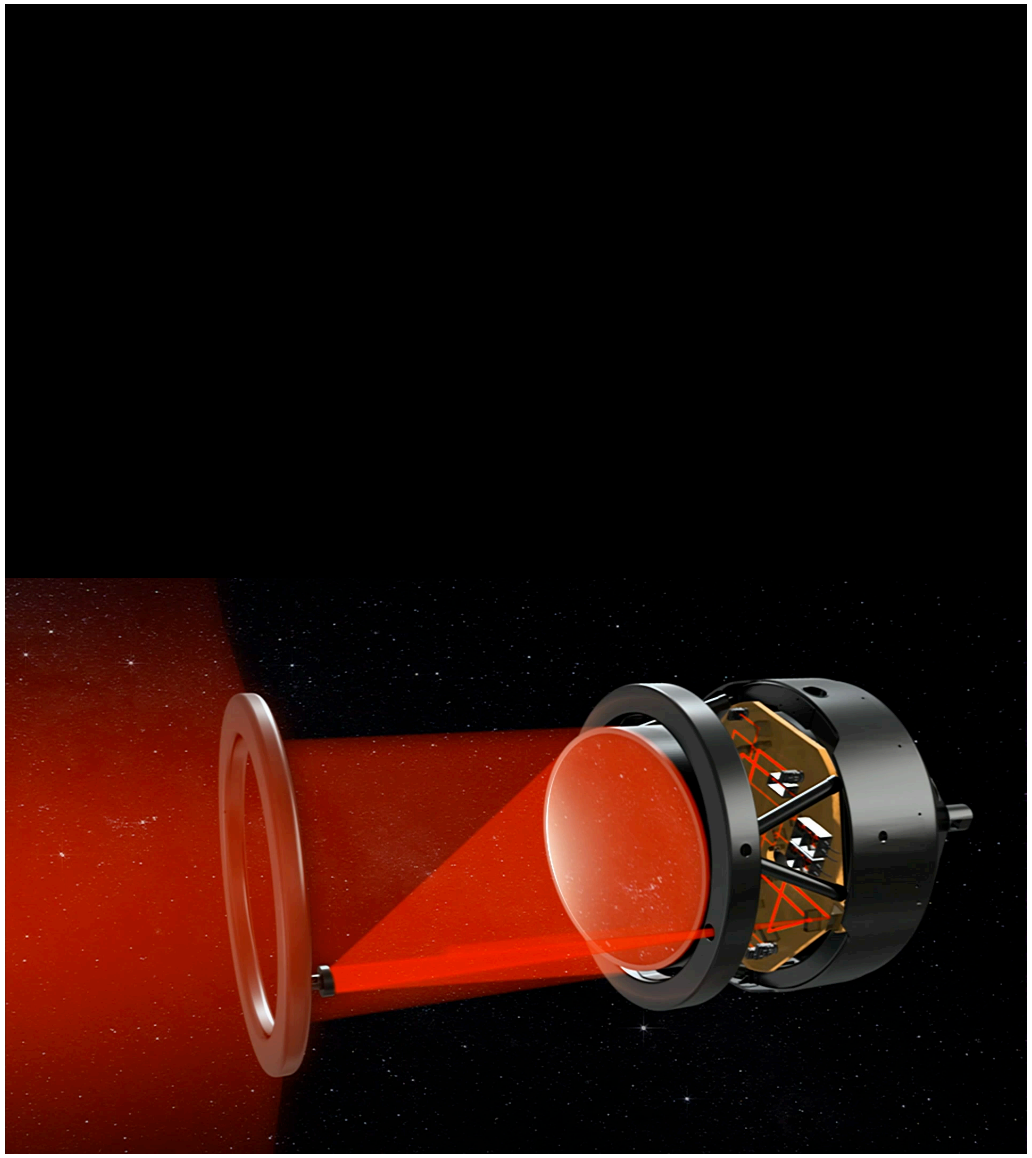

Noise Sources
- Laser Noise: $(10^{-13}$ vs $10^{-21})$
- Acceleration Noise
- Read-out noises
- Optical Path Noise

Time-Delay Interferometry (TDI): signals are combined with appropriate time shifts to **cancel laser frequency noise and extract the gravitational wave signal.**

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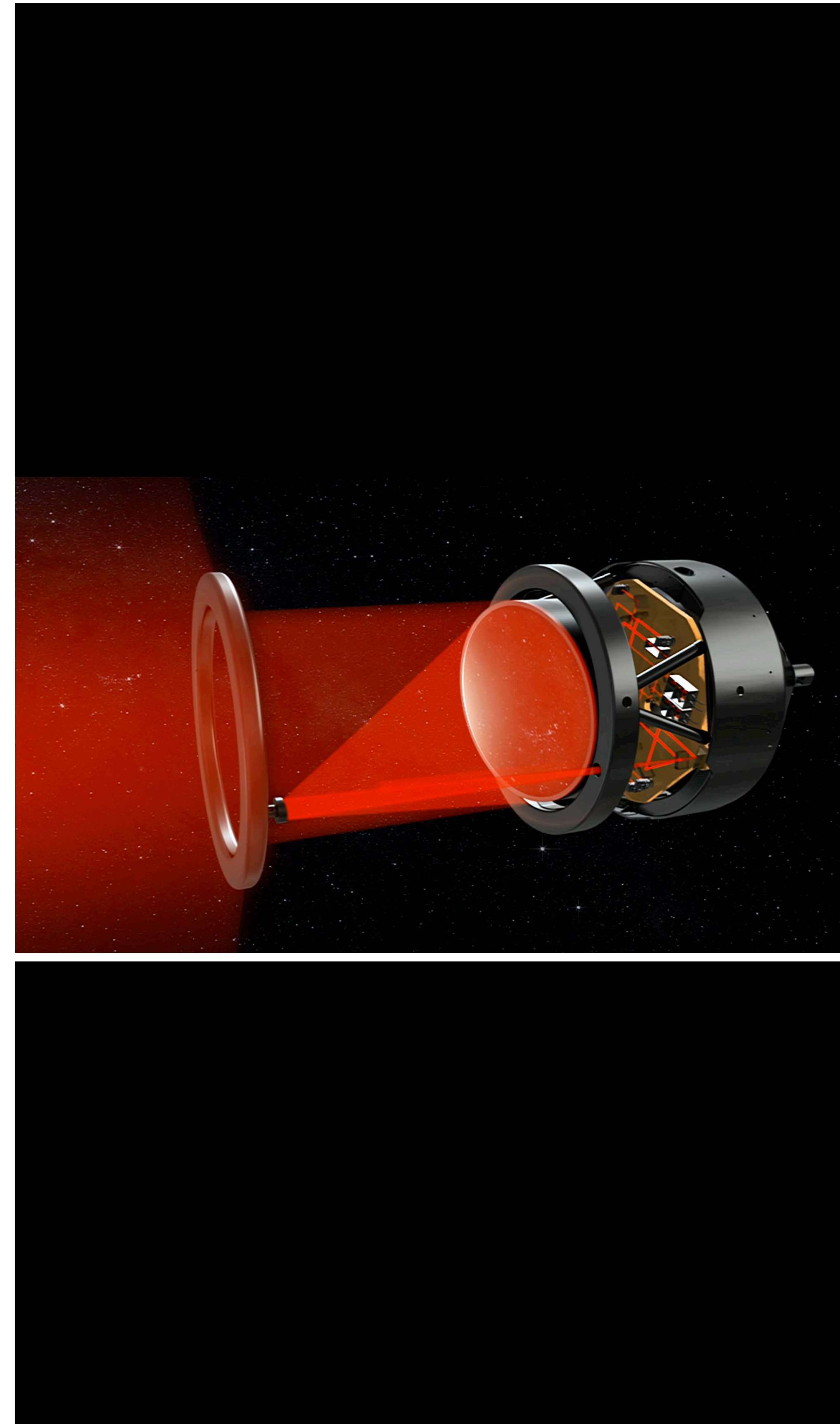
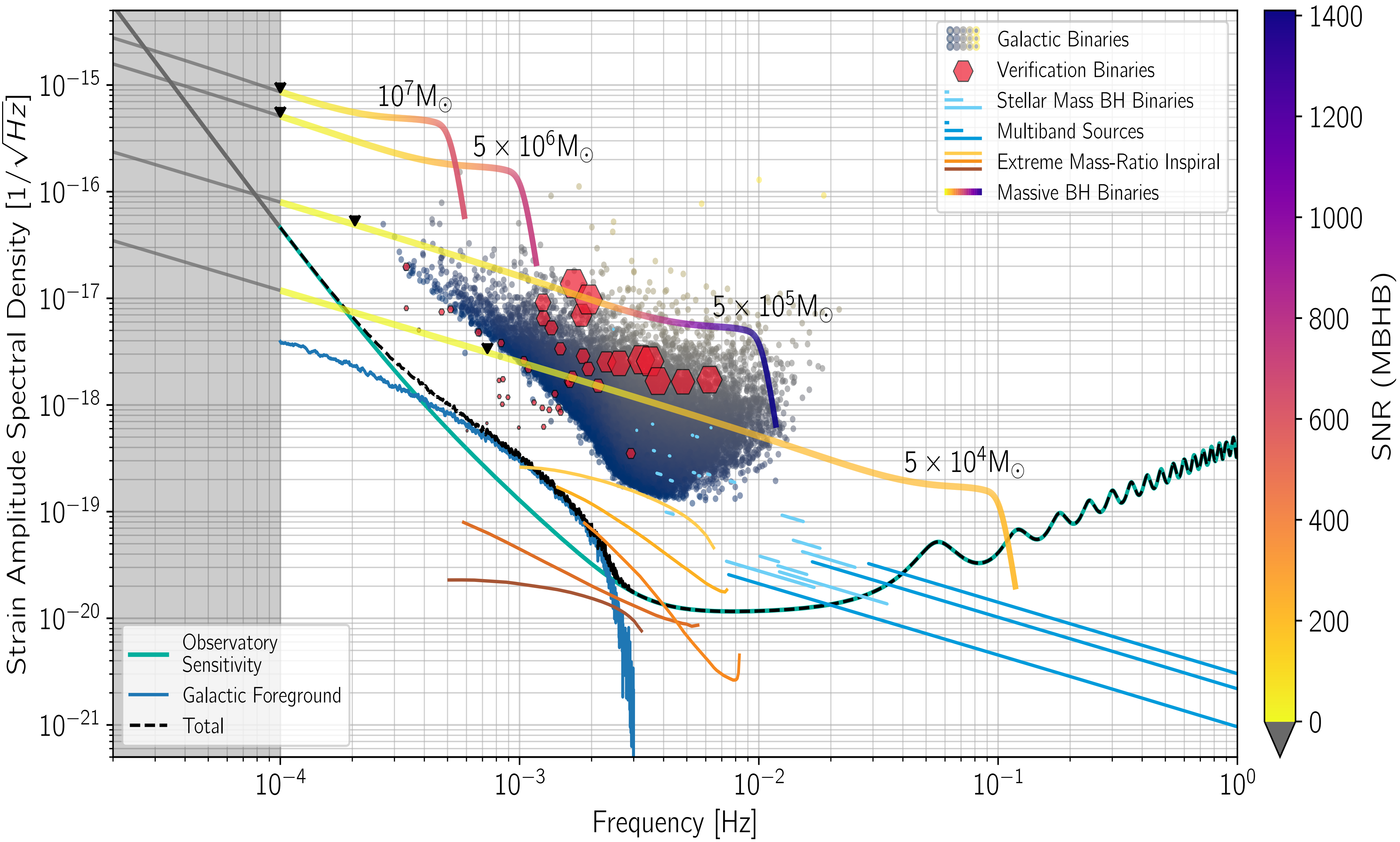
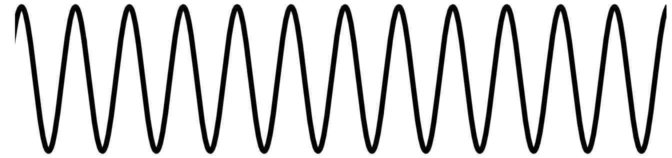


What do we expect to detect with LISA?



Sources	SNR	Duration	Event rate
Galactic binaries	10 – 500	permane nt	10000 – 30000 detectables + background
Verification binaries	7 - 100	permane nt	20 (today)
Stellar mass black hole binaries	7 - 30	1 à 10 years	1 to 20
Extreme Mass Ratio Inspirals	7 - 60	1 year	1 to 2000 / year
Massive Black Hole binaries	10 - 3000	Hours - months	10 to 100 / year

Galactic Binaries



- A large number of stars are in binary systems and evolve towards WD, NS and stellar BH binaries (expected ~60 millions in the galaxy)

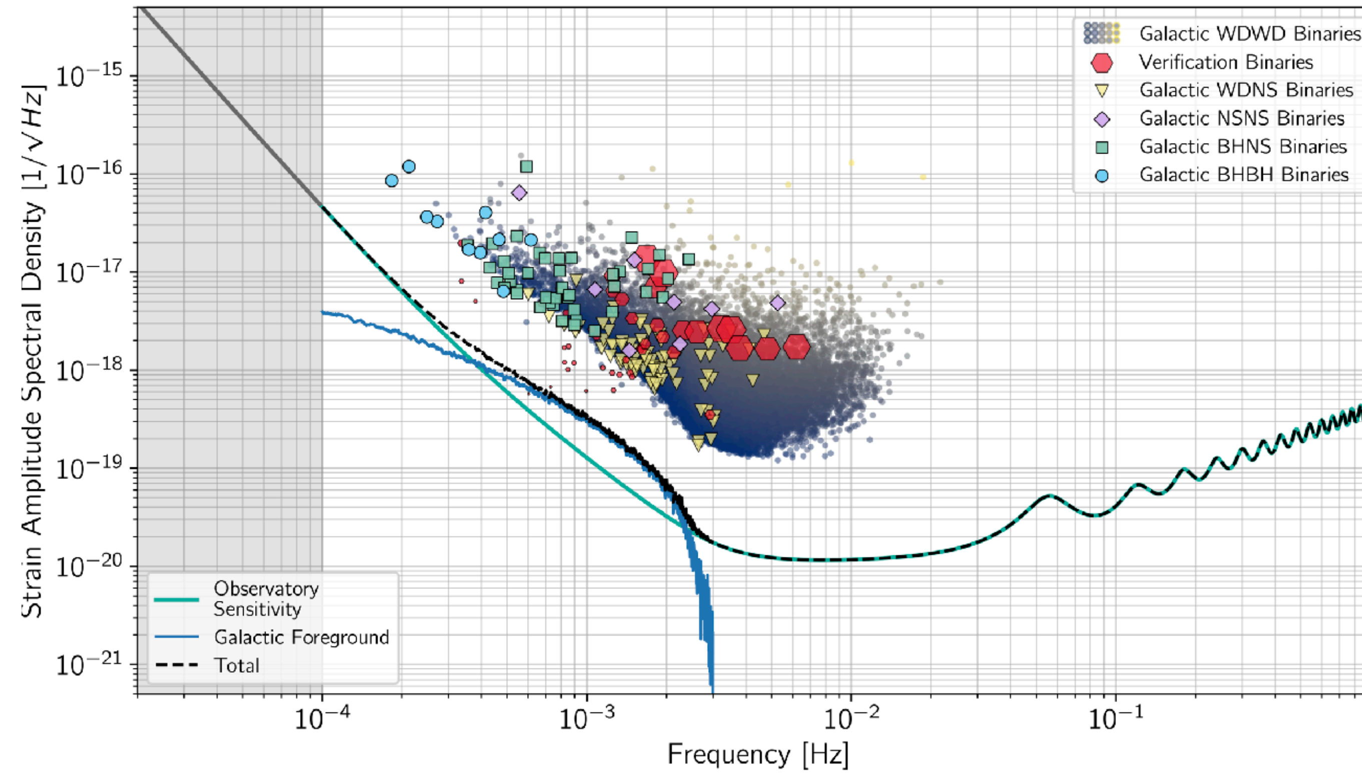
- Most are in the inspiral phase: **quasi-monochromatic**, permanent GW signal

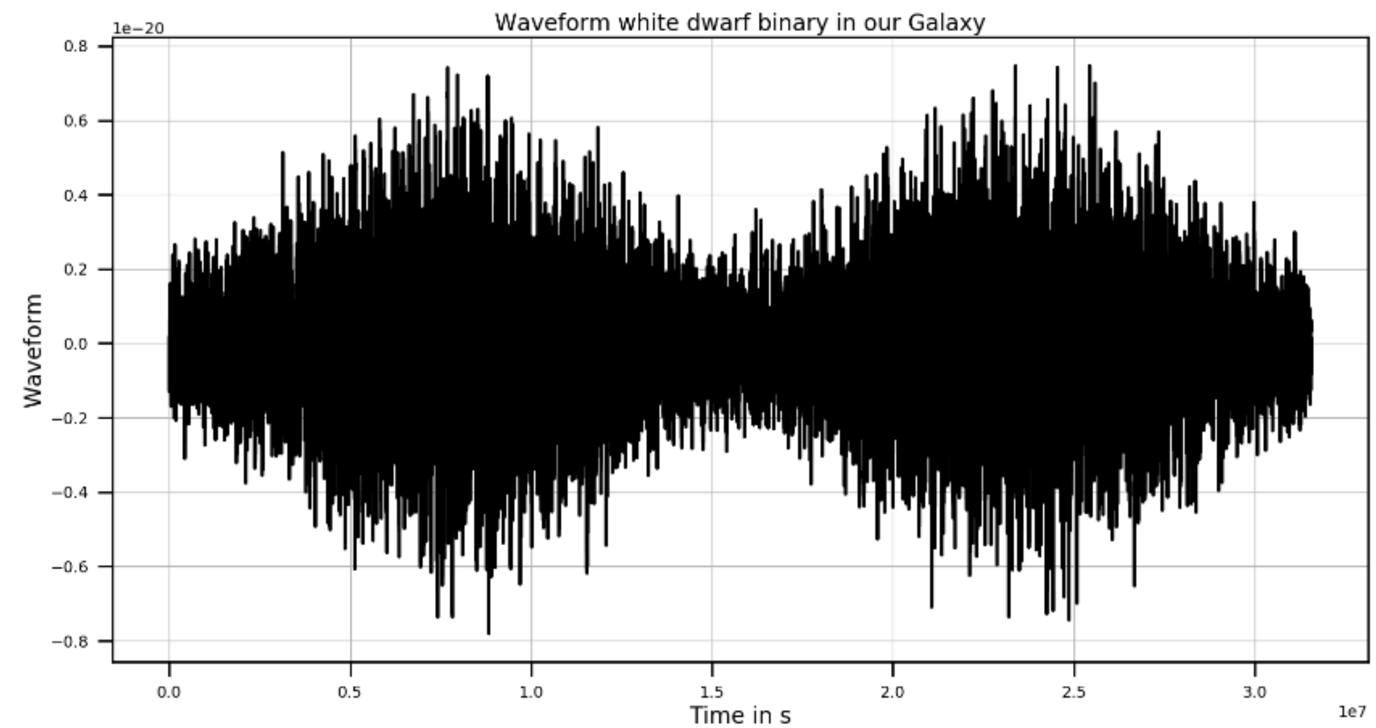
$$M_c = 1\, M_\odot \quad \tau = 10^5\,\mathrm{y} \longrightarrow f = 3\,\mathrm{mHz}, \quad \dot{f} = 10^{-16}\,\mathrm{Hz/sec}$$

- ~ 20 known WD systems are guaranteed LISA sources: verification binaries

- Resolved binaries: ~ 25000 sources are expected with SNR 7-1000

- Stochastic foreground from sources with too low SNR, yearly modulated



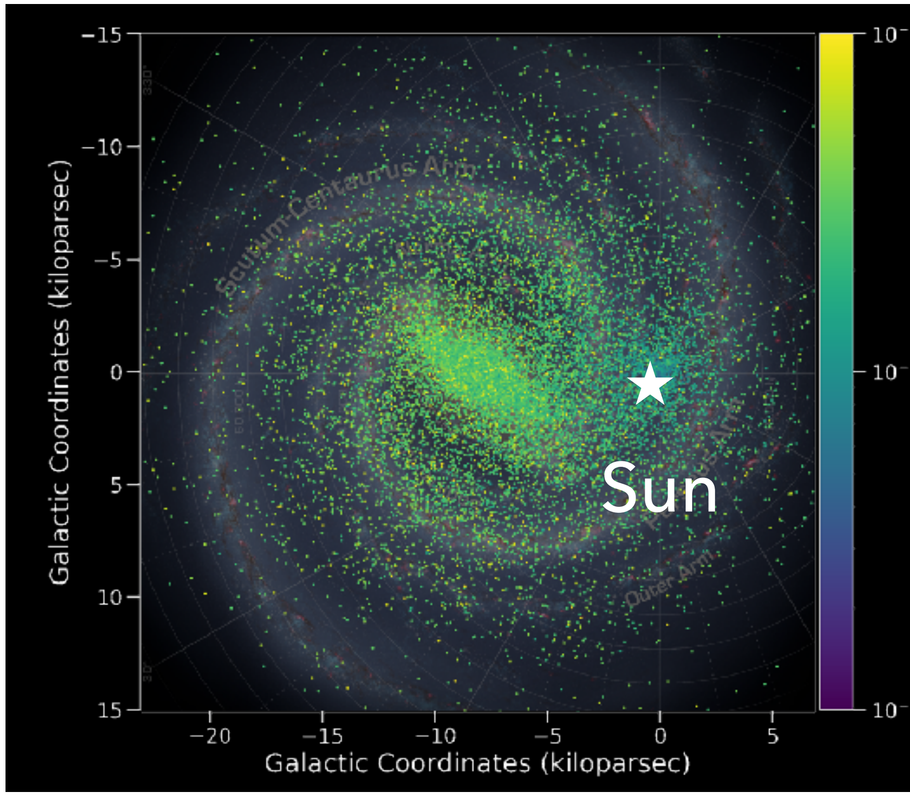


Galactic Binaries - what we will learn?

SO1: Study the formation and evolution of compact binary stars in the Milky Way Galaxy:

- Formation and evolution pathways of dark compact binary stars in the Milky Way and in neighbouring galaxies;
- The Milky Way mass distribution;
- The interplay between gravitational waves and tidal dissipation
- Spatial distribution of ultra-compact binaries detected by LISA that are too dim for detection with EM telescopes?

Spatial distribution of numerous individual double WDs



Foreground for Cosmology - to be subtracted

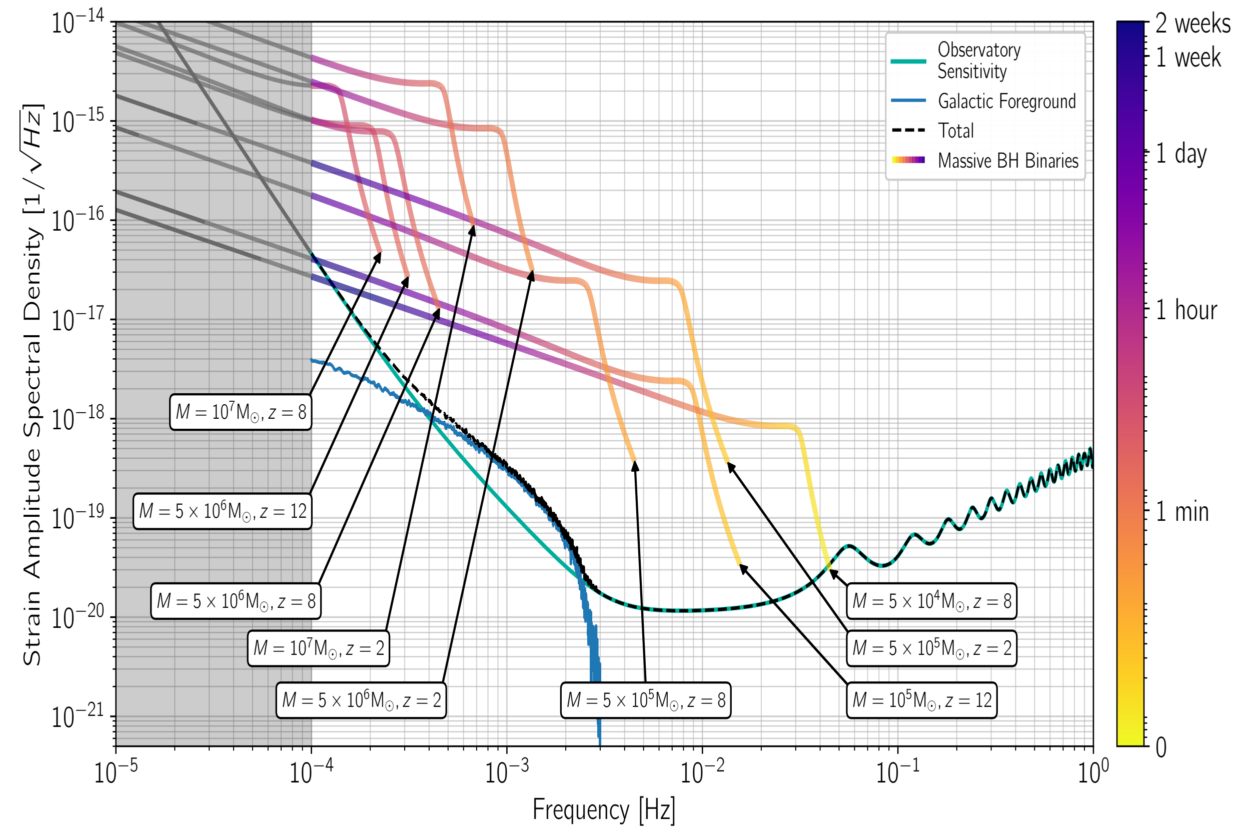
Super Massive BH Binaries

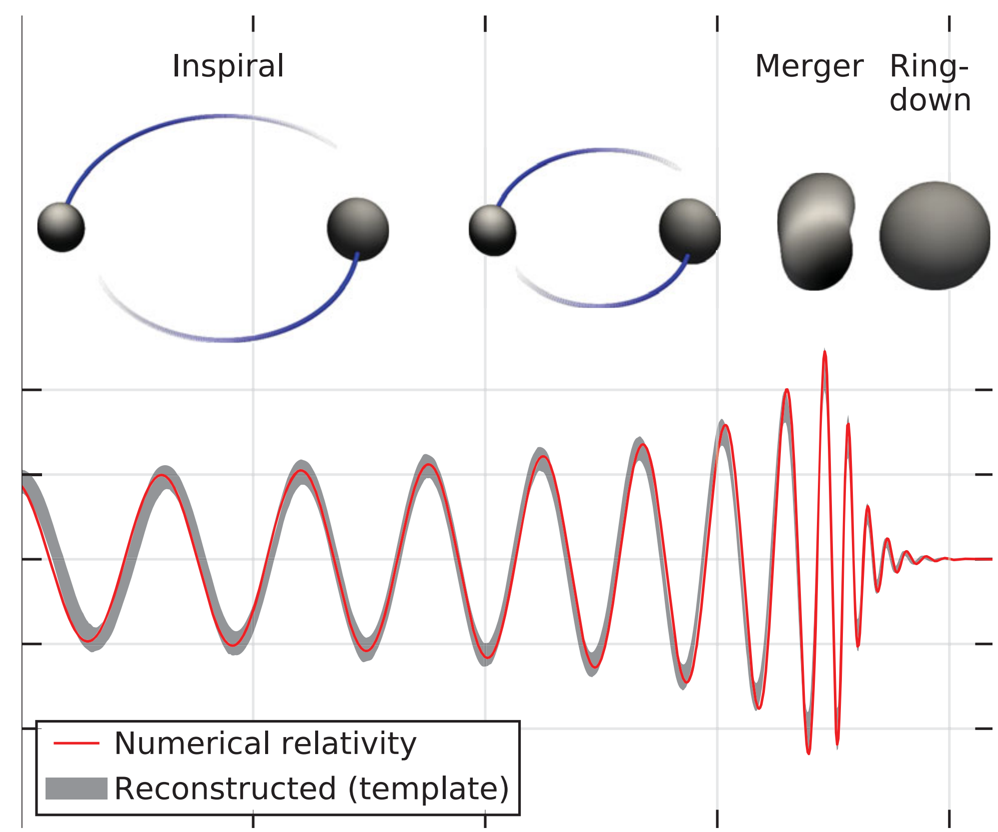
MBH are indirectly observed in the centre of many galaxies.

Galaxies collide -> MBH must exist in binaries

The loudest LISA sources
(Other than unexpected ones)

- MBHB from $10^4 M_\odot$ to $10^7 M_\odot$
- **LISA detects the inspiral, merger, ringdown**
- Signal duration: from few hours to several months prior to merger
- SNR up to few thousands
- **Expected rates: 10-100/year**
- Up to z ~ 20!



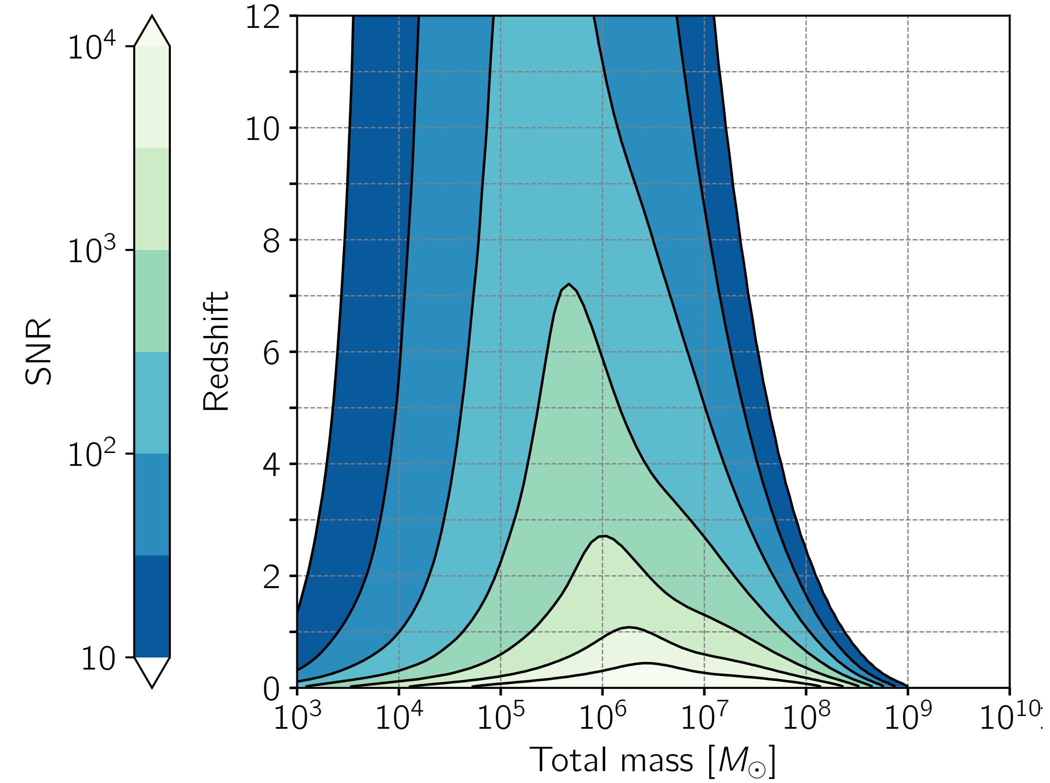


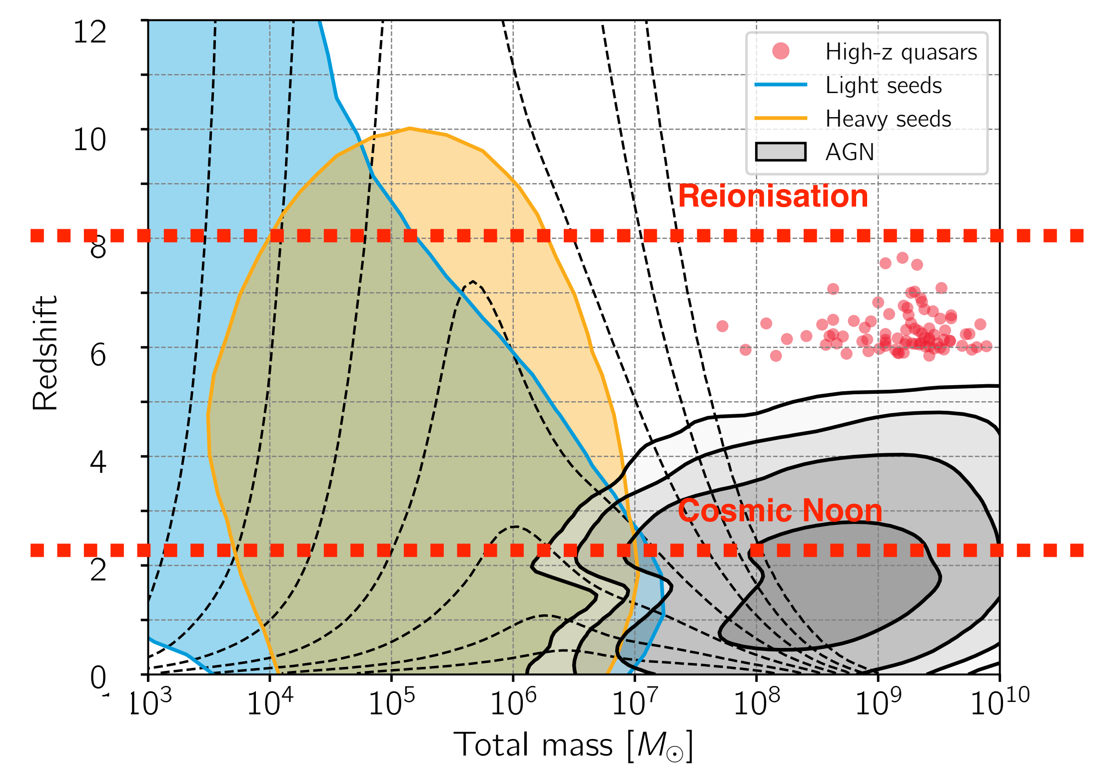
SMBHB - what we will learn?

SO2: Trace the origin, growth and merger history of massive black holes across cosmic ages:

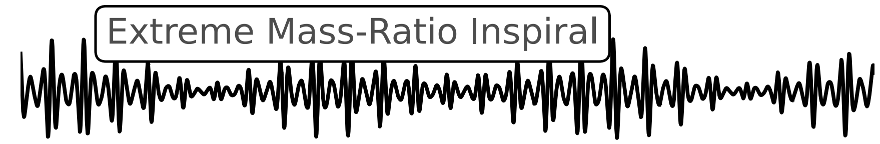
- Discover seed black holes at cosmic dawn z;
- Study the growth mechanism and merger history of massive black holes from the epoch of the earliest quasars;
- Identify the electromagnetic counterparts of massive black hole binary coalescences.
- Use SMBHB as standard candles/bright sirens to infer H0

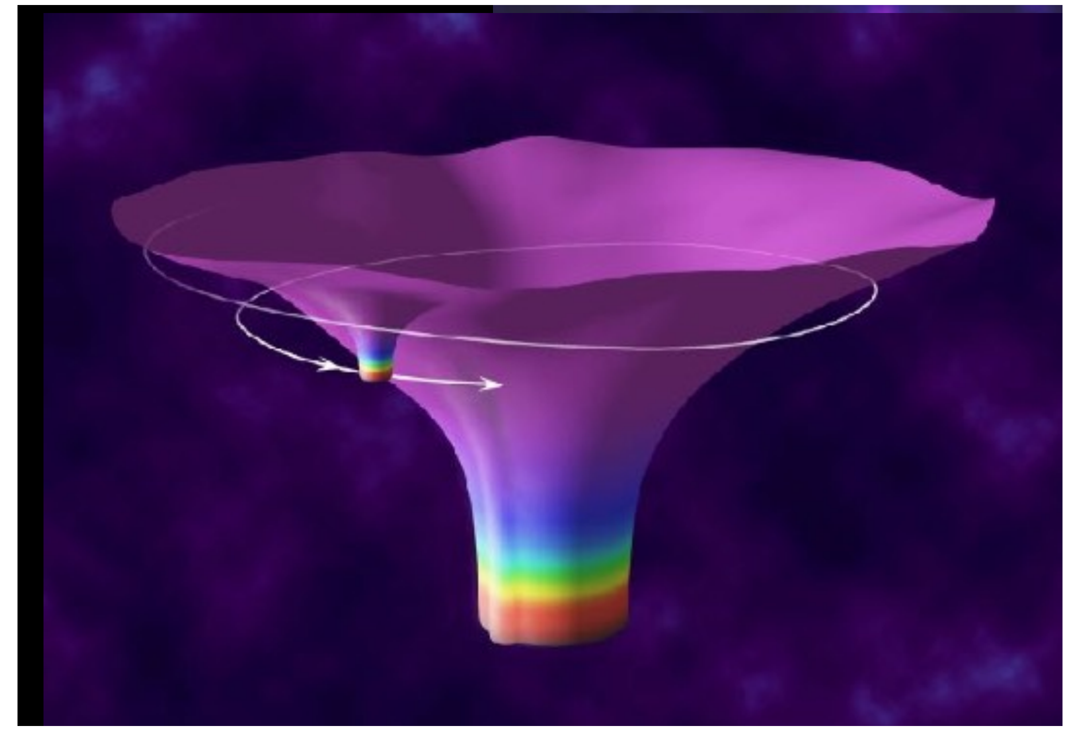
contour lines of constant SNR from MBHBs.





Extreme Mass Ratio Inspirals

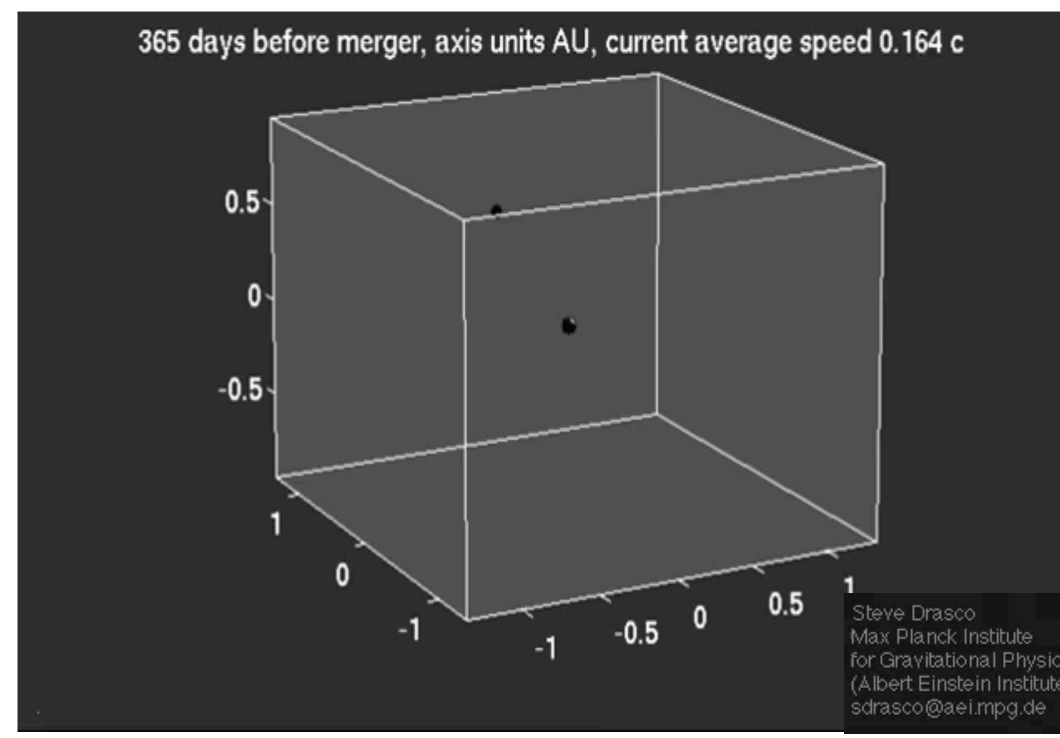




- Binaries for which the masses of the two objects are very different

$$10^{-7} < q \equiv \frac{m_2}{m_1} < 10^{-4}$$

- The waveforms are complex and the rates are highly uncertain



- The SNR can be as high as few hundreds
- They offer the opportunity to map the full BH population of the Universe
- Probe the properties and immediate environments of black holes in the local Universe
- They can be used to perform tests of General Relativity

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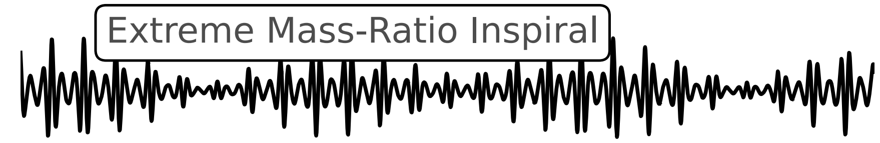

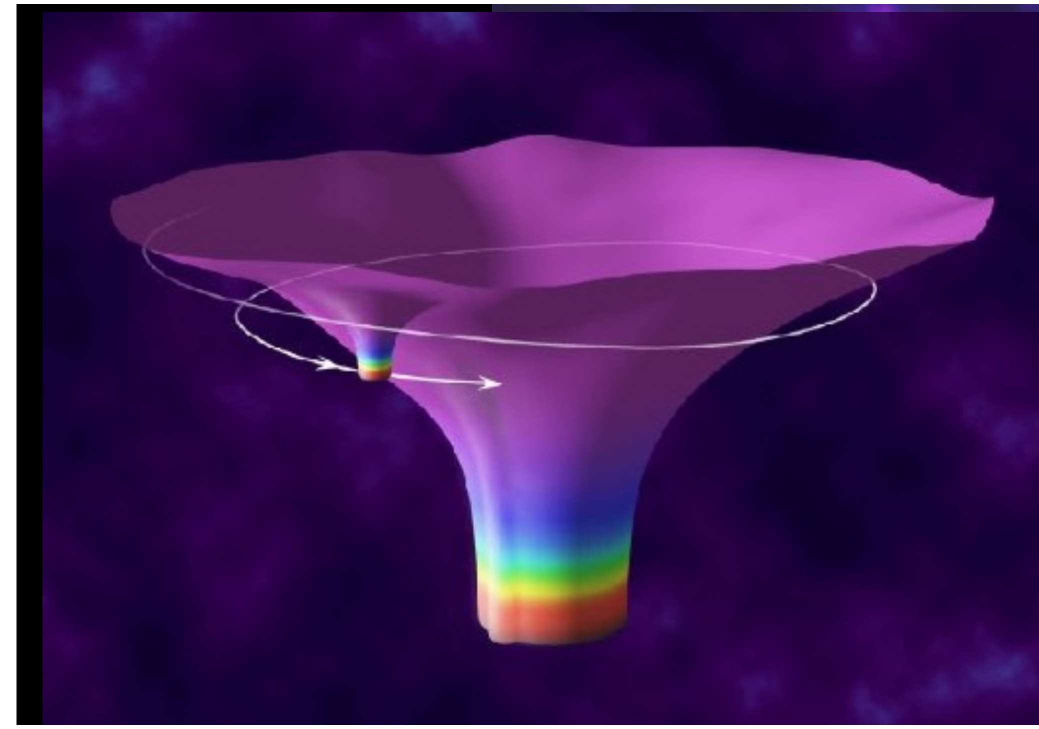

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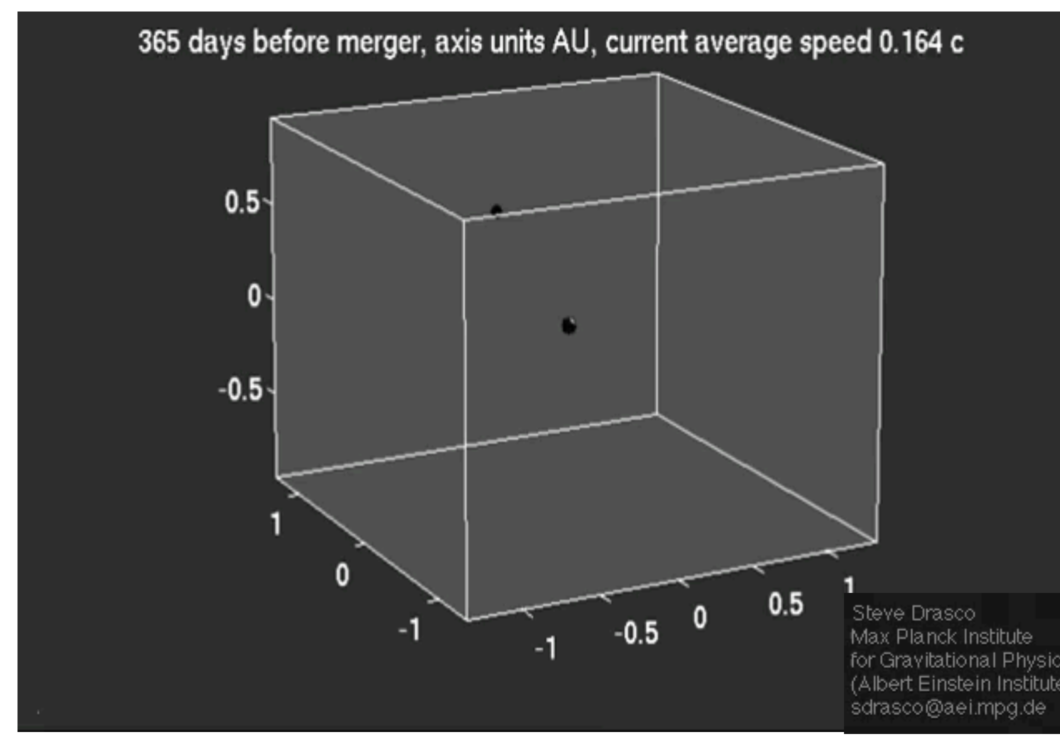

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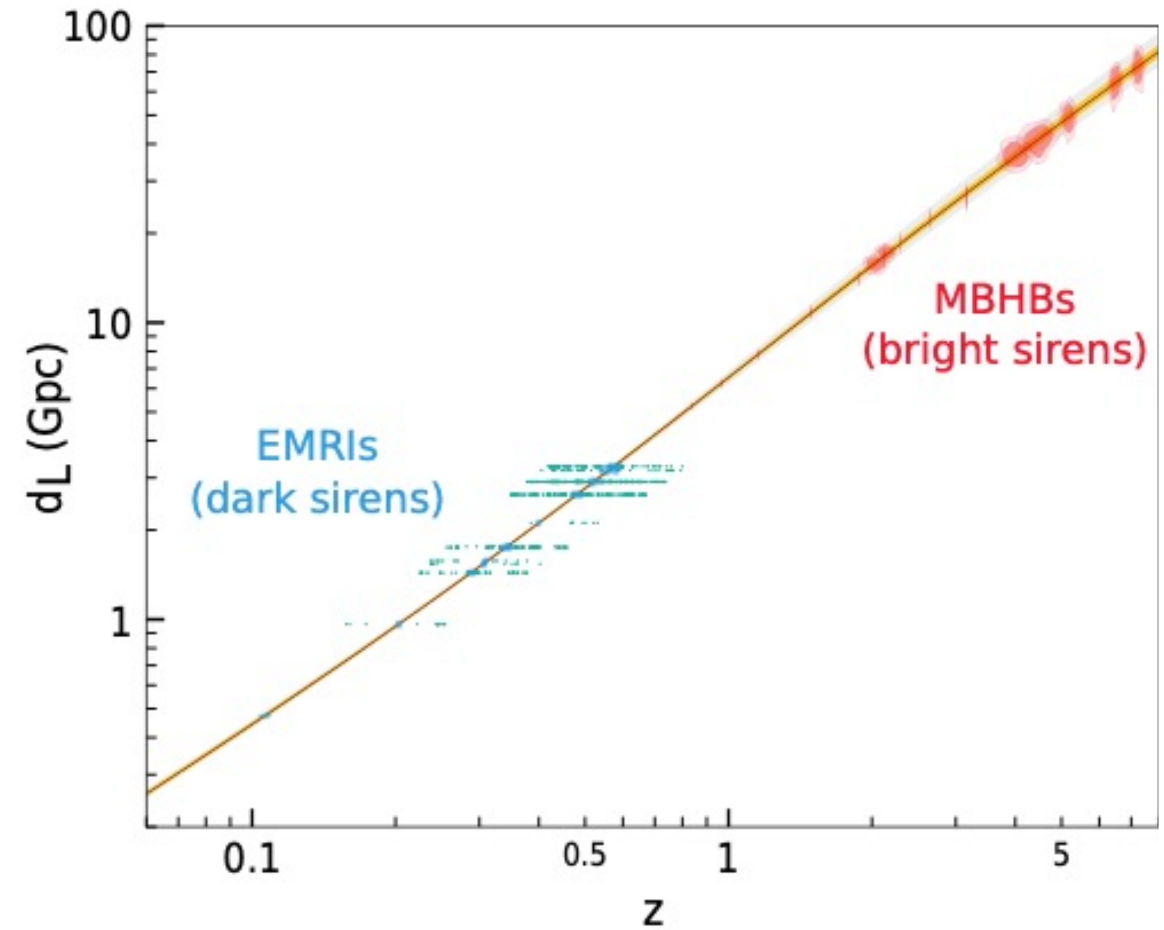
Cosmology with LISA

- **SMBHB** can be used as "**bright" sirens at high redshift** to infer cosmological parameters, e.g., **Hubble parameter**

EM counterparts and coincident GW detection: MBHB are expected to have counterparts if they occur in gaseous disks at the centre of galaxies (very uncertain rate)



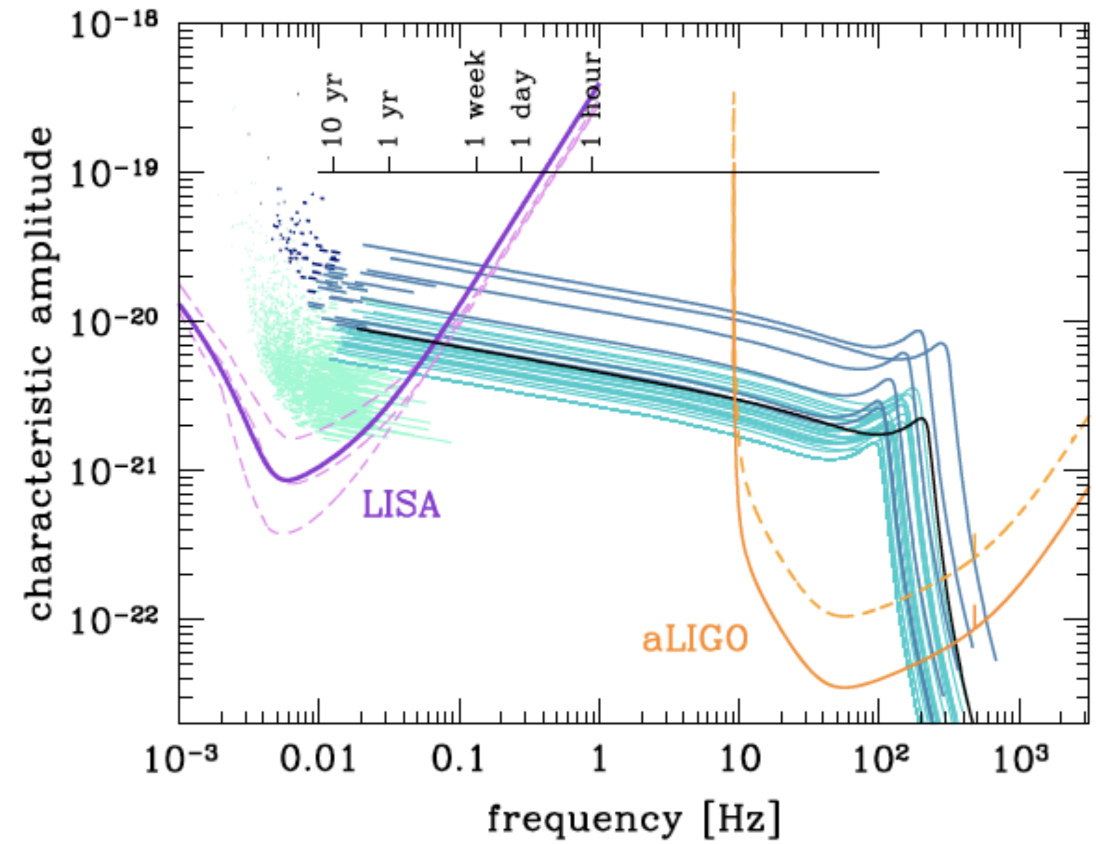
- **EMRIs and SOBHB can be used as dark sirens at smaller redshift** to infer cosmological parameters, e.g., Hubble parameter

- Both sources can be used to perform **tests of General Relativity**

Source	H_0	Ω_M	w_0
EMRIs	[1 – 6]%	[25 – 60]%	[7 – 12]%
MBHBs	[3 – 7]%	[4 – 9]%	-

Stellar Origin Black Holes

A Sesana arXiv:1702.04356



- Earlier in the inspiral phase, they can emit in the LISA band

$$M_c = 25\,M_\odot \quad \tau = 10\,\mathrm{y} \longrightarrow f = 0.01\,\mathrm{Hz}, \quad \dot{f} = 10^{-11}\,\mathrm{Hz/sec}$$

- Merging BHBs are observed by Earth-based interferometers

- Most of them will be quasi-monochromatic sources during the mission duration

- Enabling multiband and multimessenger observations at the time of coalescence.

S. Babak et al., JCAP 08 (2023) 034, ArXiv:2304.06368.

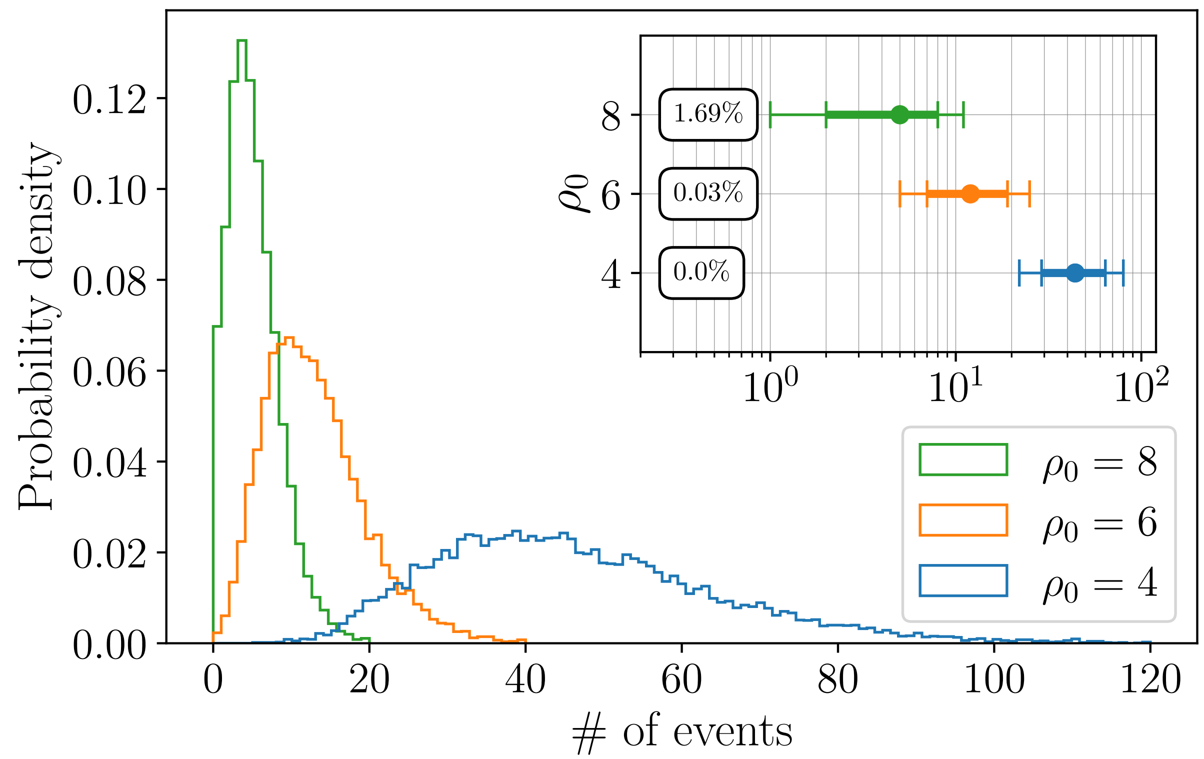
Stellar Origin Black Holes

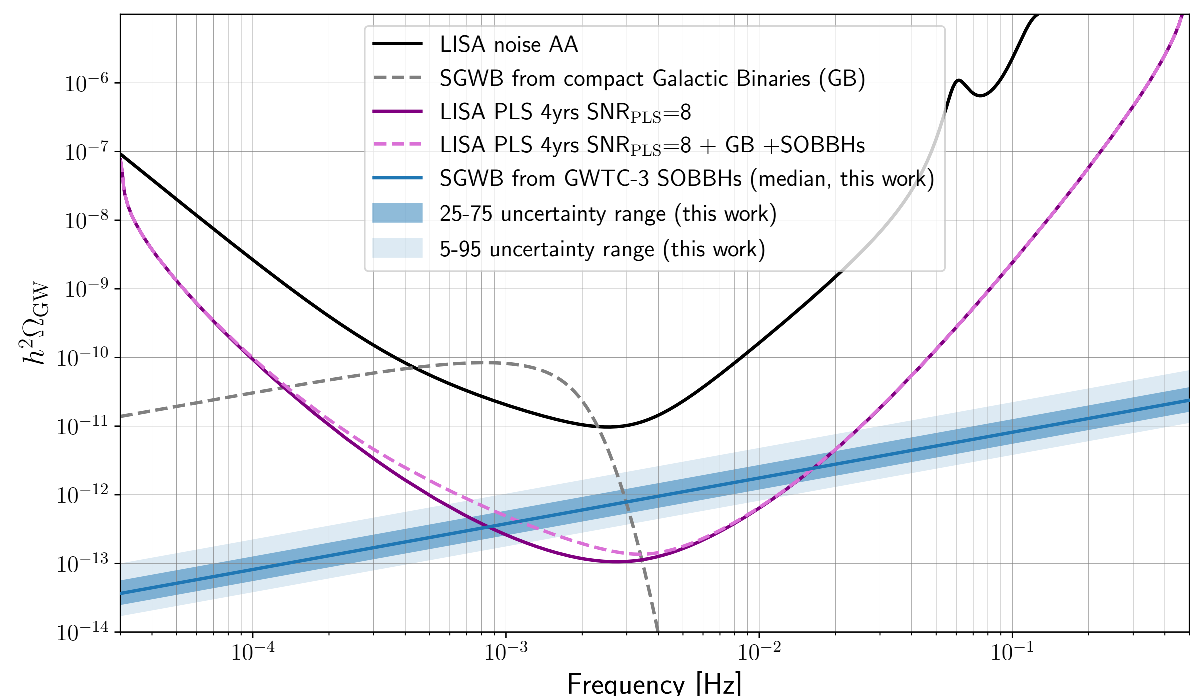
- The science that can be done with them depends on the number of resolved sources

- (Resolved: closer to merger and to us)

- Expected about 10 resolved in 4 years of mission, a few multi-band

- The rest (unresolved ones) generates a Stochastic GW Background (SGWB: distant and inspiralling)





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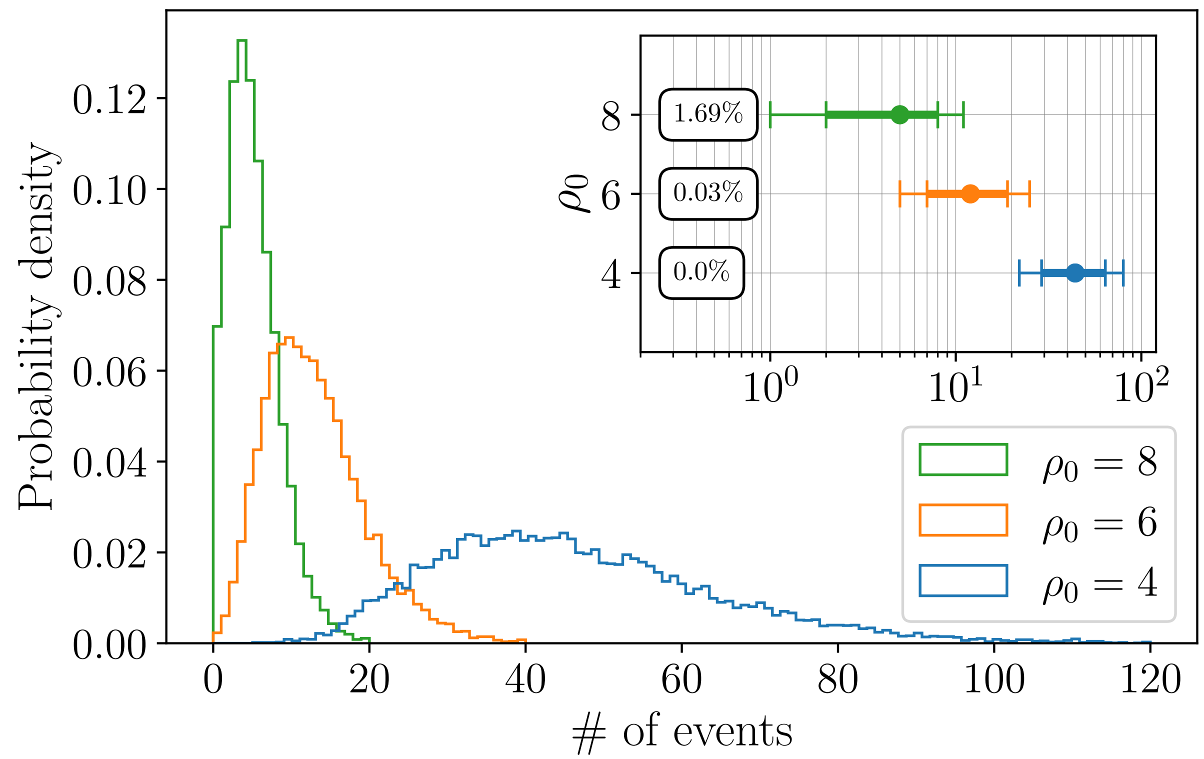

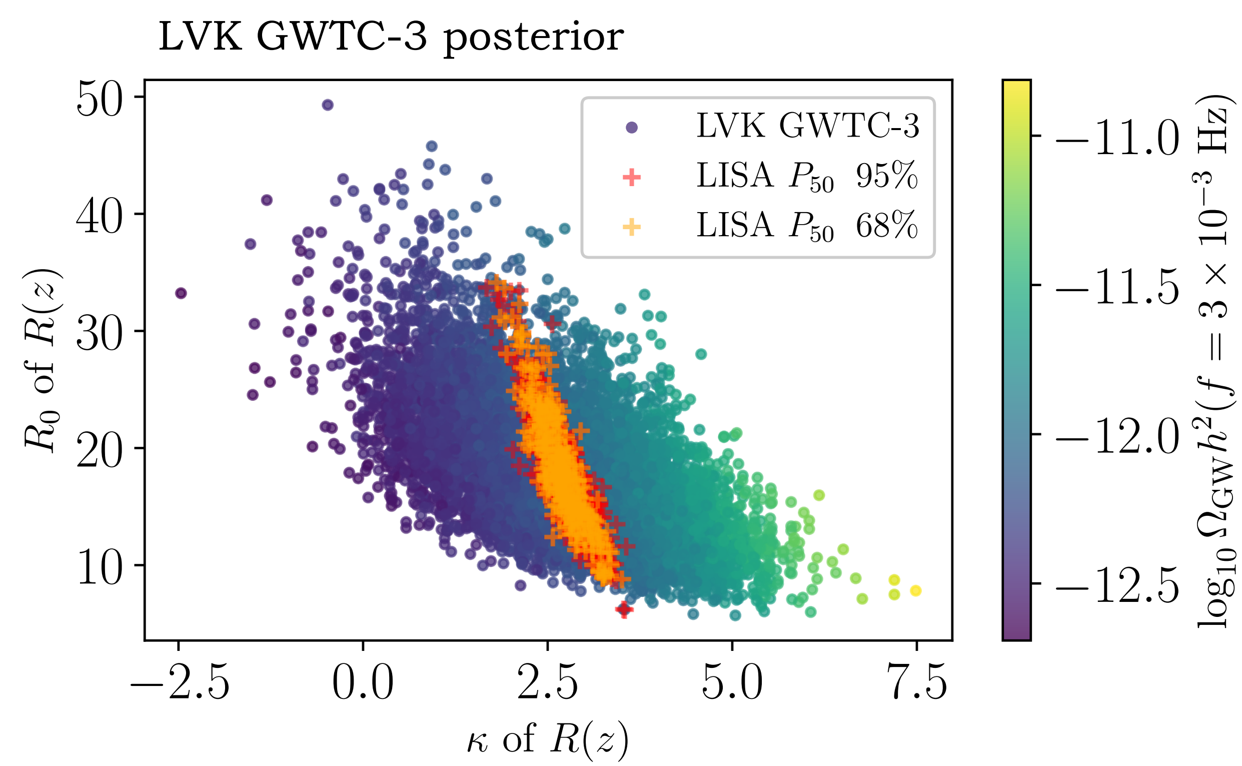

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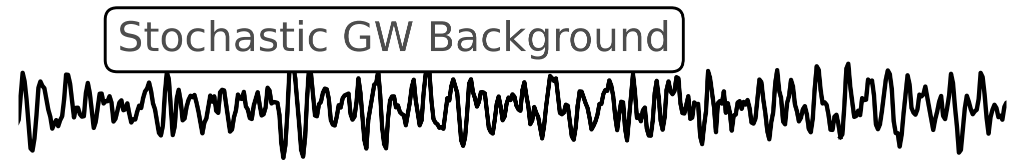
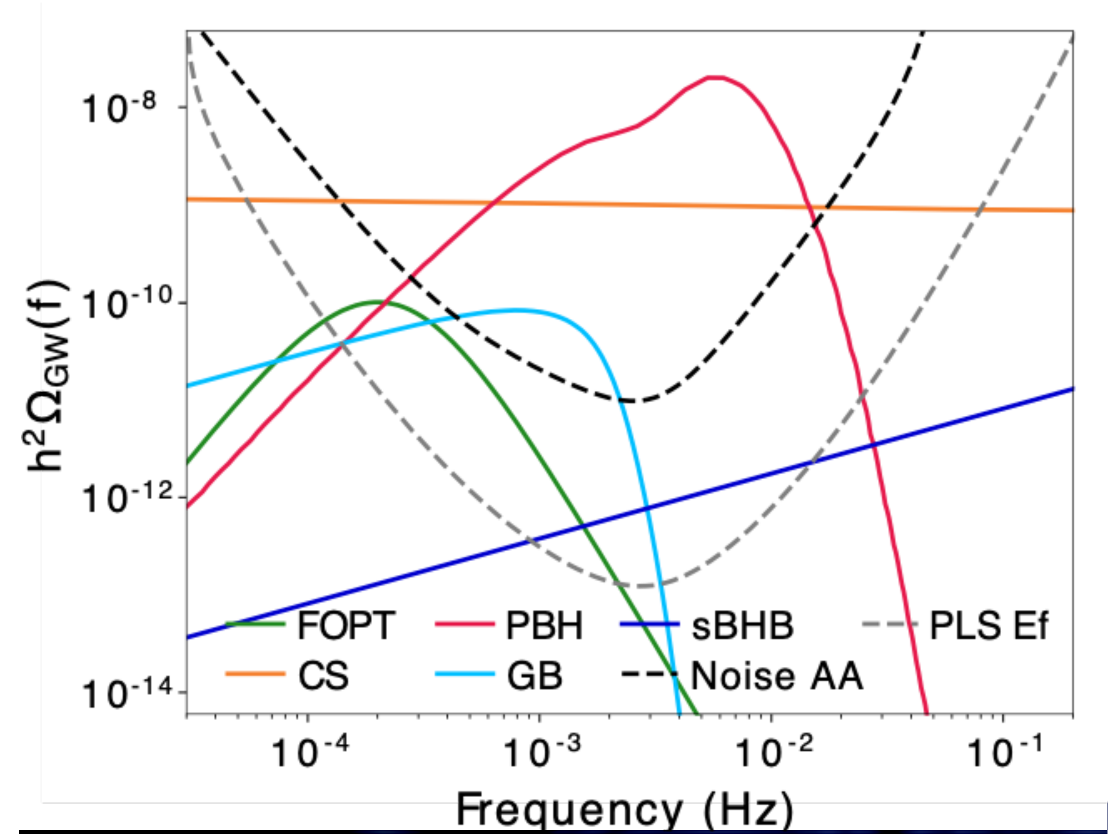


Stochastic GW backgrounds

and their implications for

- **early Universe**
- **TeV-scale particle physics (BSM physics)**



- GWs from **Inflation**
- GWs from **First Order Phase Transition (FOPT)**
- GWs from **Cosmic Strings (CS)**
- GWs from **Primordial Black Holes (PBH)**

Inflationary GW → Probe different range of scales (and of the inflationary potential), much smaller than CMB

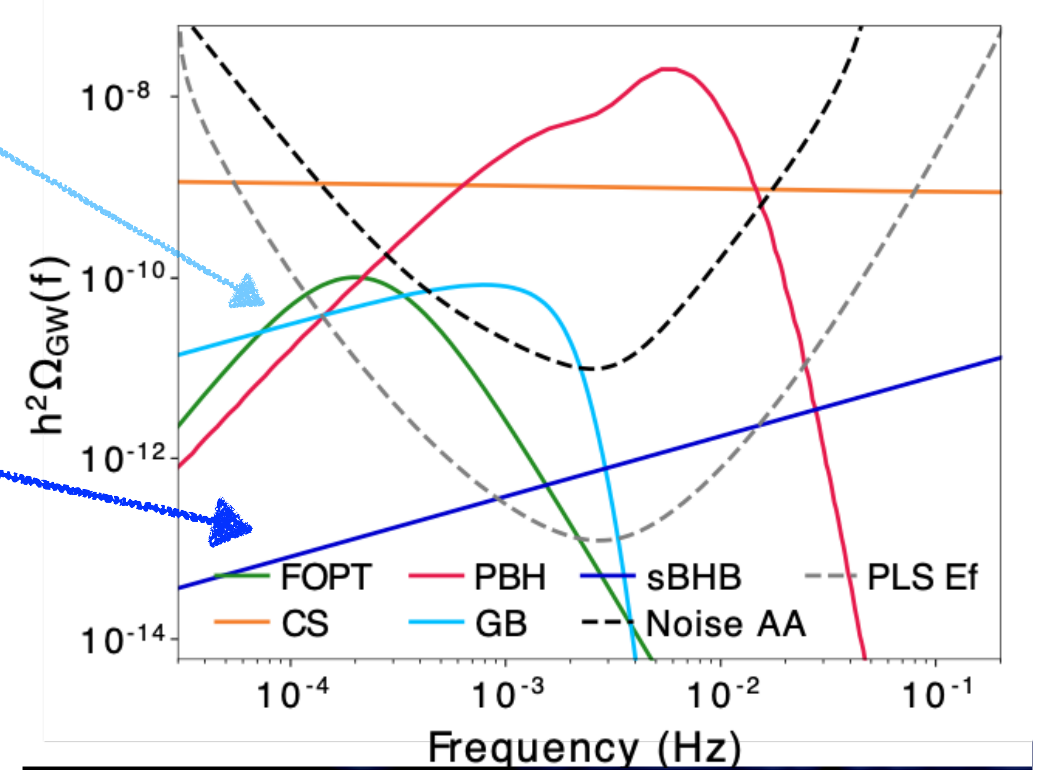
PT GWs → Test Physics Beyond the Standard Model of Particle Physics

PBH GWs → Shed light on Dark Matter candidates

Astrophysical GWB → Implication for population parameters

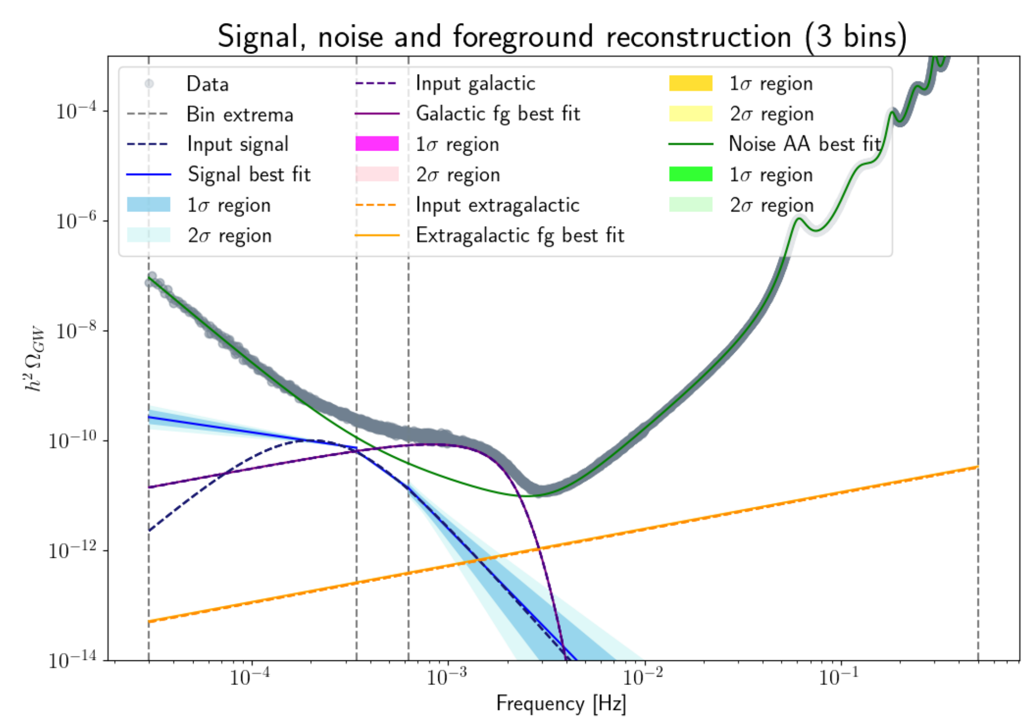
SGWB Reconstruction

- Detect and Subtract the GWB from GB?
- Characterise the astrophysical GWB from sBHBs LISA data according to LVK predictions



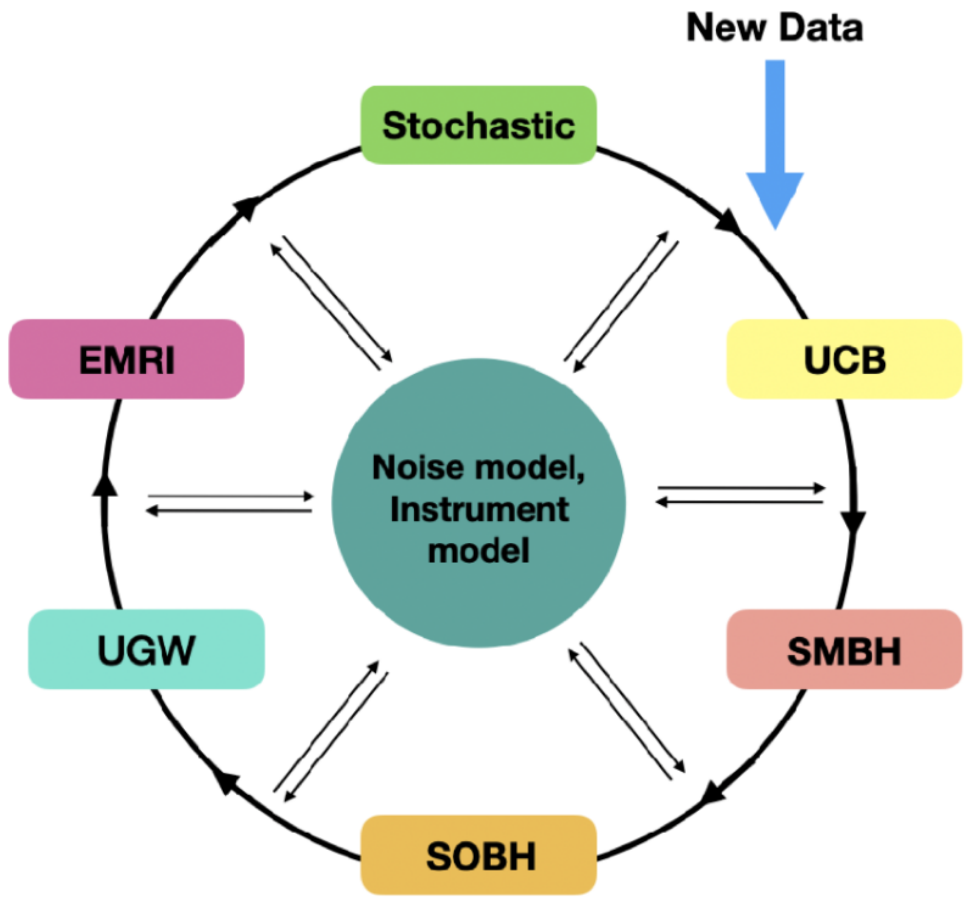
- Measure, or set upper limits on, the spectral shape of the **cosmological SGWB**

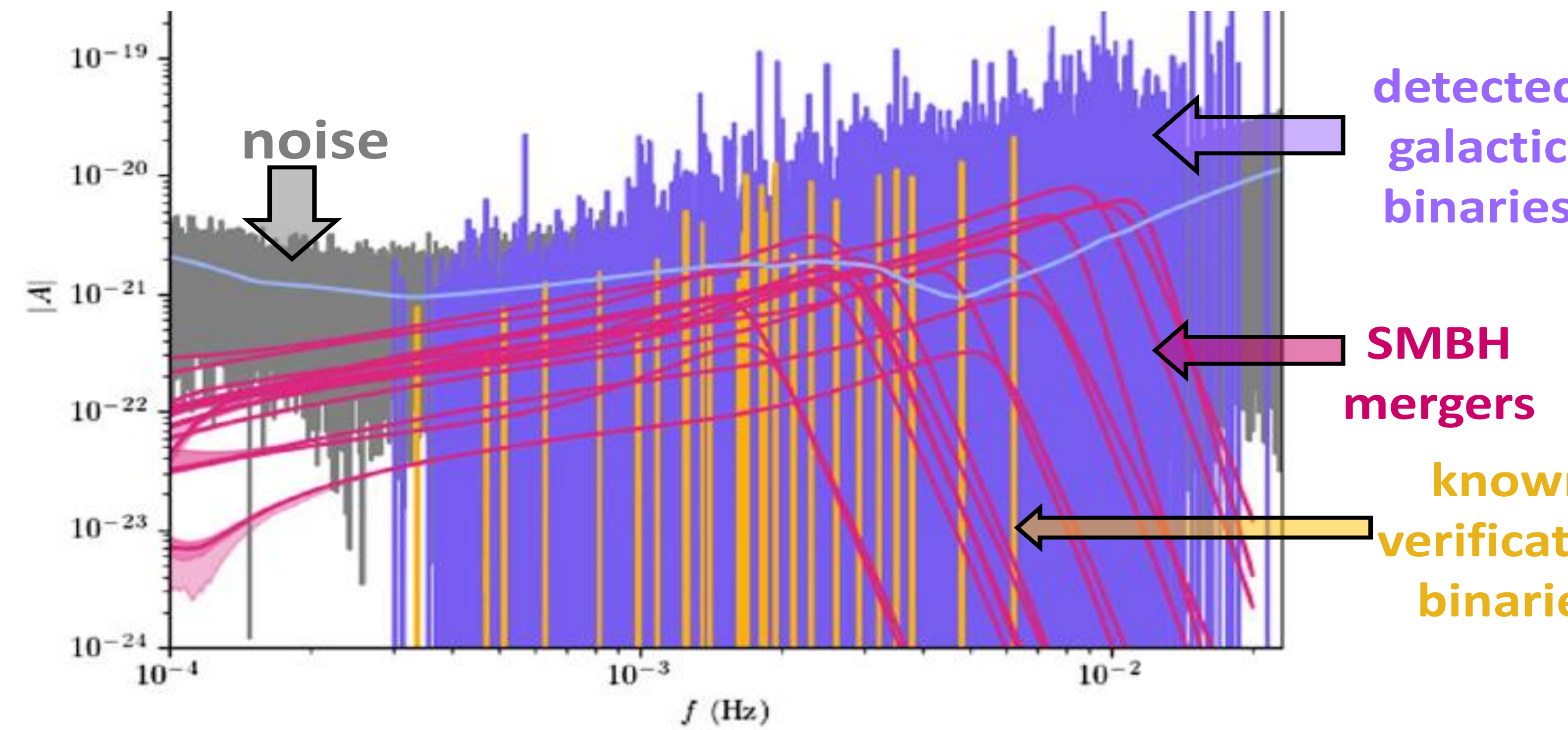
- Demonstrate that **CGWB** can be **reconstructed in LISA data under minimal assumptions** of the instrumental noise+reconstruction of spectral shape



[SGWBinner code
(LISA CosWG) '19, '20]

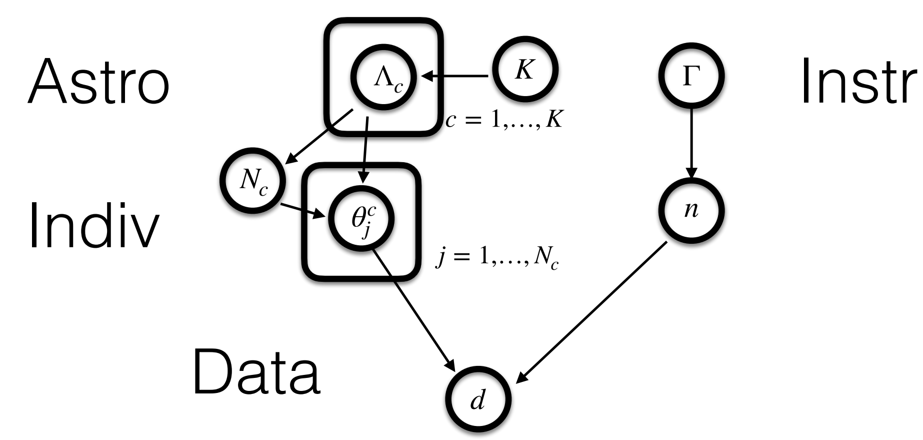
Global Fit





The data model

$$d(t) = n(t) + \sum_{c=1}^{K} \sum_{j=1}^{N_c} h_j^c(t; \theta_j^c)$$



With the **(unknown) number of classes of signals** (e.g., c= SMBH, sBH, EMRI, etc) and **N_c is the unknown number of signals per class**

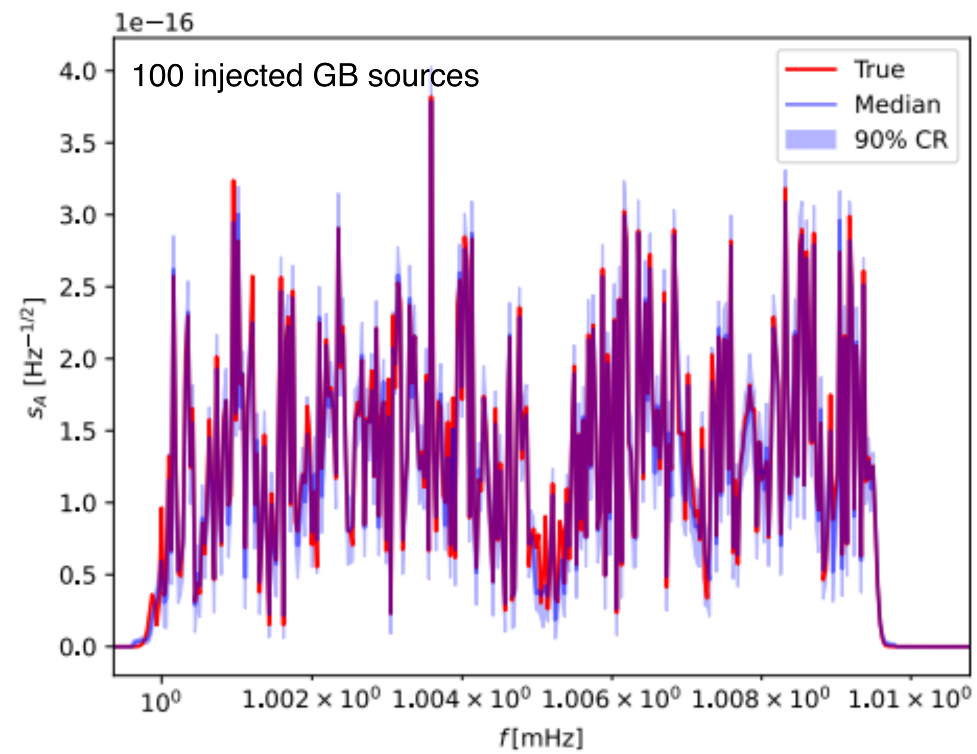
The challenge is to estimate the joint posterior for all sources, their numbers and their astrophysical distributions, jointly with the detector and noise models

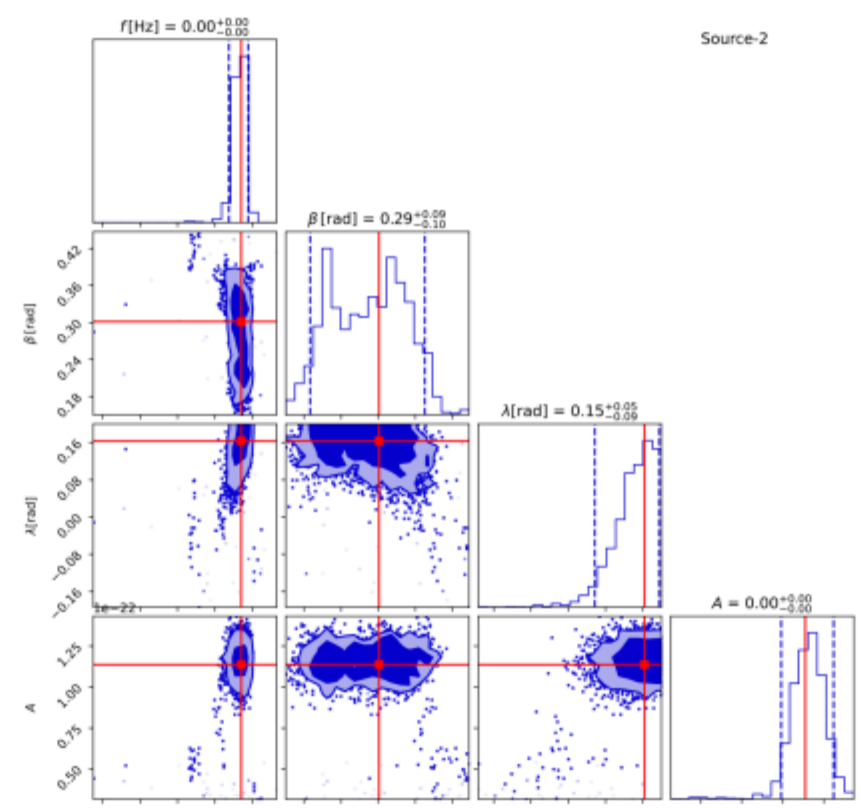
LISA Global Fit - Pisa - CTMCMC

Continuous time MCMC algorithms are Markov processes where the transitions are governed by Poisson distribution

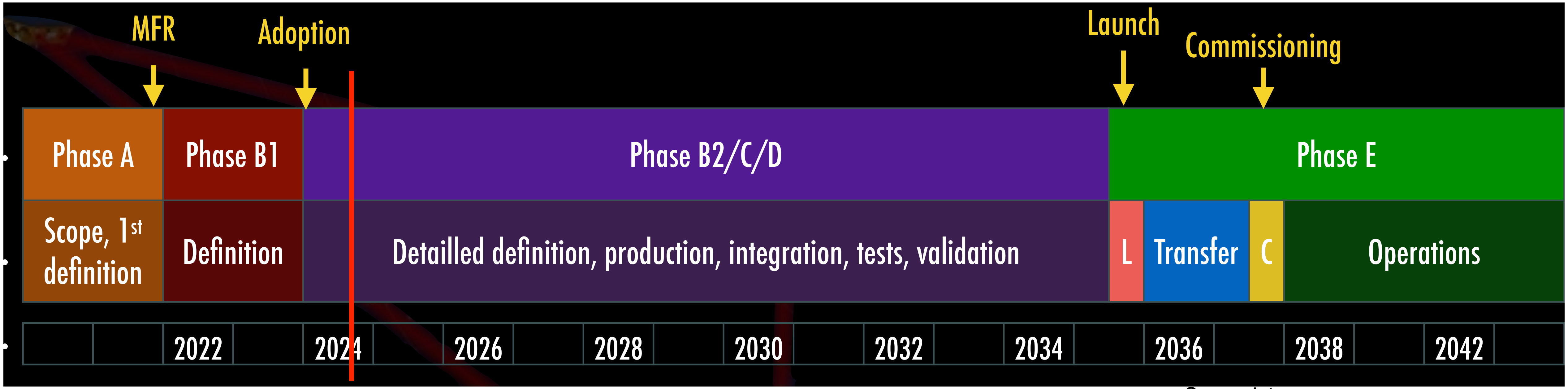
Idea is to associate the probability of a state* to the time the chain spent in that state $p(\prod_j \Omega_j | D) \propto \{\tau\}_j$

*A state is a given number of sources per class with a given set of parameters





LISA Timeline



Some plot

- 1993: first proposal ESA/NASA
- 20/06/2017: LISA mission approved by ESA Science Program Committee (SPC) after the success of LISAPathfinder and GW detection by LVK.
- 2020-2022: B1 phase success - stat industrial implementation to build the instrument
- 2022-2024: Mission Adoption
- (New) LISA Science Team in place
- 8 years: building phase (B2/C/D) of multiple MOSAs: 6 flight models + test models
- 2030-2034: launch Ariane 6.4
- 1.5 years of transfer, **4.5 years nominal mission**, 6.5 years extension → **GW Detections**

Conclusions

- **LISA is a large mission led by ESA to detect GWs in the mHz band been adopted in January by ESA**

- **LISA has been adopted in January by ESA**, i.e. it is fully supported by ESA, its member states and NASA

- It is now in its development and building phase for a **launch in 203 for 4.5 to 10 years of operations**

- **LISA** will cover a large range of domains and has a huge science case for **astrophysics, cosmology and fundamental physics.**

- LISA is a tool to explore the Universe !

Thank you!

History of LISA

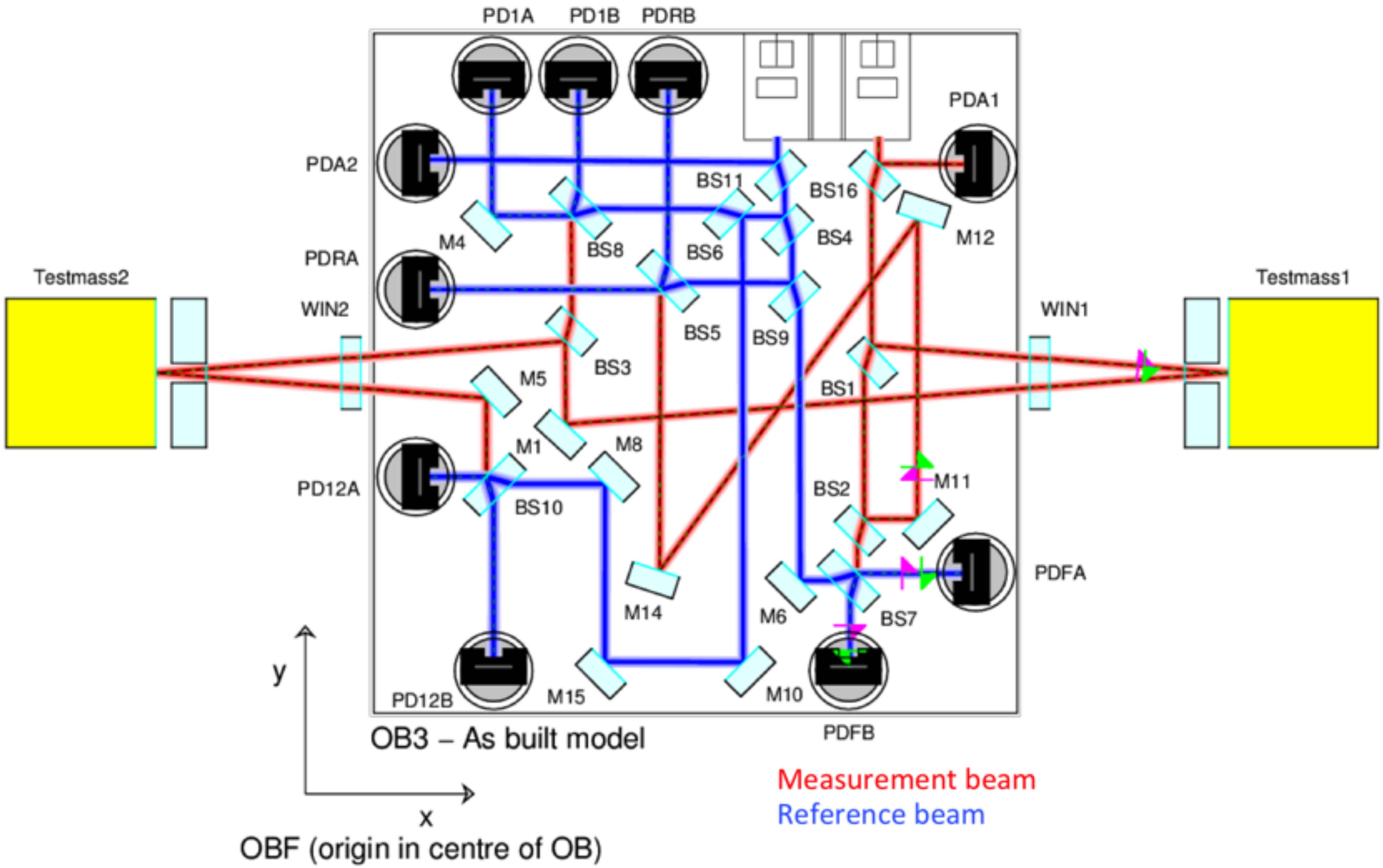
- 1978: first study based on a rigid structure (NASA)
- 1980s: studies with 3 free-falling spacecrafts (US)
- 1993: proposal ESA/NASA: 4 spacecrafts
- 1996-2000: pre-phase A report
- 2000-2010: LISA and LISAPathfinder: ESA/NASA mission
- 2011: NASA stops => ESA continue: reduce mission
- 2012: selection of JUICE L1 ESA
- 2013: selection of ESA L3 : « The gravitational Universe »
- 2015-2016: success of LISAPathfinder + detection GWs

LISA at ESA

- 25/10/2016 : Call for mission
- 13/01/2017 : submission of «LISA proposal» (LISA consortium)
- 8/3/2017 : Phase 0 mission (CDF 8/3/17 → 5/5/17)
- 20/06/2017 : LISA mission approved by SPC
- 8/3/2017 : Phase 0 payload (CDF June → November 2017)
- 2018→2020 : competitive phase A : 2 companies compete
- 2020→2022 : B1: start industrial implementation
- 2022-2024 : mission adoption
- During about 8.5 years : construction
- 2030-2034 : launch Ariane 6.4
- 1.5 years for transfert
- 4 years of nominal mission → GW observations !

LISA MISSION SUMMARY

<table>
<tr><th colspan="3">Measurement</th></tr>
<tr><td colspan="3">Gravitational waves (GWs) in the Frequency Band of 0.1 mHz - 1.0 Hz with a GW Strain Spectral Density: 10^{-21}–10^{-23}</td></tr>
<tr><th colspan="3">Payload</th></tr>
<tr><td>Lasers</td><td colspan="2">2 per spacecraft • 2 W output power • wavelength 1064 nm • frequency stability $300\,\mathrm{Hz}/\sqrt{\mathrm{Hz}}$</td></tr>
<tr><td>Optical Bench</td><td colspan="2">2 per spacecraft • double-sided use • high thermal stability (Zerodur)</td></tr>
<tr><td>Interferometry</td><td colspan="2">heterodyne interferometry • $15\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ precision • Inter-spacecraft ranging to ~1 m</td></tr>
<tr><td>Telescope</td><td colspan="2">2 per spacecraft • 30 cm off-axis telescope • high thermal stability</td></tr>
<tr><td>Gravitational Reference System</td><td colspan="2">2 per spacecraft • acceleration noise $<3\,\mathrm{fm}/(\mathrm{s}^2\sqrt{\mathrm{Hz}})$ • 46 mm cubic AuPt test mass • Faraday cage housing • electrostatic actuation in 5 degree of freedom</td></tr>
<tr><th colspan="3">Mission</th></tr>
<tr><td>Duration</td><td colspan="2">4.5 years science orbit • >82 % duty cycle • ~6.25 years including transfer and commissioning</td></tr>
<tr><td>Constellation</td><td colspan="2">Three drag-free satellites forming an equilateral triangle • 2.5×10^6 km separation • trailing/leading Earth by ~20° • inclined by 60° with respect to the ecliptic</td></tr>
<tr><td>Orbits</td><td colspan="2">Heliocentric orbits • semimajor axis ~1 AU • eccentricity $e \approx 0.0096$ • inclination $i \approx 0.96°$</td></tr>
<tr><th colspan="3">Data Analysis</th></tr>
<tr><td>Noise Reductions</td><td colspan="2">Laser noise suppression with time-delay interferometry • Ranging processing and delay estimation • Spacecraft jitter suppression and reduction to 3 lasers • Tilt-to-length effect correction • Clock noise suppression • Clock synchronisation</td></tr>
<tr><td rowspan="5">Data Levels</td><td>Level 0</td><td>Raw data, de-multiplexed, time-ordered, corruption removed</td></tr>
<tr><td>Level 0.5</td><td>Primary science telemetry, decommutated, time-stamped, unit-level calibrations applied</td></tr>
<tr><td>Level 1</td><td>Time-Delay Interferometry (TDI) variables (GW strain)</td></tr>
<tr><td>Level 2</td><td>Output from a global fit pipeline, statistical evidence for candidate sources</td></tr>
<tr><td>Level 3</td><td>Catalogue of GW sources (detection confidence, estimated astrophysical parameters)</td></tr>
</table>

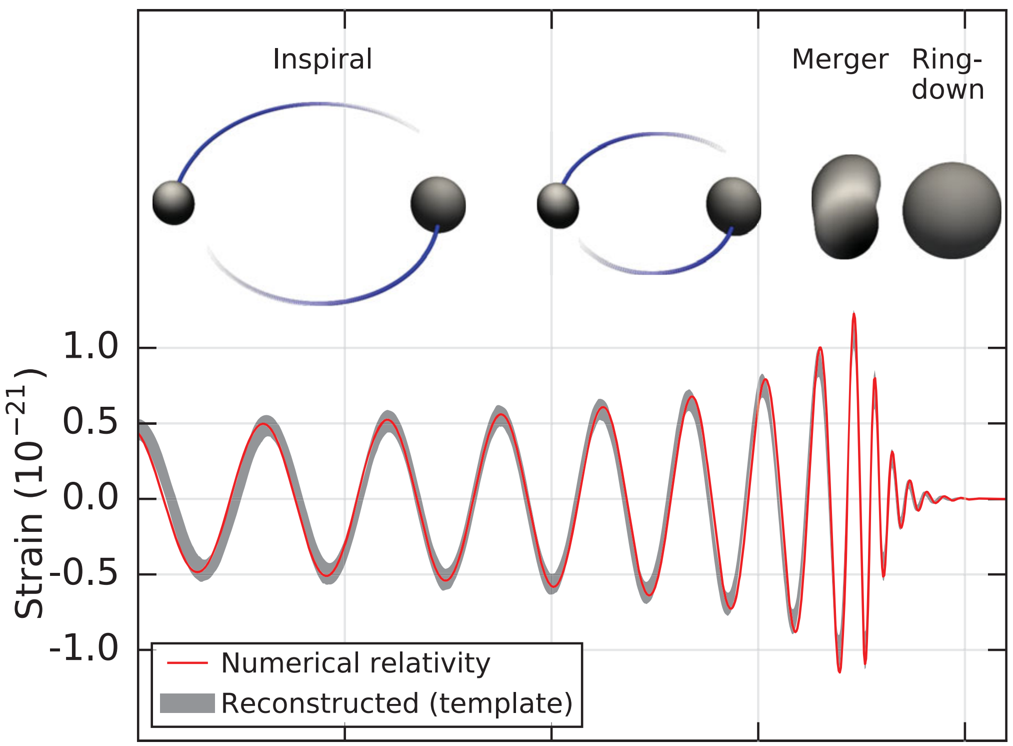

PD1A
PD1B
PDRB
PDA1
PDA2
BS11
BS16
M4
BS8
BS6
BS4
M12
PDRA
Testmass2
Testmass1
WIN2
WIN1
BS5
BS9
BS3
M5
BS1
M1
M8
PD12A
M11
BS2
BS10
PDFA
M14
M6
BS7
y
PD12B
M15
M10
PDFB
OB3 – As built model
Measurement beam
Reference beam
x
OBF (origin in centre of OB)

What is a Stochastic Gravitational Wave Background (SGWB)

A SGWB is, by definition, made up of an incoherent superposition of signals from sources that are unresolved in both the time and angular domain

Resolved Sources:

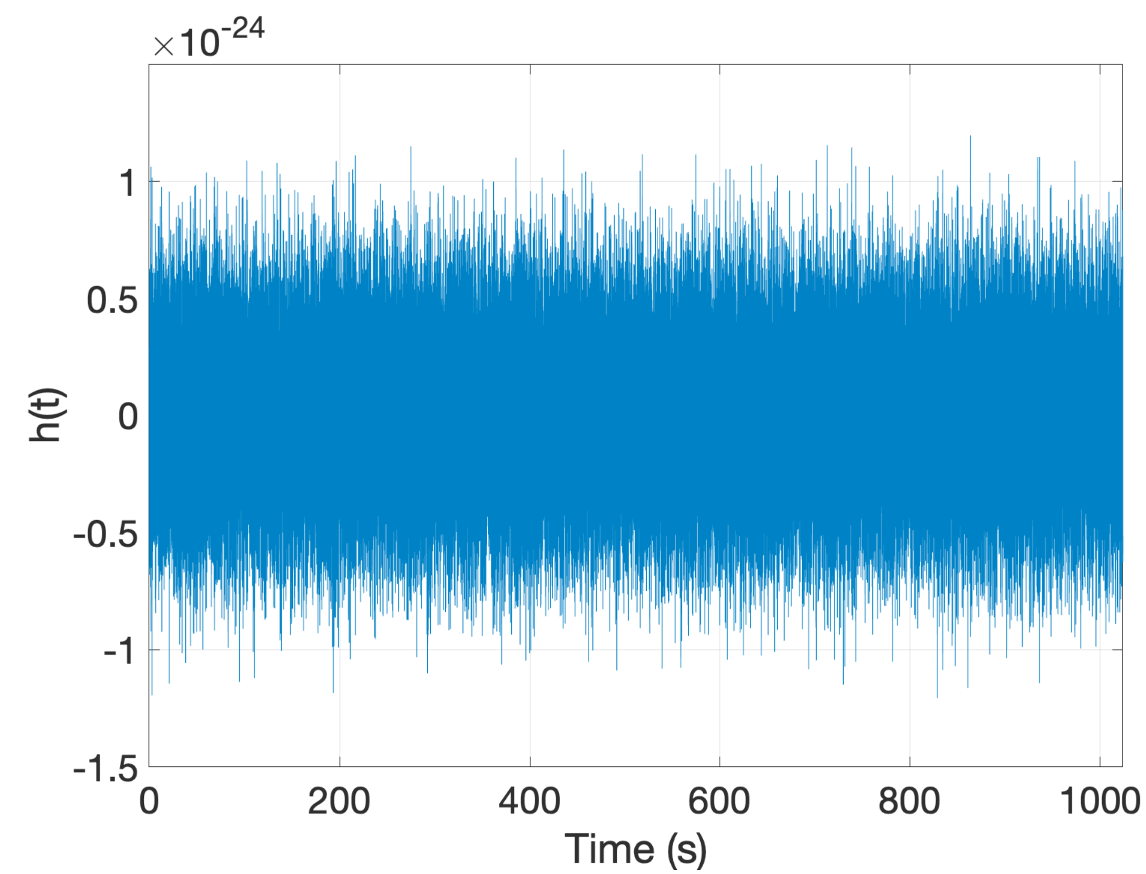
- Black Holes
- Neutron Stars
- White Dwarfs
- Supernovae
- …



Unresolved Sources:

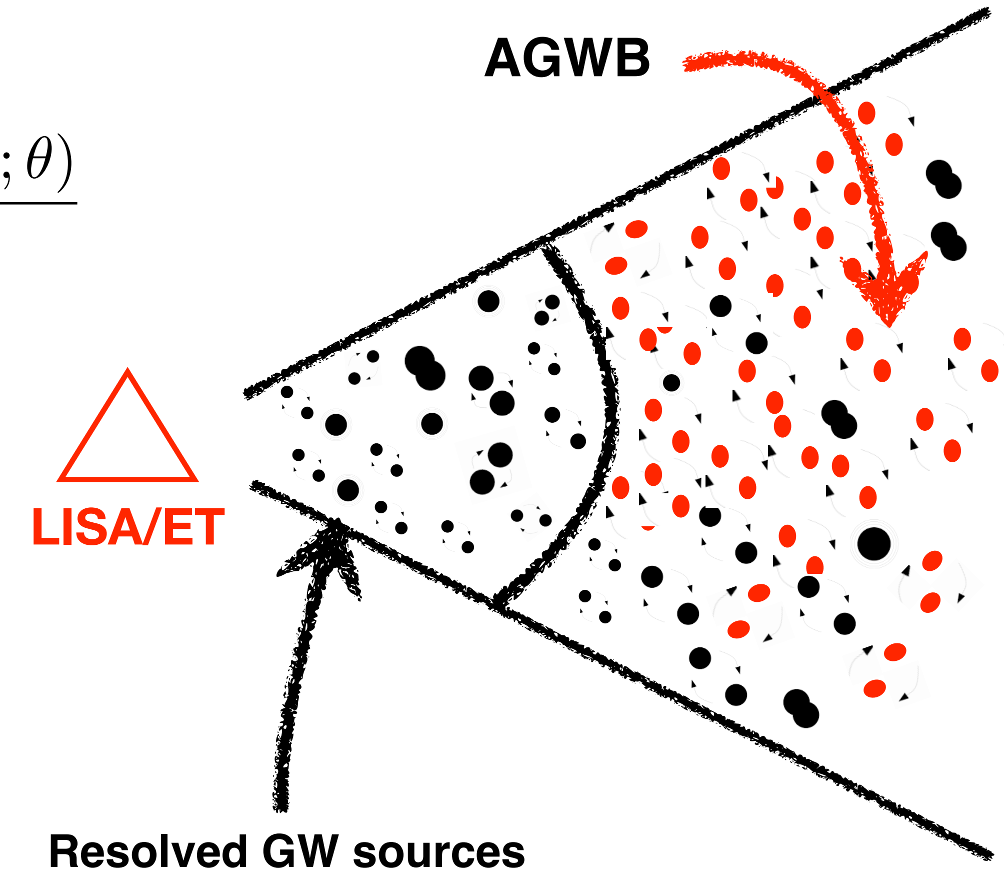
Stochastic Backgrounds

- **Astrophysical**
- **Cosmological**



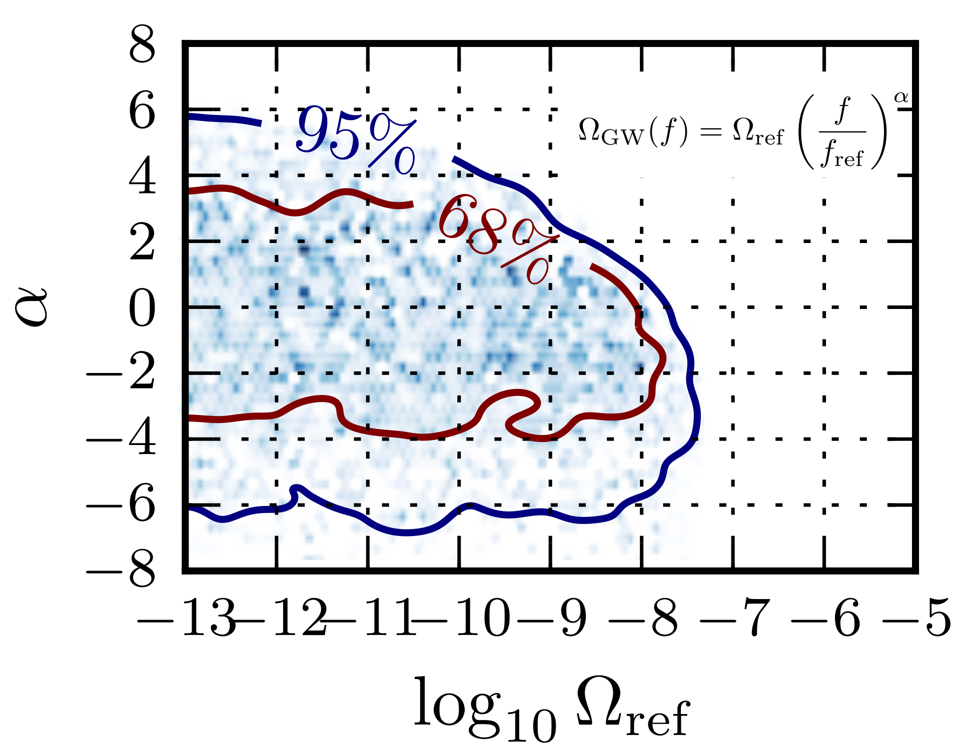
Astrophysical GW Background

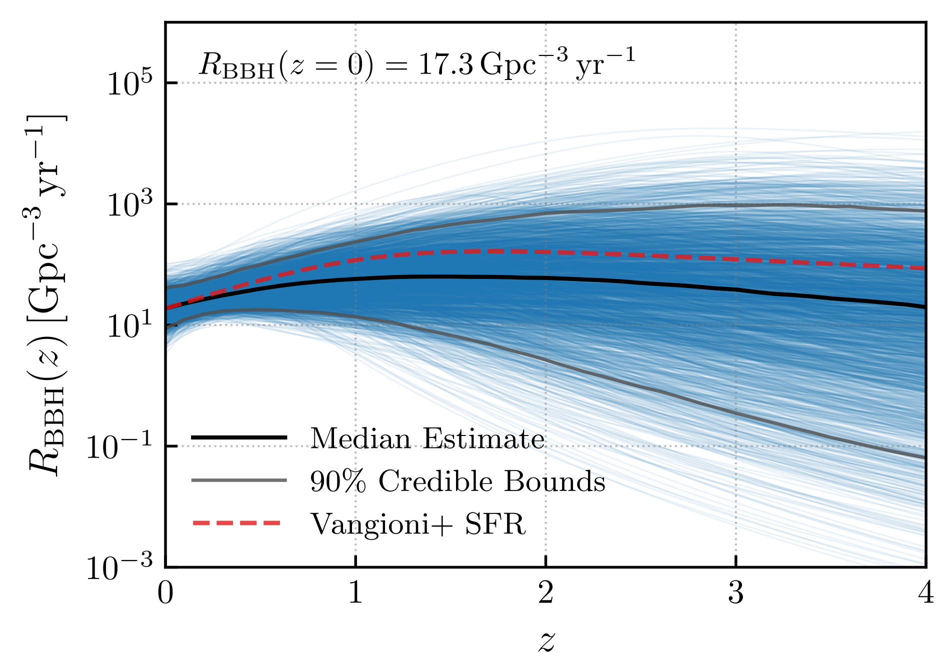
$$\Omega(f) = \frac{f}{\rho_c} \int d\theta \, p(\theta) \int_0^{z_{max}(\theta)} dz \frac{R(z;\theta)}{(1+z)H(z)} \frac{dE_{GW}(f_s;\theta)}{df_s}$$



Carry information about:

- star formation history
- statistical properties of source populations
- our cosmological model



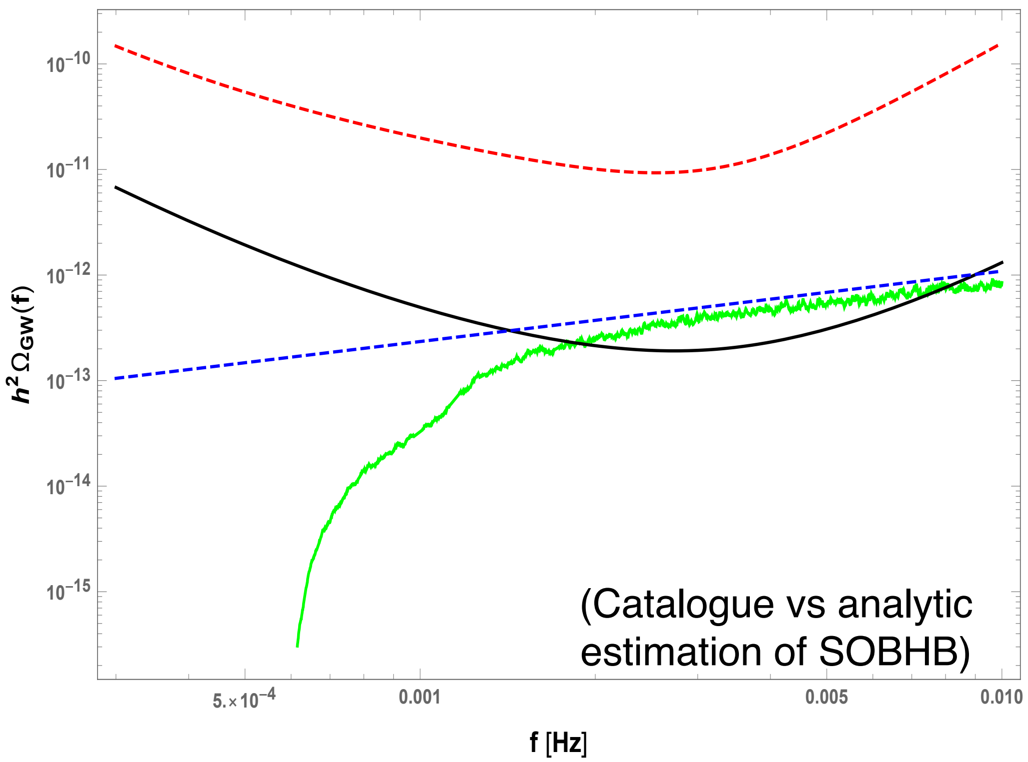


[LVK Collaboration 2021]

It is a kind of “noise” for the cosmological background, even if with different properties

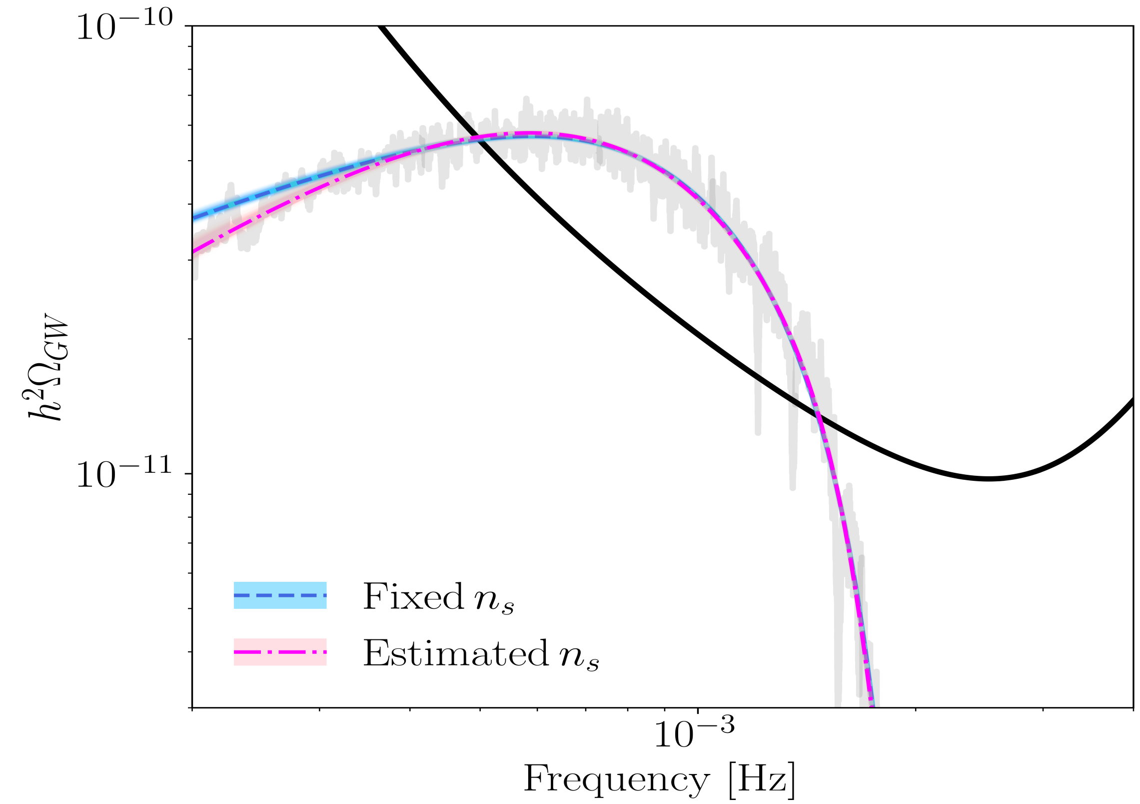
Astrophysical GW Background

LISA SOBHB



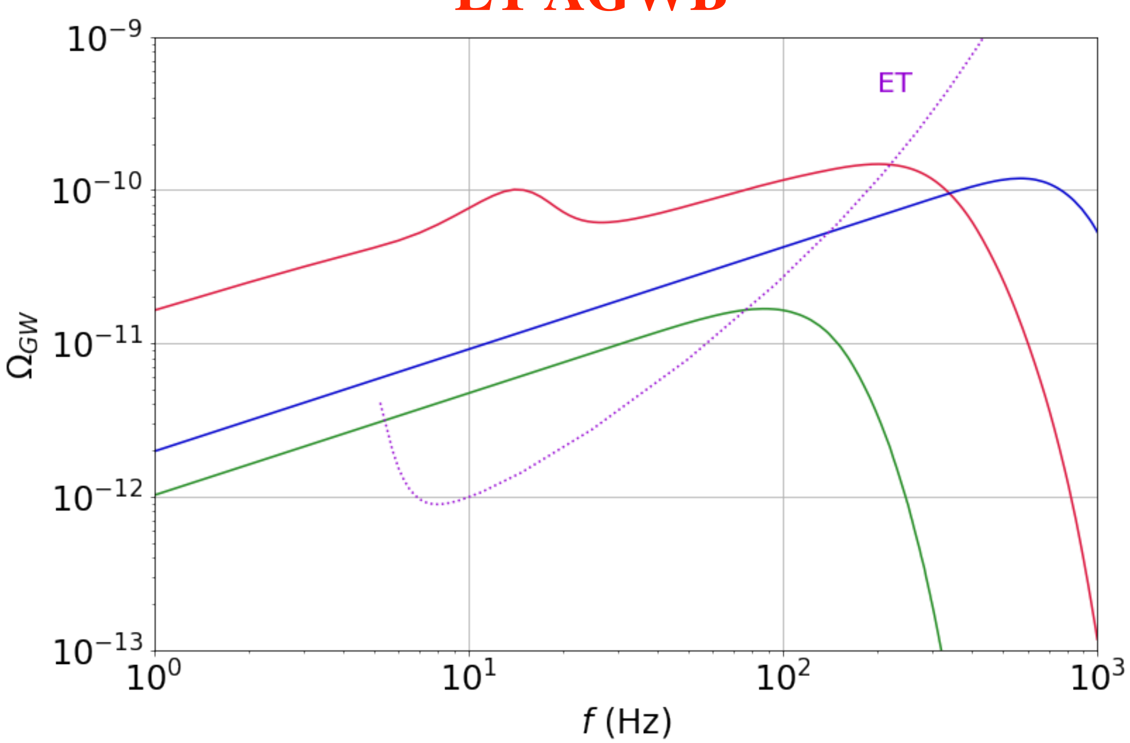
[Caprini C. et al., in preparation]

LISA GB

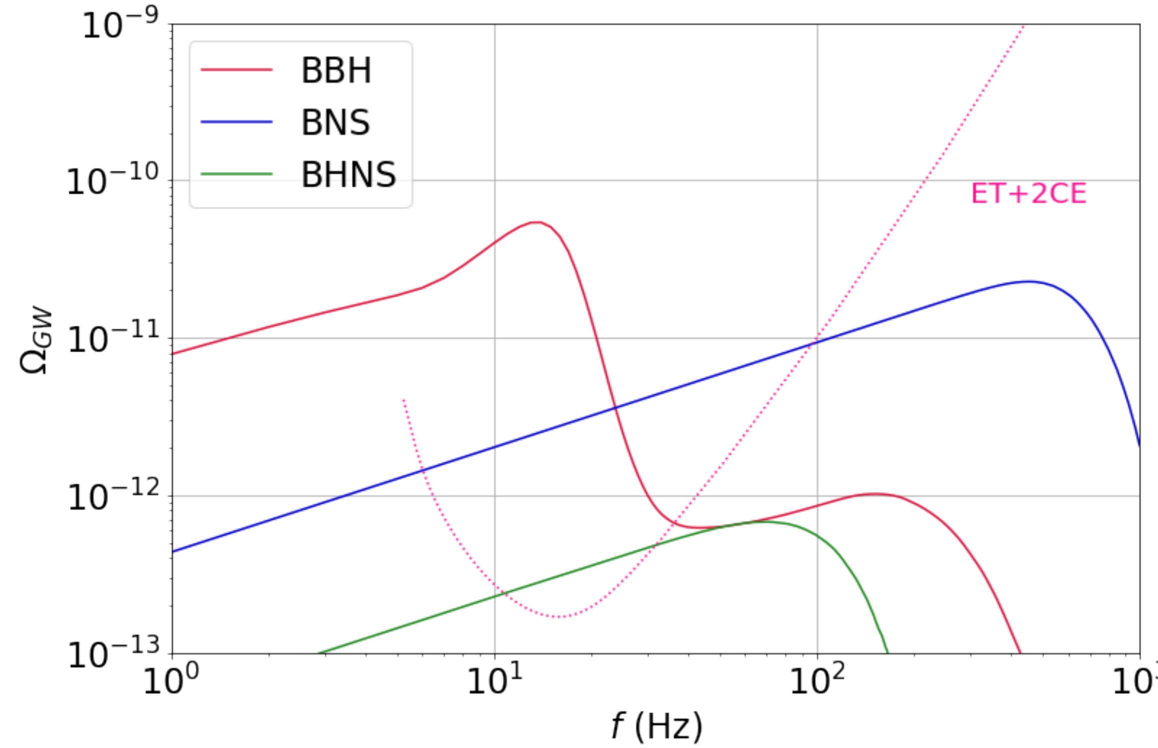


[Karnesis N., et al., 2021]

ET AGWB

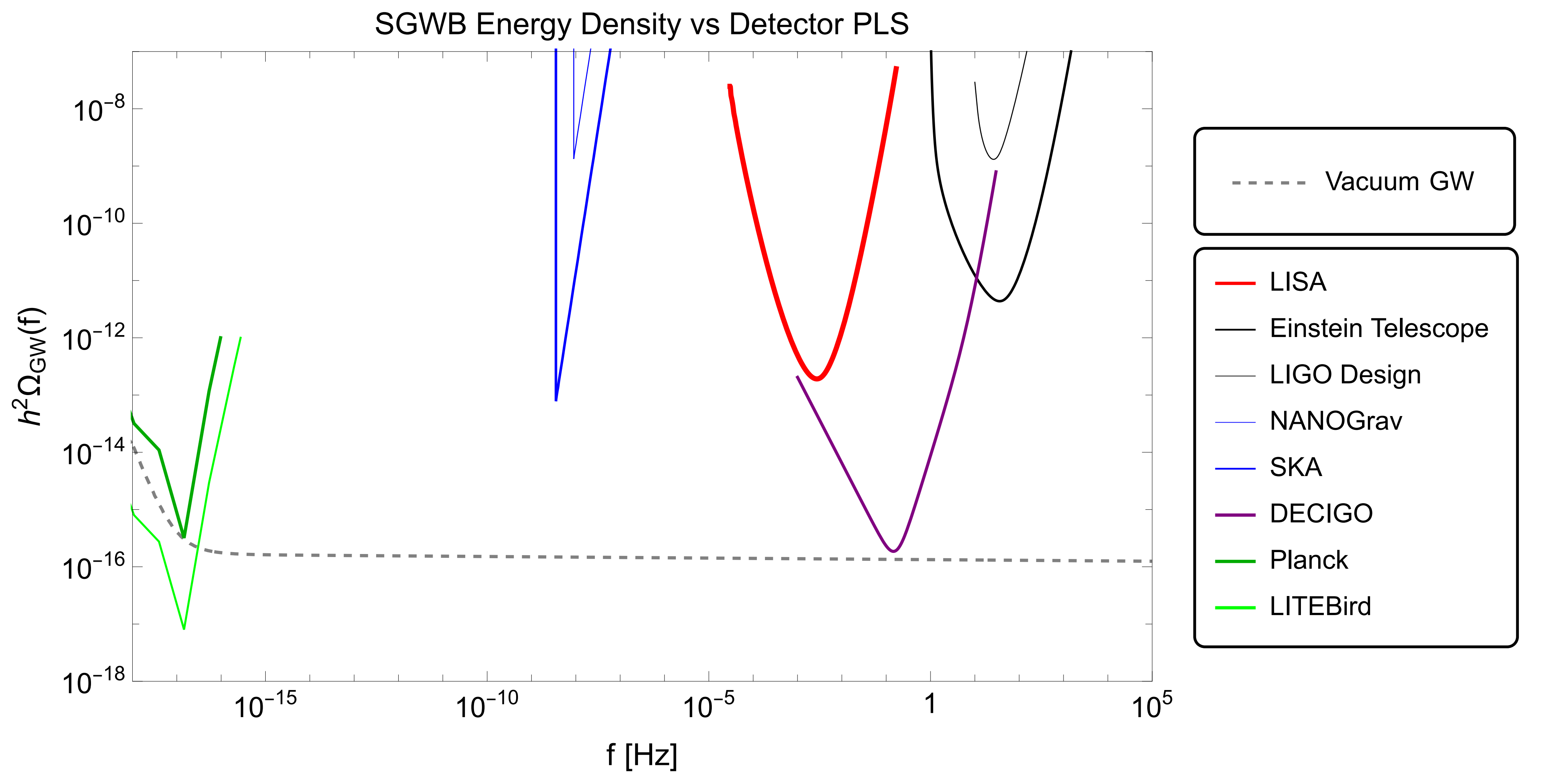


ET + 2CE



[Perigois C. et al., 2021]

Cosmological GW Background



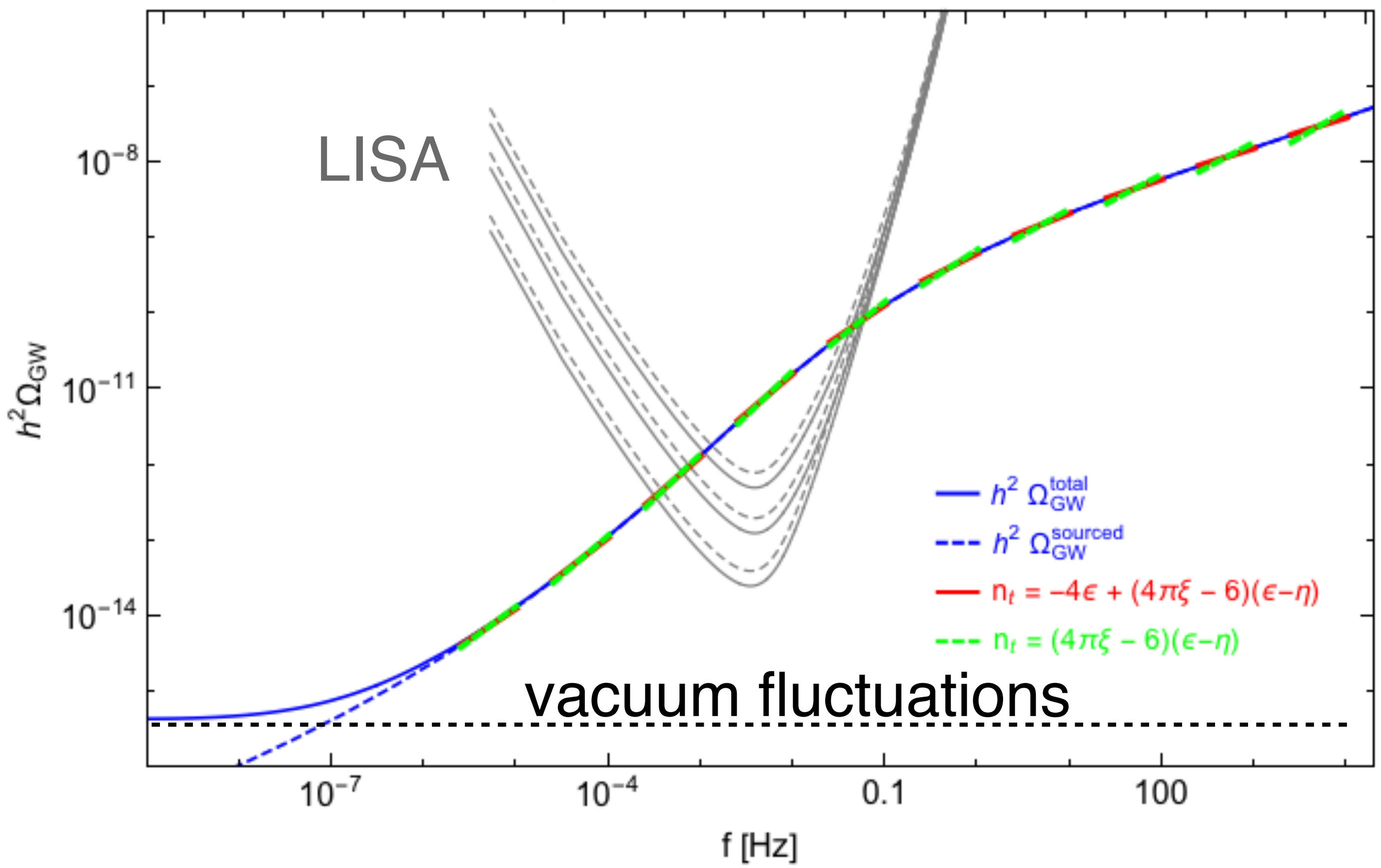
[LISA Cosmology White Paper]

Inflationary sources: Axion-inflation

$$\mathcal{L} \supset -\frac{\varphi}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\xi \equiv \frac{\dot{\varphi}}{2fH}$$

GW energy spectrum today



Peculiar features

- Blue-Tilted SGWB Spectrum
- **Chiral** SGWB spectrum
- Non-Gaussian SGWB

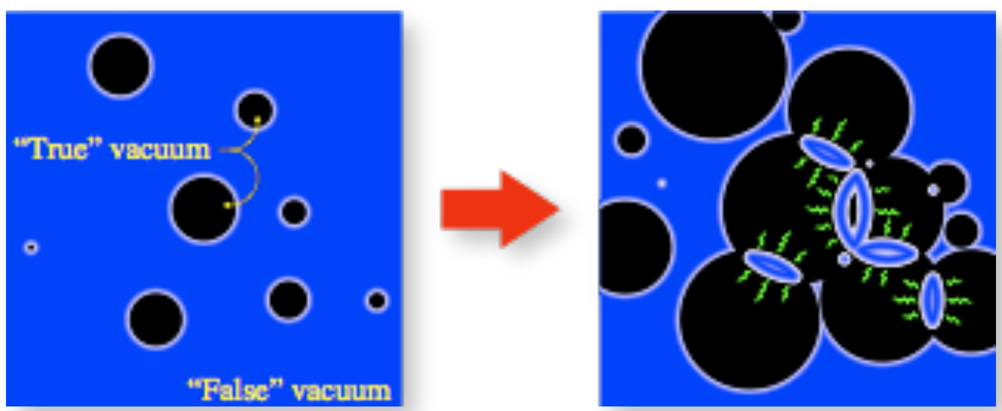
$$\Omega_{GW}^{TOT}(f) = \Omega_{GW,vacuum}(f) + \Omega_{GW,sourced}(f)$$

[Bartolo N., et al. '16 - LISA CosWG paper]
[Cook & Sorbo, '11] [Namba et al., '15]
[Domcke, Pieroni, Binetruy, '16]

SGWB from Phase transition

As the temperature in the very early universe decreases, there can be several PTs: QCD, EW....Beyond Standard Model?

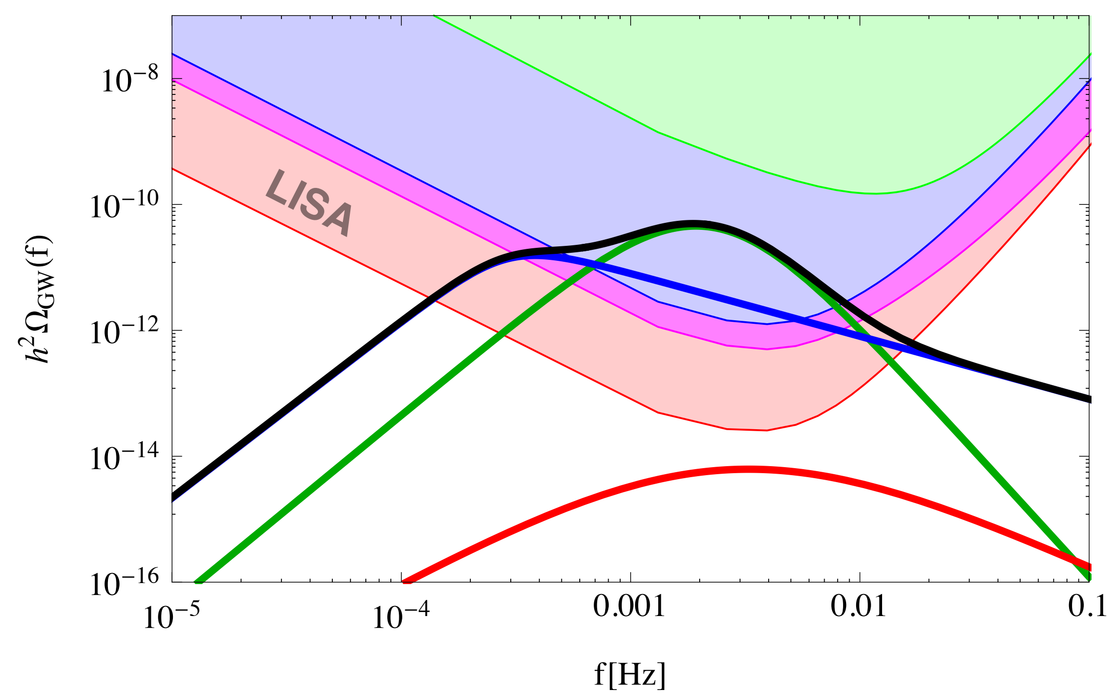
If the PT is first order, the SGWB signal could be detectable by LISA/ET



Processes

- Bubble collisions
- MHD Turbulence
- Sound Waves

[Caprini C., et al '16, '19- LISA CosWG paper]



- peaked spectrum with

$$f_{\text{peak}} \sim 10^{-3}\ \text{Hz}\ \frac{T}{100\ \text{GeV}}$$

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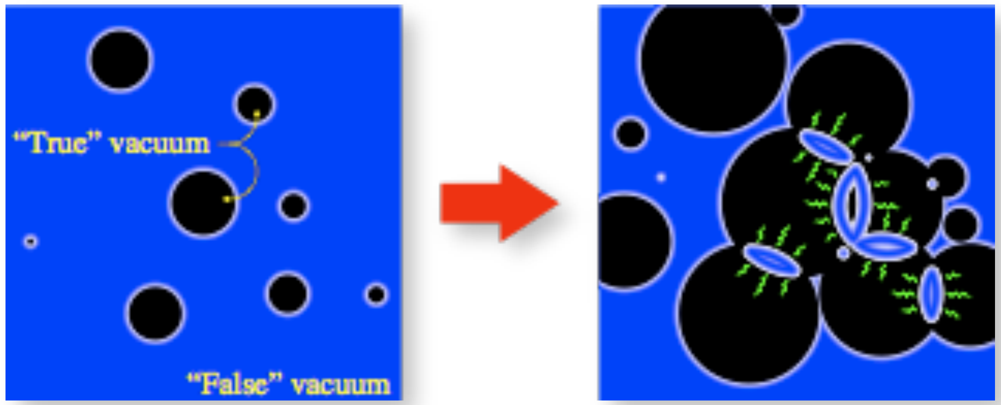

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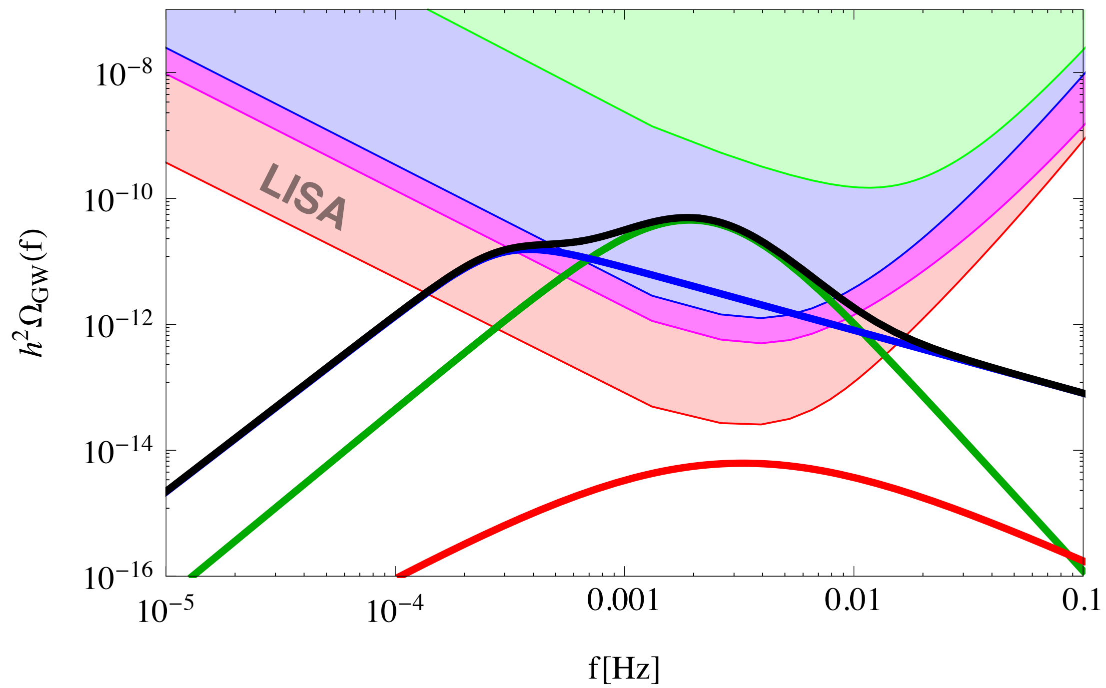

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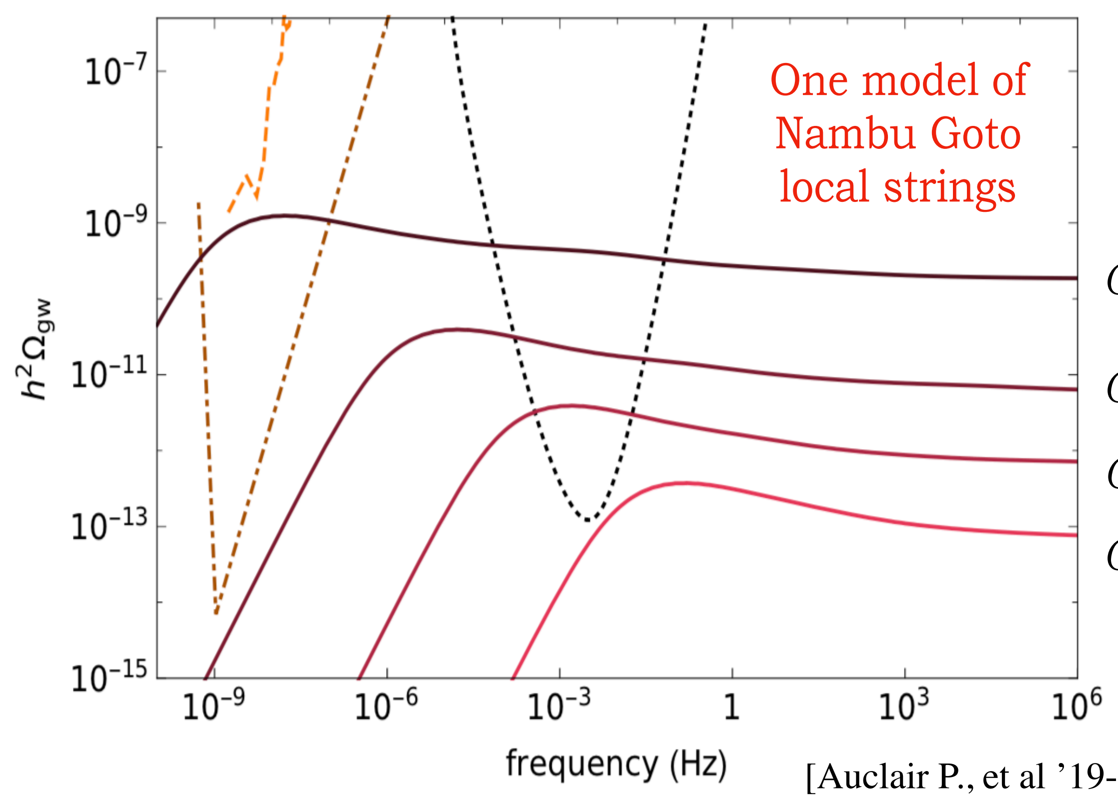
$$f_{\rm peak} \sim 10^{-3}\ {\rm Hz}\ \frac{T}{100\ {\rm GeV}}$$

SGWB from Topological Defects

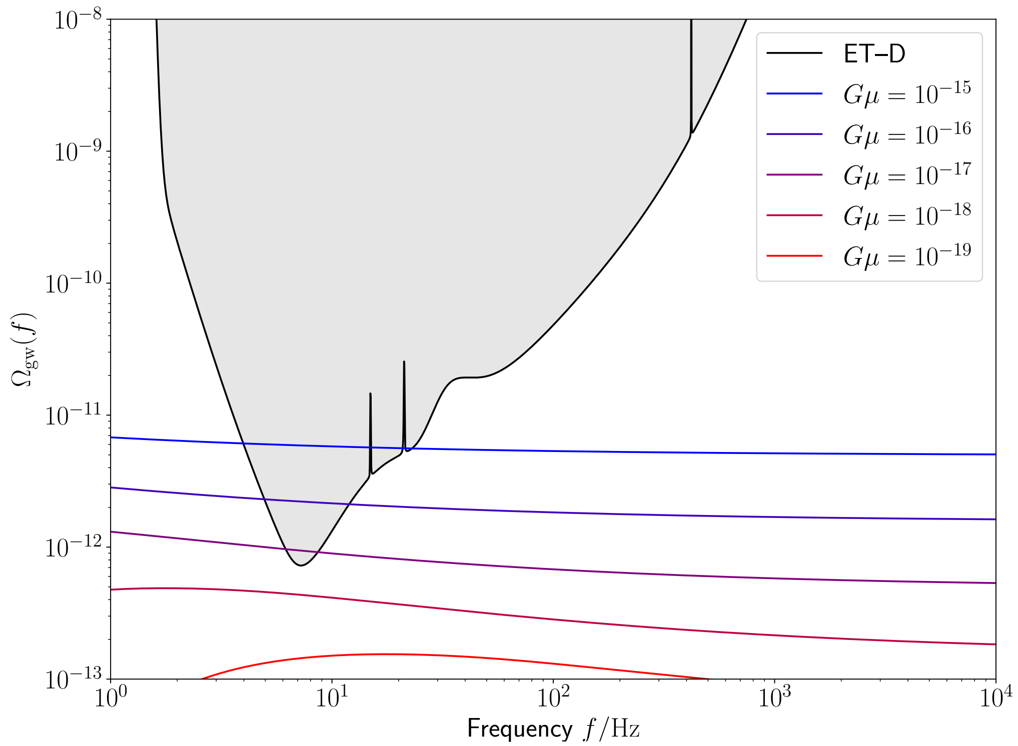
Cosmic Strings (or other kind of topological defects) are non-trivial field configurations left-over after the phase transition has completed

A network of cosmic strings emits GWs

(results are model dependent)



[Auclair P., et al '19- LISA CosWG]



[ET Science Case, 2020]

Future CMB B-mode

$$G\mu \sim 10^{-9}$$

LIGO/Virgo bound

$$G\mu \sim 10^{-14}$$

LISA or ET

$$G\mu \gtrsim 10^{-17}$$

$$G\mu \sim 10^{-6}\left(\frac{\eta}{10^{16}\ \mathrm{GeV}}\right)^2$$

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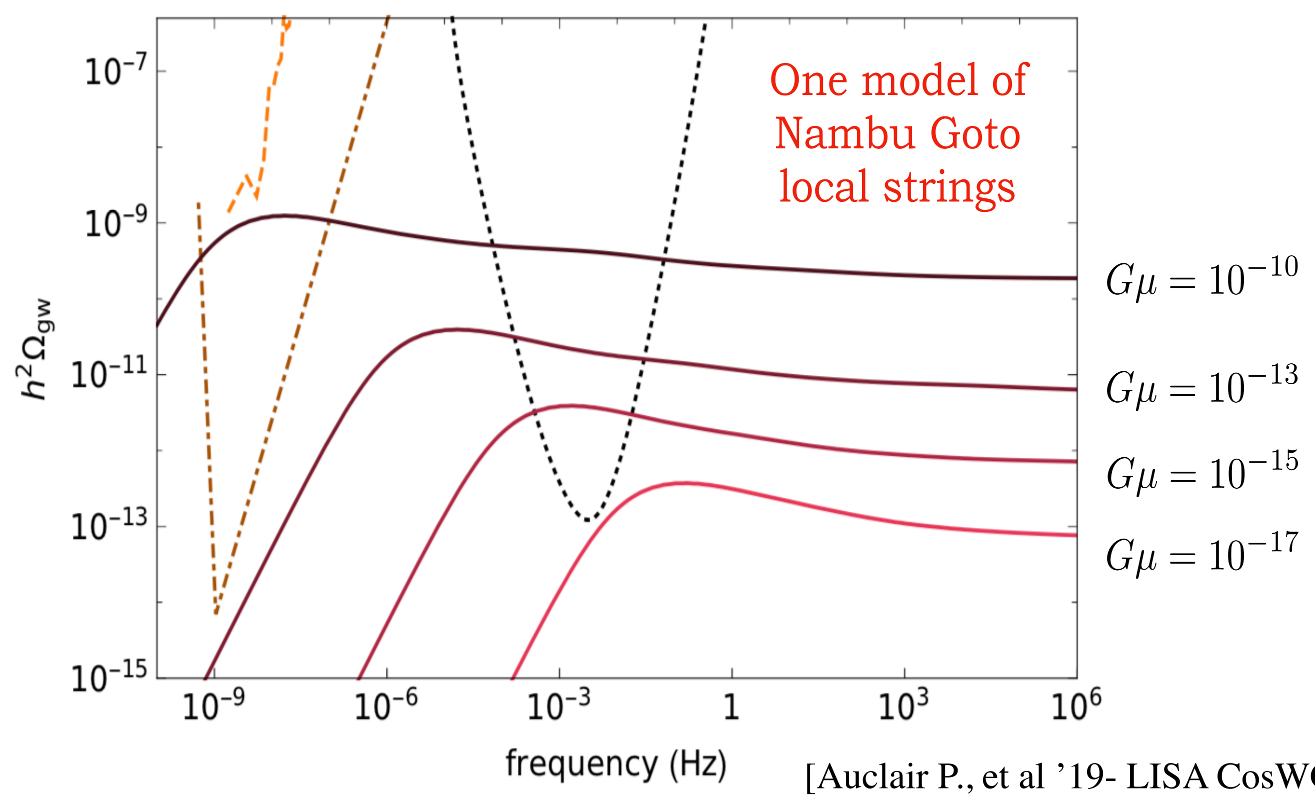

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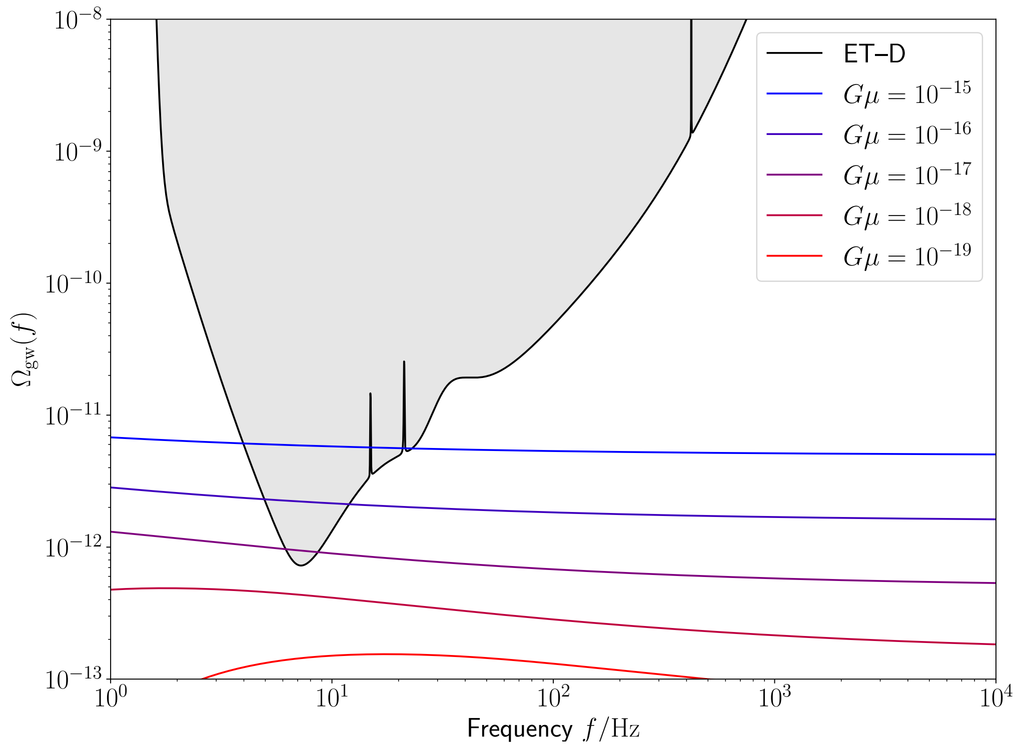

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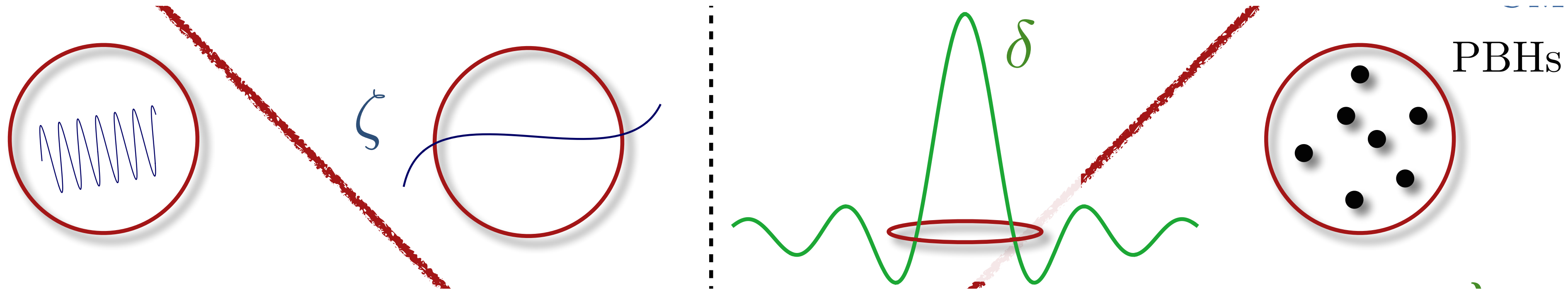
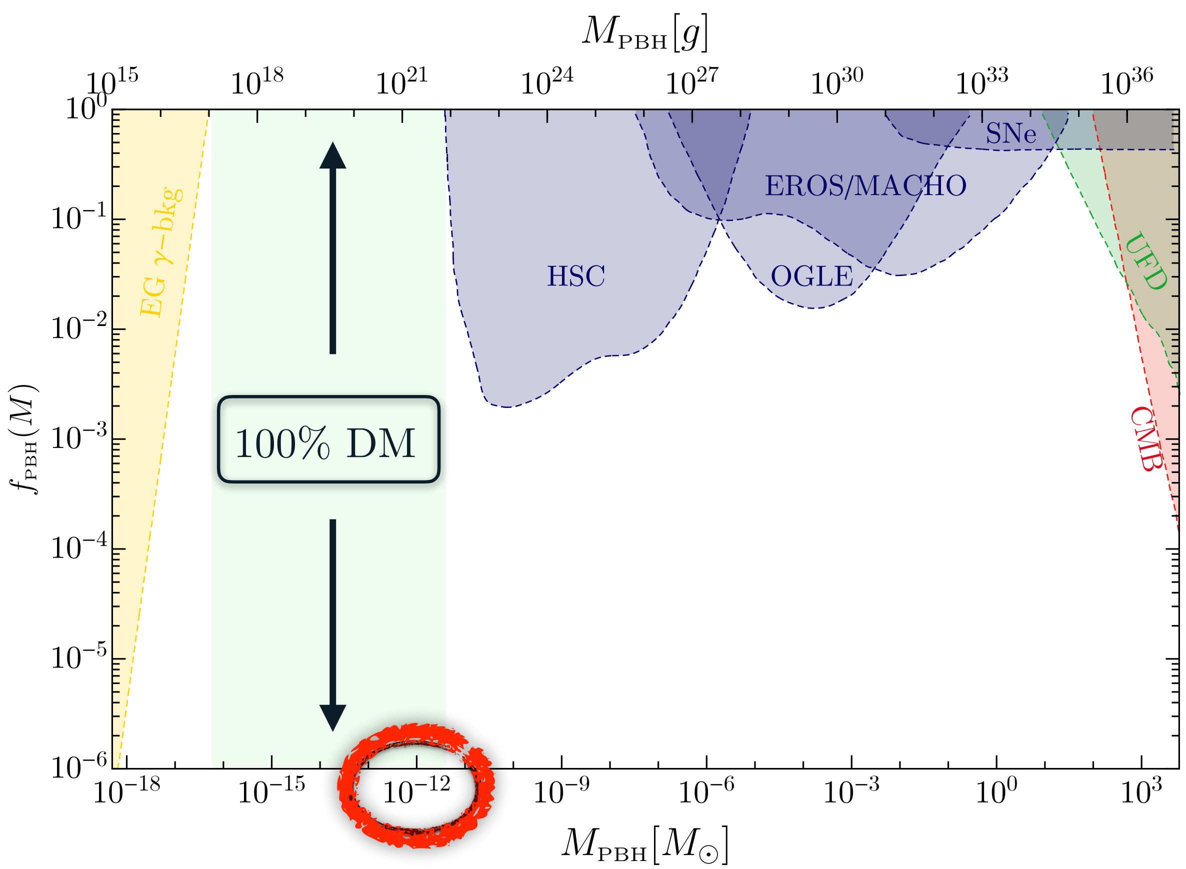
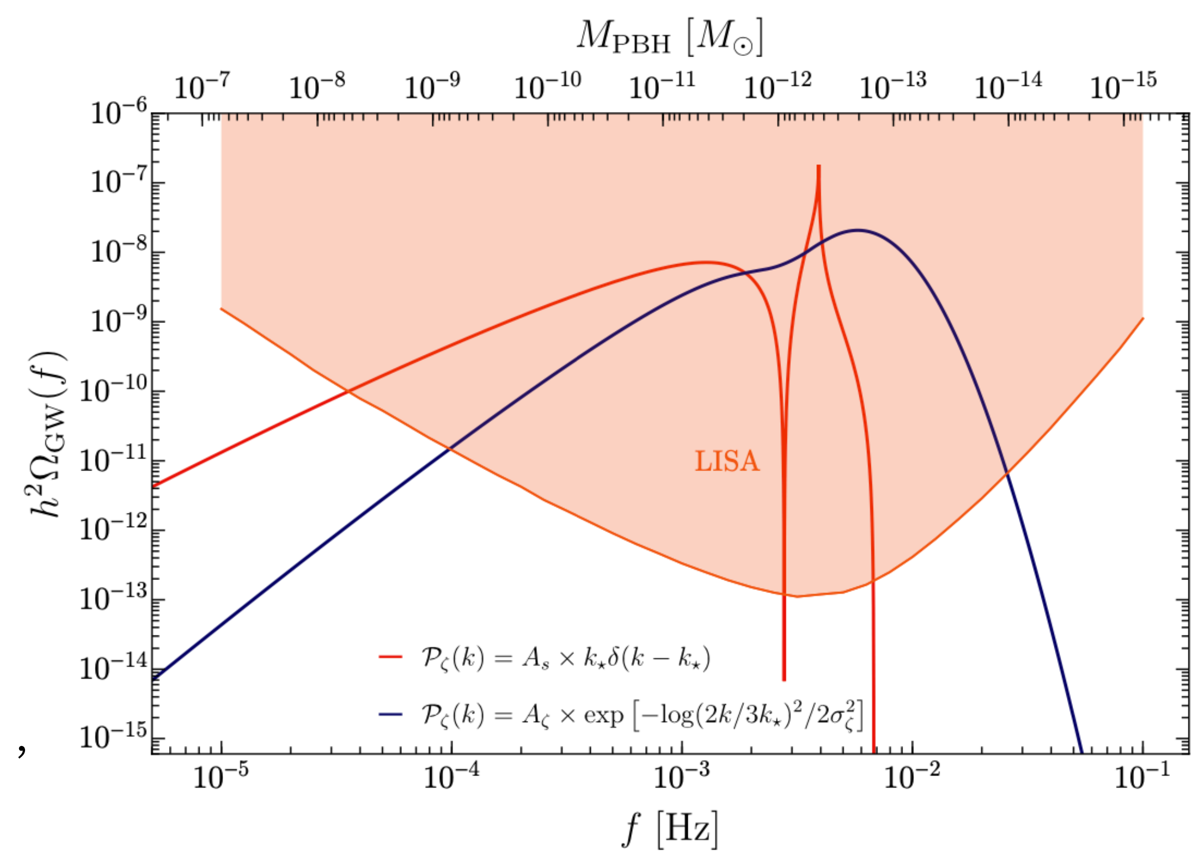
GW from Primordial Black Holes



$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \mathcal{O}(\partial_i \zeta \partial_j \zeta)$$

[Tomita, K., 1967]
[Matarrese, S., et al., 1993]
[Domenech, G., review '21]





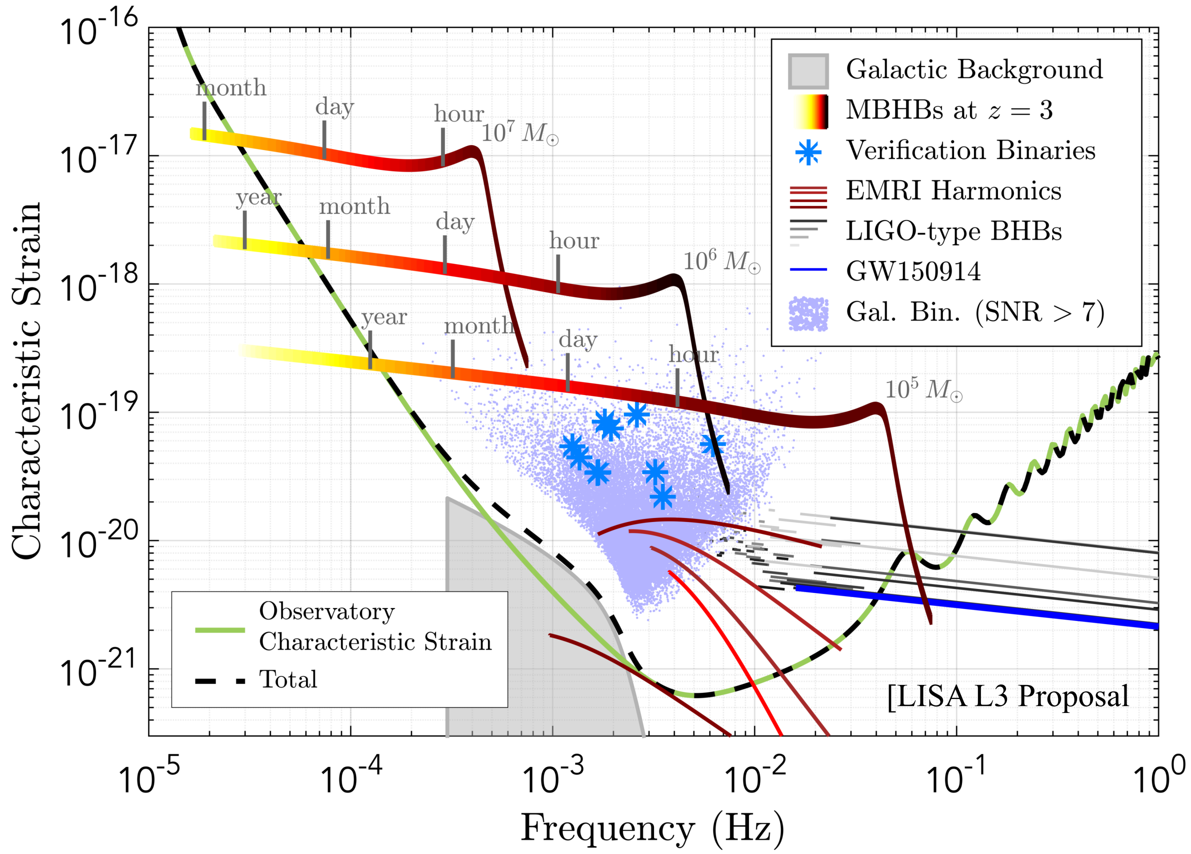
[Espinosa, et al., 2018]
[Bartolo, N., et al., PRL 2019]
[De Luca, V., et al., PRL 2021]

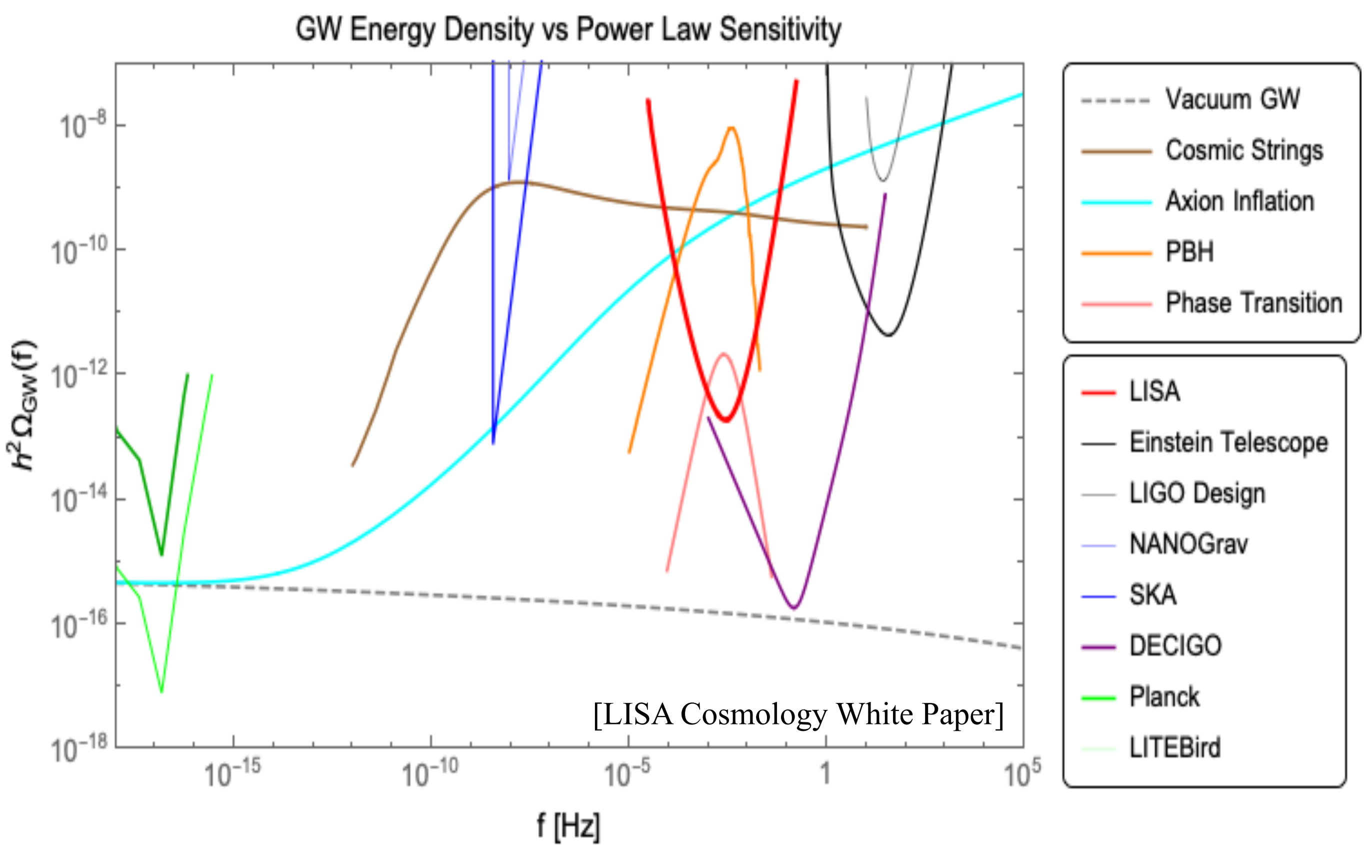
$$f \simeq 3 \cdot 10^{-9}\,\mathrm{Hz} \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{-1/2}$$

$$M \simeq 10^{-12} M_\odot \quad \Longleftrightarrow \quad f \simeq 10^{-3}\mathrm{Hz}$$

Characterization of the SGWB

GWB from cosmological sources superimposed to the Astrophysical GWB





Peculiar features to distinguish them:

- **Spectral Dependence:** $\Omega_{\rm GW}(f)$ [SGWBinner code (LISA CosWG) '19, '20]
- **Net Polarization:** $\Omega_{\rm GW,\lambda}$ $\lambda = L, R$ [Domcke, V., et al.,'20]
- **Anisotropies/Directionality:** $\Omega_{\rm GW}(f, \vec{x})$
- **Statistics:** $\langle \Omega^n_{\rm GW} \rangle$