Criticality as the origin of scales in particle physics



Martti Raidal

INFN, Rome

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My field: BSM phenomenology

- Left-right symmetric models, collider phenomenology
- $SUSY + N_R$, EDMs
- Flavour anomalies
- DM model building
- Cosmic ray data (Pamela, Fermi etc), DM indirect and direct detection
- Primordial black holes, GW detection and phenomenology
- DESI, evolving Dark Energy

Outline

Aim: Explain hierarchies in physical scales

- Coleman-Weinberg mechanism with small couplings (introduce a concept of multi-phase criticality)
- Freeze-out in DM induced dynamical symmetry breaking
- Freeze-in in DM induced dynamical symmetry breaking
- DM induced dynamical symmetry breaking in naturally hierarchical and stable model of all known particle physics

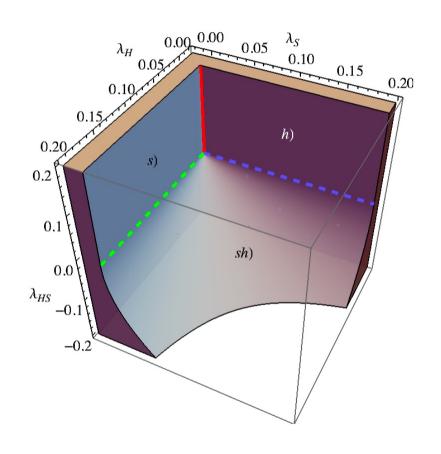
There is only one, HIGH scale for all particle physics

The SM scale is loop suppressed compared to the scale

The hierarchies in physical phenomena come from 1) loops and

2) from the hierarchy in small dimensionless couplings

Classically scale invariant Higgs-Dilaton model



$$V = \lambda_H |H|^4 + \lambda_{HS} |H|^2 \frac{s^2}{2} + \lambda_S \frac{s^4}{4}$$

• Phase s) $s \neq 0$ and h = 0

$$\lambda_S = 0$$

• Phase h) $h \neq 0$ and s = 0

• Phase sh) $s,h \neq 0$ $2\sqrt{\lambda_H \lambda_S} + \lambda_{HS} = 0$

 Multi-phase criticality: masses and mixings vanish

$$\lambda_S(\bar{\mu}) = \lambda_{HS}(\bar{\mu}) = 0,$$

CW mechanism and multi-phase criticality

Dynamical symmetry breaking around the MP criticality: GW not good

$$\begin{split} V^{(1)}|_{\overline{\rm MS}} &= \frac{1}{4(4\pi)^2} \, {\rm Tr} \bigg[M_S^4 \left(\ln \frac{M_S^2}{\bar{\mu}^2} - \frac{3}{2} \right) + \\ &\qquad (10) \\ &\qquad -2 M_F^4 \left(\ln \frac{M_F^2}{\bar{\mu}^2} - \frac{3}{2} \right) + 3 M_V^4 \left(\ln \frac{M_V^2}{\bar{\mu}^2} - \frac{5}{6} \right) \bigg] \\ &\qquad R = e^{-1/2} s_S^2 / s_{HS}^2 \end{split},$$

$$m{eta}$$
-function suppressed $m_s^2 pprox rac{2s^2eta_{\lambda_S}}{(4\pi)^2}, \qquad m_h^2 pprox rac{-s^2eta_{\lambda_{HS}} \ln R}{(4\pi)^2} = 2\lambda_H h^2$ $m{eta}$ -function suppressed

$$heta pprox \sqrt{-rac{eta_{\lambda_{HS}}^3 \ln R}{2\lambda_H}} rac{1 + \ln R}{4\pi (2eta_{\lambda_S} + eta_{\lambda_{HS}} \ln R)},$$

 β -function suppressed

For small couplings the CW must be treated with better precision than the Gildener-Weinberg approximation

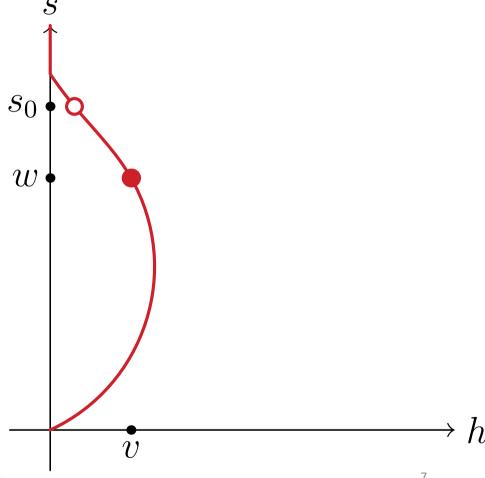
The origin of the effect

 Arrange tree-level Gildener-Weinberg flat direction along the s-axis

 Quantum effects bend the flat direction to a banana

Usually this is just neglected small effect

 Due to the multi-phase criticality, the EW scale is loop suppressed



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7

Comments

• In realistic models couplings never run to zero at the same scale:

$$\lambda_S(\bar{\mu}) = 0, \qquad \lambda_{HS}(\bar{\mu}) \approx 0$$

- Small quartic couplings from pheno:
 - Higgs $\lambda(10^{10}\text{GeV}) = 0$, inflaton $\lambda < 10^{-12}$, DM freeze-in requires $\lambda << 1$
 - Triviality?
- Top Yukawa affects perpendicular direction of the flat direction
- In realistic models one need more scalar couplings to have dynamical symmetry breaking along the multi-phase criticality direction

DM induced dynamical symmetry breaking

Assume freeze-out of DM

DM induced multi-critical dynamical symmetry breaking

The scalar model: the Higgs, a dilaton and scalar DM

$$\begin{split} V &= \lambda_H |H|^4 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_{S'}}{4} S'^4 + \frac{\lambda_{HS}}{2} |H|^2 S^2 + \frac{\lambda_{HS'}}{2} |H|^2 S'^2 + \frac{\lambda_{SS'}}{4} S^2 S'^2. \\ m_h^2 &\simeq -\frac{\beta_{\lambda_{HS}}}{(4\pi)^2} w^2 \ln R, \\ m_s^2 &\simeq 2 \frac{\beta_{\lambda_S}}{(4\pi)^2} w^2, \\ m_{s'}^2 &\simeq \frac{1}{2} \lambda_{SS'} w^2. \end{split} \qquad \lambda_{HS'} &\approx -\frac{(4\pi)^2 m_h^2}{m_{s'}^2 \ln R}. \\ \theta &\simeq \frac{2\sqrt{2} \pi m_s m_h^2 v (1 + \ln R)}{(m_h^2 - m_s^2) m_{s'}^2 \ln R}. \end{split}$$

$$w\simeq rac{\sqrt{2}m_{s'}^2}{4\pi m_s}.$$

One scale w

Scalar DM must be heavy, the dilaton can be heavier or lighter than the Higgs boson

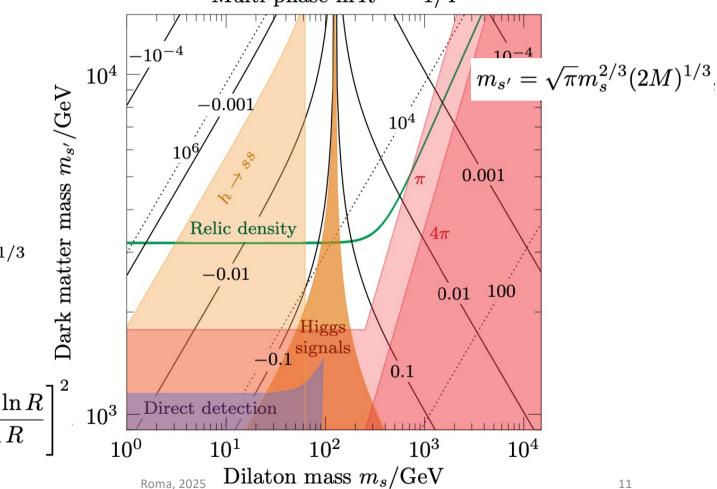
DM freeze out in this model

Multi-phase $\ln R = -1/4$

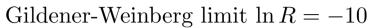
$$\sigma_{
m ann} v_{
m rel} pprox rac{\lambda_{SS'}^2 + 4\lambda_{HS'}^2}{64\pi m_{s'}^2}$$
 $\sigma_{
m ann} v_{
m rel} pprox rac{1}{M^2}$

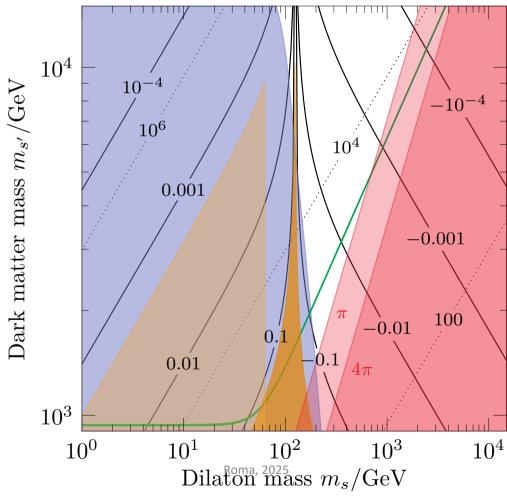
$$m_{s'} = \sqrt{\pi} (2m_h)^{2/3} M^{1/3} / (-\ln R)^{1/3}$$

$$\sigma_{
m SI} \simeq rac{f_N^2 m_N^4}{4\pi m_{s'}^2} \left[rac{\lambda_{HS'}}{m_h^2} + rac{\lambda_{SS'}}{m_s^2} rac{1+\ln R}{\ln R}
ight]^2 \quad 10^3 ext{Direct detection}$$



DM freeze-out in the Gildener-Weinberg limit





To motivate the small scalar couplings:

repeat everything for freeze-in of DM

DM freeze-in in the multi-critical framework

All scalar couplings, except the Higgs quartic, must be super small

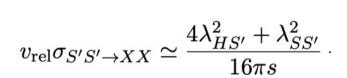
• Criticality naturally embedded:
$$\lambda_S(\bar{\mu}) = 0, \quad \lambda_{HS}(\bar{\mu}) \approx 0$$

Introduce RH neutrinos N (Seesaw Type I)

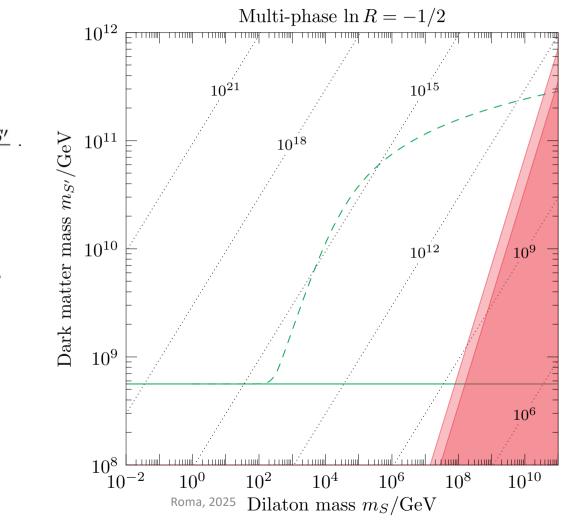
$$-\mathcal{L}_Y = y_H \bar{\ell} \tilde{H} N_R + \frac{y_S}{2} S \bar{N}_R^c N_R + \text{h.c.},$$

• Neutrino masses and leptogenesis coming from the same framework

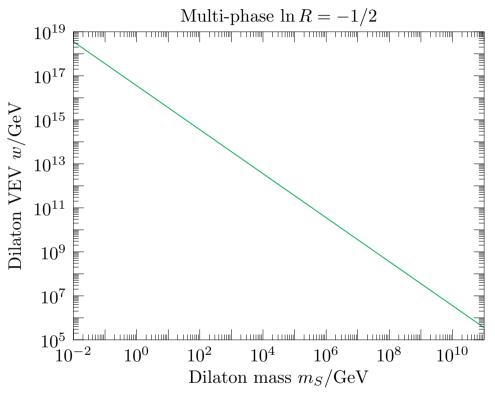
DM induced CW and freeze-in results



Dilaton never thermalizes



DM induced hierarchy in scales



$m_S/{ m GeV}$	$m_{S'}/{ m GeV}$	$w/{ m GeV}$	λ_S	λ_{HS}	$\lambda_{HS'}$	$\lambda_{SS'}$
10	5.62×10^{8}	3.55×10^{15}	-1.98×10^{-30}	-2.48×10^{-27}	1.57×10^{-11}	5.00×10^{-14}
10^{4}	5.62×10^{8}	3.55×10^{12}	-1.98×10^{-18}	-2.48×10^{-9}	1.56×10^{-11}	5.00×10^{-8}

Comments

- This models induces all the known scales in particle physics
 - There is one dynamical symmetry breaking scale w

$$10^9 \text{ GeV} < w < 10^{18} \text{ GeV}$$

- The DM and the RH neutrino mases, $m_{DM}^2 \sim \lambda_{SS} \, w^2 \sim 10^9$ GeV, $M_N = y_N w$
- The SM particle masses, $m_e=y_e$ v, $m_t=y_t$ v, where $v\sim\beta w$
- The light neutrino masses come from the seesaw, $m_{\nu 1} = y^2_H/M_N v^2$
- Standard leptogenesis from Majorana N decays
- The model is technically natural the effective potential is stable
- No hierarchy problem
- The model has two predictions:
 - 1. The existence of a new particle the dilaton with preferably small mass
 - 2. The DM is a scalar with a fixed mass 5.6 108 GeV

Renormalization of scalars after the LHC

Standard folklore

- Assume NP separate from the SM
- Use the EFT framework to claim

$$\delta m_h^2 \sim \bar{y}_t^2/(4\pi)^2 \Lambda_{\mathrm{NP}}^2$$

 Misuse of the ETF framework and non-physical bare parameters

Shaposhnikov et al

- Consider the full system
- Only physical parameters matter
- Once the physical parameters are fixed, only log running remains
- The hierarchy problem is a problem of giving physical significance to bare parameters

Possible extensions of the framework

Add non-minimal coupling to gravity

$$-\frac{1}{2}\xi R s^2 \rightarrow M^2_{PL} = \xi w^2$$

Add extra scalars or extra non-minimal couplings for inflaton

Conclusions

• The simplest scalar model containing H, S, S' with one high scale w

$$W > M_{NR} \sim M_{DM} >> V >> m_{\nu}$$

- The scalar DM couplings trigger the CW and the loop-suppressed EW scale
- Huge but technically natural hierarchy between the EW and DM scales
- Neutrino masses and leptogenesis occur in a standard way ($M_{NR} \sim M_{DM}$)
- This scenario predicts:
 - A light scalar, the dilaton, which may be lighter or heavier than the SM Higgs boson
 - The scalar DM with a fixed mass 5.6 108 GeV