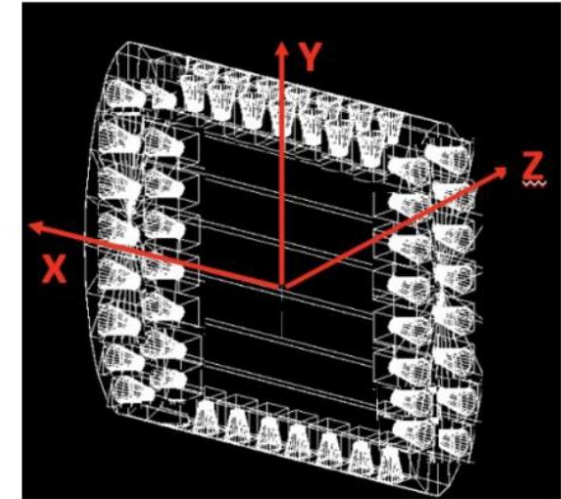


ROOT

An Object-Oriented
Data Analysis Framework

Multiple-view reconstruction in GRAIN with lenses

53 cameras in GRAIN



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DUNE Italian meeting - November 11th, 2025



UNIVERSITÀ
DEL SALENTO
L'Ateneo tra i due mari



Istituto Nazionale di Fisica Nucleare



Projective geometry based reconstruction

(from 2D images to 3D track and vice versa)

Each α camera is characterized
by a 4x3 matrix: $P_\alpha[4][3]$

example

0	0	100	6000
0	-100	0	-51200
1	0	0	649

2D track on SiPM array
associated with a 3x1 matrix: $L_\alpha[3][1]$

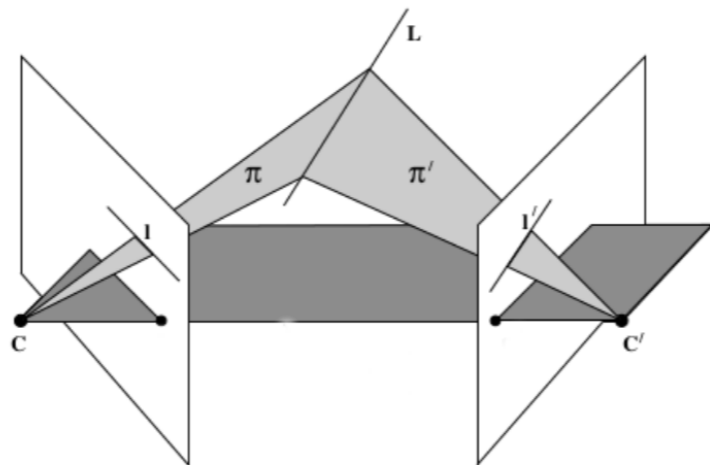
m_α -1 q_α

track parameters

The plane from the product
of two matrixes: $P_\alpha^T[3][4] L_\alpha[3][1] = \pi_\alpha[4][1]$

a_α b_α c_α d_α

plane parameters



Track in space found from the intersection of two planes
In this analysis only pairs of orthogonal lens-cameras

New functions

{ 2Dto3D (tracks on α & β cameras) $\Rightarrow$ 3D-track
Two 2D-tracks on perpendicular cameras
(vertical & horizontal)

2D SiPM local reference system
3D GRAIN reference system

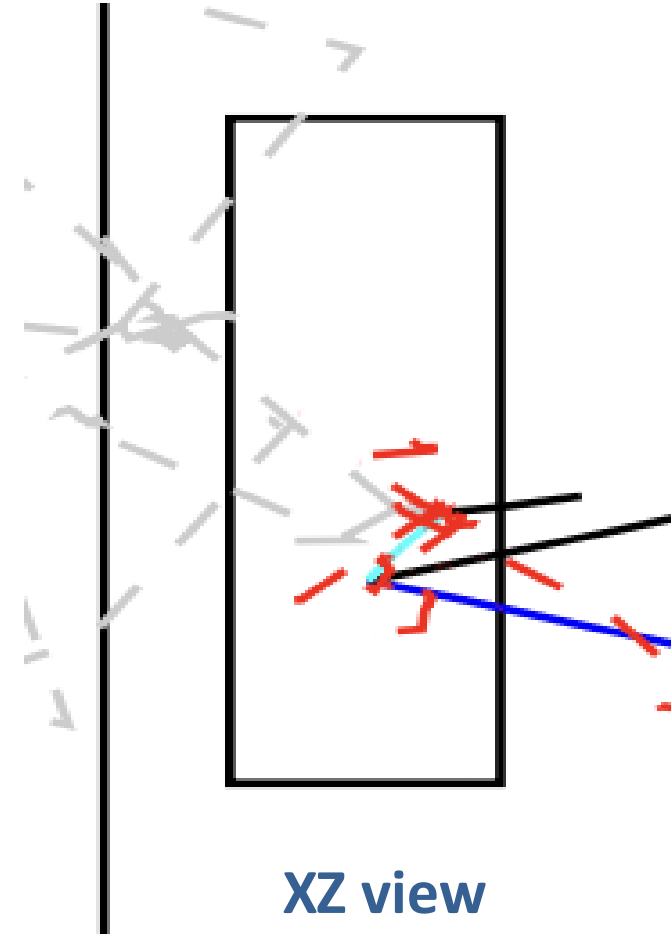
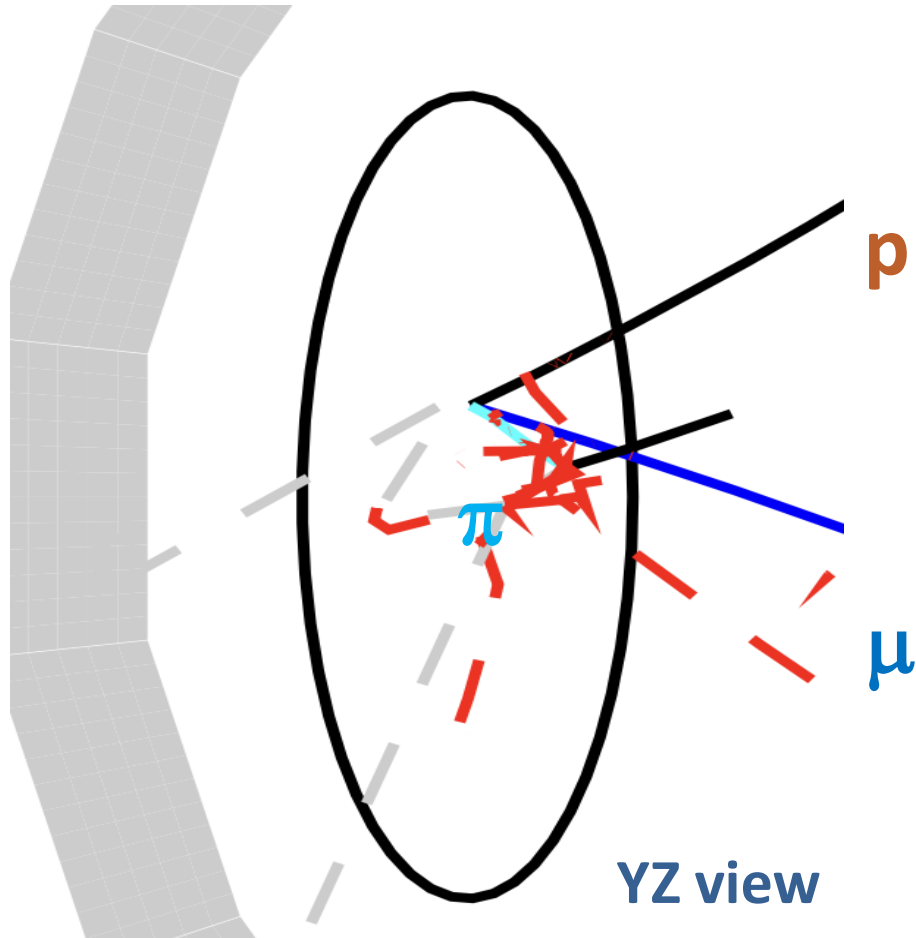
{ 3Dto2D (3D-track) $\Rightarrow$ 2D-image on a 3rd camera (γ)

flag = 0 projection NO-visible on SiPM
1 projection visible on SiPM

Event 158: $\nu \rightarrow p \pi^+ \mu^-$

Vertex position: -292.3, +183.6, -4.3 mm

Neutrino energy: 2.97 GeV

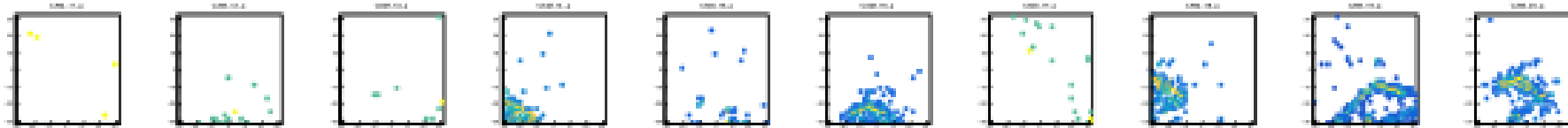


Cam_1

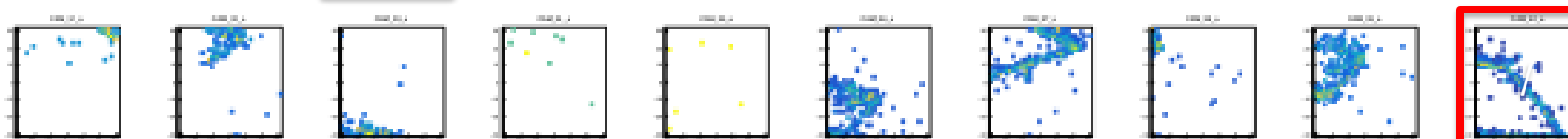
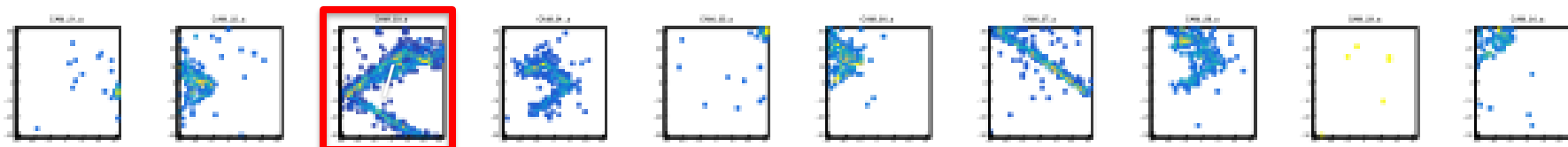


Cam_10

Cam_11

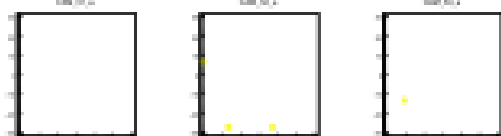


Shifted images
with same slope in
parallel cameras

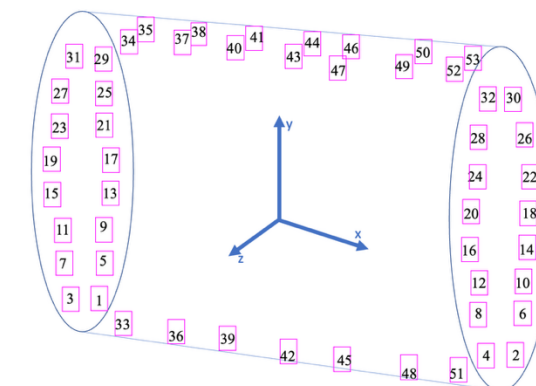


Topological associations
are possible

Cam_51



Event 158

 $p \quad \mu^- \quad \pi^+$ 

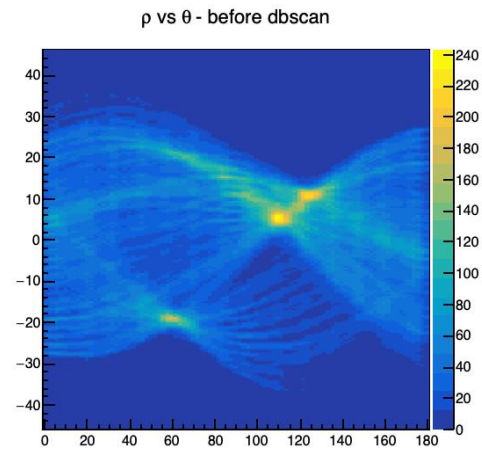
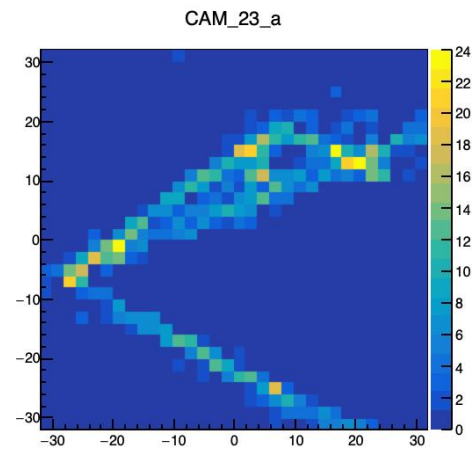
In this presentation

Rough algorithm to search for 2D-tracks

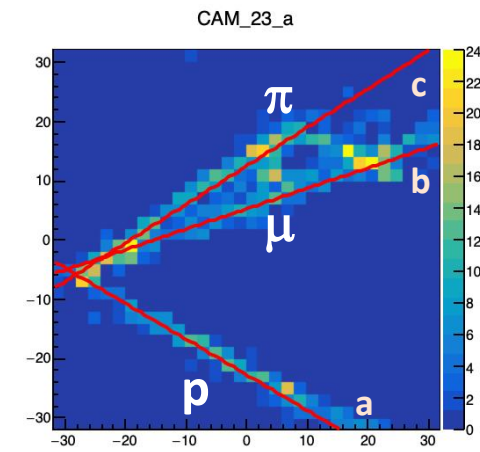
- dbscan + Hough transform
- no fit, just peaks in the Hough plane
- to be optimized

The work is focused on

- 3D reconstruction
- track association



Cam_23



Tracks

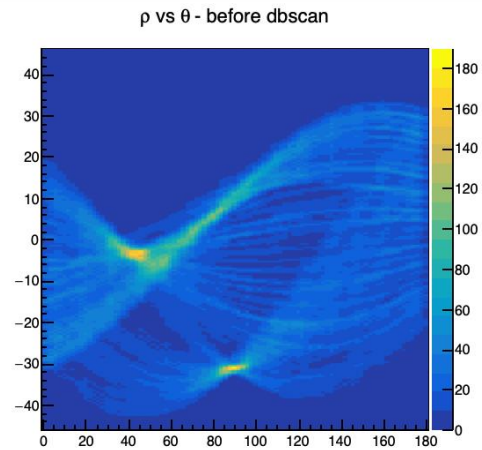
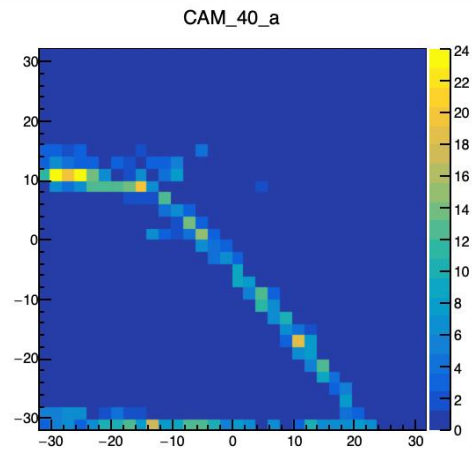
23/a

23/b

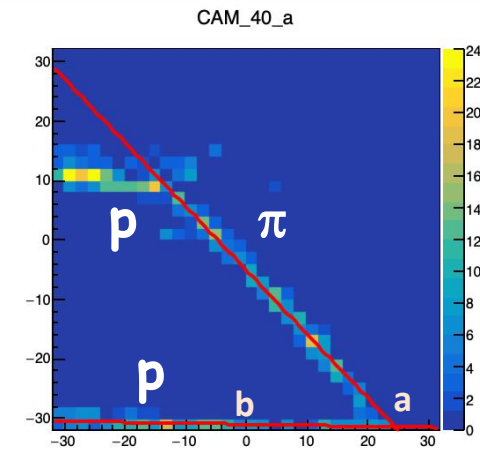
23/c

Peak choice

No track fit



Cam_40



Tracks

40/a

40/b

Event 158

 $p \mu^- \pi^+$

Event 158

True 2D-tracks  3D to 2D

Proton	slope	intercept
track 23/a	-0.560	-27.15
track 27	-0.645	+7.26
track 37/a	-0.454	+33.07
track 40/b	-0.036	-33.52

Muon		
track 23/b	+0.397	+2.17
track 37/b	+0.306	+12.56

Pion		
track 23/c	+0.692	+11.23
track 40/a	-1.128	-4.07

≈

Reconstructed 2D-tracks

	slope	intercept	
Track 23/a	-0.601	-22.75	vertical cameras
Track 23/b	+0.344	+5.29	
Track 23/c	+0.649	+12.52	
Track 27	-0.700	+13.43	

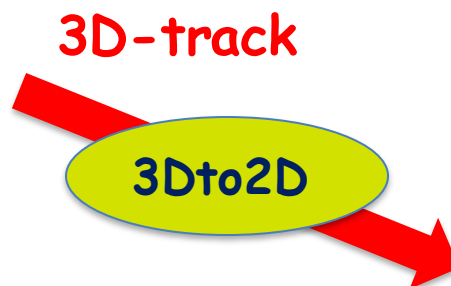
Track 37/a	-0.384	+29.45	horizontal cameras
Track 37/b	+0.306	+12.55	
Track 40/a	-1.072	-5.13	
Track 40/b	-0.017	-31.00	

Association of 2D tracks without MC information

2D-track on camera α
2D-track on camera β



2D SiPM local reference system
3D GRAIN reference system



2D-image on camera γ

If a reconstructed 2D-track on camera γ is in agreement with this image,
then tracks on α , β , and γ cameras are projections of the same track. If not ...

tracks:	40/b and 27 $\Rightarrow$ 23	
projection	-0.622 -21.09	
tracks:	37/a and 27 $\Rightarrow$ 23	
projection	-0.636 -21.33	

tracks:	40/a and 27 $\Rightarrow$ 23	
projection	-0.396 -14.24	
tracks:	37/b and 27 $\Rightarrow$ 23	
projection	-0.867 -28.28	

?

Reconstructed		
23/a	-0.600,	-22.75
23/b	+0.344,	+5.29
23/c	+0.649,	+12.52
27	-0.700,	+13.43

37/a	-0.384,	+29.46
37/b	+0.306,	+12.55
40/a	-1.072,	-5.13
40/b	-0.017,	-31.00

Projections of the
same track (proton)

40/b 27 23/a 37/a

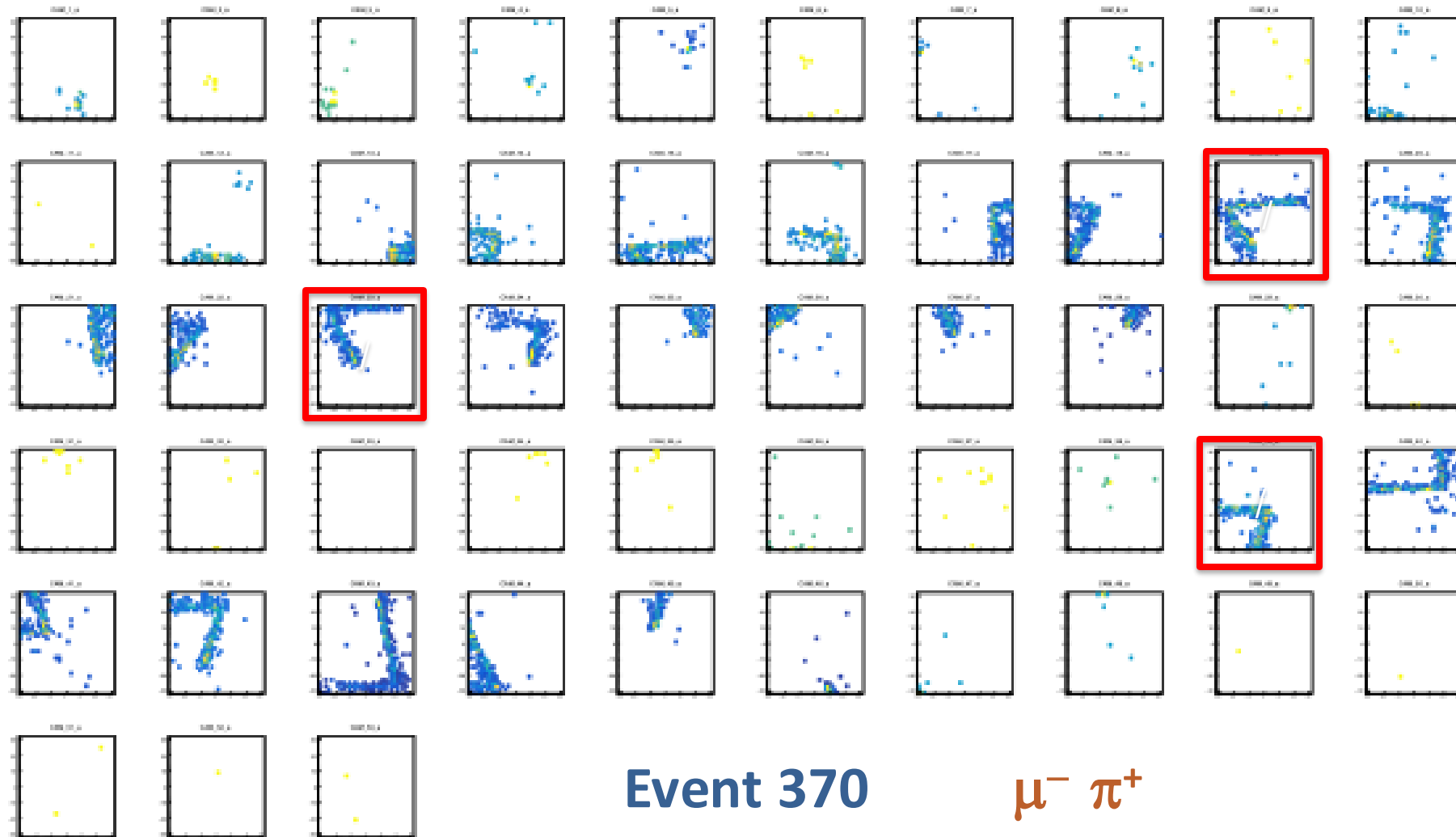
Not associated because
2 tracks are not enough

$\mu \Rightarrow$ 23/b 37/b
 $\pi \Rightarrow$ 23/c 40/a

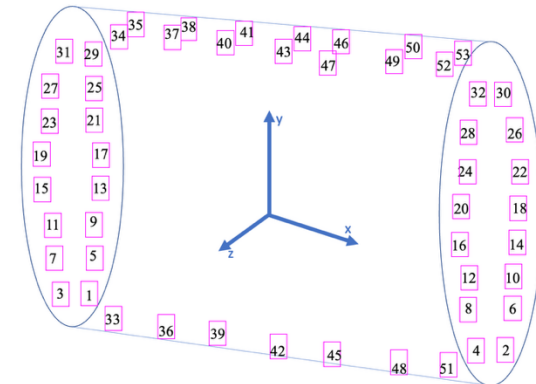
(Topological association?)

Event 158

Anyway, the 2D-track pair (α , β)
allows for the projection on all chambers
looking for another track in agreement
with the projection



Event 370

 $\mu^- \pi^+$ 

Event 370

Reconstructed

19/a -1.235, -29.40
 19/b +0.087, +5.02
 22 +1, +26.16
 23 -2.145, -18.93

 39/a -0.123, -7.56
 39/b +4.331 -13.34
 40 -0.017, +7.00
 42 +2.605, +9.77
 43 -5.145, +73.37
 44 -2.145, -56.79

tracks: 39/b and 23 $\Rightarrow$ 19

projection -1.516 -32.21

tracks: 22 and 42 $\Rightarrow$ 19

projection -0.979 -27.03

tracks: 23 and 39/b $\Rightarrow$ 22

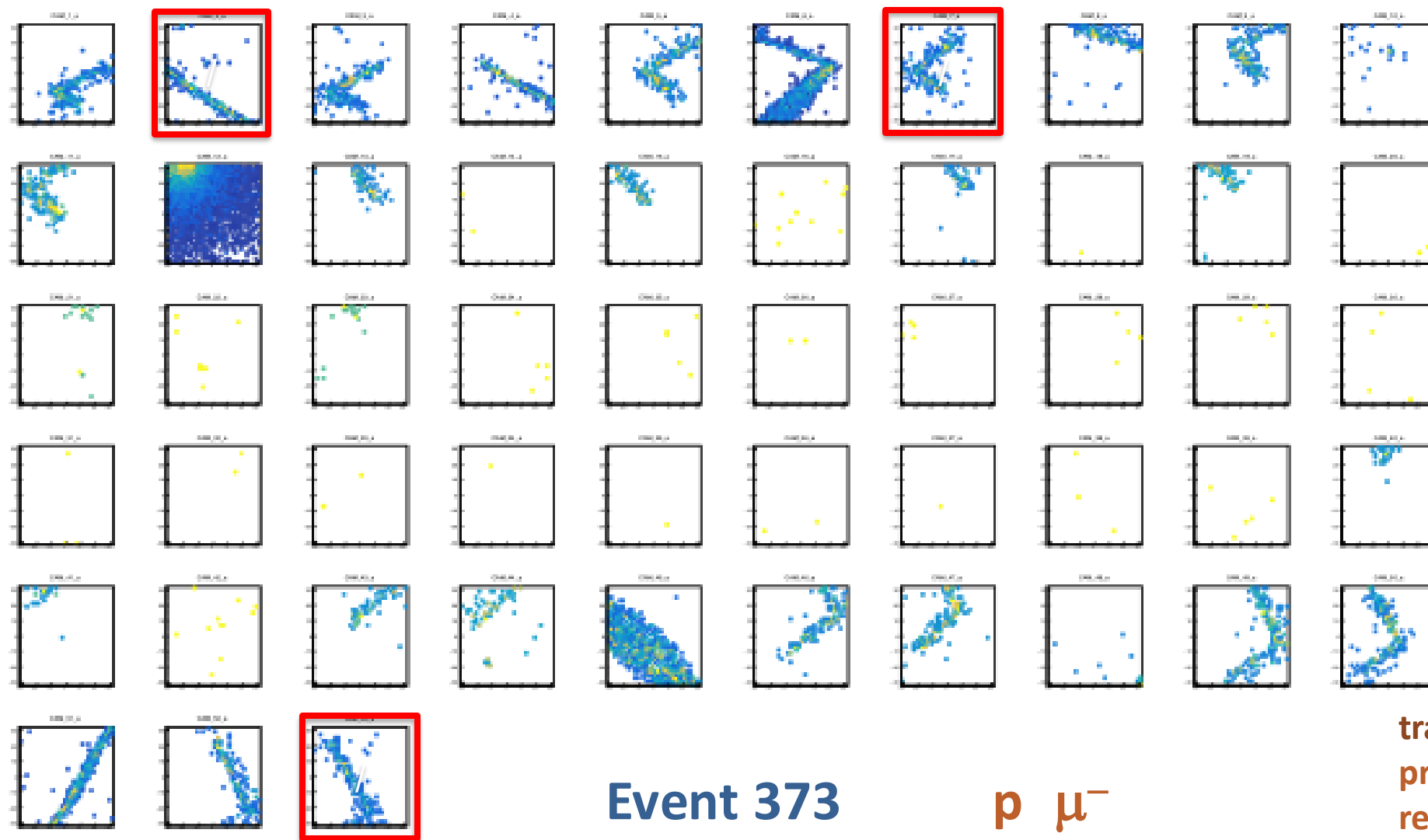
projection +1.364 +35.41

cameras: 19/b and 40 $\Rightarrow$ 39

projection 0.029 -5.60

Slanted track (muon) 19/a 22 23 39/b 42 43 44

Horizontal track (pion) 19/b 39/a 40

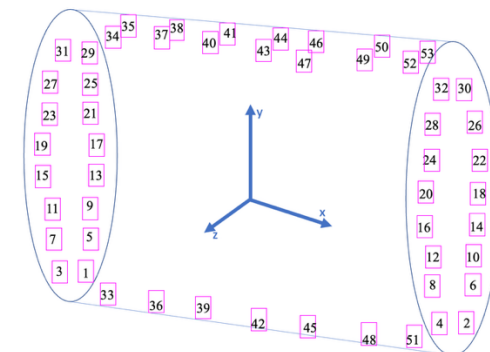


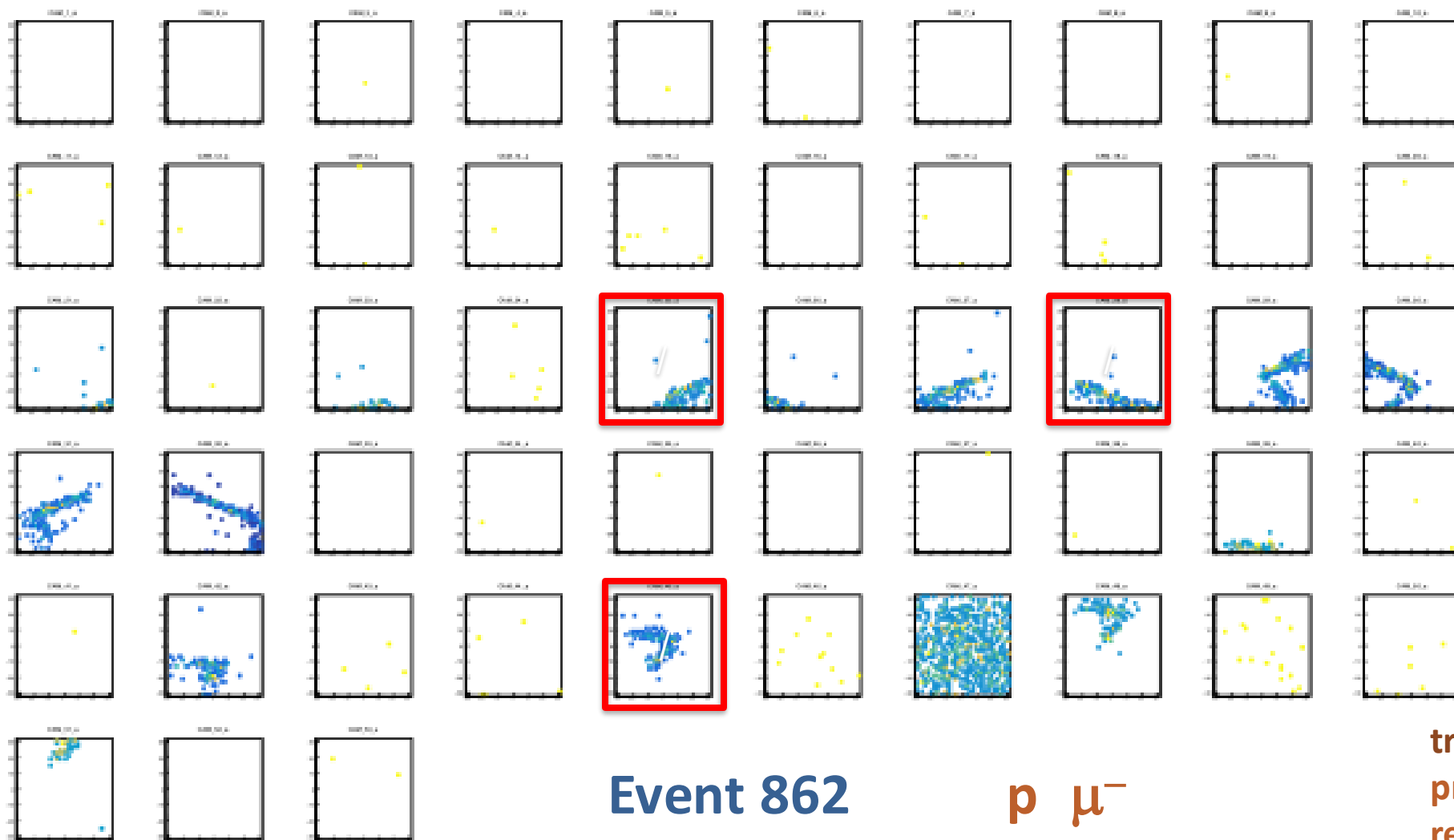
Event 373

 $p \mu^-$ tracks: 7/b and 53 $\Rightarrow$ 2

projection -0.515 -21.83

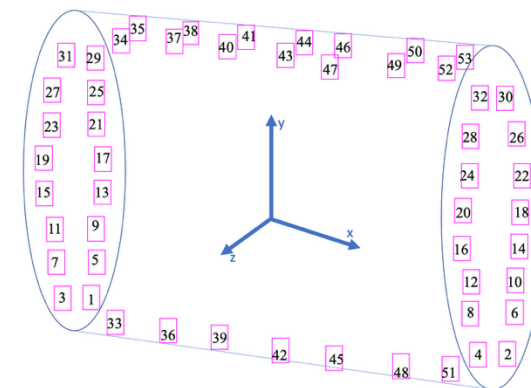
reconstruction -0.649 -18.48





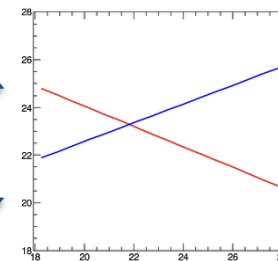
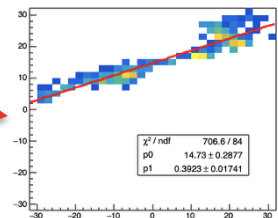
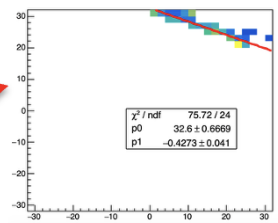
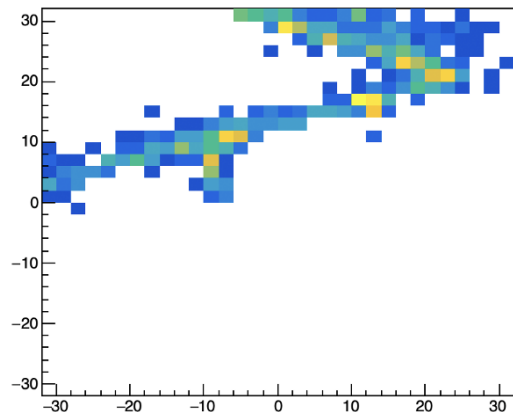
$p \mu^-$

tracks: 28 and 45/a $\Rightarrow$ 25
 projection 0.881 -26.96
 reconstruction 0.601 -28.58

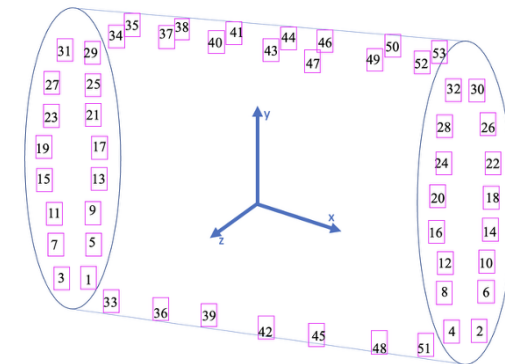


Procedure for vertex reconstruction with two-tracks neutrino events

camera 37



this example
 $\nu \rightarrow \mu^- p \pi^+$



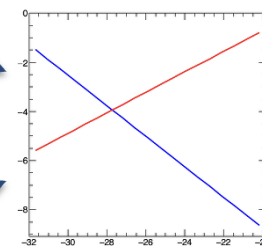
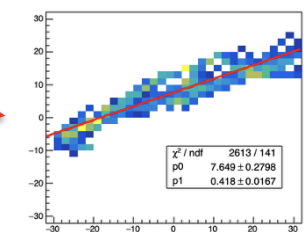
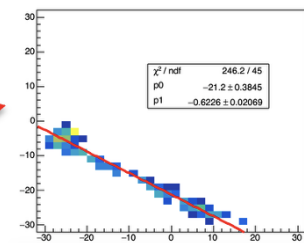
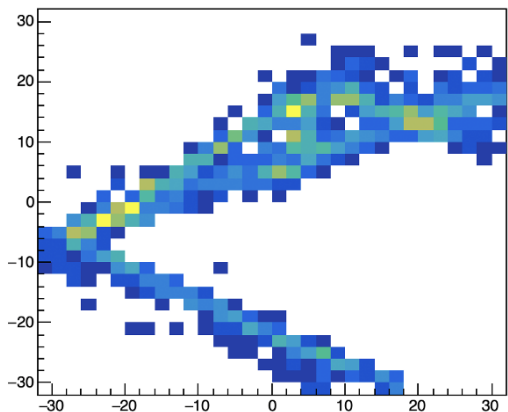
- Local 2D-vertices are obtained from tracks intersections
- 3D vertex in GRAIN frame from two local vertices on orthogonal cameras

Preliminary estimate:

few-cm uncertainty

Function 2DVto3DV

camera 23



Conclusions

- Couple of 2D-tracks on orthogonal cameras
 ⇒ 3D track
 ⇒ Its projection on a third camera
 ⇒ **IF** a reconstructed track is in agreement with the projection
 ⇒ The 3 tracks are associated
- Two 3D-tracks ⇒ Vertex as mid point of the shortest distance
- Many reconstructed 2D-tracks on the cameras ⇒ Many associations and possible checks
- Also topological criteria can be used/investigated

Code :	tracks	- 2Dto3D - 3D track from 2 perpendicular cameras - 3Dto2D - track image on a camera - association - tracks on α, β, γ cameras
	vertices	- 2DVto3DV - 3D vertex from 2 perpendicular cameras - check - agreement of 2D vertices ($V_\alpha V_\beta V_\gamma$)

BUFFER

Required

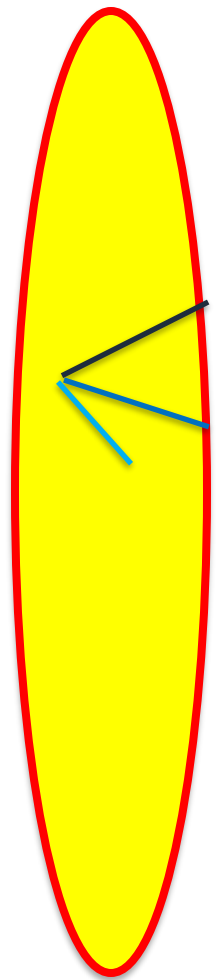
centri_CAMS.txt

coordinates of the camera centers
(GRAIN reference system, in mm)

P_alpha function

creates the 53-camera matrix.
Based on geometry and features of the lenses

Every change in the setup must be considered



MC data: first & last coordinates

↓
3D-tracks

↓
3Dto2D



True 2D-tracks

Proton	slope	intercept
track 23/a	-0.560	-27.15
track 27	-0.645	+7.26
track 37/a	-0.454	+33.07
track 40/b	-0.036	-33.52

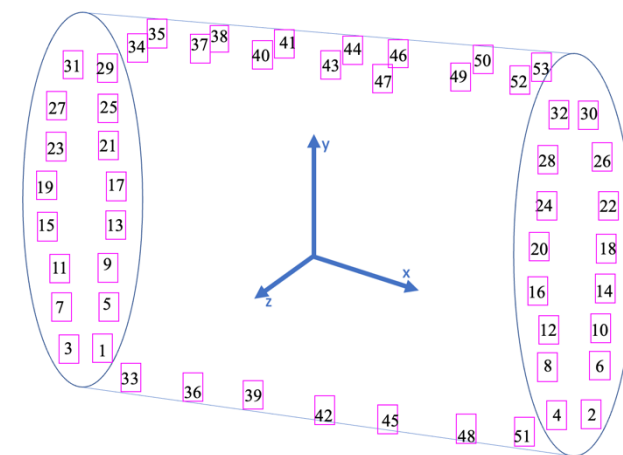
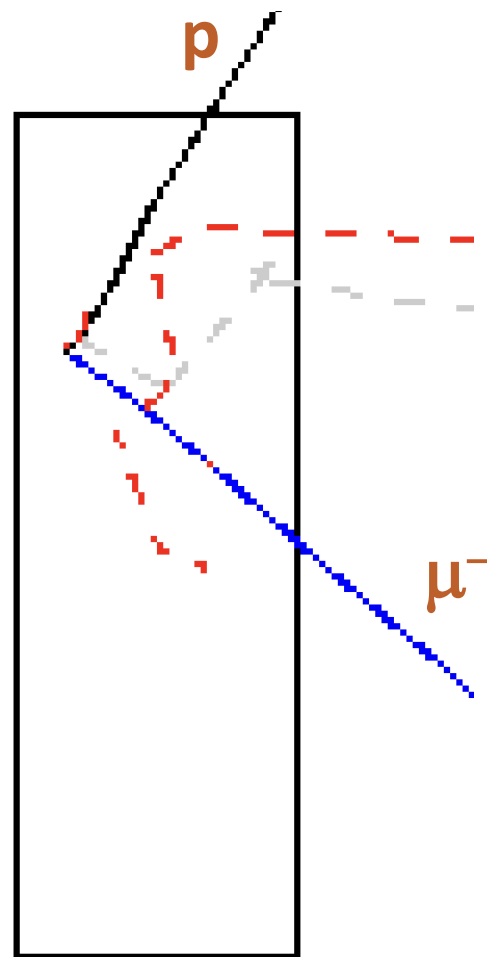
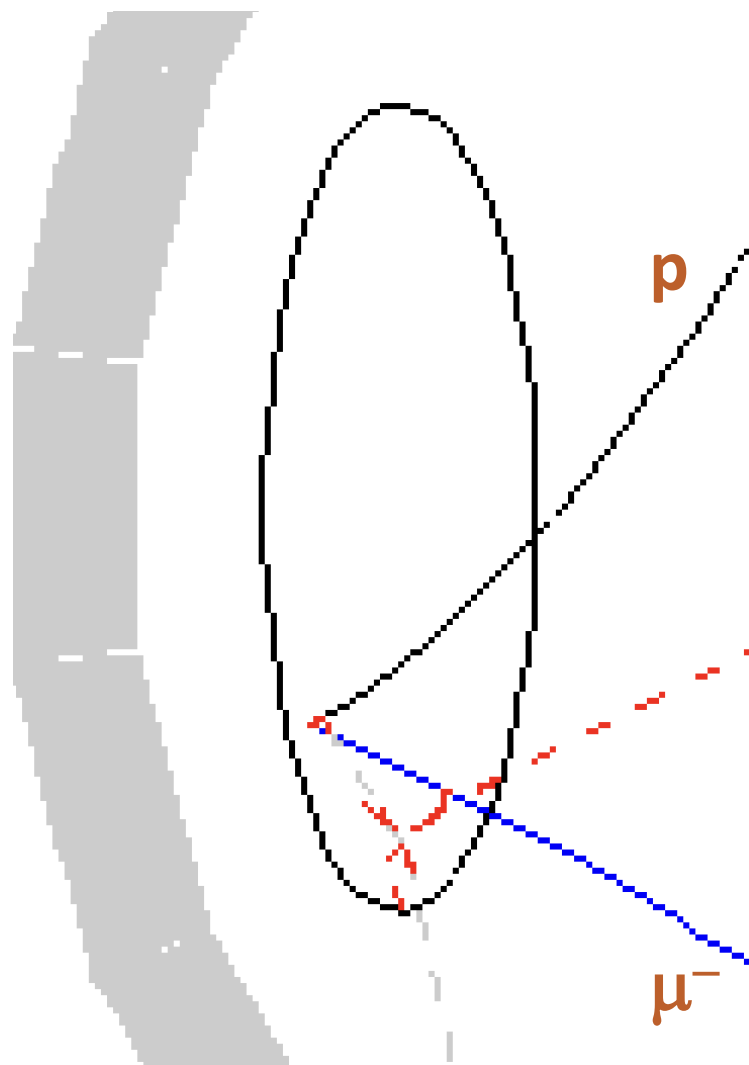
Muon		
track 23/b	+0.397	+2.17
track 37/b	+0.306	+12.56

Pion		
track 23/c	+0.692	+11.23
track 40/a	-1.128	-4.07

Association of 2D tracks without MC information

- Required 2D-tracks on 3 cameras (α, β, γ ; α, β are orthogonal)
- Use of trifocal tensor $TTT[53][53][53][3][3][3]$
- 2Dto3D (track on α + track on β) $\Rightarrow$ 3D-track
- 3Dto2D (3D track) $\Rightarrow$ 2D-track on γ camera ($m_\gamma, q_\gamma, \text{flag}$)
- If m_γ and q_γ (flag=1) are *close* to the measured values, the 3 2D-tracks are associated. If not, repeat the procedure with other tracks

* How much ? It depends on the analysis



Event 373

 $p \mu^-$