



**QUANTUM**  
Information and Matter

**MAX PLANCK INSTITUTE**  
OF QUANTUM OPTICS



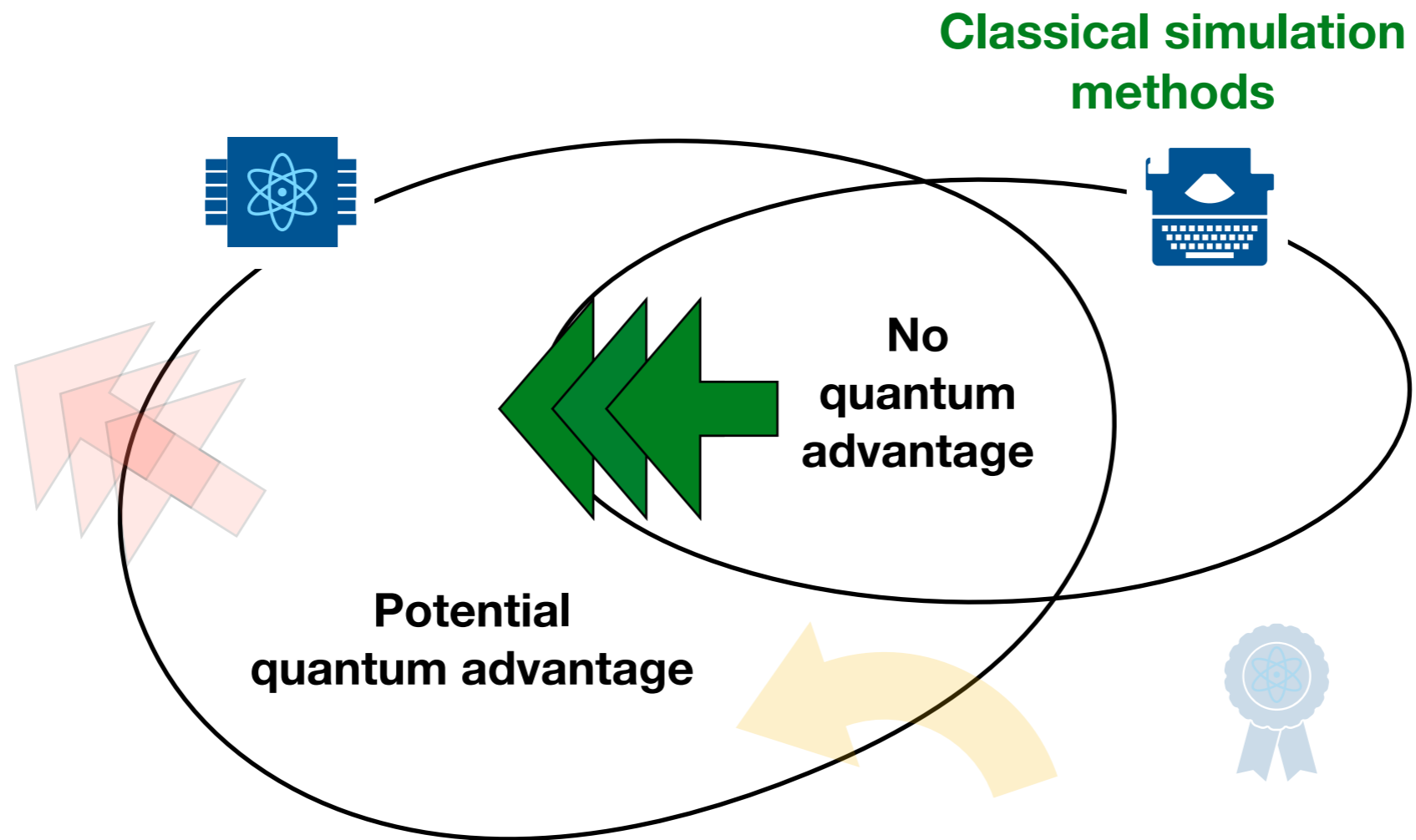
# AVERAGE-COMPUTATION BENCHMARKING FOR LOCAL EXPECTATION VALUES IN DIGITAL QUANTUM DEVICES

F. BACCARI, P. KOS, G. STYLIARIS

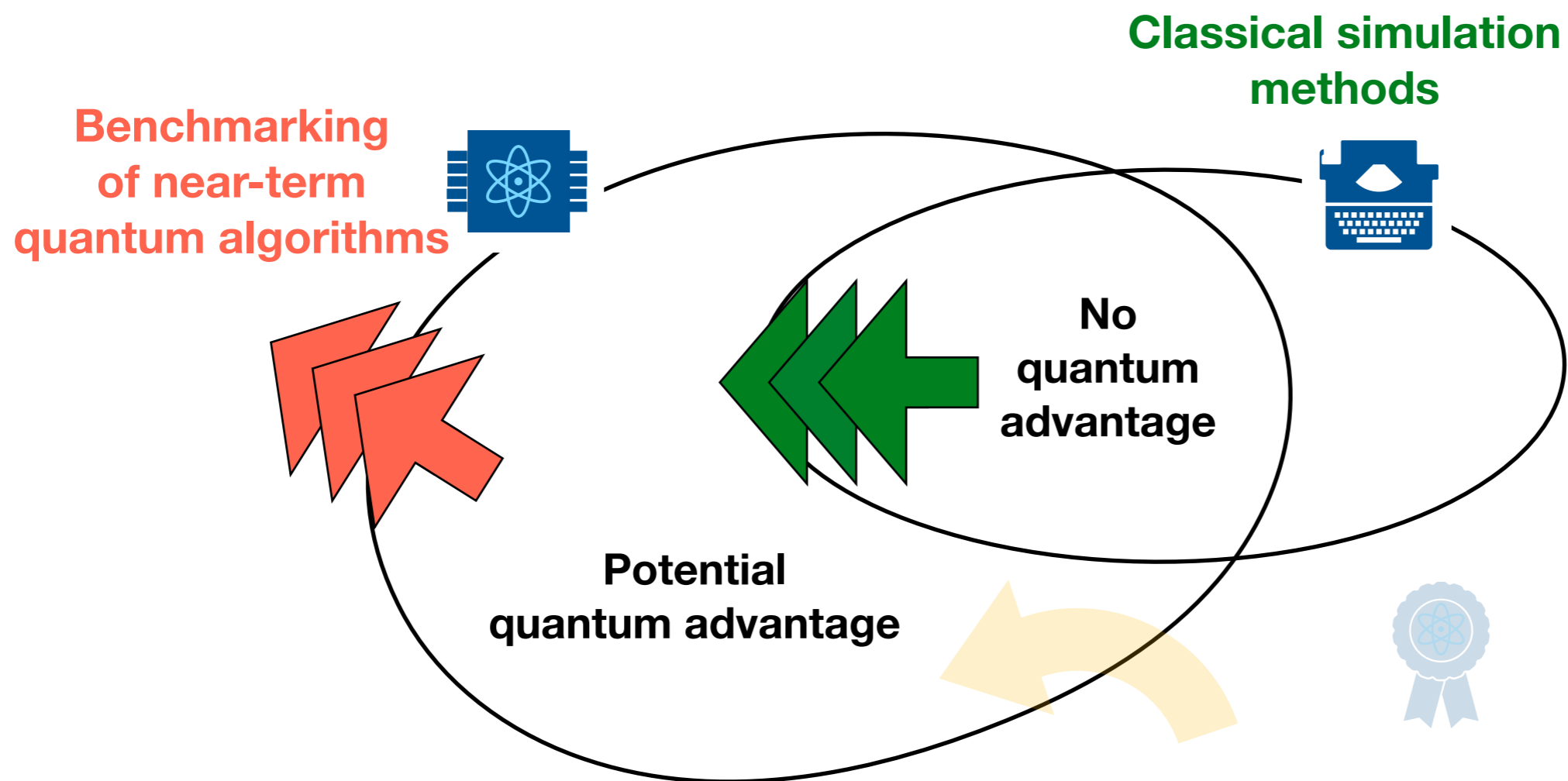


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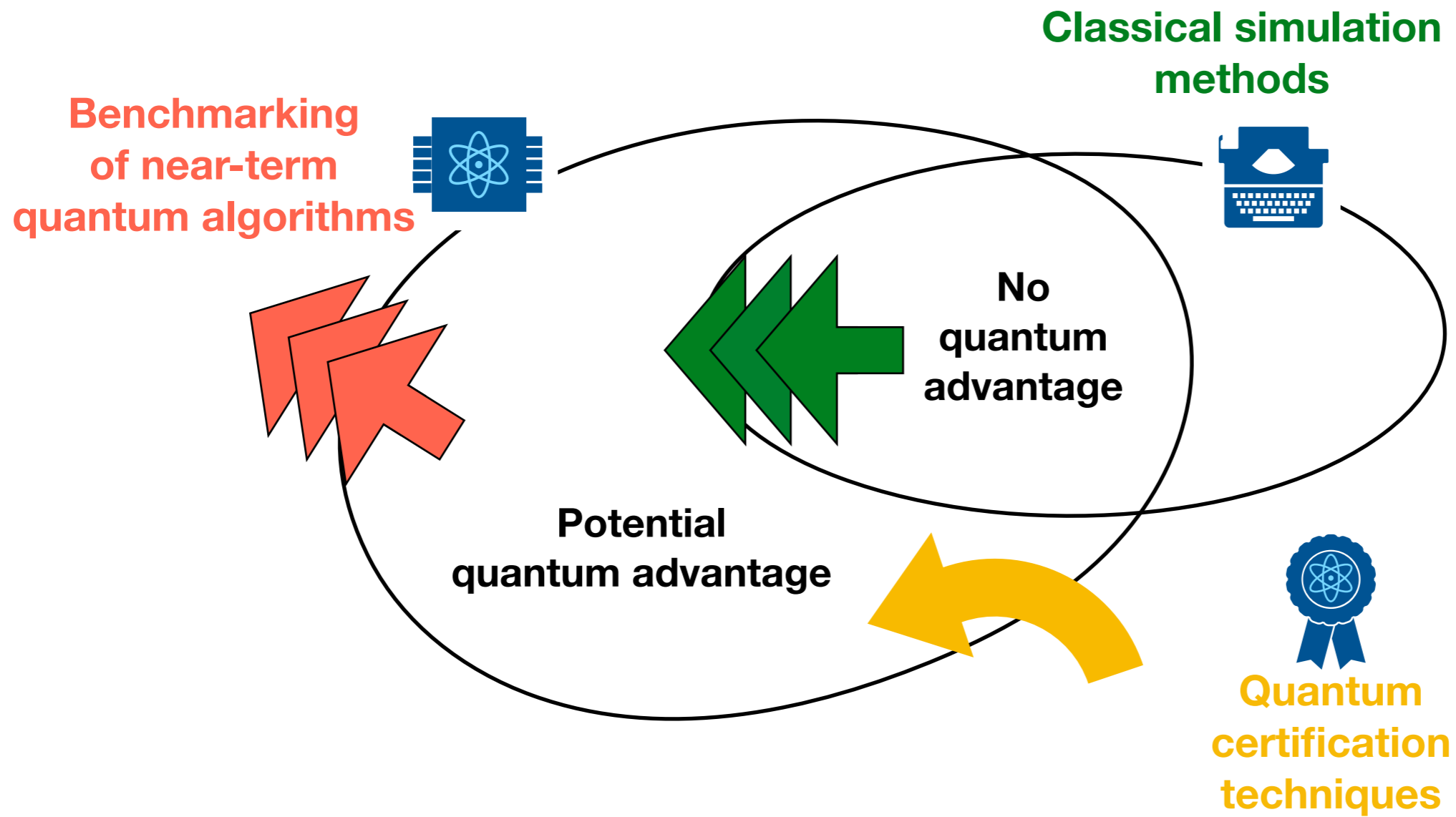
# RESEARCH INTERESTS



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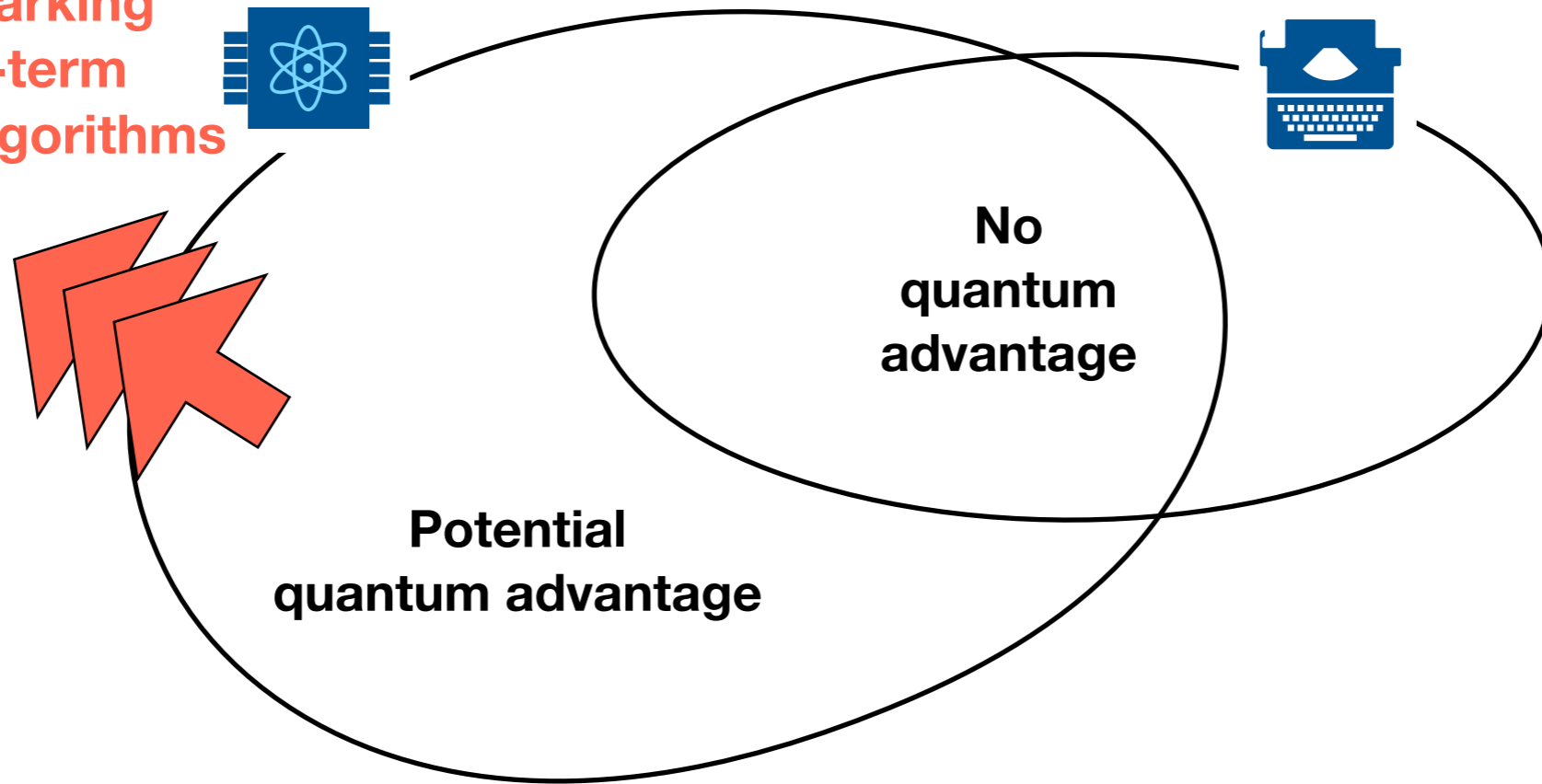
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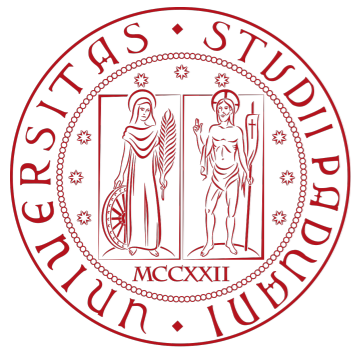
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Benchmarking  
of near-term  
quantum algorithms



Potential  
quantum advantage

No  
quantum  
advantage



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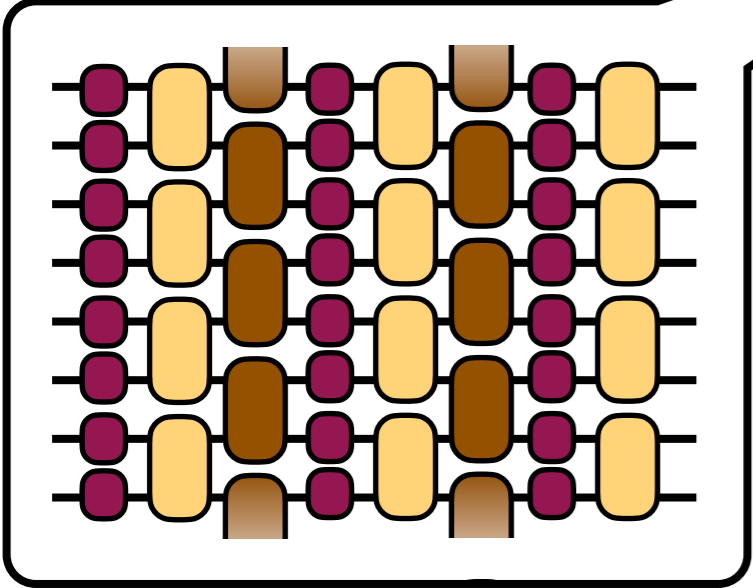
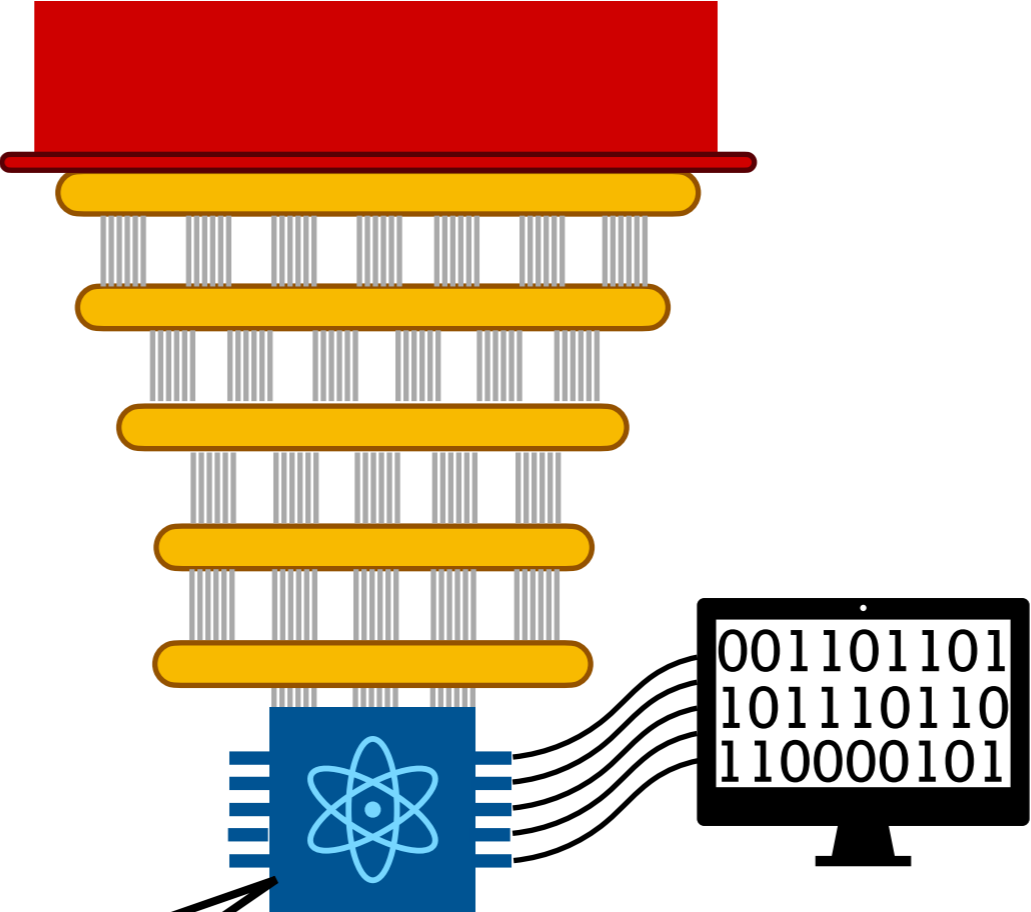
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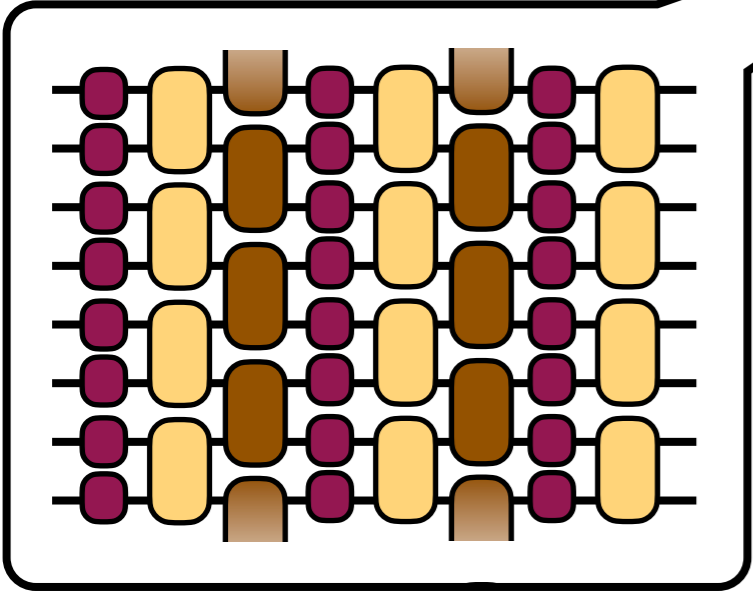
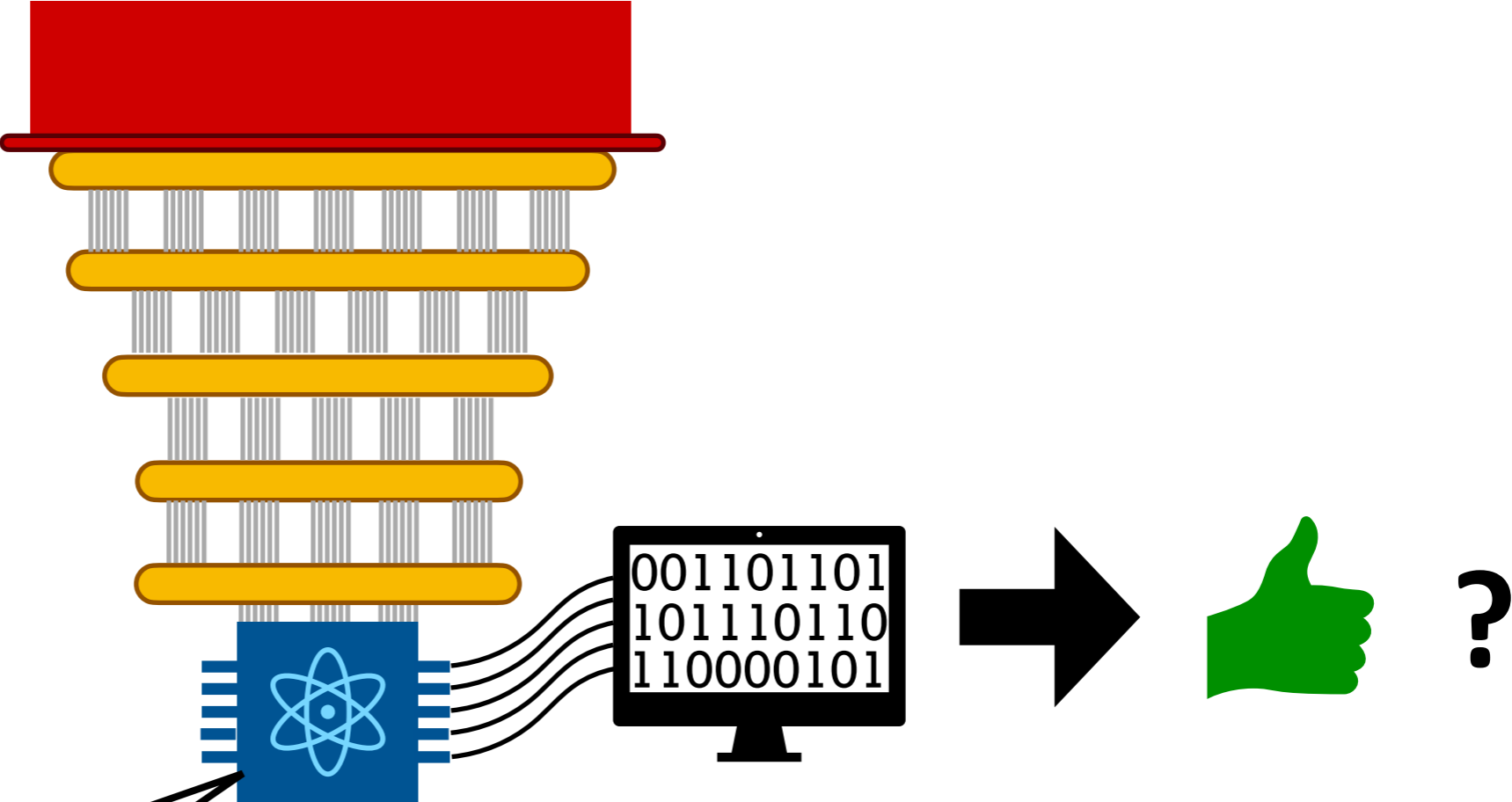
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# Motivation

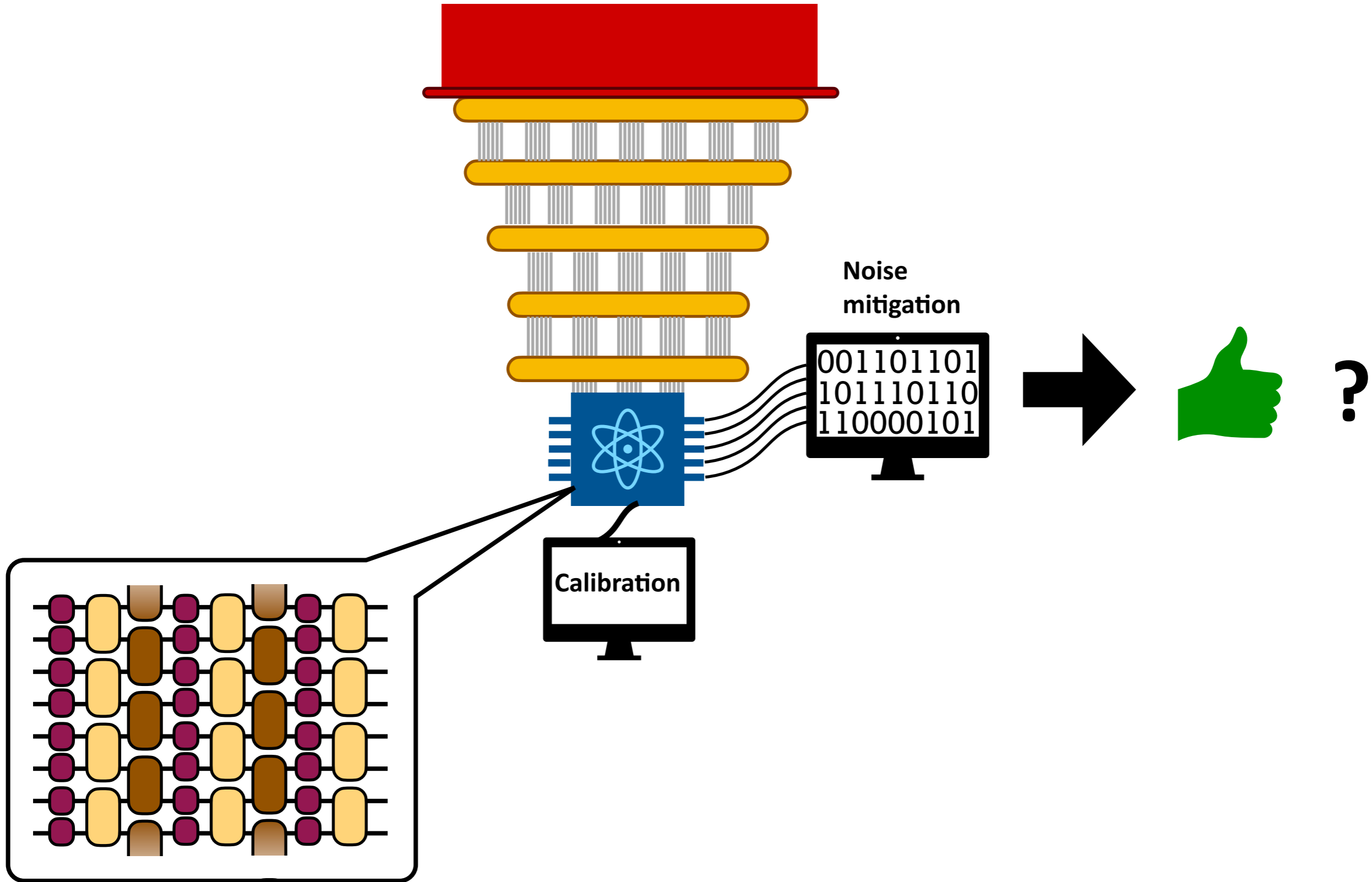
# WHAT IS BENCHMARKING?



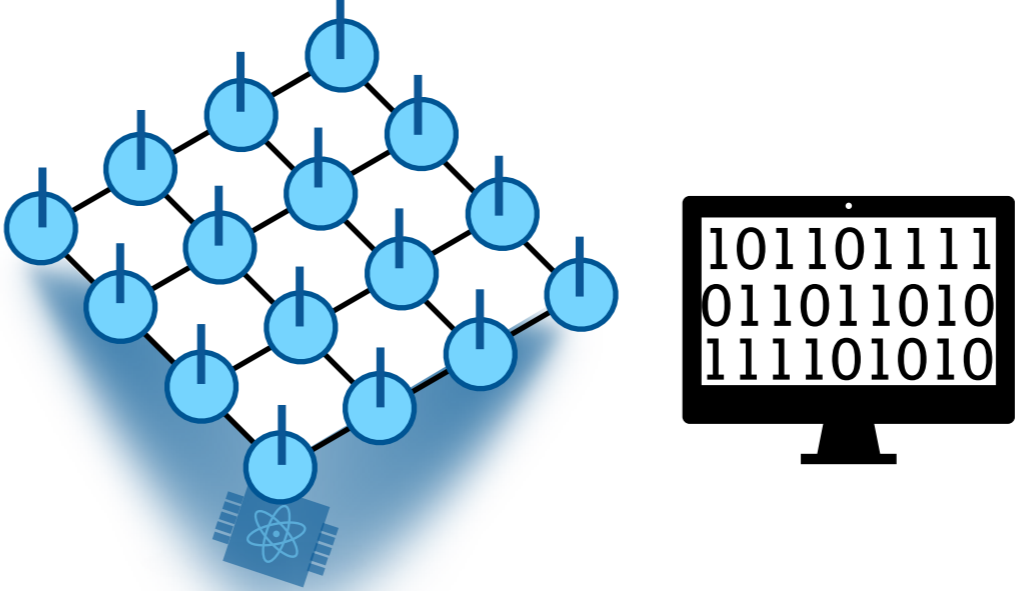
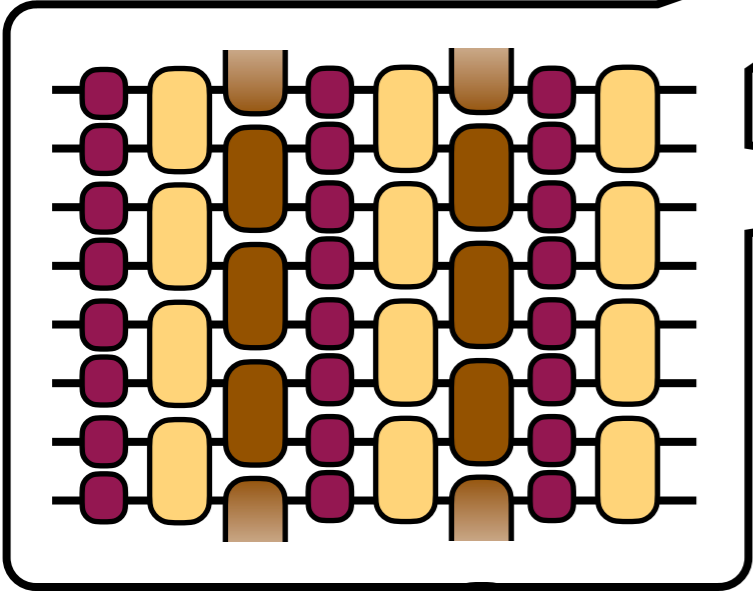
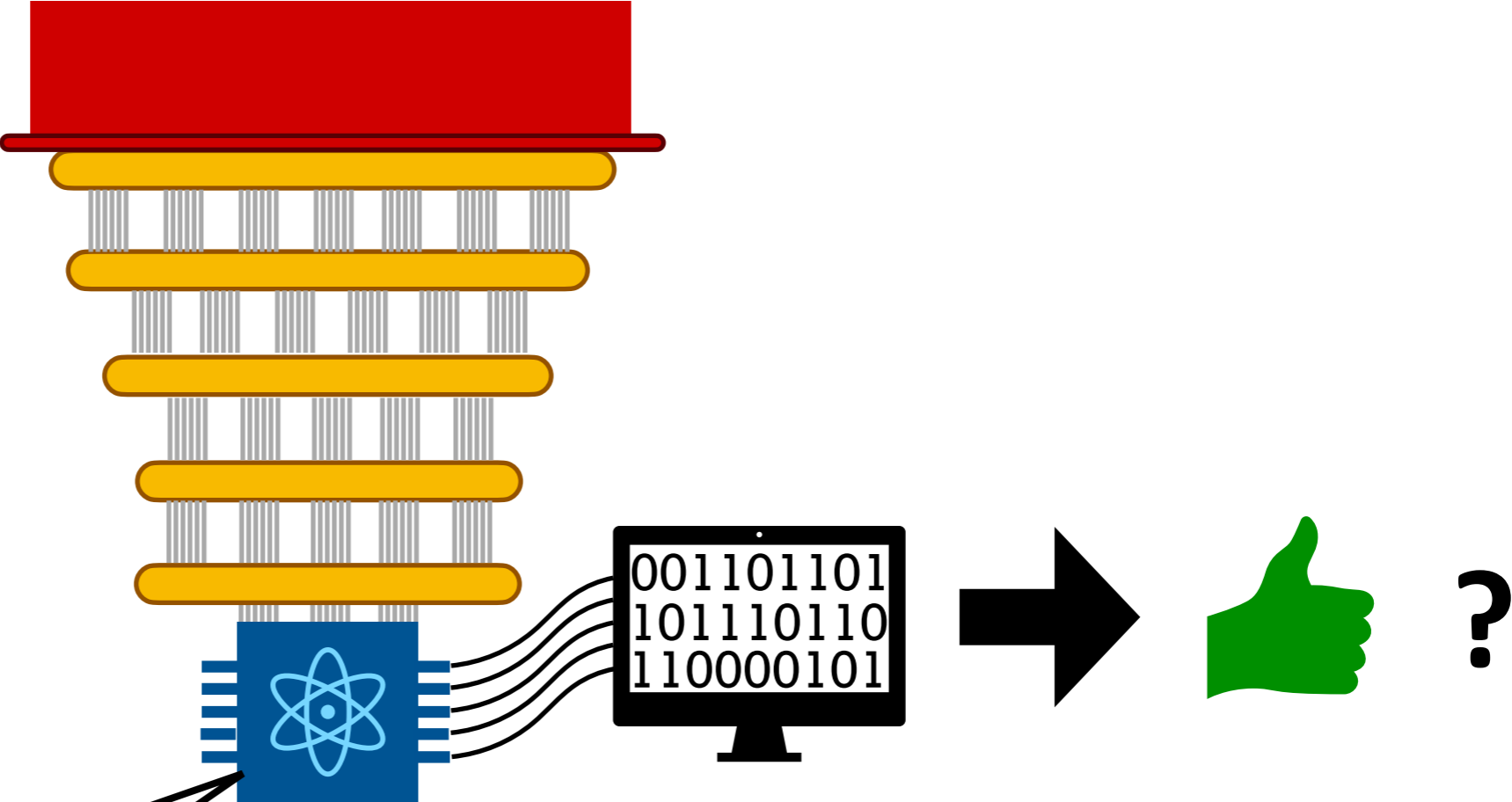
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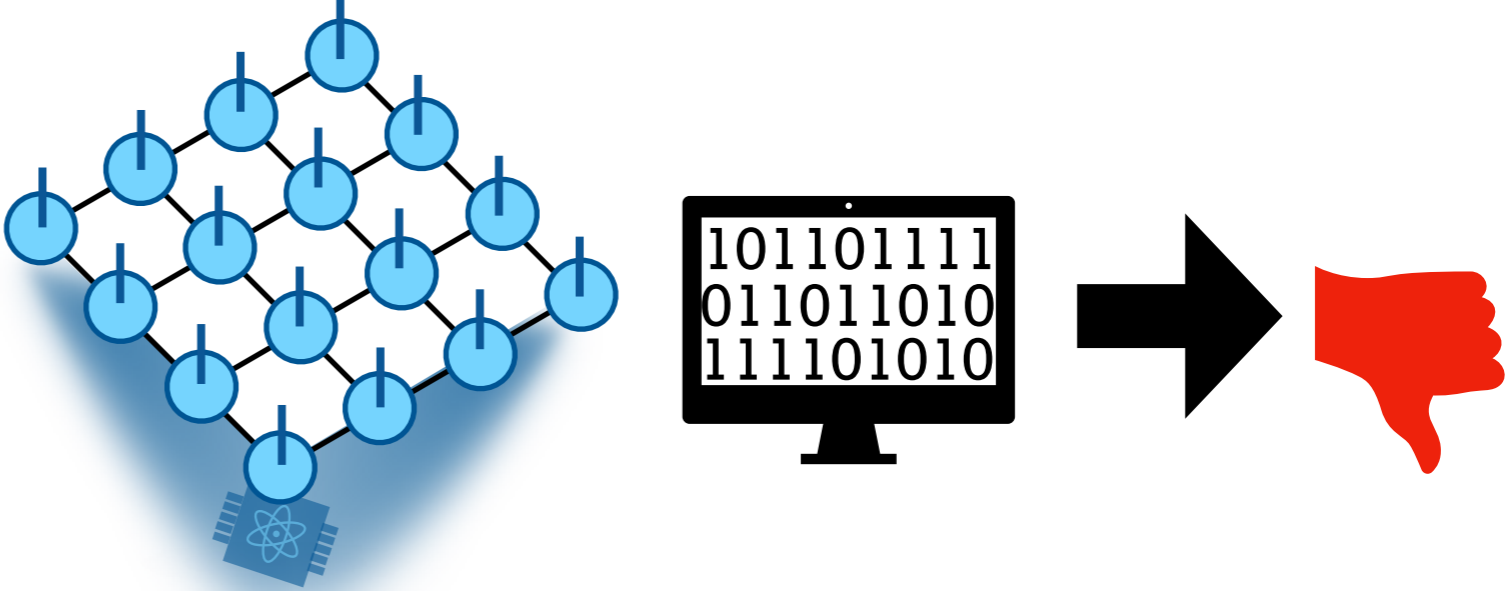
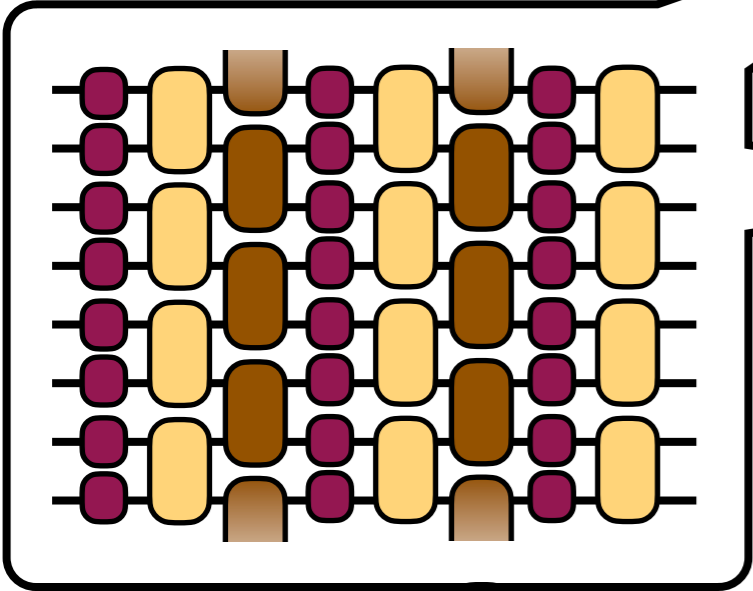
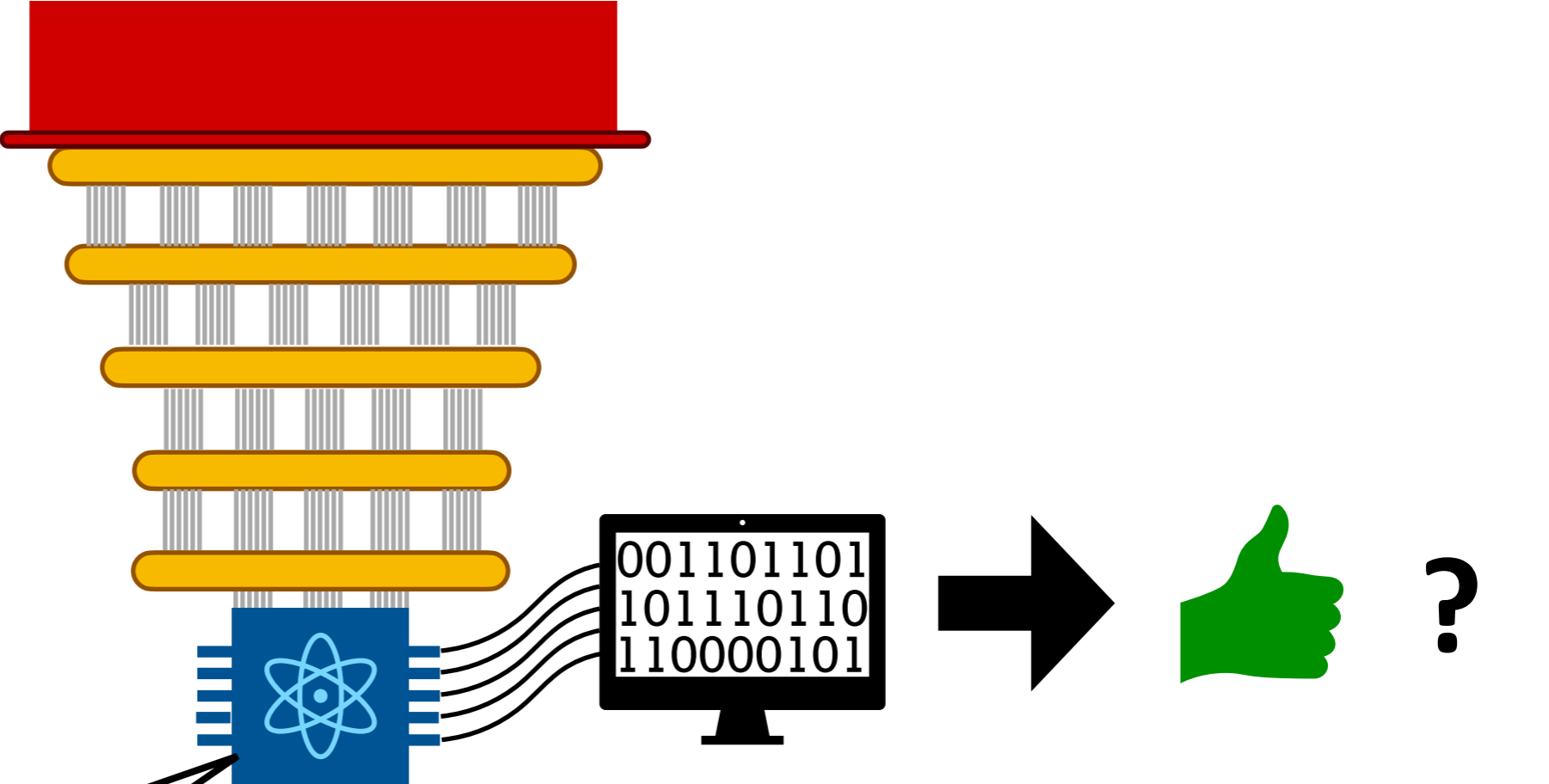
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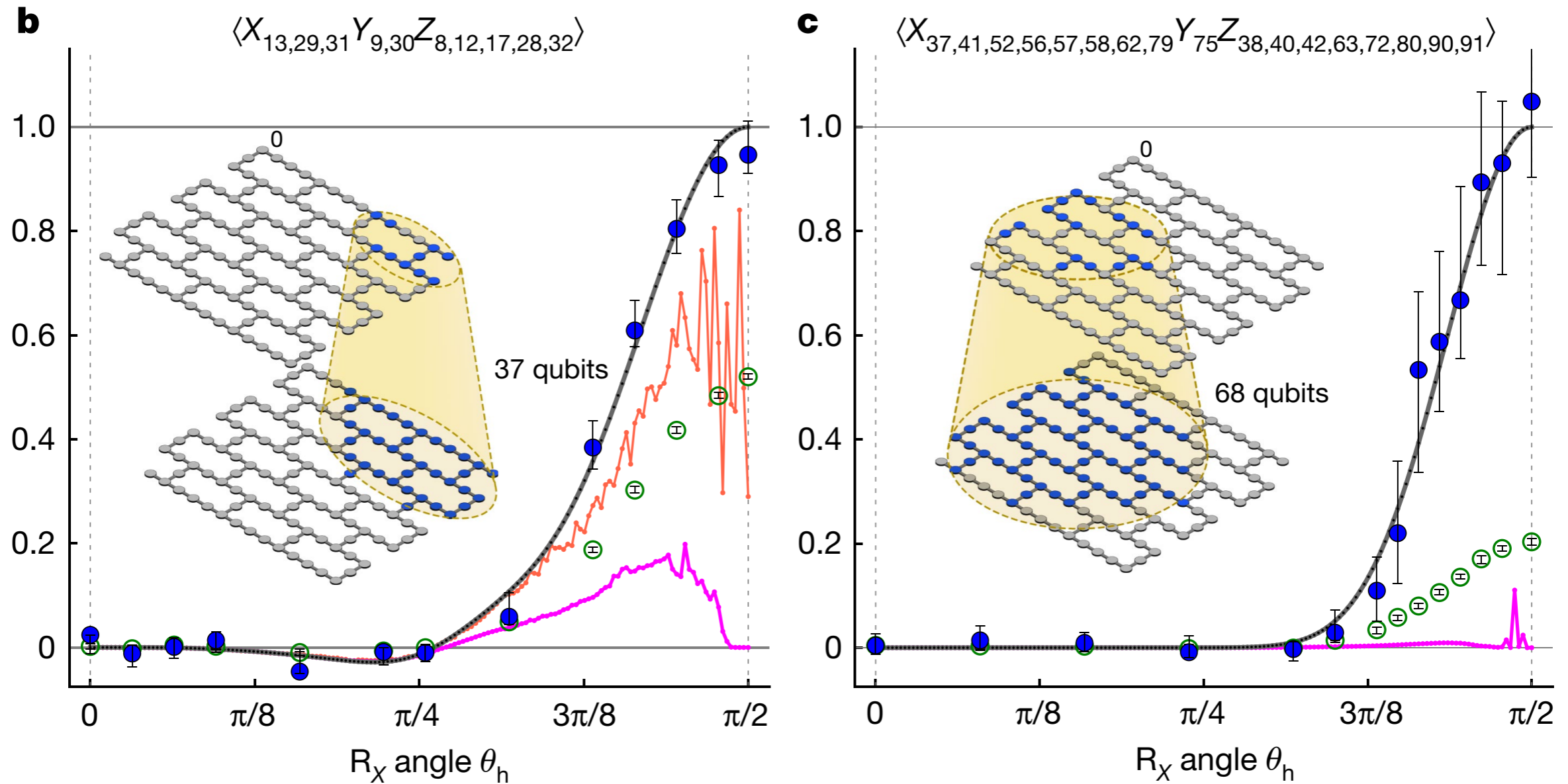
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# WHAT IS BENCHMARKING?



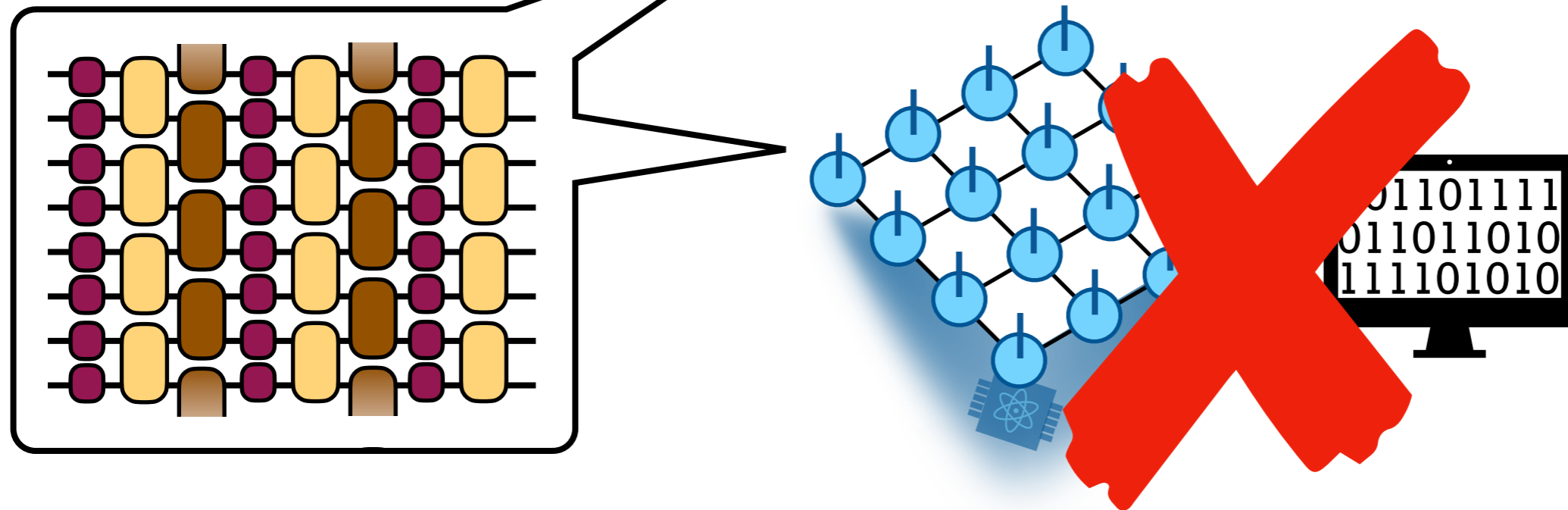
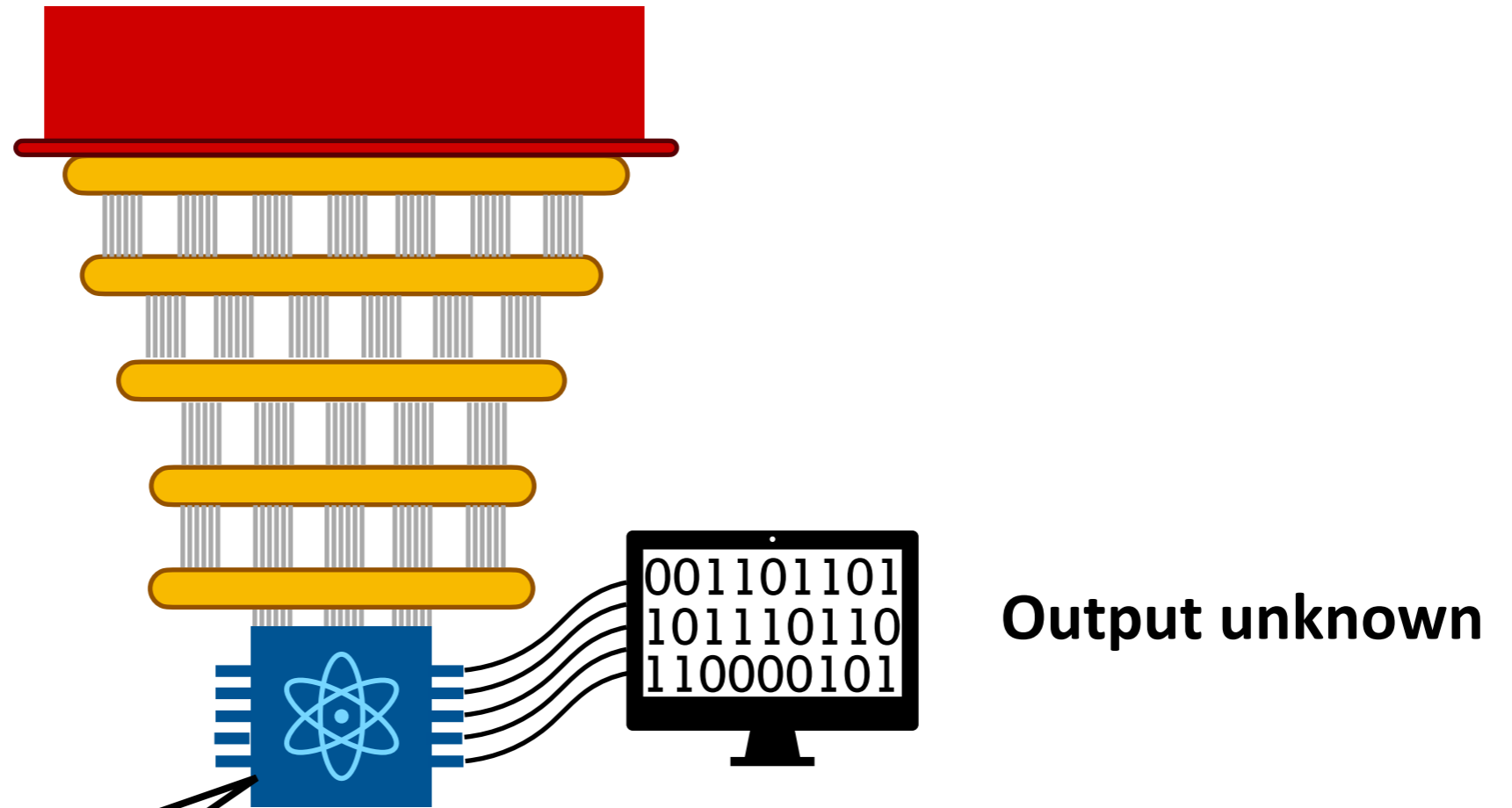
# BENCHMARKING QUANTUM COMPUTATION IS USEFUL



Kim, Y., Eddins, A., Anand, S. *et al.* Evidence for the utility of quantum computing before fault tolerance, *Nature* **618**, 500–505 (2023)

**What is the problem?**

# BENCHMARKING OF QUANTUM COMPUTATION IS LIMITED

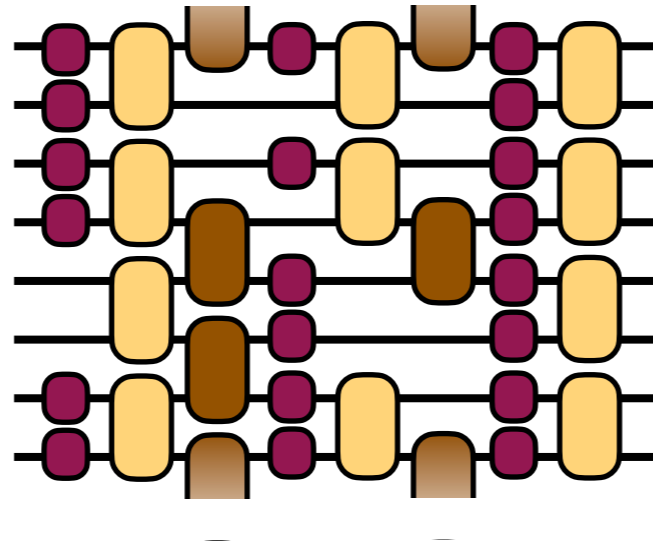


# BENCHMARKING OF QUANTUM COMPUTATION IS LIMITED

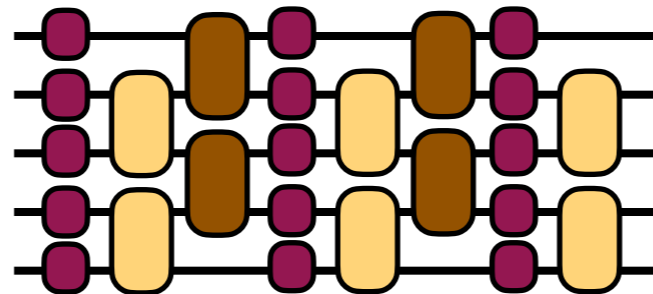
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Then I can only simplify the problem:

- Use simulable gatesets



- Reduce the size/depth

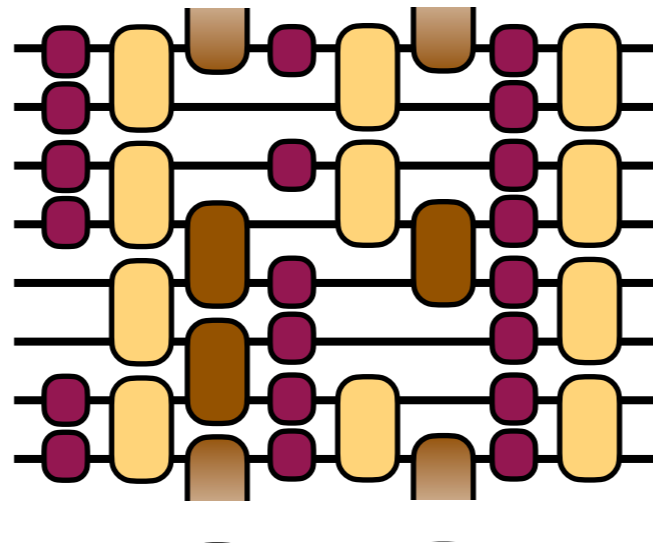


# BENCHMARKING OF QUANTUM COMPUTATION IS LIMITED

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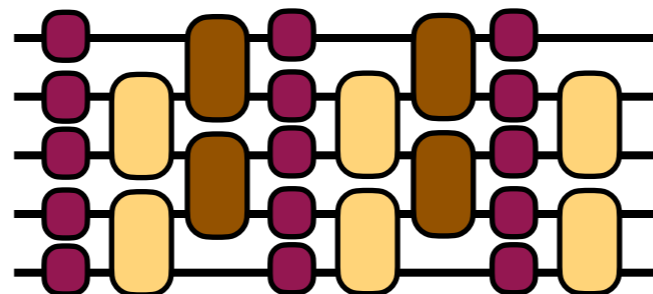
Then I can only simplify the problem:

- Use simulable gatesets



**Problem:** one never runs on the actual computation

- Reduce the size/depth



# BENCHMARKING OF QUANTUM COMPUTATION IS LIMITED

---

**Desiderata:** get some way to run a benchmark on the computation of interest

- Same size
- Same layout
- Same classical simulation complexity (more later)
- ...

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## Establishing trust in quantum computations

Timothy Proctor,<sup>1,\*</sup> Stefan Seritan,<sup>1</sup> Erik Nielsen,<sup>1</sup> Kenneth Rudinger,<sup>1</sup>  
Kevin Young,<sup>1</sup> Robin Blume-Kohout,<sup>1</sup> and Mohan Sarovar<sup>2,†</sup>


<sup>1</sup>Quantum Performance Laboratory, Sandia National Laboratories,  
Albuquerque, NM 87185, USA and Livermore, CA 94550, USA

<sup>2</sup>Sandia National Laboratories, Livermore, CA 94550

nature  
physics

ARTICLES

<https://doi.org/10.1038/s41567-021-01409-7>

 Check for updates

## Measuring the capabilities of quantum computers

Timothy Proctor <sup>1,2</sup> , Kenneth Rudinger <sup>1,2</sup>, Kevin Young <sup>1,2</sup>, Erik Nielsen<sup>1,2</sup> and  
Robin Blume-Kohout <sup>1,2</sup>

Quantum computers can now run interesting programs, but each processor's capability—the set of programs that it can run successfully—is limited by hardware errors. These errors can be complicated, making it difficult to accurately predict a processor's capability. Benchmarks can be used to measure capability directly, but current benchmarks have limited flexibility and scale poorly to many-qubit processors. We show how to construct scalable, efficiently verifiable benchmarks based on any program by using a technique that we call circuit mirroring. With it, we construct two flexible, scalable volumetric benchmarks based on randomized and periodically ordered programs. We use these benchmarks to map out the capabilities of twelve publicly available processors, and to measure the impact of program structure on each one. We find that standard error metrics are poor predictors of whether a program will run successfully on today's hardware, and that current processors vary widely in their sensitivity to program structure.

# BENCHMARKING OF QUANTUM COMPUTATION IS LIMITED

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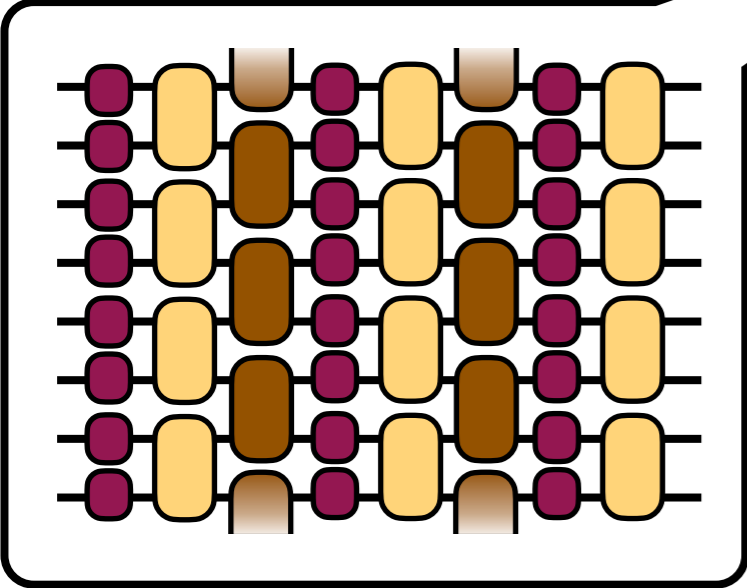
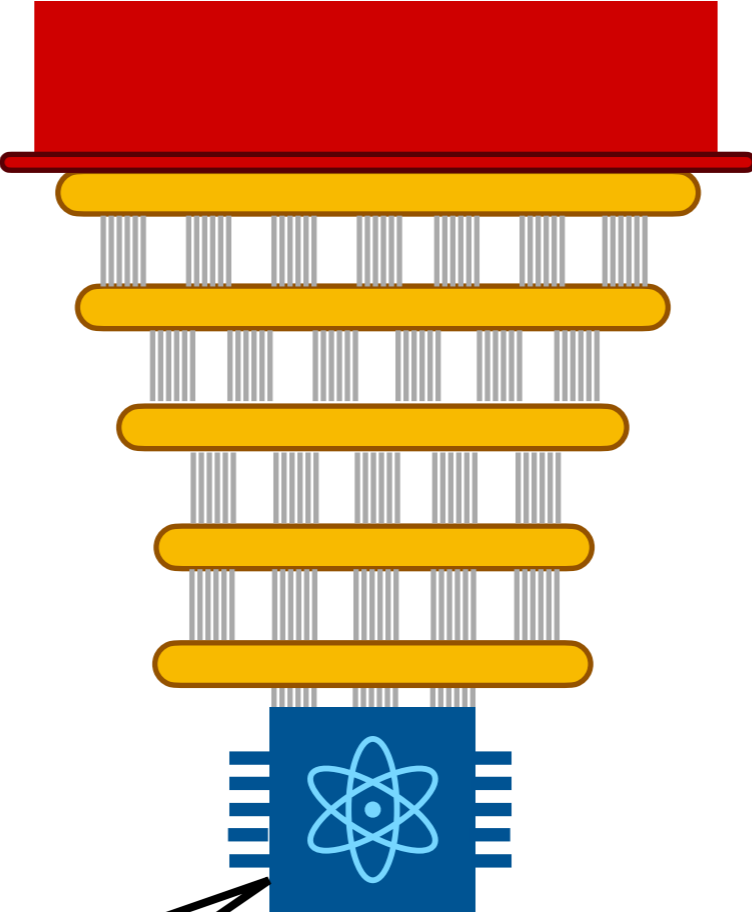
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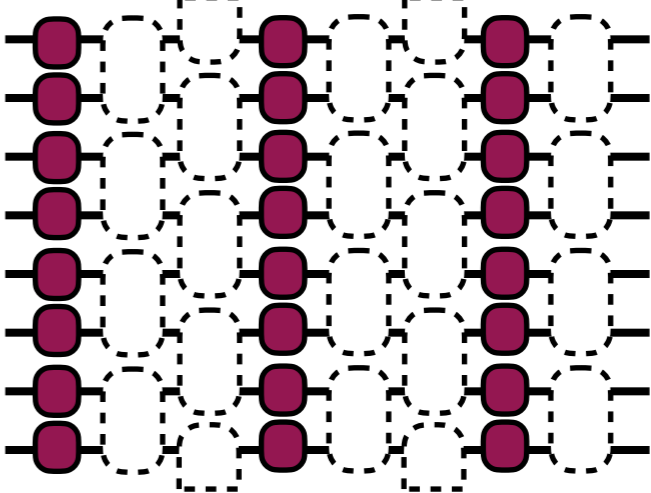
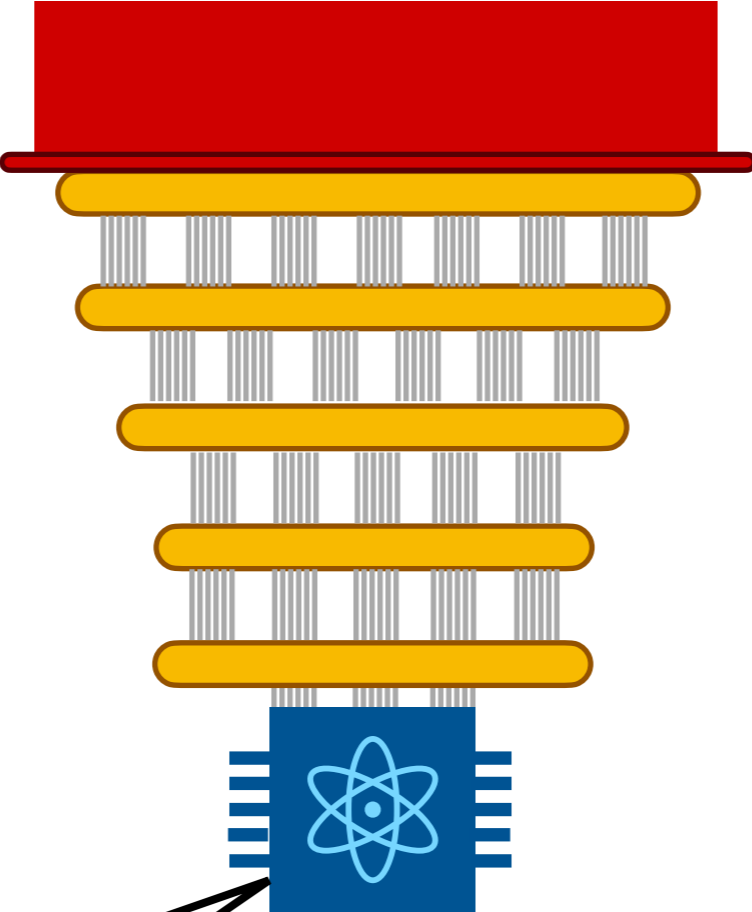
We test the performance  
on a specific task: estimating  
local observables

# **Average-computation benchmarking**

# TAKE DATA FROM A FAMILY OF CIRCUITS

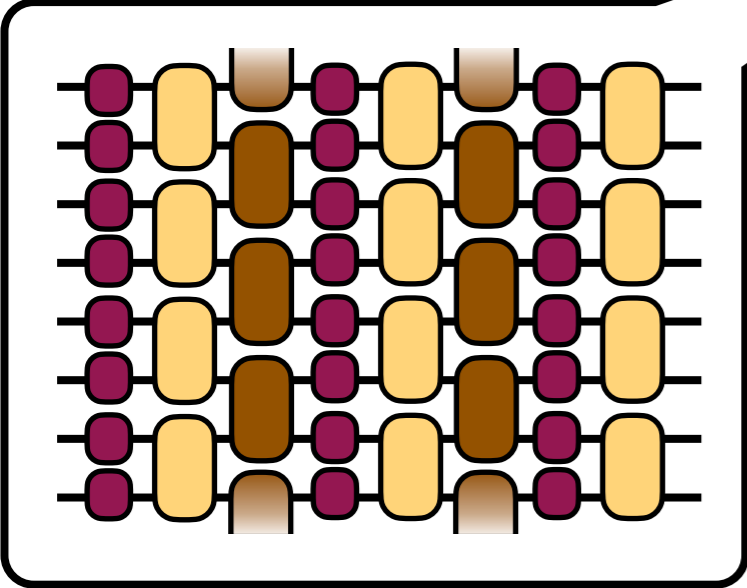


# TAKE DATA FROM A FAMILY OF CIRCUITS

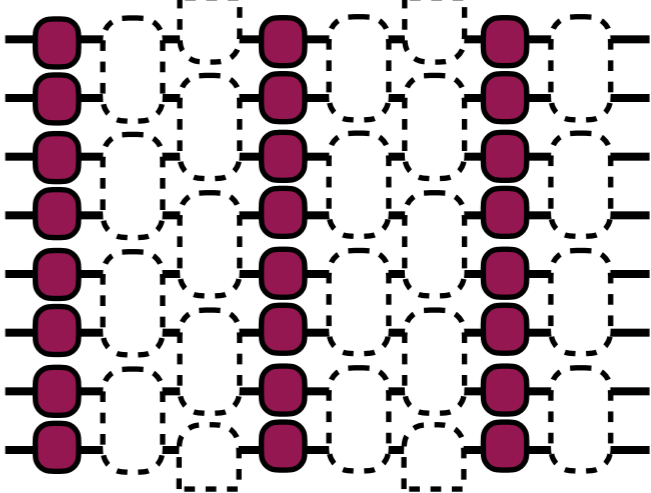
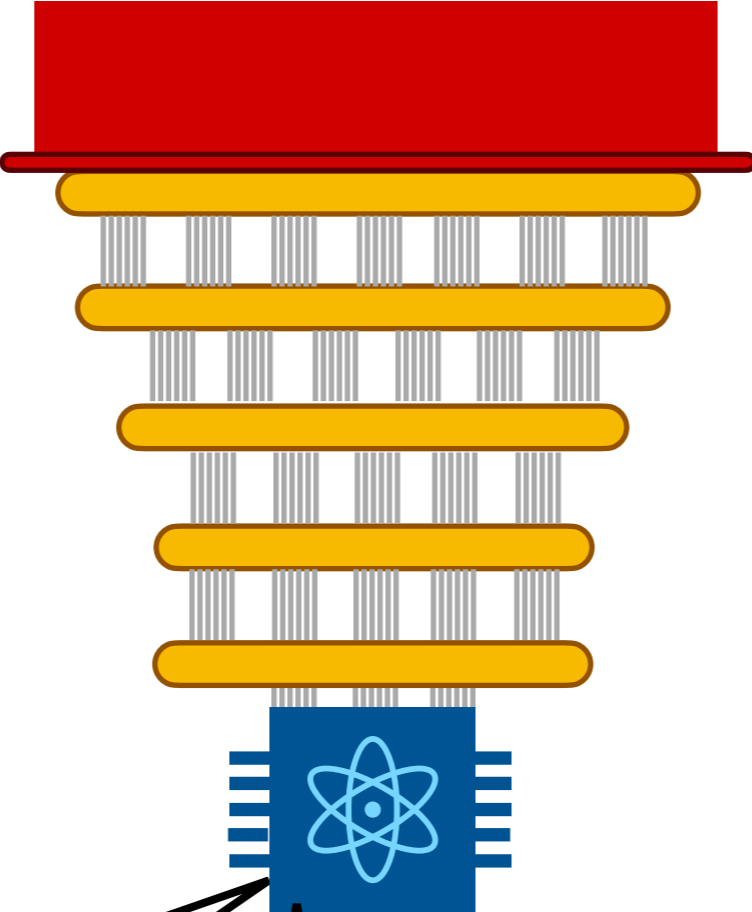


Sample each gate independently from

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- ...

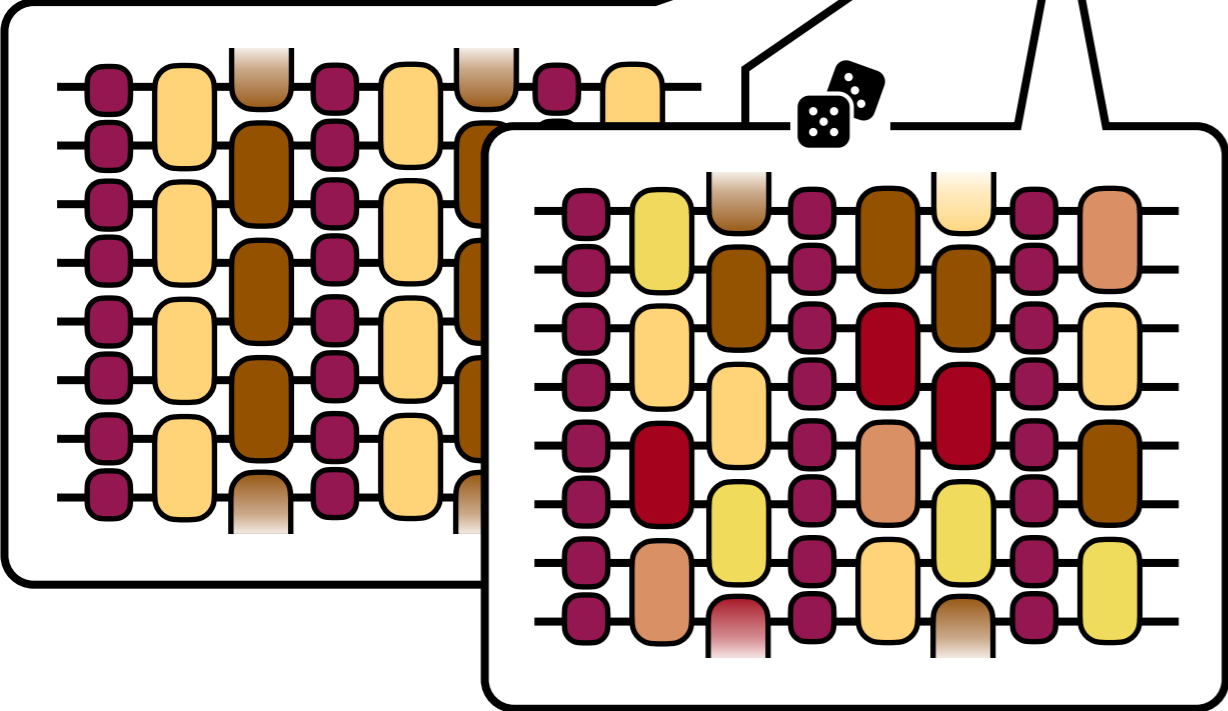


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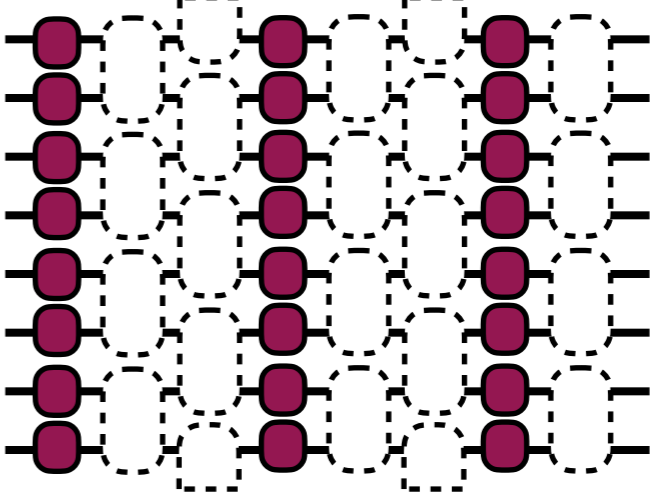
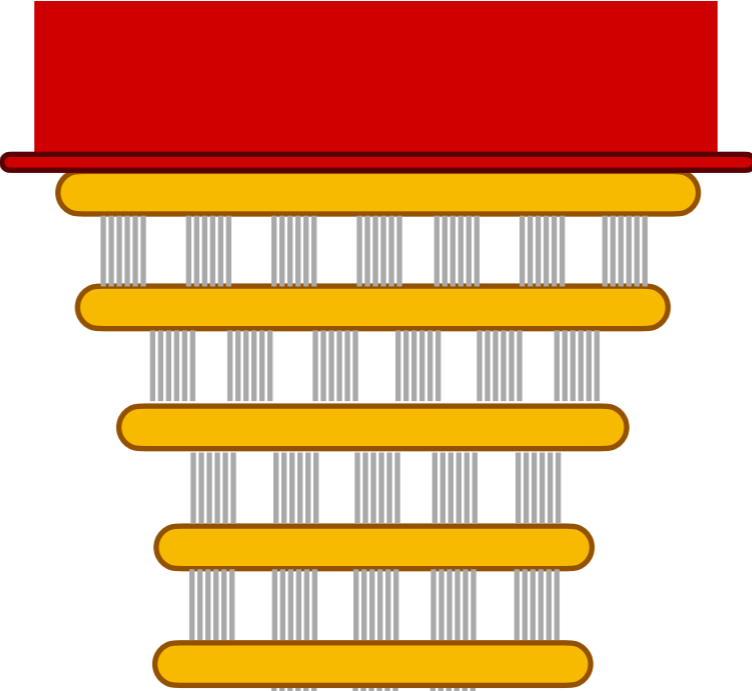


Sample each gate independently from

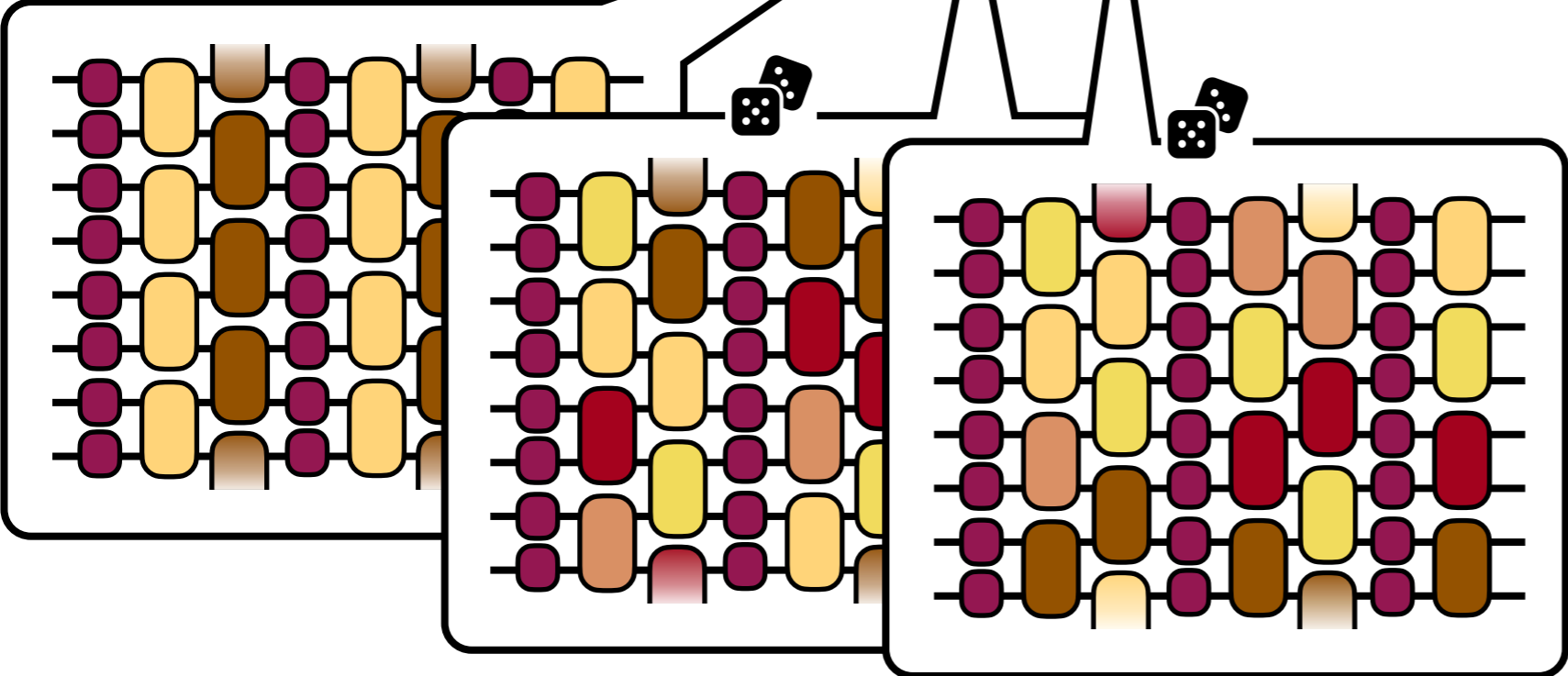
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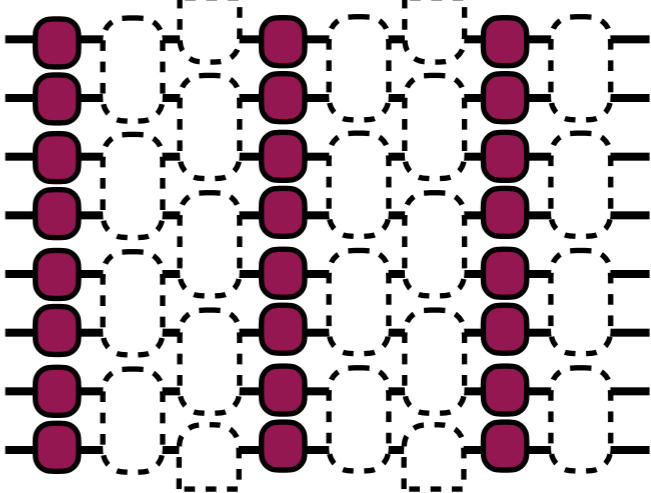
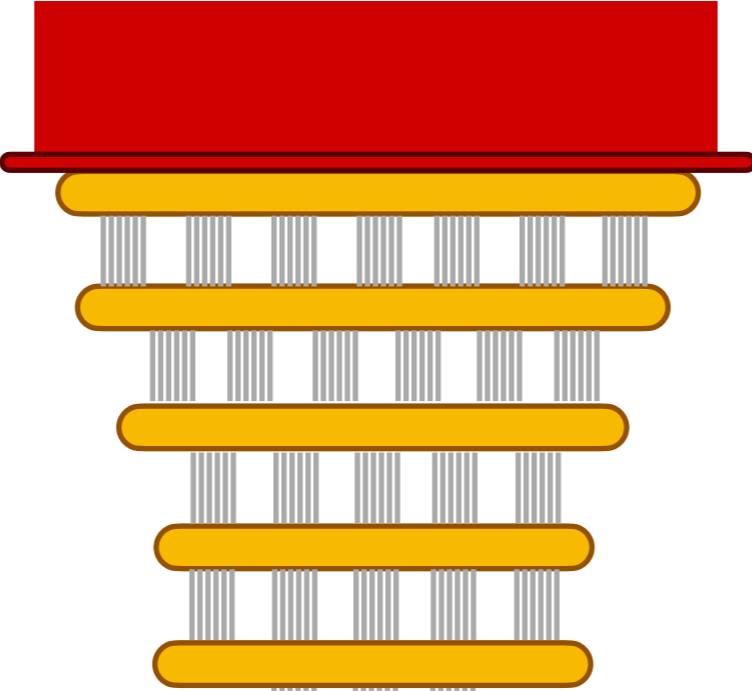
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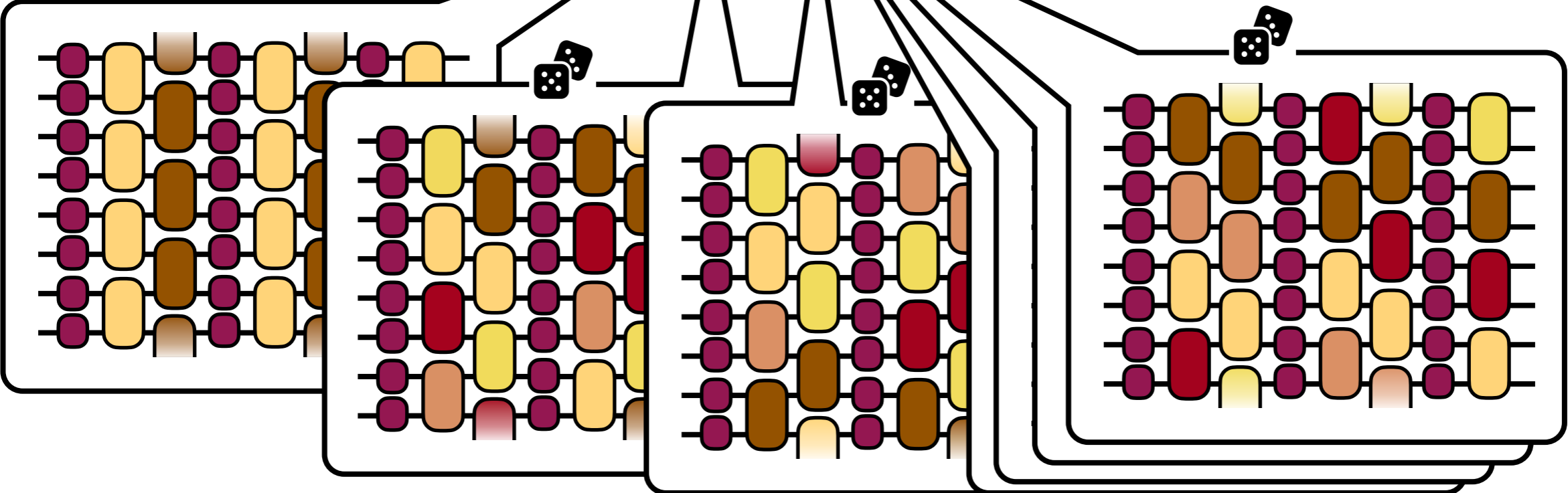
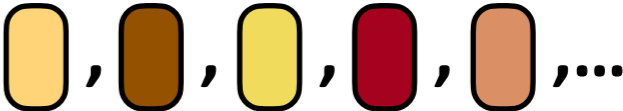
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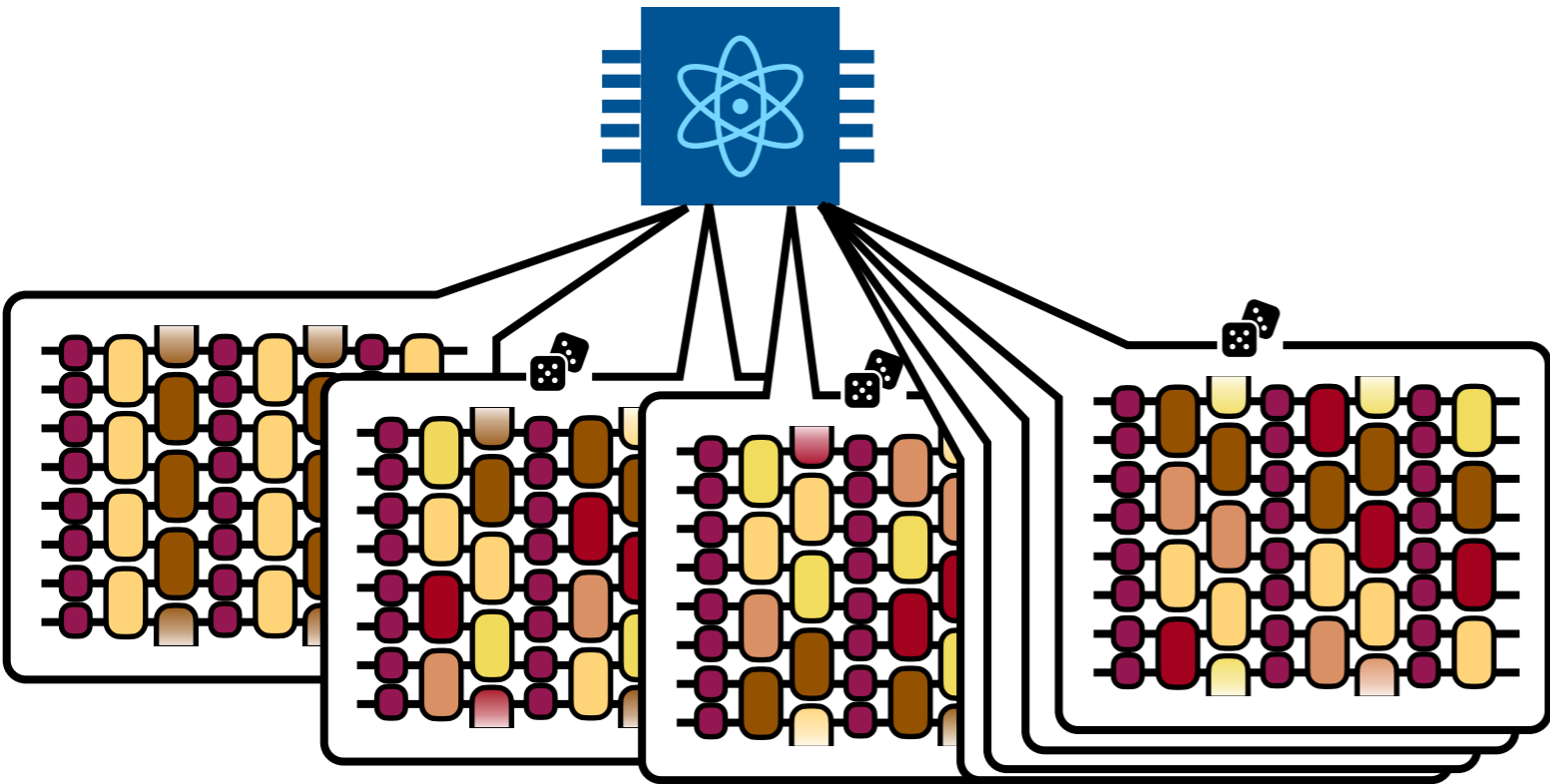
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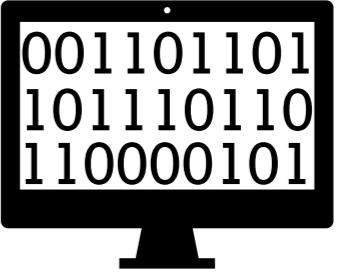
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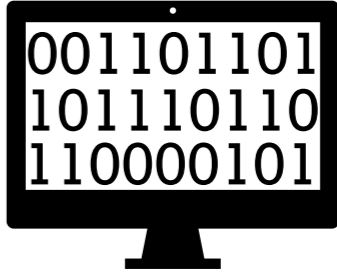
# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$U_1$

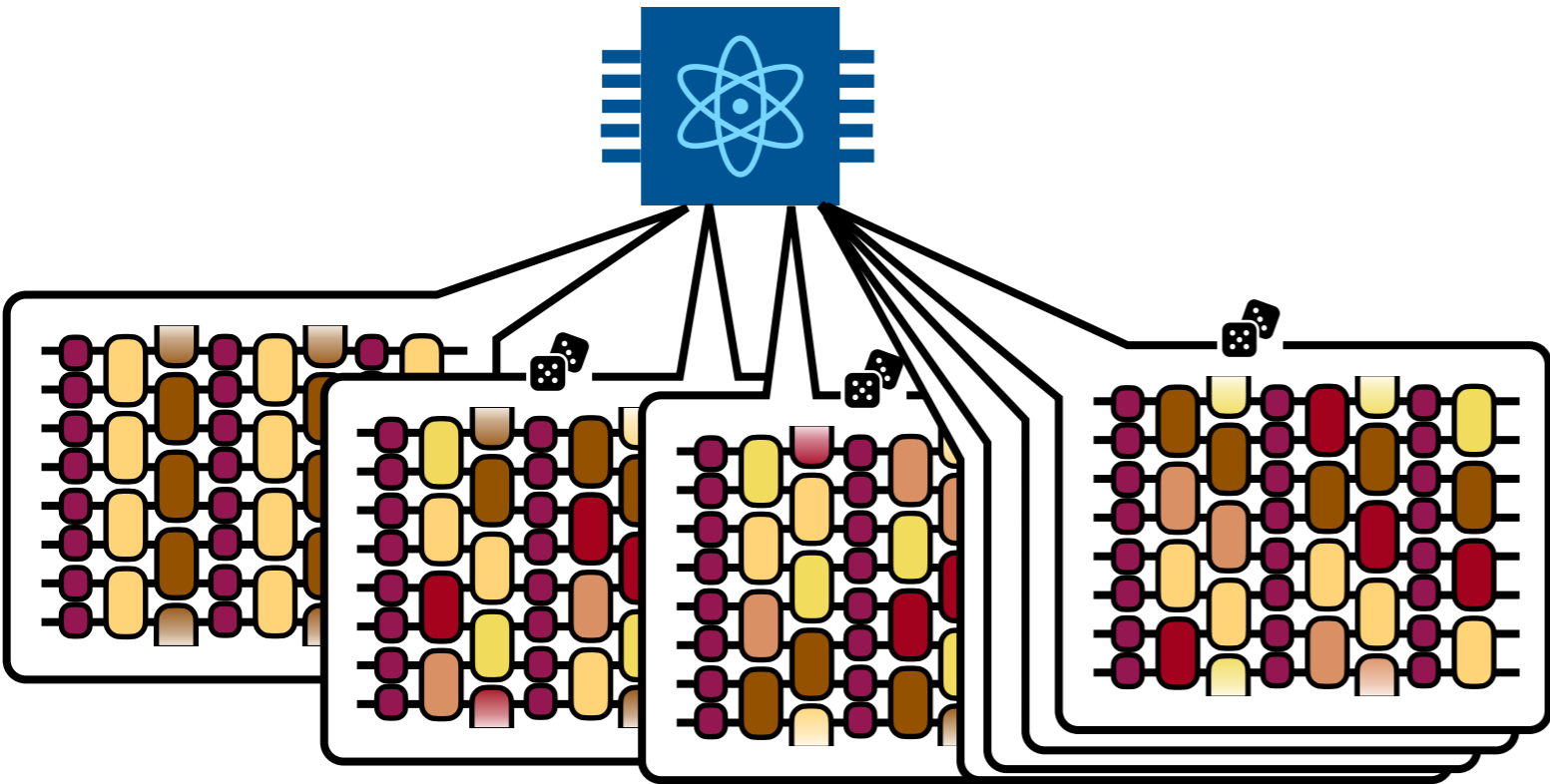


$U_2$



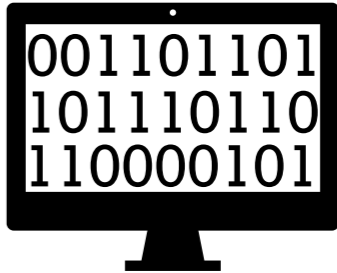
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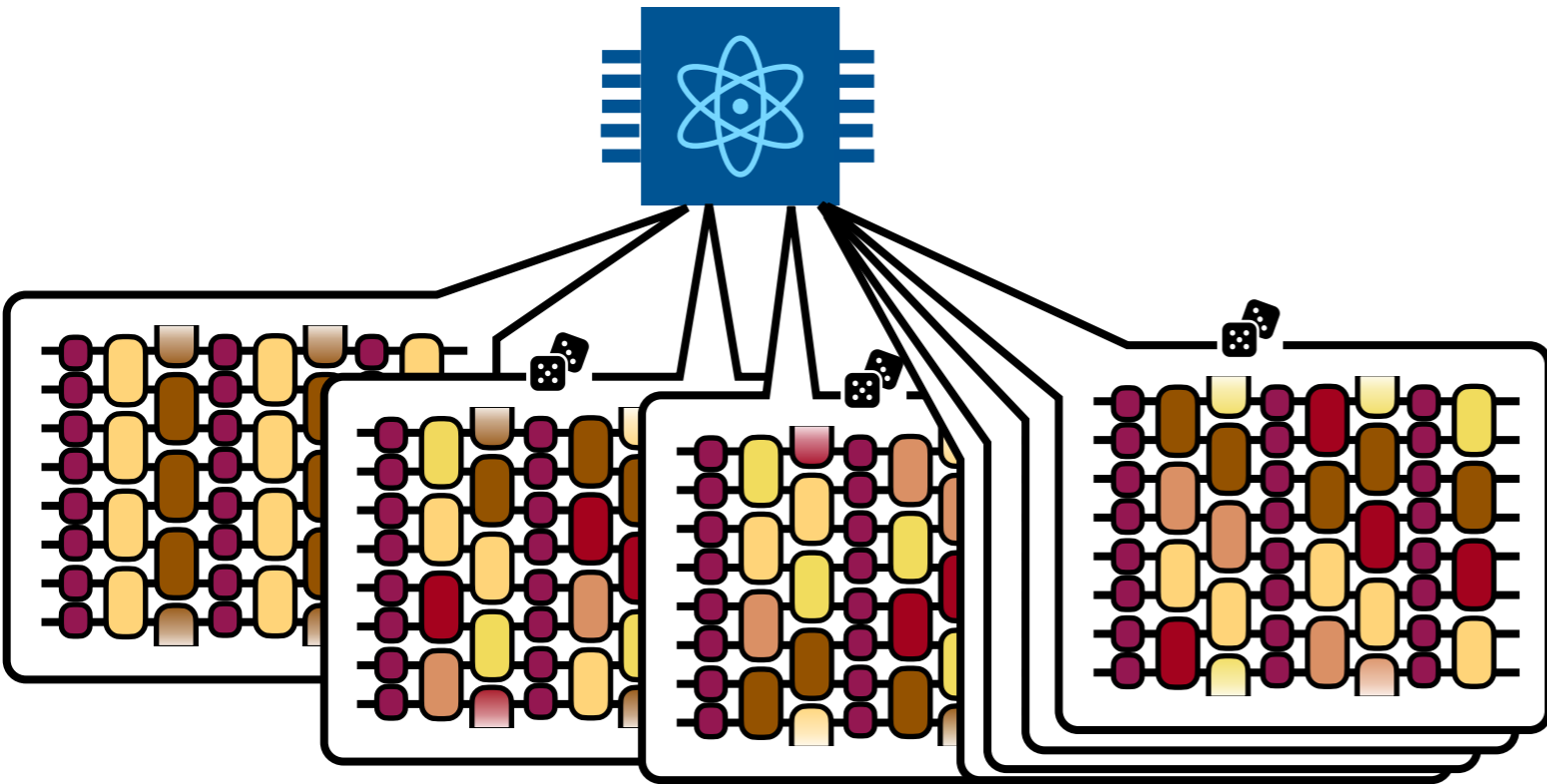
$$\text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)]$$

$U_2$



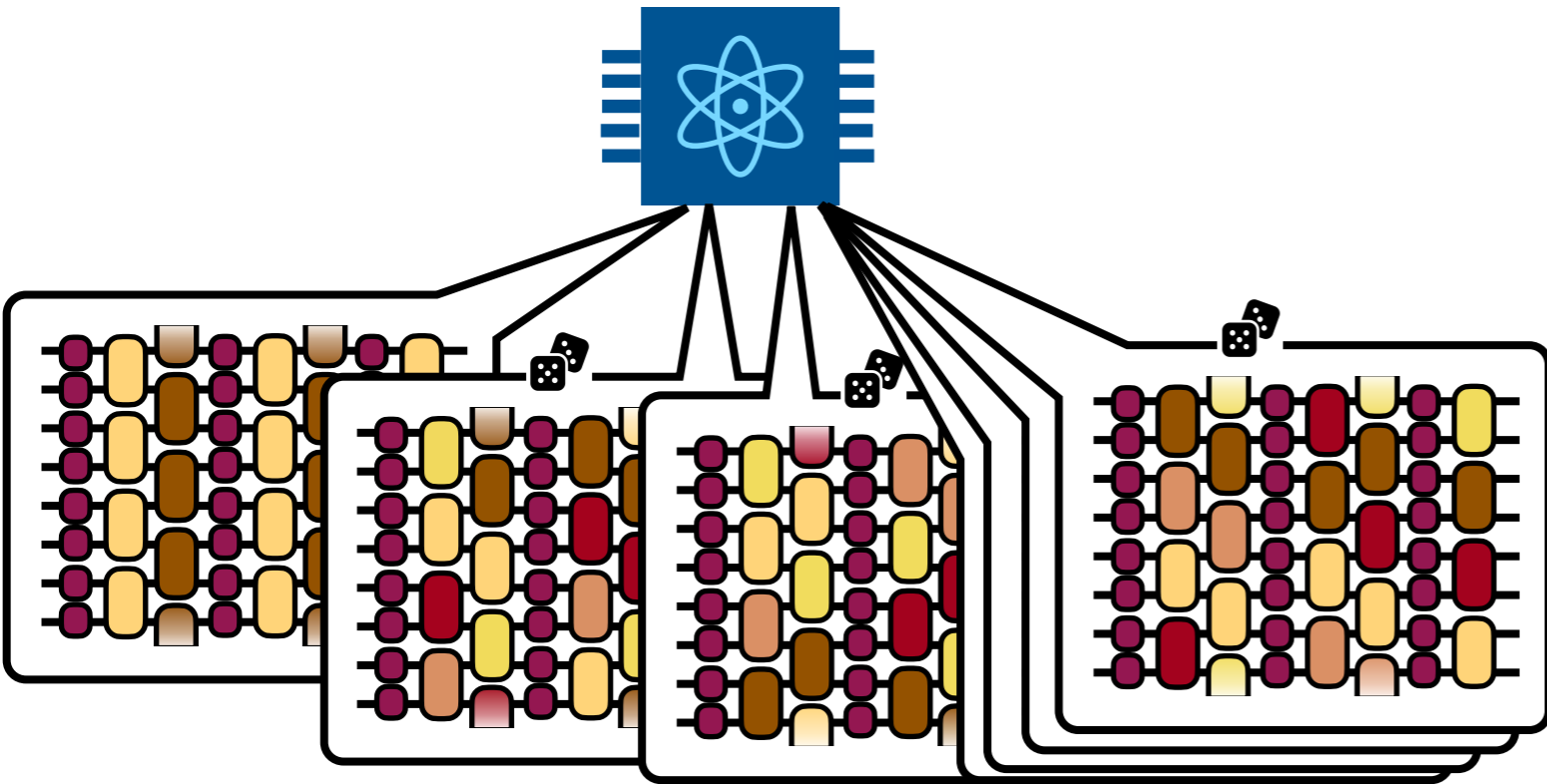
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# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$$\begin{aligned} & \text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)] \\ & \quad + \\ & \text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)] \\ & \quad + \\ & \cdot \\ & \cdot \\ & \cdot \end{aligned}$$

# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS

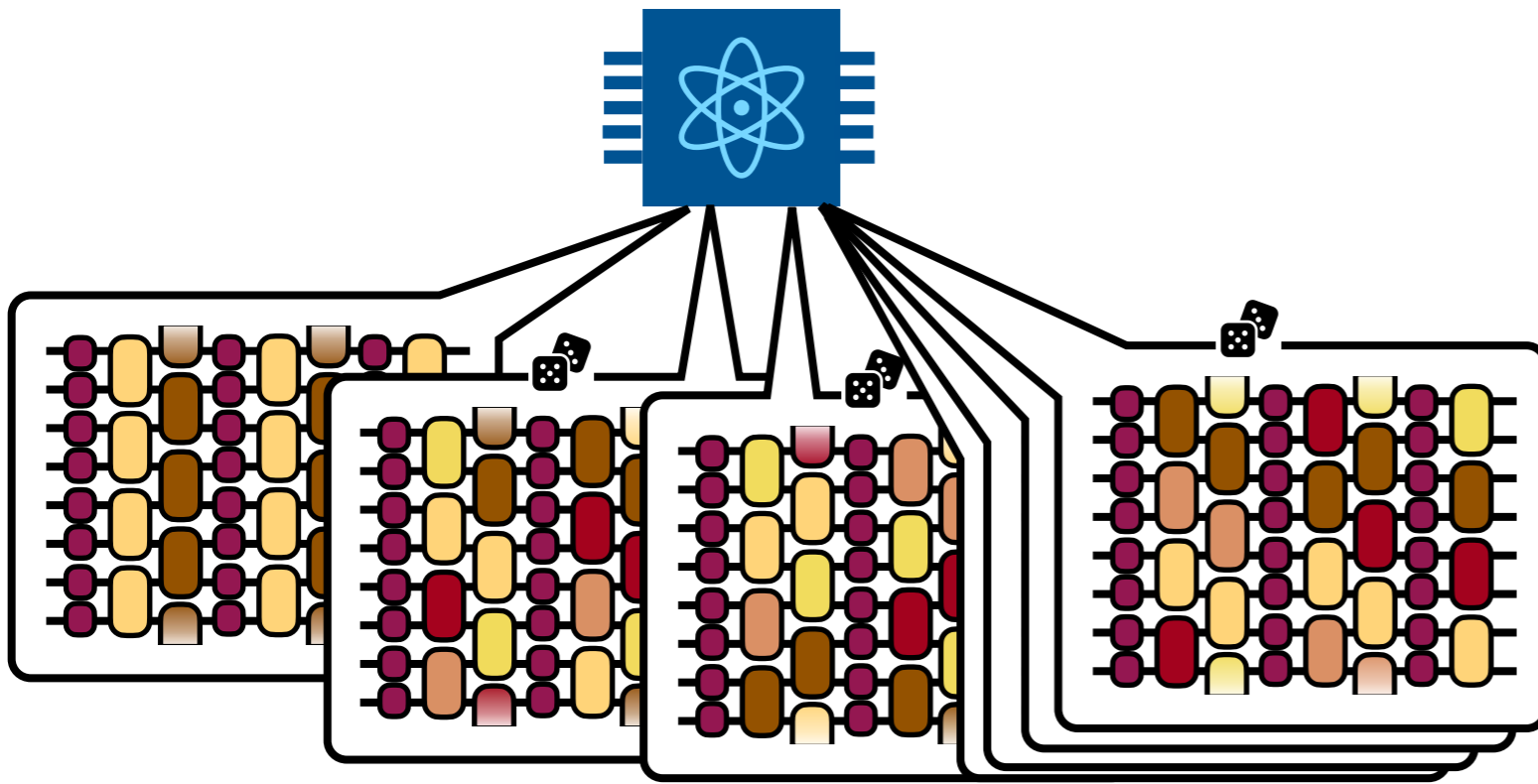


$$\begin{aligned} & \text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)] \\ & + \\ & \text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)] \\ & + \\ & \vdots \end{aligned}$$

**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$$\mathbb{E}[\text{yellow}, \text{brown}, \text{yellow}, \text{red}, \text{brown}, \dots] = \text{green } \varepsilon$$

$$\begin{aligned} & \text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)] \\ & + \\ & \text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)] \\ & + \\ & \vdots \end{aligned}$$

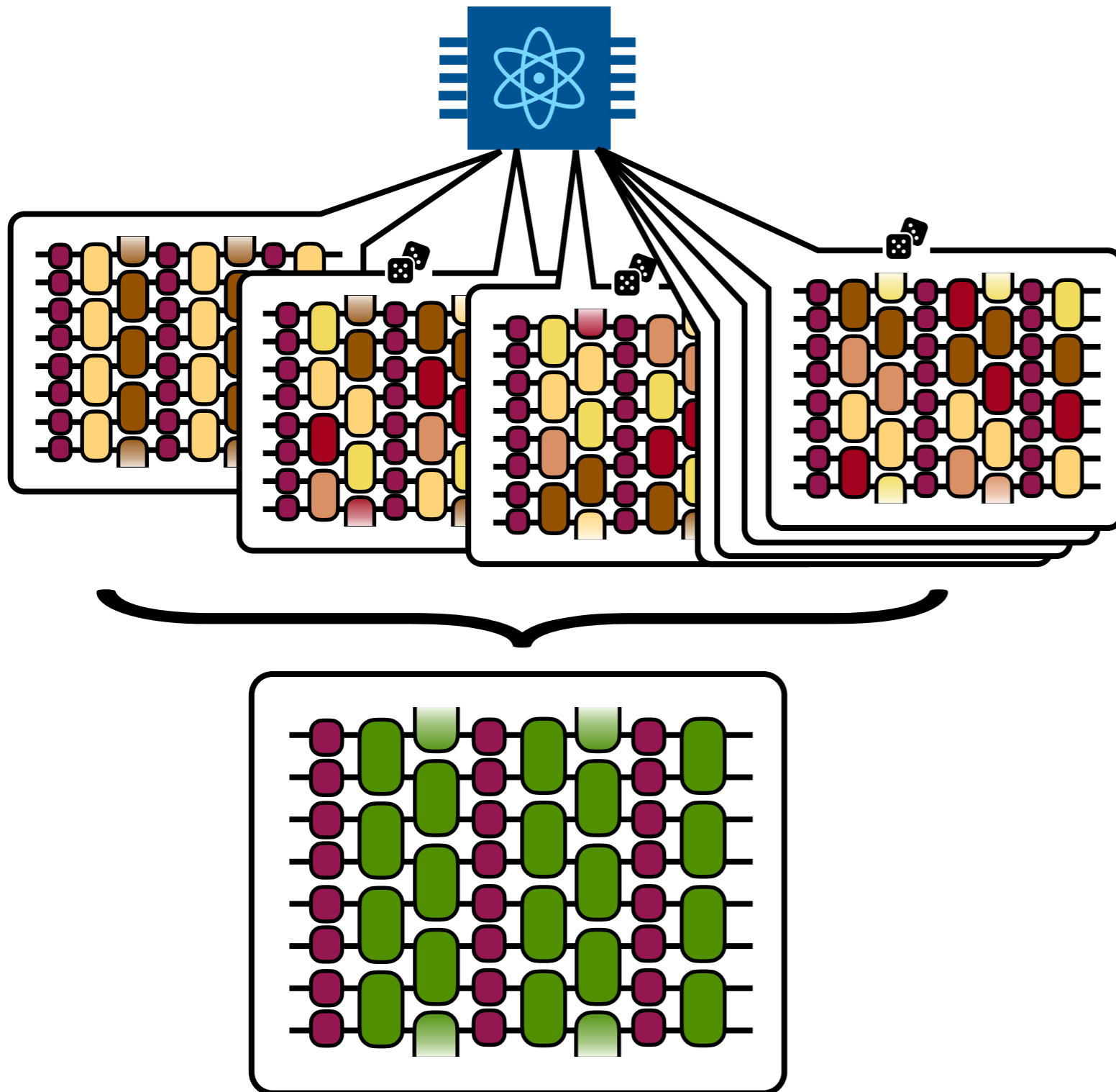
**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

||

$$\text{tr}[O_{\text{test}} \mathbb{E}_U [(U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$$\begin{aligned}
 & \text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)] \\
 & \quad + \\
 & \text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)] \\
 & \quad + \\
 & \quad \cdot \\
 & \quad \cdot \\
 & \quad \cdot
 \end{aligned}$$

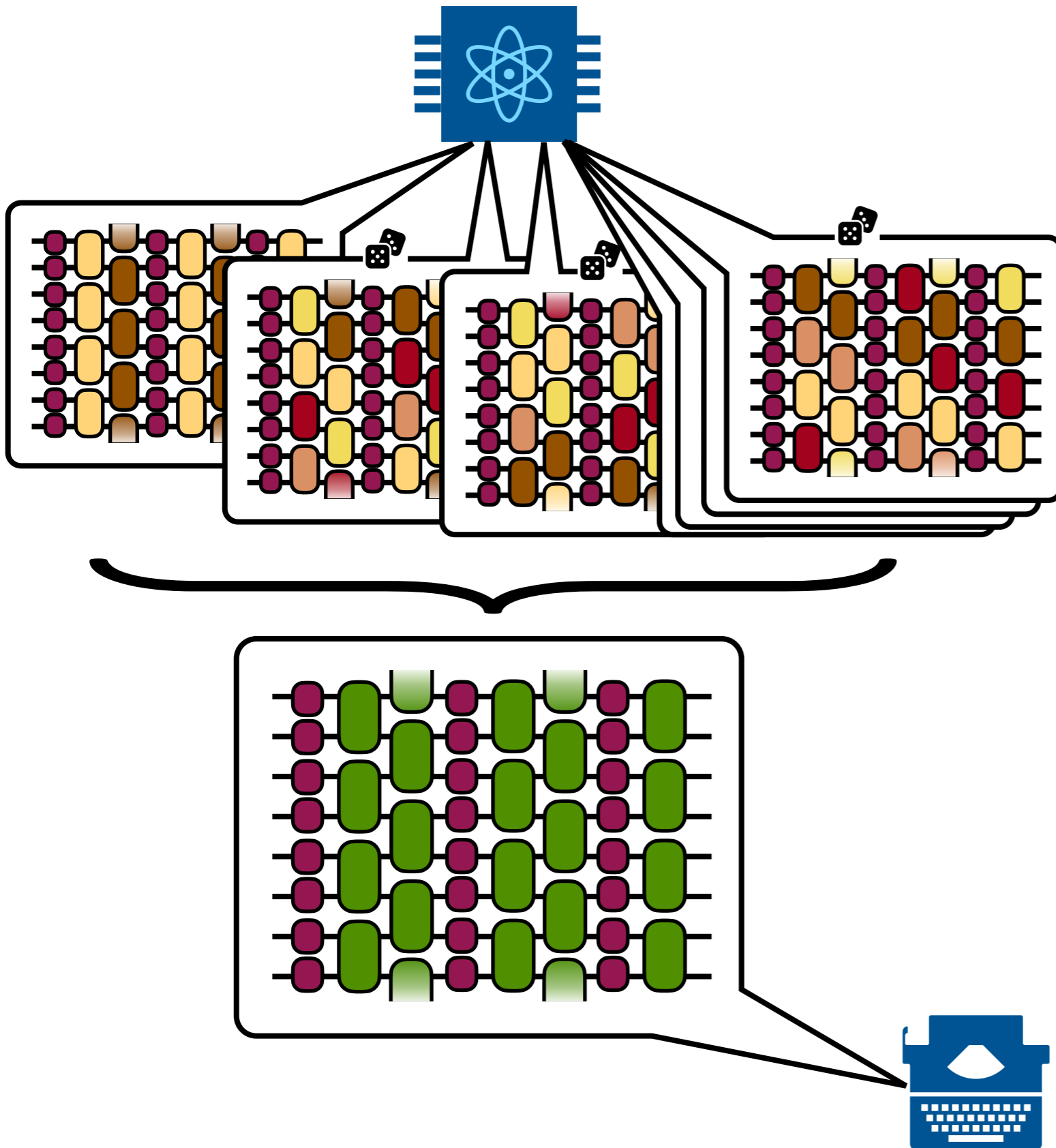
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||

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle \langle \psi_0|)]$$

# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$$\begin{aligned} & \text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)] \\ & + \\ & \text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)] \\ & + \\ & \vdots \end{aligned}$$

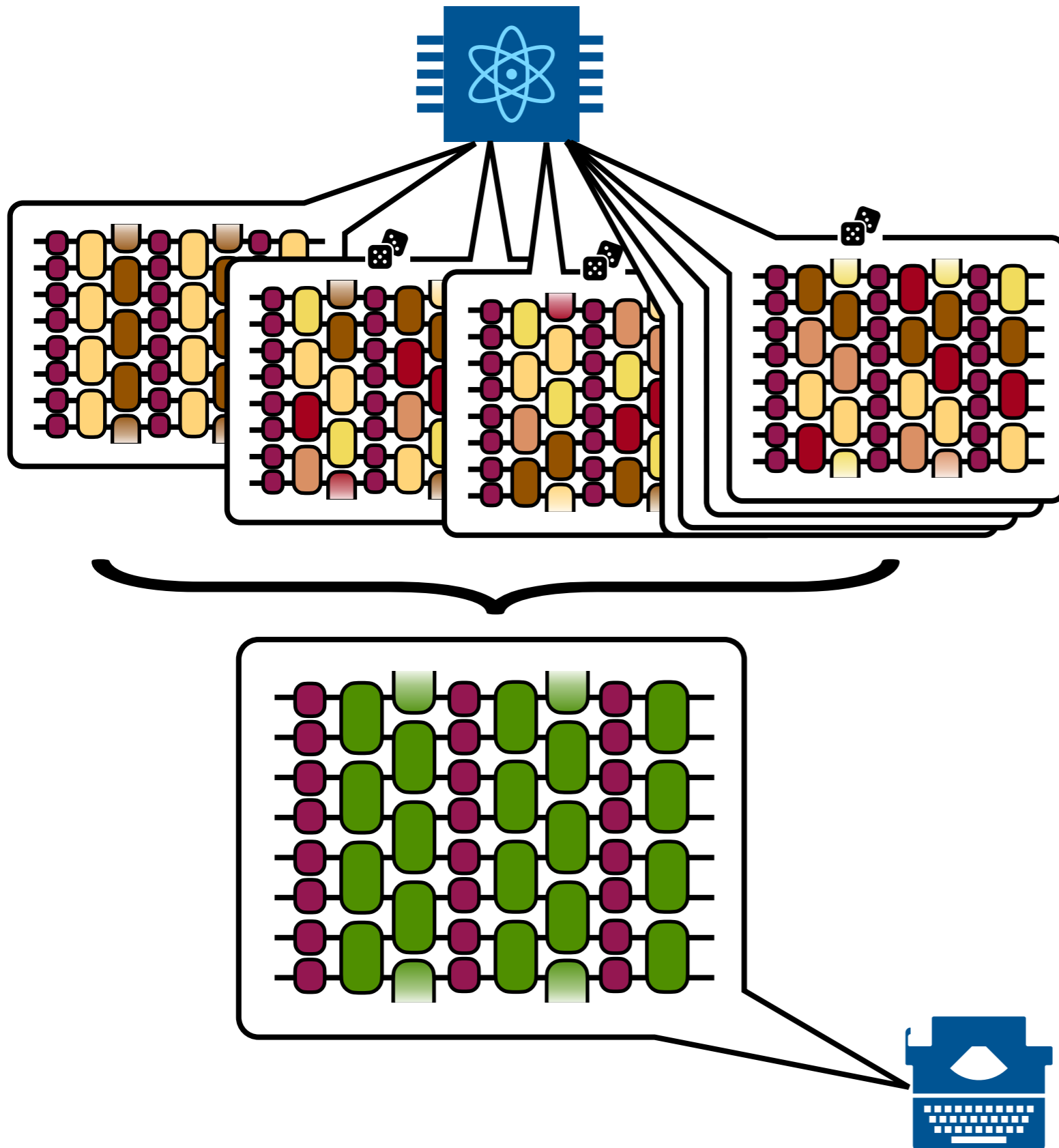
**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle \langle \psi_0|)]$$

**Classical simulation**

# COMPUTE AVERAGE DATA IN TWO DIFFERENT WAYS



$$\text{tr}[O_{\text{test}} (U_1^\dagger |\psi_0\rangle \langle \psi_0| U_1)]$$

+

$$\text{tr}[O_{\text{test}} (U_2^\dagger |\psi_0\rangle \langle \psi_0| U_2)]$$

+

⋮

**Quantum data**

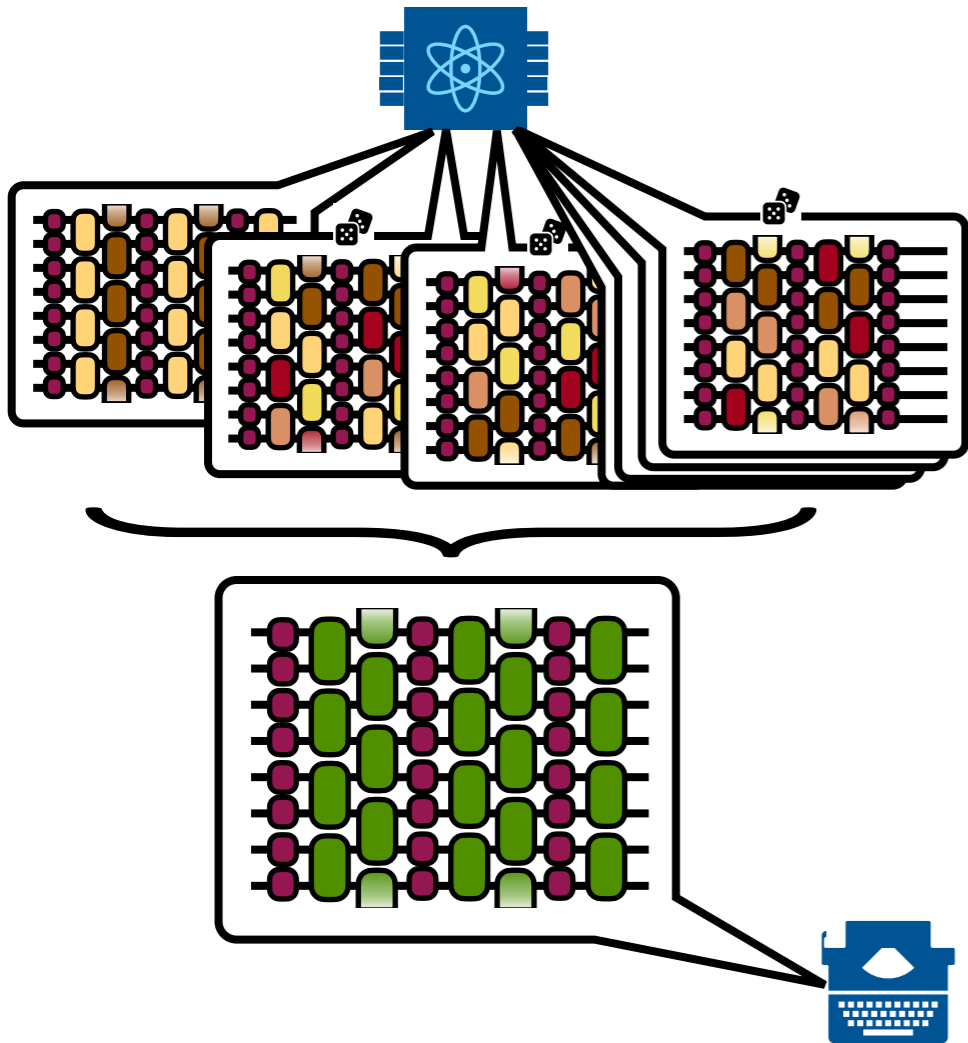
$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

**VS**

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle \langle \psi_0|)]$$

**Classical simulation**

# MAKE IT INFORMATIVE



**Quantum data**

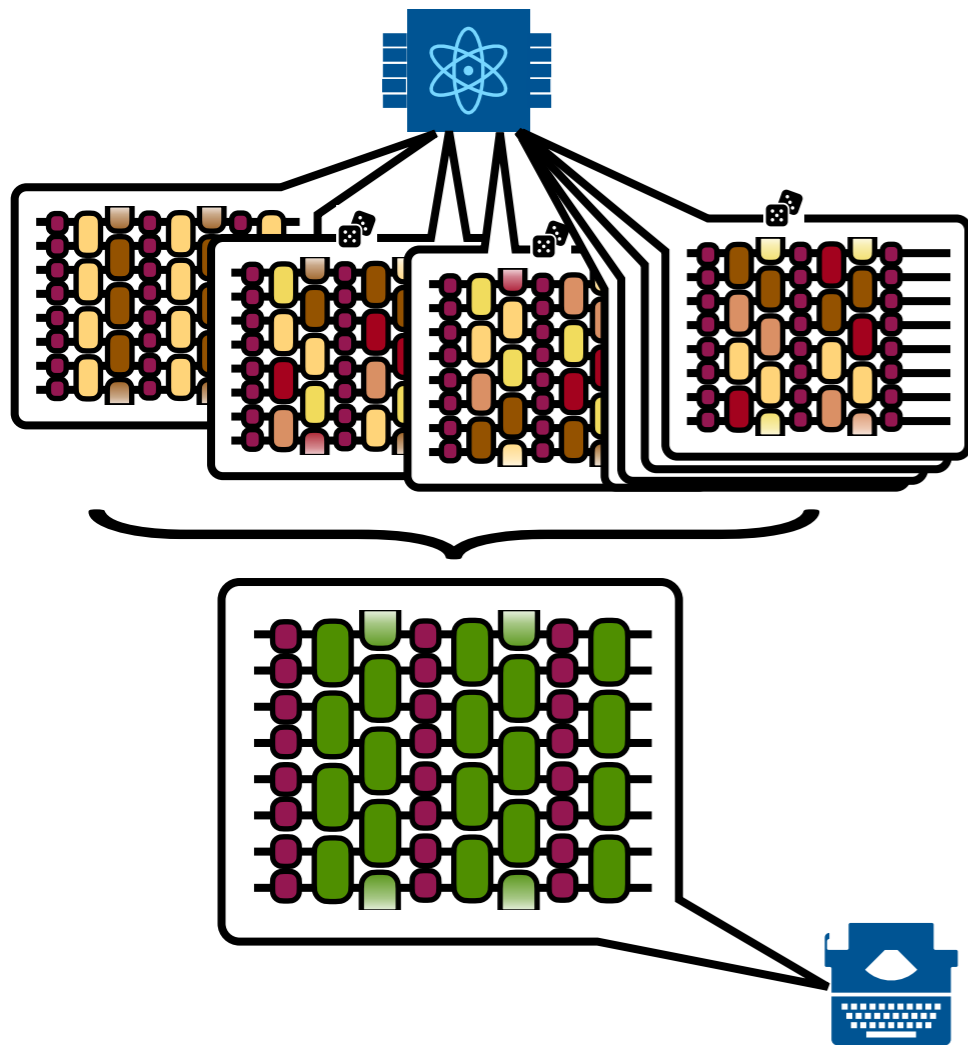
$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

**VS**

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle\langle\psi_0|)]$$

**Classical simulation**

# MAKE IT INFORMATIVE



**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

**VS**

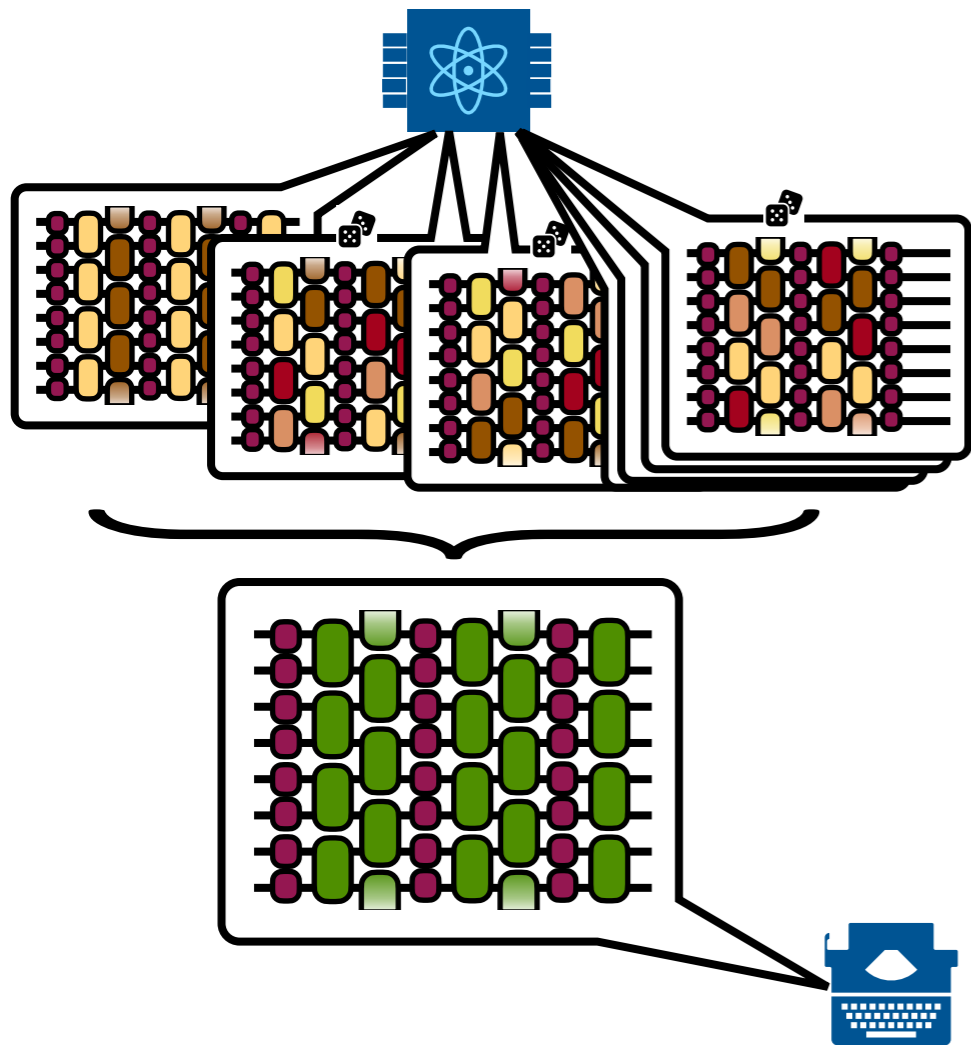
$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle\langle\psi_0|)]$$

**Classical simulation**

$O_{\text{test}}$

**Few-body expectation value**

# MAKE IT INFORMATIVE



$O_{\text{test}}$

Few-body expectation value

**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

**VS**

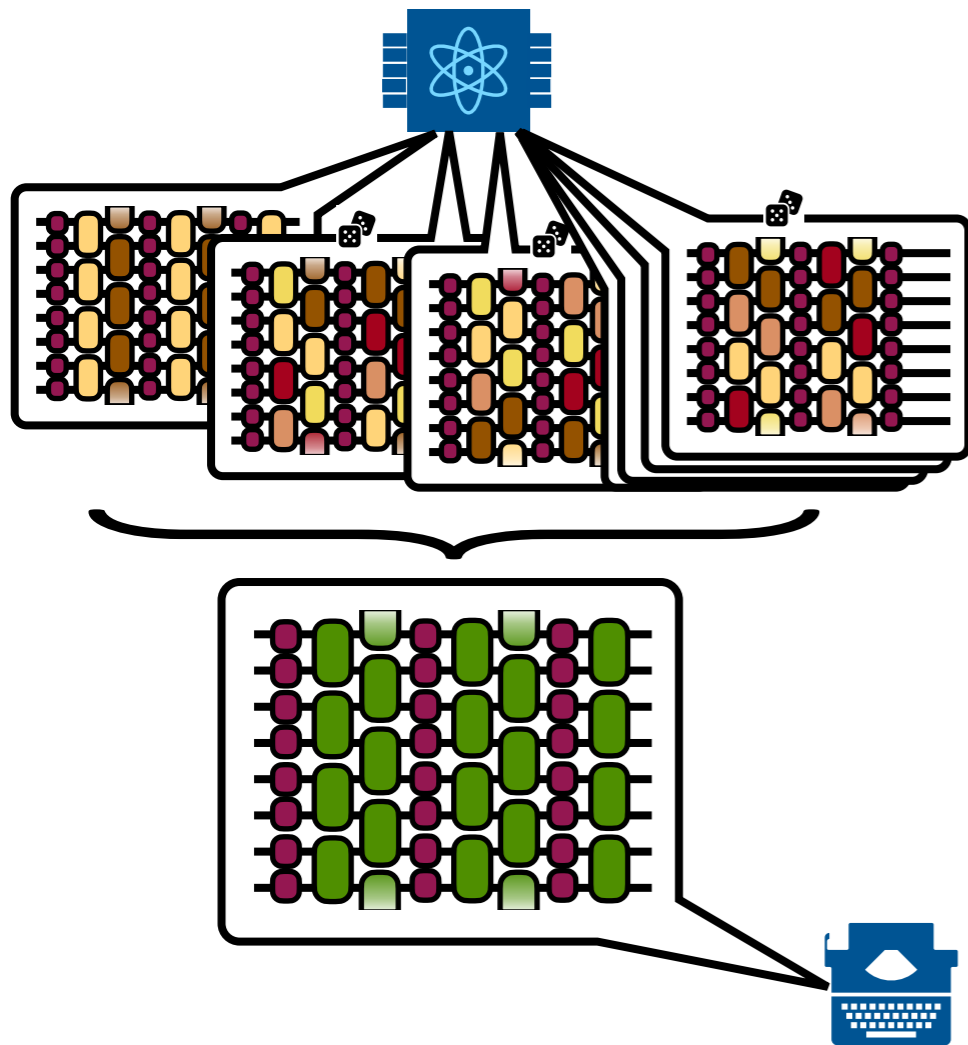
$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle\langle\psi_0|)]$$

**Classical simulation**

**Do it in a way that the average computation retains information about the original computation**

$$\mathcal{E} \rightarrow \mathcal{E}_U$$

# MAKE IT INFORMATIVE



$O_{\text{test}}$

Few-body expectation value

Quantum data

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

VS

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle\langle\psi_0|)]$$

Classical simulation

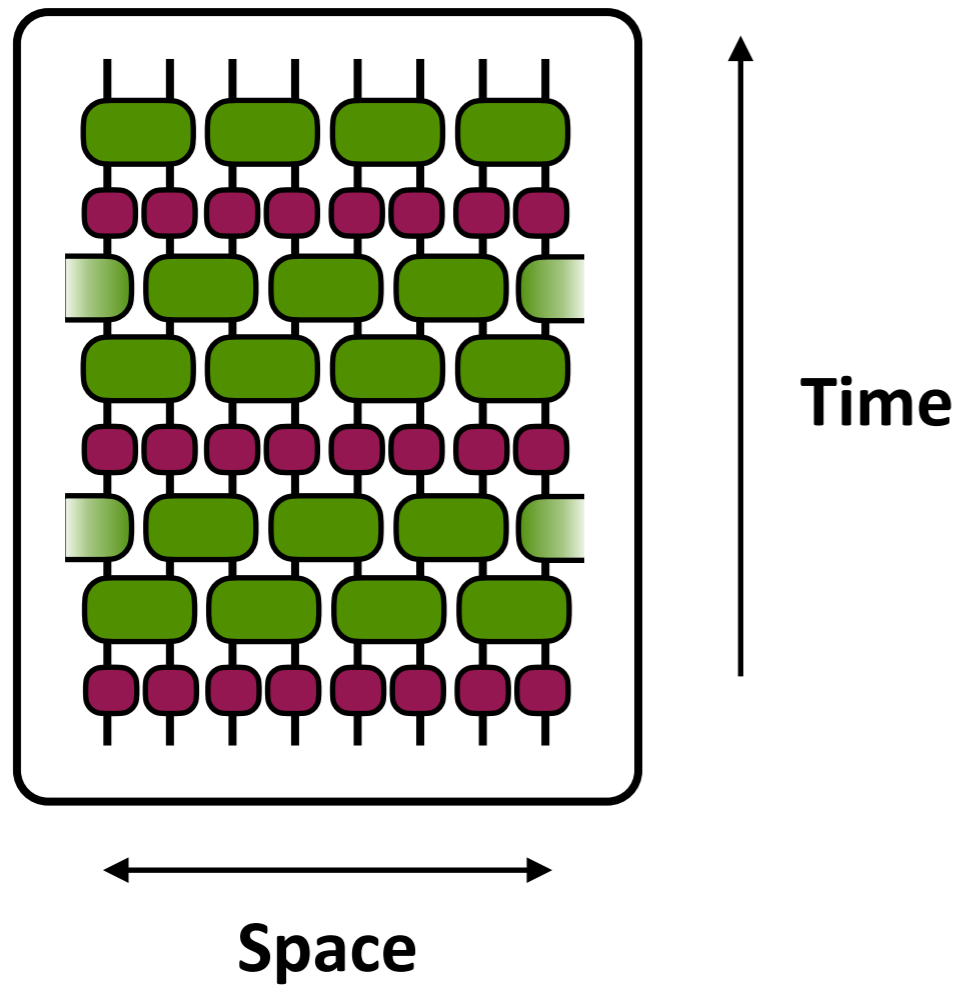
Do it in a way that the average computation retains information about the original computation

**Bad example:** Haar random unitaries

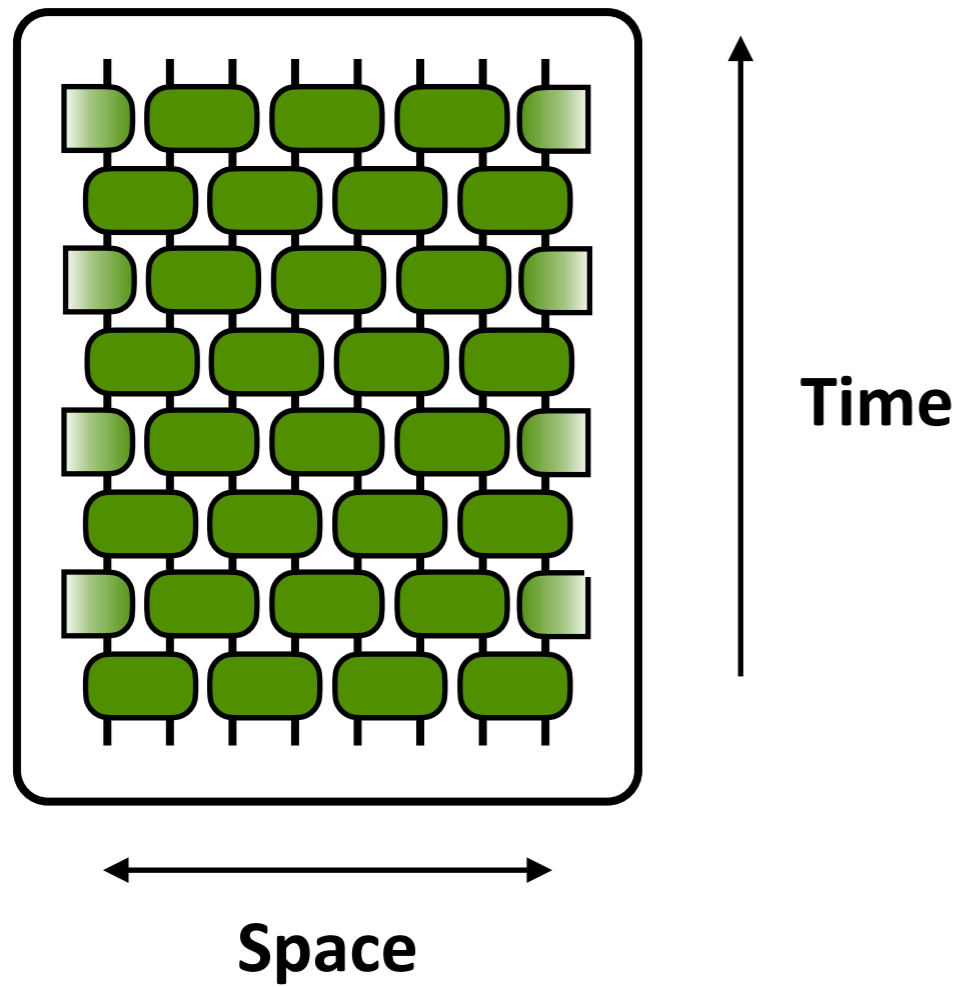
$$\mathcal{E} \rightarrow \mathcal{E}_U$$

**The method(s)**

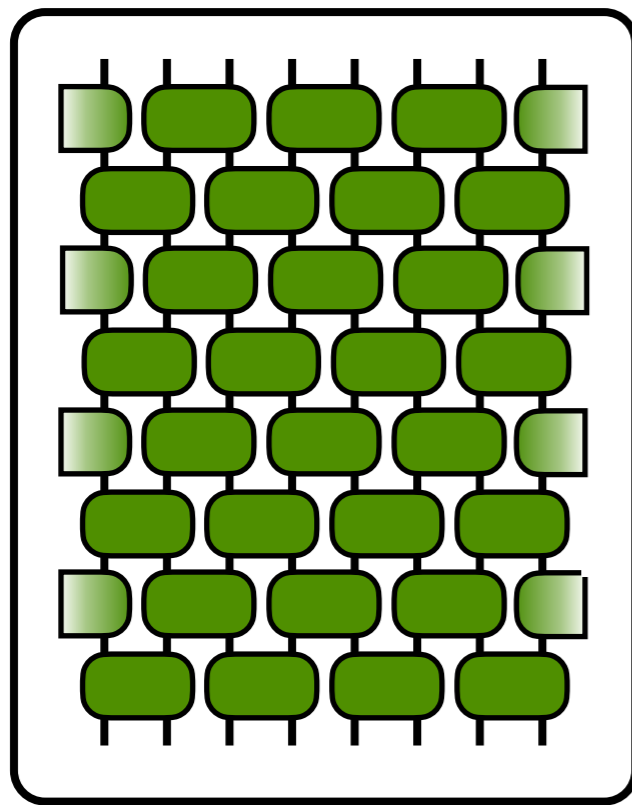
# SPACE-TIME CHANNELS



# SPACE-TIME CHANNELS

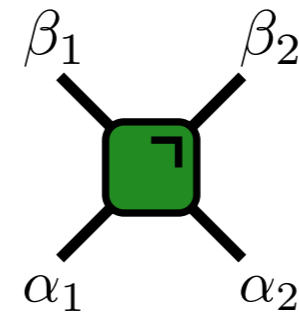
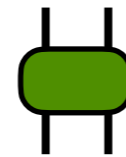


# SPACE-TIME CHANNELS



Space

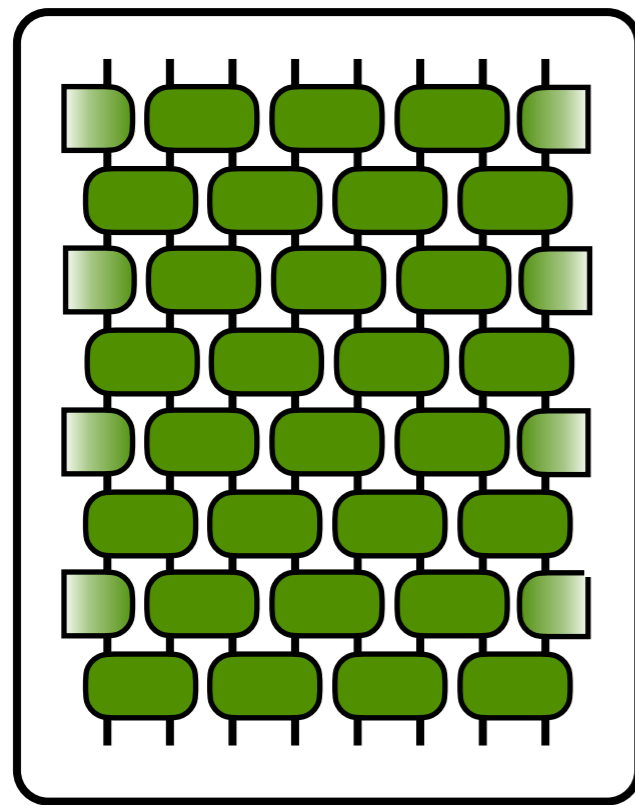
Time



$$\alpha_i, \beta_i = \mathbb{1}, x, y, z$$

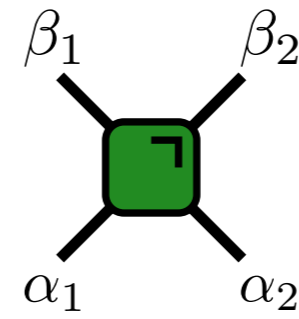
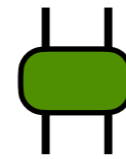
$$\text{tr}(\sigma_{\beta_1} \otimes \sigma_{\beta_2} \mathcal{E}(\sigma_{\alpha_1} \otimes \sigma_{\alpha_2}))$$

# SPACE-TIME CHANNELS



Space

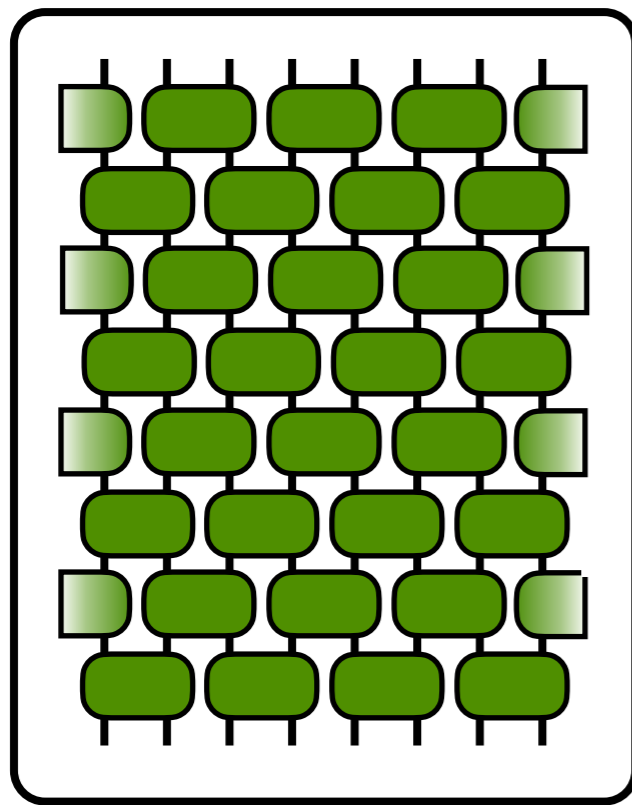
Time



$$\text{?} = \mathbb{1}$$

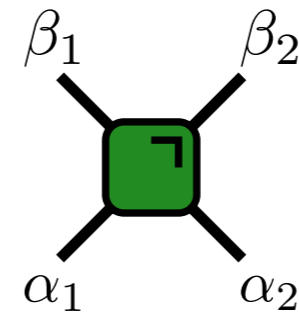
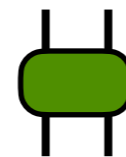
$$\text{tr}(\sigma_{\beta_1} \otimes \sigma_{\beta_2} \mathcal{E}(\sigma_{\alpha_1} \otimes \sigma_{\alpha_2}))$$

# SPACE-TIME CHANNELS



Space

Time

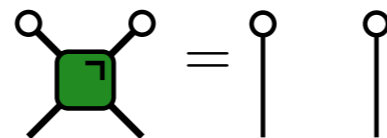


$$\text{⌋} = \mathbb{1}$$

$$\text{tr}(\sigma_{\beta_1} \otimes \sigma_{\beta_2} \mathcal{E}(\sigma_{\alpha_1} \otimes \sigma_{\alpha_2}))$$

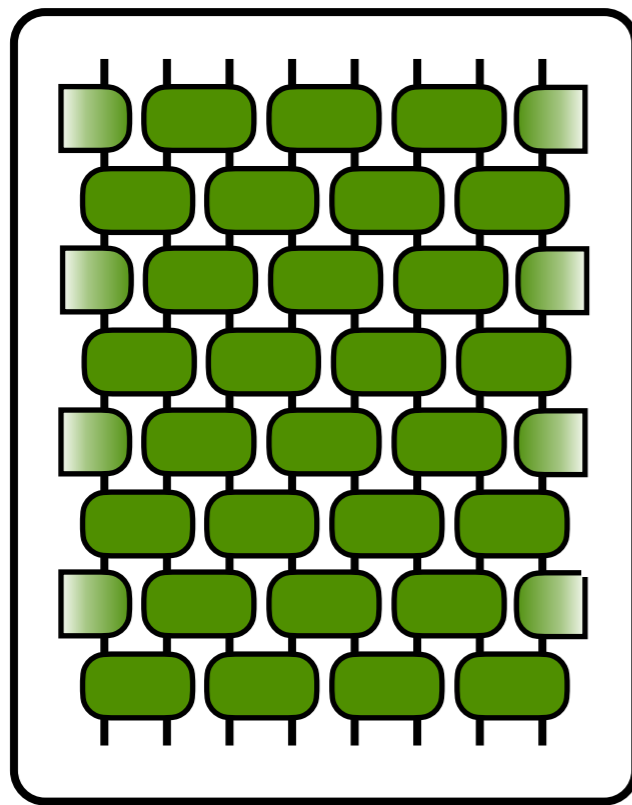
**Properties:**

- Trace preservation



$$\text{tr}(\mathcal{E}(\sigma_{\alpha_1} \otimes \sigma_{\alpha_2})) = \delta_{\alpha_1 \mathbb{1}} \delta_{\alpha_2 \mathbb{1}}$$

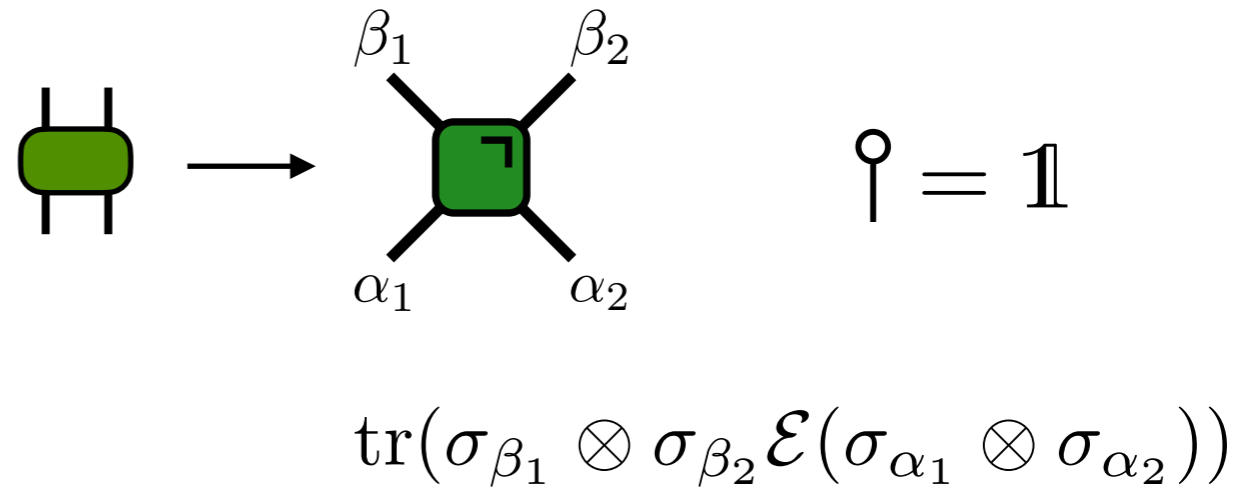
# SPACE-TIME CHANNELS



Space

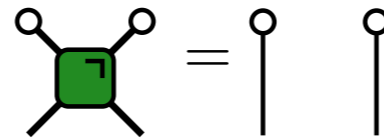
Space

Time

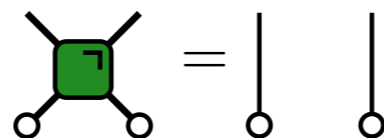


## Properties:

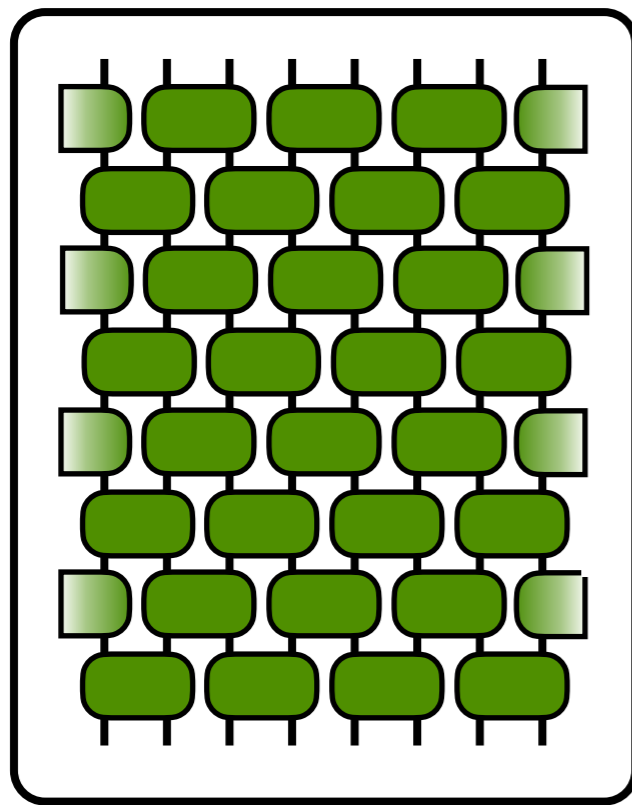
- Trace preservation



- Unitality

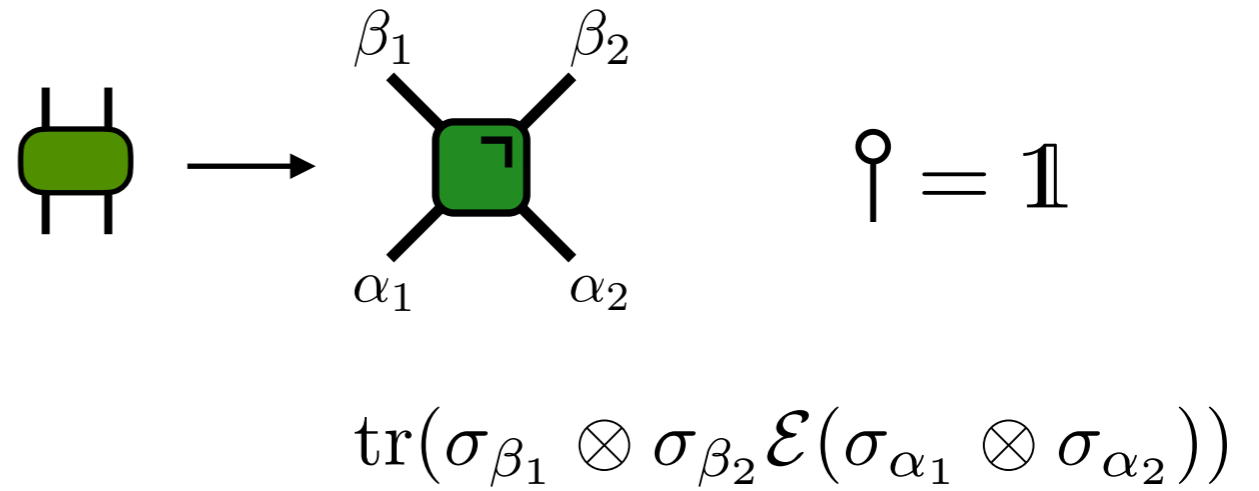


# SPACE-TIME CHANNELS



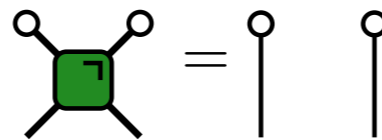
Space

Time

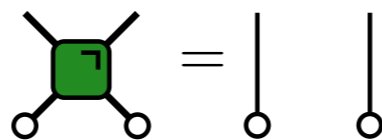


## Properties:

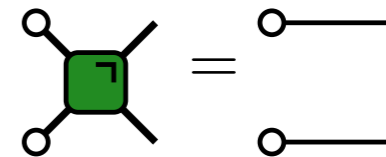
- Trace preservation



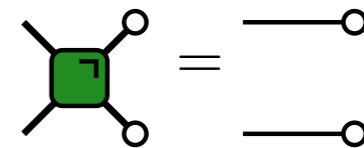
- Unitality



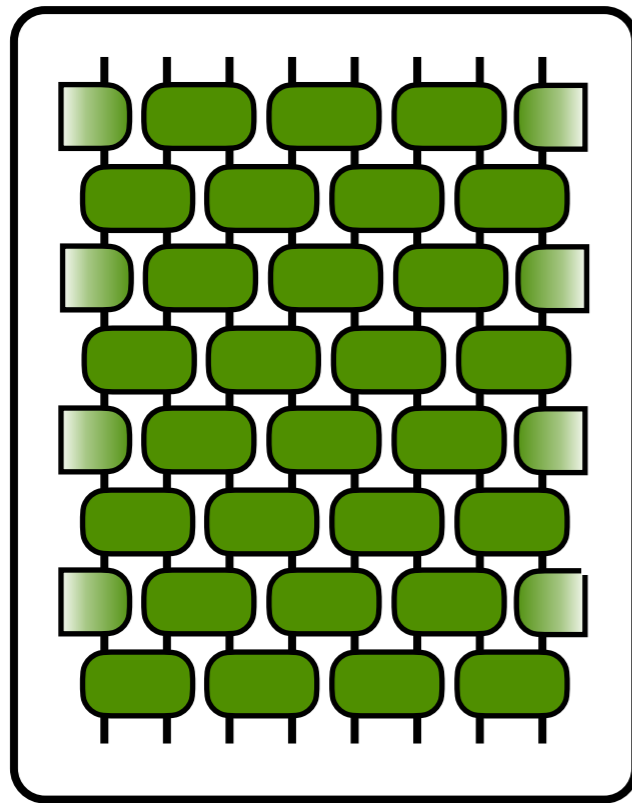
- Left unitality



- Right unitality

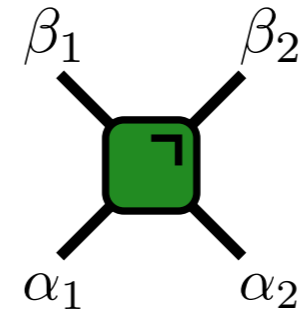
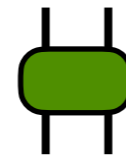


# SPACE-TIME CHANNELS



Time

Space

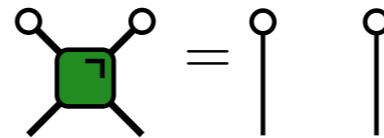


$$\text{⌋} = \mathbb{1}$$

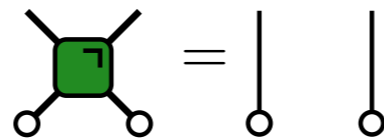
$$\text{tr}(\sigma_{\beta_1} \otimes \sigma_{\beta_2} \mathcal{E}(\sigma_{\alpha_1} \otimes \sigma_{\alpha_2}))$$

## Properties:

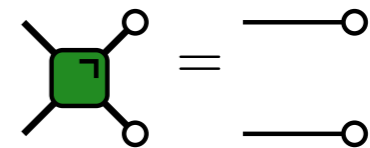
- Trace preservation



- Unitality

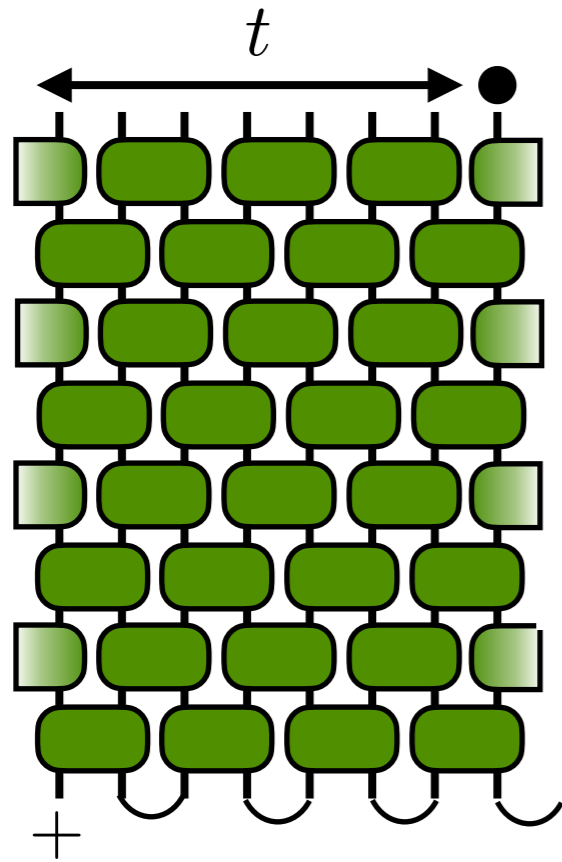


- Right unitality



3-way

# COMPUTING CORRELATIONS

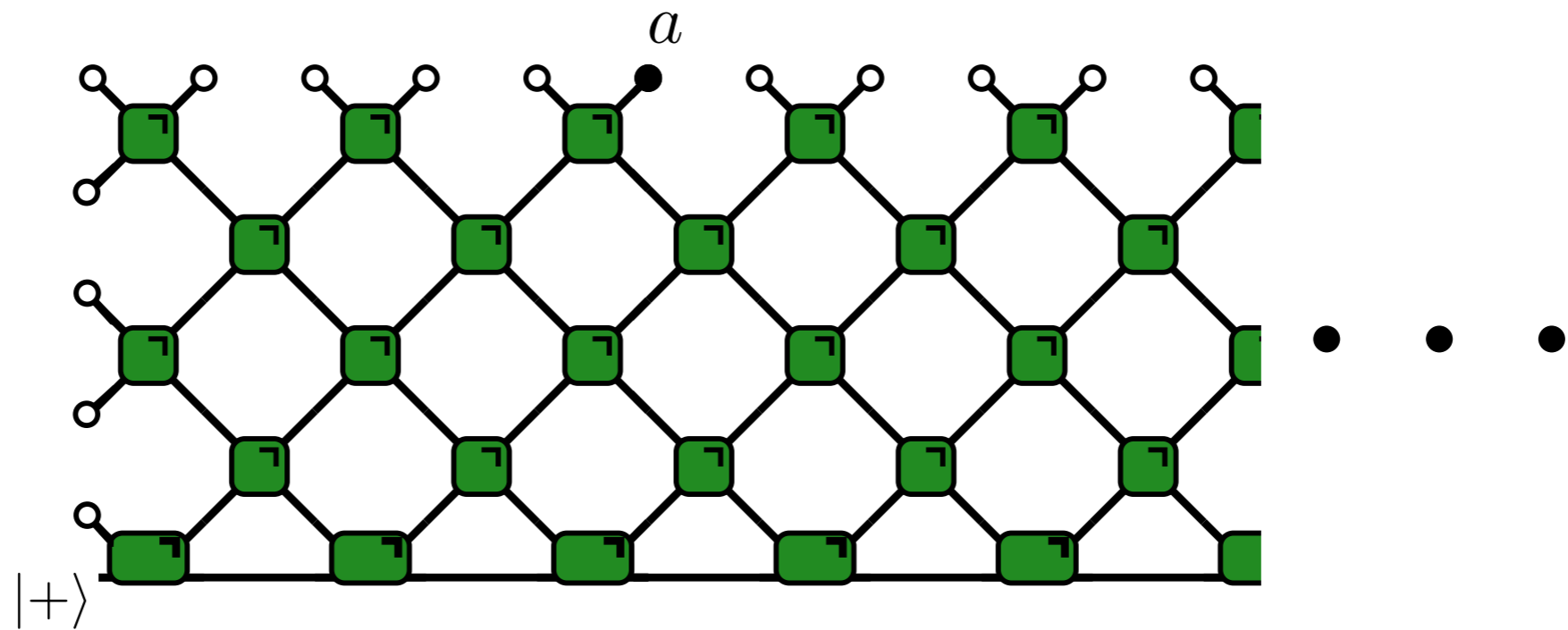


$$\text{tr}[\sigma_a^t \mathbb{E}_U [(U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

$$|\psi_{\text{in}}\rangle = |+_0\rangle |\Psi_{12}^+\rangle |\Psi_{34}^+\rangle |\Psi_{56}^+\rangle \dots$$

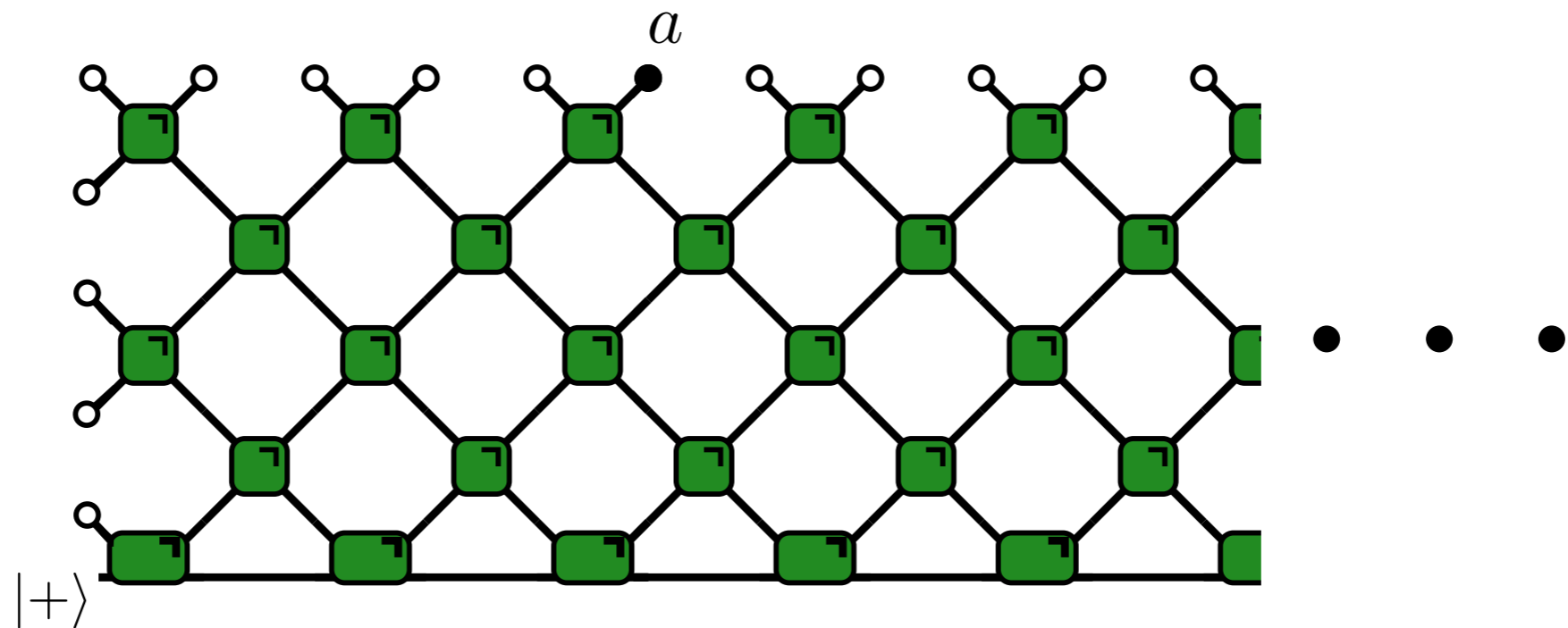
# COMPUTING CORRELATIONS

$$\text{tr}[\sigma_a^t \mathbb{E}_U[(U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

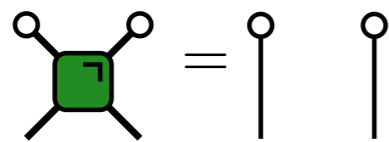


# COMPUTING CORRELATIONS

$$\text{tr}[\sigma_a^t \mathbb{E}_U[(U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

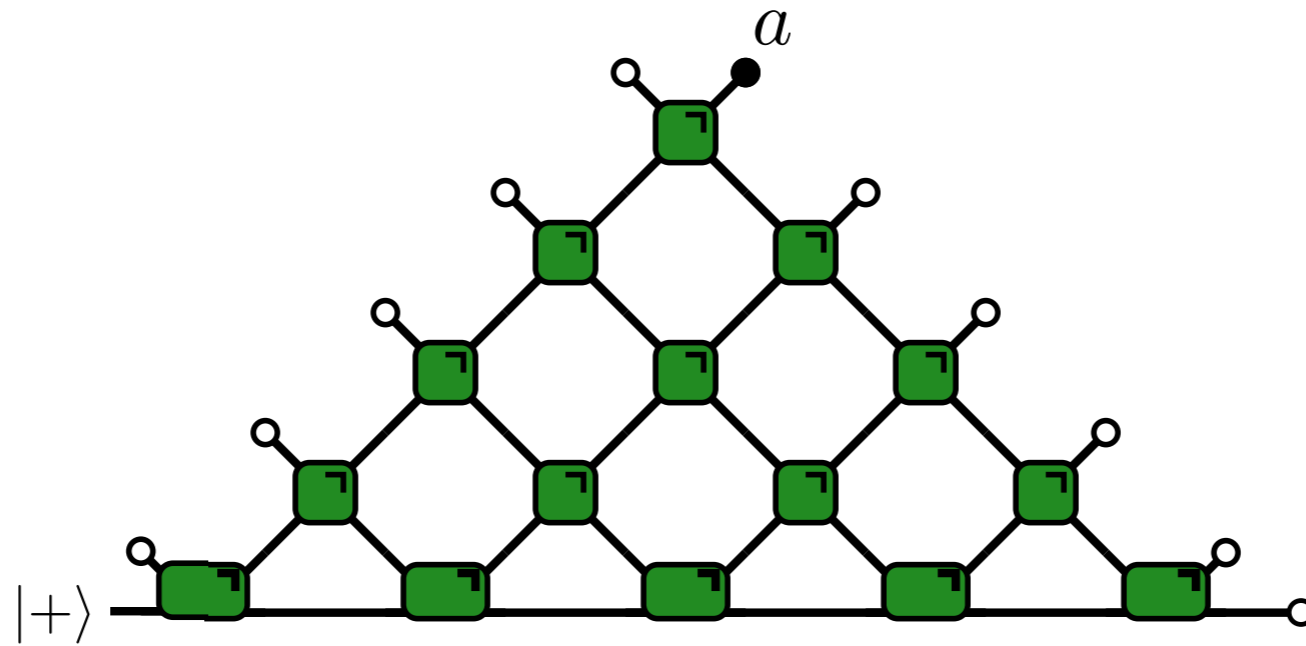


- Trace preservation



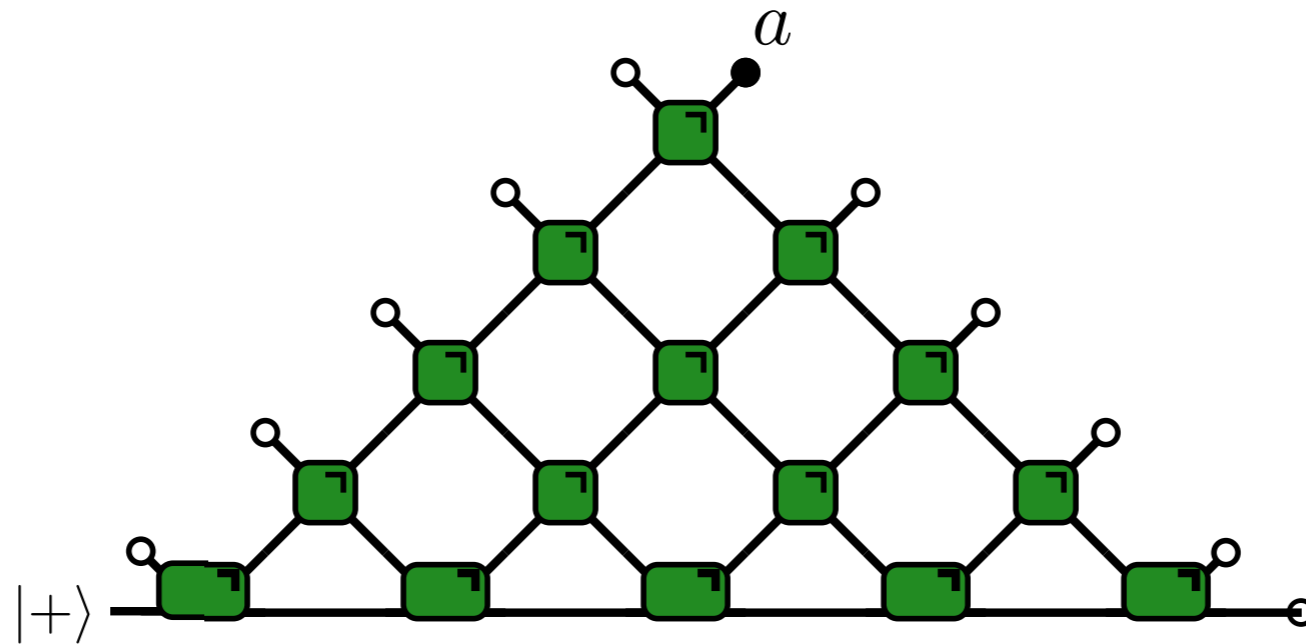
# COMPUTING CORRELATIONS

$$\text{tr}[\sigma_a^t \mathbb{E}_U [(U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

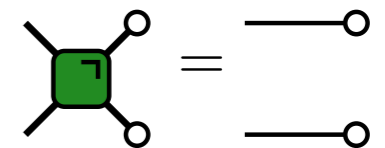


# COMPUTING CORRELATIONS

$$\text{tr}[\sigma_a^t \mathbb{E}_U [(U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$

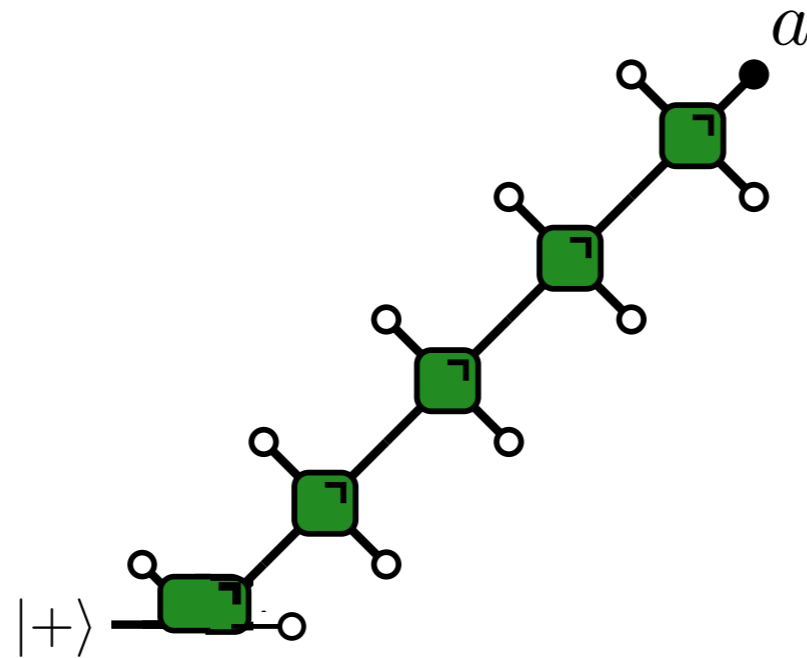


- Right unitality



# COMPUTING CORRELATIONS

$$\text{tr}[\sigma_a^t \mathbb{E}_U [(U^\dagger |\psi_0\rangle \langle \psi_0| U)]]$$



**Matrix multiplication**

$$\mathcal{M}_+ = \text{[Transfer Matrix Symbol]}$$

**Transfer matrices**

# AVERAGING TO GET SPACE-TIME CHANNELS

---

Define an averaging strategy such that the resulting channel is 3-way

$$\mathbb{E}[\text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots] = \text{green } \varepsilon \quad \text{3-way unital}$$

# AVERAGING TO GET SPACE-TIME CHANNELS

Define an averaging strategy such that the resulting channel is 3-way

$$\mathbb{E} [ \text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots ] = \text{green } \varepsilon \quad \text{3-way unital}$$

$$\frac{1}{4} \left[ \text{orange box} + \text{orange box with } X + \text{orange box with } Y + \text{orange box with } Z \right]$$

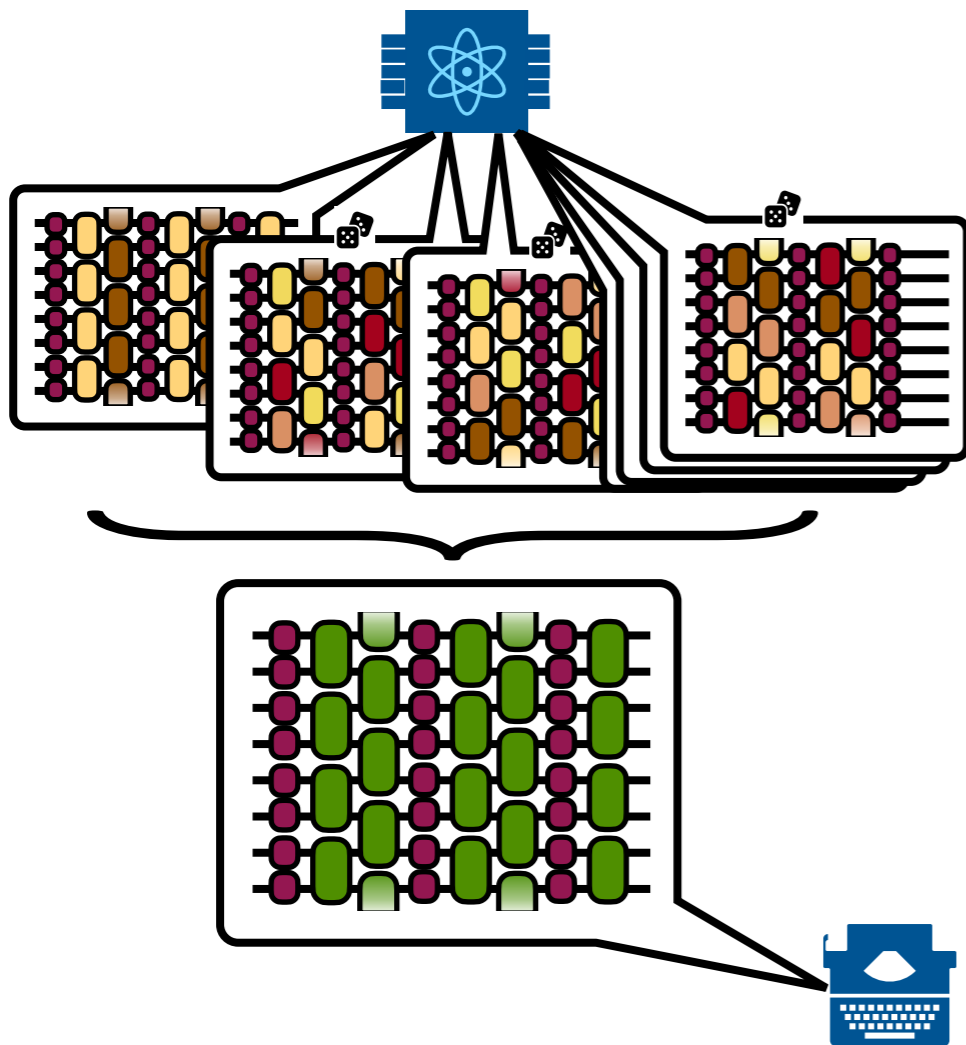
3-way for any unitary

# AVERAGING TO GET SPACE-TIME CHANNELS

Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E} [ \text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots ] = \text{green}$$

3-way unital



$$O_{\text{test}} = \sigma_a^t$$

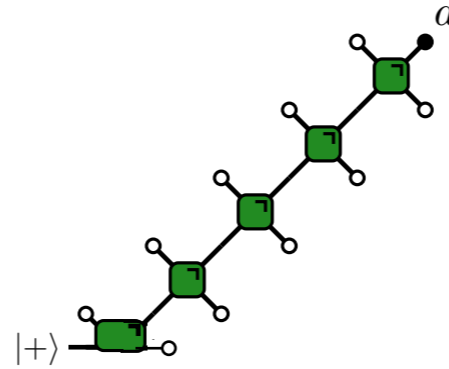
$$O_{\text{test}} = \sigma_a^i \sigma_b^{i+1} \sigma_c^{i+2}$$

# AVERAGING TO GET SPACE-TIME CHANNELS

---

1) The averaging procedure retains information about the original unitary

$$\mathcal{M}_+ = \text{[Diagram of a single green square node with four connections: two to white circles and two to black lines.]}$$

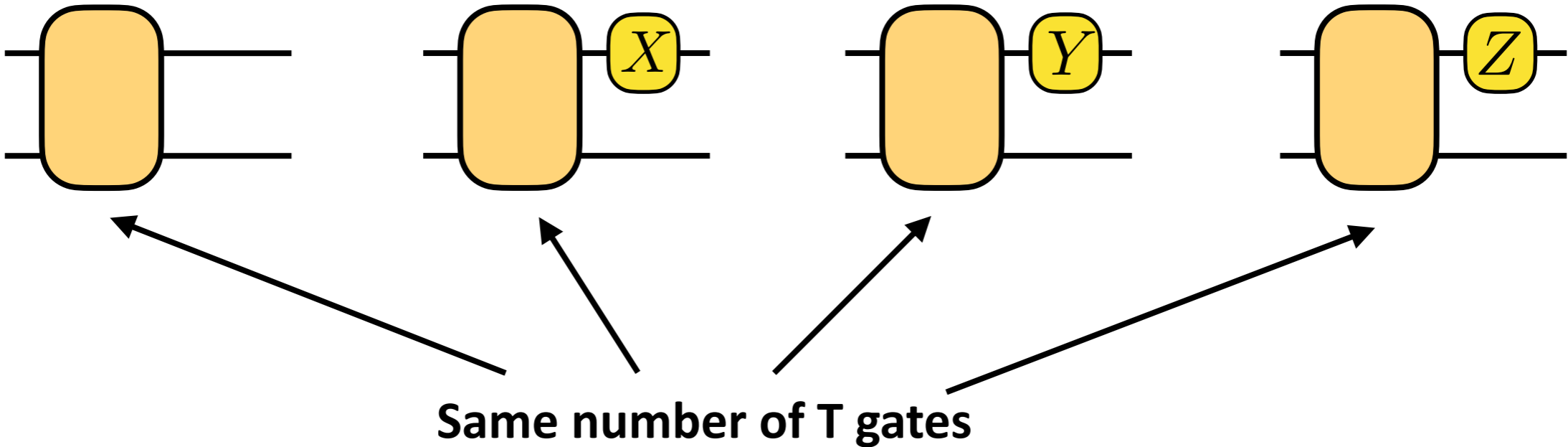


# AVERAGING TO GET SPACE-TIME CHANNELS

1) The averaging procedure retains information about the original unitary



2) Number of non-Clifford operations is preserved

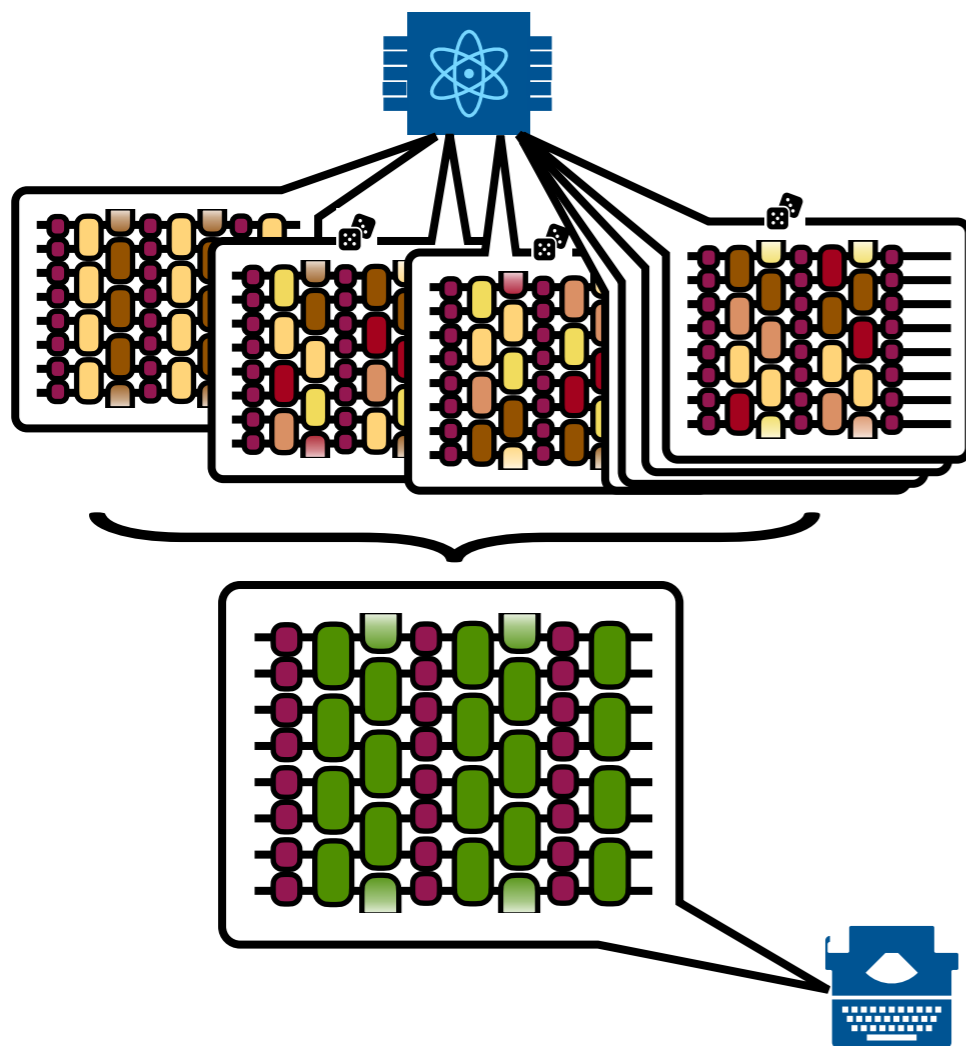


# Outlook

# OUTLOOK

Average-computation benchmarking strategy:

- involves running circuits of same layout/size/complexity



**Quantum data**

$$\mathbb{E}_U [\text{tr}[O_{\text{test}} (U^\dagger |\psi_0\rangle\langle\psi_0| U)]]$$

**VS**

$$\text{tr}[O_{\text{test}} \mathcal{E}(|\psi_0\rangle\langle\psi_0|)]$$

**Classical simulation**

# OUTLOOK

---

**Average-computation benchmarking strategy:**

- involves running circuits of same layout/size/complexity**

# OUTLOOK

---

**Average-computation benchmarking strategy:**

**- involves running circuits of same layout/size/complexity**

**Connection between benchmarking and classical simulation of open system dynamics.**

# OUTLOOK

---

**Average-computation benchmarking strategy:**

- involves running circuits of same layout/size/complexity

**Connection between benchmarking and classical simulation of open system dynamics.**

**Can we export it to other simulation methods?**

- Simulation of noisy circuits
- Pauli path
- ...

# THANKS

---



**Pavel Kos**



**Georgios  
Styliaris**

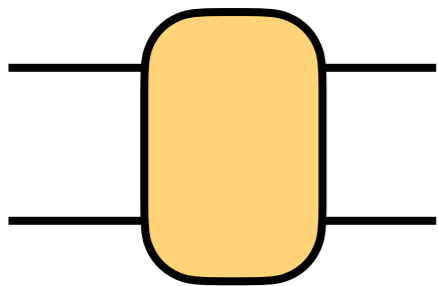
**Additional infos**

# AVERAGING TO GET SPACE-TIME CHANNELS

---

Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E}[\text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots] = \text{green } \mathcal{E} \quad \text{4-way unital}$$

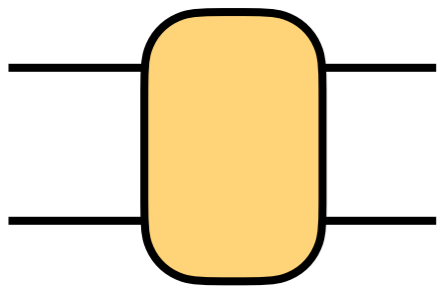


$$U_1 \otimes U_2 e^{i\theta_x \sigma_x \sigma_x} e^{i\theta_y \sigma_y \sigma_y} e^{i\theta_z \sigma_z \sigma_z} V_1 \otimes V_2$$

# AVERAGING TO GET SPACE-TIME CHANNELS

Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E}[\text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots] = \text{green } \varepsilon \quad \text{4-way unital}$$



$$U_1 \otimes U_2 e^{i\theta_x \sigma_x \sigma_x} e^{i\theta_y \sigma_y \sigma_y} e^{i\theta_z \sigma_z \sigma_z} V_1 \otimes V_2$$

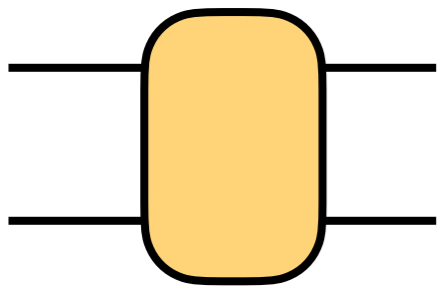
$$\theta_x^{(+)} = \frac{\pi}{4} + \delta_x$$

$$\theta_y^{(+)} = \frac{\pi}{4} + \delta_y$$

# AVERAGING TO GET SPACE-TIME CHANNELS

Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E}[\text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots] = \text{green } \varepsilon \quad \text{4-way unital}$$



$$U_1 \otimes U_2 e^{i\theta_x \sigma_x \sigma_x} e^{i\theta_y \sigma_y \sigma_y} e^{i\theta_z \sigma_z \sigma_z} V_1 \otimes V_2$$

$$\theta_x^{(+)} = \frac{\pi}{4} + \delta_x$$

$$\theta_x^{(-)} = \frac{\pi}{4} - \delta_x$$

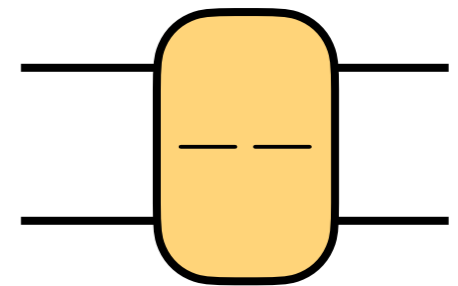
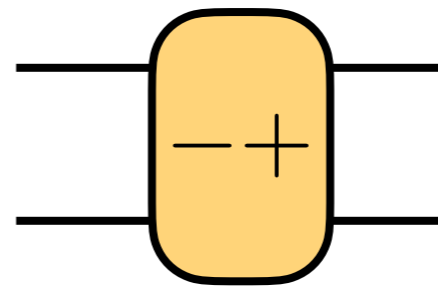
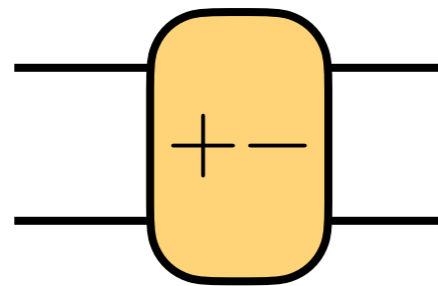
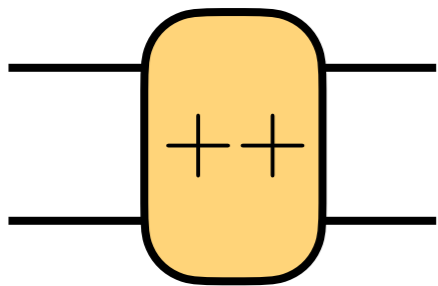
$$\theta_y^{(+)} = \frac{\pi}{4} + \delta_y$$

$$\theta_y^{(-)} = \frac{\pi}{4} - \delta_y$$

# AVERAGING TO GET SPACE-TIME CHANNELS

Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E}[\text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots] = \text{green } \varepsilon \quad \text{4-way unital}$$



$$\theta_x^{(+)} = \frac{\pi}{4} + \delta_x$$

$$\theta_x^{(-)} = \frac{\pi}{4} - \delta_x$$

$$\theta_y^{(+)} = \frac{\pi}{4} + \delta_y$$

$$\theta_y^{(-)} = \frac{\pi}{4} - \delta_y$$

# AVERAGING TO GET SPACE-TIME CHANNELS

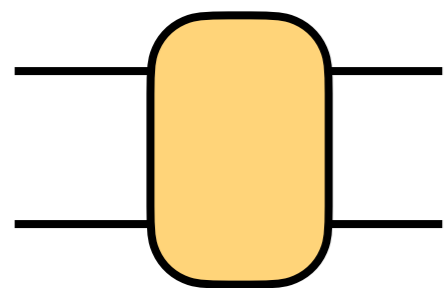
Define an averaging strategy such that the resulting channel is 4-way

$$\mathbb{E} [ \text{orange}, \text{brown}, \text{yellow}, \text{red}, \text{tan}, \dots ] = \text{green } \varepsilon \quad \text{4-way unital}$$

$$\frac{1}{4} \left[ \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{orange} \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{orange} \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{orange} \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{orange} \begin{array}{c} \text{---} \\ \text{---} \end{array} \right]$$

4-way for any unitary

# SAMPLE COMPLEXITY



$$= \text{CNOT}_{12}(u_n \otimes u_n) \text{CNOT}_{21}(u_n \otimes u_n) \text{CNOT}_{12}(u_n \otimes u_n)$$

