MAP meeting

11 Sep 2025

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NEW INSIGHTS INTO LAMBDA FRAGMENTATION FUNCTIONS

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In collaboration with
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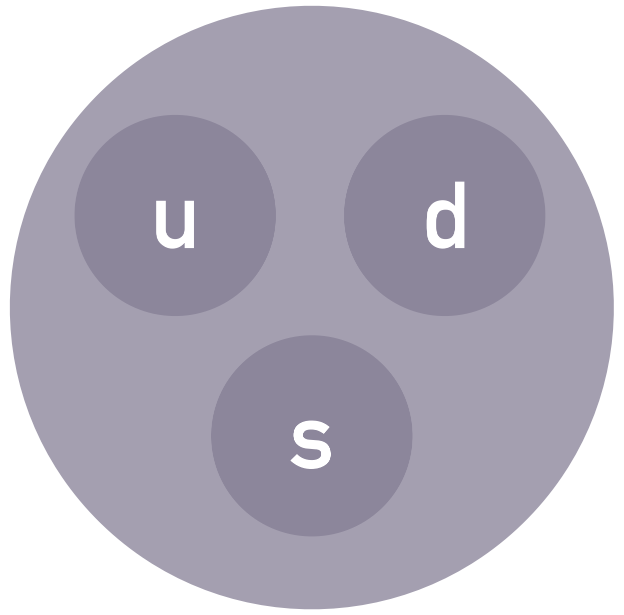
INTRODUCTION

Big amount of information collected in the last decades regarding hadron production through **fragmentation functions**.
We still need to improve our knowledge on fragmentation functions focusing on specific hadrons.

The Λ^0 **baryon** has a **simple structure:**

- u and d quarks form a spin-0 isospin singlet state
- the Λ^0 spin is carried almost entirely by the strange quark



State of the art of fits (for unpolarized Λ FF)

- D. De Florian, M. Stratmann, W. Vogelsang (1998) , SIA data from LEP, TASSO, CELLO, HRS, SLD
- S. Albino, B. A. Kniehl, G. Kramer (2006) SIA data with $\sqrt{s} = M_Z$
  (2008) SIA data with $\sqrt{s} \leq M_Z$, $pp(\bar{p})$ collisions (BRAHMS, PHENIX, STAR)
- F. A. Ceccopieri, D. Mancusi (2013), Λ in target fragmentation region
- J. Gao, C. Liu, M. Li, X. Shen, H. Xing, Y. Zhao, Y. Zhou (2025)

2025 $\longrightarrow$ **new global fit including SIA, SIDIS, pp collisions**

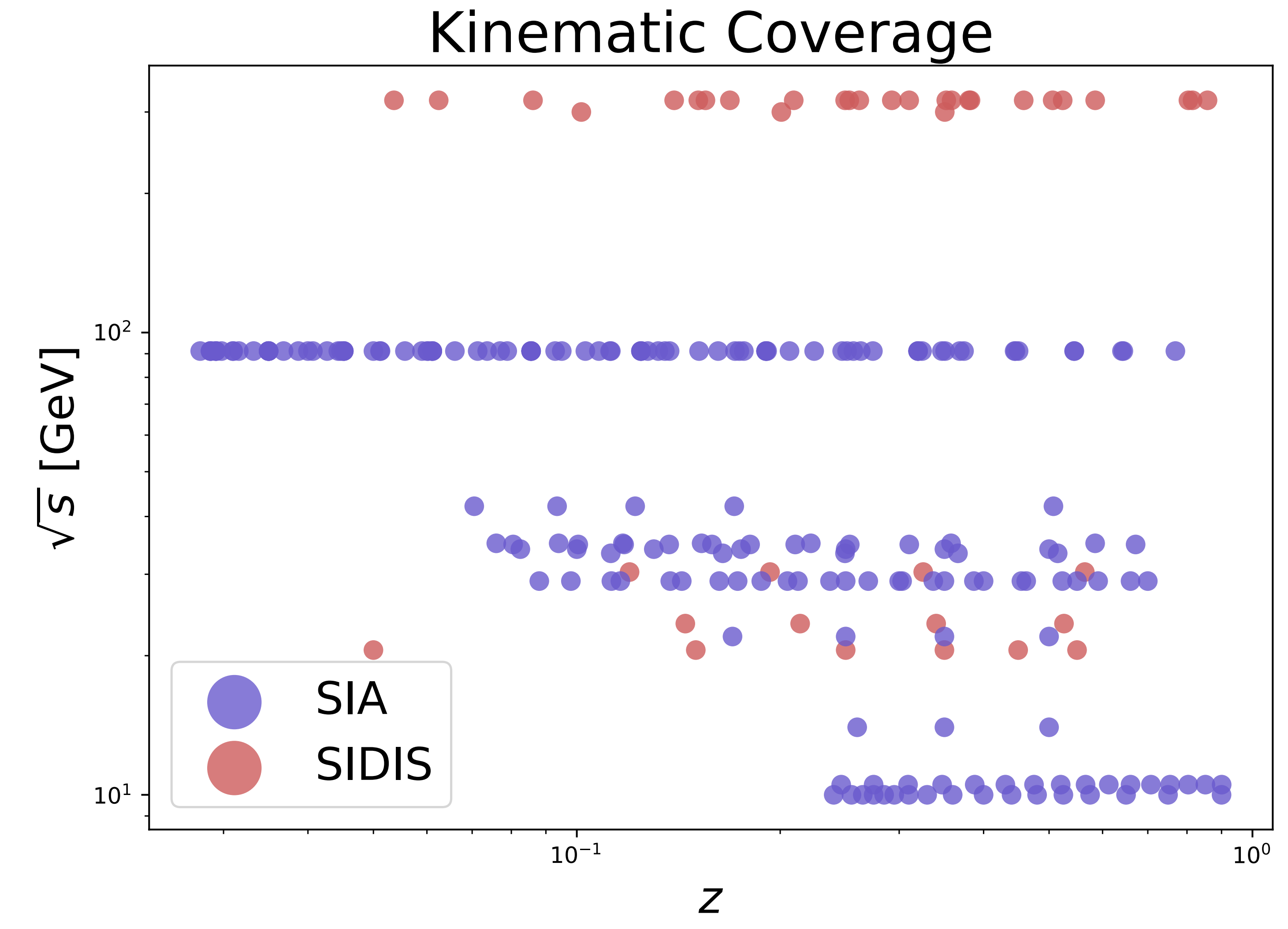
EXPERIMENTS

SIA

Experiment	Observable	$\sqrt{s}$ (GeV)
ARGUS	Multiplicity	10
BELLE	Cross section	10.52
TASSO @ 14,22,34	Cross section	14,22,34
TASSO @ 33.3	Cross section	33.3
TASSO @ 34.8,42.1	Cross section	34.8, 42.1
CELLO	Multiplicity	35
HRS	Cross section	29
MARK II	Cross section	29
ALEPH	Multiplicity	91.2
DELPHI @ 91 GeV	Multiplicity	91.2
OPAL	Multiplicity	91.2
SLD unt, uds-tag, c-tag, b-tag	Multiplicity	91.2
DELPHI @ 183,189	Multiplicity	183,189

NC SIDIS

Experiment	Observable	$\sqrt{s}$ (GeV)
Chicago-Harvard -Illinois-Oxford @ Fermilab	Multiplicity	20
EMC	Multiplicity	23.5
E665	Multiplicity	30
H1	Multiplicity	300
ZEUS	Multiplicity	318



EXPERIMENTS

CC SIDIS

Experiment	Observable	$< E_\nu >$ (GeV)
NOMAD	Cross section	45.3
ABCMO	Multiplicity	43
Chang et al. (Fermilab)	Multiplicity	>10
WA59	Multiplicity	44.2

pp collisions

Experiment	Observable	$\sqrt{s}$ (GeV)
STAR	Multiplicity	200
CDF	Multiplicity	630 & 1800
ALICE	Multiplicity	7000 & 13000

$$\frac{1}{N}\frac{d^2N_\Lambda}{dp_T dy} \quad \text{or} \quad \frac{1}{N}\frac{d^3N_\Lambda}{dp^3}$$

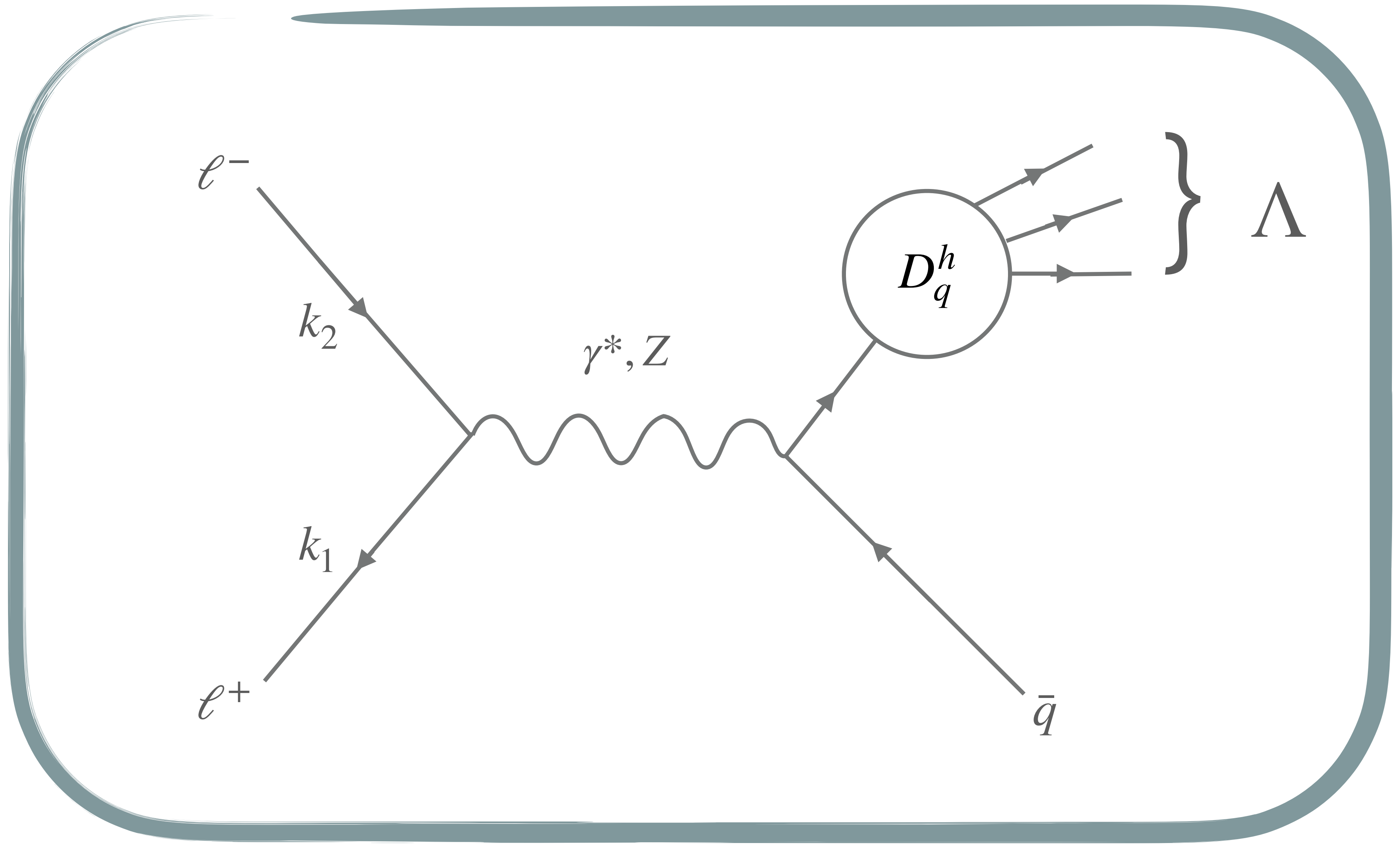
THEORETICAL BASIS

SIA kinematics

$$Q^2 = q^2$$

$$z = \frac{2P_h \cdot q}{Q^2}$$

$$\sqrt{s} = Q$$



SINGLE INCLUSIVE ANNIHILATION

$$l^+(k_1) + l^-(k_2) \to \Lambda(P_\Lambda) + X$$

Differential cross section

$$\frac{d\sigma^h}{dz}(z, Q) = \frac{4\pi\alpha^2}{Q^2} F_2^h(z, Q)$$

NNPDF Collaboration, V. Bertone et al., Eur.Phys.J.C 77 (2017) 8, 516

Fragmentation structure function

$$F_2^h(z, Q) = \frac{1}{n_f} \sum_q^{n_f} e_q^2(Q) \Big[C_{2,q}^S\left(z, \alpha_s(Q)\right) \otimes D_\Sigma^h\left(z, Q\right) + C_{2,q}^{NS}\left(z, \alpha_s(Q)\right) \otimes D_{NS}^h\left(z, Q\right) + C_{2,g}^S\left(z, \alpha_s(Q)\right) \otimes D_g^h\left(z, Q\right) \Big]$$

SIA HADRON MASS EFFECTS

SIA $\longrightarrow$ Treated as in S. Albino, B. A. Kniehl, and G. Kramer (2008) $z = z\,(x_p, m_h)$

$$z(p_h, m_h) = \frac{2}{\sqrt{s}}(p_h^2 + m_h^2)^{1/2}$$

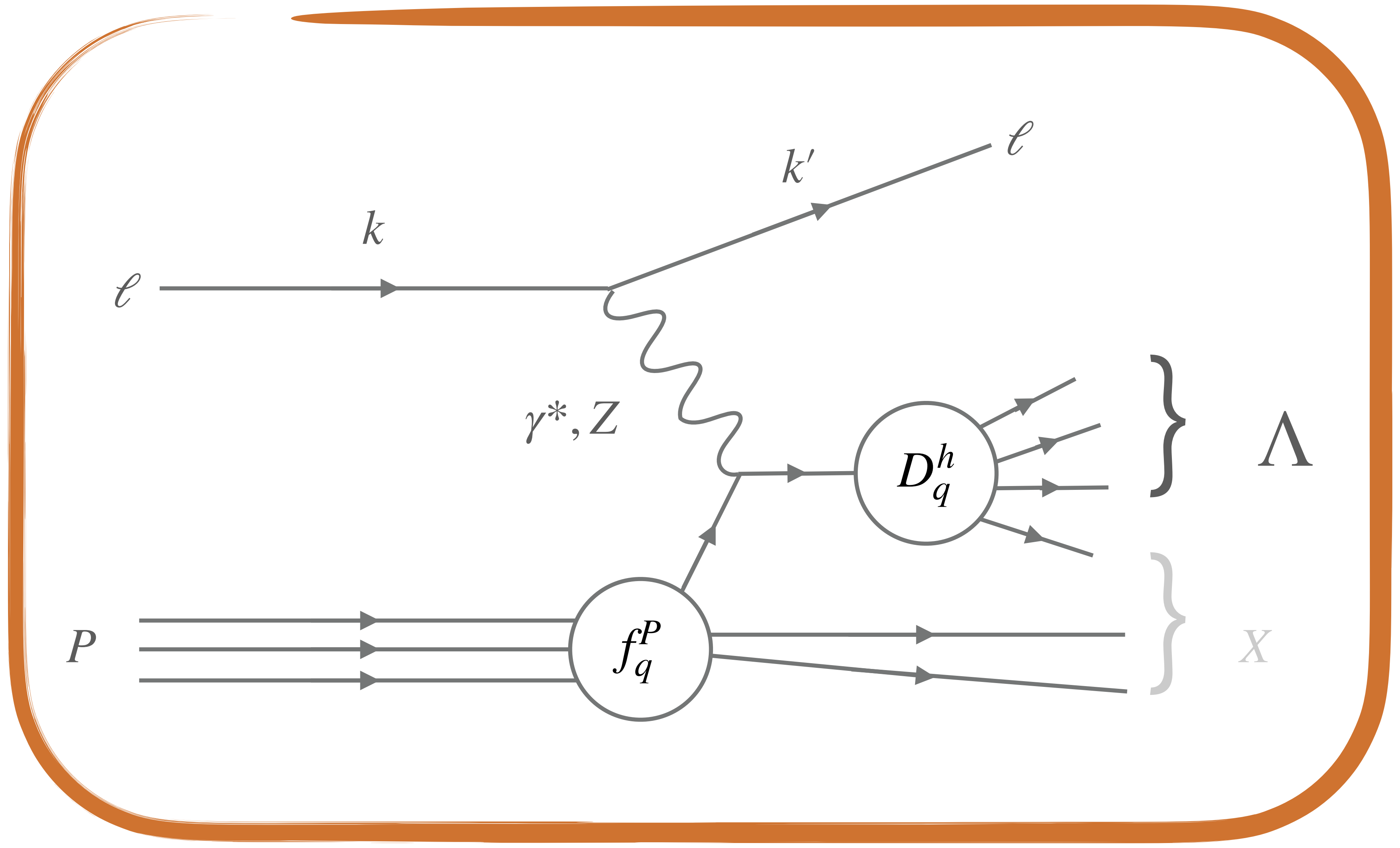
$$z = \frac{E_h}{E_b} = \frac{2E_h}{\sqrt{s}} \qquad z(x_p, m_h) = x_p\left(1 + \frac{4m_h^2}{x_p^2 s}\right)^{1/2} \qquad x_p = \frac{p_h}{p_b} = \frac{2p_h}{\sqrt{s}}$$

$$z(\xi, m_h) = e^{-\xi}\left(1 + 4e^{2\xi}\frac{m_h^2}{s}\right)^{1/2} \qquad \xi = \ln(1/x_p)$$

THEORETICAL BASIS

SIDIS Kinematics

$$Q^2 = -q^2$$
$$x = \frac{Q^2}{2p\cdot q}$$
$$z = \frac{P\cdot P_h}{P\cdot q}$$
$$y = \frac{Q^2}{xs}$$



SEMI INCLUSIVE DIS

$$\ell(k) + N(P) \to \ell(k') + \Lambda(P_\Lambda) + X$$

- **Differential cross section**

$$\frac{d^3\sigma}{dxdQdz} = \frac{4\pi\alpha^2}{xQ^3}[(1+(1-y)^2F_2(x,z,Q) - y^2F_L(x,z,Q)]$$

MAPFF1.0

R. A. Khalek et al.,
Phys.Rev.D 104 (2021) 3, 034007

- **Fragmentation structure function** within collinear factorisation ($Q \gg \Lambda_{QCD}$)

$$F_i(x,z,Q) = x\sum_{q\bar{q}} e_q^2 \Big[(C_{i,qq}(x,z,Q)\otimes f_q(x,Q) + C_{i,qg}(x,z,Q)\otimes f_g(x,Q))\otimes D_q^\Lambda(z,Q)$$
$$+ (C_{i,gq}(x,z,Q)\otimes f_q(x,Q))\otimes D_g^\Lambda(z,Q)\Big] \qquad i = 2,L$$

SIDIS HADRON MASS EFFECTS

SIDIS $\longrightarrow$ Most datasets provide $\dfrac{1}{\sigma_T}\dfrac{d\sigma^h}{dx_F}$ $\longrightarrow$

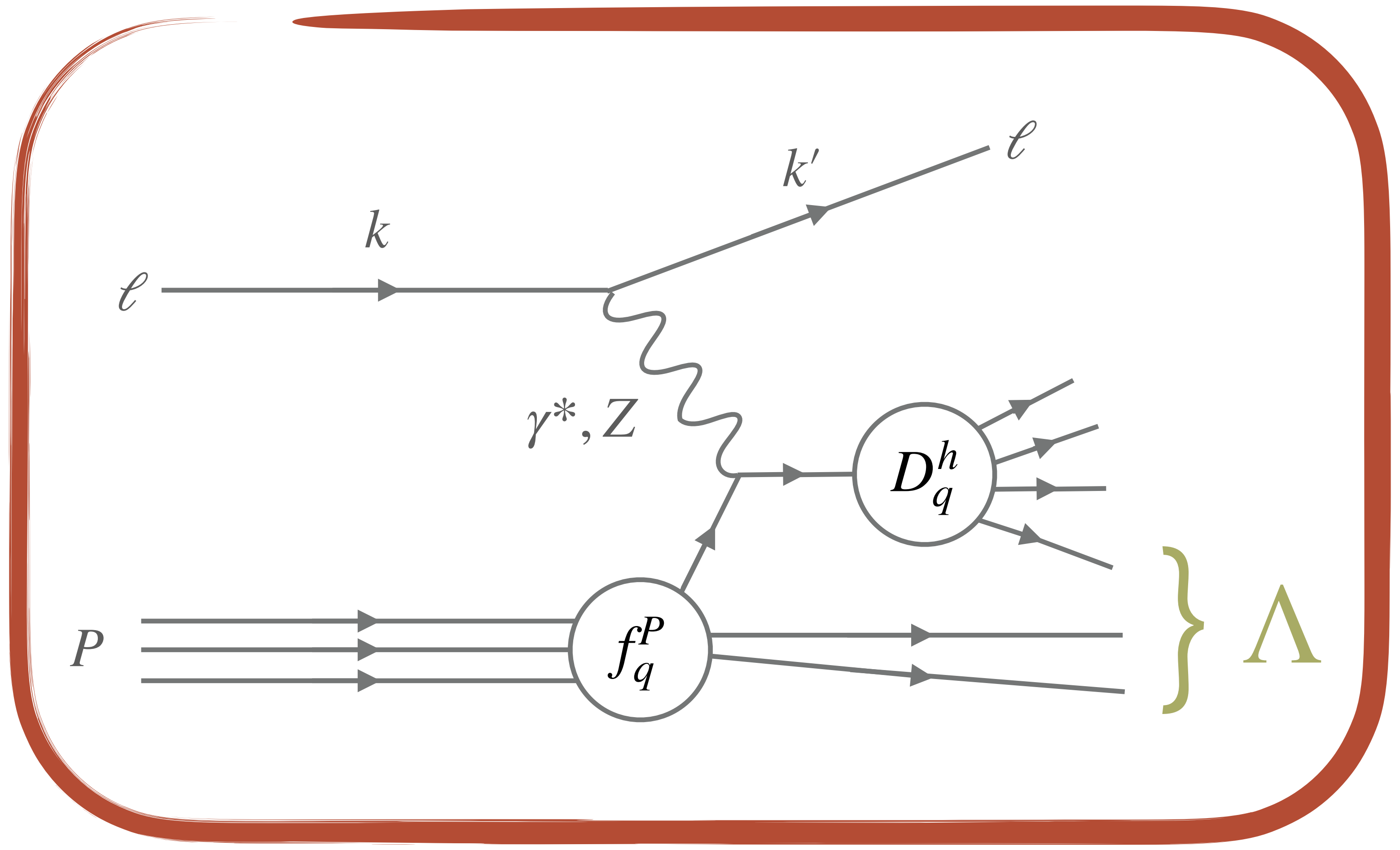
- $x_F < 0$ target fragmentation region
- $x_F > 0$ current fragmentation region

SIDIS HADRON MASS EFFECTS

Most datasets provide $\frac{1}{\sigma_T}\frac{d\sigma^h}{dx_F} \longrightarrow$

$x_F < 0$ target fragmentation region

$x_F > 0$ current fragmentation region



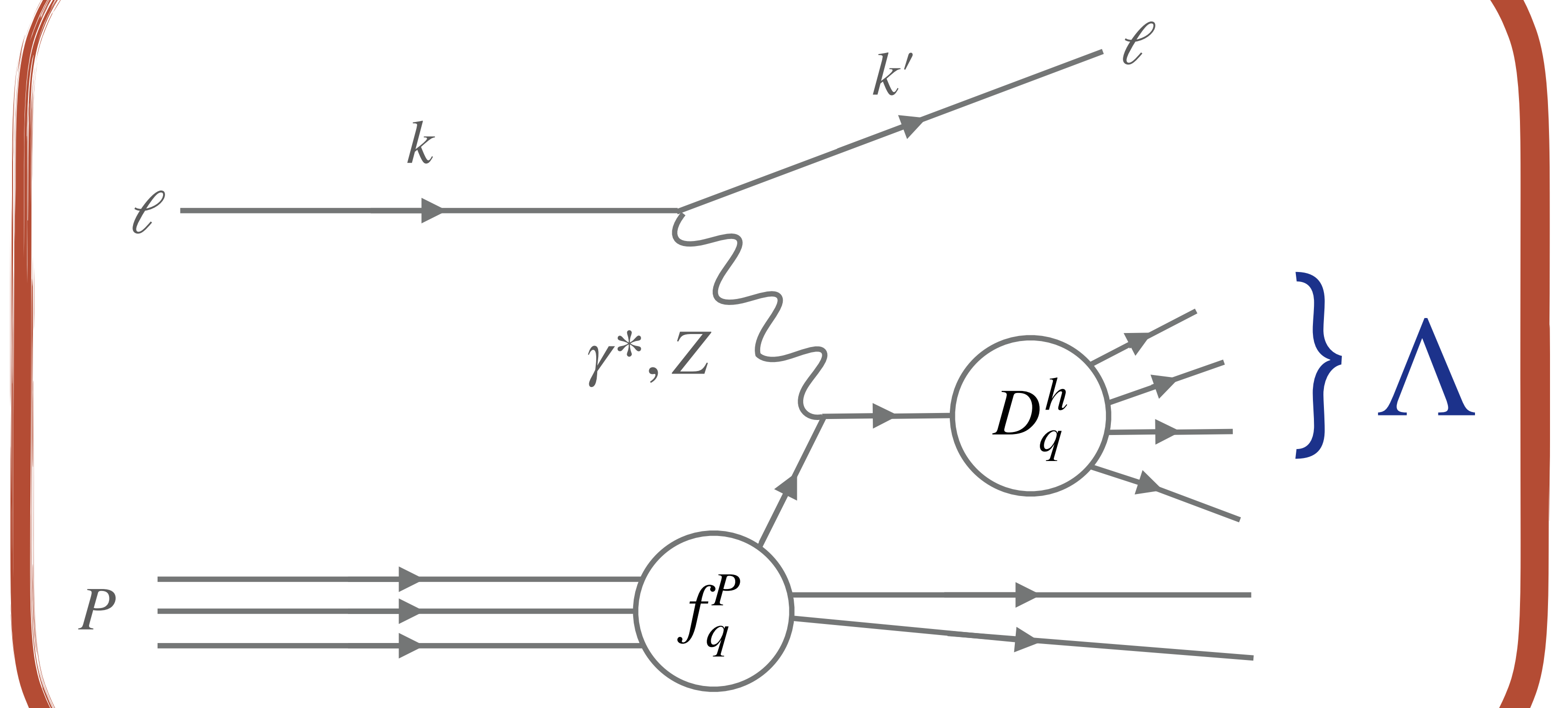
Handled with **fracture functions**
F. A. Ceccopieri, D. Mancusi (2013)

SIDIS HADRON MASS EFFECTS

Most datasets provide $\frac{1}{\sigma_T}\frac{d\sigma^h}{dx_F} \longrightarrow$

$x_F < 0$ target fragmentation region

$x_F > 0$ current fragmentation region



We restrict to $x_F > 0$ for proper SIDIS analysis

$$z = x_F \longrightarrow z = \frac{x_F}{2}\left(1 + \left(1 + \frac{4M_\Lambda^2}{x_F^2 W^2}\right)^{1/2}\right)$$

Integrated multiplicity

$$\frac{dM}{dz} = \left[\int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \int_{z_{min}}^{z_{max}} dz \frac{d^3\sigma}{dxdQdz}\right] \Big/ \left[\Delta z \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \frac{d^2\sigma}{dxdQ}\right]$$

MAPFF1.0

R. A. Khalek et al.,
Phys.Rev.D 104 (2021) 3, 034007

SIDIS HADRON MASS EFFECTS

Integrated multiplicity

$$\frac{dM}{dz} = \left[\int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \int_{z_{min}}^{z_{max}} dz \frac{d^3\sigma}{dx dQ dz} \right] \Bigg/ \left[\Delta z \int_{Q_{min}}^{Q_{max}} dQ \int_{x_{min}}^{x_{max}} dx \frac{d^2\sigma}{dx dQ} \right]$$

MAPFF1.0

R. A. Khalek et al., Phys.Rev.D 104 (2021) 3, 034007

Bin by bin integration

Experiment	Q^2 range	x range
CHIO @ Fermilab	$4 < Q^2 < 50$	$0.001 < x < 1$
EMC	$Q^2 > 4$	$0.02 < x < 1$
E665	$Q^2 > 1$	$0.003 < x < 1$
H1	$10 < Q^2 < 70$	$0.0001 < x < 0.01$
ZEUS	$10 < Q^2 < 10240$	$0.001 < x < 0.75$

Whole range of integration in x and Q
(as specified by the collaboration)

Phase space reduction

$$Q_{max} < \sqrt{s}$$

$$x_{min} = \max \left[x_{min}^{collab}, \frac{Q^2}{s y_{max}} \right]$$

$$x_{max} = \min \left[x_{max}^{collab}, \frac{Q^2}{s y_{min}}, \frac{Q^2}{Q^2 + W^2} \right]$$

METHODOLOGY

MontBlanc

`MontBlanc` is a code devoted to the extraction of collinear distributions. So far, it has been used to determine the fragmentation functions (FFs) of the pion from experimental data for single-inclusive annihilation and semi-inclusive deep-inelastic scattering. Details concerning this fit of FFs in particular and the methodology in general can be found in the reference below.

Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

METHODOLOGY

- Statistical framework relies on Monte Carlo sampling method

Data is assumed to be sampled from a multivariate Gaussian distribution:

$$\mathscr{G}\left(\mathbf{x}^{(k)}\right) \propto \exp\left[\left(\mathbf{x}^{(k)} - \mu\right)^T C^{-1} \left(\mathbf{x}^{(k)} - \mu\right)\right]$$

With $\mathbf{x}^{(k)} = \{x_1^{(k)}, x_2^{(k)}, \ldots, x_{N_{dat}}^{(k)}\}$, $k = 1,\ldots, N_{rep}$ replicas of a set of N_{dat} values

$\mu = \{\mu_1, \mu_2, \ldots, \mu_{N_{dat}}\}$ expectation values = measured data

Generation of replicas using the Cholesky decomposition $\mathbf{L}, (\mathbf{C} = \mathbf{L} \cdot \mathbf{L}^T)$

$$C_{ij} = \delta_{ij}\sigma_{i,\mathrm{unc}}^2 + \sum_\beta \sigma_{i,\mathrm{corr}}^{(\beta)}\sigma_{j,\mathrm{corr}}^{(\beta)}$$ Elements of the covariance matrix

$$\mathbf{x}^{(k)} = \mu + L \cdot \mathbf{r}^{(k)}$$

With $\mathbf{r}^{(k)}$ such that $\frac{1}{N_{\mathrm{rep}}}\sum_k^{N_{\mathrm{rep}}} x_i^{(k)} \simeq \mu_i, \quad \frac{1}{N_{\mathrm{rep}}}\sum_k^{N_{\mathrm{rep}}} x_i^{(k)}x_j^{(k)} \simeq \mu_i\mu_j + C_{ij}$

METHODOLOGY

PASSED | codefactor A | CodeQL passing | DOI 10.5281/zenodo.10522420

APFEL++

A PDF evolution library in C++

- Computation of SIA and SIDIS cross sections

$$\frac{d\sigma^h}{dz}(z,Q) = \frac{4\pi\alpha^2}{Q^2} F_2^h(z,Q)$$

$$\frac{d^3\sigma}{dxdQdz} = \frac{4\pi\alpha^2}{xQ^3}[(1+(1-y)^2 F_2(x,z,Q^2) - y^2 F_L(x,z,Q^2)]$$

- Perturbative expansion of the coefficient functions (NLO and NNLO)

$$C(x,z,Q) = \sum_n \left(\frac{\alpha_s(Q)}{4\pi}\right)^n C^{(n)}(x,z)$$

- DGLAP Evolution of FFs
- PDF for SIDIS

METHODOLOGY

- NNAD NN parameterisation and analytical derivatives

Parameterisation choice:
single one-layered feed-forward Neural Network

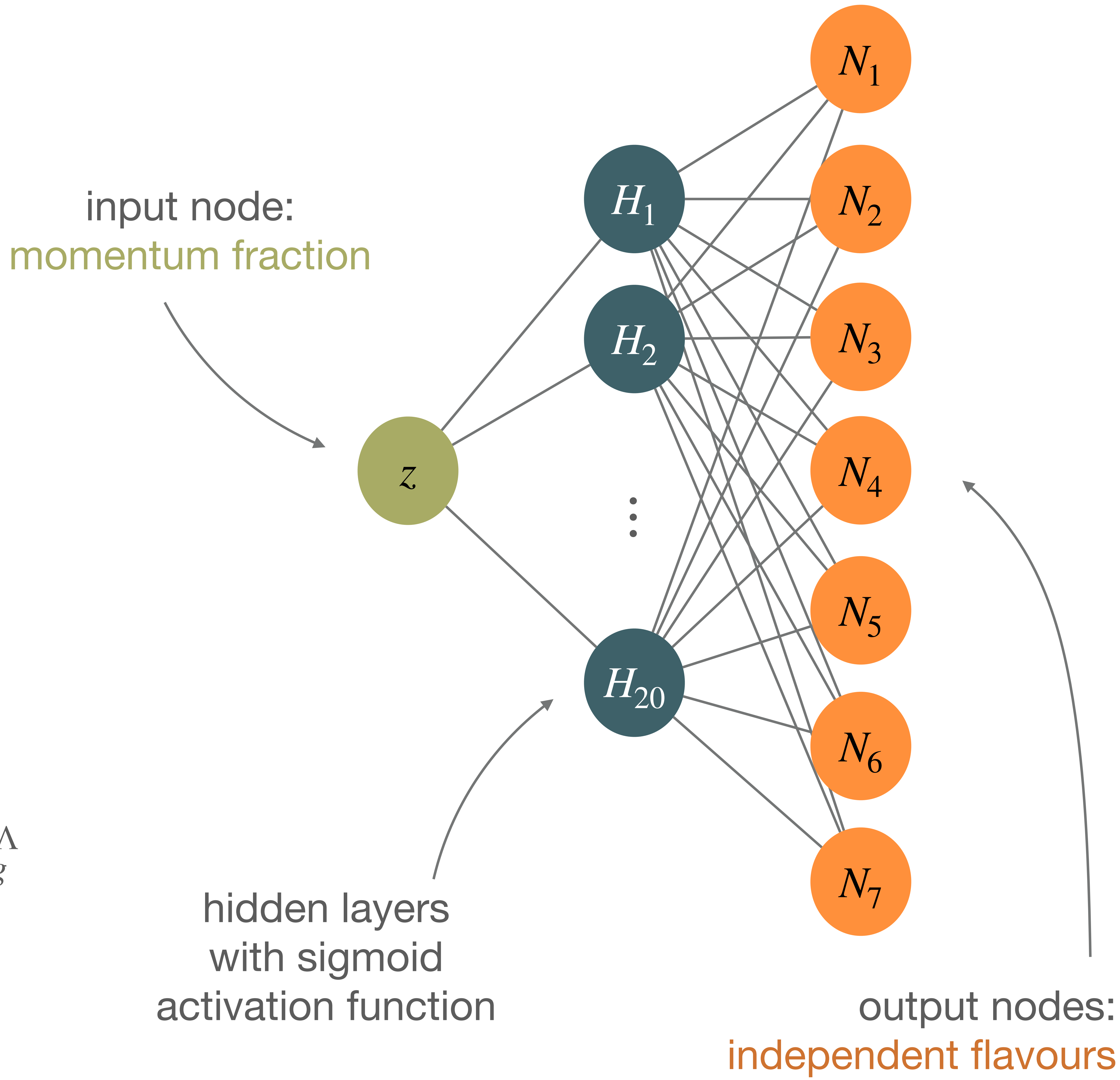
$$\mathcal{N}_i(z, \theta)$$

$$z\, D_i^{\Lambda}(z, \mu_0 = 5 \text{ GeV}) = \left(\mathcal{N}_i(z,\theta) - \mathcal{N}_i(1,\theta)\right)^2$$

7 independent flavours

$$D_u^{\Lambda}, \quad D_d^{\Lambda}, \quad D_s^{\Lambda}, \quad D_{\bar{u}}^{\Lambda} = D_{\bar{d}}^{\Lambda} = D_{\bar{s}}^{\Lambda}, \quad D_{c^+}^{\Lambda}, \quad D_{b^+}^{\Lambda}, \quad D_g^{\Lambda}$$

$$f^+ = f + \bar{f}$$



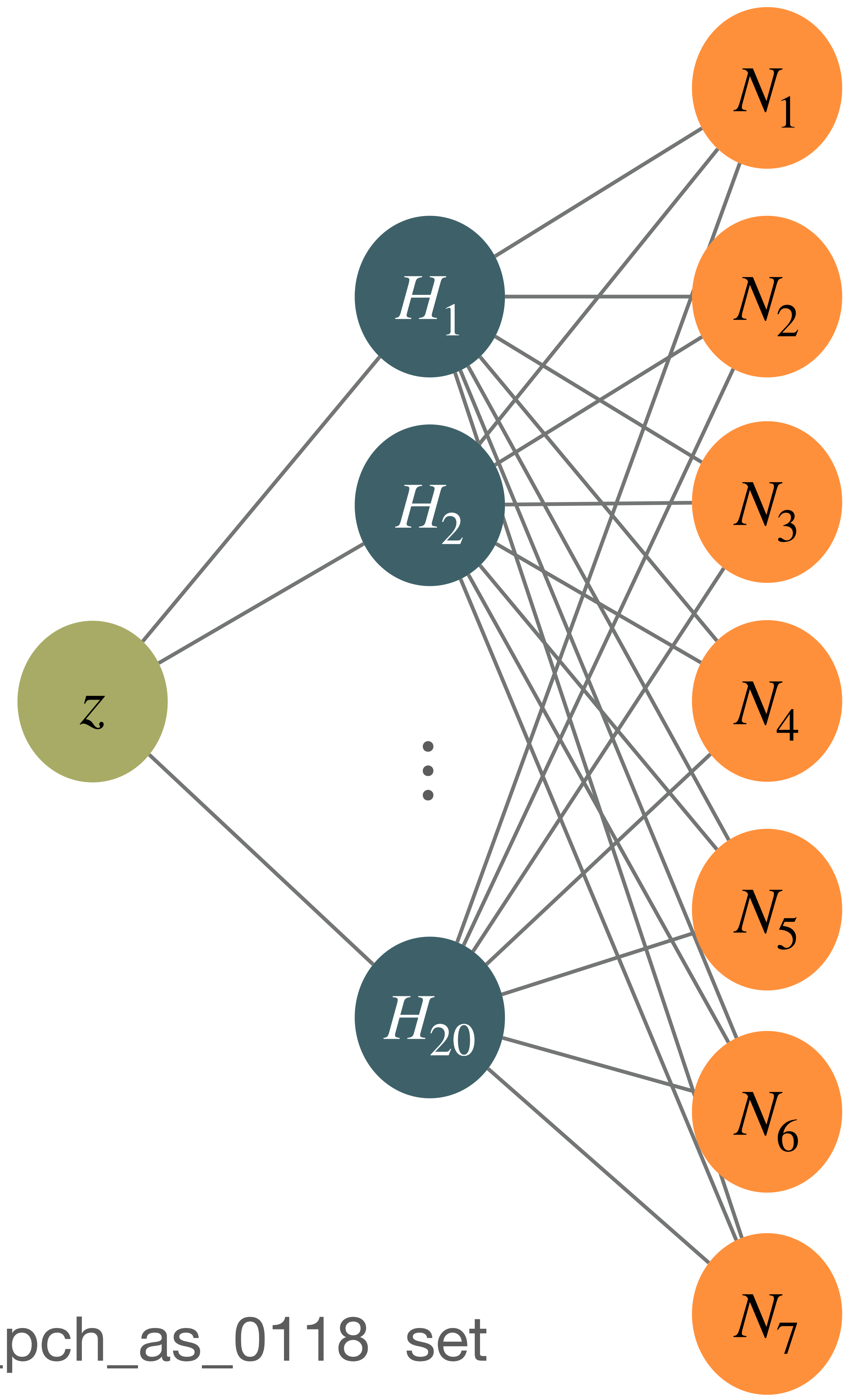
METHODOLOGY

Parameterisation

$$z\,D_i^{\Lambda}(z,\mu_0 = 5\text{ GeV}) = \left(\mathcal{N}_i(z,\theta) - \mathcal{N}_i(1,\theta)\right)^2$$

7 independent flavours

$$D_u^{\Lambda},\quad D_d^{\Lambda},\quad D_s^{\Lambda},\quad D_{\bar{u}}^{\Lambda} = D_{\bar{d}}^{\Lambda} = D_{\bar{s}}^{\Lambda},\quad D_{c^+}^{\Lambda},\quad D_{b^+}^{\Lambda},\quad D_g^{\Lambda}$$



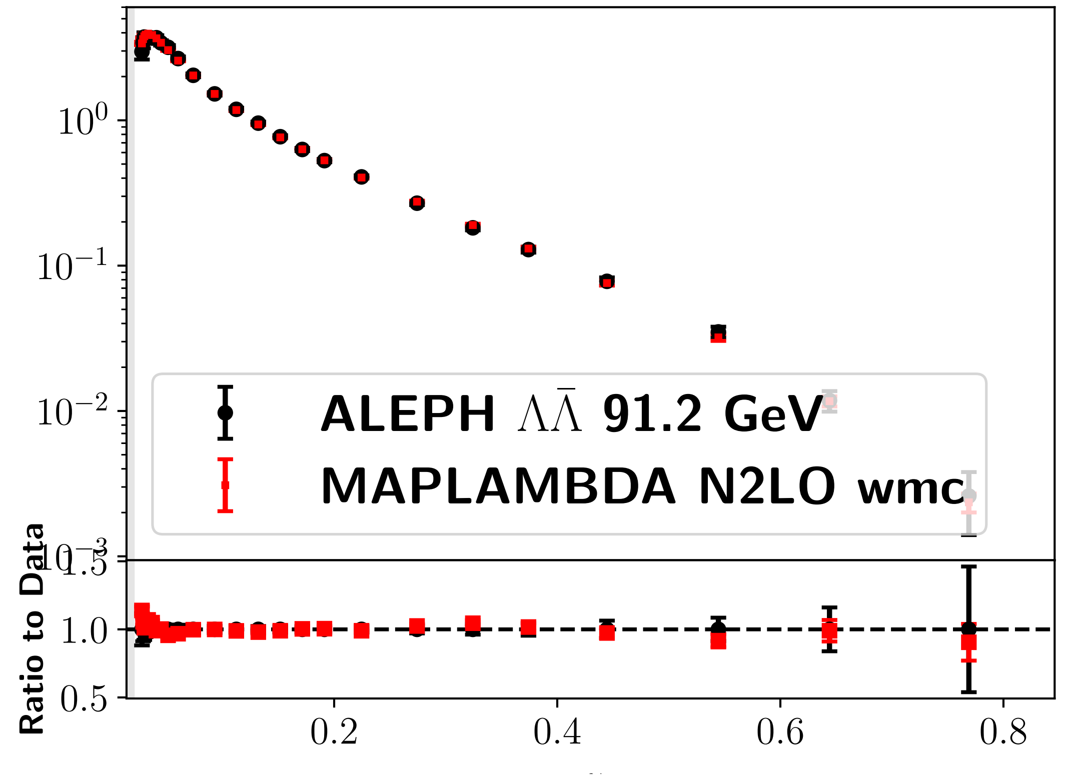
$$\chi^{2(k)} \equiv \left(T(\theta^{(k)}) - x^{(k)}\right)^T C^{-1} \left(T(\theta^{(k)}) - x^{(k)}\right)$$

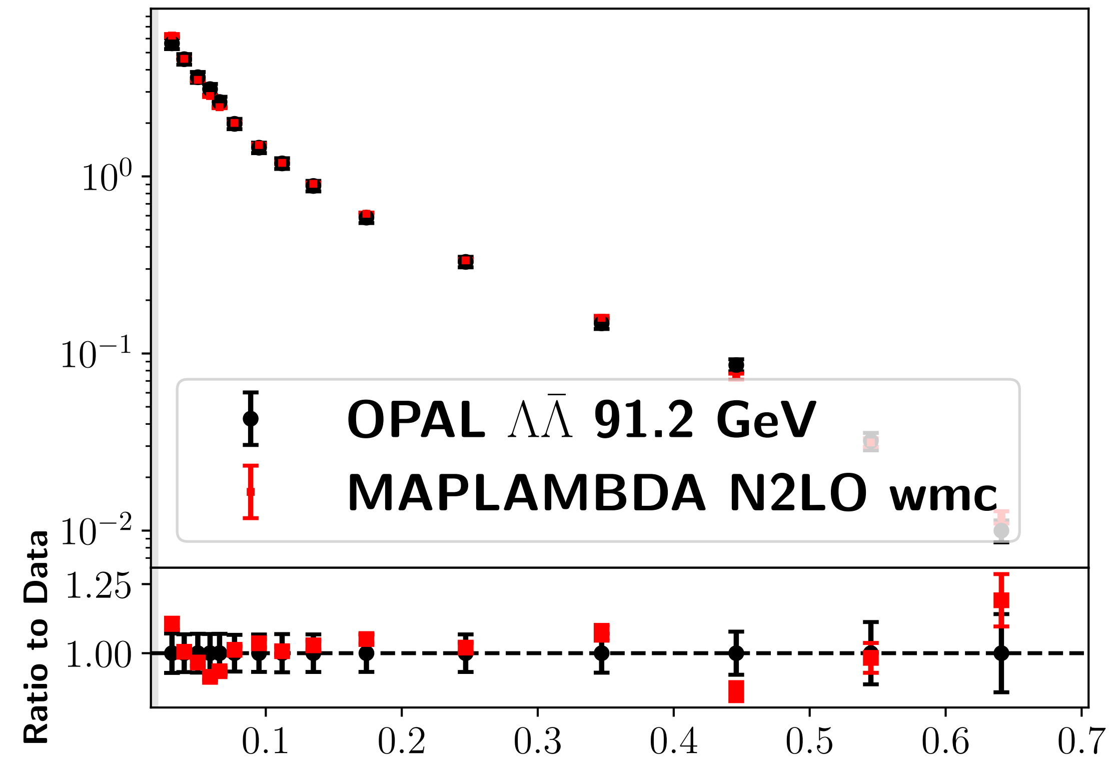
Propagation of PDF correlated uncertainties for SIDIS using the NNPDF31_n(n)lo_pch_as_0118 set

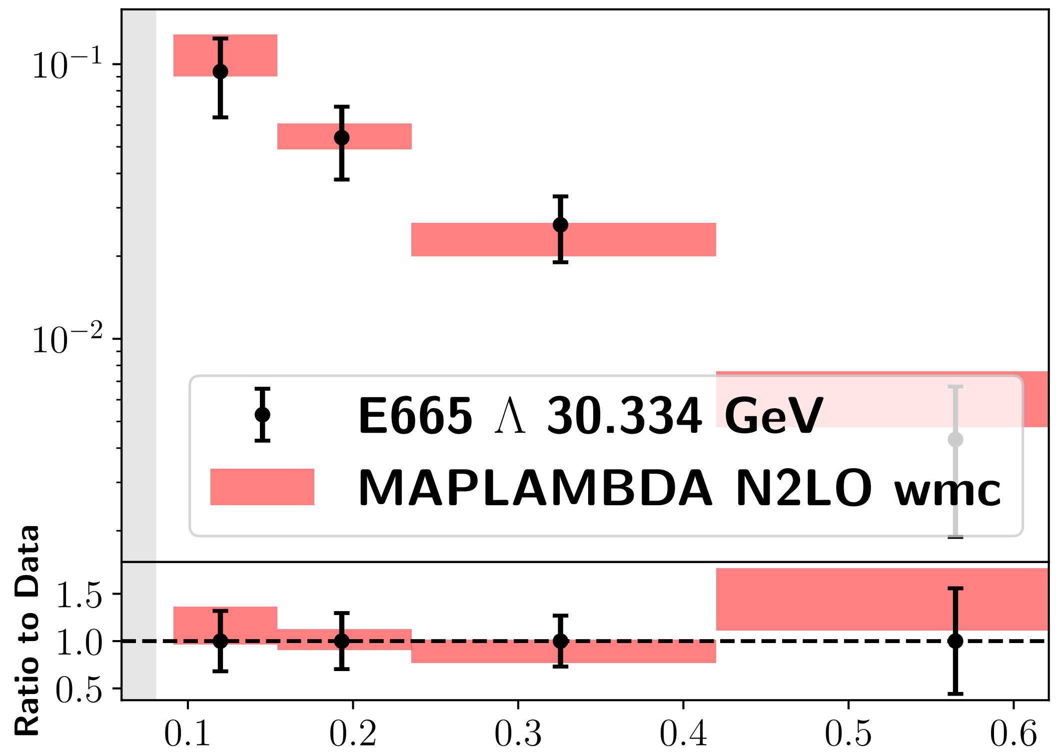
Minimisation algorithm: **Levenberg-Marquardt** implemented in **Ceres-solver**

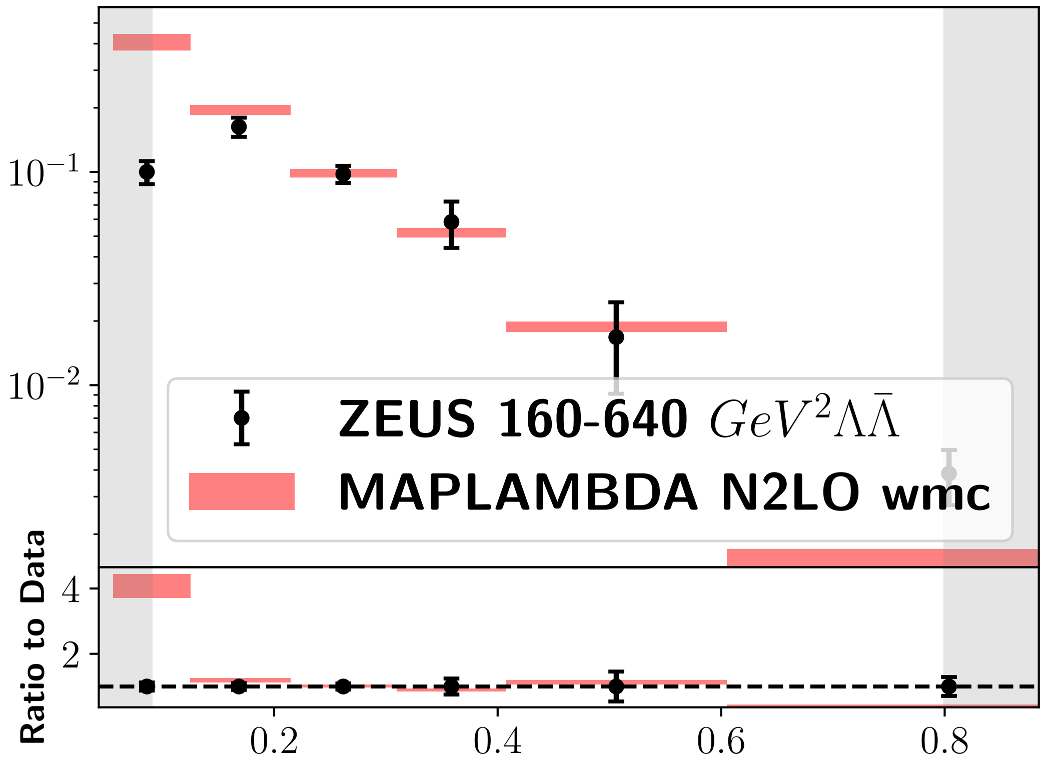
RESULTS

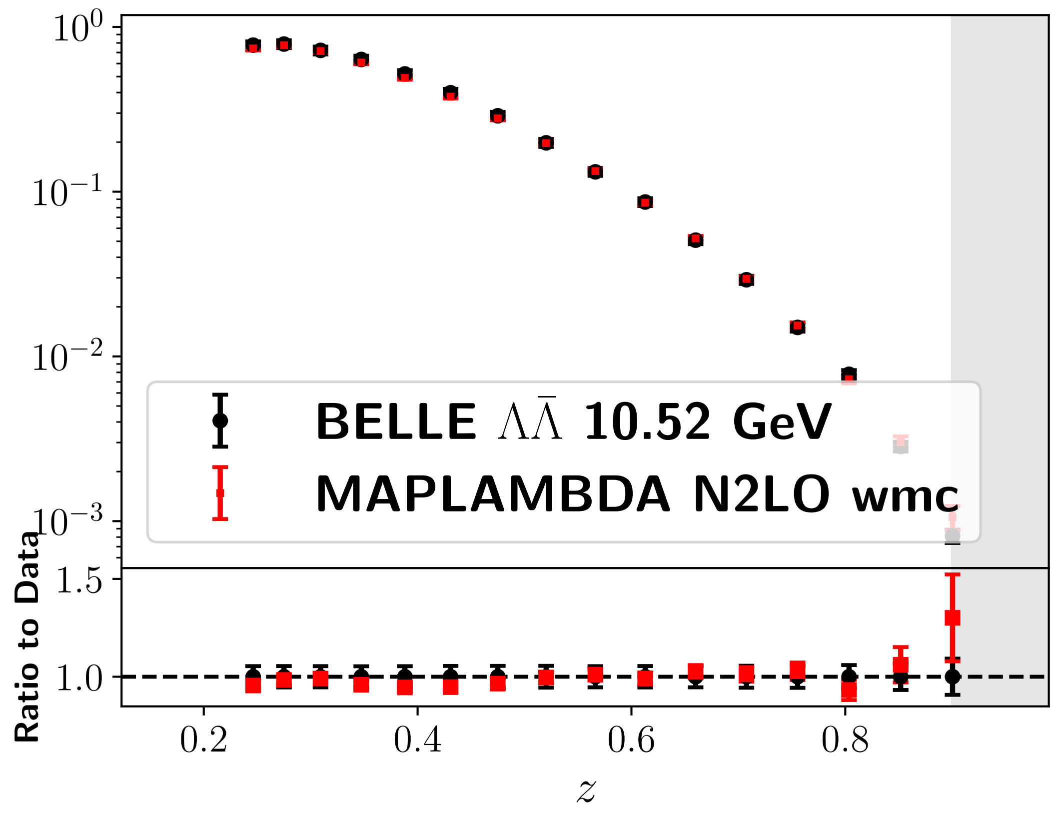
Experiment	χ^2 per point	N_{dat} after cuts
SIDIS		
E665 Λ	0.27	4
E665 $\bar{\Lambda}$	1.22	3
EMC Λ	0.92	4
EMC $\bar{\Lambda}$	0.18	3
H1 $\Lambda\bar{\Lambda}$	0.78	2
CHIO Fermilab Λ	0.02	4
CHIO Fermilab $\bar{\Lambda}$	0.01	4
ZEUS $\Lambda\bar{\Lambda}$ 10-40 GeV^2	0.02	1
40-160 GeV^2	0.33	2
160-640 GeV^2	0.65	4
640-2560 GeV^2	0.09	3
2560-10240 GeV^2	0.001	1
SIA $\Lambda + \bar{\Lambda}$		
ALEPH	0.53	25
ARGUS	1.35	16
BELLE	0.40	15
CELLO	0.59	7
DELPHI 91.2 GeV	2.31	10
HRS	0.99	12
MARK II	2.09	13
OPAL	0.81	15
SLD B	0.57	8
SLD C	1.94	8
SLD UDS	2.09	8
SLD	0.66	16
TASSO 14 GeV	0.27	3
TASSO 22 GeV	0.53	4
TASSO 33 GeV	0.77	5
TASSO 34 GeV	0.74	7
TASSO 34.8 GeV	2.74	10
TASSO 42.1 GeV	1.02	4
Total	**0.97**	**221**

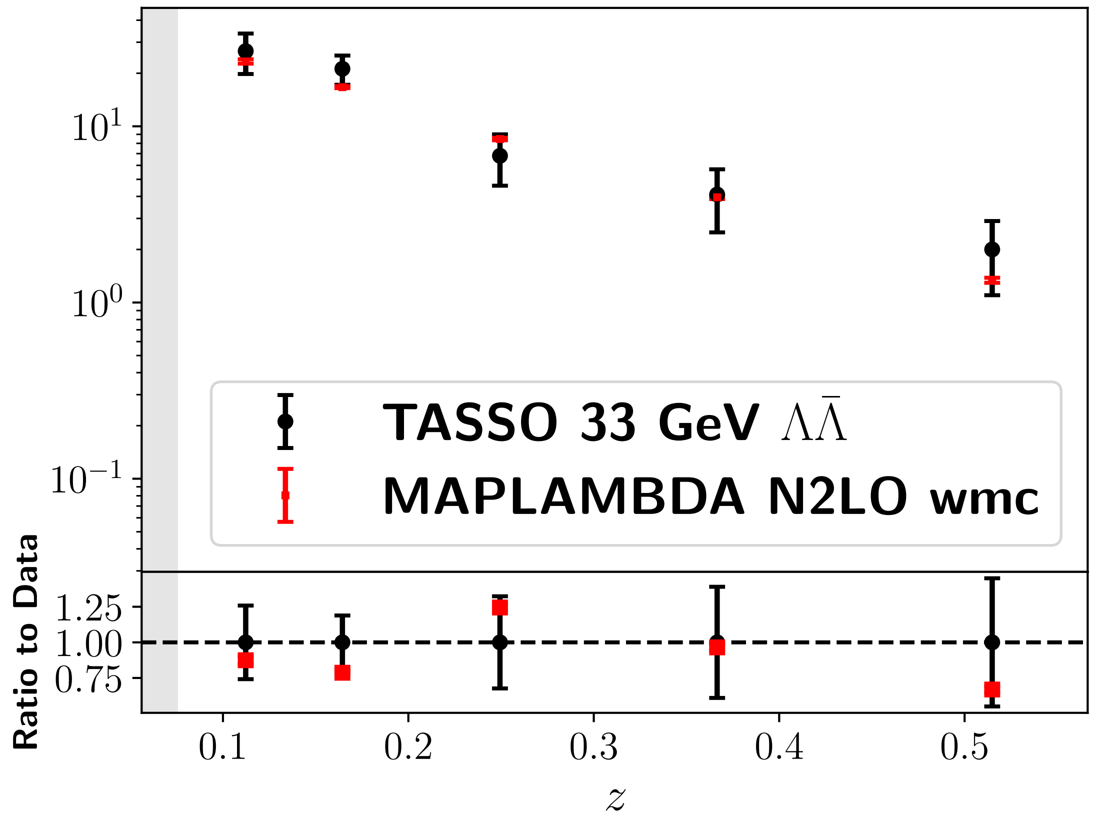










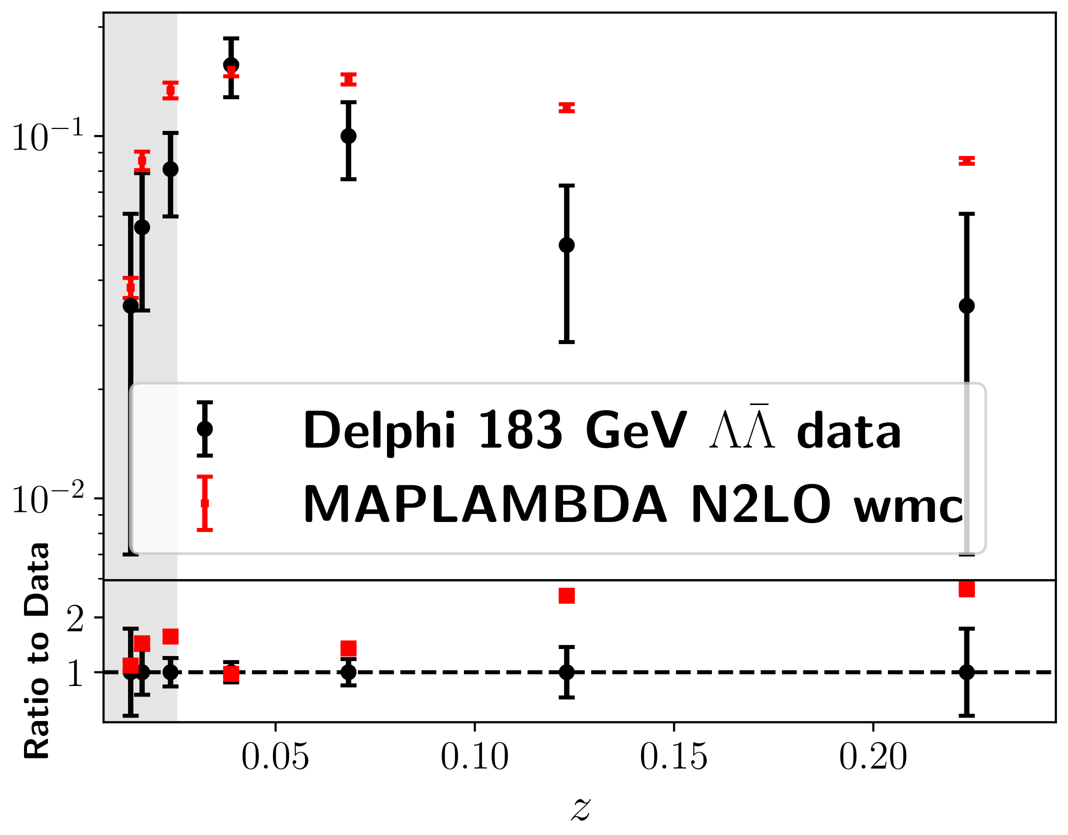


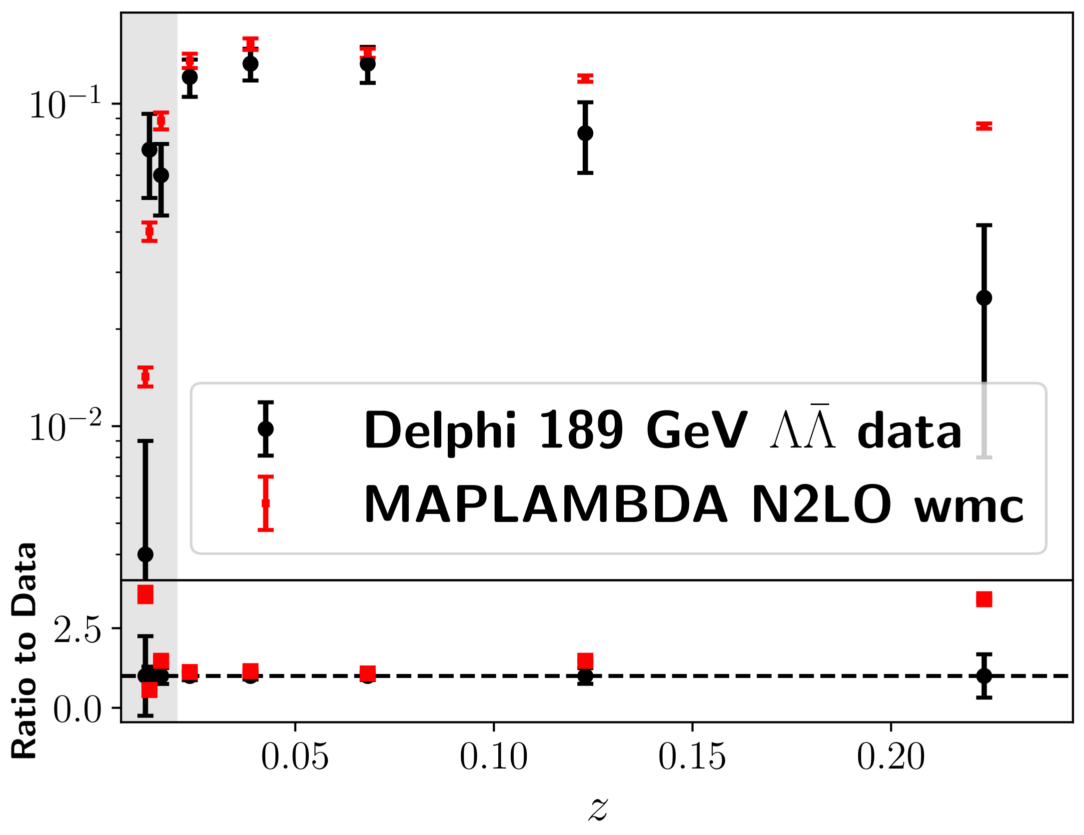
FIT COMPARISONS

Experiment	χ^2/N_{dat} NLO	χ^2/N_{dat} N2LO
E665 Λ	0.21	0.27
E665 $\bar{\Lambda}$	0.99	1.22
EMC Λ	1.05	0.92
EMC $\bar{\Lambda}$	0.19	0.18
H1 $\Lambda\bar{\Lambda}$	0.78	0.78
CHIO Fermilab Λ	0.02	0.02
CHIO Fermilab $\bar{\Lambda}$	0.01	0.01
ZEUS $\Lambda\bar{\Lambda}$ 10-40 GeV^2	0.06	0.02
40-160 GeV^2	0.33	0.33
160-640 GeV^2	0.68	0.65
640-2560 GeV^2	0.08	0.09
2560-10240 GeV^2	0.002	0.001
SIA $\Lambda + \bar{\Lambda}$		
ALEPH	0.55	0.53
ARGUS	1.39	1.35
BELLE	0.49	0.40
CELLO	0.59	0.59
DELPHI 91.2 GeV	2.30	2.31
HRS	0.99	0.99
MARK II	2.12	2.09
OPAL	0.81	0.81
SLD B	0.63	0.57
SLD C	1.92	1.94
SLD UDS	2.04	2.09
SLD	0.67	0.66
TASSO 14 GeV	0.27	0.27
TASSO 22 GeV	0.53	0.53
TASSO 33 GeV	0.77	0.77
TASSO 34 GeV	0.73	0.74
TASSO 34.8 GeV	2.77	2.74
TASSO 42.1 GeV	1.01	1.02
Total	0.98	0.97

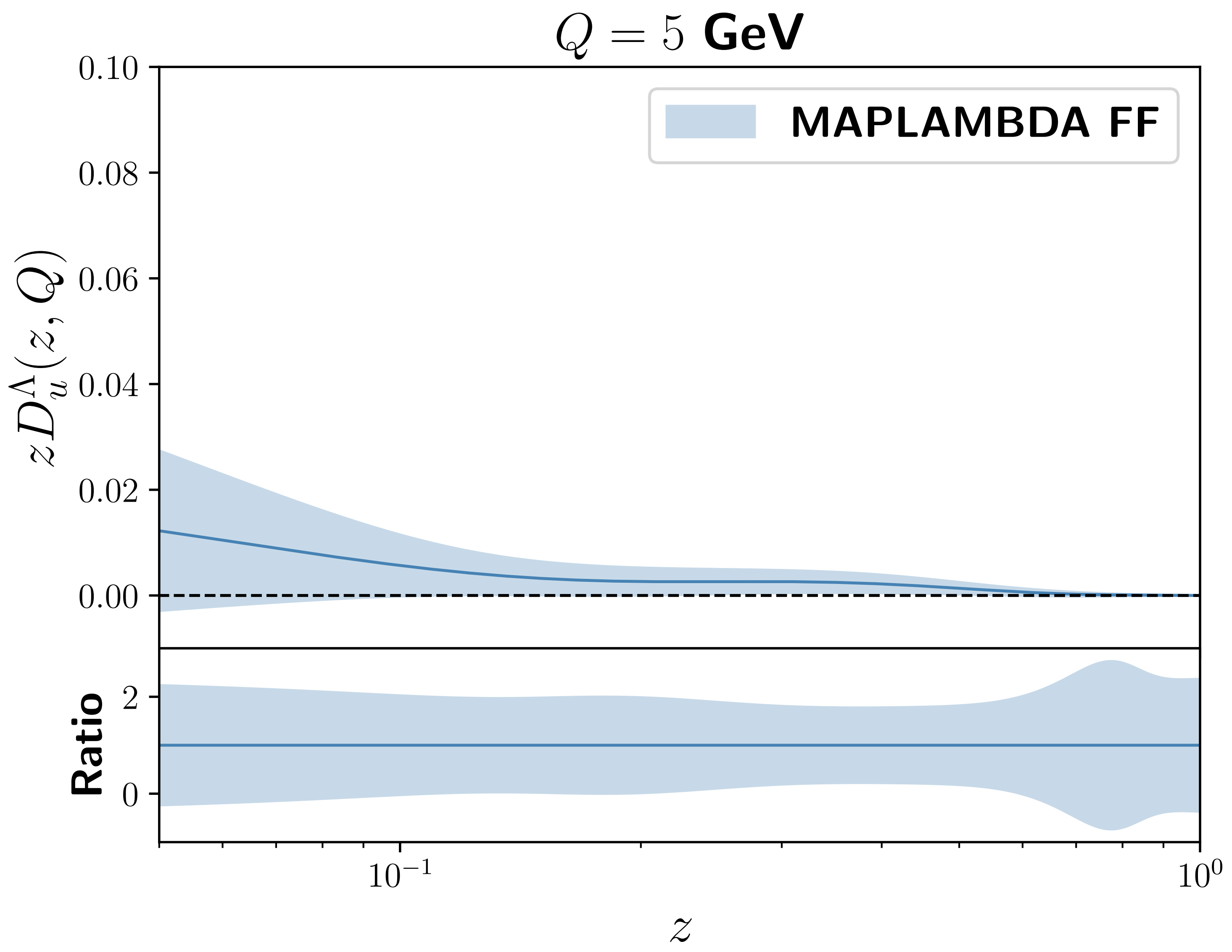
- Stable fit between NLO and N2LO
- Fit with only SIA data at N2LO $\rightarrow \chi^2 = 1.04$
- Delphi data at 183 & 189 GeV excluded

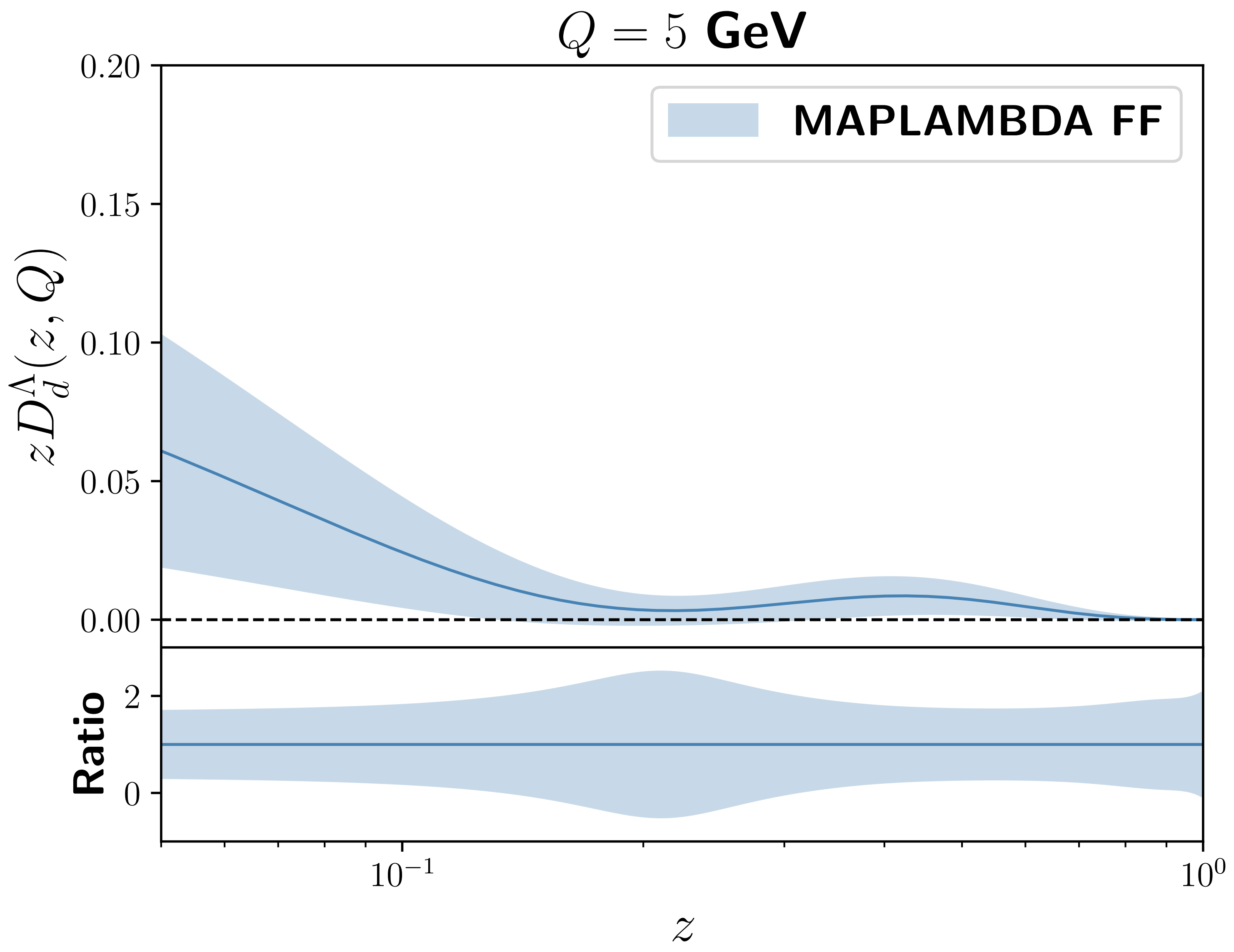
$$\chi^2_{183} = 2.3, \chi^2_{189} = 2.42, \ \chi^2_{tot} = 1.05$$

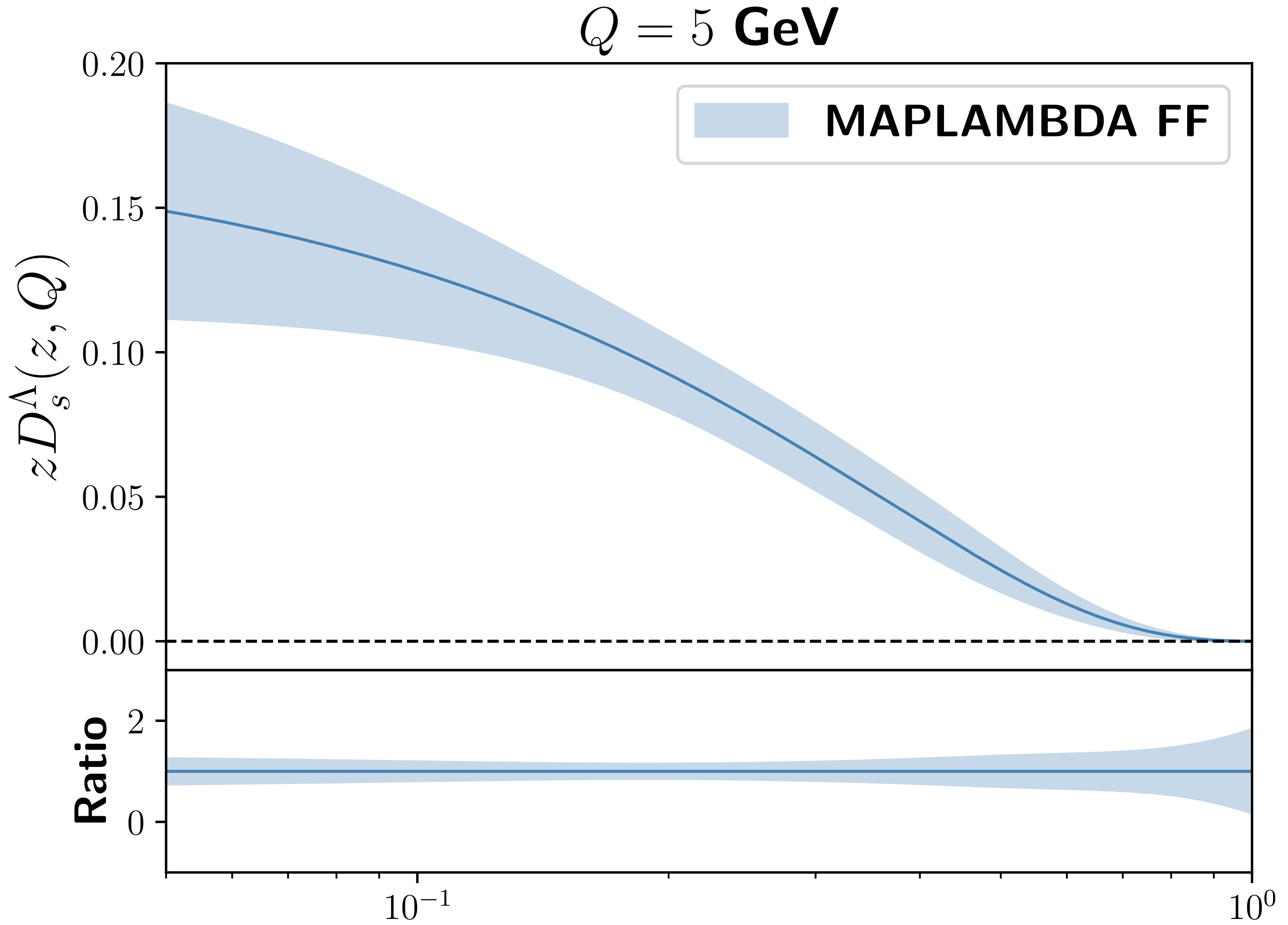


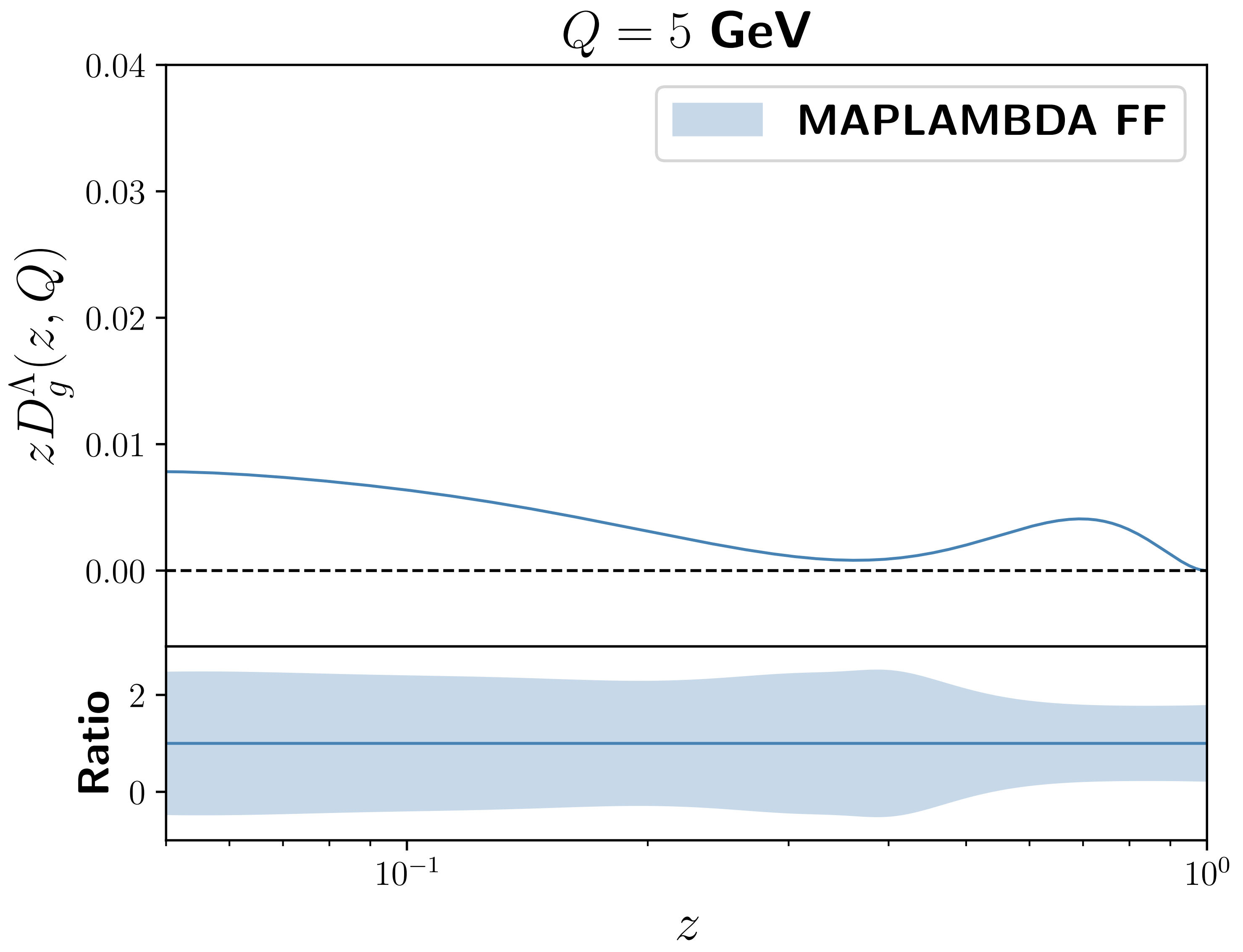


FRAGMENTATION FUNCTIONS AT N2LO









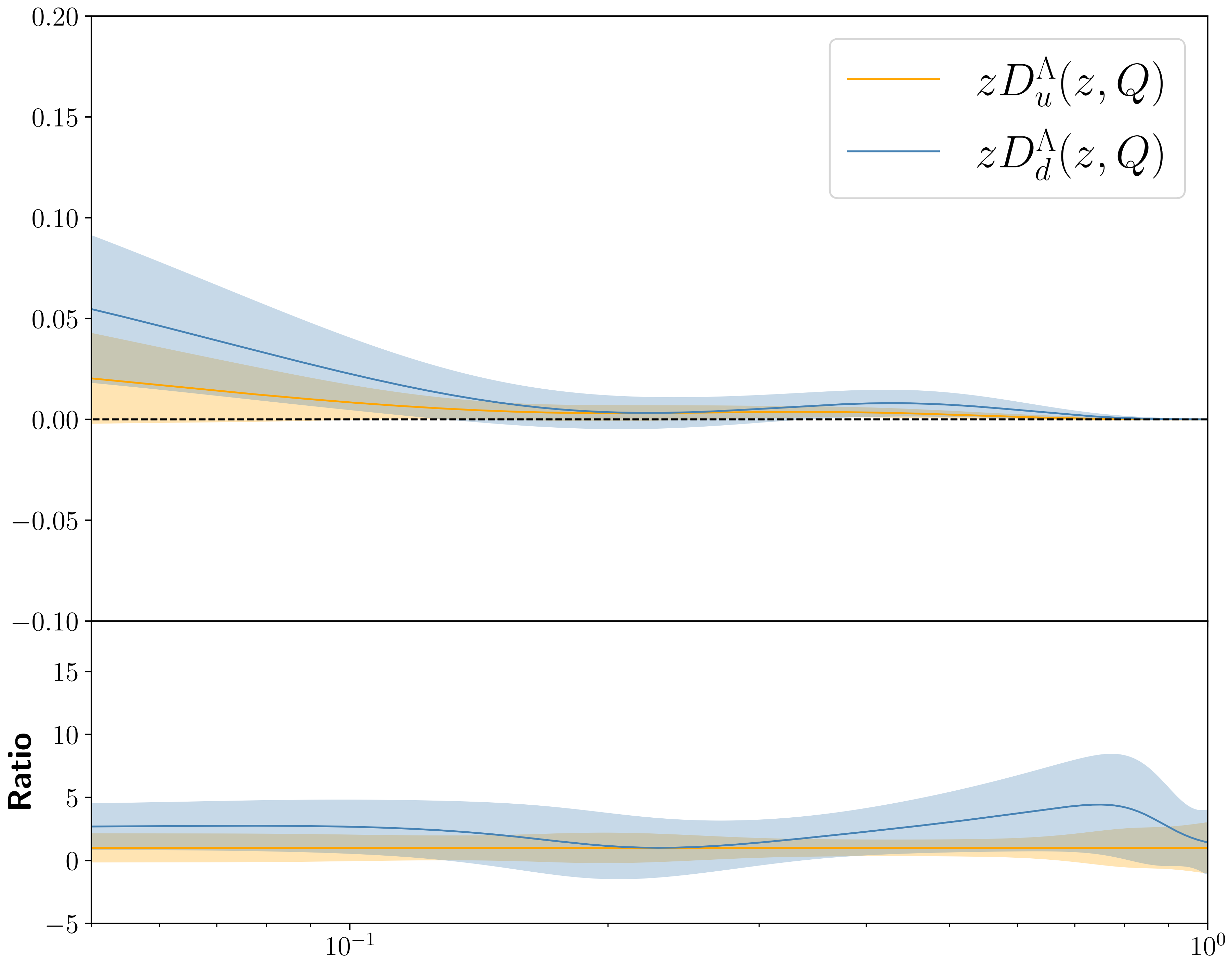
D_s^Λ dominates over u and d!

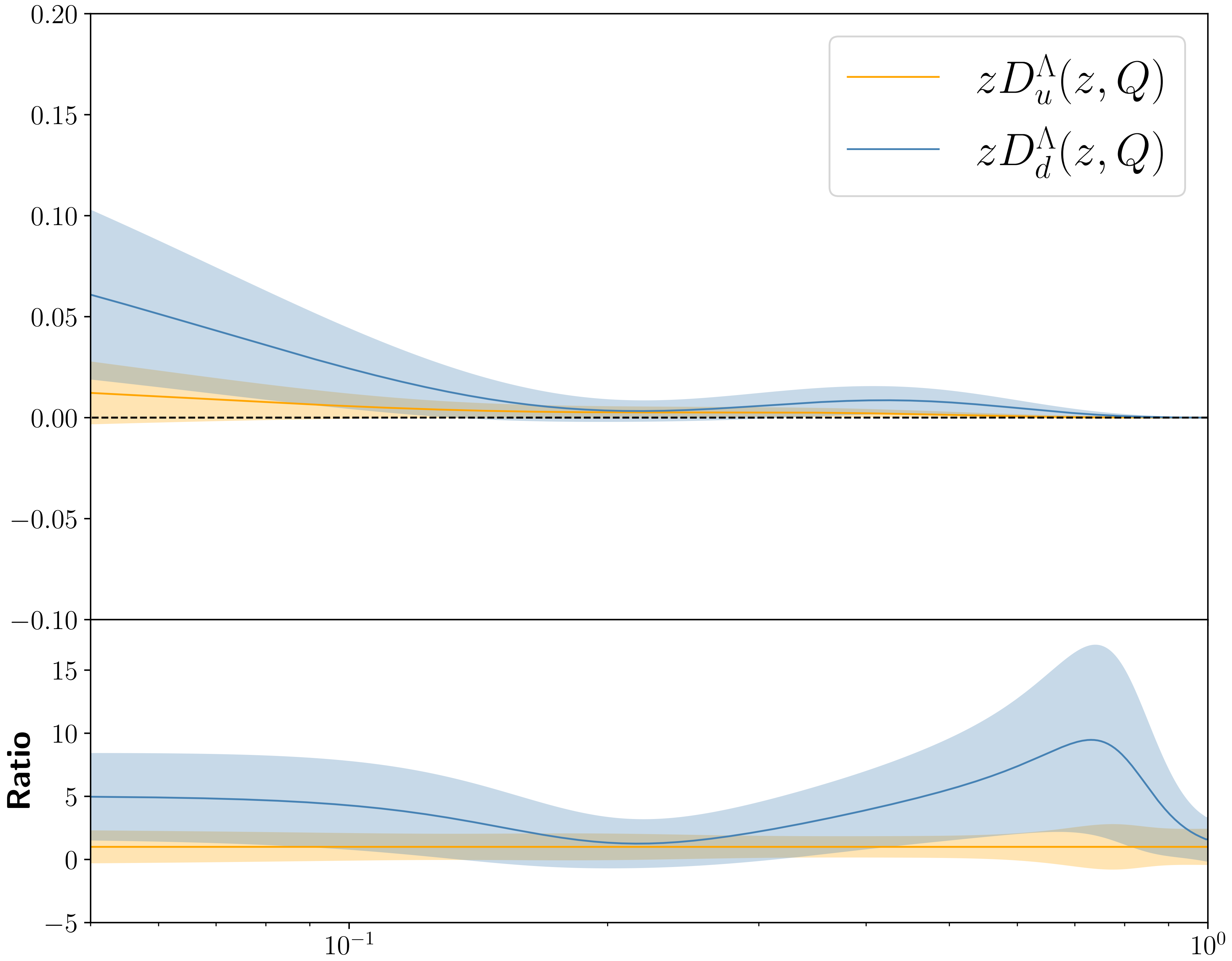
COMPATIBILITY OF D_u AND D_d

NLO

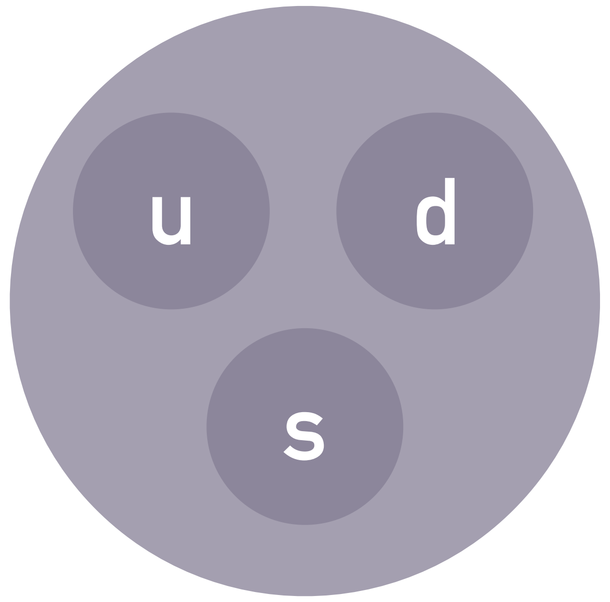
N2LO

D_u^Λ vs D_d^Λ



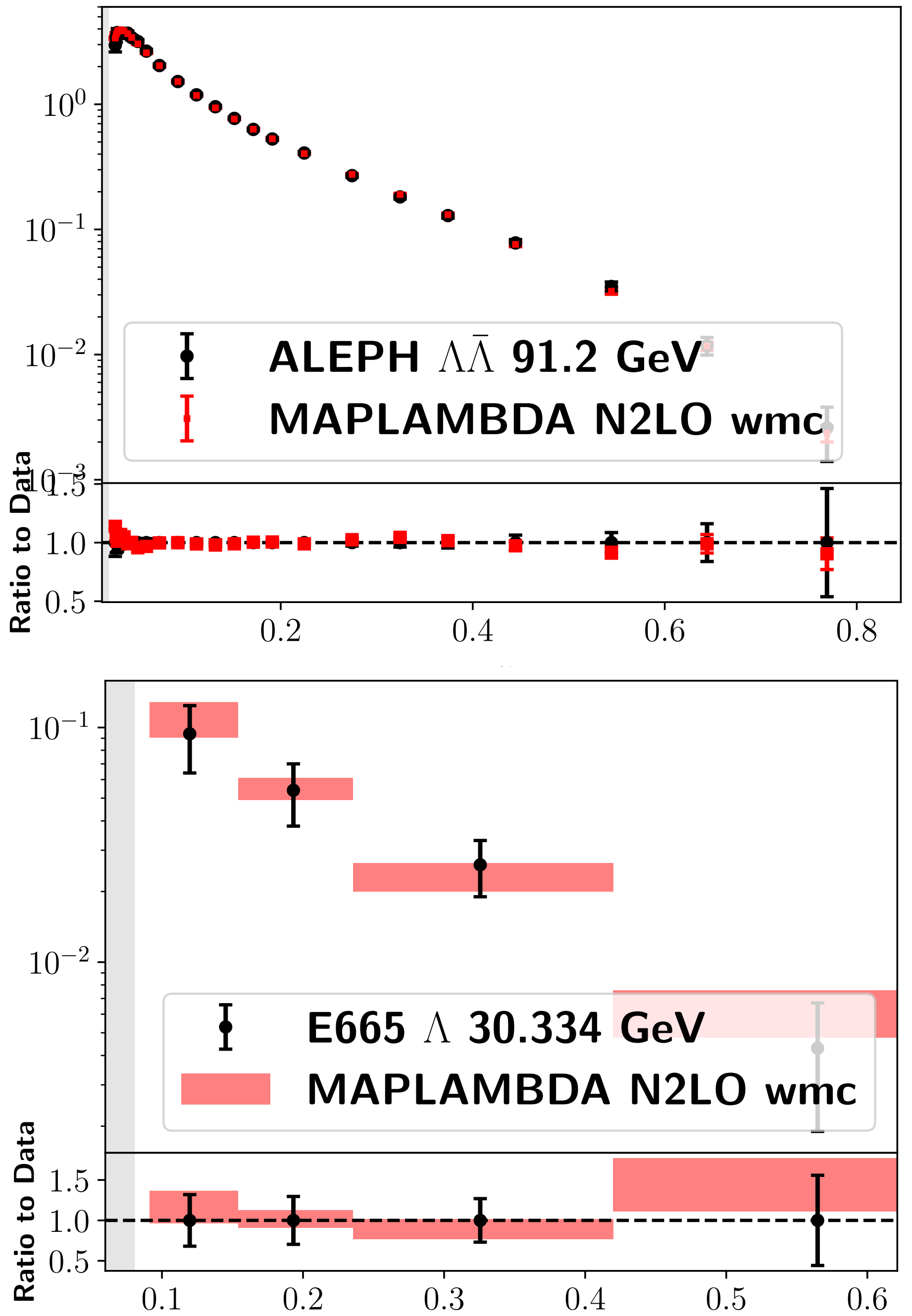


CONCLUSIONS



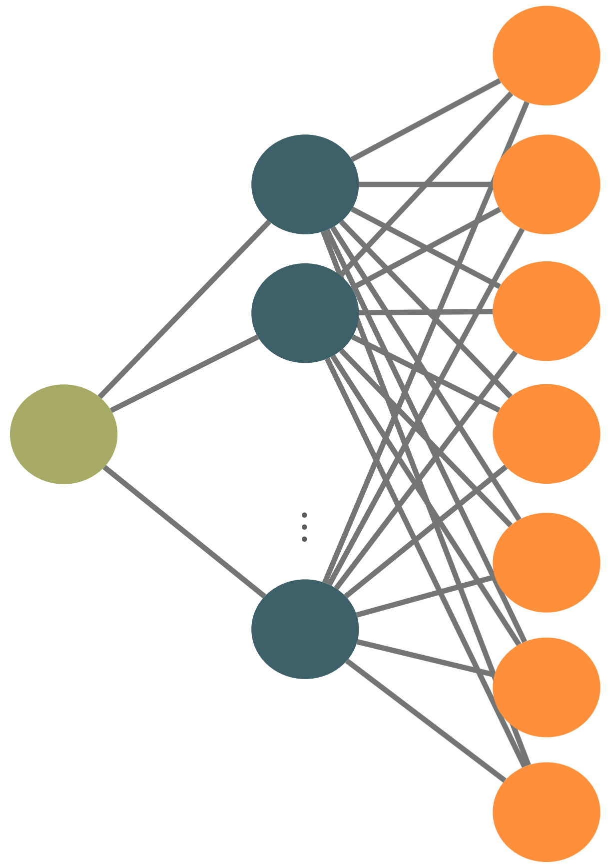
RECAP: New steps towards a better comprehension of the Λ baryon production in fragmentation

- Determination of Λ unpolarized collinear fragmentation function from SIA and SIDIS NC data by means of Neural Network and Monte Carlo sampling method as statistical framework at NLO and N2LO
- χ^2=0.97 at N2LO obtained requiring 7 independent flavours
- Hadron mass corrections included



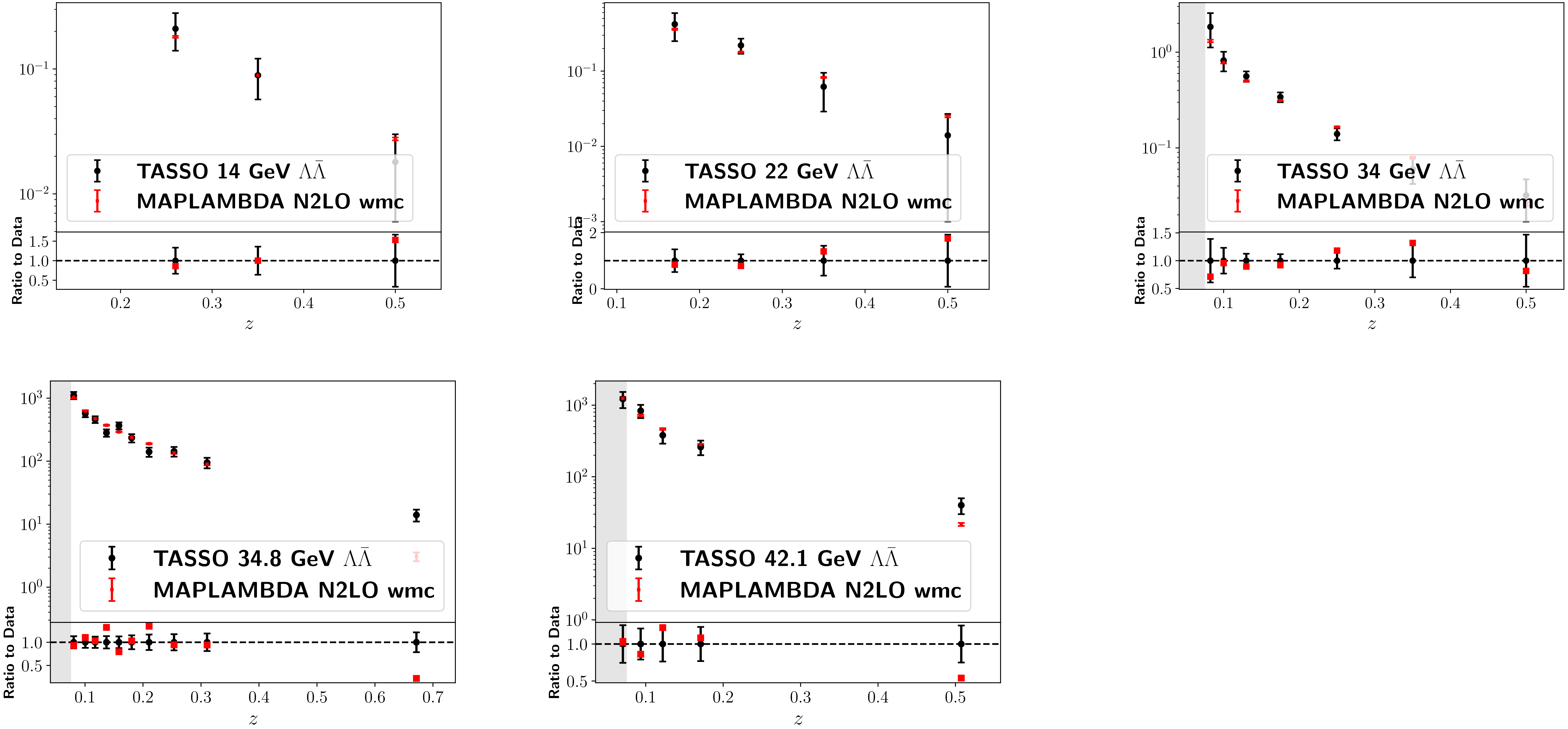
NEXT STEPS: Extend the analysis to

- CC SIDIS
- pp collisions
- Longitudinally polarised Λ



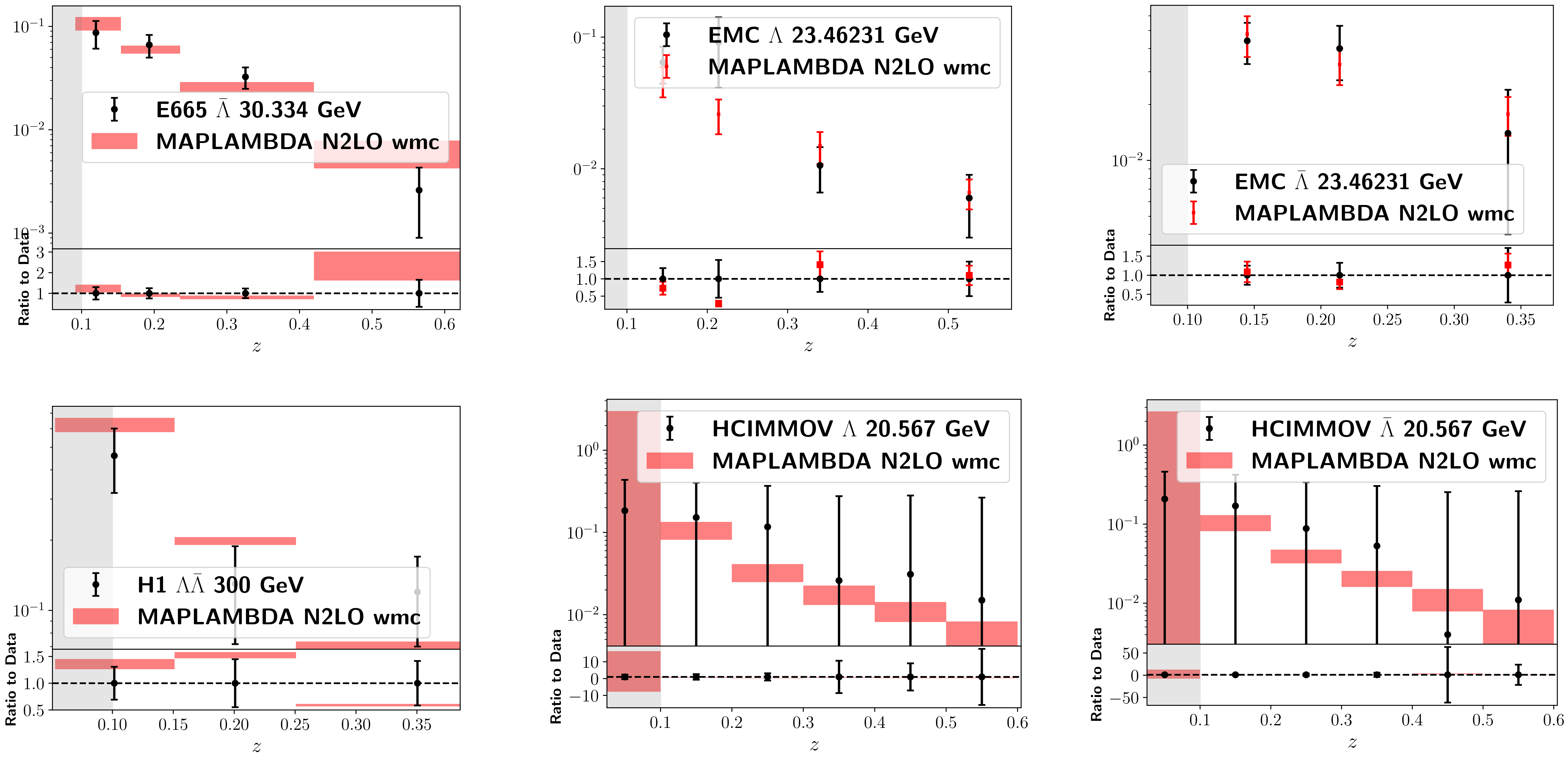
BACKUP

RESULTS

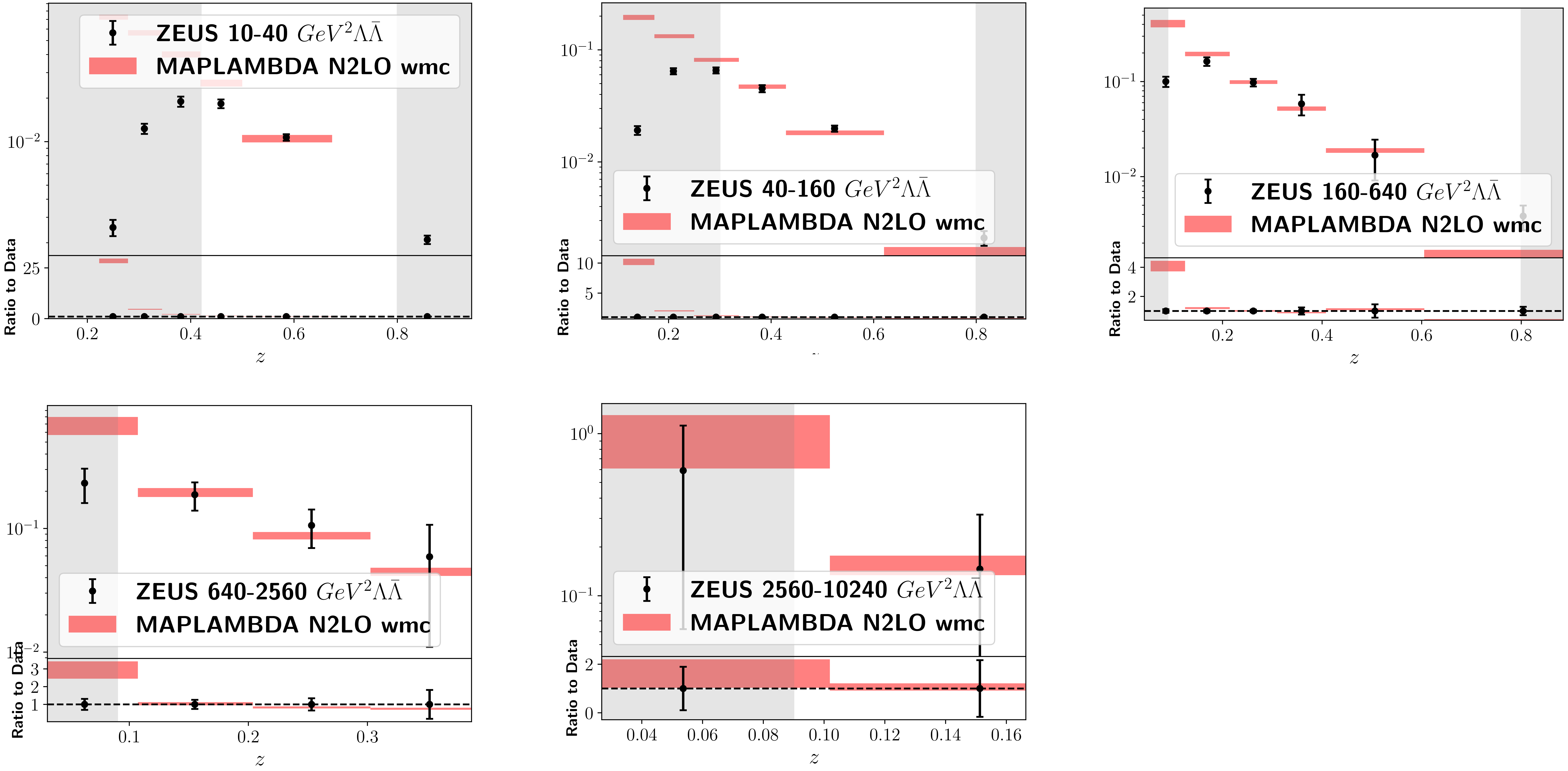


TASSO 14 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
TASSO 22 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
TASSO 34 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
TASSO 34.8 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
TASSO 42.1 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
Ratio to Data
z

RESULTS

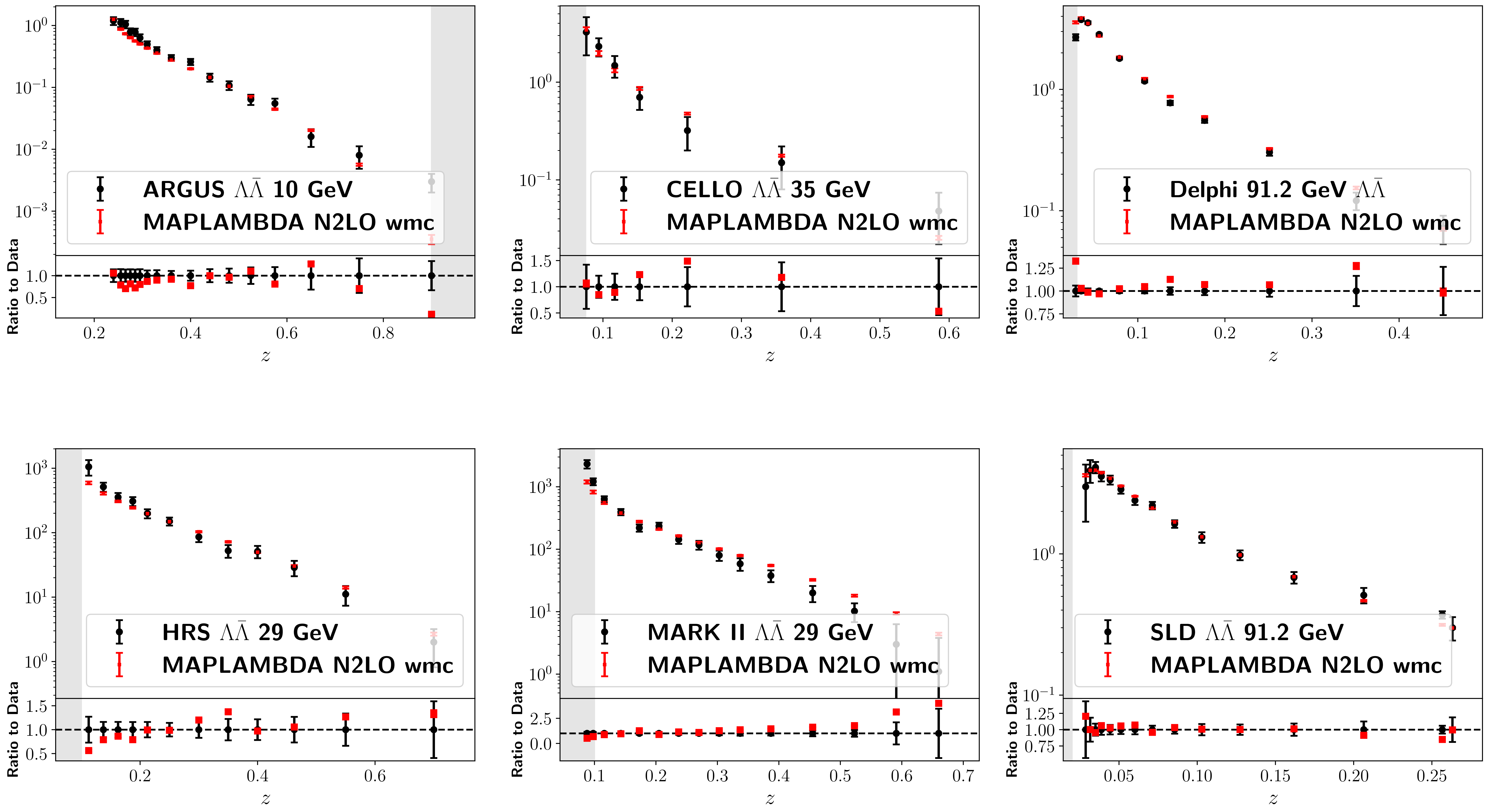

E665 $\bar{\Lambda}$ 30.334 GeV
MAPLAMBDA N2LO wmc
Ratio to Data
z
EMC Λ 23.46231 GeV
MAPLAMBDA N2LO wmc
EMC $\bar{\Lambda}$ 23.46231 GeV
MAPLAMBDA N2LO wmc
H1 $\Lambda\bar{\Lambda}$ 300 GeV
MAPLAMBDA N2LO wmc
HCIMMOV Λ 20.567 GeV
MAPLAMBDA N2LO wmc
HCIMMOV $\bar{\Lambda}$ 20.567 GeV
MAPLAMBDA N2LO wmc

RESULTS



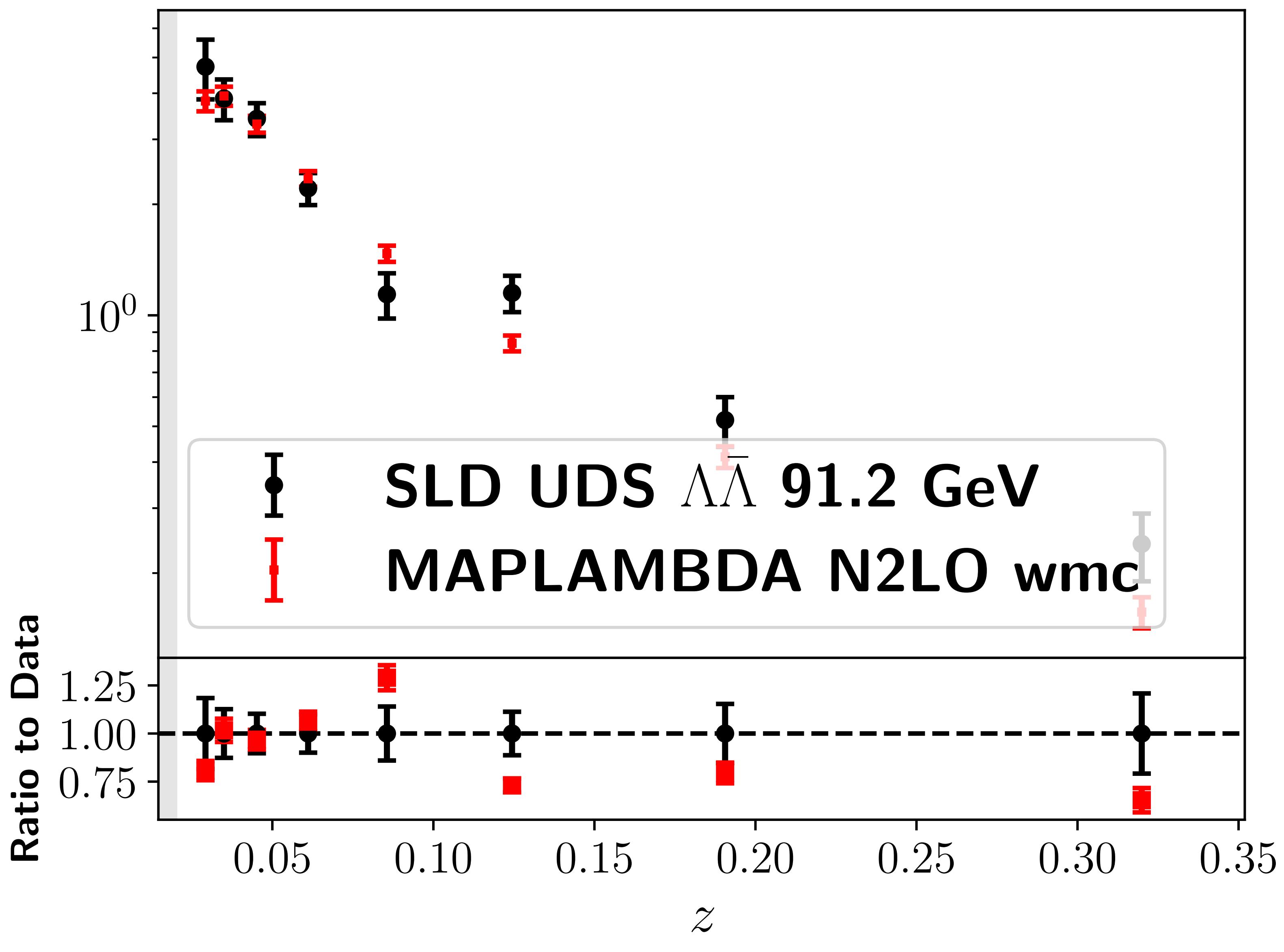
ZEUS 10-40 $GeV^2 \Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
ZEUS 40-160 $GeV^2 \Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
ZEUS 160-640 $GeV^2 \Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
ZEUS 640-2560 $GeV^2 \Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
ZEUS 2560-10240 $GeV^2 \Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
Ratio to Data
z

RESULTS

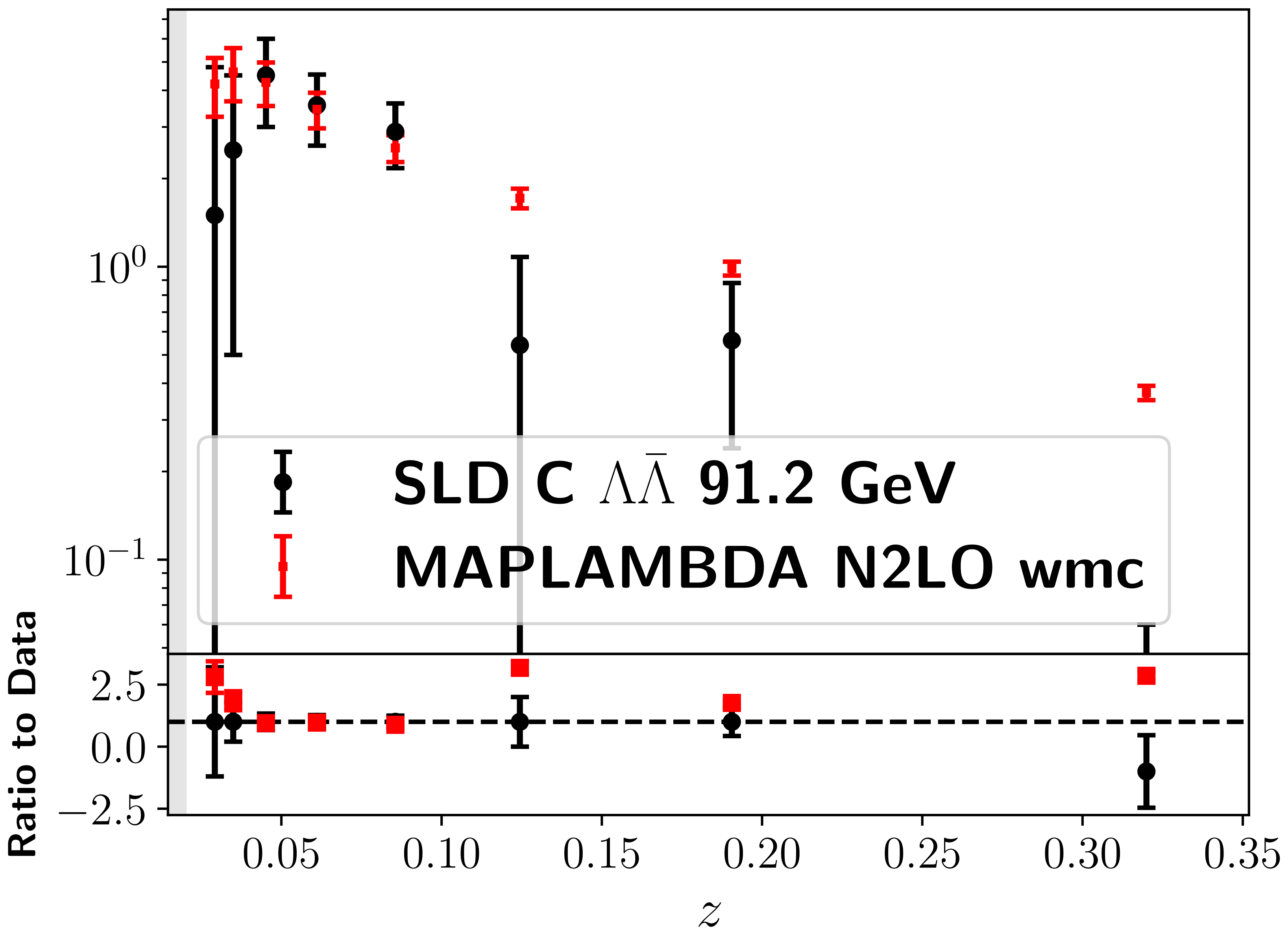


ARGUS $\Lambda\bar{\Lambda}$ 10 GeV
MAPLAMBDA N2LO wmc
CELLO $\Lambda\bar{\Lambda}$ 35 GeV
MAPLAMBDA N2LO wmc
Delphi 91.2 GeV $\Lambda\bar{\Lambda}$
MAPLAMBDA N2LO wmc
HRS $\Lambda\bar{\Lambda}$ 29 GeV
MAPLAMBDA N2LO wmc
MARK II $\Lambda\bar{\Lambda}$ 29 GeV
MAPLAMBDA N2LO wmc
SLD $\Lambda\bar{\Lambda}$ 91.2 GeV
MAPLAMBDA N2LO wmc
Ratio to Data
z

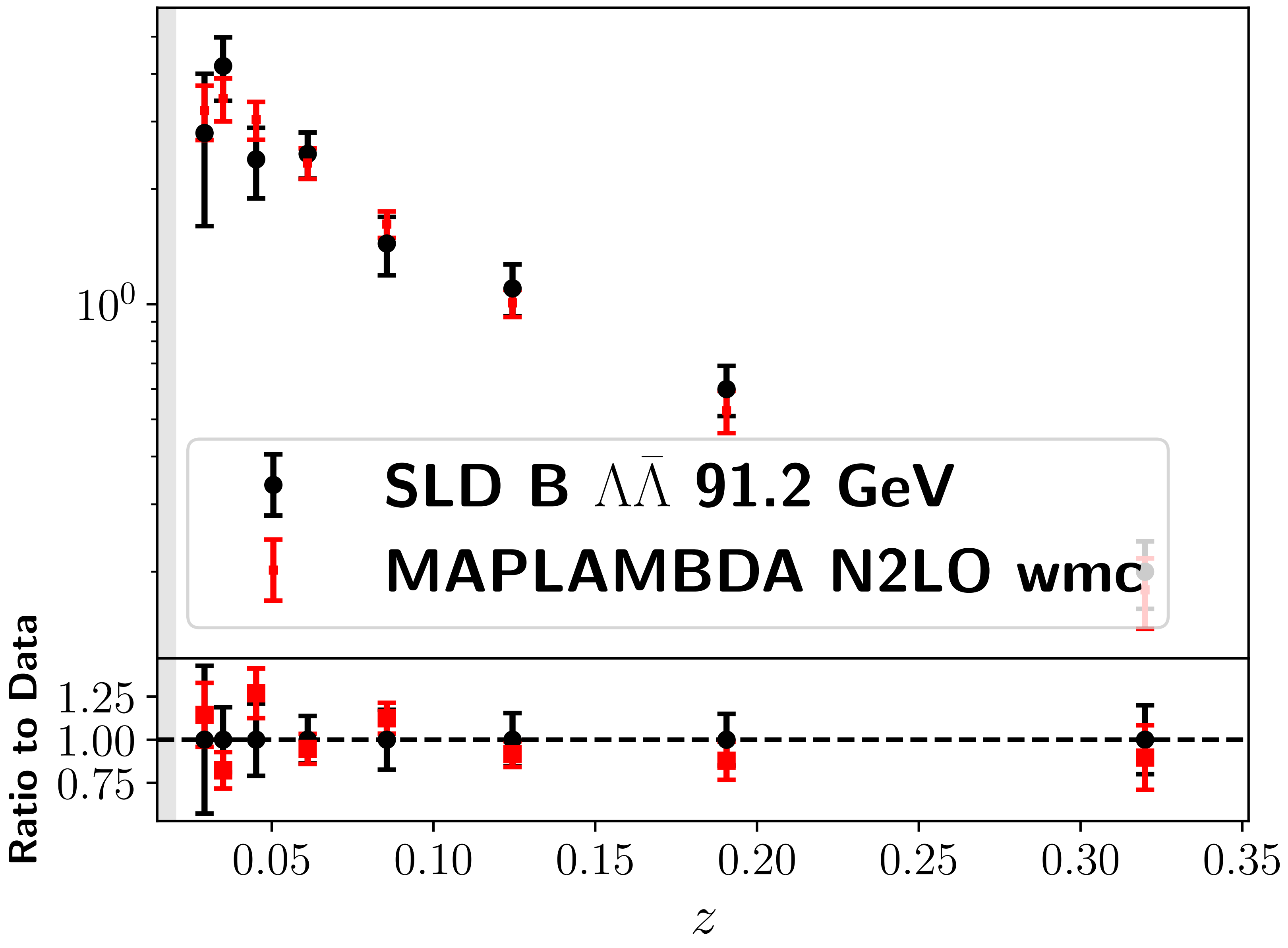
RESULTS



SLD UDS $\Lambda\bar{\Lambda}$ 91.2 GeV
MAPLAMBDA N2LO wmc
Ratio to Data
z

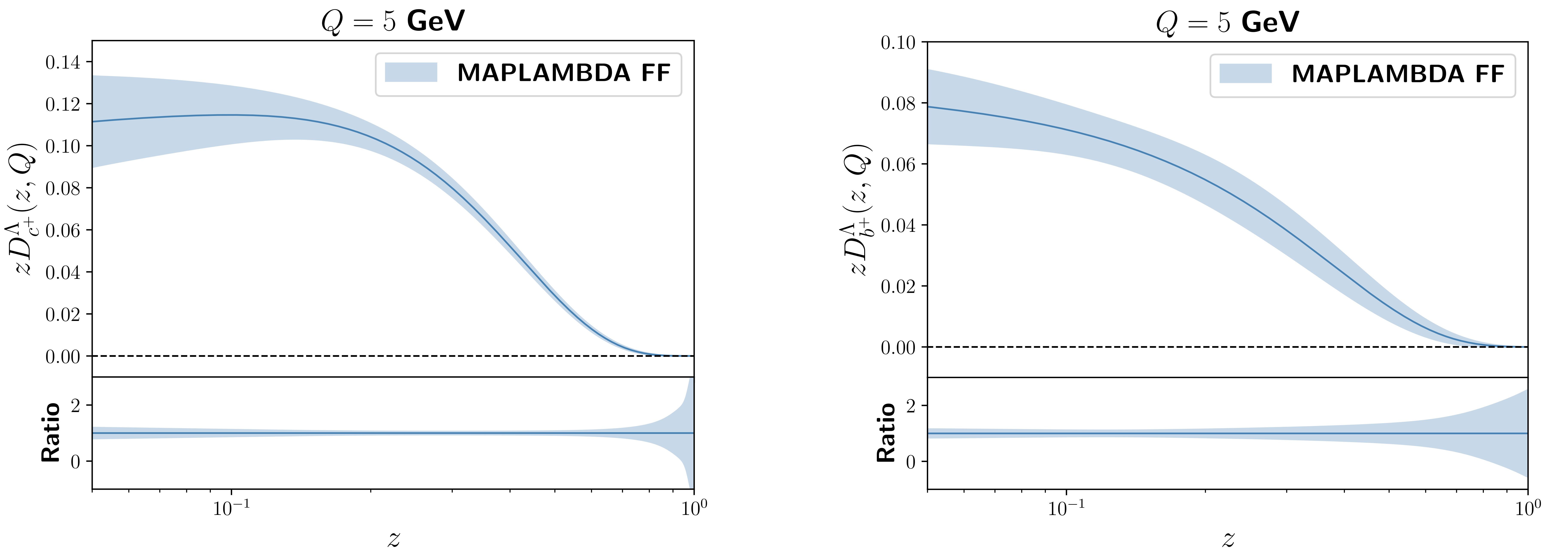


SLD C $\Lambda\bar{\Lambda}$ 91.2 GeV
MAPLAMBDA N2LO wmc
Ratio to Data
z



SLD B $\Lambda\bar{\Lambda}$ 91.2 GeV
MAPLAMBDA N2LO wmc
Ratio to Data
z

FFS


$Q = 5$ GeV
MAPLAMBDA FF
$zD^{\Lambda}_{c^+}(z, Q)$
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Ratio
2
0
10^{-1}
10^{0}
z
$Q = 5$ GeV
MAPLAMBDA FF
$zD^{\Lambda}_{b^+}(z, Q)$
0.10
0.08
0.06
0.04
0.02
0.00
Ratio
2
0
10^{-1}
10^{0}
z