Model calculation for twist-3 proton PDFs and their evolution with the energy scale

Arianna Vercesi

Università degli Studi di Pavia

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Outline

Light-front formalism

Instant form

$$x^\mu = (x^0, x^1, x^2, x^3)$$

Light-front form

$$x^\mu = (x^+, x^-, \mathbf{x}_\perp)$$

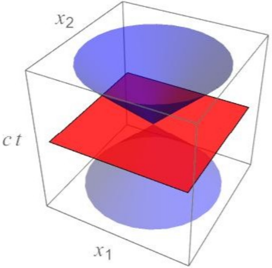
$$x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}} \qquad \mathbf{x}_\perp = (x^1, x^2)$$

Light-front formalism

Instant form

$$x^\mu = (x^0, x^1, x^2, x^3)$$

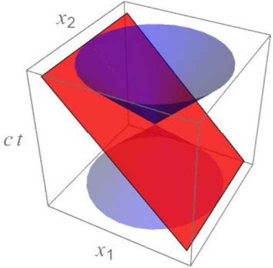
Null-time surface $x^0 = 0$



Light-front form

$$x^\mu = (x^+, x^-, \mathbf{x}_\perp)$$

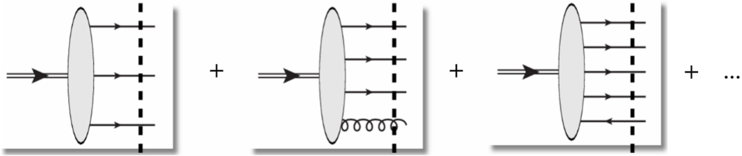
Null-time surface $x^+ = 0$



Fock expansion

Hadronic states can be expanded with Fock components weighted by **light-front wave functions** (LFWFs):

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}\,|qqq\bar{q}q\rangle + \ \ldots$$



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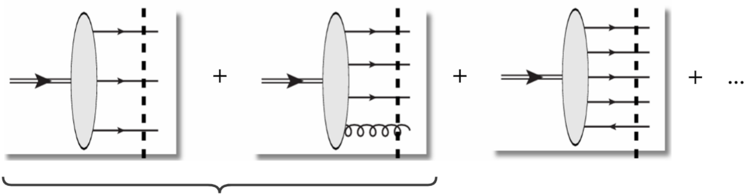

LFWFs model

The model for LFWFs is defined at low energy scale ($\mu^2 = 0.26\text{GeV}^2$)

$$\Psi_{qqq(g)}\big(\{\xi_i\},\{\mathbf{k}_{i\perp}\}\big) \propto G\big(\{\xi_i\},\{\mathbf{k}_{i\perp}\}\big)\, P\big(\{\xi_i\}\big)$$

- 2D gaussian $G\big(\{\xi_i\},\{\mathbf{k}_{\perp i}\}\big) \sim \prod_i e^{-\mathbf{k}_{i\perp}^2/\xi_i}$
- Polynomial term $P\big(\{\xi_i\}\big) \sim \; \xi_1^{r_1}\xi_2^{r_2}\xi_3^{r_3}\; (\xi_4^{r_4})$

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↑

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$$\text{PDF} \sim \int [d\xi_i][d^2\mathbf{k}_{i\perp}]\;\Psi_{qqq}\Psi^*_{qqqg}$$

Twist-3 PDFs

The generic twist-3 PDF f is given by the following Fourier transform:

$$\langle P,S|\ O(z_1,z_2,z_3)\ |P,S\rangle \sim \int [dx]\ e^{-iP^+\sum_i x_i z_i}\, f(x_1,x_2,x_3)$$

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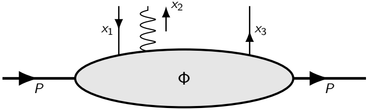
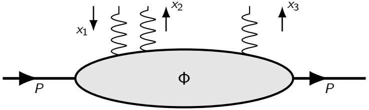


quark PDFs

$$O(z_{123}) \sim \bar{\psi}(z_1) F^{\mu+}(z_2)\ \Gamma\psi(z_3)$$



gluon PDFs

$$O(z_{123}) \sim F^{\mu+}(z_1) F^{\nu+}(z_2)\ F^{\rho+}(z_3)$$

Support of twist-3 PDFs

$$\int [dx] = \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \ \delta(x_1 + x_2 + x_3)$$

Support of twist-3 PDFs

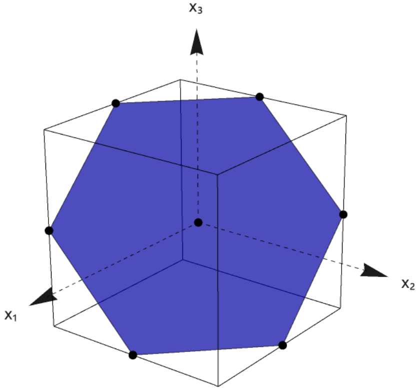
$$\int [dx] = \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \int_{-1}^{1} dx_3 \; \delta(x_1 + x_2 + x_3)$$

- x_i longitudinal momentum fractions

$$-1 \leq x_i \leq +1$$

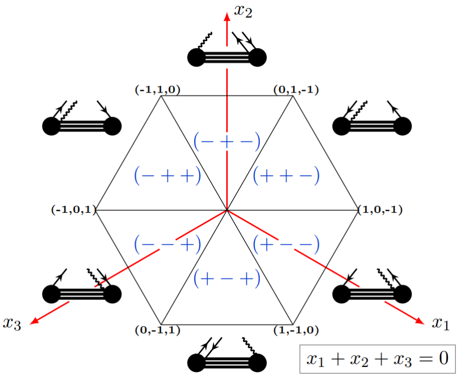
- x_i non-independent variables

$$x_1 + x_2 + x_3 = 0$$



Support of twist-3 PDFS

$x_1 > 0$	antiquark emission
$x_1 < 0$	quark absorption
$x_2 > 0$	gluon emission
$x_2 < 0$	gluon absorption
$x_3 > 0$	quark emission
$x_3 < 0$	antiquark absorption



[arXiv:2405.01162v2]

Twist-3 evolution

- Quark PDFs:

$$\mathbf{T}(\mathbf{x_{123}}), \boldsymbol{\Delta}\mathbf{T}(\mathbf{x_{123}}) \leftrightarrow \mathfrak{S}^{\pm}(x_{123})$$

- Gluon PDFs:

$$\mathbf{T}^{\pm}_{\mathbf{3F}}(\mathbf{x_{123}}) \leftrightarrow \mathfrak{F}^{\pm}(x_{123})$$

where $\mathfrak{S}^{\pm}$ and $\mathfrak{F}^{\pm}$ are definite-C-parity distributions.

- Simmetries for $(x_1, x_2, x_3) \to (-x_3, -x_2, -x_1)$:

$$T(x_{123}) = T(-x_{321})$$

$$\Delta T(x_{123}) = -\Delta T(-x_{321})$$

$$T^{\pm}_{3F}(x_{123}) = T^{\pm}_{3F}(-x_{321})$$

Twist-3 evolution

- Quark PDFs:

$$\mathbf{T}(\mathbf{x_{123}}), \mathbf{\Delta T}(\mathbf{x_{123}}) \leftrightarrow \mathfrak{S}^{\pm}(x_{123})$$

- Gluon PDFs:

$$\mathbf{T}^{\pm}_{\mathbf{3F}}(\mathbf{x_{123}}) \leftrightarrow \mathfrak{F}^{\pm}(x_{123})$$

where $\mathfrak{S}^{\pm}$ and $\mathfrak{F}^{\pm}$ are definite-C-parity distributions.

$$T(x_{123}) = \frac{1}{4}\Big(\mathfrak{S}^{+}(x_{123}) + \mathfrak{S}^{+}(x_{321}) + \mathfrak{S}^{-}(x_{123}) - \mathfrak{S}^{-}(x_{321})\Big)$$

$$\Delta T(x_{123}) = -\frac{1}{4}\Big(\mathfrak{S}^{+}(x_{123}) - \mathfrak{S}^{+}(x_{321}) + \mathfrak{S}^{-}(x_{123}) + \mathfrak{S}^{-}(x_{321})\Big)$$

$$T^{\pm}_{3F}(x_{123}) = \frac{1}{2}\Big(\mathfrak{F}^{\pm}(x_{123}) \mp \mathfrak{F}^{\pm}(x_{321})\Big)$$

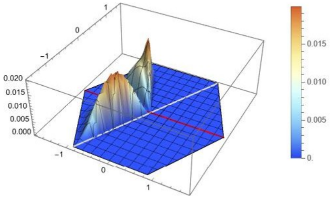
Twist-3 evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} \mathfrak{S}^{\pm}_{NSi} = -a(\mu^2)\mathbb{H}_{NS}\mathfrak{S}^{\pm}_{NSi}$$

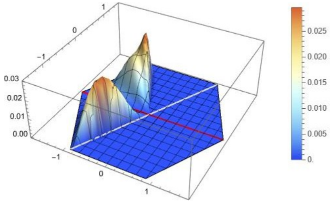
$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \mathfrak{S}^{\pm}_{S} \\ \mathfrak{F}^{\pm} \end{pmatrix} = -a(\mu^2) \begin{pmatrix} \mathbb{H}^{\pm}_{qq} & \mathbb{H}^{\pm}_{qg} \\ \mathbb{H}^{\pm}_{qg} & \mathbb{H}^{\pm}_{gg} \end{pmatrix} \begin{pmatrix} \mathfrak{S}^{\pm}_{S} \\ \mathfrak{F}^{\pm} \end{pmatrix}$$

- Non-singlet (NS) combinations evolve independently
- Singlet (S) contribution mixes with gluon ($\mathfrak{S}^{\pm}_{S} = \sum_f \mathfrak{S}^{\pm}_{f}$)

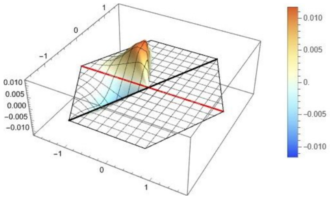
Initial scale functions (0.26 GeV^2)



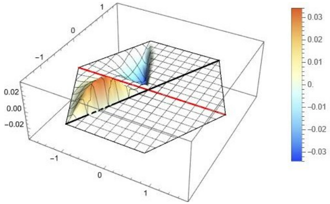
$T_d(x_{123})$



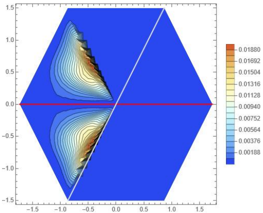
$-T_u(x_{123})$



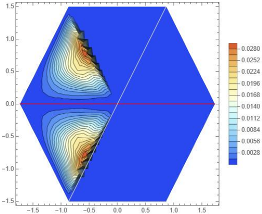
$\Delta T_d(x_{123})$



$\Delta T_u(x_{123})$



$T_d(x_{123})$



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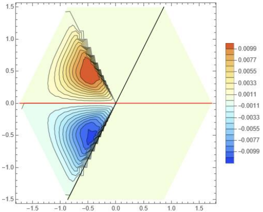
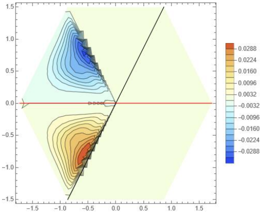
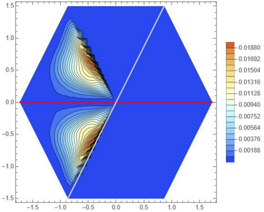


$\Delta T_d(x_{123})$

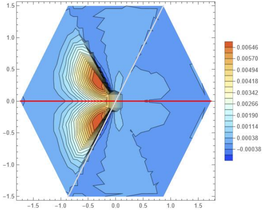


$\Delta T_u(x_{123})$

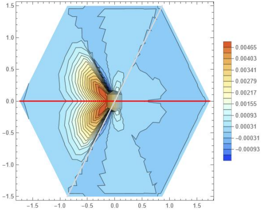
Evolution of T_f



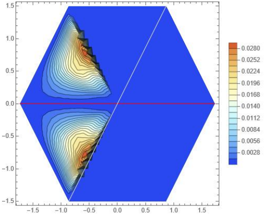
$T_d(x_{123})$ $Q^2 = 0.26\ \text{GeV}^2$



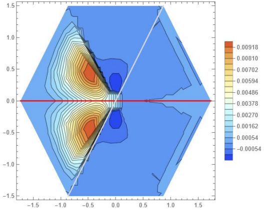
$T_d(x_{123})$ $Q^2 = 25\ \text{GeV}^2$



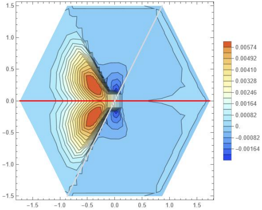
$T_d(x_{123})$ $Q^2 = 10^4\ \text{GeV}^2$



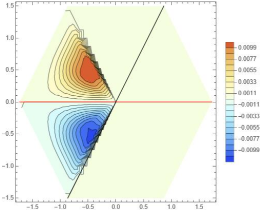
$-T_u(x_{123})$ $Q^2 = 0.26\ \text{GeV}^2$



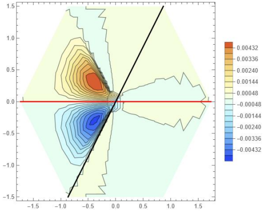
$-T_u(x_{123})$ $Q^2 = 25\ \text{GeV}^2$



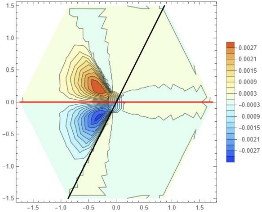
$-T_u(x_{123})$ $Q^2 = 10^4\ \text{GeV}^2$



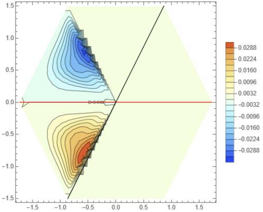
$\Delta T_d(x_{123})$ $Q^2 = 0.26$ GeV^2



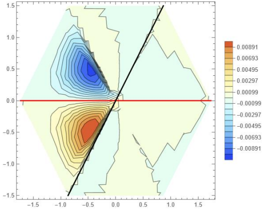
$\Delta T_d(x_{123})$ $Q^2 = 25$ GeV^2



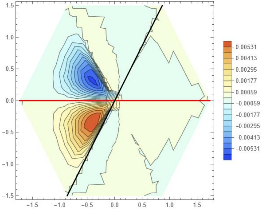
$\Delta T_d(x_{123})$ $Q^2 = 10^4$ GeV^2



$\Delta T_u(x_{123})$ $Q^2 = 0.26$ GeV^2

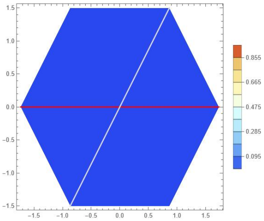


$\Delta T_u(x_{123})$ $Q^2 = 25$ GeV^2

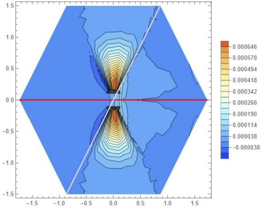


$\Delta T_u(x_{123})$ $Q^2 = 10^4$ GeV^2

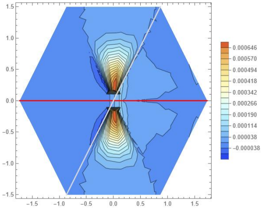
Evolution for other flavours



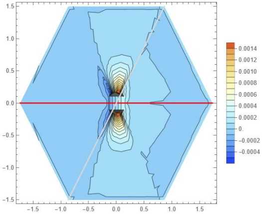
$T_s(x_{123}) \qquad Q^2 = 0.26\ \text{GeV}^2$



$T_s(x_{123}) \qquad Q^2 = 4\ \text{GeV}^2$

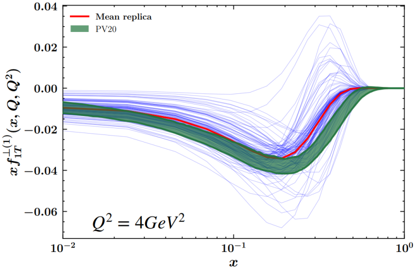


$T_s(x_{123}) \qquad Q^2 = 100\ \text{GeV}^2$

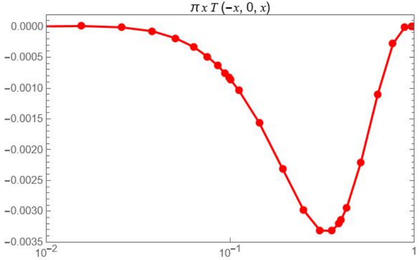


$T_s(x_{123}) \qquad Q^2 = 10^4\ \text{GeV}^2$

Frame Title



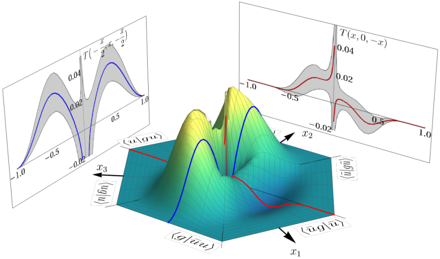
Mean replica
PV20
$x f_{1T}^{\perp(1)}(x, Q, Q^2)$
$Q^2 = 4GeV^2$
x
0.04
0.02
0.00
−0.02
−0.04
−0.06
10^{-2}
10^{-1}
10^{0}

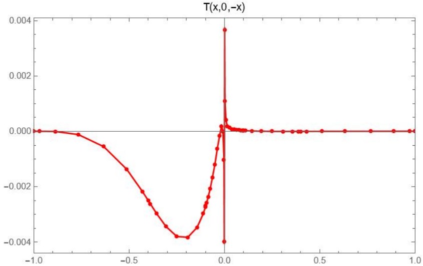


$\pi\, x\, T\,(-x, 0, x)$
0.0000
−0.0005
−0.0010
−0.0015
−0.0020
−0.0025
−0.0030
−0.0035
10^{-2}
10^{-1}
1

[–]

Frame Title


$T(x,0,-x)$
0.04
0.02
-1.0
-0.5
-0.02
0.5
1.0
x_2
x_3
x_1


T(x,0,–x)
0.004
0.002
0.000
–0.002
–0.004
–1.0
–0.5
0.0
0.5
1.0

[–]

Thank you
for your attention!