Electroweak Corrections at the TeV Scale

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work done during many years in collaboration with
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Outlines in High energy EW physics

- Infrared (IR) Problem & the eikonal current
- Asymptotic states for **QED-QCD-EW**
- Leading Logs, $\alpha \log^2 \frac{Q^2}{M^2}$, Cancellation Theorems:
  KLN (Kinoshita-Lee-Nauenberg) theorem
  BN (Block-Nordsieck) theorem
  EX: Structure of double logs in Sudakov form factors in EW
  IN: Structure of double logs in EW BN violation observables
- EW DGLAP: evolution equations for structure functions
  EW DGLAP Sum Rules
- EW physics in **Cosmology** (**heavy** DM annihilation or decay)
- Exotics : *Power Suppressed amplitudes* $\left(\alpha \frac{m^2}{Q^2} \log^2 \frac{Q^2}{M^2}\right)$
  IN: Revised KLN theorem at one loop (real + virtual emission)
  EX: *Anomalous Sudakov* at all orders

Observables: Inclusive $\equiv$ *IN* & *Exclusive* $\equiv$ *EX*

IR Problem: a long history...

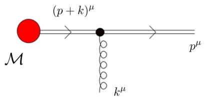
- (1937) Bloch and Nordsieck (BN) introduced a practical way to handle IR-divergences. They showed that while *individual cross sections* can be IR-divergent, the physically relevant *inclusive cross sections* where one sums over all possible emissions of soft radiation below a detector threshold remain IR-finite.
- (1962-64) Kinoshita-Lee-Nauenberg (KLN) theorem which states that sufficiently inclusive sums over degenerate *initial and final states* yield IR-finite probabilities.
- During the late 1960s and early 1970s: Chung, Kibble, and Dollard culminated in what became known as the *Faddeev-Kulish (FK)* approach. The FK approach modifies the S-matrix using asymptotic states surrounded by soft bosonic particles.
  See also M.Ciafaloni, Marchesini, Catani for a FK approach to QCD.
- Infrared triangle: relationship between soft theorems, memory effects, and asymptotic symmetries (Strominger, *Lectures on the Infrared Structure of Gravity and Gauge Theory*, arXiv:1703.05448.)

For very high energy processes with $\boxed{Q \gg M_W}$, the massive EW gauge bosons behave as *approximately* massless gauge bosons

massless gauge field $\sim$ massive gauge field (transverse)
where the mass (M_W) is becoming the physical cut off of the uncanceled IR divergences.

We can translate the resummation techniques of the IR structures in QED and QCD to the EW sector with some *interesting results !*

Eikonal structure for Leading Log corrections



$$\overline{u}(p)T^a\gamma^\mu \frac{\not p + \not k}{(k+p)^2}\mathcal{M} \sim \overline{u}(p)T^a\gamma^\mu \frac{\not p}{2k\cdot p}\mathcal{M}$$

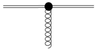
Dirac algebra for the numerator

$$\overline{u}(p)\gamma^\mu \not p = \overline{u}(p)\left[-\not p + 2p^\mu\right] = 2p^\mu \overline{u}(p)$$

$$\overline{u}(p)\mathcal{M} \times T^a \frac{p^\mu}{k\cdot p}$$

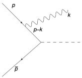
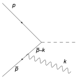
The emission of a soft gluon is *factorized*.

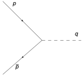
We extract the *eikonal* Feynman rules.



Vertex : $g_s\, p^\mu T^a$

Propagator : $\frac{1}{p\cdot k}$

$\sigma_{LP} \sim \int d\mathrm{PS}\,\Big|$  $+$  $\Big|^2$

$\sim \int d\mathrm{PS}\,\Big($  $-$  $\Big)^2 \Big|$  $\Big|^2$

$$\sim \int d\mathrm{PS}\,\underbrace{\left(\frac{p^\mu}{p\cdot k} - \frac{\bar{p}^\mu}{\bar{p}\cdot k}\right)^2}_{\text{eikonal current}} \quad |\mathcal{A}_2(p,\bar{p})|^2$$

$$\text{Factorization :} \quad \mathcal{M} = \mathcal{M}_{soft}\cdot\mathcal{M}_{Hard}$$

$$\mathcal{M}^{a,\mu}_{n+1}(p_1 \ldots p_n;\ k) = g\left(\sum_i^n T^a_i \frac{p_i^\mu}{p_i\cdot k}\right)\cdot \mathcal{M}_n(p_1\ldots p_n)$$

Emission amplitude of a soft gauge boson (γ, g, W) with with charge/color/isospin index **a**, $\mathbf{T}^{\mathbf{a}}_i$ "*charge/color/isospin*" operator for the i-th particle.

$$\boxed{J^{\mathbf{a}}{}_\mu(k) = g\sum_i \mathbf{T}^{\mathbf{a}}_i \frac{p_{i\mu}}{p_i\cdot k}}$$

Eikonal current features

$$\mathcal{M}^{a,\mu}_{n+1}(p_1 \ldots p_n;\ k) = g \left(\sum_i^n T_i^a \frac{p_i^\mu}{p_i \cdot k} \right) \cdot \mathcal{M}_n(p_1 \ldots p_n) \qquad J^{a}{}_{\mu}(k) = g \sum_i \mathbf{T}_i^{a} \frac{p_{i\mu}}{p_i \cdot k}$$

- Amplitude **factorization** (soft factors decouple from the hard dynamics).
- The eikonal current is **universal** in the sense that it is independent of the specific details of the hard scattering process: **kinematics** & **spins** of the hard partons.
- The soft current depends on the **momentum** and **charges** of all hard partons involved in the scattering process.
- **Soft** & **Collinear** Singularity: Poles at $k \to 0$ & $p_i \cdot k \to 0$
- The eikonal current captures both **real** (soft) and **virtual** (loop) corrections.
- The eikonal current is **conserved (gauge invariant)** for hard processes where *charge/color/isospin* are conserved!

$$k^\mu\ J^a_\mu(k) \cdot \mathcal{M}_n = g \sum_i \mathbf{T}_i^a \cdot \mathcal{M}_n = 0$$

- However, for QCD (EW) in contrast to QED, the soft emission of a gluon (W) carries away some colour (isospin) charge. Soft emission *does not factorize exactly* and leads to colour/isospin correlations.

Asymptotic States in SM: QED, QCD, EW

$\boxed{QED}$:

Abelian	U(1)	Unbroken
Photons	Singlets	
fermions	Charged	

$\boxed{QCD}$:

Non Abelian	SU(3)	Unbroken-Confining
Hadrons	Singlets	

$\boxed{EW}$:

Non Abelian	SU(2)	Spontaneously Broken
W, Z	Charged	
Left fermions	Charged	
h	Charged	

Cancellation Theorems for Leading IR divergencies

K.L.N. Theorem In a theory with massless fields, transition rates are free of IR divergences IF the summation over INITIAL and FINAL degenerate (*in Energy*) states is carried out.

B.N. in QED: IR divergences cancel out after summation over all degenerate **final** soft photons compatible with experimental detection.

B.N. in QCD : Leading IR singularities cancel after:

- summation over **final soft gluons**
- average over **final color** and **initial color** or for **color singlet** initial states (like protons)

QCD IR singularities in inclusive cross sections are cancelled when:

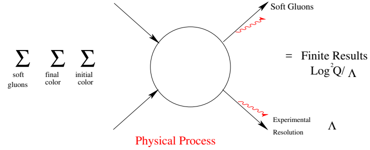
LO: **colours** *of hard partons are arranged in singlets*

NLO: **momenta** *of partons within the same singlet must be equal.* *(Perturbative indication for colour confinement)*

EW violation of the Block-Nordsiek Theorem

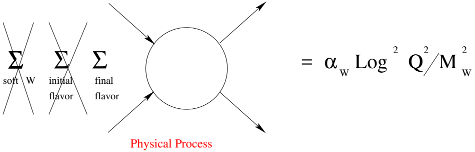
From QCD to EW: $SU(3) \to SU(2)$, (*Color* $\to$ Flavor), M_W physical IR cutoff

BN in QCD:



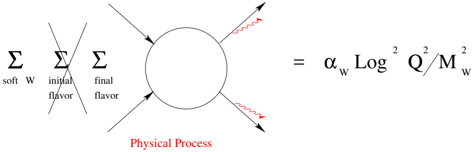
EW Sudakov

the Initial flavor is dictated by the accelerator



BN violation

the Initial flavor is fixed by the accelerator



From EW Perturbation Theory→large double logs in

$\boxed{ALL}$ high energy cross sections $\sigma(Q \gg M_W)$ with EW charged initial states

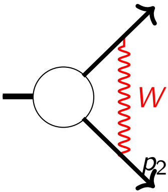
$$\frac{\Delta\sigma}{\sigma} = \alpha_W \left(\underbrace{Log^2 \frac{Q^2}{M_W^2} + Log \frac{Q^2}{M_W^2}}_{LHC+Next...?} + \underbrace{1 + o(\frac{m^2}{Q^2})}_{LEP} \right)$$

IN QCD and QED only single logs ($\alpha\ Log \frac{Q^2}{m^2}$) for sufficient inclusive observables! Typical size of the one loop logs ($Q = 1\ TeV$):

$$\frac{\alpha_W}{4\pi} Log^2 \frac{Q^2}{M_W^2} = 6.7\%, \qquad \frac{\alpha_{W/S}}{4\pi} Log \frac{Q^2}{M_W^2} = 1.4\,/\,3.6\ \%$$

High energy limit ($Q \to \infty$) $\equiv$ *Infrared limit ($M_W \to 0$)*

Sudakov Form Factor:



$$J^a_\mu(k) = g\left(T^a_1 \frac{p_{1\mu}}{p_1 \cdot k} + T^a_2 \frac{p_{2\mu}}{p_2 \cdot k}\right), \quad T^a_1 = -T^a_2$$

$$-\alpha_W \boxed{\sum_a (T^a_1)^2} \int d^3k \, \frac{p_1 \cdot p_2}{p_1 \cdot k \; p_2 \cdot k} \propto -\alpha_W \, t_1(t_1+1) \, Log^2 \frac{Q^2}{M^2}$$

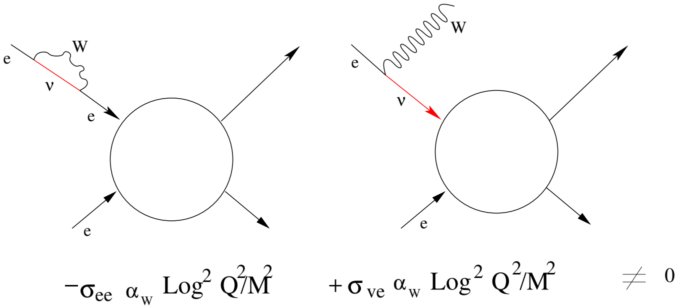
$$\sigma(s) = \sigma_H \; e^{-\frac{\alpha_W}{4\pi} \sum_i (C_i^2 + y_i^2 \tan^2\theta_W) \, Log^2 \frac{Q^2}{M_W^2}} \underbrace{\rightarrow}_{Q/M_W \to \infty} 0$$

$C_i^2 = t_i\,(t_i+1)$ is the external leg Casimir isospin (e.g., $t_i = \frac{1}{2}$ for a left fermion, $t_i = 1$ for a W)

Sudakov corrections always depress the Hard cross section.

EW violation of the Block-Nordsiek Theorem

One loop example of EW BN violation



Different coefficients (the hard cross sections) for virtual ($\propto \sigma_{e\bar{e}}$) and real ($\propto \sigma_{\nu\bar{e}}$) corrections

($\sigma_{\nu\bar{e}\to\sum_q q\bar{q}} \simeq 2\ \sigma_{e\bar{e}\to\sum_q q\bar{q}}$ for $Q^2 \gg M_W^2$)

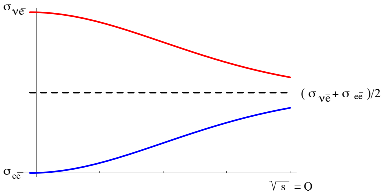
EW violation of the Block-Nordsiek Theorem

EW BN violation structure: Resumation Leading EW Virtual plus Real radiative corrections

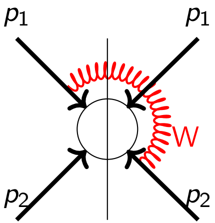
$$\sigma_{e\bar{e}}^{inclusive} = \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} + \frac{\sigma_{e\bar{e}}^{H} - \sigma_{\nu\bar{e}}^{H}}{2}\, e^{-\frac{\alpha_W}{2\pi} Log^2 \frac{Q^2}{M_W^2}} \underbrace{\rightarrow}_{Q\gg M_W} \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2}$$

$$\sigma_{\nu\bar{e}}^{inclusive} = \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2} - \frac{\sigma_{e\bar{e}}^{H} - \sigma_{\nu\bar{e}}^{H}}{2}\, e^{-\frac{\alpha_W}{2\pi} Log^2 \frac{Q^2}{M_W^2}} \underbrace{\rightarrow}_{Q\gg M_W} \frac{\sigma_{e\bar{e}}^{H} + \sigma_{\nu\bar{e}}^{H}}{2}$$

$(ex : \sigma_{\nu\bar{e}\rightarrow q\bar{q}}^{H} = 2\ \sigma_{e\bar{e}\rightarrow q\bar{q}}^{H})$ Effectively e_L becomes indistinguishable from ν_e



Eikonal current applied to the Overlap Matrix (a squared amplitude)



$$J^a_\mu(k) = g\left((T^a_1 - T^{a'}_1)\frac{p_{1\mu}}{p_1\cdot k} + (T^a_2 - T^{a'}_2)\frac{p_{2\mu}}{p_2\cdot k}\right),$$

$$\text{conservation} \qquad (T^a_1 - T^{a'}_1) = -(T^a_2 - T^{a'}_2) \equiv \vec{\mathbf{T}}$$

$$-\alpha \boxed{\sum_a (T^a_1 - T^{a'}_1)^2} \int d^3k \frac{p_1\cdot p_2}{(p_1\cdot k)(p_2\cdot k)} \rightarrow -\alpha\, \vec{\mathbf{T}}^2\, Log^2\frac{Q^2}{M^2},$$

$$\vec{\mathbf{T}}^2 = T_1(T_1+1)$$

$$\text{for fermions:}\ \frac{1}{2}\otimes\frac{1}{2} \rightarrow T_1 = 0,\, 1, \quad \mathbf{T}^2_f = 0,\, 2$$

$$\text{for bosons } W:\ 1\otimes 1 \rightarrow T_1 = 0,\, 1,\, 2, \ \mathbf{T}^2_W = 0,\, 2,\, 6$$

The hard cross section is first decomposed in total t-channel isospin basis:

$$\sigma(s) = \sum_T e^{-\frac{1}{2}\frac{\alpha_W}{4\pi}T(T+1)Log^2\frac{Q^2}{M_W^2}}\ \sigma^H_T \underbrace{\rightarrow}_{Q/M_W\rightarrow\infty} \sigma^H_{T=0}, \qquad \sigma^H_{T\neq 0} \lessgtr 0$$

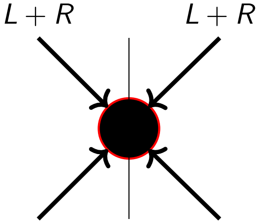
T is the total isospin obtained by composing two single leg isospins in $t-$channel.

$\sigma^H_{T=0}$ is the average cross section.

BN violating corrections can be negative or positive.

Abelian BN violation

The effect is present also for a chiral U(1) Hypercharge gauge group where, due to the spontaneous symmetry breaking, *mass eigenstate $\neq$ gauge eigenstate*.



examples:

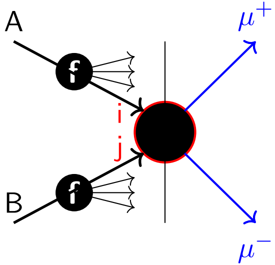
Fermionic transverse polarized beams $(\alpha\,|L>_{y_L}+\beta\,|R>_{y_R})$

higgs/goldstone state $(h/\phi=|H>_{y_h=1/2}\pm|H^*>_{y_h^*=-1/2})$

$$\sigma_{L+R}\sim\sigma_L+\sigma_R+(\mathcal{M}_L\,\mathcal{M}_R^*+h.c.)\;e^{-\alpha_y(y_L-y_R)^2\log^2 Q^2}$$

Factorization & **Structure Function** (fragmentation functions) approach

Resummation of **IR** & **collinear** logs



$$\sigma_{AB\to\mu^+\mu^-+X} = \sum_{i,j}^{f,W,\Phi,g} \int dx_1\ dx_2\ \mathbf{f}_{iA}(x_i, Q, M_W)\ \mathbf{f}_{jB}(x_j, Q, M_W)\ \ \sigma^H_{ij}(x_i\, x_j\, Q^2)$$

$$\sigma = f^t \otimes \sigma^H \otimes f \qquad\qquad (a\otimes b)\,(x) \equiv \int_0^1 dx_1\ dx_2\ a(x_1)\ b(x_2)\ \delta(x_1\ x_2 - x)$$

QCD : $f_{q_i},\ f_{\bar q_i},\ f_g \ :\ 2\,n_f\,N_f+1,\ n_f=2$ quark flavours, $N_f=3$ families

EW: $\underbrace{f_{l_{L\,i}},\ f_{l_{R\,i}},\ f_{\bar l_{L\,i}},\ f_{\bar l_{R\,i}},\ f_{q_{L\,i}},\ f_{q_{R\,i}},\ f_{\bar q_{L\,i}},\ f_{\bar q_{R\,i}}}_{2\,n_f\,N_f,\ \ N_f=3,\ n_f=8},\ \underbrace{f_h,\ f_{\phi^a}}_{4},\ \underbrace{f_{B_\mu},\ f_{W^a_\mu},\ f_g}_{5},\ \underbrace{f_{BW^3},\ f_{h\phi^3}}_{mixing},\ \underbrace{f_{LR\,i}}_{trans\ pol.}$

DGLAP running scales: $\underbrace{\Lambda_{QCD} < \mu < M_W}_{QED+QCD}\ \underbrace{matching}_{\Leftrightarrow}\ \underbrace{M_W < \mu < ?}_{QCD+EW(QED)}$

$$-\frac{\partial f(x,\mu)}{\partial \log \mu^2} = \{\alpha_{QED}\ f\otimes P_{QED}\}\ \theta(m_e^2<\mu^2<M_W^2)+\{\alpha_S\ f\otimes P_{QCD}\}\ \theta(\Lambda^2<\mu^2<M_W^2)+$$

$$\left\{\alpha_W\ f\otimes\left(\underbrace{\log\frac{Q^2}{\mu^2}\ P_{EW}^{(IR)}}_{New}+P_{EW}\right)+\alpha_S\ f\otimes\ P_{QCD}\right\}\ \theta(M_W^2<\mu^2<Q^2)$$

QCD: $P^V_{ii}+P^R_{ii},\ P^R_{ij}$, μ-independent

EW: $P^V_{ii},\ P^R_{ii},\ P^R_{ij}$, explicitly μ-dependent

Schrödinger like evolution equation

$$\partial_t\, \mathbf{f}(t) = \mathbf{P}(t) \otimes \mathbf{f}(t) \to \mathbf{f}(t) = \mathbf{U}(t,\, t_0) \otimes \mathbf{f}(t_0), \quad \mathbf{U}(t,\, t_0) = T_t\, e^{\int_{t_0}^{t} dt'\ \mathbf{P}(t')}$$

$$\text{basis } \mathbf{f} \equiv \left|\ \mathbf{l}_{e,\nu,\ldots} \quad \mathbf{w}_{W^{\pm,3},B} \quad \mathbf{q}_{u,d,\ldots} \quad \mathbf{g}\ \right|$$

$$\text{"}\mathit{Hamiltonian}\text{"} \quad \mathbf{P}(t) = \alpha_s(t)\, P_{QCD} + \alpha_w\, P_{EW}(t)$$

$$P_{QCD} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & P_{qq} & P_{qg} \\ 0 & 0 & P_{gq} & P_{gg} \end{vmatrix}, \qquad P_{EW}(t) = \begin{vmatrix} P_{ll} & P_{lw} & 0 & 0 \\ P_{wl} & P_{ww} & P_{wq} & 0 \\ 0 & P_{qw} & P_{qq} & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$e^{\alpha_s\, P_{QCD} + \alpha_w\, P_{EW}} = \left(1 - \frac{\alpha_s\, \alpha_w}{2} [P_{QCD},\, P_{EW}] + \ldots\right)\, e^{\alpha_s\, P_{QCD}}\, e^{\alpha_w\, P_{EW}}$$

$$[P_{QCD},\, P_{EW}] = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & P_{qq}\, P_{qw} & 0 & -P_{qg}\, P_{qq} \\ 0 & P_{gq}\, P_{qw} & P_{gq}\, P_{qq} & 0 \end{vmatrix}$$

EW and QCD sectors are connected through the channels $l \leftrightarrow \boxed{W \leftrightarrow q} \leftrightarrow g$

- gauge basis $f_{\nu_L},\ f_{e_L},\ f_{W^\pm},\ f_{W^3}$

$$\underbrace{\partial f}_{N\ eqs} = \alpha \underbrace{f}_{N} \otimes \underbrace{P}_{NxN\ matrix} , \qquad N = 5$$

- In total T channel isospin the new $P^{(T)}$ kernel is block diagonal

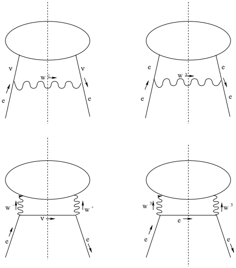
$$\boxed{f_L^{(0)} = \frac{f_\nu + f_e}{2},\ f_W^{(0)} = \frac{f_+ + f_3 + f_-}{3}}$$

$$\boxed{f_L^{(1)} = \frac{f_\nu - f_e}{2},\ f_W^{(1)} = \frac{f_+ - f_-}{2}}$$

$$\boxed{f_W^{(2)} = \frac{f_+ + f_- - 2f_3}{6}}$$

$$\underbrace{\partial f(T)}_{N_T\ eqs} = \alpha \underbrace{f(T)}_{N_T} \otimes \underbrace{P^{(T)}}_{N_T x N_T\ matrix} \qquad \underbrace{N_0 = 2,\ N_1 = 2,\ N_2 = 1}_{5}$$

Example DGLAP in the gauge base $(\nu,\ e)_L$ and $(W^{\pm},\ W^3)$:



$$
\begin{aligned}
-\frac{4\pi}{\alpha_W}\frac{\partial f_\nu}{\partial t} &= f_\nu \otimes (3P_{ff}^V + P_{ff}^R) + 2f_e \otimes P_{ff}^R + 2f_+ \otimes P_{gf}^R + f_3 \otimes P_{gf}^R \\
-\frac{4\pi}{\alpha_W}\frac{\partial f_e}{\partial t} &= f_e \otimes (3P_{ff}^V + P_{ff}^R) + 2f_\nu \otimes P_{ff}^R + 2f_- \otimes P_{gf}^R + f_3 \otimes P_{gf}^R \\
-\frac{2\pi}{\alpha_W}\frac{\partial f_+}{\partial t} &= \frac{f_{\bar e} + f_\nu}{2} \otimes P_{fg}^R + f_3 \otimes P_{gg}^R + f_+ \otimes (P_{gg}^R + 2P_{gg}^V) \\
-\frac{2\pi}{\alpha_W}\frac{\partial f_-}{\partial t} &= \frac{f_{\bar \nu} + f_e}{2} \otimes P_{fg}^R + f_3 \otimes P_{gg}^R + f_- \otimes (P_{gg}^R + 2P_{gg}^V) \\
-\frac{2\pi}{\alpha_W}\frac{\partial f_3}{\partial t} &= \frac{f_{\bar e} + f_e + f_{\bar \nu} + f_\nu}{4} \otimes P_{fg}^R + (f_- + f_+) \otimes P_{gg}^R + 2f_3 \otimes P_{gg}^V
\end{aligned}
$$

$t = \log \mu$

Example DGLAP in the t-channel base

$$\textit{Fermions}\,(f)\ :\ \frac{1}{2}\otimes\frac{1}{2} = 0\oplus 1 \qquad \textit{Gauge bosons}\,(g)\ :\ 1\otimes 1 = 0\oplus 1\oplus 2$$

$$\underset{L}{f}(0) = \frac{f_\nu + f_e}{2},\quad \underset{L}{f}(1) = \frac{f_\nu - f_e}{2},\quad \underset{W}{f}(0) = \frac{f_+ + f_3 + f_-}{3},\quad \underset{W}{f}(1) = \frac{f_+ - f_-}{2},\quad \underset{W}{f}(2) = \frac{f_+ + f_- - 2f_3}{6}$$

$$\boxed{T=0}\quad \begin{cases} \frac{\partial}{\partial t}\underset{L}{f}(0) = \frac{\alpha_W}{2\pi}\left(\frac{3}{4}\underset{L}{f}(0)\otimes(P^R_{ff}+P^V_{ff}) + \frac{3}{4}\underset{W}{f}(0)\otimes P^R_{gf}\right) \\ \frac{\partial}{\partial t}\underset{W}{f}(0) = \frac{\alpha_W}{2\pi}\left(2\,\underset{W}{f}(0)\otimes(P^R_{gg}+P^V_{gg}) + \frac{1}{2}\big(\underset{L}{f}(0)+\underset{\bar{L}}{f}(0)\big)\otimes P^R_{fg}\right) \end{cases}$$

$$\boxed{T=1}\quad \begin{cases} \frac{\partial}{\partial t}\underset{L}{f}(1) = \frac{\alpha_W}{2\pi}\left(\underset{L}{f}(1)\otimes P^V_{ff} - \frac{1}{4}\underset{L}{f}(1)\otimes(P^R_{ff}+P^V_{ff}) + \frac{1}{2}\underset{W}{f}(1)\otimes P^R_{gf}\right) \\ \frac{\partial}{\partial t}\underset{W}{f}(1) = \frac{\alpha_W}{2\pi}\left(\underset{W}{f}(1)\otimes P^V_{gg} + \underset{W}{f}(1)\otimes(P^R_{gg}+P^V_{gg}) + \frac{1}{2}\big(\underset{L}{f}(1)+\underset{\bar{L}}{f}(1)\big)\otimes P^R_{fg}\right) \end{cases}$$

$$\boxed{T=2}\quad \left\{\frac{\partial}{\partial t}\underset{W}{f}(2) = \frac{\alpha_W}{2\pi}\left(3\,\underset{W}{f}(2)\otimes P^V_{gg} - \underset{W}{f}(2)\otimes(P^R_{gg}+P^V_{gg})\right)\right.$$

DGLAP in t-channel isospin base

$$f_i^{(\mathbf{T})} = Tr[\mathcal{P}_{\mathbf{T}}\ f_i] \qquad i = g,\ B,\ W,\ \Phi,\ R,\ L,\ \bar{R},\ \bar{L}, \qquad \mathcal{P}_{\mathbf{T}} \text{ are isospin projectors}$$

$$-\frac{\partial}{\partial \log \mu^2} f_i^{(\mathbf{T})} = \alpha \left(\frac{\mathbf{T}^2}{2}\ f_i^{(\mathbf{T})} \otimes \underbrace{P_{ii}^V(\mu)}_{singular} + (C_i - \frac{\mathbf{T}^2}{2})\ f_i^{(\mathbf{T})} \otimes \underbrace{(P_{ii}^V + P_{ii}^R)}_{regular} + \sum_j\ f_j^{(\mathbf{T})} \otimes P_{ji}^R \right)$$

$$P_{ii}^V(\mu) = \boxed{-\log \frac{Q^2}{\mu^2}} + \bar{P}_{ii}^V \qquad \bar{P}_{ii}^V = \frac{3}{2}\ (i = f),\ 2\ (i = \Phi),\ \frac{11}{6} - \frac{n_f}{6} - \frac{n_s}{24}\ (i = W)$$

- $\boxed{Sudakov}$: $P^R = 0$

$$-\frac{\partial}{\partial \log \mu^2} f_i^{(\mathbf{T})} = \alpha\ C_i\ f_i^{(\mathbf{T})} \otimes P_{ii}^V \to f_i^{(\mathbf{T})} = e^{-\alpha\ C_i \left(\frac{1}{2} \log^2 \frac{Q^2}{\mu^2} + \bar{P}_{ii}^V\ \log \frac{Q^2}{\mu^2} \right)}$$

- $\boxed{BN}$ at LL: $(P_{ii}^R + P_{ii}^V)_{IR} \to 0,\ P_{ij}^R \to 0$

$$-\frac{\partial}{\partial \log \mu^2} f_i^{(\mathbf{T})} = \alpha\ \frac{\mathbf{T}^2}{2}\ f_i^{(\mathbf{T})} \otimes (P_{ii}^V)_{IR}(\mu) \to f_i^{(\mathbf{T})} = e^{-\alpha\ \frac{\mathbf{T}^2}{2} \left(\frac{1}{2} \log^2 \frac{Q^2}{\mu^2} \right)}$$

DGLAP Sum Rules

$$\frac{\partial}{\partial \log \mu^2} f(x,\mu) = (P \otimes f)(x,\mu) \quad \underbrace{\rightarrow}_{\mathbf{f}(\mathbf{N},\epsilon)=\int_0^1 dz\; f(z,\epsilon)\, z^{\mathbf{N}-1}} \quad \frac{\partial}{\partial \log \mu^2} \mathbf{f}(\mathbf{N},\mu) = \mathbf{P}(\mathbf{N},\mu)\, \mathbf{f}(\mathbf{N},\mu)$$

two kind of singularities in the z integration of $P_{ij}^{(R/V)}(z)$:

$$\boxed{\text{for } z \to 1} \quad \int^{1-\epsilon} \frac{dz}{1-z} \sim -\log \epsilon, \qquad \boxed{\text{for } z \to 0} \quad \int_{\kappa} \frac{dz}{z} \sim -\log \kappa$$

$$P_{ff}^V = \left(\frac{3}{2} + \log \epsilon^2\right) \delta(1-z) \quad \rightarrow \quad \mathbf{P}_{ff}^V(1,\epsilon) = \mathbf{P}_{ff}^V(2,\epsilon) = \left(\frac{3}{2} + \log \epsilon^2\right)$$

$$P_{ff}^R = \frac{1+z^2}{1-z} \quad \rightarrow \quad \begin{cases} \mathbf{P}_{ff}^R(1,\epsilon) = -\frac{3}{2} - \log \epsilon^2, \\ \mathbf{P}_{ff}^R(2,\epsilon) = -\frac{17}{6} - \log \epsilon^2 \end{cases}$$

$$P_{gf}^R = \frac{1+(1-z)^2}{z} \quad \rightarrow \quad \mathbf{P}_{gf}^R(1,\kappa) = -\frac{3}{2} - \log \kappa^2$$

$$P_{gg}^V = \left(\frac{5}{3} + \log \epsilon^2\right) \delta(1-z) \quad \rightarrow \quad \mathbf{P}_{gg}^V(1,\epsilon) = \mathbf{P}_{gg}^V(2,\epsilon) = \left(\frac{5}{3} + \log \epsilon^2\right),$$

$$P_{gg}^R = 2\left(z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right) \rightarrow \begin{cases} \mathbf{P}_{gg}^R(1,\epsilon,\kappa) = -\frac{11}{3} - \log \epsilon^2 - \log \kappa^2, \\ \mathbf{P}_{gg}^R(2,\epsilon) = -\frac{11}{6} - \log \epsilon^2 \end{cases}$$

DGLAP Sum Rules

probabilistic interpretation of the Parton Distribution Functions & quantum numbers conservation (symmetries of the theory).

- $\boxed{\textbf{fermion number conservation}}$:

$\sum_a \int_0^1 dx\; f_{ae}(x,\mu) = 1 \leftrightarrow \frac{\partial}{\partial \mu^2}\tilde{f}_L^{(0)}(1,\mu) = 0$

$$\mathbf{P}_{ff}^R(1,\epsilon) + \mathbf{P}_{ff}^V(1,\epsilon) = 0$$

- $\boxed{\textbf{momentum conservation}}$:

$\sum_a \int_0^1 dx\; x\; f_{aj}(x,\mu) = 1,\; j = f,\; W \leftrightarrow \frac{\partial}{\partial \mu^2}\tilde{f}_j^{(0)}(2,\mu) = 0$

$$\mathbf{P}_{ff}^R(2,\epsilon) + \mathbf{P}_{ff}^V(2,\epsilon) + \mathbf{P}_{gf}^R(2) = 0$$

$$\mathbf{P}_{gg}^R(2,\epsilon) + \mathbf{P}_{gg}^V(2,\epsilon) + \frac{1}{2}\mathbf{P}_{fg}^R(2) + \frac{1}{2}\mathbf{P}_{\phi g}^R(2) = 0$$

Charges Conservation

bfcharges conservation : $\boxed{\sum_a \int_0^1 dx\ q_a\ f_{ai}(x,\mu) = q_i}$

Weak Isospin conservation: T^3

- Fermion: $\sum_a^{f,w} \int_0^1 dx\ T^3_a\ f_{a\nu}(x,\mu) = T^3_\nu = \frac{1}{2}$

$$3\ \mathbf{P}^V_{ff}(1,\epsilon) - \mathbf{P}^R_{ff}(1,\epsilon) + 4\ \mathbf{P}^R_{gf}(1,\kappa) = 0 \to \mathbf{P}^R_{ff}(1,\epsilon) = \ \mathbf{P}^R_{gf}(1,\kappa)$$

- Gauge boson: $\sum_a^{w,f} \int_0^1 dx\ T^3_a\ f_{aw^+}(x,\mu) = 1$

$$\frac{1}{2}\ \mathbf{P}^R_{fg}(1) + \mathbf{P}^R_{gg}(1,\epsilon,\kappa) + 2\ \mathbf{P}^V_{gg}(1,\epsilon) = 0$$

EW Kernels

$$\mathbf{P}^R_{ff}(2,\epsilon)+\mathbf{P}^V_{ff}(2,\epsilon)=\mathcal{O}(1),\ \mathbf{P}^R_{gg}(2,\epsilon)+\mathbf{P}^V_{gg}(2,\epsilon)=\mathcal{O}(1),\ \mathbf{P}^R_{ff}(1,\epsilon)+\mathbf{P}^V_{ff}(1,\epsilon)=0$$

$$\boxed{\mathbf{P}^R_{ff}(1,\epsilon)=\mathbf{P}^R_{gf}(1,\kappa)} \qquad \boxed{\mathbf{P}^R_{gg}(1,\epsilon,\kappa)+2\,\mathbf{P}^V_{gg}(1,\epsilon)=-\frac{1}{2}\,\mathbf{P}^R_{fg}(1)} \quad \rightarrow \quad \boxed{\boxed{\kappa\equiv\epsilon}}$$

$$P^R_{ff}(x,\epsilon)=\frac{1+x^2}{1-x}\,\boxed{\theta(1-x-\epsilon)},\quad P^V_{ff}(x,\epsilon)=-\delta(1-x)\left(\log\frac{1}{\epsilon^2}-\frac{3}{2}\right)$$

$$P^R_{gf}(x,\epsilon)=\frac{1+(1-x)^2}{x}\,\boxed{\theta(x-\epsilon)}$$

$$P^V_{gg}(x,\epsilon)=-\delta(1-x)\left(\log\frac{1}{\epsilon^2}-\frac{5}{3}\right)$$

$$P^R_{gg}(x,\epsilon)=2\left(x\,(1-x)+\frac{x}{1-x}+\frac{1-x}{x}\right)\boxed{\theta(x-\epsilon)}\,\boxed{\theta(1-x-\epsilon)}$$

$$P^R_{fg}(x,\epsilon)=\left(x^2+(1-x)^2\right)$$

Parton Splitting Relations

- Parton exchange:

$$P^R_{ff}(x,\epsilon) = P^R_{gf}(1-x,\epsilon), \quad P^R_{gg}(x,\epsilon) = P^R_{gg}(1-x,\epsilon)$$

- Crossing relation:

$$P^R_{fg}(x) = x\,P^R_{gf}\left(\frac{1}{x},\epsilon\right), \quad P^R_{gg}(x) \neq -x\,P^R_{gg}\left(\frac{1}{x},\epsilon\right)$$

- Supersymmetry relation:

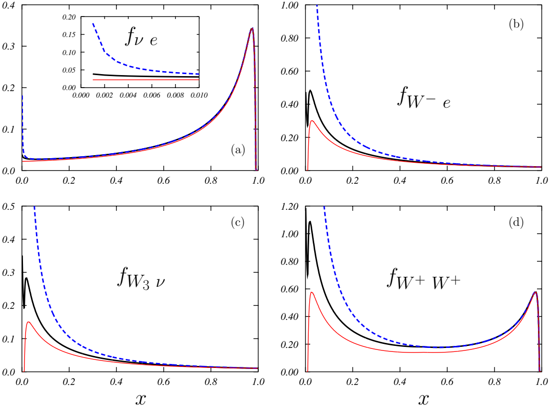
$$P^R_{ff}(x,\epsilon) + P^R_{gf}(x,\epsilon) = P^R_{fg}(x) + P^R_{gg}(x,\epsilon)$$

- Conformal Invariance

$$\left(x\frac{d}{dx}-2\right)P^R_{fg}(x) \neq \left(x\frac{d}{dx}+1\right)P^R_{gf}(x,\epsilon)$$

$$\hat{P}^R_{gf}(x,\epsilon) = \underbrace{P^R_{gf}(x)}_{\frac{1+(1-x)^2}{x}}\ \theta(x-\epsilon), \quad f_{gf}(x,\epsilon) \sim \alpha \int_\epsilon^1 \frac{d\epsilon'}{\epsilon'} P^R_{fg}(x,\epsilon') \to \begin{cases} f_{gf}(x,\epsilon) \sim \alpha\ P^R_{gf}(x)\ \boxed{\log\frac{1}{\epsilon}} \\ \hat{f}_{gf}(x,\epsilon) \sim \alpha\ P^R_{gf}(x)\ \boxed{\theta(x-\epsilon)\log\frac{x}{\epsilon}} \end{cases}$$

$$\epsilon = 0.01$$



Black lines: full calculations. **Red lines**: only the first order $\mathcal{O}(\alpha)$. **Blue dashed lines**: full calculations without constrains on the splitting functions.

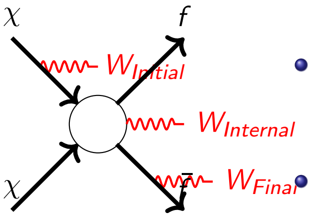
EW versus QCD DGLAP

Main Leading order differences...

Properties	*EW*	*QCD*
chirality *flavour*	*chiral* *Flavour changing*	*vector − like* *Flavour blind*
evolution range *initial conditions* μ *dependence* *diagonal* $P_{ii}^{R/V}$ *off − diagonal* P_{ij}	$\mu > M_W$ $f(M_W)$ $\alpha_W\ P(\mu)$ $\kappa_1\, P_{ii}^V(\mu) + \kappa_2\, P_{ii}^R(\mu)$ $P_{gf}^R(\mu)$	$\mu > \Lambda_{QCD}$ $f(\Lambda_{QCD})$ $\alpha_s(\mu)\ P$ $P_{ii}^V + P_{ii}^R$ P_{gf}^R
Log resummation	$\alpha_w^n\,(\log^{2n} Q^2 \div \log^n Q)$	$(\alpha_s\ \log Q^2)^n$
Mixed − distributions	$f_{Z_T\gamma},\ f_{Z_L h},\ f_{LR}$	

Plus "subleading" differences...

Annihilation or decay of heavy neutral relicts $\chi\chi \to f\bar{f} + W$ with
$\boxed{M_\chi \gg M_W}$

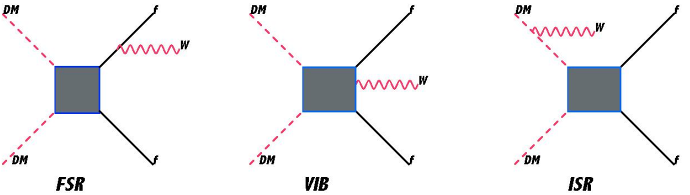


- *Final* state W emission: modification of the final state spectra (SM physics)
- *Internal* W emission: Change of chirality of the effective operators for the annihilation processes
- *Initial* state W emission: only for $SU(2)$ charged DM (wino, etc.) change of leading order effective operators for to the annihilation process

From $< \upsilon\, \sigma >_{\chi\chi \to f\, f}$ to $< \upsilon\, \sigma >_{\chi\chi \to f\, f\, W}$

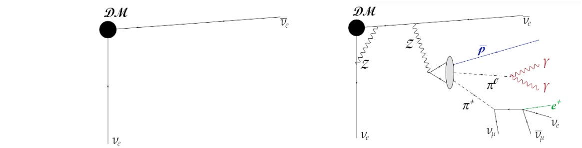
Indirect detection sensitive to the non relativistic DM velocity

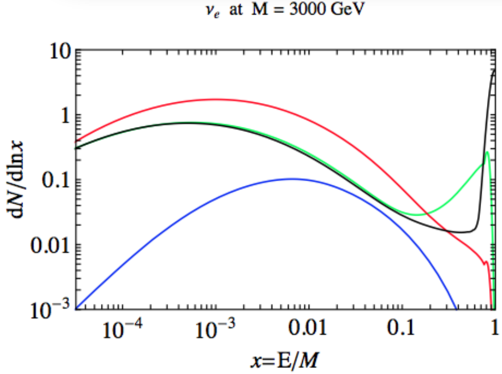
$< \upsilon\, \sigma > = \underbrace{\mathbf{a}}_{s-wave} + \underbrace{\mathbf{b}}_{p-wave} \upsilon^2$ with $\upsilon = 10^{-3}$, $\sigma(Majorana\ DM \to f\bar{f}) \propto \mathbf{b}\ v^2$



	$s - wave$ $\quad \sigma_{\chi\chi \to ffW} / \sigma_{\chi\chi \to ff}$	$p - wave$ $\quad \sigma_{\chi\chi \to ffW} / \sigma_{\chi\chi \to ff}$
FSR	$\alpha\ \log^{(n)} \sim \#\%$	$\alpha\ \log^{(n)} \sim \#\%$
VIB	$\alpha\, \frac{M_\chi^4}{\Lambda^4} \ll 1$	$\frac{\alpha}{\upsilon^2}\, \frac{M_\chi^4}{\Lambda^4} \sim \frac{10^4\ M_\chi^4}{\Lambda^4}$
ISR	$\alpha \sim \#\%$	$\frac{\alpha}{\upsilon^2} \geq 10^4$

$$\bar{\chi}\chi \to \nu + \bar{\nu} \quad \text{with } EW \to \quad \bar{\chi}\chi \to \nu + \bar{\nu} + Z$$





γ, e, ν, p .

Partially inclusive cross sections at LHC

Cross sections $PP \to Q\bar{Q} + X$ with $Q\bar{Q} = t\bar{t}, t\bar{b}, b\bar{t}, b\bar{b}$ tagged

$\sigma_H(g\ g \to Q\ \bar{Q})$ (2 EW legs),

$\sigma_H(q\ \bar{q} \to Q\ \bar{Q})$ (is a 3 EW legs because the **the proton sea** is, *almost*, an EW singlet)

Two kinds of observables:

$\boxed{EW\ Sudakov}$: $(PP \to \text{tagged final state} + X)$ with $\underline{W, Z \notin X}$

$\boxed{EW\ BN}$: $(PP \to \text{tagged final state} + X)$ with $\underline{W, Z \in X}$

Cross sections	Flavor	σ_H Tree Level	σ_{BH}	σ_{Sud}
$PP \to Q_i \bar{Q}_j + X$	$\delta_{ij}\ \ (t\ \bar{t},\ \ b\ \bar{b})$	$\alpha_s^2 + \mathcal{O}(\alpha_w^2)$	$\alpha_s^2\ \boxed{\alpha_w\ log^2 Q^2}$	$\alpha_s^2\ \boxed{\alpha_w\ log^2 Q^2}$
	$i \neq j\ \ (t\ \bar{b},\ \ b\ \bar{t})$	α_w^2	$\alpha_w^2\ \boxed{\frac{\alpha_s^2}{\alpha_w}\ log^2 Q^2}$	$\alpha_w^2\ \boxed{\alpha_w\ log^2 Q^2}$

$\alpha_w \to 0$ we have $d\sigma^{BN}_{t\bar{t}} \sim d\sigma^{H}_{t\bar{t}}\ \frac{1}{4}(3 + e^{-2\ L_W})$, $d\sigma^{BN}_{t\bar{b}} \sim d\sigma^{H}_{t\bar{t}}\ \frac{1}{4}(1 - e^{-2\ L_W})$

In the high-energy regime hard cross sections are *isospin-invariant*. **Vev insertions** (masses or interactions) generates mass suppressed terms $\mathcal{O}\left(\frac{v^2}{Q^2}\right)$ that **break isospin flow**.

What happens to the IR dynamics for Hard amplitudes with isospin not conserved?

Playground: chiral $U_{Z'}(1) \otimes U_Z(1)$ with $Q^2 \sim M_{Z'} \gg M_Z$.

- One Loop verification of KLN cancellation theorem (**real+virtual**) for IR terms $\alpha \left(\frac{m^2}{Q^2}\right)^n \log^2 Q^2$ for Z' decay.
- All order Sudakov Form factors (**only virtual**) for the amplitude $Z' \to f\ \bar{f}$

$$Z'^{\mu}\ \bar{u}(p_1)\left(\gamma_\mu \underbrace{(F_L\, P_L + F_R\, P_R)}_{\text{isospin conserving}} + \underbrace{\frac{m}{Q^2}(p_{1\,\mu} - p_{2\,\mu})\, F_M + \frac{m}{Q^2}(p_{1\,\mu} + p_{2\,\mu})\,\gamma_5\ F_P}_{\text{isospin breaking}}\right) v(p_2)$$

Power Suppressed IR double logs

Structure of power suppressed double logs corrections

$$\alpha \left(\frac{m_i^2}{Q^2}\right)^n Log^2 \frac{Q^2}{m_j^2} \Rightarrow \begin{cases} \to 0 & for \quad Q \to \infty \\ \to 0 & for \quad m_i \to 0 \;\&\; m_j = m_i \\ \to \infty & for \quad m_j \to 0 \;\&\; m_j \neq m_i \end{cases}$$

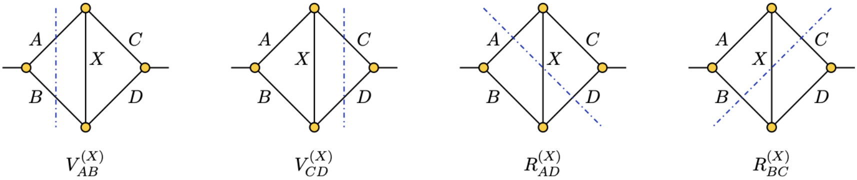
$n = 1, ..., 3$ at one-loop order.

All the one loop terms $\alpha \left(\frac{m^2}{Q^2}\right)^n Log^2 Q^2$ with $n = 0,\ 1,\ 2,\ 3.$

Can we define a combination of observables which is free from these power-suppressed terms?
i.e an "improved" KLN?

Power Suppressed KLN theorem

The sum over all possible cuts gives $0 \times \mathcal{O}\left(\alpha \left(\frac{m^2}{Q^2}\right)^n \log^2 Q^2\right) \; \forall n$



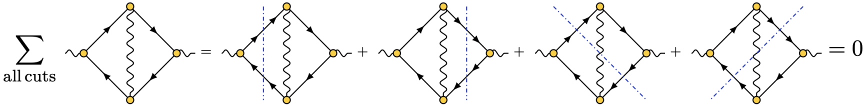
$$V_{AB}^{(X)} + V_{CD}^{(X)} + R_{AD}^{(X)} + R_{BC}^{(X)} = 0$$

$$R_{AD}^{(X)} = R_{BC}^{(X)} = -V_{CD}^{(X)} = -V_{AB}^{(X)}$$

NB: these equalities hold at the double log level and includes **all** power suppressed terms.

Power Suppressed KLN theorem in QED

In QED, power-suppressed double-logarithmic corrections at one loop cancel out, if we sum over all possible cut diagrams!

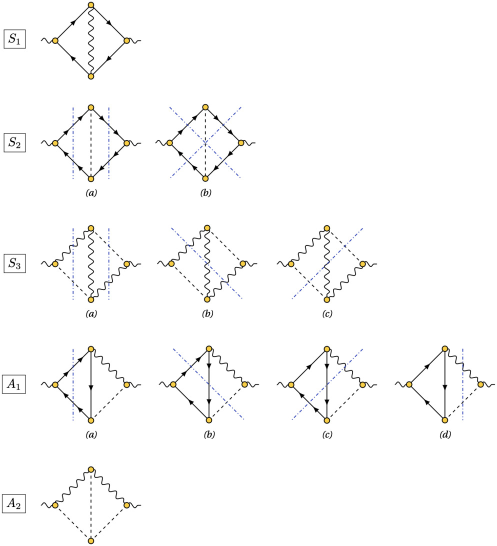


$$\Gamma_{Virtual_\gamma}(Z' \to \bar{\psi}\,\psi) \quad \sim \quad \left(\Gamma_0\,(1+\epsilon_\gamma^2)^2 + \Gamma_1\,\epsilon_\psi^2 + \Gamma_2\,\epsilon_\gamma^2\,\epsilon_\psi^2 + \Gamma_3\,\epsilon_\psi^4\right)\boxed{\log^2 \epsilon_\gamma^2},$$

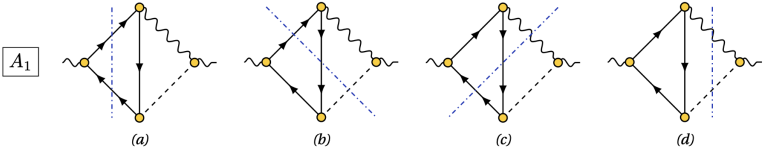
$$\epsilon_\gamma = \frac{\lambda_{IR}}{Q},\ \epsilon_\psi = \frac{m_\psi}{Q}$$

$$\Gamma_{Virtual_\gamma}(Z' \to \bar{\psi}\,\psi) + \Gamma_{Real}(Z' \to \bar{\psi}\,\psi\,\gamma) \underbrace{=}_{Q\to\infty} 0$$

Different cuts for each bubble generate different physical process



The presence of the A_1 bubble is the element that relate the decay rate of the Z' into fermionic and purely bosonic channels



Physical processes

Virtual $\rightarrow$ (a) : $Z' \rightarrow \boxed{\bar{\psi}\ \psi}$, (d) : $Z' \rightarrow \boxed{h\ Z}$

Real $\rightarrow$ (c) : $Z' \rightarrow \boxed{\bar{\psi}\ \psi\ Z}$, (b) : $Z' \rightarrow \boxed{\bar{\psi}\ \psi\ h}$

- *"Standard"* KLN cancellation mechanism:
  $\Gamma(Z' \to \boxed{\bar{f} f}) + \sum_X \Gamma(Z' \to \boxed{\bar{f} f} + \underbrace{X}_{soft})$ is IR safe.
- To cancel all **leading** and **power suppressed** terms we have to include **all possible decay channels**

$$\boxed{\Gamma(Z' \to \bar{f} f) + \sum_X^{Z,h} \Gamma(Z' \to \bar{f} f\ X) + \Gamma(Z' \to Zh) + \Gamma(Z' \to ZZZ) + \Gamma(Z' \to Zhh)} = \boxed{\textbf{IR safe}}$$

- it happens when $\begin{cases} m_f \neq 0 \\ U_Z(1): \quad y_h = y_R - y_L \neq 0 \\ U_{Z'}(1): \quad f_h = f_R - f_L \neq 0 \end{cases}$

heavy **chiral** Z' gauge boson produced at future colliders....

Improved degenerate states definition for KLN

The KLN theorem states that the transition amplitude squared is finite once we sum over initial and final degenerate states
Hypothesis: perturbation theory with degenerate states + unitarity of the theory.

$$|\bar{S}|^2 = \sum_{\phi_i, \phi_f \in \mathcal{D}(E)} | < \phi_f | S | \phi_i > |^2 < \infty$$

$\mathcal{D}(E)$ is given by all the states degenerate in energy in a $\Delta > 0$ range, i.e. $|E_{i/f} - E| < \Delta$

In the case of the decay of a "Neutral" particle Φ

$$\boxed{\sum_{\phi_f \in |E_f - E| < \Delta}} | < \phi_f | S | \Phi > |^2 = \boxed{\sum_{Kin} \sum_{Quantum\,Num} \sum_{Channel}} | < \phi_f | S | \Phi > |^2$$

Look inside the higher twist operators...

Why? Growing Sudakov Form Factors: $e^{+\alpha\, Log^2 \frac{Q^2}{M^2}}$

$$\sigma_H \sim \frac{\alpha^2}{Q^2}(1+\frac{M^2}{Q^2}) \underbrace{\rightarrow}_{\text{IR Virtual Cloud}} \sigma_{Sud} \sim \frac{\alpha^2}{Q^2}\left(e^{-\alpha\, Log^2 \frac{Q^2}{M^2}} + \frac{M^2}{Q^2} e^{+\alpha\, Log^2 \frac{Q^2}{M^2}}\right)$$

NB: $\frac{M^2}{Q^2} e^{+\alpha\, Log^2 \frac{Q^2}{M^2}} = \left(\frac{Q^2}{M^2}\right)^{\alpha \log \frac{Q^2}{M^2} - 1} \rightarrow \sigma_H \sim Q^{2\,(\alpha \log \frac{Q^2}{M^2} - 2)}$

depressing Sudakov $\boxed{e^{-\alpha\, Log^2 \frac{Q^2}{M^2}} \sim \frac{M^2}{Q^2}\; e^{+\alpha\, Log^2 \frac{Q^2}{M^2}}}$ *anomalous Sudakov*

Scale of overtaking beyond Planck scale....

$$Q \sim M\, e^{\frac{1}{4\,\alpha}}$$

Resummed Sudakov Effective $Z' \to \bar{f}\, f$

$$Z'^{\nu}\ \bar{u}(p_1)\left(\gamma_\mu(F_L\, P_L + F_R\, P_R) + \frac{m}{Q^2}(p_{1\,\mu} - p_{2\,\mu})F_M + \frac{m}{Q^2}(p_{1\,\mu} + p_{2\,\mu})\gamma_5\ F_P\right) v(p_2)$$

Form Factors: $F_{L,R}$ conserve chirality, $F_{M,P}$ violate chirality.
All order resummed form factors

$$\begin{aligned}
F_L &= \left(e^{-y_L^2 L^2} - \frac{\rho}{2}\ \left(e^{-y_R^2 L^2} - e^{-y_L^2 L^2}\right)\right) \\
F_R &= \left(e^{-y_R^2 L^2} - \frac{\rho}{2}\ \left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right)\right) \\
F_M &= \frac{1}{2}\left(e^{-y_L^2 L^2} +\ e^{-y_R^2 L^2}\right) - e^{-y_L\, y_R\, L^2} \\
F_P &= \frac{1}{2}\left(e^{-y_L^2 L^2} - e^{-y_R^2 L^2}\right);
\end{aligned}$$

$$\frac{\alpha}{4\pi}\log^2\frac{Q^2}{m_Z^2} \equiv L^2, \quad \rho = \frac{m^2}{p_1 \cdot p_2}$$

In the *magnetic dipole moment form factor* F_M we have an *Anomalous Sudakov* ($e^{-y_L\, y_R\, L^2}$) whose exponent can be *positive* if $\boxed{y_L\, y_R < 0}$.

From the quantum number of the SM fields we see that $U(1)$ "anomalous" Sudakov form factors are presents for the down quark sector where $y_L = \frac{1}{6}$ and $y_R = -\frac{1}{3}$ so that $y_L\, y_R = -\frac{1}{18} < 0$.

Conclusions

At very high scales ($Q > 10$ TeV), the QCD and EW interactions become of comparable strength. A unified DGLAP framework that includes both QCD and EW splittings is necessary.

- For extremely very high energies, the IR structure of EW physics seams worst than QCD because of the "**charged**" asymptotic states
- EW BN violation means an IR sensitivity for **any** observable of an high energy collider.
- EW versus QCD DGLAP evolution equations
- Many Collider & Cosmological implications...

Elitzur ' s theorem: Gauge symmetries can never be broken spontaneously
composite states rather than elementary ones as asymptotic in and out states, very much like hadrons in QCD.
The Inadequacy of the Gauge-Variant Field: They stress that the Higgs field $\Phi(x)$, being in the fundamental representation of the gauge group, is not a physical, observable operator. Only gauge-invariant quantities are physical. Therefore, its vacuum expectation value Φ is not a valid order parameter. one should consider the correlation functions of gauge-invariant composite operators that have the appropriate quantum numbers. in a certain regime (the "physical region" or scaling limit), the spectrum of this gauge-invariant correlator is related to the spectrum of the correlator of the gauge-variant elementary field in a fixed gauge (like the unitary gauge): The physical, gauge-invariant spectrum "shadows" the spectrum of the gauge-variant formalism. The masses calculated perturbatively in the unitary gauge (like the

Gauge group SU(2)
Fundamental Field (Φ) This field transforms under the fundamental representation of SU(2). This means it's gauge-variant.
Gauge-Invariant Composite Field (H):

$$H = \Phi^{+}\ \Phi = \sum_i |\phi_i|^2 \tag{1}$$

This is a scalar quantity. No matter how you perform an SU(2) gauge transformation, the value of Φ does not change. It is physically observable. NB: the famous "Mexican hat" potential. It is manifestly invariant under the full SU(2) gauge group because it depends only on the gauge-invariant $V = V(H)$ We characterize the phase of the theory by the expectation value of a gauge-invariant operator H. We do not need to point to a specific gauge-variant VEV Φ $< H >= 0$

IR versus UV evolution eqs

$$IR: \quad \frac{\partial}{\partial \log \epsilon}\mathbf{f} = \alpha\, \mathbf{P}(\epsilon) \otimes \mathbf{f}, \quad \epsilon = \frac{\mu}{Q}, \quad \mathbf{f}(Q) = f_0\, \mathbf{I}$$

$$UV: \quad \frac{\partial}{\partial \log e}\mathbf{f} = \alpha\, \mathbf{f} \otimes \mathbf{P}(e)\,, \quad e = \frac{M}{\mu}, \quad \mathbf{f}(M) = f_0\, \mathbf{I}$$

$$\mathbf{f}_{IR} = T_\epsilon e^{\alpha \int d\log\epsilon' \; P(\epsilon')} \otimes \mathbf{f}(Q), \qquad \mathbf{f}_{UV} = \mathbf{f}(M) \otimes \bar{T}_e e^{\alpha \int d\log e' \; P(e')}$$

$$\text{simplest parametrization}: \quad P(\chi) \equiv P^R + P^V \log\chi, \quad \chi = \epsilon,\, e$$

$$f_{UV} - f_{IR} \propto \alpha^2 \; \log^3\frac{Q}{M}\; \left[P^R,\, P^V\right] \to 0 \quad P_V \propto \mathbf{I}$$

Ingredient for one loop Power Suppressed KNL

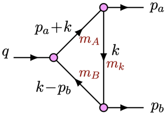
Virtual (tree point functions) + Real (three body dacays) corrections

$$\Gamma^{Virtual} = \underbrace{V\left(\frac{m_i^2}{Q^2}\right)}_{Polynomial\ dim\ \leq 6} Log^2 Q^2, \qquad \Gamma^{Real} = \underbrace{R\left(\frac{m_i^2}{Q^2}\right)}_{Polynomial\ dim\ \leq 6} Log^2 Q^2$$

- k integrals for virtual and real corrections
- IR unitarity theorem
- $p + k$ theorem: the algebraic manipulations done for the cut of a given bubble lead to the same result (apart from a sign) for virtual and real contributions.

Virtual corrections: the double-log structure of the scalar three-point integral

$$\mathcal{C}_0\left(q^2, p_a^2, p_b^2, m_A^2, m_B^2, m_k^2\right) =$$



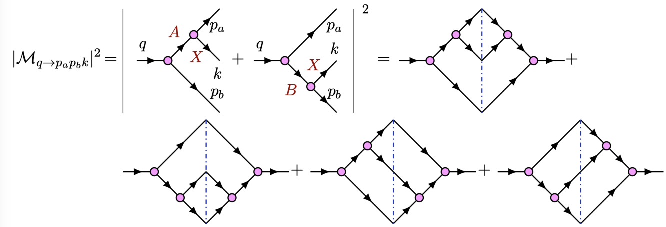
$$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_k^2 + i\epsilon)\left[(p_a+k)^2 - m_A^2 + i\epsilon\right]\left[(p_b-k)^2 - m_B^2 + i\epsilon\right]}$$

$$\mathcal{C}_0(\underbrace{Q^2, m_a^2, m_b^2}_{external\ parameters}, \underbrace{m_A^2, m_B^2, m_k^2}_{internal}) \underbrace{=}_{Q \to \infty} \underbrace{\frac{1}{\Phi_2(m_a^2, m_b^2)} \frac{i}{32\,\pi^2\,Q^2} Log^2 Q^2}_{only\ external\ parameters}$$

$$\Gamma^V \propto V_{AB}^X\left(\frac{m^2}{Q^2}\right) \Phi_2\ \mathcal{C}_0$$

$$\Phi_2(m_a^2, m_b^2) \equiv \frac{1}{16\,\pi\,Q}\sqrt{1 - \frac{2\,(m_a^2 + m_b^2)}{Q^2} + \frac{2\,(m_a^2 - m_b^2)}{Q^4}}$$

Real corrections: the double-log structure of a particle emission

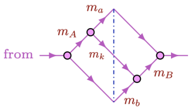


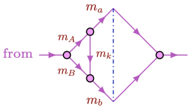
$$\Gamma^R \propto \int |\mathcal{M}_{q\to p_a p_b k}|^2 \, d\Phi_3(q; p_a, p_b, k) \propto R^X_{AB}\left(\frac{m^2}{Q^2}\right) \int \frac{dE_a\, dE_k}{D_A\, D_B}$$

$D_{A,B}$: propagators of the intermediate states

Virtual versus real corrections

$$\underbrace{\int_{\rm BC} \frac{dE_a dE_k}{[(p_a+k)^2 - m_A^2][(p_b+k)^2 - m_B^2]}} = \underbrace{-\frac{1}{4}\Phi_{\rm LIPS}(m_a^2, m_b^2)\mathcal{C}_0(Q^2, m_a^2, m_b^2, m_A, m_B, m_k)} = \frac{1}{8Q^2}\log^2 Q^2\,,$$





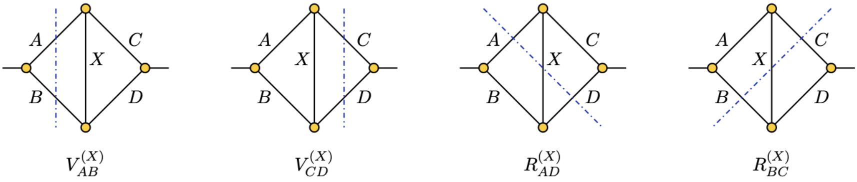
where we define the two-body Lorentz invariant phase space

$$\Phi_{\rm LIPS}(m_a^2, m_b^2) \equiv \sqrt{1 - \frac{2(m_a^2 + m_b^2)}{Q^2} + \frac{(m_a^2 - m_b^2)^2}{Q^4}}\,,$$

$$\Gamma^R = R^X_{AB}\left(\frac{m^2}{Q^2}\right)\ Log^2 Q^2, \qquad \Gamma^V = V^X_{AB}\left(\frac{m^2}{Q^2}\right)\ Log^2 Q^2$$

IR unitarity theorem

The sum over all possible cuts of a given diagram gives $0 \log^2 Q^2$



$$V^{(X)}_{AB} + V^{(X)}_{CD} + R^{(X)}_{AD} + R^{(X)}_{BC} = 0$$

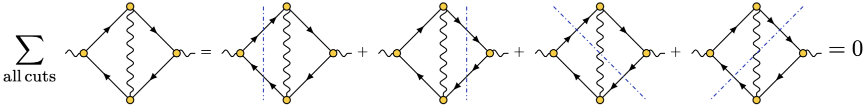
"p + k theorem"

$$R^{(X)}_{AD} = R^{(X)}_{BC} = -V^{(X)}_{CD} = -V^{(X)}_{AB}$$

NB: this equality holds at the <u>double log level</u> and includes <u>all power suppressed terms</u>.

IR unitarity theorem

In QED, power-suppressed double-logarithmic corrections at one loop cancel out, if we sum over all possible cut diagrams!



$$\Gamma_{Virtual_\gamma}(Z' \to \bar{\psi}\,\psi) \quad \sim \quad \left(\Gamma_0\,(1+\epsilon_\gamma^2)^2 + \Gamma_1\,\epsilon_\psi^2 + \Gamma_2\,\epsilon_\psi^3\right)\log^2\epsilon_\gamma^2,$$

$$\epsilon_\gamma = \frac{\lambda_{IR}}{Q},\ \epsilon_\psi = \frac{m_\psi}{Q}$$

$$\Gamma_{Virtual_\gamma}(Z' \to \bar{\psi}\,\psi) + \Gamma_{Real}(Z' \to \bar{\psi}\,\psi\,\gamma) \underbrace{=}_{Q\to\infty} 0$$

Toy Model: Heavy Z' decays

Chiral $U'(1) \otimes U(1)$ gauge theory (in Feynman gauge) with:
heavy Z' gauge boson
light: fermion ψ, higgs h and gauge boson Z $\qquad \epsilon_{z,\phi,\psi,h} \ll 1$

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\,\left[\partial_\mu + ig\left(y_L P_L + y_R P_R\right) Z_\mu + ig'\left(f_L P_L + f_R P_R\right) Z'_\mu\right]\gamma^\mu \psi\,, \\
\mathcal{L}_s &= \left|\left(\partial_\mu + ig' f_{\varphi'} Z'_\mu\right)\varphi'\right|^2 + \left|\left(\partial_\mu + ig' f_\phi Z' + igy_\phi Z_\mu\right)\varphi\right|^2 + V(\varphi) + \mathcal{V}(\varphi')\,, \\
\mathcal{L}_m &= \left(h_f \varphi\,\bar{\psi}\,P_L\,\psi + h.c.\right)\,.
\end{aligned}
$$

field	U(1) charge	$U'(1)$ charge
$\psi_{L/R} = P_{L/R}\,\psi$	$y_{L/R}$	$f_{L/R}$
φ	$y_\phi = y_R - y_L$	$f_\phi = f_R - f_L$
φ'	0	$f_{\phi'}$

fields	mass spectrum
$Z,\ \phi$	M
$Z',\ \phi'$	Q
ψ	m_ψ
h	m_h

$$
\epsilon = \epsilon_\phi \equiv \frac{M}{Q}, \qquad \epsilon_\psi \equiv \frac{m_\psi}{Q}, \qquad \epsilon_h \equiv \frac{m_h}{Q}
$$

Tree level decays: $Z' \to \bar{\psi}\,\psi$ and $Z' \to Z\ h$

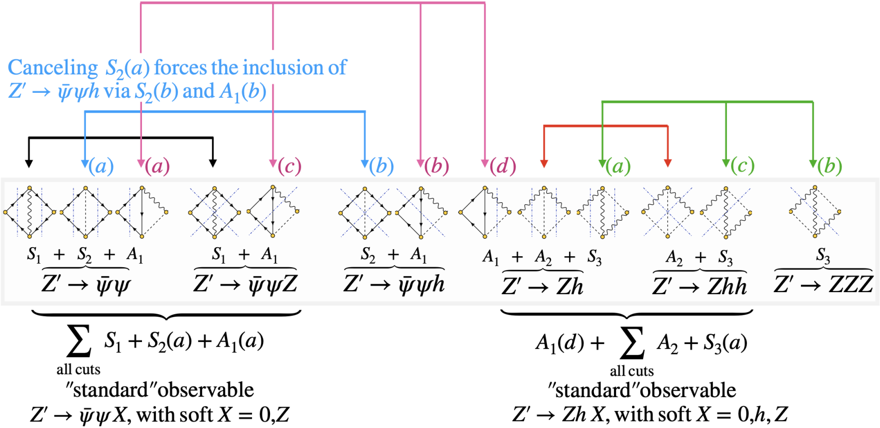
Limits

- $\boxed{y_\phi = 0}$ $(A_1 = 0)$
  Z gauge boson is massless $M = 0$,
  Fermions are $U(1)$ vector like $y_\phi = y_R - y_L = 0$.
  Mixing angle $Z' - Z$: $c_\theta = 1$ (diagonal mass matrix)
  The couplings $Z'\ Z\ h$ and $Z\ Z\ h$ are null.
  Yet the coupling with goldstone mode $Z'\ \phi\ h$, $\phi'\ \phi\ h$, $\phi\ \phi\ h$ are non zero.
- $\boxed{f_\phi = 0}$ $(A_1 = 0)$
  Fermions are $U'(1)$ vector like.
  The decay channel $Z' \to Zh$ is zero so the heavy Z' can decay only in light fermions.
- Massless fermions $\boxed{\epsilon_\psi = 0}$ $(S_2 = A_1 = 0)$
  Higgs and goldstone fields do not interact with massless fermions.
- Massless higgs $\boxed{\epsilon_h = 0}$ $(A_2 = 0)$
- Zero vev $\boxed{v = 0}$ $(S_1 \neq 0$ and $S_3 \neq 0)$
  All the light spectrum is massless $\epsilon_i = 0$.
  Only the leading log (LL) corrections, proportional to the Sudakov double logs, remains.

The domino effect

Once $A_1(b)$ is included,
the cancellation of power suppressed double logs forces
the inclusion of $A_1(d)$ which, in turn, further enlarges the observable by including the Zh final state

Canceling $S_2(a)$ forces the inclusion of
$Z' \to \bar{\psi}\psi h$ via $S_2(b)$ and $A_1(b)$



Again the KLN theorem: degenerate states

KLN Th: The theorem states that the transition amplitude squared is finite once we sum over initial and final degenerate states
Hypothesis: perturbation theory with degenerate states + unitarity of the theory.

$$|\bar{S}|^2 = \sum_{\phi_i,\phi_f \in \mathcal{D}(E)} | < \phi_f | S | \phi_i > |^2 < \infty$$

$\mathcal{D}(E)$ is given by all the states degenerate in energy in a $\Delta > 0$ range, i.e. $|E_{i/f} - E| < \Delta$

In the case of the decay of a "Neutral" particle Φ

$$\boxed{\sum_{\phi_f \in |E_f - E| < \Delta} | < \phi_f | S | \Phi > |^2 = \sum_{Kin} \sum_{Quantum\ Num} \sum_{Channel} | < \phi_f | S | \Phi > |^2}$$

KLN: kinematical degenerate states

$$\boxed{\sum_{\color{red}{Kin}}} : \quad |f_1\, f_2,\ f_3 > \equiv |f_1\, f_2 >_{Hard} + \int_{Collinear}^{Infrared} (dk)\, |f_3(k) >$$

kinematically degenerate states Infrared and collinear.

$\boxed{\textit{In QED}}$ the decay of a neutral Φ into charged particles $q\bar{q}$ that interact with photons γ:

$$\text{Kinematical Sum} : \begin{cases} |\langle \bar{q}\, q|\, S\, |\Phi\rangle|^2 \to LL + LL_{PS} \\ \sum_k |\langle \bar{q}\, q,\, \gamma(k)|\, S\, |\Phi\rangle|^2 \to LL +\ LL_{PS}, \\ |\langle \bar{q}\, q|\, S\, |\Phi\rangle|^2 + \sum_k |\langle \bar{q}\, q,\, \gamma(k)|\, S\, |\Phi\rangle|^2 \to NLL, \end{cases}$$

$$LL = Log^2 \frac{Q^2}{m^2},\ LL_{PS} = \frac{m^2}{Q^2} Log^2 \frac{Q^2}{m^2},\quad NLL = \text{next to leading log}$$

KLN: quantum number degenerate states

$\boxed{\displaystyle\sum_{\textit{Quantum Number}}}$: quantum number degenerate states

$\boxed{\textit{In the SM}}$: *color* SU(3) quantum number, or *isospin* SU(2)

$$\text{Quantum Numbers Sum}: \begin{cases} |\langle \bar{q}_\alpha\, q_\beta|\, S\, |\Phi\rangle|^2 \to LL + LL_{PS} \\ |\langle \bar{q}_\alpha\, q_\beta|\, S\, |\Phi\rangle|^2 + \sum_{k,a} |\langle \bar{q}_\alpha\, q_\beta,\, g_a(k)|\, S\, |\Phi\rangle|^2 \to LL + \; LL_{PS}, \\ \sum_{\alpha,\beta} |\langle \bar{q}_\alpha\, q_\beta|\, S\, |\Phi\rangle|^2 + \sum_{k,a,\alpha,\beta} |\langle \bar{q}_\alpha\, q_\beta,\, g_a(k)|\, S\, |\Phi\rangle|^2 \to NLL, \end{cases}$$

In QCD, to cancel all the LL logs, we need to sum over all the colours of the quarks and over the emission of IR gluons of all possible colours.

A similar effects is present also for SU(2) multiples with the Isospin sum. The difference is that the gauge group is spontaneously broken and the gauge mediators are massive

$LL = Log^2 \frac{Q^2}{m^2}$, $LL_{PS} = \frac{m^2}{Q^2} Log^2 \frac{Q^2}{m^2}$, $NLL = \text{next to leading log}$

KLN: channel degenerate states

$\boxed{\sum_{Channel}}$: all possible different (Lorentz spin) channels

$\boxed{\textit{In Spontaneously broken } U'(1) \otimes U(1)}$ the heavy Z' can decay into two light states: a fermion-antifermion state ($Z' \to |q\ \bar{q}>$) and a light Z gauge boson-light higgs state ($Z' \to |Zh>$).

$$\text{Channel sum}: \left\{ \begin{array}{l} \left. \begin{array}{l} \left. \begin{array}{l} |\langle \bar{q}q|\, S\, |\Phi\rangle|^2 \to LL + LL_{PS} \\ \sum_{X=Z,h} \sum_k |\langle q\,\bar{q},\, X(k)|\, S\, |Z'\rangle|^2 \to LL + LL_{PS} \end{array} \right\} \to LL_{PS} \\ \sum_{X=Z,h} \sum_k \left(|\langle X\,\bar{q},\, q(k)|\, S\, |\Phi\rangle|^2 + |\langle X\, q,\, \bar{q}(k)|\, S\, |\Phi\rangle|^2 \right) \to LL_{PS} \end{array} \right\} \to LL_{PS} \\ \left. \begin{array}{l} \left. \begin{array}{l} |\langle Zh|\, S\, |Z'\rangle|^2 \to LL + LL_{PS} \\ \sum_k |\langle Zh,\, h(k)|\, S\, |\Phi\rangle|^2 \to LL_{PS} \end{array} \right\} \to LL + LL_{PS} \\ \sum_k \left(|\langle Z\, Z,\, Z(k)|\, S\, |\Phi\rangle|^2 + |\langle h\, h,\, Z(k)|\, S\, |\Phi\rangle|^2 \right) \to LL + LL_{PS} \end{array} \right\} \to LL_{PS} \end{array} \right\} \to NLL$$

$$LL = Log^2 \frac{Q^2}{m^2}, \quad LL_{PS} = \frac{m^2}{Q^2} Log^2 \frac{Q^2}{m^2}, \quad NLL = \text{next to leading log}$$

One loop KLN with Power suppressed double logs included

- *standard* KLN cancellation mechanism: If $\Gamma(Z' \to \bar{f}f)$ is IR then $\Gamma(Z' \to \bar{f}f) + \sum_X \Gamma(Z' \to \bar{f}f \underbrace{X}_{soft})$ is IR safe.
- the only way of cancelling all leading and power suppressed terms is to include all possible decay channels, and this must include channels very different from the starting one like $\Gamma(Z' \to \bar{f}f) + \sum_X \Gamma(Z' \to \bar{f}f\ X) + \Gamma(Z' \to Zh) + \Gamma(Z' \to ZZZ) + \Gamma(Z' \to Zhh)$ IR safe
- necessary requirements ($A_1 \neq 0$): $\begin{cases} \epsilon_\psi \neq 0 \\ y_\phi = y_R - y_L \neq 0 \\ f_\phi = f_R - f_L \neq 0 \end{cases}$

The only possible phenomenological models where the above three requirements can be present is in a new physical scenario where an extra heavy Z' gauge boson can be produced at future colliders.