Muon $g-2$: issues with data driven approaches

F. Piccinini

INFN, Pavia and GGI

Muon $g-2$ or stress testing the SM, Rome, 14 October 2025

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani
INFN

SM contributions to a_μ

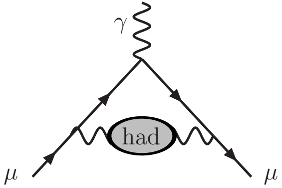
- **QED**: it accounts for more than 99.99% of the total, with negligible uncertainty at the present precision
- **ElectroWeak**: calculated up to three loops, with negligible uncertainty $(\sim 153(1) \cdot 10^{-11})$
- **QCD**: the largest source of uncertainty, due to non-perturbative effects

$$a_\mu^{\mathrm{SM}} = 116\,592\,033(62) \cdot 10^{-11}$$

R. Aliberti et al., Phys. Rept. 1143 (2025) [arXiv:2505.21476]

QCD contributions

Hadronic Vacuum Polarization (HVP)

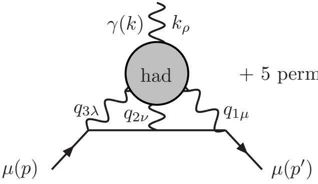


F. Jegerlehner, arXiv:0902.3360

- starts at $\mathcal{O}(\alpha^2)$ $\sim 7000(60) \cdot 10^{-11}$

‘<

Hadronic Light-by-Light (HLxL)



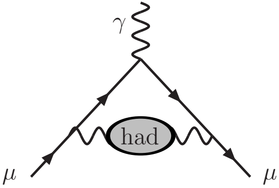
$+\ 5$ permutations of the q_i

F. Jegerlehner, arXiv:0902.3360

- starts at $\mathcal{O}(\alpha^3)$ $\sim 100(10) \cdot 10^{-11}$

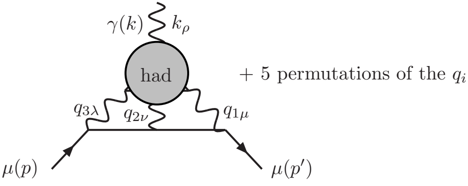
QCD contributions

Hadronic Vacuum Polarization (HVP)



F. Jegerlehner, arXiv:0902.3360

Hadronic Light-by-Light (HLxL)



F. Jegerlehner, arXiv:0902.3360

- starts at $\mathcal{O}(\alpha^2)$ $\quad \sim 7000(60) \cdot 10^{-11}$

‘<

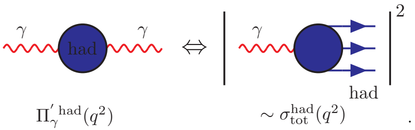
- starts at $\mathcal{O}(\alpha^3)$ $\quad \sim 100(10) \cdot 10^{-11}$

- **two approaches for both contributions:**
  - **first principle calculations with LQCD**
  - **data driven approach**

see previous talk by G. Gagliardi

$\Longrightarrow$ focus on HVP, since it is the largest source of uncertainty

The (time-like) dispersive approach for HVP

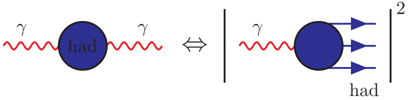


- **at Leading Order**

$$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha^2}{3\pi^2}\right) \int_{m_\pi^2}^{\infty} ds \, \frac{K^{\mathrm{LO}}(s) R(s)}{s}$$

$$K^{\mathrm{LO}}(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\left(\frac{s}{m^2}\right)}$$

$$R(s) = \frac{\sigma^0(e^+e^- \to \mathrm{hadrons}(+\gamma))}{\sigma_{\mathrm{pt}}} \qquad \sigma_{\mathrm{pt}} = \frac{4\pi\alpha^2}{3s}$$



$\Pi_\gamma^{\prime\,\text{had}}(q^2)$ $\quad \sim \sigma_{\text{tot}}^{\text{had}}(q^2)$.

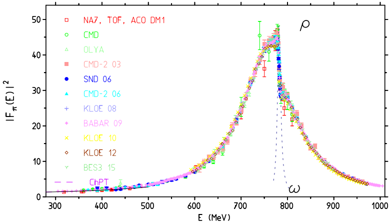
- **at Leading Order**

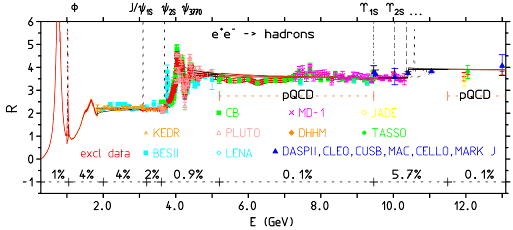
$$a_\mu^{\text{HLO}} = \left(\frac{\alpha^2}{3\pi^2}\right)\int_{m_\pi^2}^{\infty} ds \frac{K^{\text{LO}}(s)R(s)}{s}$$

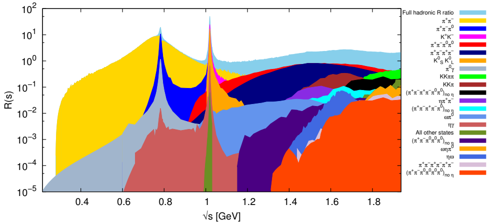
$$K^{\text{LO}}(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)\left(\frac{s}{m^2}\right)}$$

$$R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{\sigma_{\text{pt}}} \qquad \sigma_{\text{pt}} = \frac{4\pi\alpha^2}{3s}$$

- **at Higher Orders**
  - $K^{\text{LO}}(s) \to K^{\text{NLO}}(s) \Longrightarrow a_\mu^{\text{HNLO}}$
  - $K^{\text{LO}}(s) \to K^{\text{NNLO}}(s) \Longrightarrow a_\mu^{\text{HNNLO}}$





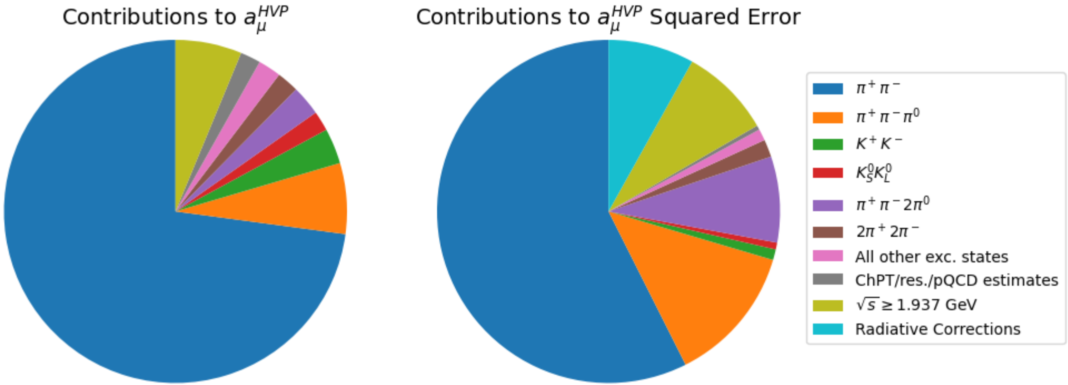


- **Pion form factor**

$$\langle \pi^{\pm}(p')|j^{\mu}(0)|\pi^{\pm}(p)\rangle = \pm(p'+p)^{\mu} F_{\pi}^{V}((p'-p)^2)$$

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3s}\beta_{\pi}^3 |F_{\pi}^{V}(s)|^2$$

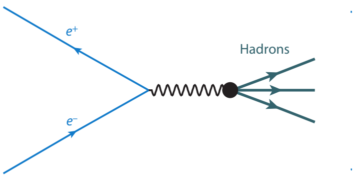
Contributions to integral and to its error



talk by A. Wright, FCCP 2025, Capri, 29 Setember - 1 October 2025

Methods for the hadronic cross section measurement

Energy scan

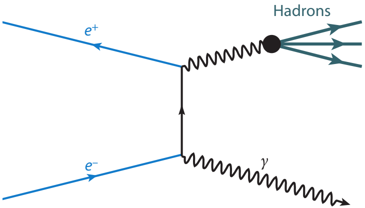


$$\sigma^0(e^+e^- \to X) = \frac{N_X}{\epsilon_X (1 + \delta_{RC}) L_{ee}}$$

- dependence on the radiative corrections δ_{RC} in generators, photonic and full vac. pol.
- dependence on the absolute luminosity

$*$ $\Longrightarrow$ theoretical systematics

Radiative return



- **tagged analysis**
- **untagged analysis**
- possible normalization to $\mu^+\mu^-\gamma$ events
  - independence of the absolute normalization and of the vacuum polarization
  - high stat $\mu^+\mu^-\gamma$ required

Overview of present experiments

Experiment	Published Method	Normalization	Separation π - μ - e	Future
KLOE	ISR untagged ISR tagged ISR untagged	Luminosity Luminosity μ+μ-γ	Kinematics Track Kinematics Track Kinematics Track	ISR untagged μ+μ-γ statistics x 7
BABAR	ISR tagged	μ+μ-γ	Particle ID	ISR tagged, separation by polar angle, statistics x 2
BESIII	ISR tagged	Luminosity	Particle ID (ML)	ISR tagged, μ+μ-γ, statistics x 7, 1C kin. fit
BELLE-II				ISR tagged, μ+μ-γ, Particle ID
CMD-3	Energy scan	e+e-	Kinematics Track Kinematics EMC	overall improvements
SND	Energy scan	e+e-	Kinematics EMC	overall improvements ML for π – e separation

A. Denig, talk at FCCP 2025, Capri

MC tools: radiative corrections and the pion structure

Eur. Phys. J. C (2010) 66: 585–686
DOI 10.1140/epjc/s10052-010-1251-4

THE EUROPEAN PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

S. Actis[38], A. Arbuzov[9,e], G. Balossini[32,33], P. Beltrame[13], C. Bignamini[32,33], R. Bonciani[15], C.M. Carloni Calame[35], V. Cherepanov[25,26], M. Czakon[1], H. Czyż[19,a,f,i], A. Denig[22], S. Eidelman[25,26,g], G.V. Fedotovich[25,26,e], A. Ferroglia[23], J. Gluza[19], A. Grzelińska[8], M. Gunia[19], A. Hafner[22], F. Ignatov[25], S. Jadach[8], F. Jegerlehner[3,19,41], A. Kalinowski[29], W. Kluge[17], A. Korchin[20], J.H. Kühn[18], E.A. Kuraev[9], P. Lukin[25], P. Mastrolia[14], G. Montagna[32,33,b,d], S.E. Müller[22,f], F. Nguyen[34,d], O. Nicrosini[33], D. Nomura[36,h], G. Pakhlova[24], G. Pancheri[11], M. Passera[28], A. Penin[10], F. Piccinini[33], W. Płaczek[7], T. Przedzinski[6], E. Remiddi[4,5], T. Riemann[41], G. Rodrigo[37], P. Roig[27], O. Shekhovtsova[11], C.P. Shen[16], A.L. Sibidanov[25], T. Teubner[21,h], L. Trentadue[30,31], G. Venanzoni[11,c,i], J.J. van der Bij[12], P. Wang[2], B.F.L. Ward[39], Z. Was[8,g], M. Worek[40,19], C.Z. Yuan[2]

Recently used generators: general features

- for luminosity (Bhabha, $\mu^+\mu^-$, $\gamma\gamma$ production): NLO with multiphoton resummation (Parton Shower in BabaYaga, YFS in KKMC, BHWIDE)

- for $\pi^+\pi^-$ production: NLO with IS multiphoton resummation with structure functions (MCGPJ)

- for radiative processes: NLO (Phokhara). LO $2 \to 3$ with collinear LL Structure functions (AfkQED)

- Pion form factor in F$\times$QED approximation: diagrams calculated in sQED with $F_\pi(q^2)$ factorized with a scale q^2 able to guarantee the IR cancellation

From R-ratio to a_μ

- The evaluation of the dispersion relation for a_μ is performed by different groups
  - DHMZ — M. Davier, A. Hoecker, B. Malaescu, Z. Zhang
  - KNTW — A. Keshavarzi, D. Nomura, T. Teubner, A. Wright
  - CHKLS — G. Colangelo, M. Hoferichter, B. Kubis, T.P. Leplumery, P. Stoffer

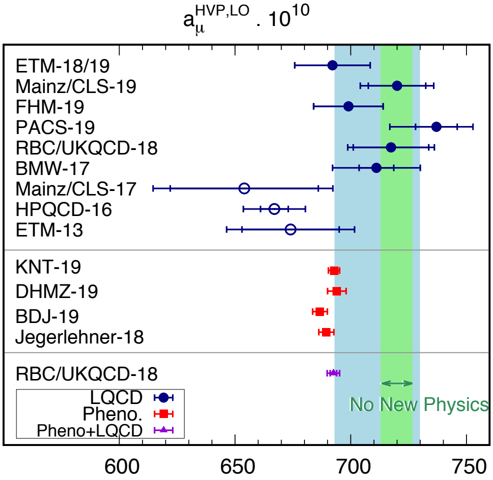
- interpolation between different energy points

- combination of all exclusive channels, considering the correlations between channels and between experiments

- using general constraints, from analyticity, unitarity and crossing symmetry

having different approaches crucial to have more reliable results

Situation at the time of WP20



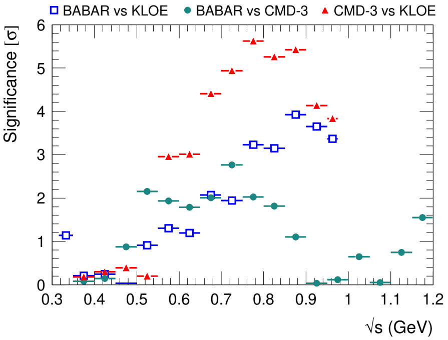
T. Aoyama et al., Phys. Rept. 887 (2020) 1

$$\mathbf{a_\mu^{HVP-LO} = 6931(40) \cdot 10^{-11}}$$

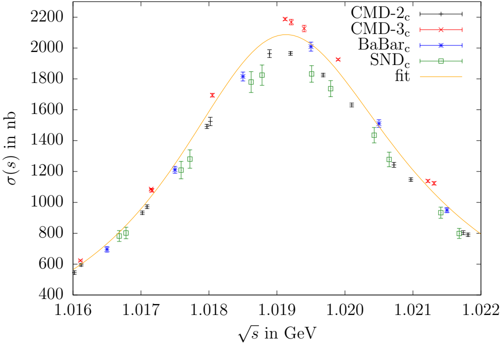
Results on the experimental side after WP20: CMD-3

- new measurement of $\sigma(e^+e^- \to \pi^+\pi^-)$ @VEPP-2000 in strong tension with previous experiments (even CMD-2!)

F.V. Ignatov et al., Phys. Rev. Lett. 132 (2024) 23; Phys. Rev. D109 (2024) 11

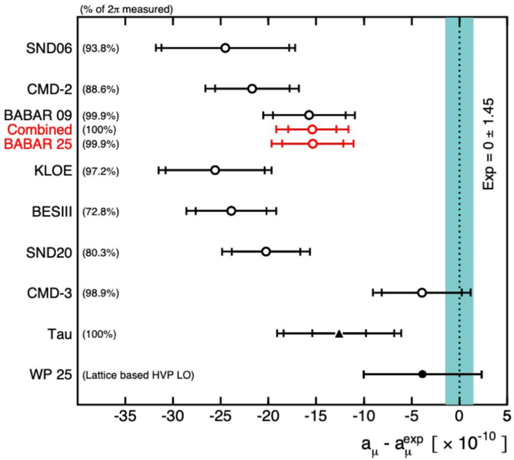


- At present no explanation found for the difference of results of CMD-3 w.r.t. other experiments
- tensions also in other channels



Recent news on the experimental side: BaBaR

- New blind analysis of $460\ \mathrm{fb}^{-1}$ integrated luminosity confirms the results of 2009

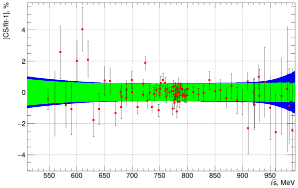


Davier, Lutz, Malaescu, Zhang, Polat, Pinto, talk at Muon $g-2$ Theory Initiative meeting, IJCLab, Orsay, 8-12 September 2025

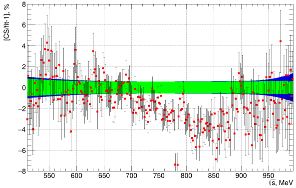
Very recent SND analysis of 2018 data $\rightarrow$ tension with BaBaR and KLOE

A. Kupich, talk at Muon $g-2$ Theory Initiative, IJCLab, Orsay, 8-12 September 2025

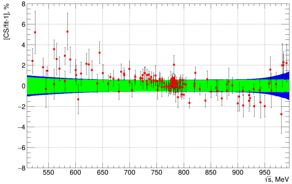
- SND data vs fit



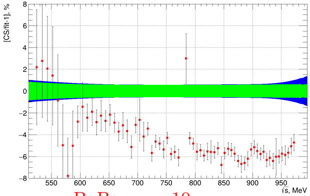
- BaBaR data vs SND



- CMD-3 vs SND



- KLOE vs SND



$a_\mu^{\pi\pi}\cdot 10^{10} = 431.11 \pm 3.52$ vs $a_\mu^{\pi\pi\ \mathrm{CMD-3}}\cdot 10^{10} = 433.62 \pm 3.76$, $a_\mu^{\pi\pi\ \mathrm{BaBar}}\cdot 10^{10} = 423.87 \pm 2.06$

- **RadioMonteCarLow 2** is a community effort focused on Monte Carlo tools and radiative corrections for e^+e^-collisions at low energies ($\sqrt{s} <$ few GeV)
- The goal is to assess the current state of MC codes, make them accessible, and further improve them where needed
  $\hookrightarrow$ A living repository of MC generators and benchmark results
- Close collaboration between theorists and experimental collaborations
  $\hookrightarrow$ BESIII, CMD-3, KLOE . . .
- 7 codes: AfkQed, BabaYaga@NLO, KKMC, MCGPJ, McMule, Phokhara, Sherpa
- 3+3 processes (both for energy scan and radiative return):
  - $e^+e^- \to e^+e^-(\gamma)$
  - $e^+e^- \to \mu^+\mu^-(\gamma)$
  - $e^+e^- \to \pi^+\pi^-(\gamma)$

radiomontecarlow2.gitlab.io

(A. Gurgone, FCCP 2025, Anacapri, 8-12 September 2025)

SciPost

SciPost Phys. Comm. Rep. 9 (2025)

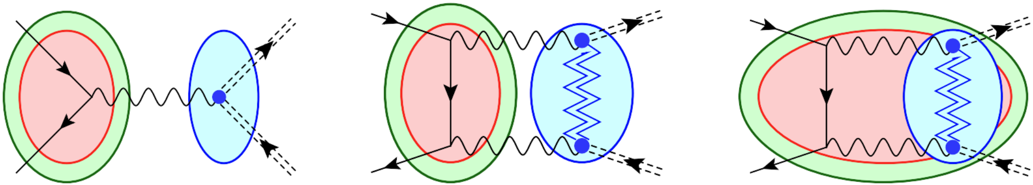
Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Riccardo Aliberti[1], Paolo Beltrame[2], Ettore Budassi[3,4], Carlo M. Carloni Calame[4], Gilberto Colangelo[5], Lorenzo Cotrozzi[2], Achim Denig[1], Anna Driutti[6,7], Tim Engel[8], Lois Flower[2,9], Andrea Gurgone[3,6,7], Martin Hoferichter[5], Fedor Ignatov[2], Sophie Kollatzsch[10,11], Bastian Kubis[12], Andrzej Kupść[13,14⋆], Fabian Lange[10,11], Alberto Lusiani[7,15], Stefan E. Müller[16], Jérémy Paltrinieri[2], Pau Petit Rosàs[2], Fulvio Piccinini[4], Alan Price[17], Lorenzo Punzi[7,15], Marco Rocco[10,18], Olga Shekhovtsova[19,20], Andrzej Siódmok[17], Adrian Signer[10,11⋆], Giovanni Stagnitto[21], Peter Stoffer[10,11], Thomas Teubner[2], William J. Torres Bobadilla[2], Francesco P. Ucci[3,4], Yannick Ulrich[2,5⋆] and Graziano Venanzoni[2,7⋆]

(RadioMonteCarLow 2 working group)

(A. Gurgone, FCCP 2025, Anacapri, 8-12 September 2025)

- WP1 & WP2: fixed-order QED
- WP3: hadronic final states (mainly pions)
- WP3: all-order QED (resummation)
- WP5: experimental inputs



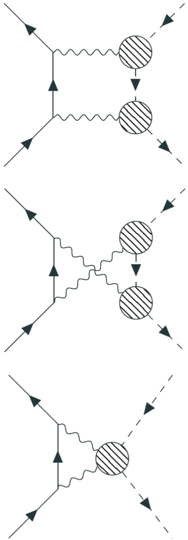
- **F$\times$sQED**: Diagrams are computed in scalar QED and multiplied by a global form factor $F_\pi(q^2)$, where q^2 is chosen to ensure the cancellation of IR divergences $\hookrightarrow$ $q^2 = m^2_{\pi\pi}$ for ISC, $q^2 = s$ for FSC and mixed

- **GVMD**: The form factor is written as a sum of Breit-Wigner functions. The propagator-like form allows one to solve the loop integral with standard techniques

$$F_\pi(q^2) = \sum_{v=0}^{N} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2} \qquad \text{with} \qquad \Lambda_v^2 = m_v^2 - i m_v \Gamma_v$$

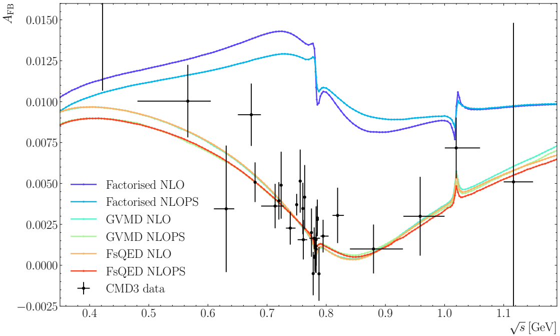
- **FsQED**: Under the general assumptions unitarity and analyticity, the form factor is decomposed using the dispersion relation

$$\frac{F_\pi(q^2)}{q^2} = \frac{1}{q^2 - \lambda^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\mathrm{Im} F_\pi(s')}{s'(q^2 - s')}$$



di-pion final state: charge asymmetry

$$A_{\mathrm{FB}}\left(\sqrt{s}\right) = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



E. Budassi et al., JHEP05 (2025) 196 [arXiv:2409.03469]

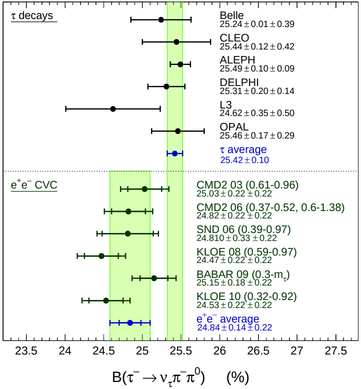
- FsQED and GVMD approaches give very similar predictions
- Relevant differences with the FxsQED approach (for exclusive angular observables)

Another possible way: τ decay data

- In the limit of isospin invariance, the normalized invariant mass spectrum of the τ decay for $I = 1$ channel $\tau^\pm \to X^\pm \nu_\tau$ is proportional to the corresponding isovector final state production cross section $e^+e^- \to X^0$

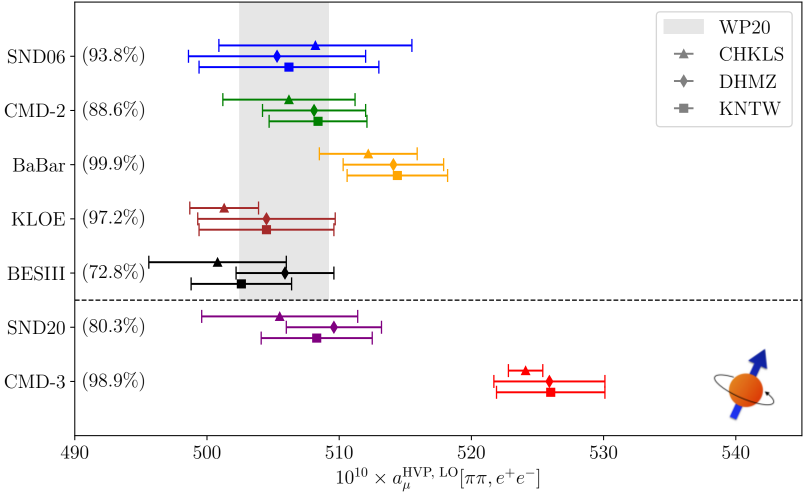
- measured differential $\mathcal{B}(\tau \to \pi^-\pi^0\nu_\tau) \Longrightarrow d\sigma(e^+e^- \to \pi^+\pi^-)/dm_{\pi^+\pi^-}$

- Isospin Breaking effects source of large uncertainties



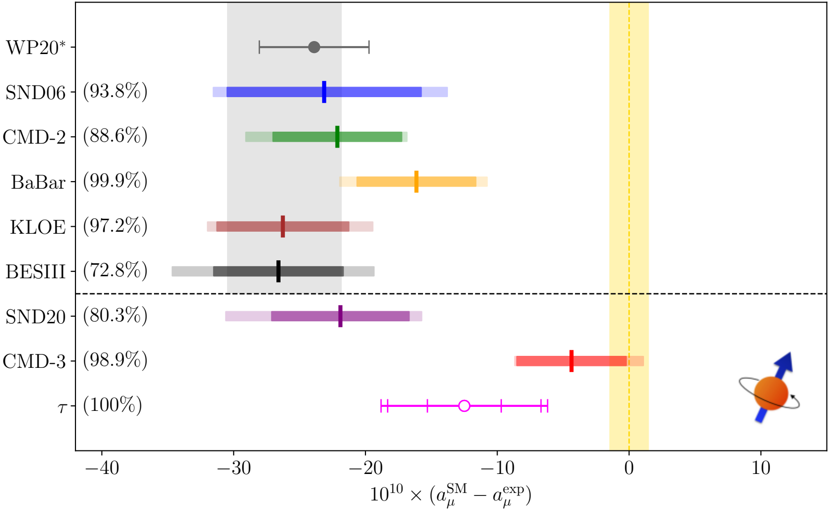
R. Aliberti et al., Phys. Rept. 1143 (2025) [arXiv:2505.21476]

Results on a_μ of different groups



R. Aliberti et al., Phys. Rept. 1143 (2025) [arXiv:2505.21476]

Situation of WP25



R. Aliberti et al., Phys. Rept. 1143 (2025) [arXiv:2505.21476]

Looking ahead, the space-like approach (MUonE)

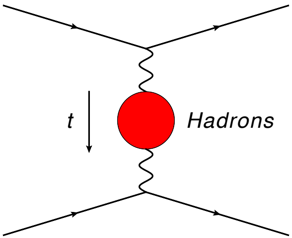
- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe *scattering*
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



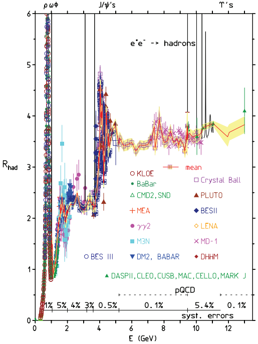
- ⋆ **$\Delta\alpha_{\mathrm{had}}(\mathrm{t})$ can be directly measured in a (single) experiment involving a space-like scattering process and $\mathrm{a}_\mu^{\mathrm{HLO}}$ obtained through numerical integration**

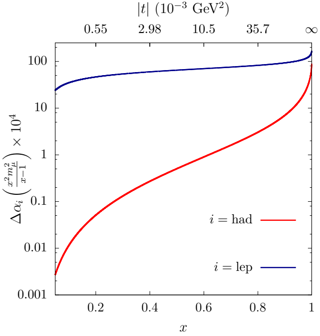
Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

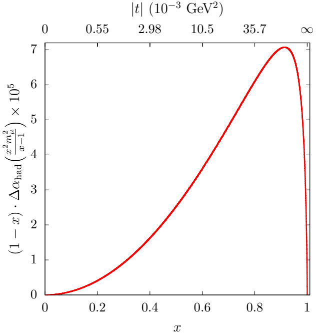
- ⋆ **A data-driven, inclusive evaluation of a_μ^{HLO}, but with space-like data**

From time-like to space-like evaluation of a_μ^{HLO} (see next talk by G. Cantatore)

Time-like $\mapsto$ **Space-like**





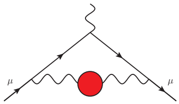


Smooth function

$\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required

$\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

- LO: $\frac{\alpha}{\pi}(1-x)$

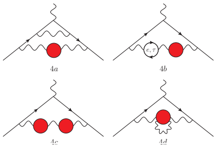


- NLO

E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462

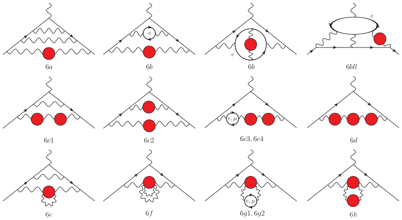
A.V. Nesterenko, J. Phys. G49 (2022) 5, 055001;

J. Phys. G50 (2022) 2, 029401

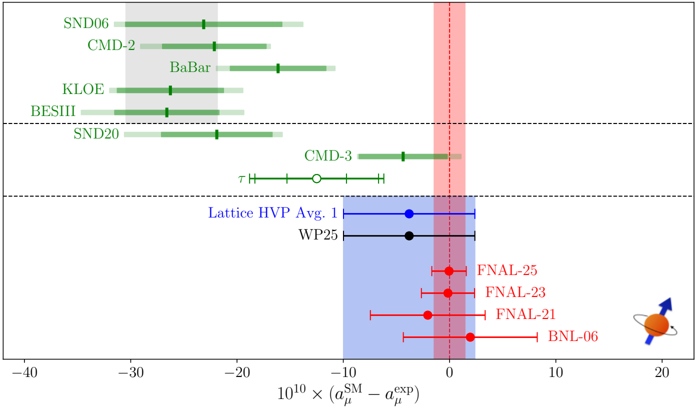


- NNLO

E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462



Summary



- the puzzle(s) of the data driven approaches
- new data, new analysis and new simulation tools will be crucial to clarify the situatio