# The HVP contribution to the Muon g-2 in the SM from lattice QCD

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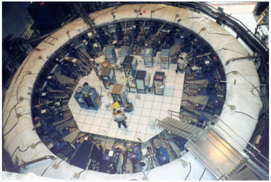
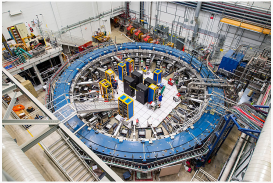
Muon g-2 or stress testing the SM

October 14th 2025, Roma Tor Vergata

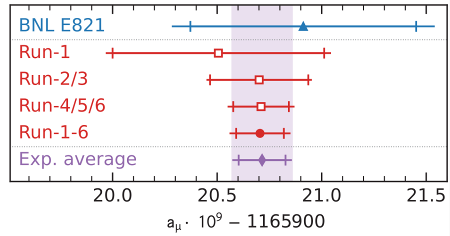
# The Muon $g-2$ experiment @FNAL



$g_\mu - 2$ @BNL (up to 2006) $\Longrightarrow$ transfer to Fermilab $\Longrightarrow$ $g_\mu - 2$ @FNAL

Muon $g-2$ Coll., PRL 135 (2025)




$$a_\mu^{\text{exp}} = 1\,165\,920\,715(145) \times 10^{-12} \quad [124\,\text{ppb}]$$

4-fold improvement thanks to the Muon $g-2$ Collaboration!

$a_\mu$ exp. @FNAL ended! A fully independent meas. of $a_\mu$ expected at JPARC.

Can we match, on the theory side, the experimental accuracy on $a_\mu$?

# The Muon $g-2$ Theory Initiative

The muon $g-2$ TI has been established in 2017 with the aim of matching the precision of the SM-theory prediction for $a_\mu$ with the experimental one.

https://muon-gm2-theory.illinois.edu

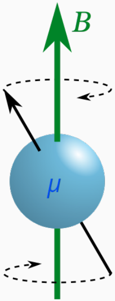
- Composed by experts in lattice QCD, dispersive approach, perturbative calculations. . .
- First white paper (WP20) out in 2020 [Physics Reports 887 (2020)].
- An update (WP25) has been published in Sep. 2025 [Physics Reports 1143 (2025)] $\Longrightarrow$
- Last TI meeting at IJCLab (Orsay) in September.

The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti[1,2], T. Aoyama[3], E. Balzani[4,5], A. Bashir[6,7], G. Benton[8,9], J. Bijnens[10], V. Biloshytskyi[1,2], T. Blum[11,12], D. Boito[13], M. Bruno[14,15], E. Budassi[16,17], S. Burri[18], L. Cappiello[19], C. M. Carloni Calame[17], M. Cè[14,15], V. Cirigliano[20,21], D. A. Clarke[22], G. Colangelo[†18], L. Cotrozzi[23], M. Cottini[18], I. Danilkin[1,2], M. Davier[†24], M. Della Morte[25], A. Denig[1,2,26,27], C. DeTar[22], V. Druzhinin[28], G. Eichmann[29], A. X. El-Khadra[†8,9], E. Estrada[30], X. Feng[31,32,33], C. S. Fischer[34,35], R. Frezzotti[36], G. Gagliardi[37], A. Gérardin[38], M. Ghilardi[16,17], D. Giusti[39,40], M. Golterman[41], S. Gonzàlez-Solís[42,43], S. Gottlieb[44], R. Gruber[45], A. Guevara[46], V. Gülpers[47], A. Gurgone[48,49], F. Hagelstein[1,2], M. Hayakawa[50,51], N. Hermansson-Truedsson[10,47], A. Hoecker[52], M. Hoferichter[†18], B.-L. Hoid[1,2], S. Holz[18], R. J. Hudspith[53], F. Ignatov[23], L. Jin[11], N. Kalntis[18], G. Kanwar[47], A. Keshavarzi[54], J. Komijani[45], J. Koponen[1,2], S. Kuberski[55], B. Kubis[56], A. Kupich[28], A. Kupść[57,58], S. Lahert[22], S. Laporta[4,5], C. Lehner[†40], M. Lellmann[1], L. Lellouch[†38], T. Leplumey[59,60], J. Leutgeb[61], T. Lin[31], Q. Liu[62], I. Logashenko[28], C. Y. London[13], G. López Castro[30], J. Lüdtke[63], A. Lusiani[49,64], A. Lutz[24], J. Mager[61], B. Malaescu[65], K. Maltman[66,67], M. K. Marinković[45], J. Márquez[30], P. Masjuan[68,69], H. B. Meyer[1,2,26,27], T. Mibe[†70], N. Miller[26,27], A. Miramontes[71,72], A. Miranda[68], G. Montagna[16,17], S. E. Müller[73], E. T. Neil[74], A. V. Nesterenko[28], O. Nicrosini[17], M. Nio[51,75], D. Nomura[76], J. Paltrinieri[23], L. Parato[45], J. Parrino[40], V. Pascalutsa[1,2], M. Passera[5,77], S. Peris[68,69], P. Petit Rosàs[23], F. Piccinini[17,78], R. N. Pilato[23], L. Polat[65,24], A. Portelli[47], D. Portillo-Sánchez[30], M. Procura[63], L. Punzi[49,64], K. Raya[7], A. Rebhan[61], C. F. Redmer[1,2], B. L. Roberts[†79], A. Rodríguez-Sánchez[72], P. Roig[30,72], J. Ruiz de Elvira[80], P. Sánchez-Puertas[81], A. Signer[82,60], J. W. Sitison[74], D. Stamen[56], D. Stöckinger[83], H. Stöckinger-Kim[83], P. Stoffer[60,82], Y. Sue[70], P. Tavella[45], T. Teubner[†23], J.-N. Toelstede[60,82], G. Toledo[84], W. J. Torres Bobadilla[23], J. T. Tsang[55], F. P. Ucci[16,17], Y. Ulrich[23], R. S. Van de Water[85], G. Venanzoni[23,49], S. Volkov[86], G. von Hippel[1,2], G. Wang[38], U. Wenger[18], H. Wittig[†1,2,26,27], A. Wright[23], E. Zaid[23], M. Zanke[56], Z. Zhang[24], M. Zillinger[18],

C. Alexandrou[87,88], A. Altherr[45], M. Anderson[1,2], C. Aubin[89], S. Bacchio[88], P. Beltrame[23], A. Beltran[1,2], P. Boyle[90], I. Campos Plasencia[91], I. Caprini[92], B. Chakraborty[93], G. Chanturia[56], A. Crivellin[60], A. Czarnecki[94], L.-Y. Dai[95], T. Dave[23], L. Del Debbio[47], K. Demory[24], D. Djukanovic[26,27], T. Draper[96], A. Driutti[49], M. Endo[97], F. Erben[55], K. Ferraby[23], J. Finkenrath[55], L. Flower[23], A. Francis[98], E. Gámiz[99], J. Gogniat[18], A. V. Grebe[85], S. Gündogdu[82,60], M. T. Hansen[47], S. Hashimoto[97], H. Hayashii[100], D. W. Hertzog[20], L. A. Heuser[56], L. Hostetler[44], X. T. Hou[101], G. S. Huang[102], T. Iijima[103,50], K. Inami[50], A. Jüttner[55,93], R. Kitano[104], M. Knecht[38], S. Kollatzsch[82,60], A. S. Kronfeld[85], T. Lenz[1,2], G. Levati[18], Q. M. Li[101], Y. P. Liao[101], J. Libby[105], K. F. Liu[96], V. Lubicz[37], M. T. Lynch[8,9], A. T. Lytle[8,9], J. L. Ma[101], K. Miura[97], K. Möhling[83], J. Muskalla[1,2], F. Noël[18], K. Ottnad[1,2], P. Paradisi[4,5], C. T. Peterson[106], A. Pich[72], S. Pitelis[1], S. Plura[1,2], A. Price[107], D. Radic[82,60], A. Radzhabov[28], A. Risch[108], S. Romiti[18], S. Sahoo[109], F. Sannino[25,19], H. Schäfer[56], Y. Schelhaas[1,2], S. I. Serednyakov[28], O. Shekhovtsova[110,111], J. N. Simone[85], S. Simula[112], E. P. Solodov[28], F. M. Stokes[67], M. Vanderhaeghen[1,2], A. Vaquero[113,114], N. Vestergaard[23], W. P. Wang[1,2], Z. Wąs[115], K. Yamashita[116], Y. B. Yang[117], T. Yoshioka[118], C. Z. Yuan[101], A. S. Zhevlakov[28]

# Introduction: the magnetic moment of a lepton




The magnetic moment $\boldsymbol{\mu}$ of a charged object parameterizes the torque that a static magnetic field exerts on it.

For a charged spin-$1/2$ particle:

$$\boldsymbol{\mu} = g\frac{e}{2m}\boldsymbol{S}$$

g is the well-known gyromagnetic factor.

In QFT the response of a charged lepton (say a muon $\mu$) to a static and uniform e.m. field is encoded in ($k = p_1 - p_2$)

$$\langle \mu(p_2)|J^{\nu}_{\rm em}(0)|\mu(p_1)\rangle = -ie\bar{u}(p_1)\Gamma^{\nu}(p_1,p_2)u(p_2)$$

Lorentz invariance and e.m. current conservation constrain $\Gamma^{\nu}$-structure:

$$\Gamma^{\nu}(p_1,p_2) = F_1(k^2)\gamma^{\nu} + \frac{i}{2m_{\mu}}F_2(k^2)\sigma^{\nu\rho}k_{\rho} + \text{P-violating terms}$$

# The muon anomalous magnetic moment

Gyromagnetic factor $g_\mu$ related to form-factors $F_1(k^2)$ and $F_2(k^2)$ through

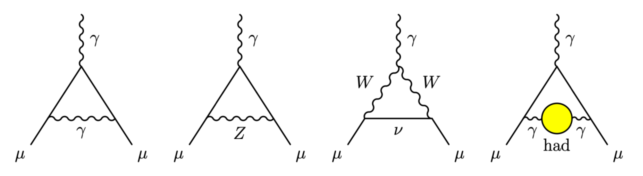
$$g_\mu = 2\left[F_1(0) + F_2(0)\right]$$

- Electric charge conservation $\implies F_1(0) = 1$.
- At tree level in the SM: $F_2(0) = 0 \implies g_\mu = g_\mu^{\text{Dirac}} \equiv 2$.

The muon anomalous magnetic moment:

$$a_\mu = \frac{g_\mu - 2}{2} = F_2(0)$$

non-zero only at loop level. Contributions from all SM (and BSM) fields. E.g.



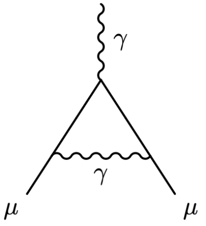

Since it is very precisely measured it is a crucial probe of the completeness of the SM.

# The muon magnetic moment in the SM

$a_\mu$ can be decomposed into QED, weak and hadronic contributions

$$a_\mu = \underbrace{a_\mu^{\text{QED}}}_{>99.99\%} + a_\mu^{\text{weak}} + \underbrace{a_\mu^{\text{had}}}_{\text{non-perturbative}}$$

- The QED contribution to $a_\mu$ is completely dominant. LO (1-loop) contribution evaluated by J. Schwinger in 1948

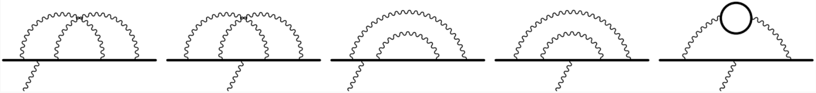



$$\Longrightarrow a_\mu^{\text{QED,1-loop}} = \frac{\alpha}{2\pi}$$


- Since Schwinger's calculation many more QED-loops included...

# The QED contribution $a_\mu^{\rm QED}$

Two-loops QED contributions to $a_\mu$



To match experimental accuracy $\Delta a_\mu^{\rm exp} \simeq \mathcal{O}(10^{-10})$ several orders in the perturbative $\alpha$ expansion need to be considered

$$\boxed{a_\mu^{\rm QED} = \frac{\alpha}{2\pi} + \sum_{n=2}^{\infty} C_\mu^n \left(\frac{\alpha}{\pi}\right)^n}$$

- Number of Feynman diagrams quickly rises with $n$: $1, 7, 72, 891, 12672, \ldots$
- Heroic effort to compute them up to five-loops [T. Aoyama et al. PRLs, 2012]

$C_\mu^6 \left(\frac{\alpha}{\pi}\right)^6 \simeq C_\mu^6 \times 10^{-16}$ requires unnaturally large $C_\mu^6 \simeq \mathcal{O}(10^6)$ to be relevant!!

$$\boxed{a_\mu^{\rm QED} = 116\,584\,718.931(104) \times 10^{-11} \quad \checkmark}$$

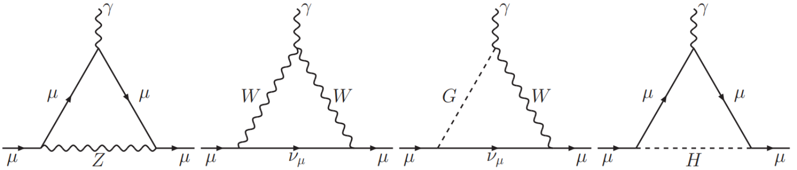
## The weak contribution $a_\mu^{\text{weak}}$

$a_\mu^{\text{weak}}$ defined as the sum of all loop diagrams containing at least a $W, H, Z$.

- Smallest of the three contributions due to Fermi-scale suppression:

$$\boxed{a_\mu^{\text{weak}} \propto \alpha_W^2 \frac{m_\mu^2}{M_W^2} \simeq \mathcal{O}(10^{-9})}$$

Sample of one-loop weak diagrams:




- At target precision of $\sim 0.1$ ppm two-loops calculation is sufficient [Czarnecki et al PRD (2006), Gnendiger et al PRD (2013)]. 3-loop contribution totally negligible.

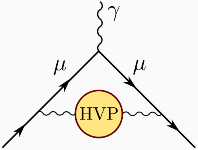
$$\boxed{a_\mu^{\text{weak}} = 154.4\,(4) \times 10^{-11} \;\checkmark}$$

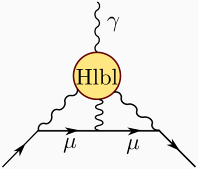
# The hadronic contribution $a_\mu^{\text{had}}$

Contributions to $a_\mu^{\text{had}}$ at target accuracy of $\mathcal{O}(10^{-10})$:

$$a_\mu^{\text{had}} = \underbrace{a_\mu^{\text{HVP,LO}}}_{\mathcal{O}(7\times10^{-8})} + \underbrace{a_\mu^{\text{Hlbl}}}_{\mathcal{O}(10^{-9})} + \underbrace{a_\mu^{\text{HVP,NLO}}}_{\mathcal{O}(10-9)} + \underbrace{a_\mu^{\text{HVP,NNLO}}}_{\mathcal{O}(10-10)}$$

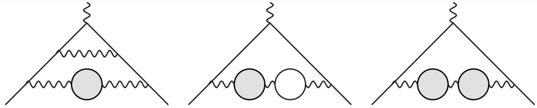
HVP, LO $\Longrightarrow$




$\Longleftarrow$ Hlbl

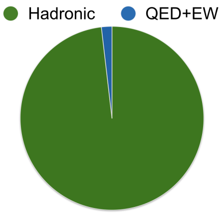
- NLO and NNLO HVP contributions relevant at target accuracy. At NLO:



- However, they can obtained from same non-perturbative input of $a_\mu^{\text{HVP,LO}}$. Hence we shall discuss only the latter.

# How important are hadronic contributions?

The uncertainty in the theory prediction for $a_\mu$ dominated by the hadronic contribution, despite its smallness




Dominant source of uncertainty is $a_\mu^{\mathrm{HVP,LO}}$

- Hadronic contributions are fully non-perturbative.
- Two main approaches to evaluate them:

**Dispersive approach**:

- Relates full $a_\mu^{\mathrm{HVP,LO}}$ to $e^+e^- \to$ hadrons cross-section via optical theorem.
- For Hlbl (only) low-lying intermediate-states contributions can expressed in terms of transition form-factors TFFs.

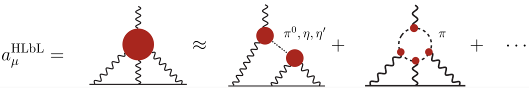
**Lattice QCD**:

- Only known first-principles SM method to evaluate both $a_\mu^{\mathrm{HVP}}$ and $a_\mu^{\mathrm{Hlbl}}$.
- In the past the accuracy of the predictions were not good enough. The situation changed in the last years.

# Summary of current status for $a_\mu^{\rm Hlbl}$ from WP25

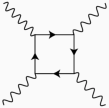
$a_\mu^{\rm Hlbl}$ occurs at $\mathcal{O}(\alpha^3)$. Related to $2 \to 2$ (generally virtual) photon scattering

- In the dispersive framework [Colangelo et al. JHEP09 (2015)] one isolates the dominant intermediate-states contributions:




- parameterized by transition form-factors. E.g. for the $\pi^0$-pole

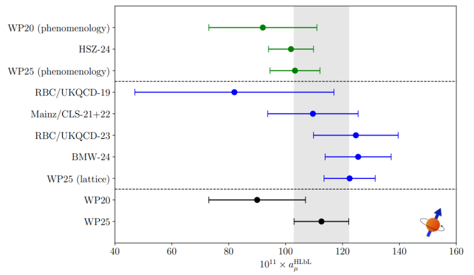
$$T\langle 0|J^\mu(q)J^\nu(0)|\pi^0(p)\rangle = -i\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta F_{\pi^0\gamma^*\gamma^*}(q^2,(q-p)^2)$$



In lattice QCD one evaluates directly:

$$\Pi^{\mu\nu\rho\sigma} = T\langle 0|J^\mu J^\nu J^\rho J^\nu|0\rangle$$

Very complex calculation, but only $O(10\%)$ precision needed.

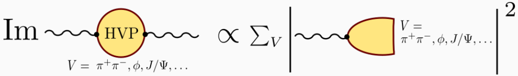



- Since WP20, three new lattice results for the Hlbl appeared.
- LQCD calculations of $a_\mu^{\rm Hlbl}$ $\approx$in line with the dispersive result. WP25 average has $< 10\%$ errors!

# The LO hadronic-vacuum-polarization (HVP) contribution

$a_\mu^{\mathrm{HVP,LO}}$ is the largest of the hadronic contributions.

- Until '20 LQCD calculations well above percent level accuracy.
- However, $a_\mu^{\mathrm{HVP,LO}}$ is related to $\sigma(\gamma^* \to \text{hadrons})$ through optical theorem...

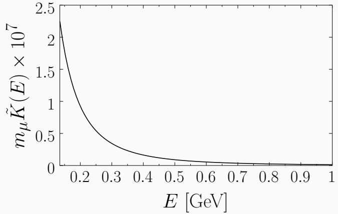



- In terms of the $e^+e^- \to$ hadron cross-section or actually the R-ratio:

$$R(E) = \frac{\sigma(e^+e^-(E) \to \text{hadrons})}{\sigma(e^+e^-(E) \to \mu^+\mu^-)}$$

- one has a very simple formula for $a_\mu^{\mathrm{HVP,LO}}$

$$a_\mu^{\mathrm{HVP,LO}} = \int_{m_\pi}^{\infty} dE\, R(E)\, \underbrace{\tilde{K}(E)}_{\text{analytic function}}$$



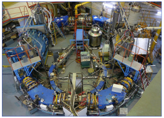
# $a_\mu^{\mathrm{HVP,LO}}$ from the dispersive approach

The central idea is to replace $R(E) \to R^{\mathrm{exp}}(E)$ and use previous formula.

$e^+e^- \to$ hadrons measured since '60 in various experiments

KLOE @ DAΦNE
FRASCATI

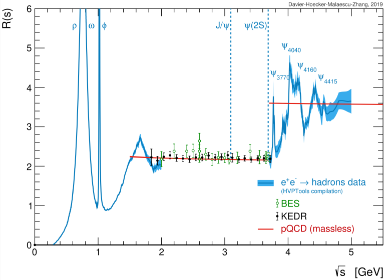
BABAR @ SLAC
STANFORD



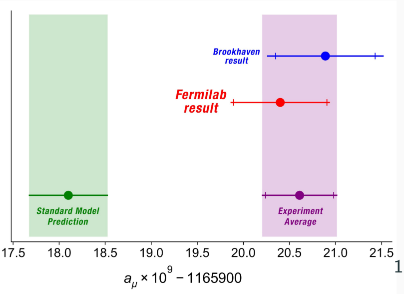
CMD3 @ VEPP-2000
NOVOSIBIRSK

Inclusive measurement of $R^{\mathrm{exp}}(E)$ obtained summing more than fourty exclusive channel measurements (comb. of various exp. , dominated by $\pi^+\pi^-$).

Experimental R-ratio



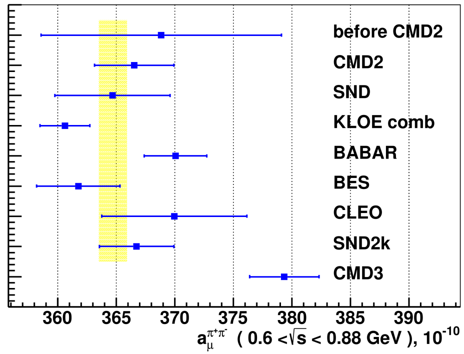

Status in 2020/2021



# The CMD-3 result [Phys. Rev. Lett. 132 (2024)]

A new measurement of $e^+e^- \to \pi^+\pi^-$ with CMD detector at VEPP-2000 in 2023,

found significant deviations from previous measurements




- Systematic uncertainty underestimated? (Talk by F. Piccinini this afternoon).
- At the moment the situation of exp. $e^+e^- \to$ hadrons needs to be clarified.
- However, since 2020 LQCD calculations reached the subpercent precision level...

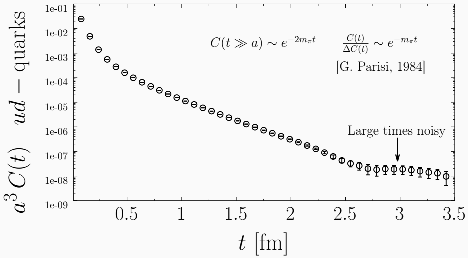
# $a_\mu^{\mathrm{HVP,LO}}$ from lattice QCD

On the lattice, evaluating $a_\mu^{\mathrm{HVP,LO}}$ is easier than $a_\mu^{\mathrm{Hlbl}}$, but $< 1\%$ accuracy needed!

The QCD input is the 2-point Euclidean correlation function of e.m. currents:

$$\boxed{C(t) = \frac{1}{3}\int d^3x\, \langle 0|J^i_{\mathrm{em}}(t,\boldsymbol{x})J^i_{\mathrm{em}}(0)|0\rangle \qquad J^i_{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma^i u - \frac{1}{3}\bar{d}\gamma^i d - \frac{1}{3}\bar{s}\gamma^i s + \frac{2}{3}\bar{c}\gamma^i c}$$

$$a_\mu^{\mathrm{HVP,LO}} = \int_0^\infty dt\ \underbrace{K(t)}_{\text{analytic kernel}}\ C(t) \qquad K(t) \overset{t\gg m_\mu^{-1}}{\rightarrow} t^2 \qquad \text{[Enhancement of } C(t) \text{ tail]}$$




**Main difficulties for subpercent accuracy**:

- Exponential S/N problem at large $t$.
- Large lattice volumes $V = L^3$ required to fit the light $\pi\pi$ states.
- Isospin-breaking effects $\alpha^3, \alpha^2(m_d - m_u)$ needs to be computed at target accuracy.

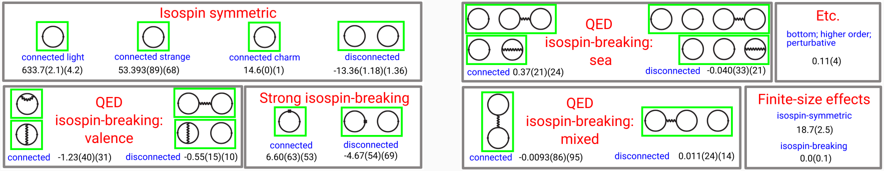
# First LQCD-result with $< 1\%$ errors by BMWc [Nature 593 (2021)]

**Leading hadronic contribution to the muon magnetic moment from lattice QCD**

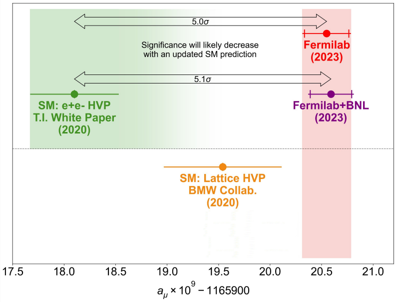
Sz. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato, K. K. Szabo, F. Stokes, B. C. Toth, Cs. Torok & L. Varnhorst

*Nature* **593**, 51–55 (2021) | Cite this article

**21k** Accesses | **403** Citations | **962** Altmetric | Metrics




$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$




- It quickly became evident that, to clarify the differences with the data-driven approach, a detailed examination of $R(E)$ was essential.
- In 2022-2023, our efforts primarily focused on the LQCD computation of the so-called Euclidean-time windows of the HVP.

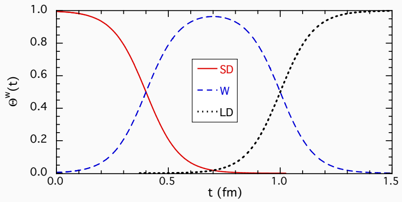
# The Euclidean windows to test $e^+e^- \to$ hadrons

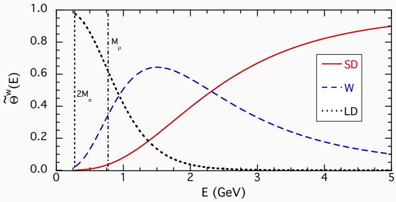
To perform stringent tests of $R(E)$ we are not bound to $a_\mu^{\mathrm{HVP,LO}}$

$$\underbrace{\int_0^\infty dt\ K(t)\ C(t)}_{\text{lattice, SM}} = a_\mu^{\mathrm{HVP,LO}} = \underbrace{\int_{M_\pi}^\infty dE\ \tilde{K}(E)\ R^{\exp}(E)}_{\text{dispersive, experimental}}$$

$$\Downarrow \qquad \Downarrow$$

$$\underbrace{\int_0^\infty dt\ K(t)\ C(t)\,\underline{\Theta^w(t)}}_{\text{lattice, SM}} = a_\mu^w = \underbrace{\int_{M_\pi}^\infty dE\ \tilde{K}(E)\ R^{\exp}(E)\,\underline{\tilde{\Theta}^w(E)}}_{\text{dispersive, experimental}}$$



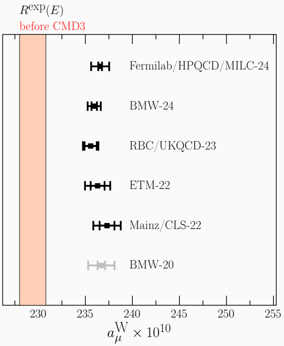




- $\Theta^{\mathrm{SD}} + \Theta^{\mathrm{W}} + \Theta^{\mathrm{LD}} = 1$. $w = \{\mathrm{SD}, \mathrm{W}, \mathrm{LD}\}$ probe $R(E)$ at different energies.
- $a^{\mathrm{SD/W}}$ very precise on the lattice $\implies$ may enhance differences with $R^{\exp}(E)$.

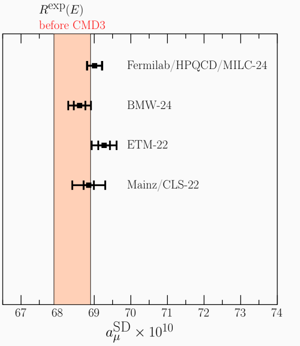
# The short- and intermediate-distance windows

In 22-24 several LQCD results for $a_\mu^{\mathrm{W}}$ and $a_\mu^{\mathrm{SD}}$. Many appeared before CMD3.

intermediate-distance $\Longrightarrow$ $E \lesssim 1$ GeV $(\pi\pi, \pi\pi\pi)$




short-distance $\Longrightarrow$ Large $E \gtrsim 1$GeV

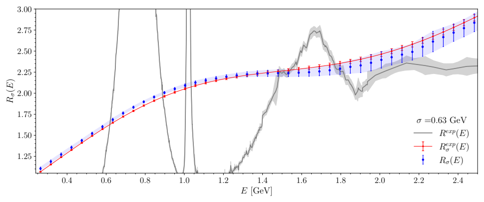



- A big achievement for the lattice community.
- Striking tension with $R^{\mathrm{exp}}(E)$-based results for $a_\mu^{\mathrm{W}}$ which is dominated by $e^+e^- \to \rho \to \pi^+\pi^-$. High-energy part of R-ratio in line with experiments.
- In PRL 130 (2023), we (ETMC) used the HLT method to compute the energy-smeared $R(E)$, reaching conclusions consistent with $a_\mu^{\mathrm{W}}$ analysis.

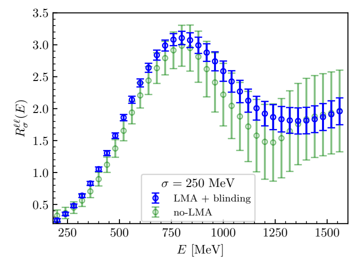
# The short- and intermediate-distance windows

In 22-24 several LQCD results for $a_\mu^{\mathrm{W}}$ and $a_\mu^{\mathrm{SD}}$. Many appeared before CMD3.

$$R_\sigma(E) = \int_0^\infty d\omega\, R(\omega)\, \underbrace{N(E-\omega,\sigma)}_{\text{Gaussian}}$$




Preliminary results with new gen. data (blinded)

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# $a_\mu^{\text{HVP,LO}}$ in the WP25

**Fall 2024: surge of new LQCD results!**

RBC/UKQCD – Phys. Rev. Lett. **134** (2025)
Mainz/CLS – JHEP **04** (2025) 098
Fermilab/HPQCD/MILC – Phys. Rev. Lett. **135** (2025)
ETMC – Phys. Rev. D **111** (2025)
BMW/DMZ – ePrint: 2407.10913

The **Muon** $g-2$ **Theory Initiative** combined all published LQCD results, yielding a robust lattice prediction for the LO–HVP contribution to $a_\mu$.

**Common decompositions of $a_\mu^{\text{HVP,LO}}$ adopted:**

- Flavour-based:

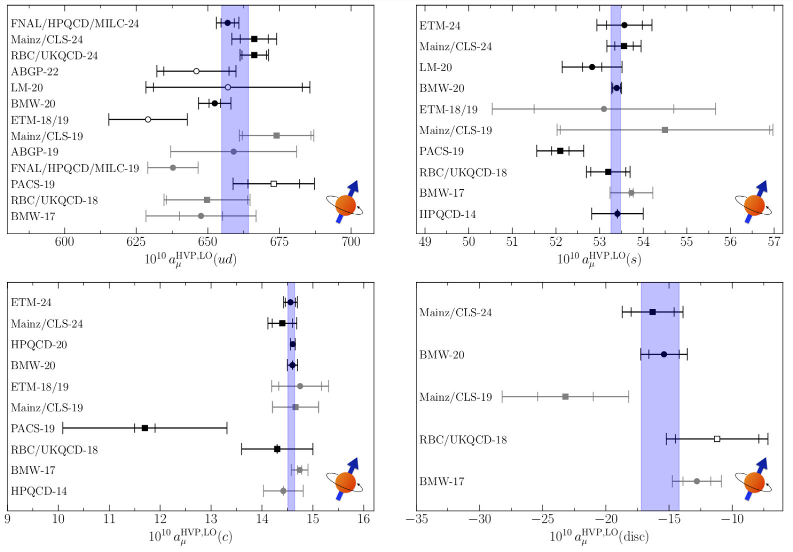
$$a_\mu^{\text{HVP,LO}} = \underbrace{a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_\mu^{\mathrm{HVP,LO}}(\text{disc})}_{a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso})} + \underbrace{\delta a_\mu^{\mathrm{HVP,LO}}}_{\text{isospin breaking}}$$

- Isospin-based: $a_\mu^{\text{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(I{=}1) + a_\mu^{\mathrm{HVP,LO}}(I{=}0) + \delta a_\mu^{\mathrm{HVP,LO}}$

- Window-based: $a_\mu^{\text{HVP,LO}} = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$ (also decomposed in flav./isospin)

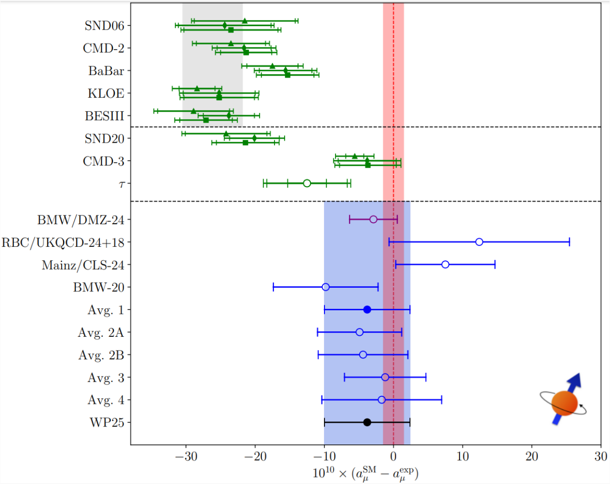
As some groups provide only partial results, the averaged value can vary slightly with the chosen decomposition. The **WP25 working groups** have thoroughly tested different combinations to ensure the stability of the global average.

# Collection of partial results from WP25




$$a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso}) = \mathrm{a}_\mu^{\mathrm{HVP,LO}}(\mathrm{ud}) + \mathrm{a}_\mu^{\mathrm{HVP,LO}}(\mathrm{s}) + \mathrm{a}_\mu^{\mathrm{HVP,LO}}(\mathrm{c}) + \mathrm{a}_\mu^{\mathrm{HVP,LO}}(\mathrm{disc})$$

# Final results from WP25




Adopting the lattice results for $a_\mu^{\text{HVP,LO}}$ leads to an upward shift of the SM prediction, bringing it into full agreement with the current world-average value for $a_\mu^{\text{exp}}$. The WP25 result (black point) still carries substantially larger uncertainties than the experimental measurement.

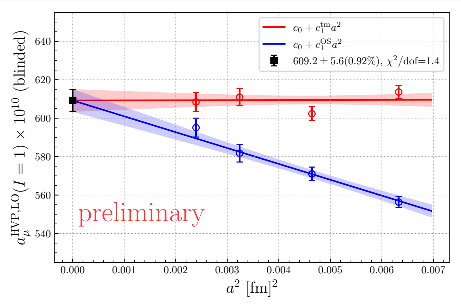
# A new result for $a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso})$ from ETMC to appear soon!

We employ the isospin-based decomposition:

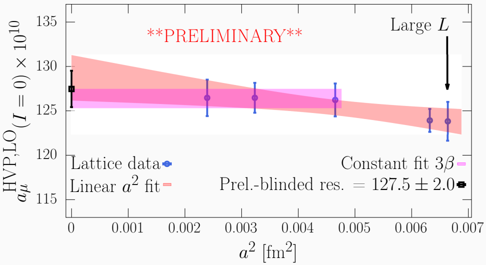
$$a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso}) = a_\mu^{\mathrm{HVP,LO}}(I=1) + a_\mu^{\mathrm{HVP,LO}}(I=0)$$

$I = 1$: Continuum extrapolation performed at fixed

$$V = L^3 = (5.46 \text{ fm})^3$$




charmless $I = 0$ contribution




- Achieved competitive accuracy on both the $I = 0$ and $I = 1$ contributions. We will likely end up with a $< 1\%$ precision for $a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso})$.
- Technical details on the lattice QCD calculation are in backup, if you are curious!

# Summary

Where are we?

$a_\mu^{\mathrm{HVP-LO}}$

- In 2020, the BMW collaboration reported a discrepancy between its result for $a_\mu^{\mathrm{HVP,LO}}$ and the dispersive determination.
- Recent independent LQCD calculations have substantially confirmed the BMW findings.
- The updated SM prediction for $a_\mu^{\mathrm{HVP,LO}}$ from WP25, based on LQCD inputs, is now consistent with the experimental value $a_\mu^{\mathrm{exp}}$.
- Further improvements in lattice QCD precision are required to match experimental accuracy.
- All major collaborations are actively pursuing this goal. Within ETMC, we will soon release a new result for $a_\mu^{\mathrm{HVP,LO}}(\mathrm{iso})$ and are currently working on $\delta a_\mu^{\mathrm{HVP,LO}}$.

$e^+e^- \to$ hadrons

- Lattice QCD has revealed an inconsistency between previous $e^+e^- \to$ hadrons measurements and the SM prediction.
- The 2022/2023 LQCD window results have been instrumental in highlighting this issue.
- Possible explanations include unaccounted systematic effects in the experimental measurement — more will be discussed in the following talks.
- The recent CMD-3 result may offer valuable insight into this discrepancy.
- The situation remains open and requires further clarification.

Thank you for the attention

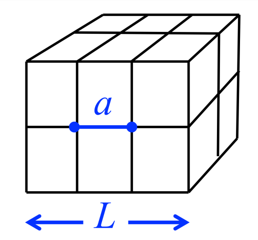
# Backup slides

# Basics of LQCD

The theoretical framework for lattice calculations is QFT in Euclidean time (obtained through Wick-rotation $t \to -i\tau$)

$$\langle \phi(x_1)\phi(x_2)\ldots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int [d\phi]\ \phi(x_1)\phi(x_2)\ldots\phi(x_n)\exp(-S_E[\phi])$$

The infinite-dimensional path integral is discretized on a 4-dimensional grid (the lattice) : $x_\mu \to n_\mu a$, which provides an UV ($1/a$) and IR ($1/L$) cut-off.




- We evaluate lattice path integral using MC methods.
- In QCD generate a stream of gauge configurations $\{U_1, \ldots, U_N\}$ distributed according to $e^{-S_E[U]}$, then...

$$\langle\bar{\mathcal{O}}\rangle = \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}[U_i] \implies \sigma_{\bar{\mathcal{O}}} \propto \frac{1}{\sqrt{N}}$$

- Repeat the calculation for different $L$ and lattice spacings $a$ and extrapolate to $a, 1/L \to \infty$.

# Generating gauge configurations

Generating state-of-the-art gauge-field configurations is an extremely expensive task, which requires massive HPC resources.

GPU-cluster Marconi100 at CINECA, Bologna.
Ceased its activities in 2023. . .

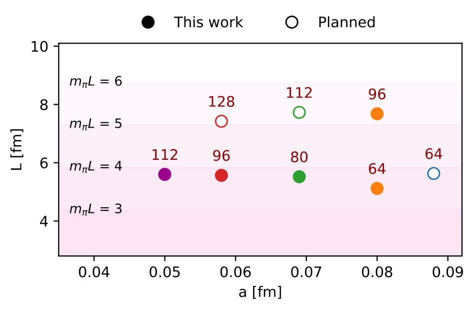

. . . replaced by LEONARDO,
the 4th fastest supercomputer in the world.

- Within the LQCD community, it is customary for researchers to form collaborations where gauge configurations are produced and then shared among the members.
- Each collaboration has its own favoured lattice discretization: Wilson-clover, Twisted-mass, Staggered, Domain Wall, Overlap...
- $\Longrightarrow$ Important for checks of universality.

The Extended Twisted-Mass Collaboration (ETMC) has recently produced a "luxury" set of gauge configurations, corresponding to (five) lattice spacings $a \in [0.049, 0.09]$ fm, spatial volumes $L^3$ up to $L \simeq 7.6$ fm and $N_f = 2+1+1$ physical flavours.

# Simulation details

Four physical-point $N_f = 2+1+1$ ensembles, with $a \in [0.049 \text{ fm} - 0.080 \text{ fm}]$. $L \sim 5.1$ fm and $L \sim 7.6$ fm to control Finite Size Effects (FSEs).

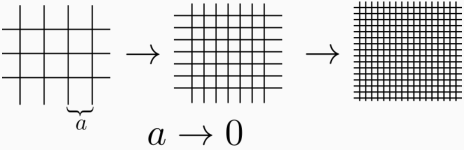



| ID | $V/a^4$ | $a$ (fm) | $L$ (fm) |
|---|---|---|---|
| B64 | $64^3 \times 128$ | 0.0795 | 5.09 |
| B96 | $96^3 \times 192$ | 0.0795 | 7.64 |
| C80 | $80^3 \times 160$ | 0.0682 | 5.46 |
| D96 | $96^3 \times 192$ | 0.0569 | 5.46 |
| E112 | $112^3 \times 224$ | 0.0489 | 5.46 |

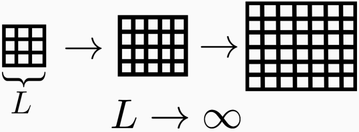
- Iwasaki action for gluons.
- Wilson-clover twisted mass fermions at maximal twist for quarks (automatic $\mathcal{O}(a)$ improvement).

# Our strategy

We perform the continuum-limit extrapolation ($a \to 0$) at fixed volume ($L \simeq 5.46$ fm):




and then, the infinite-volume extrapolation ($L \to \infty$):

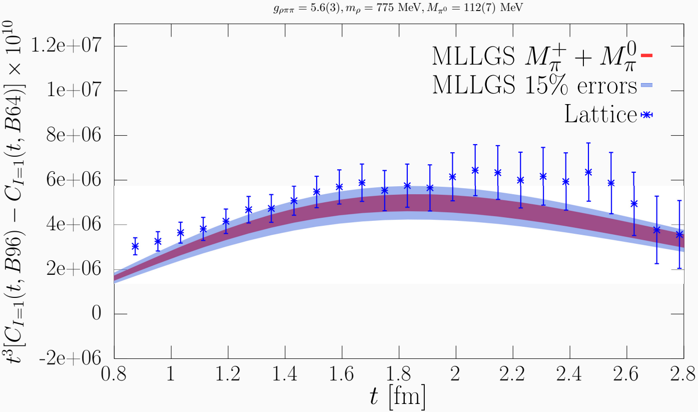



using the ensembles B64-B96, corresponding to $L \simeq 5.1$ fm and $L \simeq 7.6$ fm.

- In the case of $I = 0$ contribution, finite-size effects (FSE) are extremely small.
- For $I = 1$, dominated by $\pi^+\pi^-$ states, FSEs are sizable! Our strategy is to use the Meyer-Lellouch-Luscher-Gounaris-Sakurai (MLLGS) model to describe the finite-size effects, after checking that the model describes the B64-B96 data reasonably (model-validation).

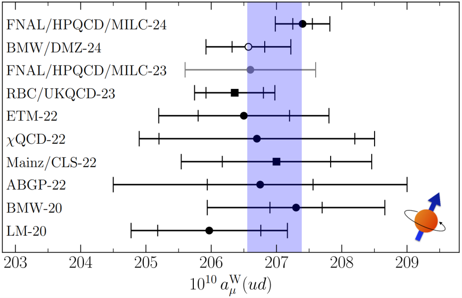
# Validation of the MLLGS model

We have compared our predictions for the MLLGS model against our data for the correlator corresponding to the $I = 1$ contribution.




The lattice MLLGS model we employ takes into account the distortion of the $\pi\pi$ spectrum occurring in twisted-mass LQCD.

# Collection of partial results from WP25 (II)




Many independent results for $a_\mu^{\mathrm{W}}(ud)$.

All in very good agreement!

Separation of $a_\mu^{\mathrm{HVP,LO}}$ into an isospin-symmetric term + $\delta a_\mu^{\mathrm{HVP,LO}}$ is scheme-dependent. Great effort in the WP25 to match all results to a reference scheme!