Muon $g-2$ or stress-testing the SM

legacy and implications of the FNAL $\mathrm{g}-2$ experiment and theory progresses

Introduction

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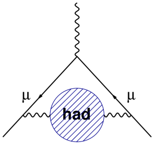
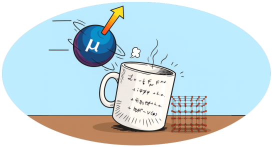
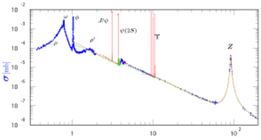
Why is muon $g - 2$ physics a stress-test for the Standard Model (SM) of fundamental interactions?

$\Longrightarrow$ $g_\mu - 2$ probes the strong interaction sector of the SM and its extensions to 0.2% precision ...

$\Longrightarrow$ a consistency check requires theory predictions and experimental data with similar accuracy







$\sigma(e^- e^+ \to \text{hadrons})$

What do we learn from the FNAL experiment and the related experimental and theoretical activity?

$\Longrightarrow$ A few key lessons about *precision physics of the SM*: feasibility conditions, cost and reach

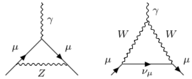
Muon $g-2$ ... or stress-testing the SM

- Experiments with muons (μ) in a magnetic field $\vec{B}$: observable $a_\mu \equiv \frac{g_\mu - 2}{2}$
  - Magnetic moment $\quad \vec{\mathcal{M}} = g\,\frac{q}{2m}\;\vec{S} \qquad \vec{T} = \vec{\mathcal{M}} \times \vec{B}$, $\quad$ no quantum effects: $\quad g_{\,\mathrm{Dirac-fermion}} = 2$
  - Quantum fluctuations affect $\vec{S}_\mu - \vec{B}$ coupling $\Longrightarrow$ anomalous magnetic moment: $a_\mu = O(\frac{\alpha_{em}}{\pi})$

- In the SM various contributions to a_μ: in terms of Feynman diagrams (much more than a cartoon)

$$a_\mu = \; a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had} \;\simeq\; 0.0011659207 + 0.0000000015 + 0.0000000715 \quad \text{(errors: few } 10^{-10}\text{)}$$

(a) Leading order (Dirac theory) (b) First order QED (Schwinger)



(a) Neutral Current (b) Charged Current (c) Higgs boson

(a) Hadronic vacuum polarization (b) Hadronic Light by Light

- Probing via muons the ground state (vacuum) of Nature – i.e. its content of particle-antiparticle pairs
  - at energy scales where gravity effects are fully negligible
  - is the vacuum particle-antiparticle content seen in experiment the same as expected from SM theory ?

a_μ as a probe for "beyond the SM" (BSM) physics

- a_μ exp. relative precision, $\delta^{exp} a_\mu \sim 10^{-7}$, is to be compared with potential impact on it of BSM physics
- $\delta^{BSM} a_\mu = \dfrac{a_\mu^{BSM} - a_\mu^{SM}}{a_\mu^{SM}}$, e.g. $\delta^{BSM} a_\mu \sim \dfrac{m_\mu^2}{M_{BSM}^2} \simeq 10^{-8} \dfrac{1\mathrm{TeV}^2}{M_{BSM}^2}$ (if tree-level muon–BSM coupling)
- good potential for setting strong SM-constraints or even for BSM-discovery

$\Longrightarrow$ provided the prediction a_μ^{SM} is known with similar relative accuracy ... i.e. one needs

- relative precision $\sim 10^{-7}$ in QED (5 loop PT) and ~ 0.1–0.01 in EW (2 loop PT) sector: OK
- relative precision 10^{-3} in the hadronic sector of SM (due to $a_\mu^{Had} \sim 10^{-4} a_\mu$): challenging

$\Longrightarrow$ a SM prediction of a_μ^{Had} with subpercent accuracy has been / can be achieved via

- Lattice QCD+QED + Hadron Mass exp. input $\rightarrow$ Euclidean correlators $G(\tau) = \int d^3x \langle J_k^{em}(\tau, \vec{x}) J_k^{em}(0) \rangle$
- dispersive analysis of $e^- e^+ \rightarrow$ hadrons exp. cross section data $\rightarrow$ $R^{Had}(E)$ ratio

On "good potential for SM-constraints or BSM-discovery": let us be realistic !

2010 - 2025: from hyper-optimism (SUSY, etc.) to over-pessimism (nothing) about Beyond-SM physics

It could have been / should now be different: SM is very nice & successful - but INCOMPLETE

⋆ Nice & successful : mathematically consistent (renormalizable) $\rightarrow$ hides what is beyond it !

(unlike the Fermi theory of weak interactions: $G_F/\sqrt{2} = g_W^2/(8M_W^2) \Rightarrow$ breakdown at $E \sim M_W$!)

⋆ Incomplete : *many facts* ... still we ignore *where (in energy & couplings space)* BSM physics lies !

- *matter-antimatter asymmetry* (EW phase transition? CP-violation? B-violation ...)
- *no particle candidates for Dark Matter* (astrophysics: hard to explain via modified gravity)
- *no quantum gravity & unsolved dark energy puzzles* (astrophysics, cosmology)
- *tiny ν masses*: may be included in SM framework; do they hint at $m_\nu \sim g_{BSM} M_{SM}^2/M_{BSM} \sim 10^{-12} M_{SM}$?

On "good potential for SM-constraints or BSM-discovery": *other relevant processes* !

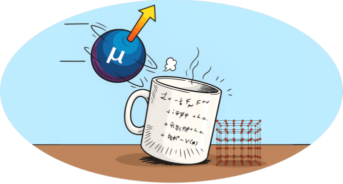
- ⋆ SM effective theory viewpoint appears "convenient" to describe the impact on physics at $E \ll M_{BSM}$ of BSM physics at scale $\sim M_{BSM}$ entailing very massive and/or accidentally (quasi-GB) light new particles

- ⋆ $L_{BSM}^{eff} = O_{d5}^{\nu\nu\phi\phi} / M_{BSMa} + O_{d5}^{\mathrm{axion}} / M_{BSMb} + O_{d6}^{SMEFT} / M_{BSMc}^2$ with unknown couplings in $O_{d..}^{..}$ terms and mass scales $M_{BSM..}$ that are to be constrained or directly revealed

- ⋆ A very incomplete list:
  - $R^{Had}(E;\epsilon)$ for $e^- e^+ \to$ hadrons , ... $a_\mu^{win} \leftrightarrow$ smeared $R^{Had}(1.5GeV, 1.0GeV)$ *(predicted with* $\sim 0.2\%$*prec.)*
  - isospin rotated: $\tau^+ \to X_{Incl}^{Had}\, l^+ \nu_l$, ... flavour singlet: $\pi^0\,(\eta\,,\,\eta') \to \gamma\gamma$
  - leptonic hadron decays: ... $\pi^+(K^+) \to l^+\nu_l$ in QCD + QED *(predicted with* $\sim 0.2\%$ *prec.)*
  - semileptonic hadron decays: $K \to \pi l \nu_l$ .... $B_s \to D_s^- l^+ \nu$
  - rare processes (quantum effects in SM): $K - \bar{K}$ oscillations .... $B_s \to l^- l^+ (\gamma)$
  - high-E processes involving Higgs boson (y_f's, triple-h, indir.), EW (α, $\sin^2\theta_W$), α_S couplings ...

Key: a few observables with high & robust precision in both experimental data and theory prediction

THANK YOU FOR PARTICIPATION !



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