Next-to-Next-to-Leading Order Predictions from Monte Carlo Event Generators Combined with Parton Showers

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NNLO Monte Carlo event generators

Why Monte Carlo event generators?

- The **fully differential events** are the closest simulations of the scattering processes observed at colliders
- They combine
  - **Multi-loop matrix elements** $\to$ perturbative accuracy
  - (**Analytic resummation** $\to$ logarithmic accuracy)
  - **Parton showers** $\to$ multiplicity of the final state
  - **Hadronization models** $\to$ from colored partons to colorless hadrons

Why NNLO?

- The **LHC** is turning into a precision machine

  $\to$ More and more accurate theoretical predictions are needed

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The Geneva framework

GENEVA is a **Monte Carlo event generator** that provides

- Fully differential **events** up to **NNLO QCD** accuracy
- **NNLL′ resummation** of the 0-jet resolution variable (e.g. the zero-jettiness $\mathcal{T}_0$, the color-singlet transverse momentum q_{T} or the transverse momentum $p_{\mathrm{T}}^{\mathrm{jet}_1}$ of the hardest jet)
- **NLL′ resummation** of the 1-jet resolution variable (e.g. the one-jettiness $\mathcal{T}_1$ or the transverse momentum $p_{\mathrm{T}}^{\mathrm{jet}_2}$ of the second hardest jet)
- **LL** interface to **parton showers**

Examples of 0-jet resolution variables

Given a process of production of a color singlet

$$p\,p \to \mathrm{CS}(q) + X$$

the phase space regions where the matrix elements present QCD infrared divergences at NLO can be equivalently identified as those with low

- **Color-singlet transverse momentum** q_{T}
- **Zero-jettiness** $\mathcal{T}_0$
- **Hardest-jet transverse momentum** $p_{\mathrm{T}}^{\mathrm{jet}_1}$

0-jet resolution variables

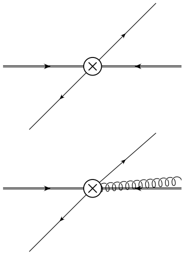
We can equivalently use q_{T}, $\mathcal{T}_0$ and $p_{\mathrm{T}}^{\mathrm{jet}_1}$ as **0-jet resolution variables**

If X is a massless parton of momentum p

$$q_{\mathrm{T}} = \sqrt{q_x^2 + q_y^2} \qquad \mathcal{T}_0 = \min(\hat{p}_0 - \hat{p}_z,\, \hat{p}_0 + \hat{p}_z) \qquad p_{\mathrm{T}}^{\mathrm{jet}_1} = \sqrt{p_x^2 + p_y^2}$$

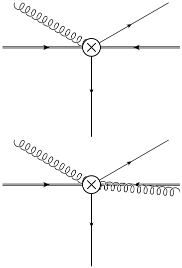
Slicing of the phase space

$$\mathcal{T}_0 < \mathcal{T}_0^{\mathrm{cut}}$$



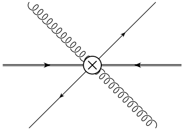
No resolved partons in the final state

$$\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}$$
$$\mathcal{T}_1 < \mathcal{T}_1^{\mathrm{cut}}$$



One resolved parton in the final state

$$\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}$$
$$\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}$$

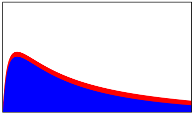


Two resolved partons in the final state

Matching the resummation and the fixed order calculation

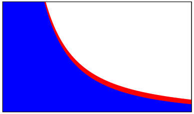
We write the **NNLO differential $\mathcal{T}_0$ spectrum** as the sum of a **resummed singular** and **non-singular** spectrum

$$\frac{d\sigma^{\mathrm{NNLO}}_{\mathrm{CS}}}{d\Phi_0\, d\mathcal{T}_0} = \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0} + \frac{d\sigma^{\mathrm{nonsing}}_{\mathrm{CS}}}{d\Phi_0\, d\mathcal{T}_0}$$



where the **non-singular** spectrum is given by the difference between the **fixed-order** and the α_{S} **expansion of the resummed singular** spectrum

$$\frac{d\sigma^{\mathrm{nonsing}}_{\mathrm{CS}}}{d\Phi_0\, d\mathcal{T}_0} = \int \frac{d\Phi_1}{d\Phi_0\, d\mathcal{T}_0} \frac{d\sigma^{\mathrm{NLO}_1}_{\mathrm{CS+jet}}}{d\Phi_1} - \left.\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0}\right|_{\mathrm{NLO}_1}$$



0-jet exclusive and 1-jet inclusive differential cross sections

- The phase space points with $\mathcal{T}_0 < \mathcal{T}_0^{\rm cut}$ contribute to the **NNLO 0-jet exclusive differential cross section**

$$\frac{d\sigma_{\rm CS}^{(0)}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) = \frac{d\sigma^{\rm NNLL'}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) + \frac{d\sigma_{\rm CS}^{\rm nonsing}}{d\Phi_0}(\mathcal{T}_0^{\rm cut})$$

- The phase space points with $\mathcal{T}_0 > \mathcal{T}_0^{\rm cut}$ contribute to the **NLO_1 1-jet inclusive differential spectrum**

$$\frac{d\sigma_{\rm CS}^{(\geq 1)}}{d\Phi_0\, d\mathcal{T}_0} = \frac{d\sigma^{\rm NNLL'}}{d\Phi_0\, d\mathcal{T}_0} + \frac{d\sigma_{\rm CS}^{\rm nonsing}}{d\Phi_0\, d\mathcal{T}_0}$$

- The integration recovers the total **NNLO differential cross section**

$$\frac{d\sigma_{\rm CS}^{\rm NNLO}}{d\Phi_0} = \frac{d\sigma_{\rm CS}^{(0)}}{d\Phi_0}(\mathcal{T}_0^{\rm cut}) + \int d\mathcal{T}_0\, \frac{d\sigma_{\rm CS}^{(\geq 1)}}{d\Phi_0\, d\mathcal{T}_0}\, \theta(\mathcal{T}_0 - \mathcal{T}_0^{\rm cut})$$

The splitting functions

To generate Φ_1 events, we need to spread the **1-jet inclusive differential spectrum** over the entire Φ_1 phase space

$$\frac{d\sigma^{(\geq 1)}_{\mathrm{CS}}}{d\Phi_1} = \left(\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0} - \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0\, d\mathcal{T}_0}\bigg|_{\mathrm{NLO}_1} \right) P_{0\to 1}(\Phi_1) + \frac{d\sigma^{\mathrm{NLO}_1}_{\mathrm{CS+jet}}}{d\Phi_1}$$

- The **fixed-order contribution** has a natural Φ_1 dependence
- We multiply the **resummed contributions** by a **splitting function** $P_{0\to 1}(\Phi_1)$ defined so that, for every function $f(\Phi_0, \mathcal{T}_0)$ of the underlying phase space Φ_0 and the resolution variable $\mathcal{T}_0$,

$$\int d\Phi_1\, f(\Phi_0, \mathcal{T}_0)\, P_{0\to 1}(\Phi_1) = \int d\Phi_0 \int d\mathcal{T}_0\, f(\Phi_0, \mathcal{T}_0)$$

Combination with the parton shower

- Ideally, we would like to feed in the generated NNLO events to a **$\mathcal{T}_N$-ordered shower** where

$$\mathcal{T}_1(\Phi_2) > \mathcal{T}_2(\Phi_3) > \ldots > \mathcal{T}_n(\Phi_{n+1}) > \ldots$$

- If that is not available, we simulate it by mean of a **vetoed p_{T}-ordered shower**

Vetoed shower

- We compute the p_{T} starting scale of the shower
- We pass the event to the shower and let it generate the additional QCD (and QED) radiation
  - → If $\mathcal{T}_2(\Phi_n) > \mathcal{T}_1(\Phi_2)$, we re-shower the event (**veto**)
  - → Otherwise, we add MPI and hadronization effects

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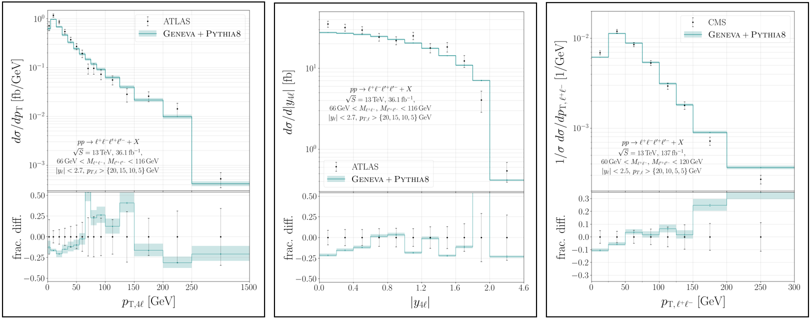
ZZ distributions in Geneva



- GENEVA predictions from S. Alioli, A. Broggio, AG, S. Kallweit, M. A. Lim, R. Nagar, D. Napoletano Phys.Lett.B 818 (2021) 136380
- ATLAS and CMS data at 13 TeV from Phys.Rev.D 97 (2018) 3, 032005 and Eur.Phys.J.C 81 (2021) 3, 200
- See also L. Buonocore, G. Koole, D. Lombardi, L. Rottoli, M. Wiesemann, G. Zanderighi JHEP 01 (2022) 072 for MINNLO$_{\text{PS}}$ predictions

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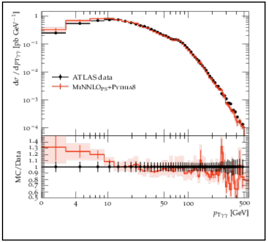
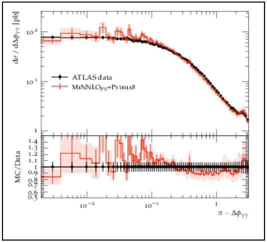
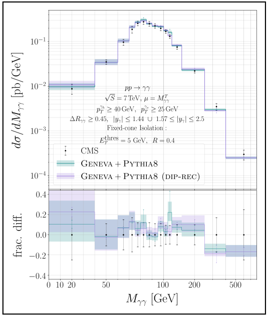
$\gamma\gamma$ distributions in MiNNLO$_{\rm PS}$ and Geneva







- MiNNLO$_{\rm PS}$ predictions from AG, C. Oleari, E. Re JHEP 09 (2022) 061
- Geneva predictions from S. Alioli, A. Broggio, AG, S. Kallweit, M. A. Lim, R. Nagar, D. Napoletano, L. Rottoli JHEP 04 (2021) 041
- ATLAS data (at 13 TeV) and CMS data (at 7 TeV) from JHEP 11 (2021) 169 and Eur. Phys. J. C 74 (2014) 3129

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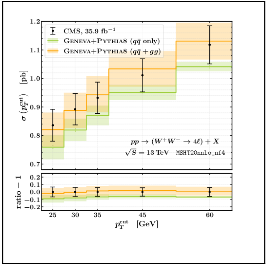
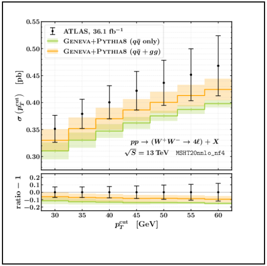
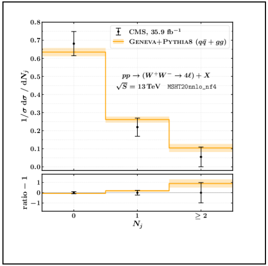
WW distributions in Geneva

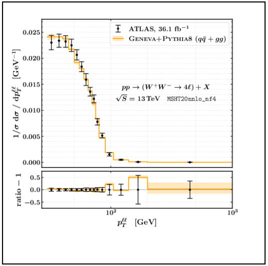


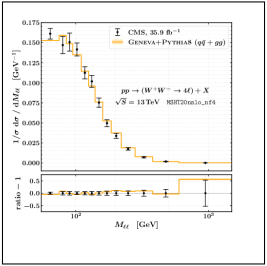


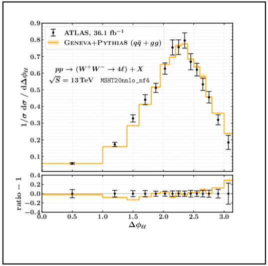


- Geneva predictions with **hardest-jet transverse momentum resummation** from AG, M. A. Lim, S. Alioli, F. Tackmann JHEP 12 (2023) 069
- ATLAS and CMS data at 13 TeV from Eur. Phys. J. C 79 (2019) 884 and Phys. Rev. D 102, 092001 (2020)

WW distributions in Geneva







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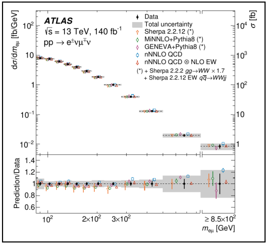
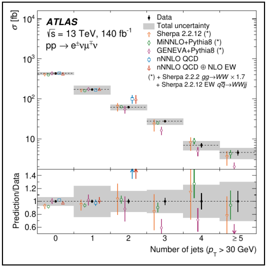
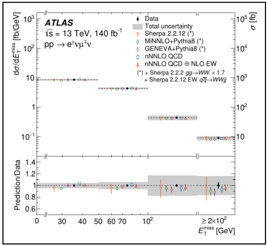
- ATLAS and CMS data at 13 TeV from Eur. Phys. J. C 79 (2019) 884 and Phys. Rev. D 102, 092001 (2020)

Comparison among different *WW* predictions in Geneva







- Comparison among data and several different theoretical predictions (Sherpa, MiNNLO$_{\text{PS}}$, Geneva and Matrix) from ATLAS (JHEP 08 (2025) 142)
- See D. Lombardi, M. Wiesemann, G. Zanderighi JHEP 11 (2021) 230 for MiNNLO$_{\text{PS}}$ predictions

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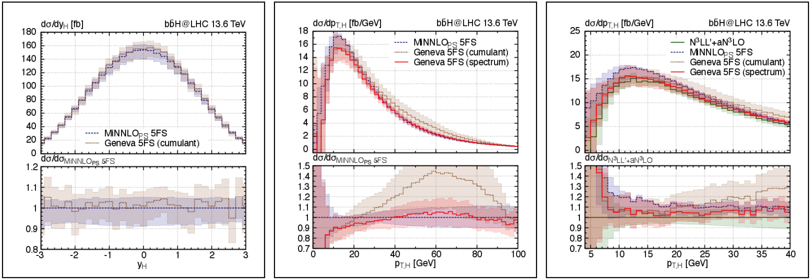
$b\bar{b} \to H$ distributions in MiNNLO$_{\text{PS}}$ and Geneva



- MiNNLO$_{\text{PS}}$ vs. Geneva comparisons from the bbH/bH contribution to the LHCHWG Report 5 arXiv:2510.18815 [hep-ph]
- See AG, R. von Kuk, M. A. Lim arXiv:2505.14773 [hep-ph] for Geneva predictions
- See C. Biello, A. Sankar, M. Wiesemann, G. Zanderighi Eur.Phys.J.C 84 (2024) 5, 479 for MiNNLO$_{\text{PS}}$ predictions

Thanks for your attention!

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