Exact results in QFTs

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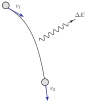
INFN retreat

S. Stefano Belbo, 15 November 2025

FONDO ITALIANO PER LA SCIENZA

Journey to S. Stefano

- 2016-2020: PhD at **Universitá di Torino**, advisor Marco Billó

  Main interests: 

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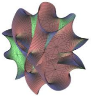
  Main interests: 

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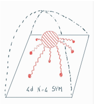
  Main interests: 

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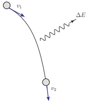
  Main interests:  ()

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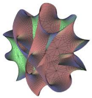
  Main interests: 

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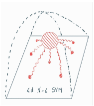
  Main interests: 

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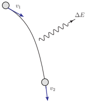
  Main interests: 

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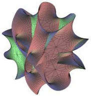
  Main interests:  ( )

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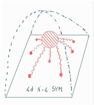
  Main interests: 

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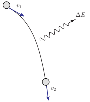
  Main interests: 

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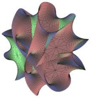
  Main interests: 

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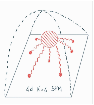
  Main interests: 

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Research interests: QFTs with extra symmetries

- Any well-defined Quantum Field Theory (QFT) possesses a spacetime **Poincaré symmetry** $\{P_\mu, M_{\mu\nu}\}$.
- In high energy theoretical physics, possible extensions have been explored to improve analytical power:
  - **conformal symmetry:** Scale invariance at the quantum level $x^\mu \to \lambda x^\mu$.
  - **supersymmetry:** Fermionic generators Q acting as:

$$Q\,|\text{Boson}\rangle \propto |\text{Fermion}\rangle \quad \text{and} \quad Q\,|\text{Fermion}\rangle \propto |\text{Boson}\rangle$$

  Fundamental fields organized into supermultiplets.
- **WHY?** Additional symmetries provide **more constraints**, allowing us to go beyond (and test) perturbative approximations to get exact results.

Main Tool: solving path integrals

- Technique: **Supersymmetric Localization** [Pestun, 2007]

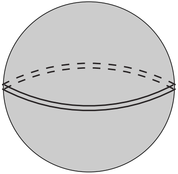
$$\int \left[\mathcal{D}\Phi(x)\right] \xrightarrow{Q_{SUSY}} \int da_{\,[N\times N]}$$

- Path integral localized to a finite dimensional integral that can be solved exactly.
- **Example**: supersymmetric circular Wilson loop in $\mathcal{N}=4$ SYM.

$$W(C) = \frac{1}{N}\mathrm{Tr}\,\mathcal{P}\exp\left\{\oint_C d\tau\left[\mathrm{i}\,A_\mu\,\dot{x}^\mu + \phi_1\right]\right\}$$

- The localization procedure allows for an exact function of the coupling $g_{\rm YM}$

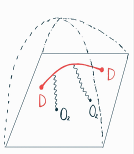
$$\langle W(C)\rangle \to \left\langle W_{\rm eq}\right\rangle = \left\langle \frac{1}{N}\mathrm{Tr}\,e^{2\pi a}\right\rangle_{\text{gaussian}}$$
$$= \frac{1}{N}\,L^1_{N-1}\left(-\frac{g^2_{\rm YM}}{4}\right)\exp\left[\frac{g^2_{\rm YM}}{8}\right]$$



Several recent results for integrated correlators and the holographic dual scattering amplitudes:

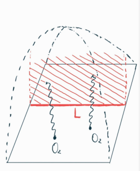
- Four-point correlator with **determinant operators**, dual to D3-brane/graviton scattering.
  [Brown, Dorigoni, FG, Wen, '24-'25]

$$\langle O_2(x_1)O_2(x_2)D(x_3)D(x_4)\rangle$$



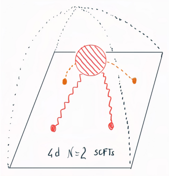
- Two-point integrated correlator with a **Line defect**, dual to graviton scattering off a long string
  [Billó, Frau, FG, Lerda, '23-'24]

$$\langle O_2(x_1)O_2(x_2)\,L\rangle$$



- Four-point integrated correlator in special $\mathcal{N}=2$ SCFTs, dual to mixed gluon/graviton scattering.
  [De Lillo, Frau, FG, Lerda et al. '25]

$$\langle O_2(x_1)O_2(x_2)\Phi_2(x_3)\Phi_2(x_4)\rangle$$



We study QFTs with a global charge (e.g. scalar ϕ^4 theory with $O(2)$ global symmetry)

- State created by an operator $\mathcal{O}_J$ carrying a large charge $J \gg 1$.
- Energy of the state $\Delta(J)$ grows with J.
- The path integral is dominated by a classical configuration (saddle point):

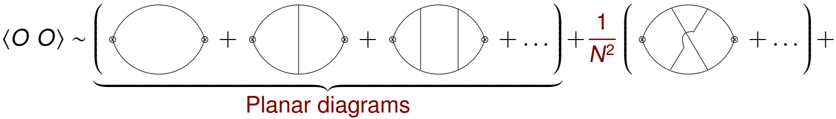
$$\langle \mathcal{O}_J(x_1)\mathcal{O}_J(x_2)\ldots\rangle \sim \int \mathcal{D}\Phi\, e^{-S[\Phi]}\mathcal{O}_J(x_1)\mathcal{O}_J(x_2) \xrightarrow{J\gg 1} e^{-S_{\text{EFT}}[\Phi_{\text{classical}}]}$$

- The EFT for the low-energy fluctuations around $\Phi_{\text{classical}}$ allows to explore the strong coupling dynamics with several **universality properties**. Ask Domenico for many more details!

Large charge in gauge theories: Feynman diagrams

- YM theories with $SU(N)$ gauge group: play with the two parameters N and J.

- When considering the usual 't Hooft limit $N \to \infty\,,\ \ \lambda = g^2_{\mathrm{YM}} N$ fixed:



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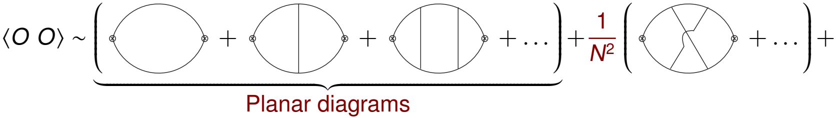
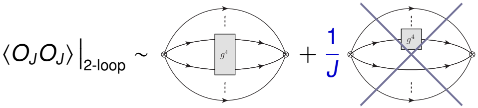


- One can consider the large charge 't Hooft limit $J \to \infty\,,\quad \lambda = g^2_{\mathrm{YM}} J$ fixed (and $J \gg N$)



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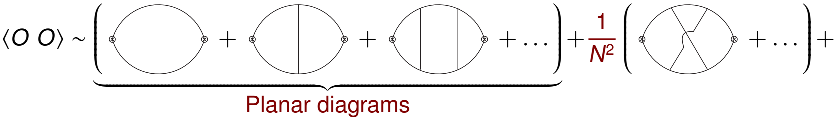

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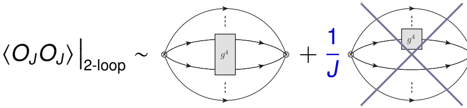
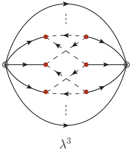


- It turns out large-J selects the maximally non-planar diagrams in N: (tested in $\mathcal{N} = 2, 4$ theories) [Beccaria, FG, Hasan '20 - Brown, FG, Wen, '24]



Large charge and semiclassics in SUSY gauge theories

- we can choose constrained large charge insertions: $\langle O_H(x_1) O_H(x_2) \ldots \rangle$ preserving (part of) superconformal symmetry

- Path integral dominated by saddle point equations

$$\delta_\Phi \Big(-S[\Phi] + \log O_H(x_1) + \log O_H(x_2) + \ldots \Big) = 0$$

- Solutions of the saddle point equations are non-trivial profiles for the scalar fields in the theory: $\phi^I \sim \phi^I_{\rm cl} \propto \sqrt{J}$

- EFT of fluctuations = Coulomb branch physics

- Large charge in $\mathcal{N} = 4$ SYM $\Rightarrow$ simplest example of **non-conformal QFT**.

- Universal properties linking SUSY theories to more realistic theories.

HHLL correlators in $\mathcal{N} = 4$ SYM

- Define L operators as $L = O_2$ [Brown, Grassi, FG, Iossa, Wen, '25]

  Choose H operators as $H = O_K^m \sim (\operatorname{tr} \phi^K)^m$ such that $m \to \infty$ (so $\Delta_H \gg N$)

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- Result from semiclassics + localisation

$$\langle O_K^m(x_1) O_K^m(x_2) O_2(x_3) O_2(x_4) \rangle = \frac{1}{x_{34}^2} \sum_s \left(t_s^2(u, v, \lambda) - 1 \right) \,,$$

where • $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$

• $t_s = \sum_{\ell=0}^{\infty} (-M_s^2)^\ell P^{(\ell)}(u, v) = \frac{u}{\sqrt{v}} \sum_{r=1}^{\infty} \frac{r\, e^{-\sigma \sqrt{r^2 + 4M_s^2}}}{\sqrt{r^2 + 4M_s^2}} \frac{\sin(r\varphi)}{\sin(\varphi)}$

• $M_s(\lambda)$ masses in the EFT

• $P^{(\ell)}(u, v)$ ladder Feynman integrals [Davydichev, Usyukina - Broadhurst, '93]

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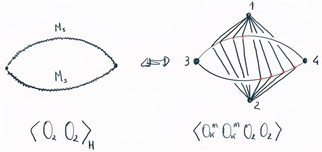
- Interpretation: scalar particles propagating in a heavy background



Connection with scalar field theories

- the function $t(u, v, \lambda)$ is associated to the scalar massive propagator
- same result as the heavy correlators in critical $O(N)$ ϕ^4-model [Giombi, Hyman ’20]
- Universal behavior for HHLL correlators across different theories

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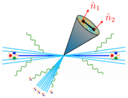
Connection with QCD-like theories



- 4pt correlators in $\mathcal{N} = 4$ SYM can be mapped to Energy-Energy correlators $\left\langle E(\vec{n}_1)E(\vec{n}_2)\right\rangle$ [Belitsky, Korchemsky, Sokatchev, Zhiboedov '13]
- $E(\vec{n})$ is the integrated energy flux measured by a detector pointed in the direction $\vec{n}$.
- Our HHLL result can provide some fully non-perturbative prediction of EEC in heavy states ($\sim$ hadron regime).