

EOS from effective Lagrangians and Neutron Stars

Passarella Luca, Polytechnic of Turin and INFN Turin
Iniziativa Specifica: NEUMATT

Santo Stefano Belbo, 15/11/2025



Istituto Nazionale di Fisica Nucleare



**Politecnico
di Torino**

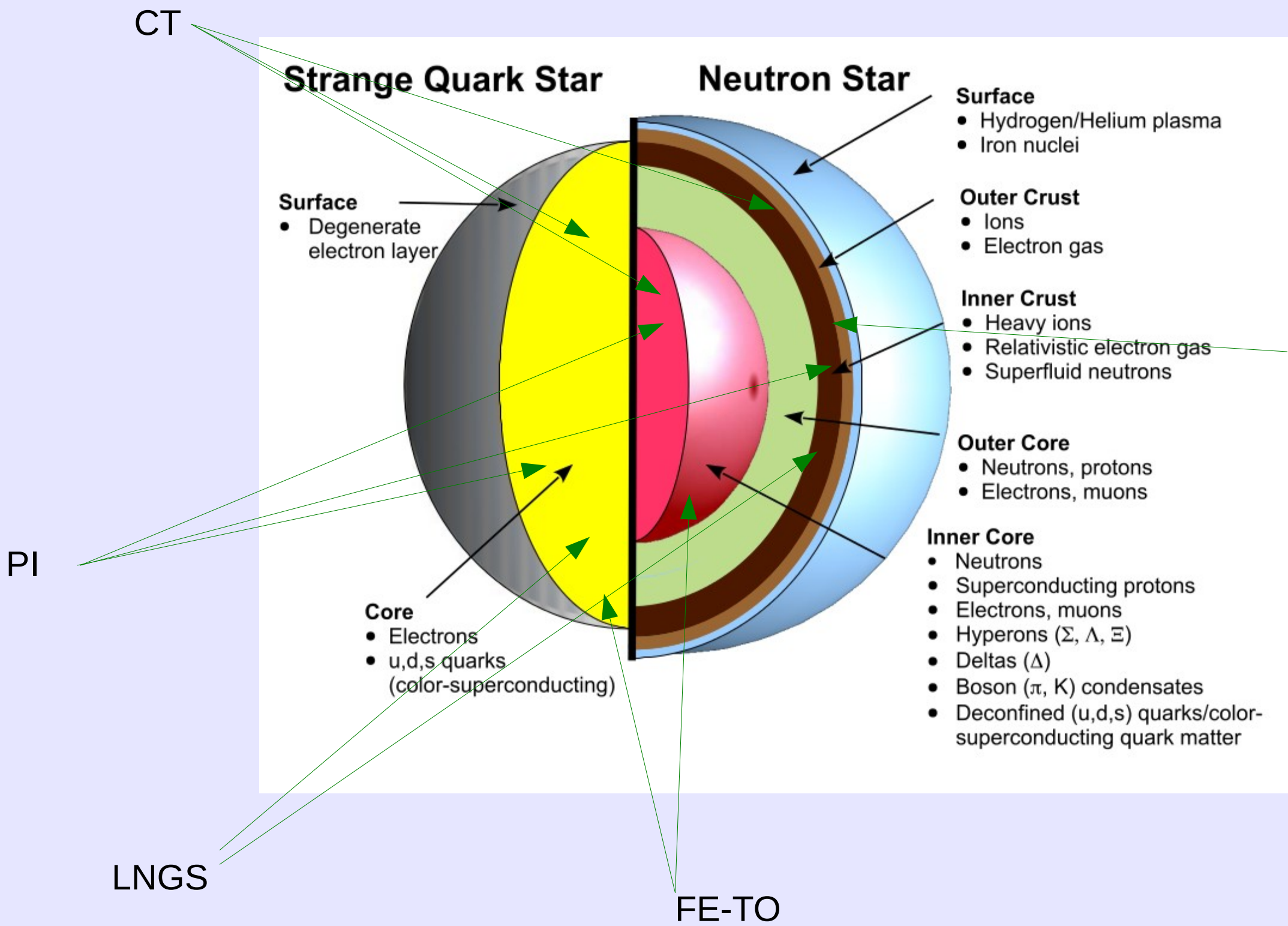
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1 Introduction

- **Introduction**
- RMF predictions through Bayesian analysis
- Modeling finite-temperature EoS:
 - Pure Gauge $SU(3)_c$ sector
 - Meson and baryon sector
- Summary
- Outlooks

NEUMATT: NEUtron star MATTer

The research topics of NEUMATT

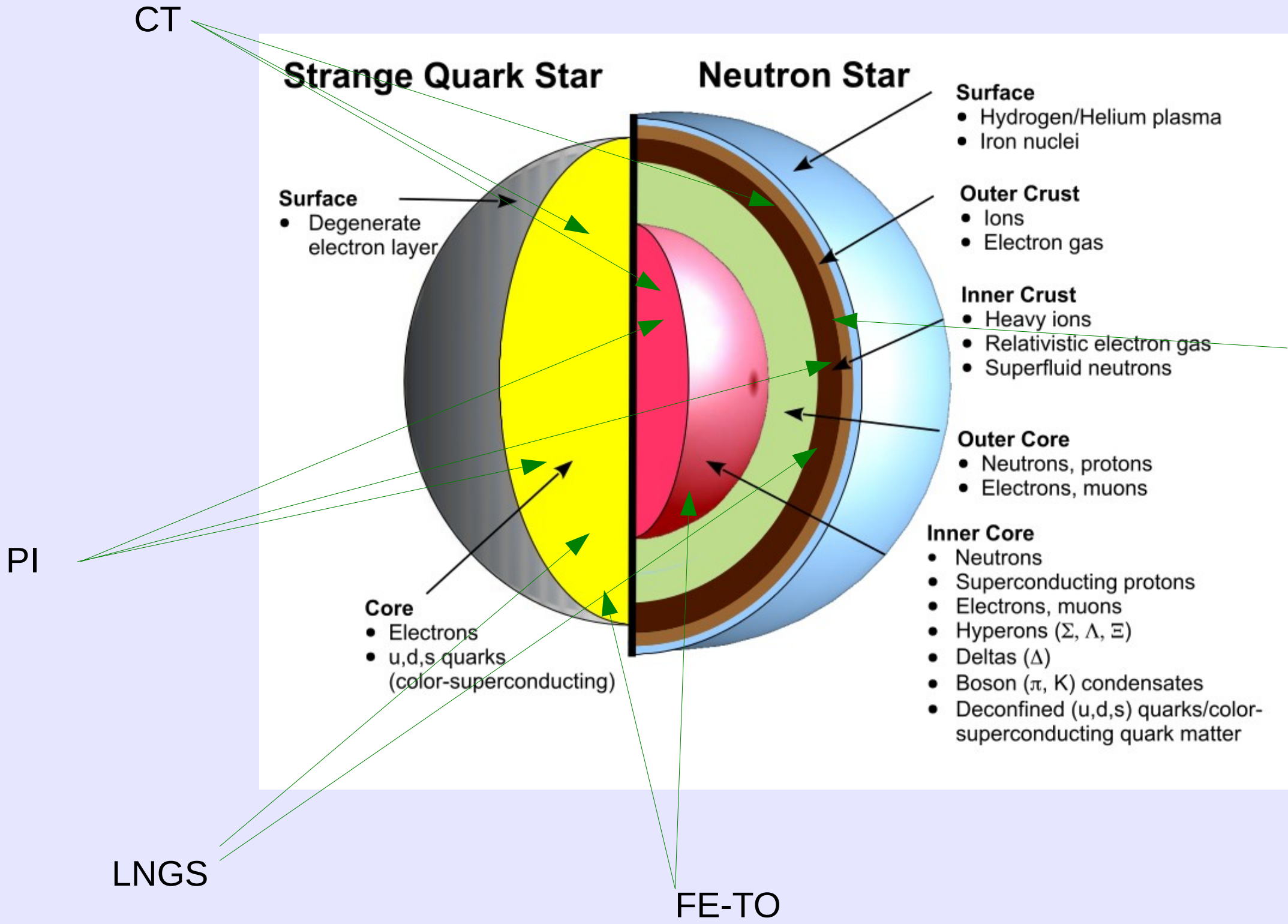


MI (no more active, unfortunately... Pierre Pizzochero)

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Goal of NEUMATT: understanding/modeling the properties of high density strongly interacting matter through the rich phenomenology of compact stars

The research topics of NEUMATT



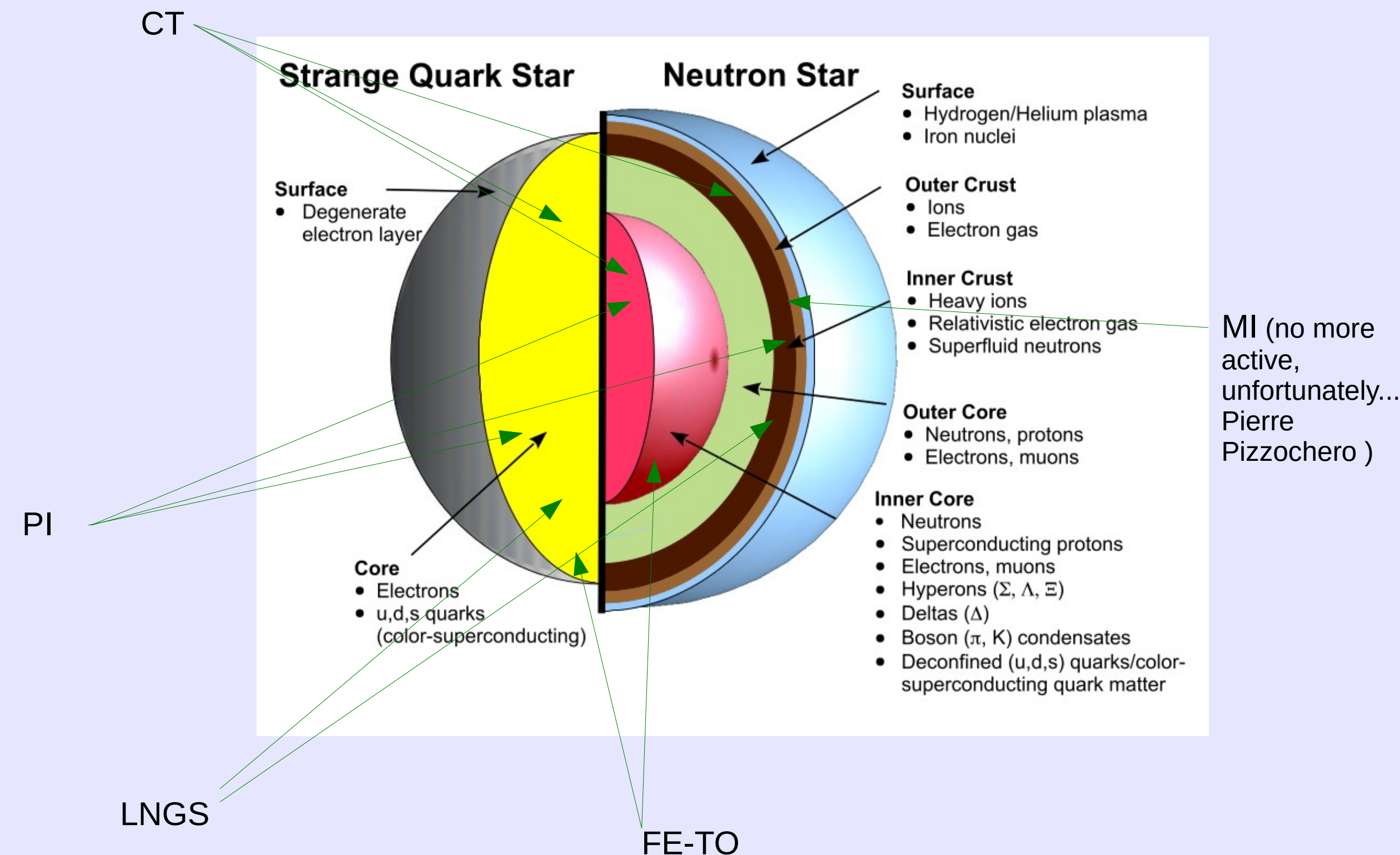
NEUMATT: NEUtron star MATTer

Goal of NEUMATT: understanding/modeling the properties of high density strongly interacting matter through the rich phenomenology of compact stars

The main areas in which the Turin group is involved are:

- *Investigation of hyperon and delta resonance effects.*
- *Analyzing quark deconfinement in compact stars: scenarios and phenomenological implications.*
- *Modeling finite-temperature Equations of State (EoSs).*

The research topics of NEUMATT



Equation of State for neutron stars

Neutron stars, the compact remnants of core-collapse supernova, are the densest macroscopic objects in the Universe.

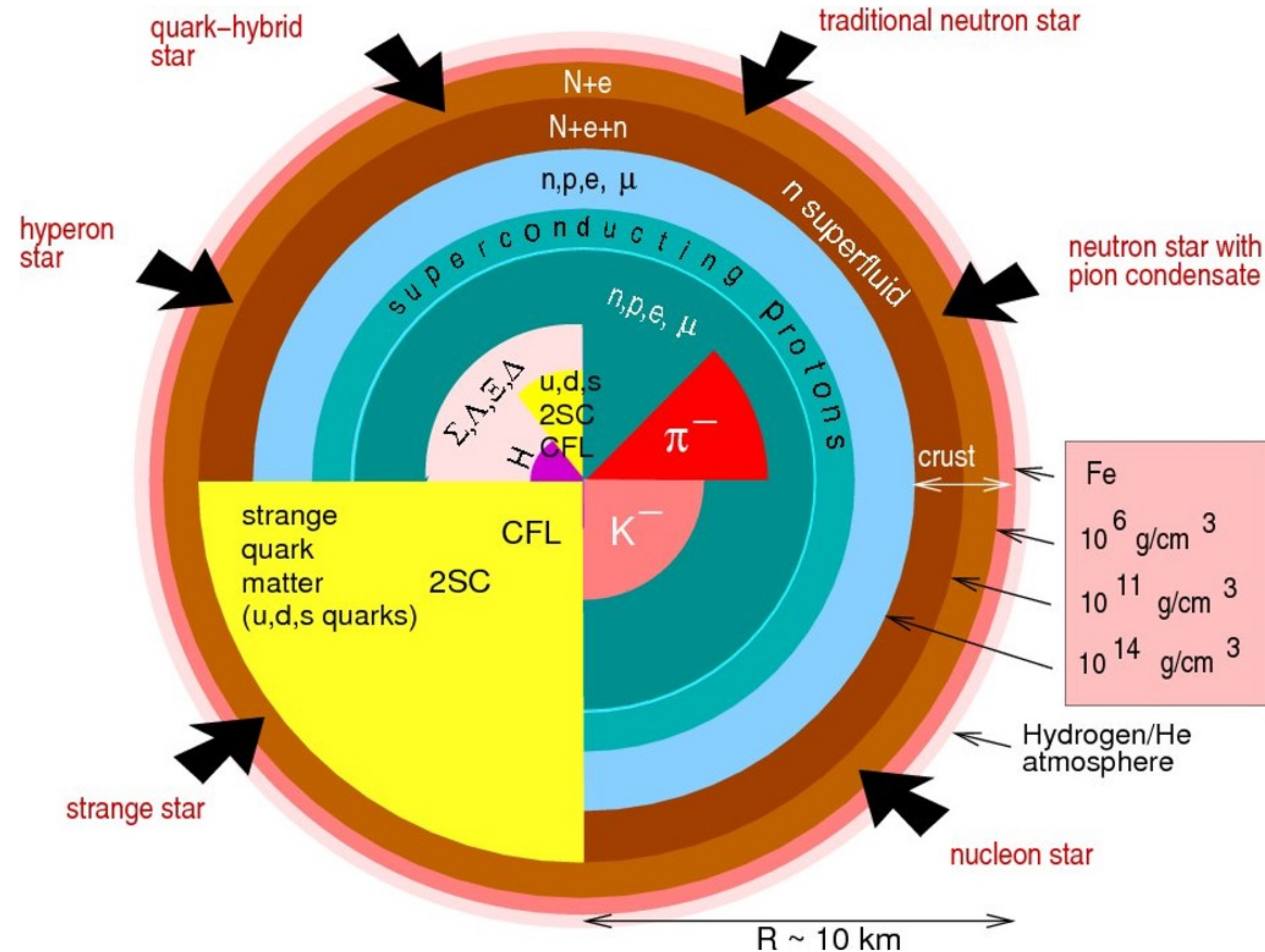
$$M_{\text{NS}} \sim 1.4 - 2.6 M_{\odot}$$

$$R_{\text{NS}} \sim 10 \text{ km}$$

$$n_{\text{NS}} \approx n_{\text{sat}}$$

$$\downarrow$$

$$n_{\text{NS}_{\text{core}}} \approx 5 - 10 n_{\text{sat}}$$



Equation of State for neutron stars

Nuclear properties:

$$B/A, E_{\text{sym}}, K, M^*, \dots$$



Many body theory
of strong interaction

Microscopic models:

$$\mathcal{L}, \sigma, \omega, \rho, \dots$$



QFT &
Statistical
mechanics

Macroscopical variables

$$p, V, T, \rho, \dots$$

Equation of State for neutron stars

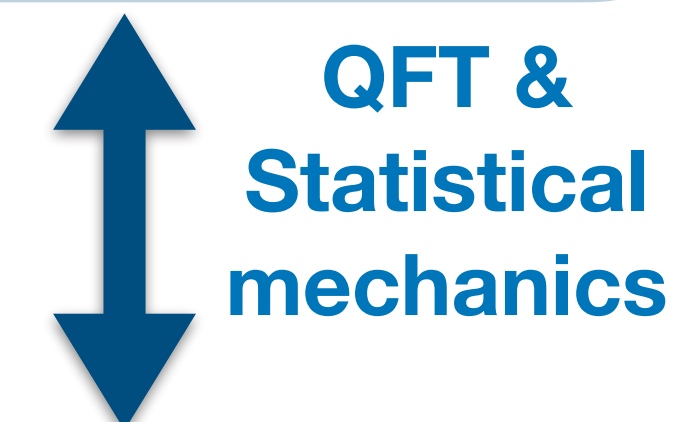
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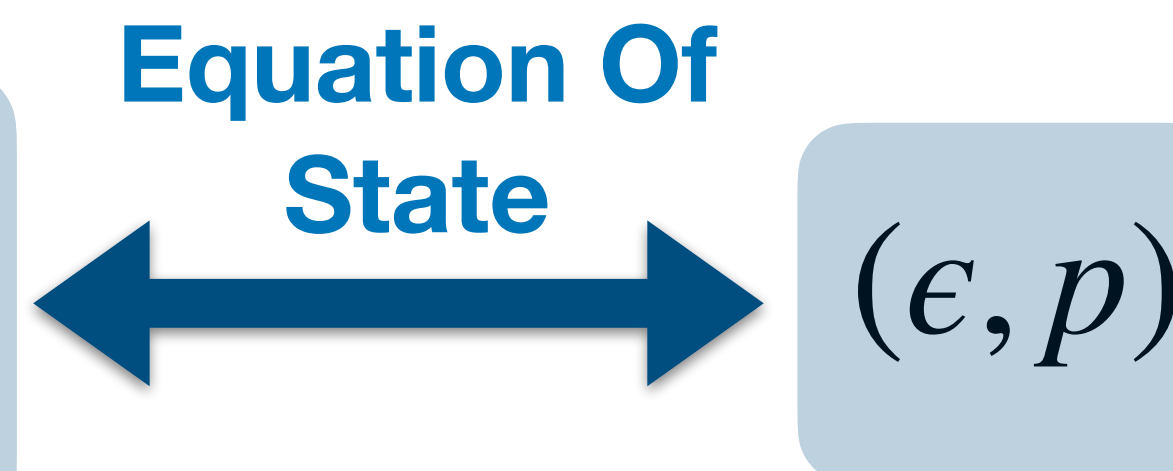
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$$(\epsilon, p)$$

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T.O.V. equations

$$\begin{cases} \frac{dm}{dr} = 4\pi r^2 \epsilon \\ \frac{dp}{dr} = -\frac{4\pi r^3 p + m}{r^2 \left(1 - \frac{2m}{r}\right)} (\epsilon + p) \end{cases}$$

(M, R)

T.O.V.
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Equation Of
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(ϵ, p)

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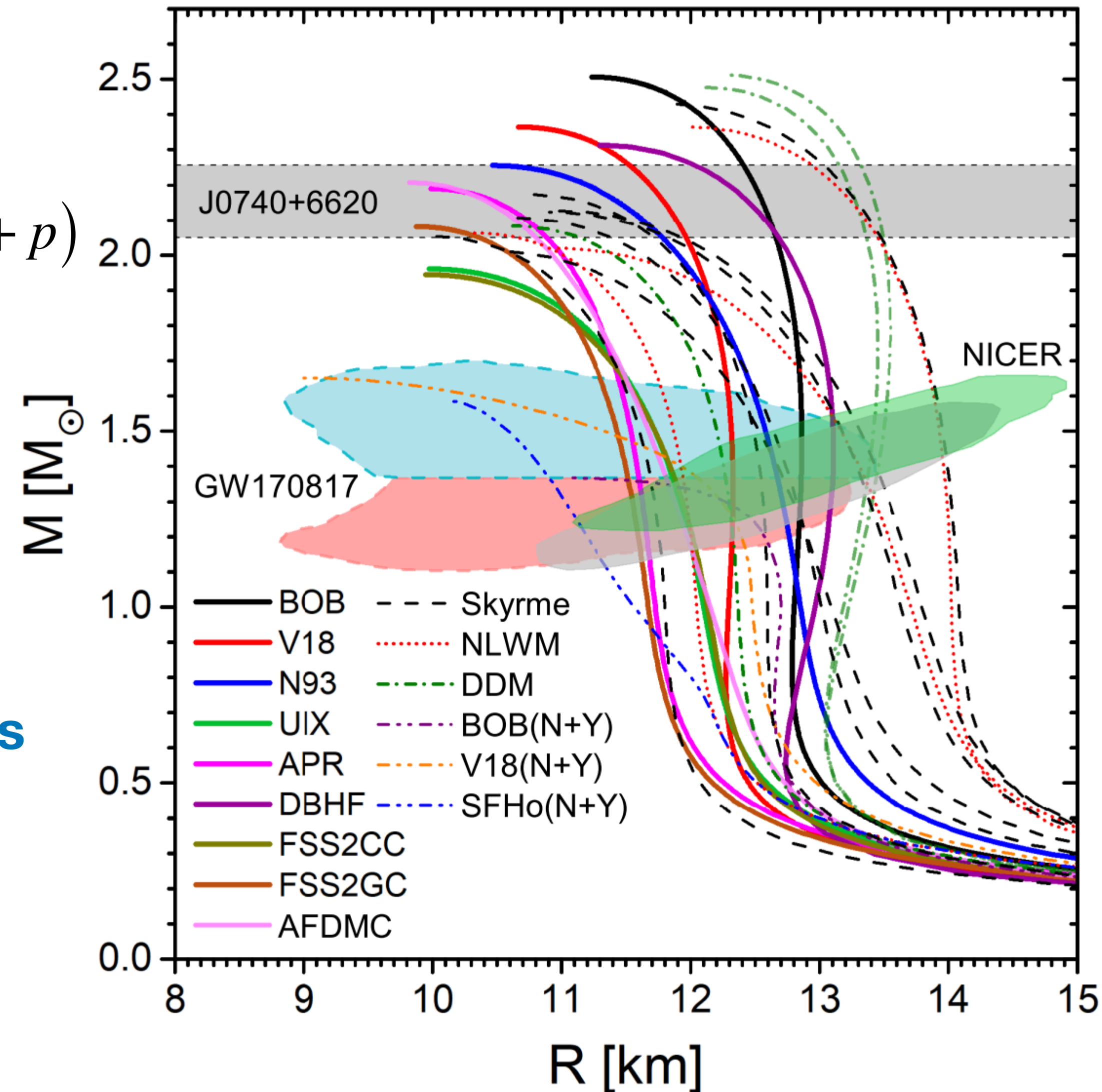


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RMF predictions for the dense-matter EOS and application to NSs

$$\begin{aligned}\mathcal{L} = & \sum_N \bar{\psi}_N (i\gamma_\mu \partial^\mu - m_N + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu \\ & - \frac{1}{2} g_\rho (n_b) \gamma_\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu) \psi_N + \frac{1}{2} (\partial_\mu \sigma \partial^\mu - m_\sigma^2 \sigma^2) \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} \\ & - \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{3} b m_N (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4\end{aligned}$$

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- Six couplings constants:

$$g_\sigma, g_\omega, g_\rho, b, c, a_\rho$$

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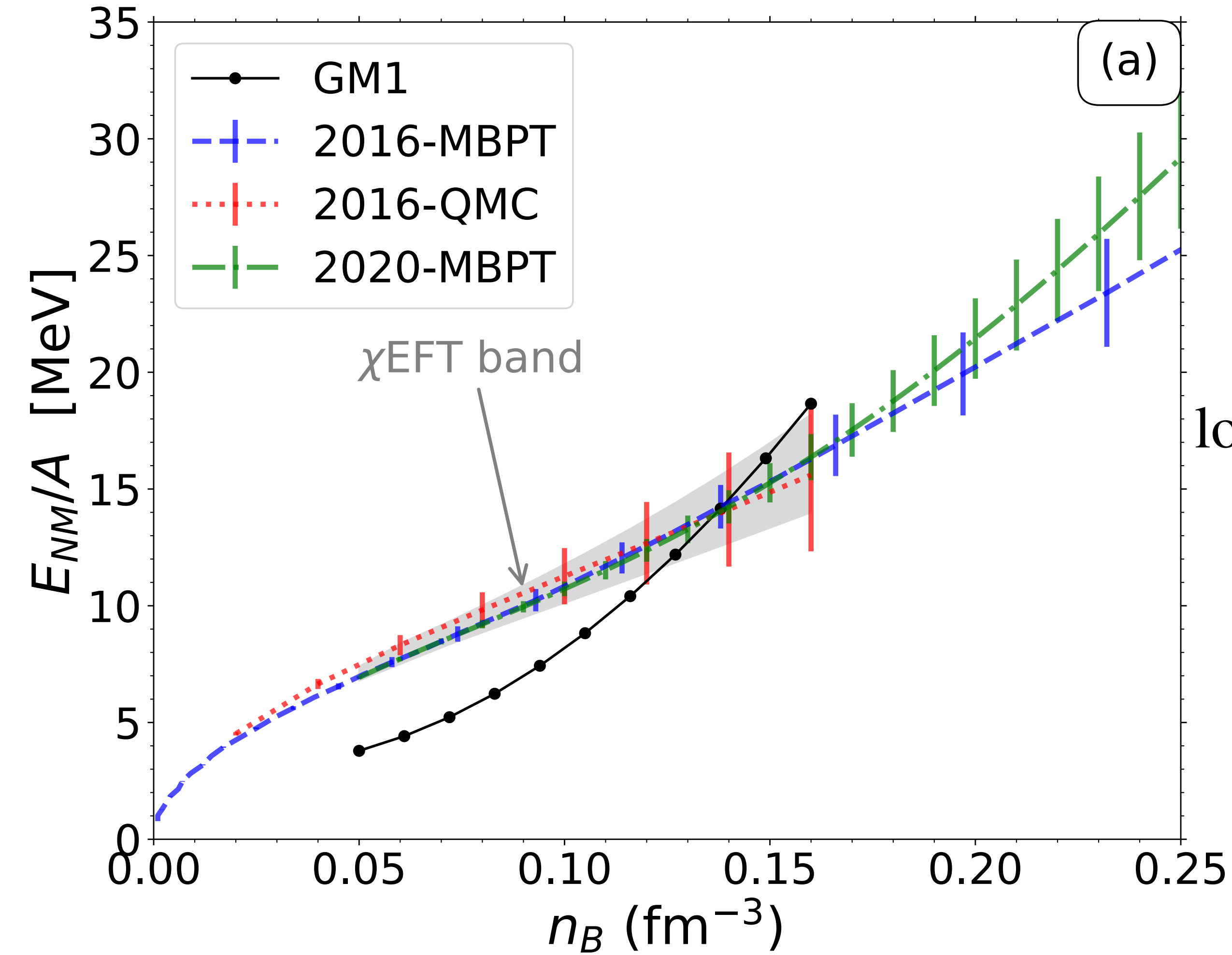
$$g_\sigma, g_\omega, g_\rho, b, c, a_\rho$$

- Six Nuclear Empirical Parameters (NEPs):

$$E_{\text{sat}}, n_{\text{sat}}, K_{\text{sat}}, E_{\text{sym},2}, L_{\text{sym}}, m_{D,\text{sat}}^*$$

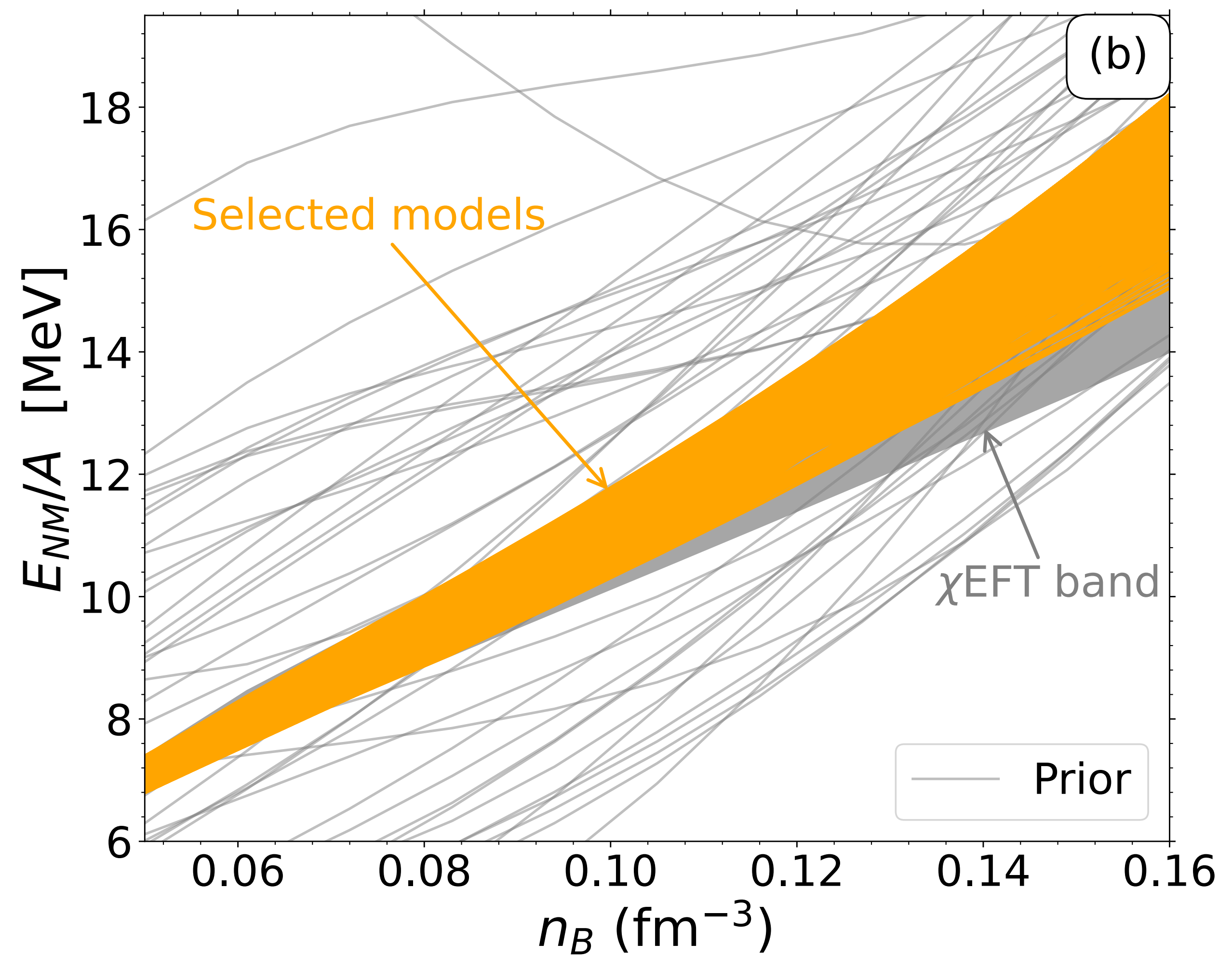
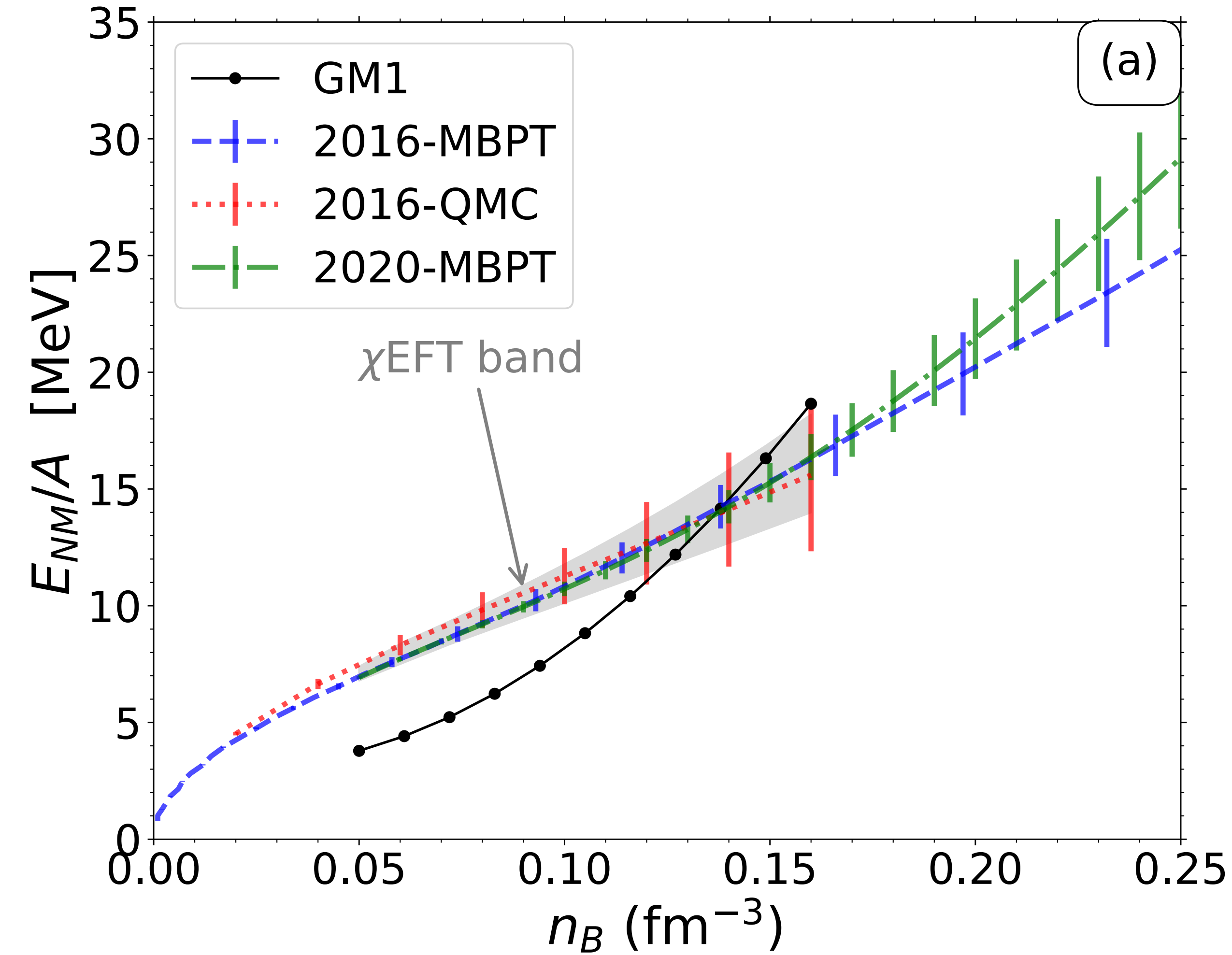
NEPs	E_{sat}	n_{sat}	K_{sat}	$E_{\text{sym},2}$	L_{sym}	$m_{D,\text{sat}}^*$
unif.dist.	$[-16.4, -15.2]$	$[0.145, 0.165]$	$[180, 280]$	$[28, 36]$	$[30, 90]$	$[0.45, 1.05]$

RMF predictions for the dense-matter EOS and application to NSs



$$\log p_{\text{MCMC}} = -\frac{1}{2} \sum_{i=1}^N \left(\frac{E(\text{model}, n_i) - E(\chi\text{EFT}, n_i)}{\Delta E(\chi\text{EFT}, n_i)} \right)^2$$

RMF predictions for the dense-matter EOS and application to NSs



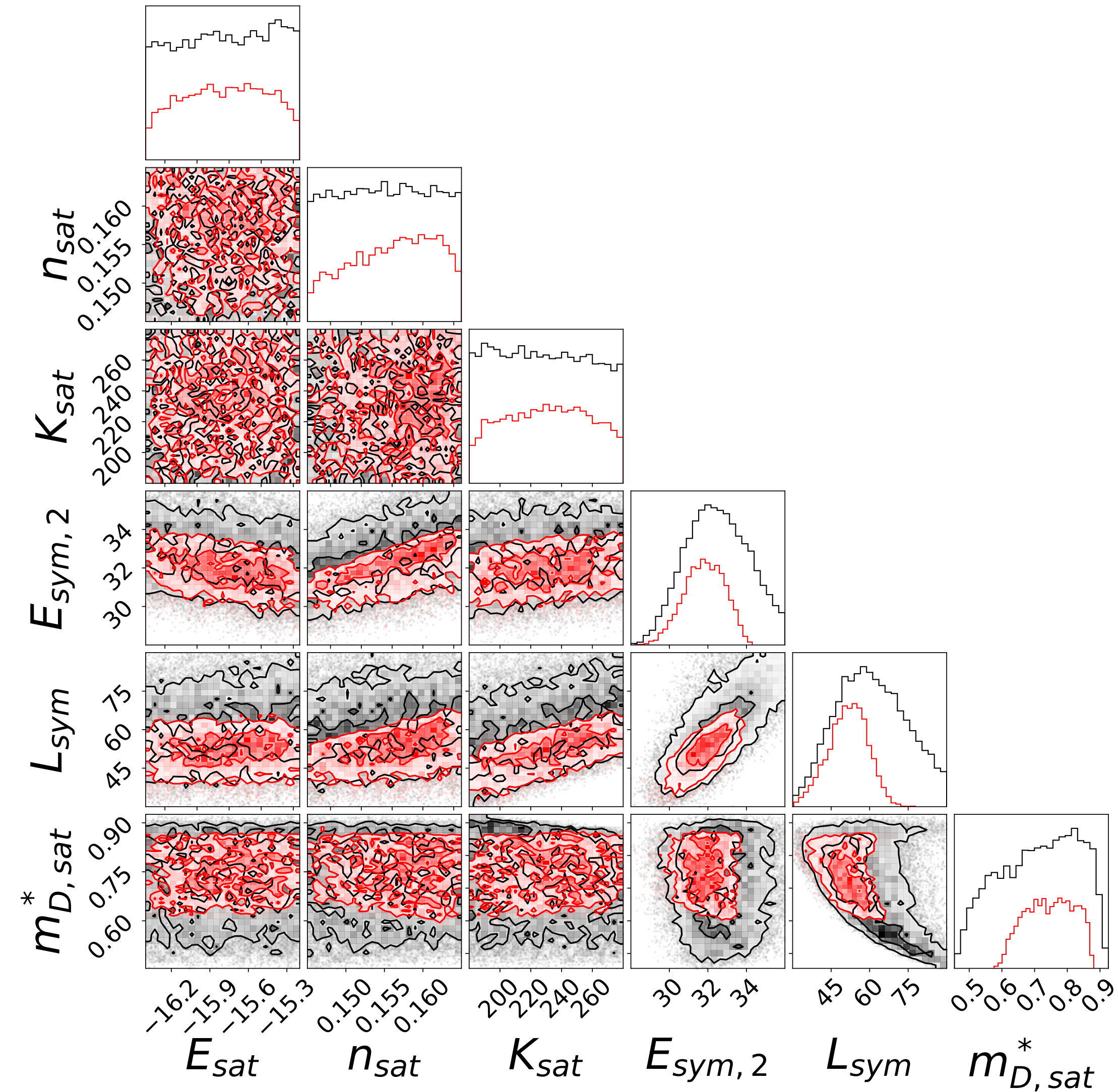
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Astrophysical constraint:

- **Tidal deformability** from GW170817

$$\Lambda \in [70, 770]$$



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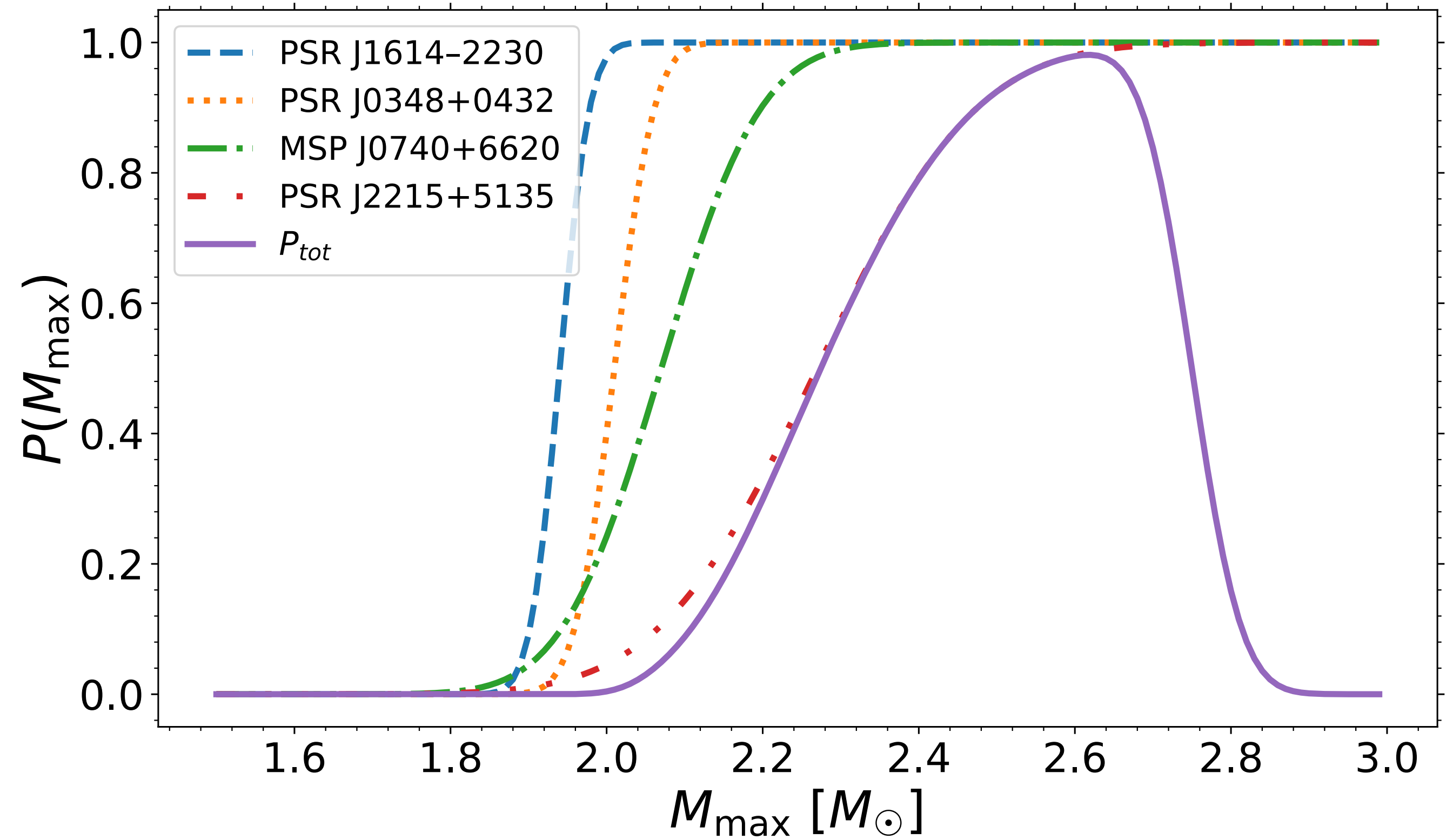
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- **Maximum mass**

$$P_{\text{tot}}(M_{\text{max}}) = P_{\text{GW170817}}(M_{\text{max}}) \prod_{i=1}^4 P_i(M_{\text{max}})$$

$$P_i(M_{\text{max}}) = \frac{\text{erf}[f_i(M_{\text{max}})] + 1}{2} \quad f_i(M) = \frac{M - M_{\text{obs},i}}{\sqrt{2} \sigma_{M,i}}$$



RMF predictions for the dense-matter EOS and application to NSs

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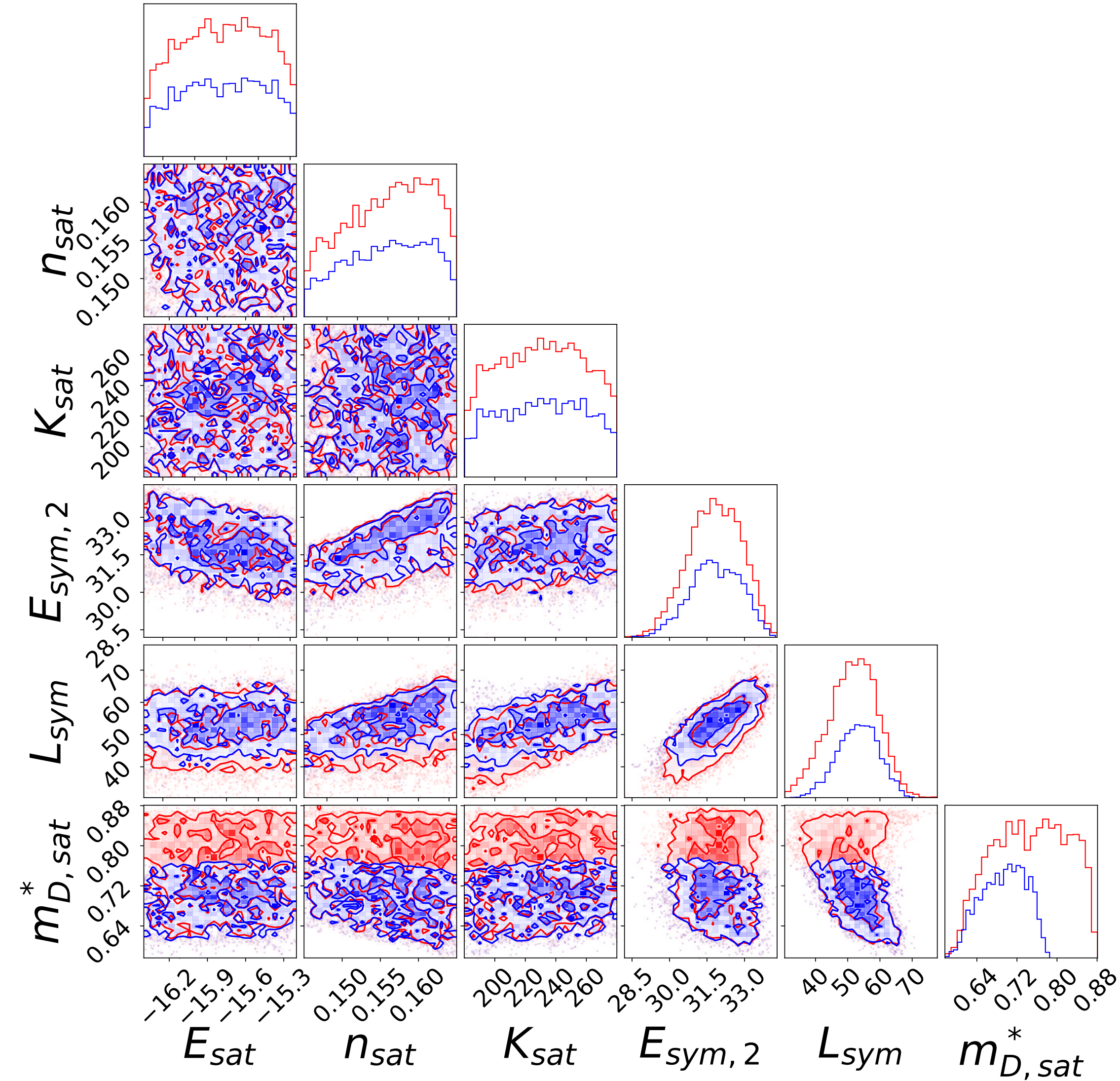
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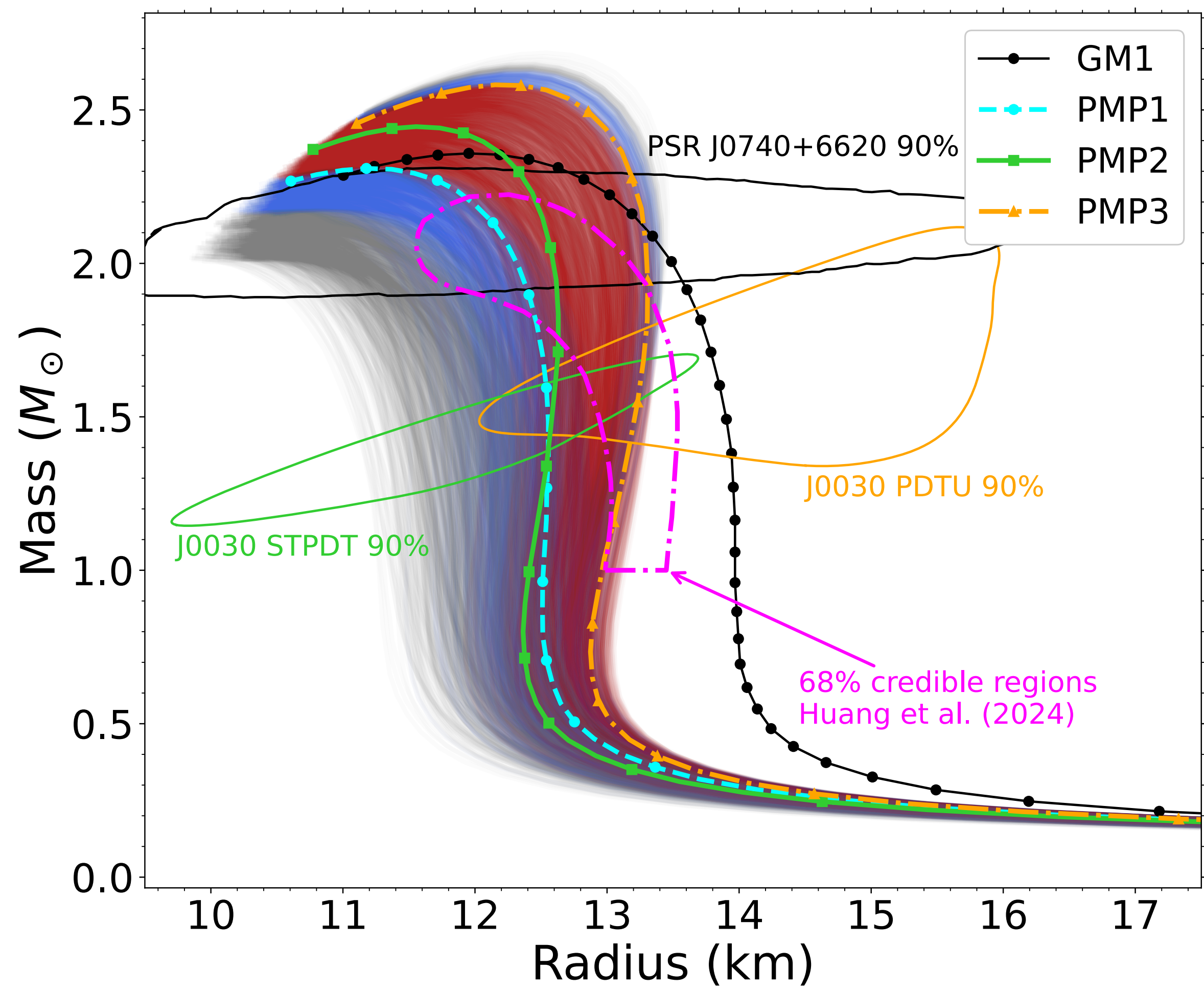
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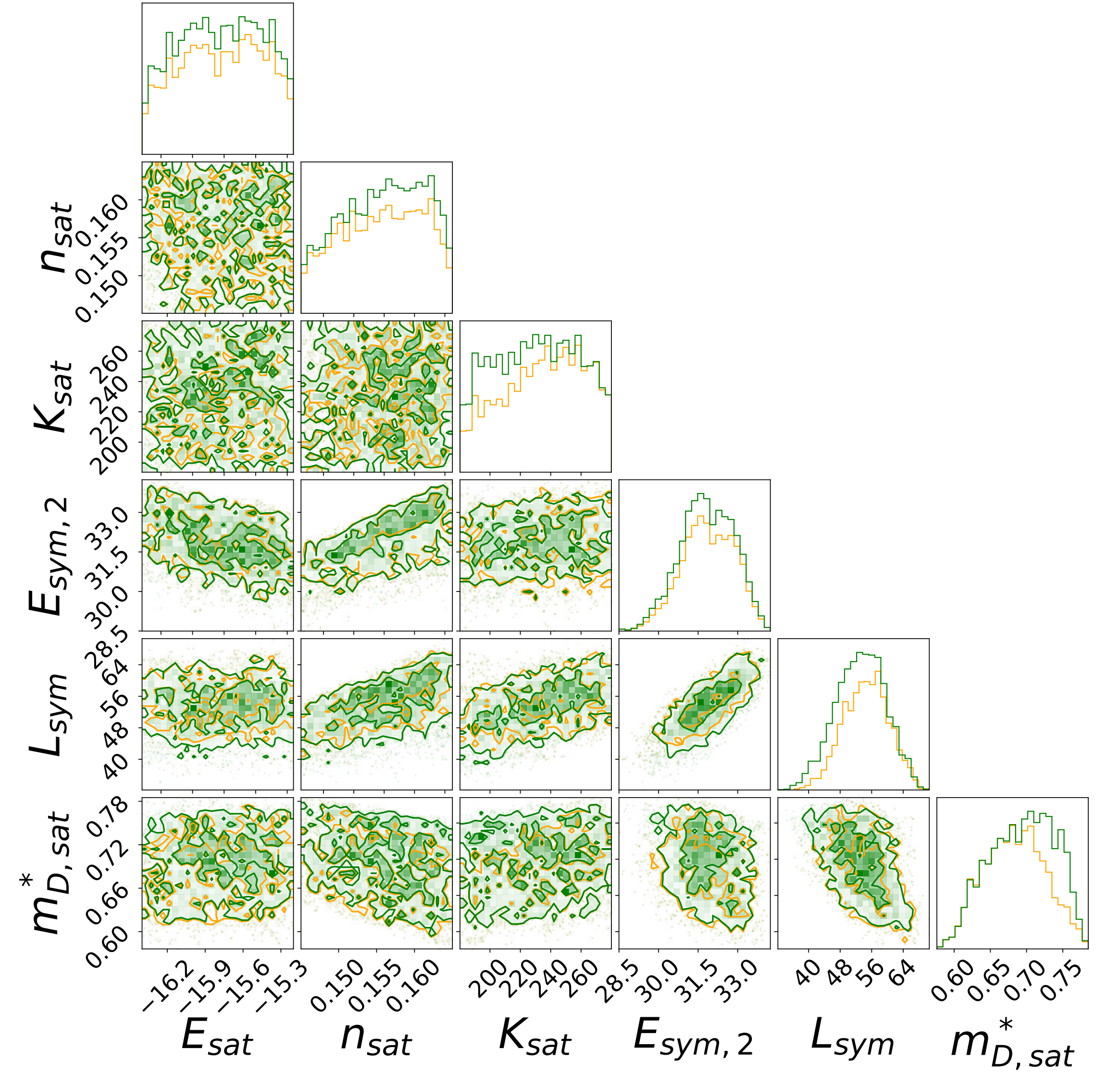
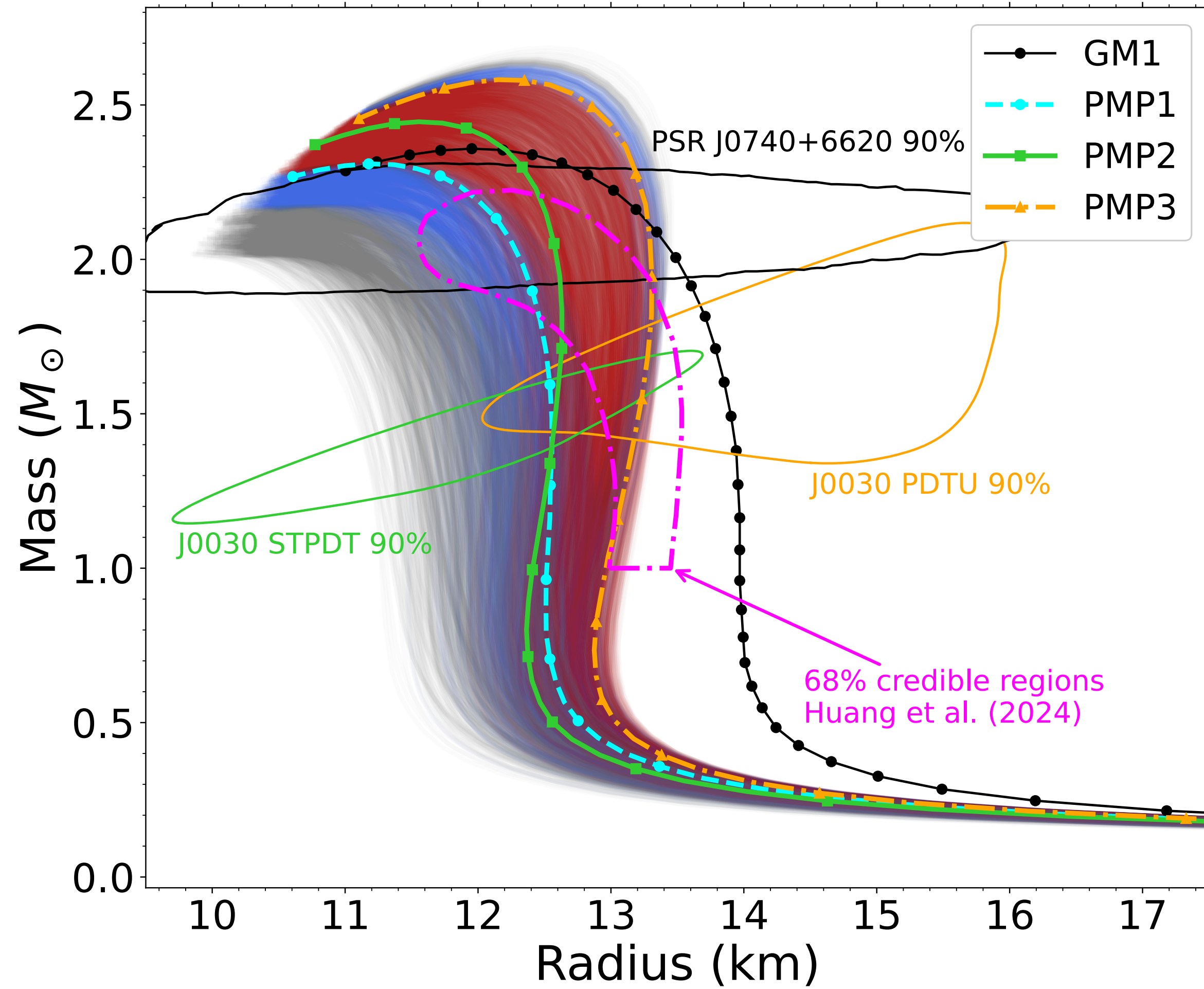
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RMF predictions for the dense-matter EOS and application to NSs



RMF predictions for the dense-matter EOS and application to NSs



Two family scenario

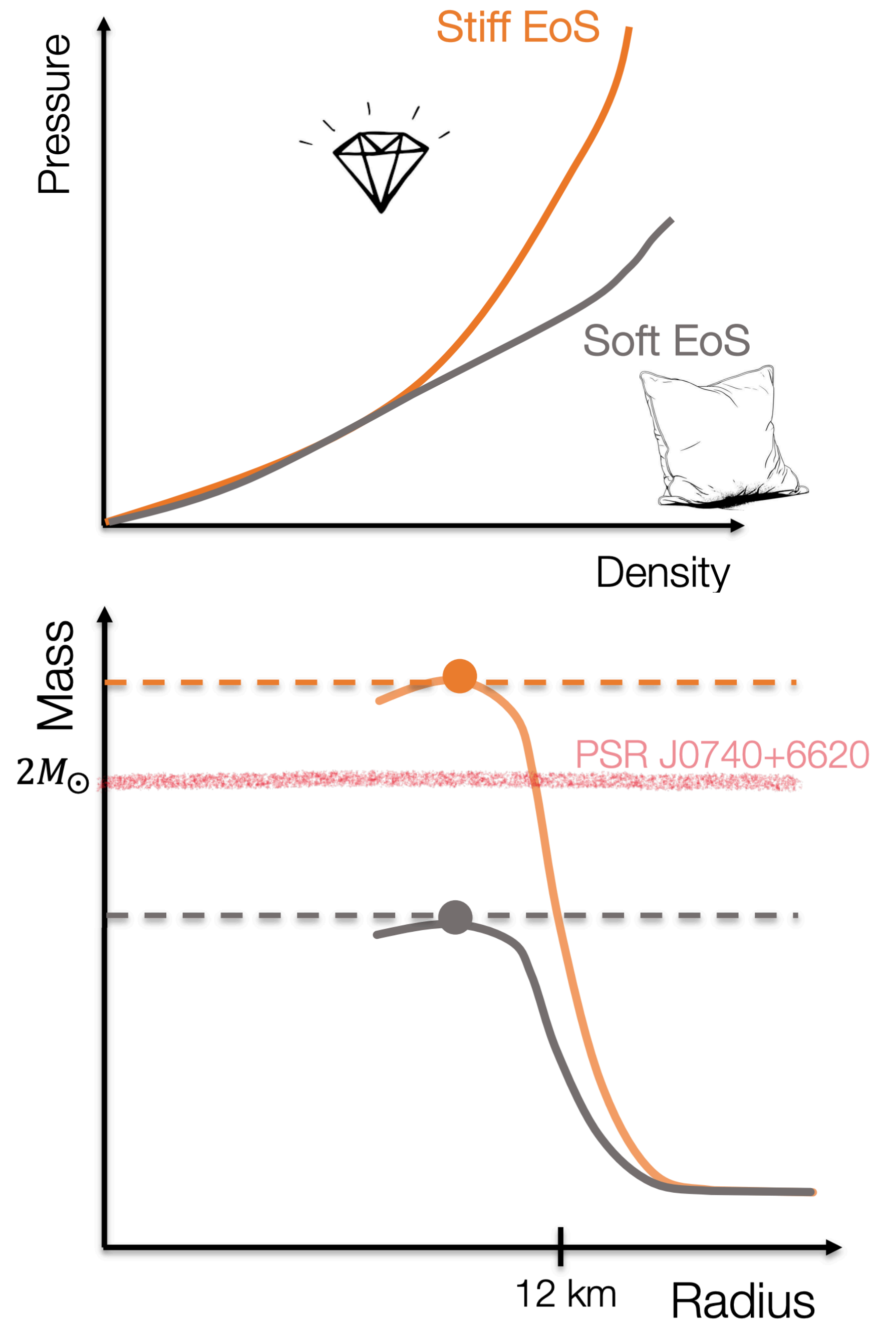
Two family scenario

- **Hyperon and Delta Puzzle:**

expected appearance of hyperons ($> 2 - 3 n_{\text{sat}}$) and Δ -resonance significantly softens the EoS [*Lavagno et al. Phys.Rev.C (2014)*].

- **Observational Conflict:**

soft EoSs fail to support observed $\sim 2M_{\odot}$ NS, which mandate a stiff EoS.



Two family scenario

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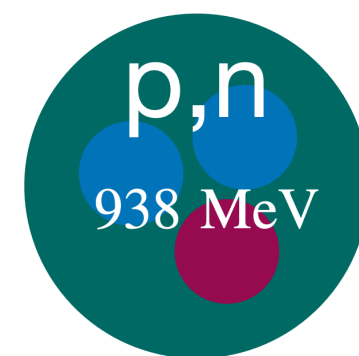
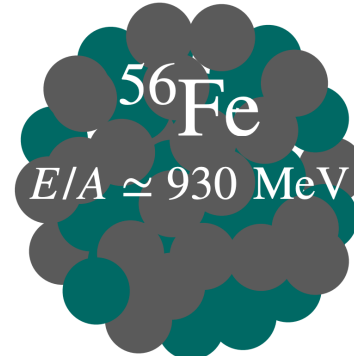
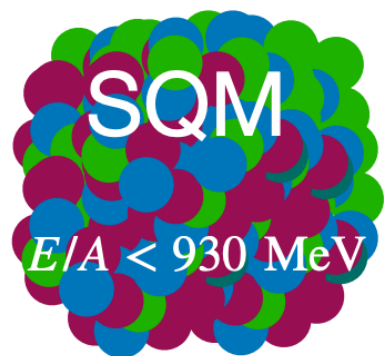
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The Two-Family Scenario

- Based on Bodmer-Witten hypothesis



- Mechanism: a strong, **first-order phase transition** in the core.
- Creates two disconnected, stable Mass-Radius branches:
 - **Hadronic:** Soft branch, $M_{\text{max}} < 2M_{\odot}$.
 - **Quark:** Stiff branch, $M_{\text{max}} \geq 2M_{\odot}$.
- Once reached deconfinement conditions, **HS** converts to **QS**

[Torino-Ferrara collaboration, recent papers: *Astrophys.J.* 974 (2024); *Universe* 11 (2025) 258; *JHEAp* 50 (2026)]

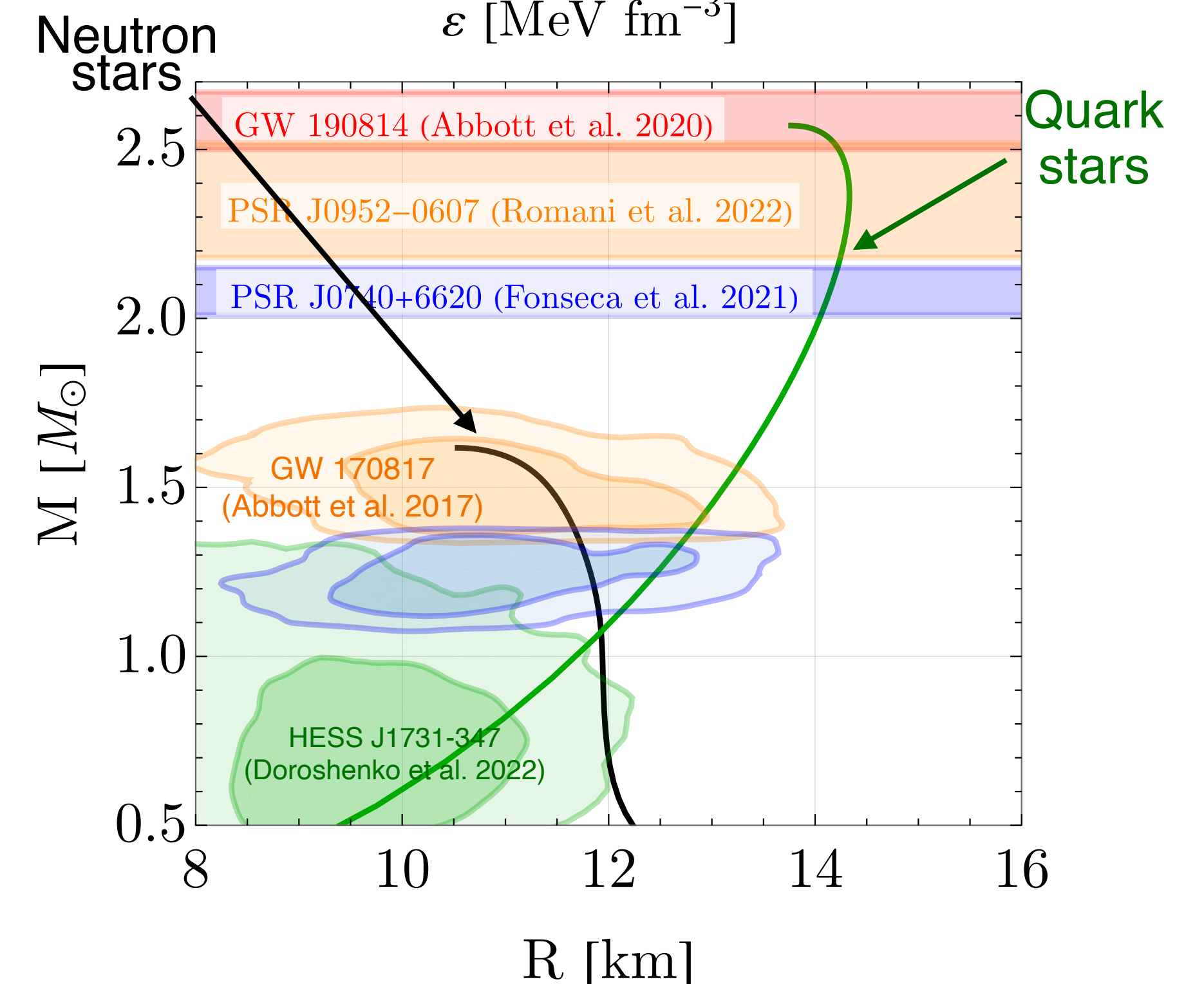
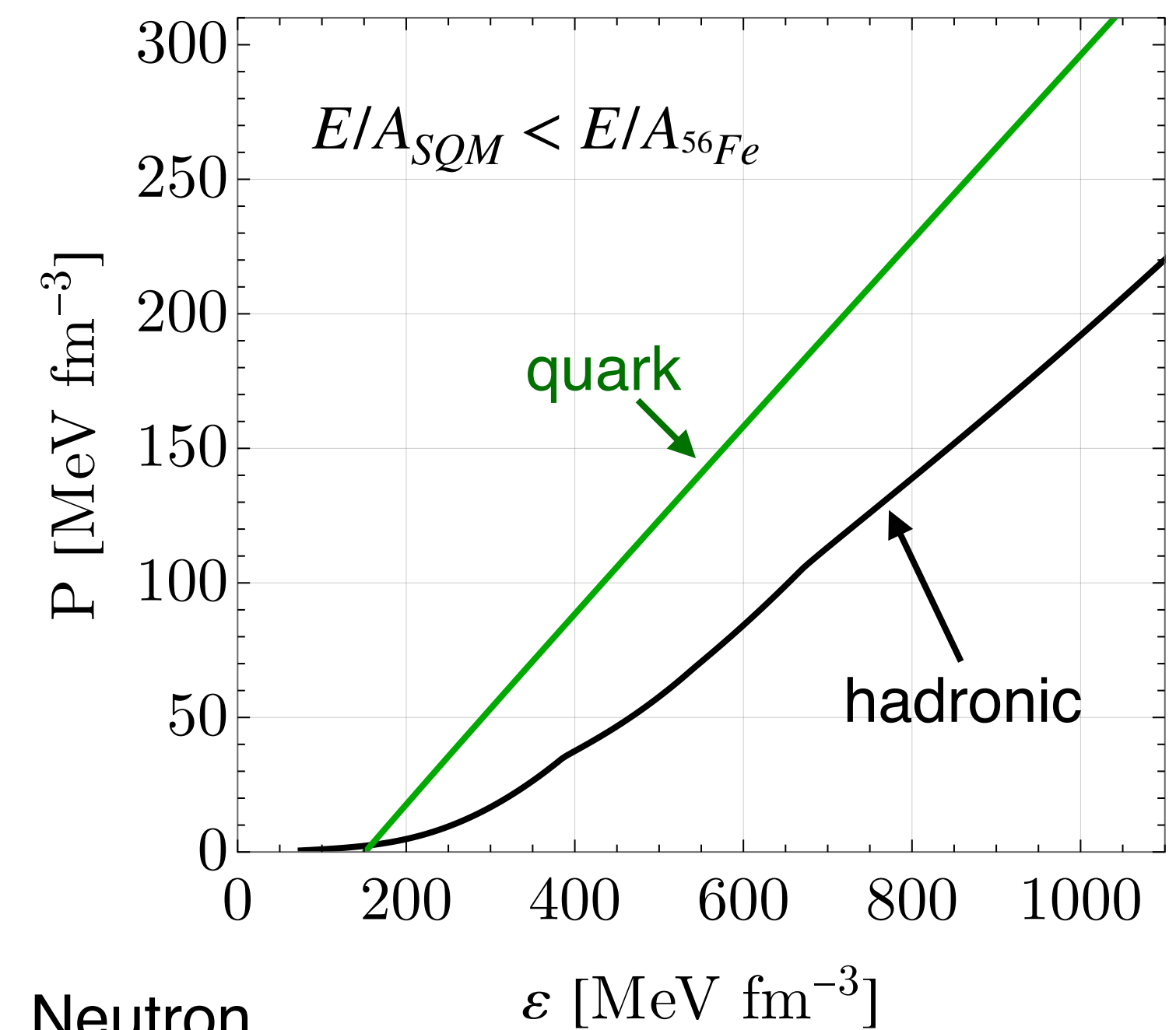
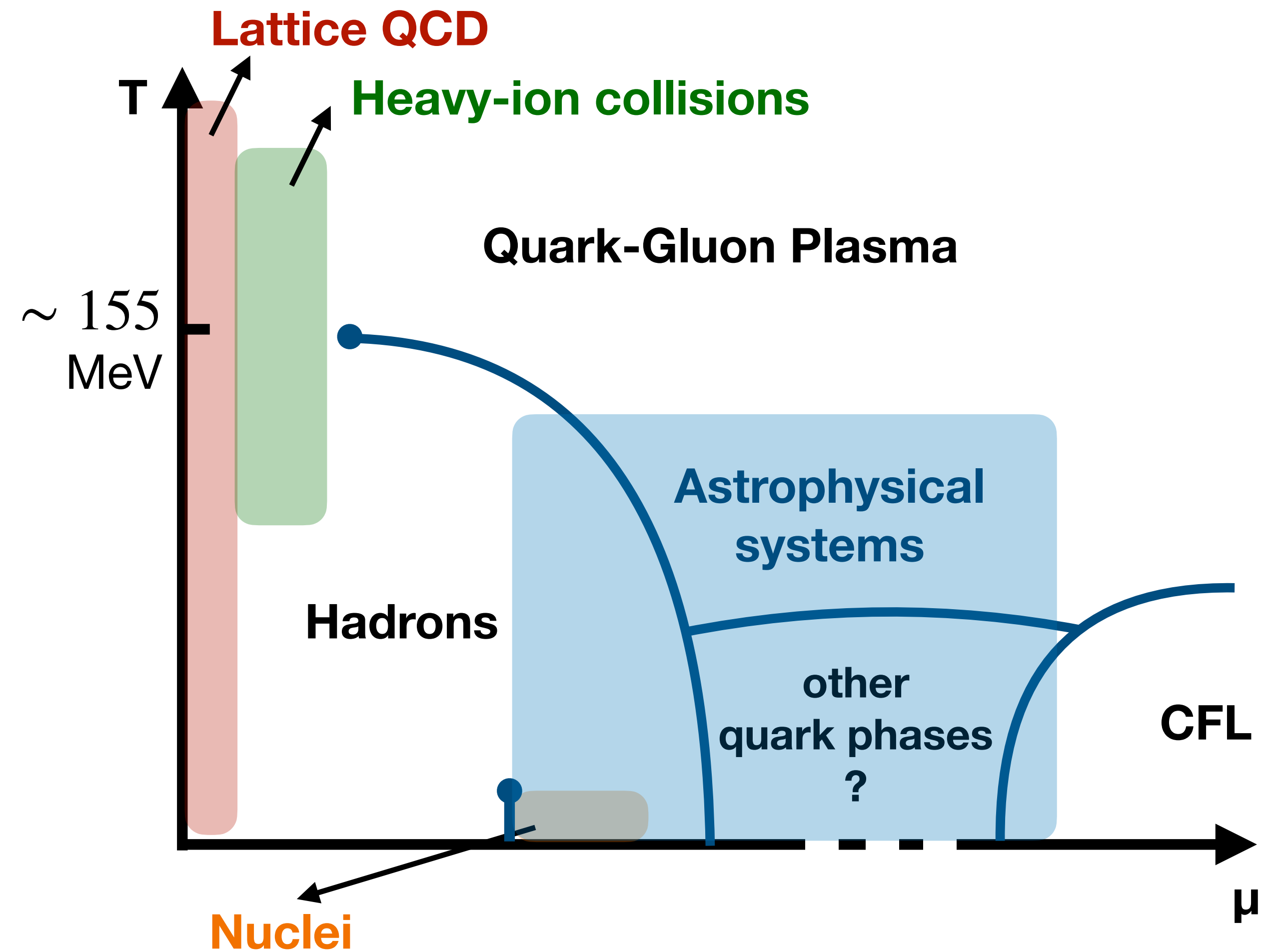


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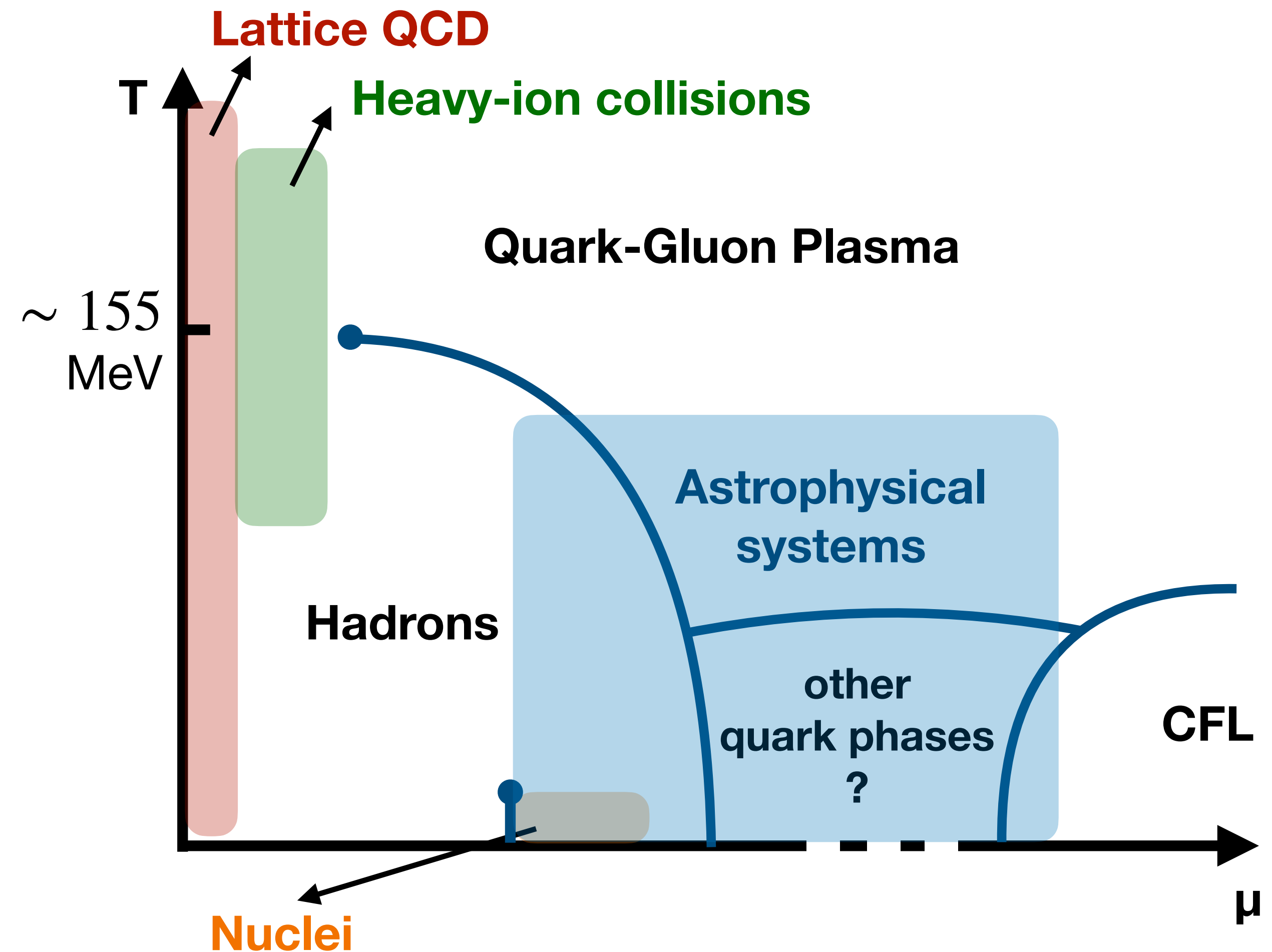
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QCD phase diagram



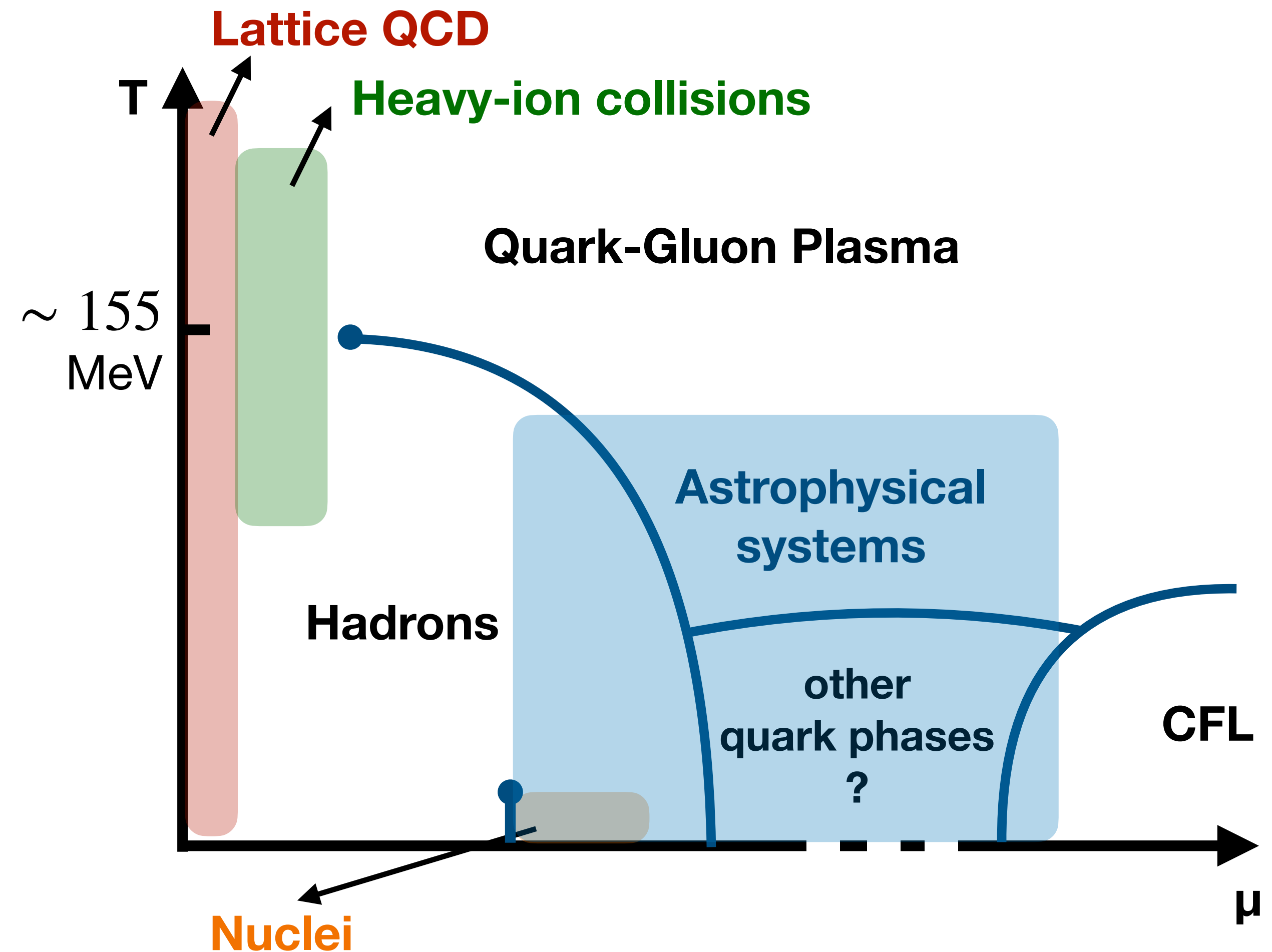
QCD phase diagram

- Regimes of low temperatures and high densities: neutron stars and compact stars



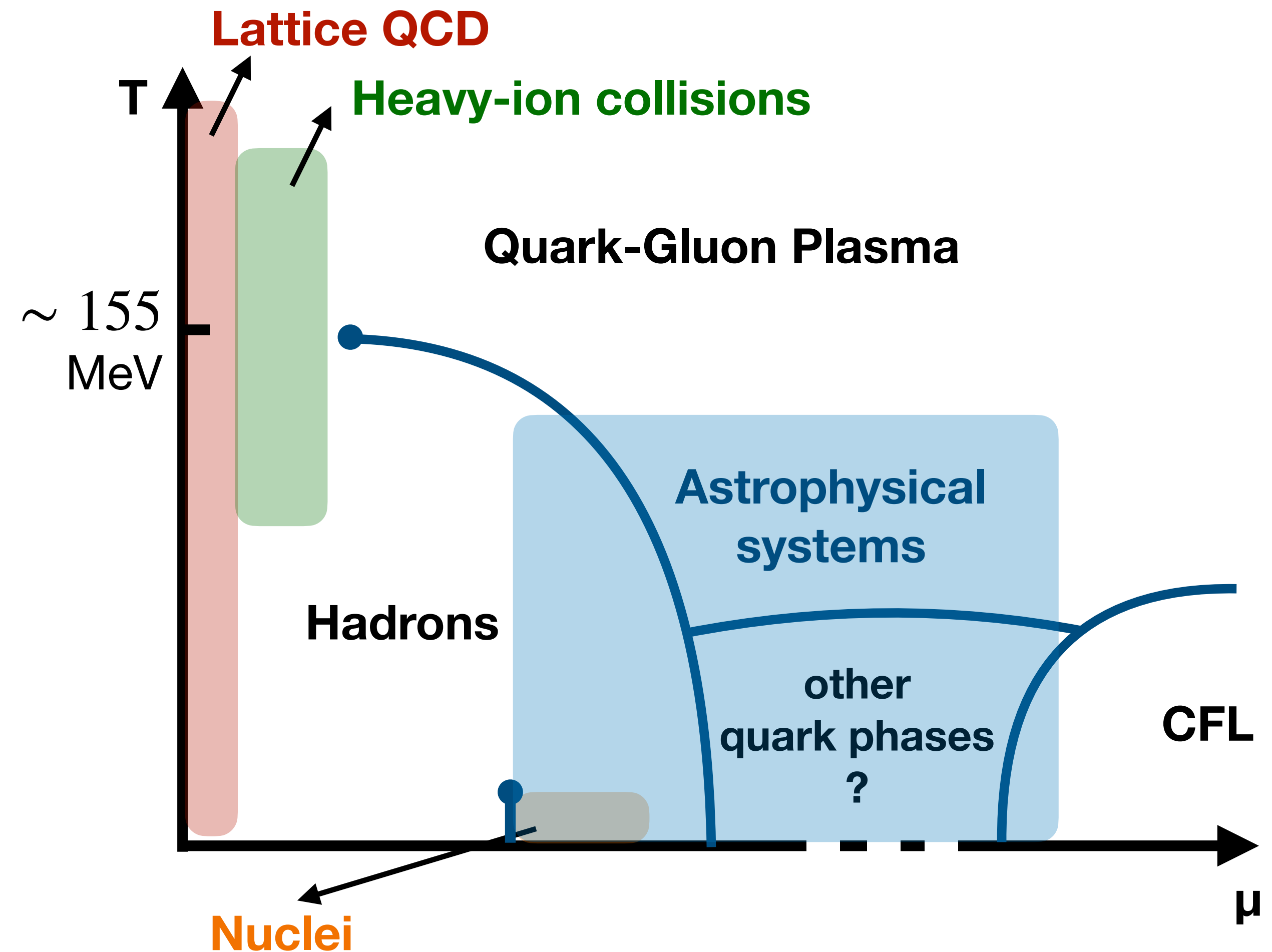
QCD phase diagram

- Regimes of low temperatures and high densities: neutron stars and compact stars
- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions



QCD phase diagram

- Regimes of low temperatures and high densities: neutron stars and compact stars
- Regime of high temperatures and low densities: Lattice QCD and heavy ions collisions
- Only a few EOS are studied in both regimes simultaneously and with several model-dependent parameters: a complete description is still challenging.



Research Overview

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- Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:

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 1. Extension of previous work in the pure gauge $SU(3)_c$ sector:
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Research Overview

- Lagrangian which is able to mimic the scale anomaly of QCD at mean field level:
 1. Extension of previous work in the pure gauge $SU(3)_c$ sector:
 - Inclusion of thermal fluctuation and excited glueballs
 - Comparison of results with lattice QCD data
 2. Contributions meson and baryons sector: σ , π , ω , ρ meson field and nucleons N
 - Investigation of the impact of thermal fluctuations for all fields
 - Exploration of phase diagram in the meson–baryon sector

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Pure gauge $SU(3)_c$ sector

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The **thermodynamical potential** is

$$\frac{\Omega(\chi, T)}{V} = \langle \mathcal{V}(\chi) \rangle - P_{\text{dil}}(\chi, T) - P_{\text{q-free}}(\chi, T) - P_{\text{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle.$$

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**Infrared
cut-off**

$$K(\chi) = \frac{A}{1 - \chi}$$

P.L., Lavagno, A. and Drago, EPJ Web Conf. 314,00038 (2024)

P.L., Lavagno, A. and Drago, Phys.Rev.C (in preparation) (2025)

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It is possible to divide the glueball into two distinct components: the mean field $\bar{\chi}$ and the fluctuating part Δ_χ , with $\langle \Delta_\chi \rangle = 0$.

$$\chi = \bar{\chi} + \Delta_\chi$$

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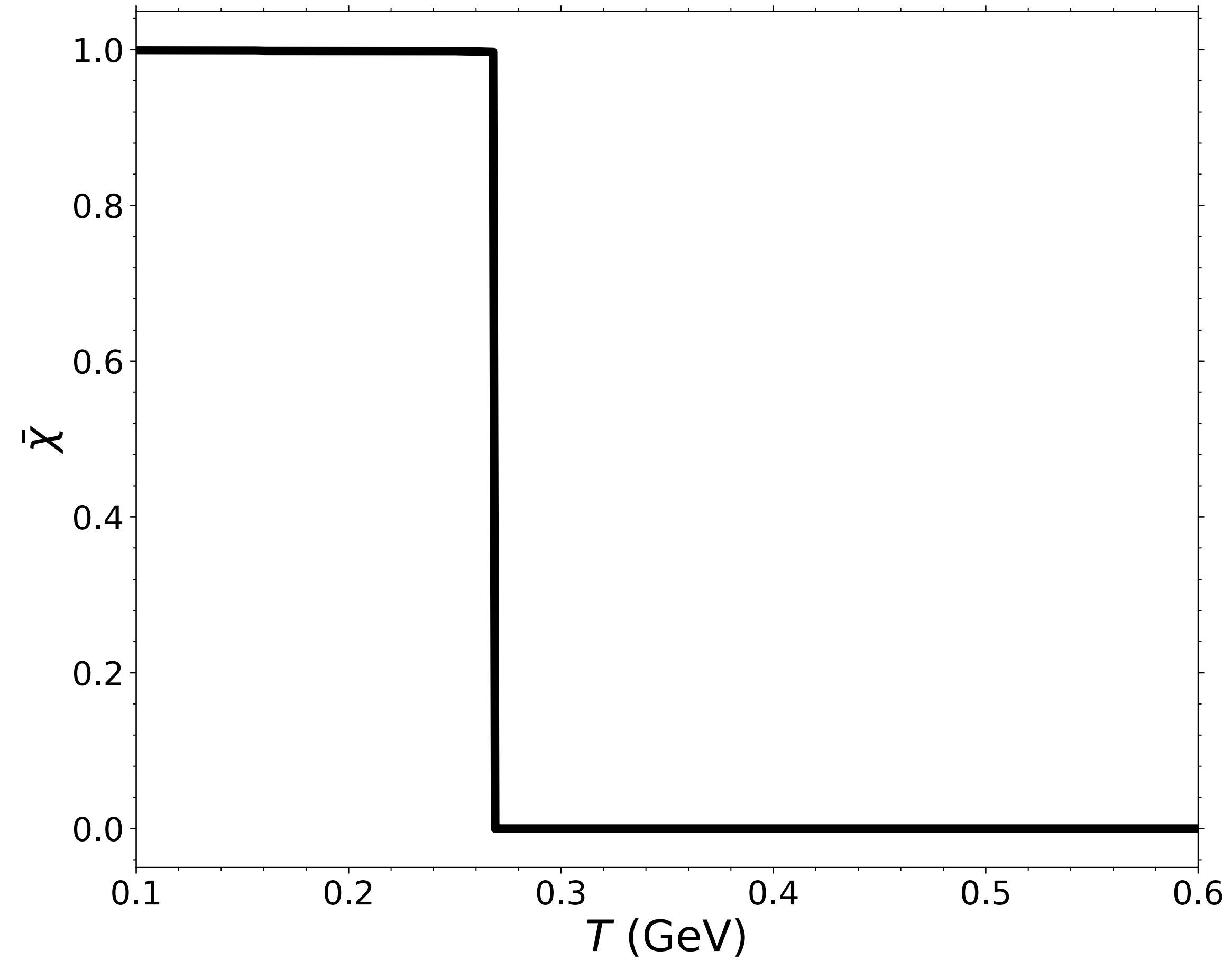
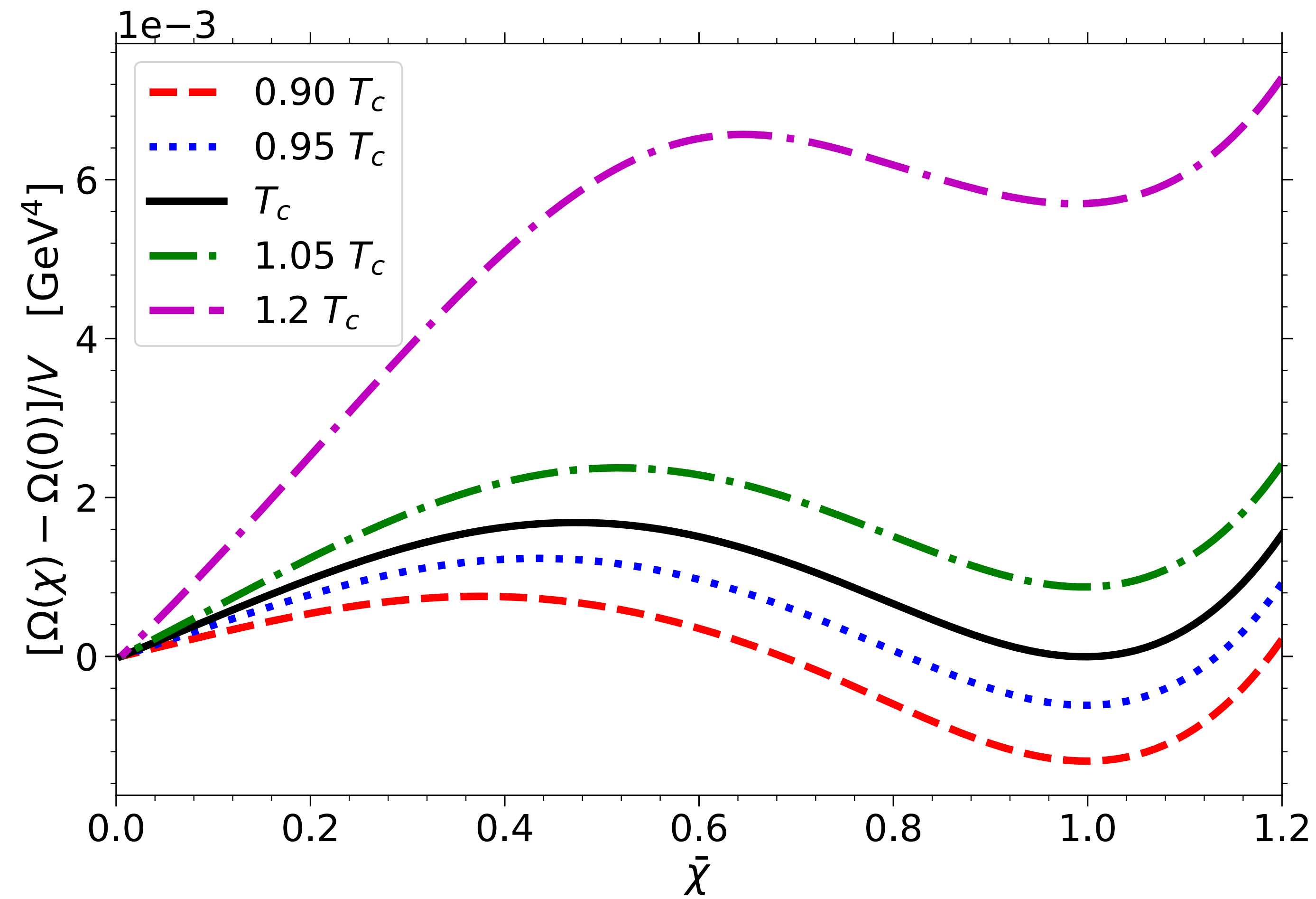
$$\chi = \bar{\chi} + \Delta_\chi$$

$$\langle \Delta_\chi^2 \rangle = \frac{1}{2\pi^2 \phi_0^2} \int_0^\infty \frac{k^2}{\epsilon_\chi} \frac{1}{e^{\beta \epsilon_\chi} - 1} dk$$

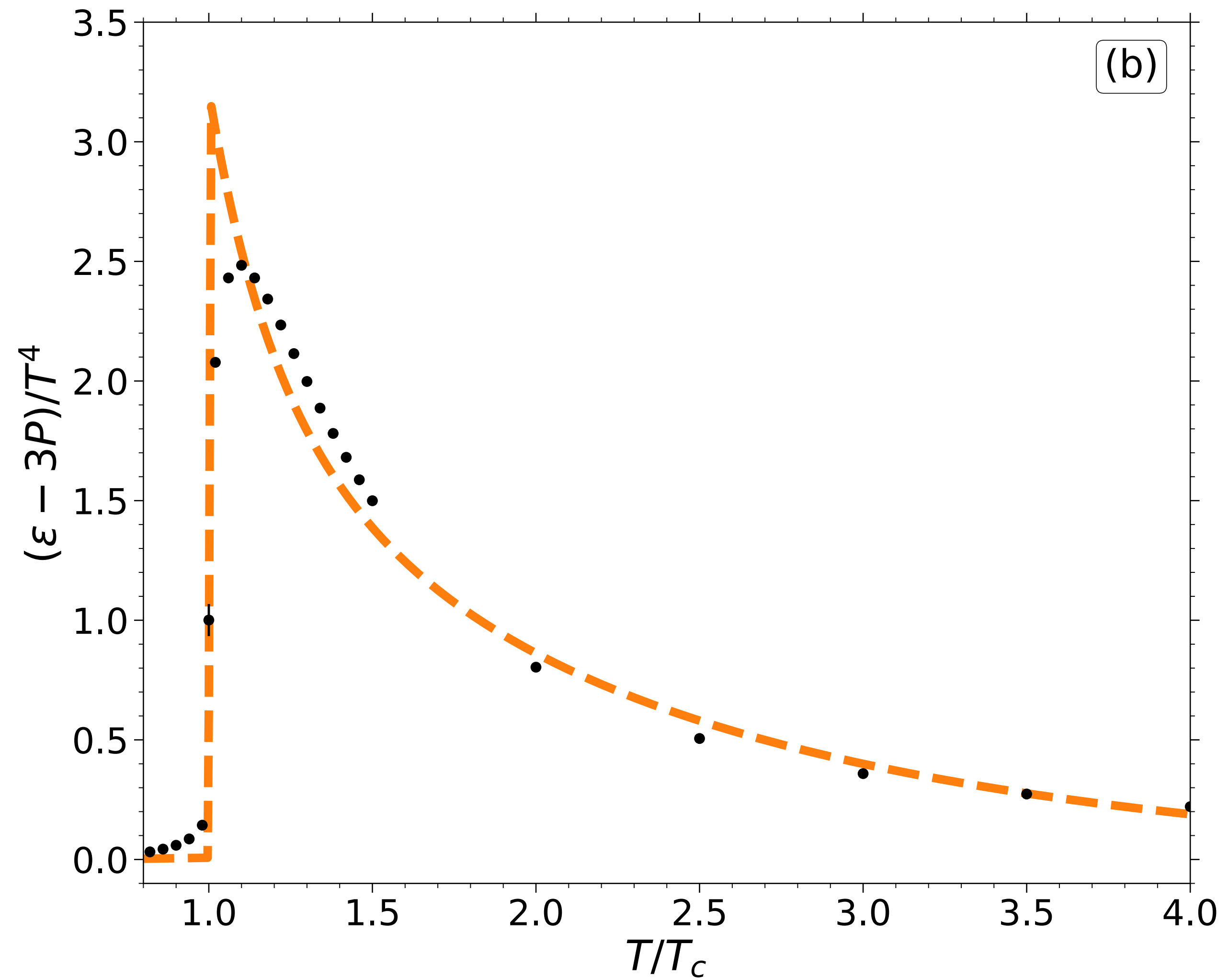
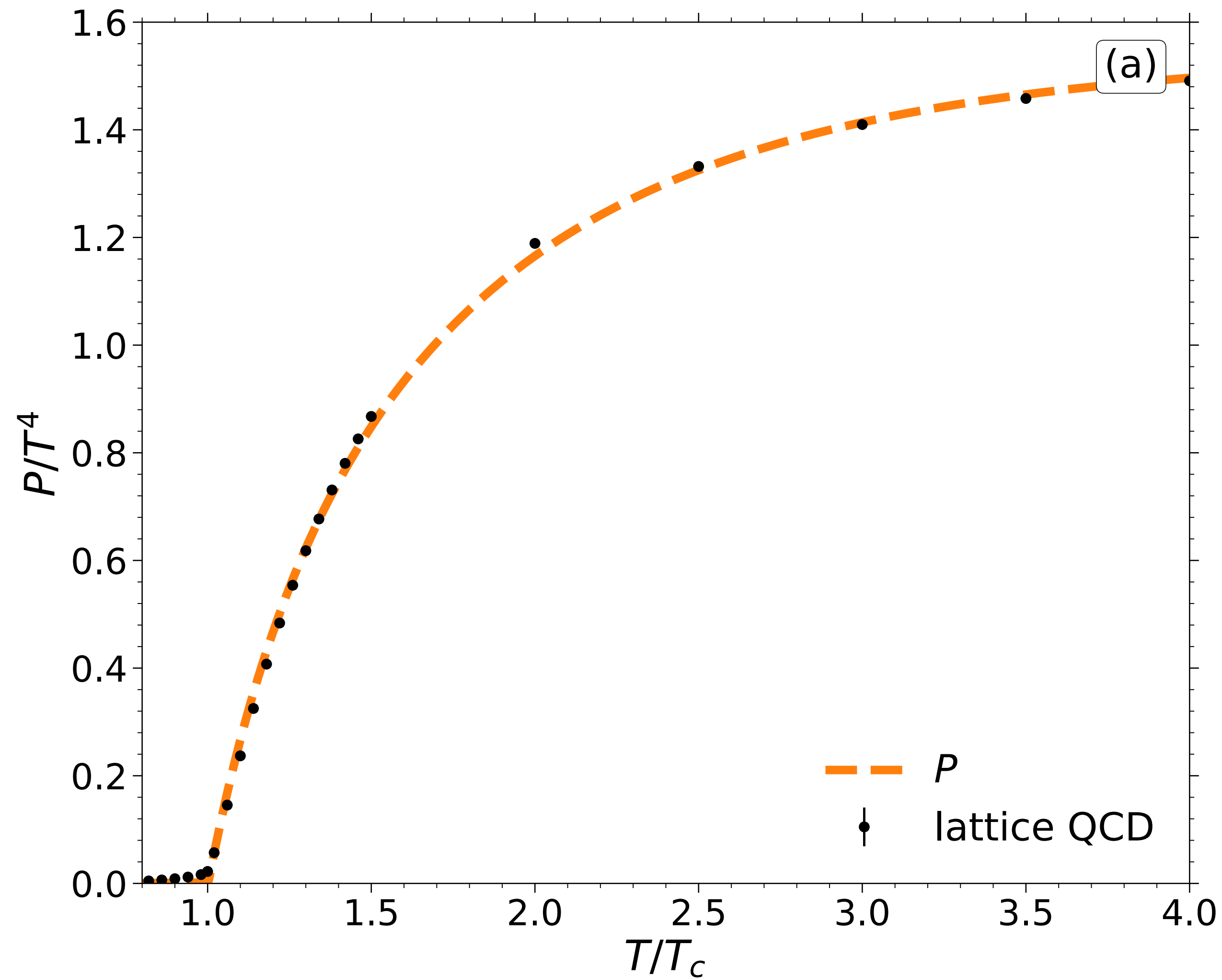
$$P_\chi(z) = \sqrt{\frac{1}{2\pi \langle \Delta_\chi^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\chi^2 \rangle}\right)$$

$$\langle f(\chi) \rangle = \int_{-\infty}^{\infty} P_\chi(z) f(\bar{\chi} + z) dz$$

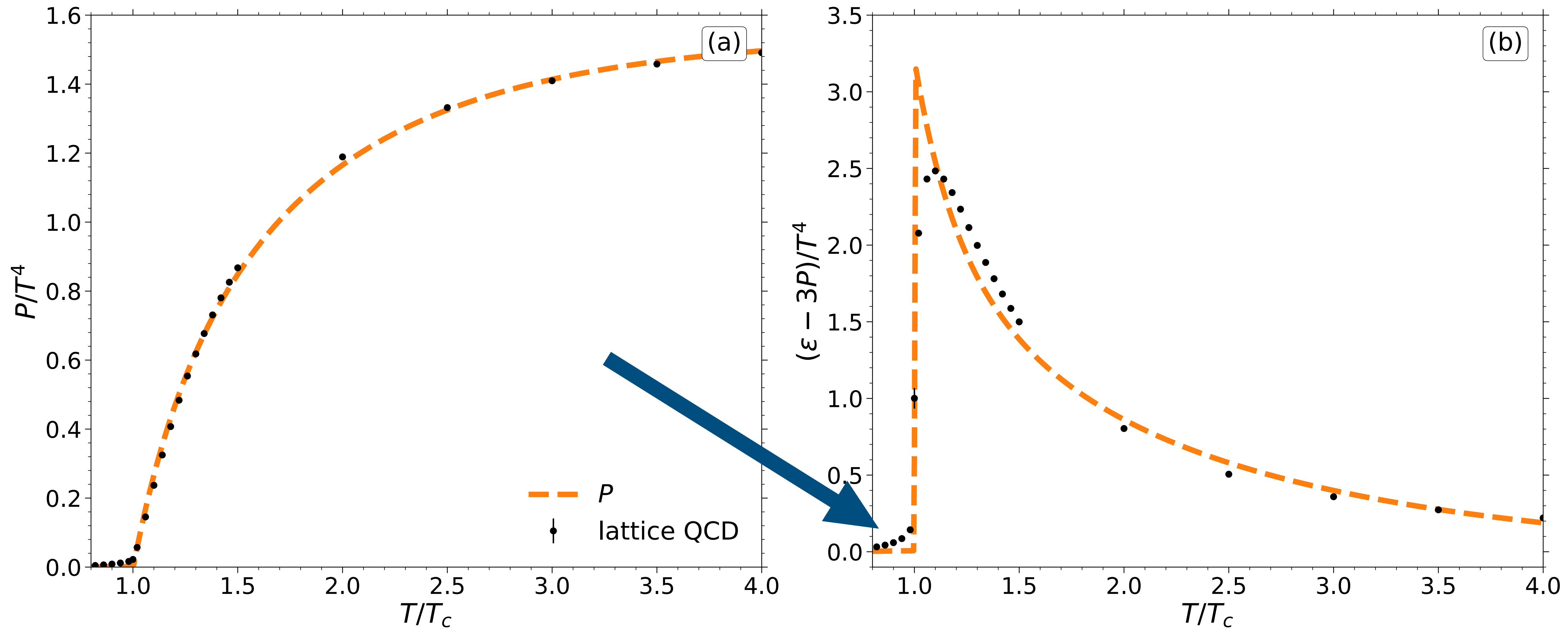
Pure gauge $SU(3)_c$ sector



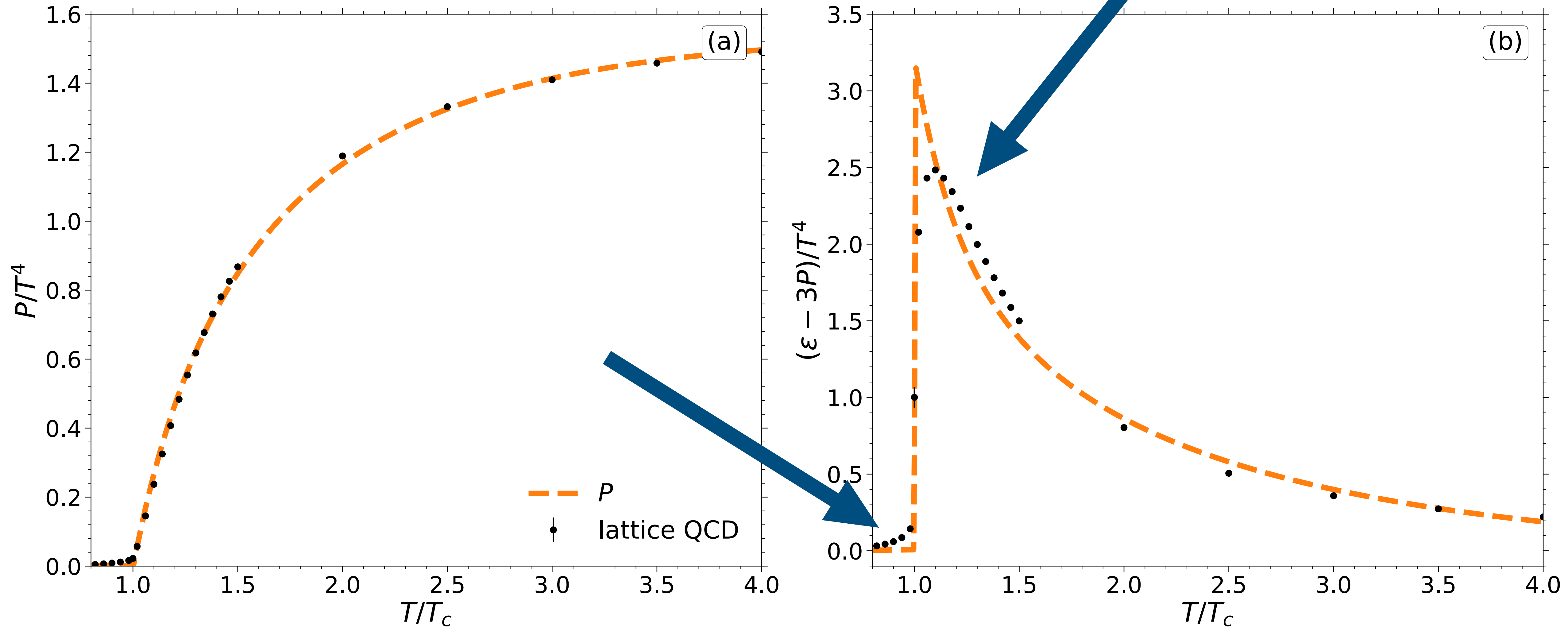
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Pure gauge $SU(3)_c$ sector



Pure gauge $SU(3)_c$ sector

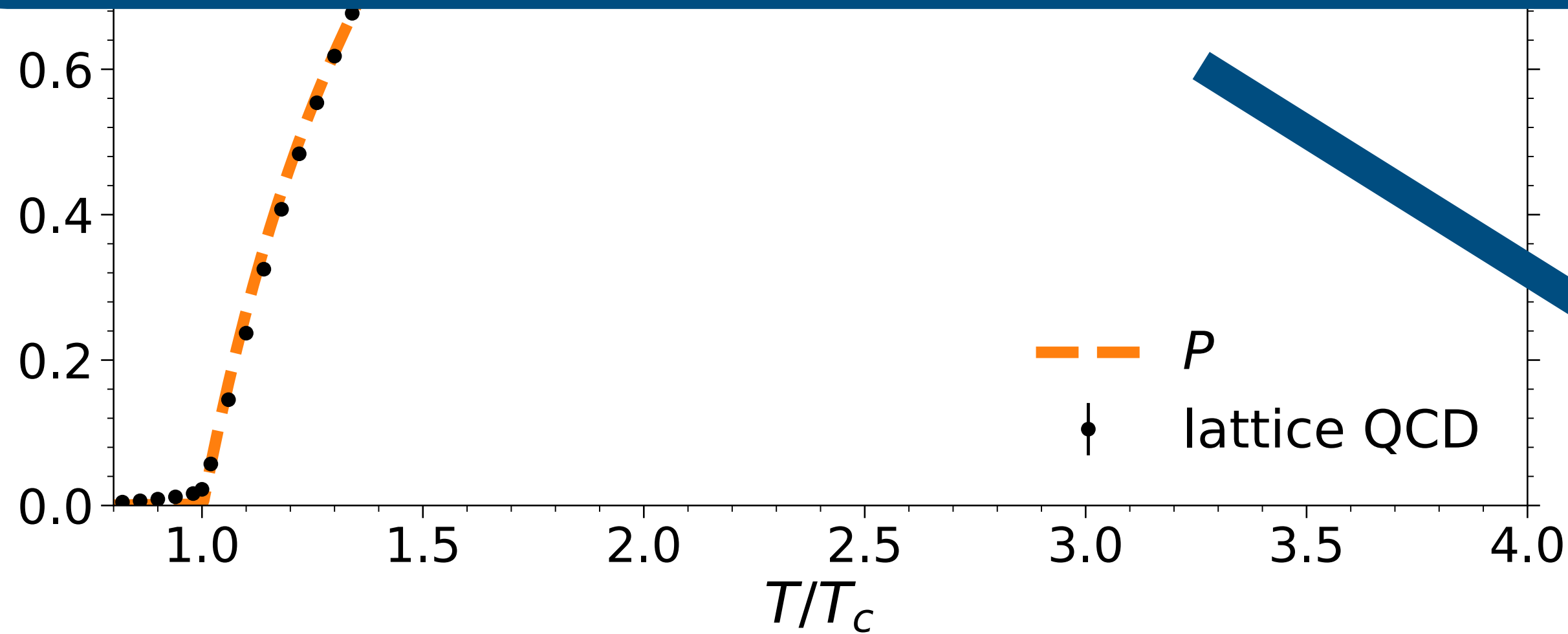


Pure gauge $SU(3)_c$ sector

$$P_{\text{dil}_J} = - (2J + 1)T \int \frac{dk^3}{(2\pi)^3} \ln \left(1 - e^{-\beta \sqrt{k^2 + m_J^{*2}}} \right)$$

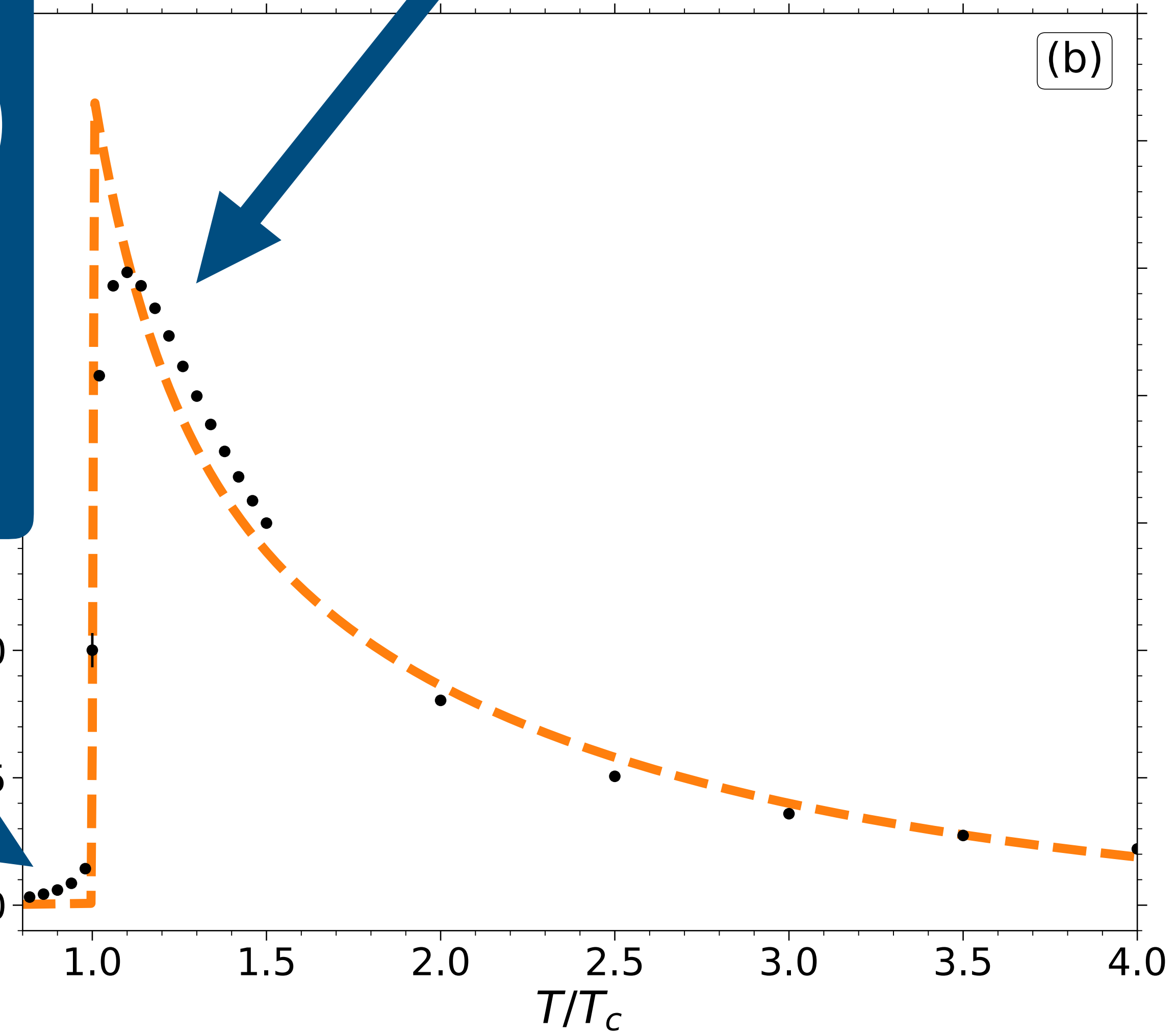
$$\frac{\tilde{\Omega}(\chi, T)}{V} = \frac{\Omega(\chi, T)}{V} - \sum_J^N P_{\text{dil}_J}$$

D/T^4



(3)

1.0
0.5
0.0



(b)

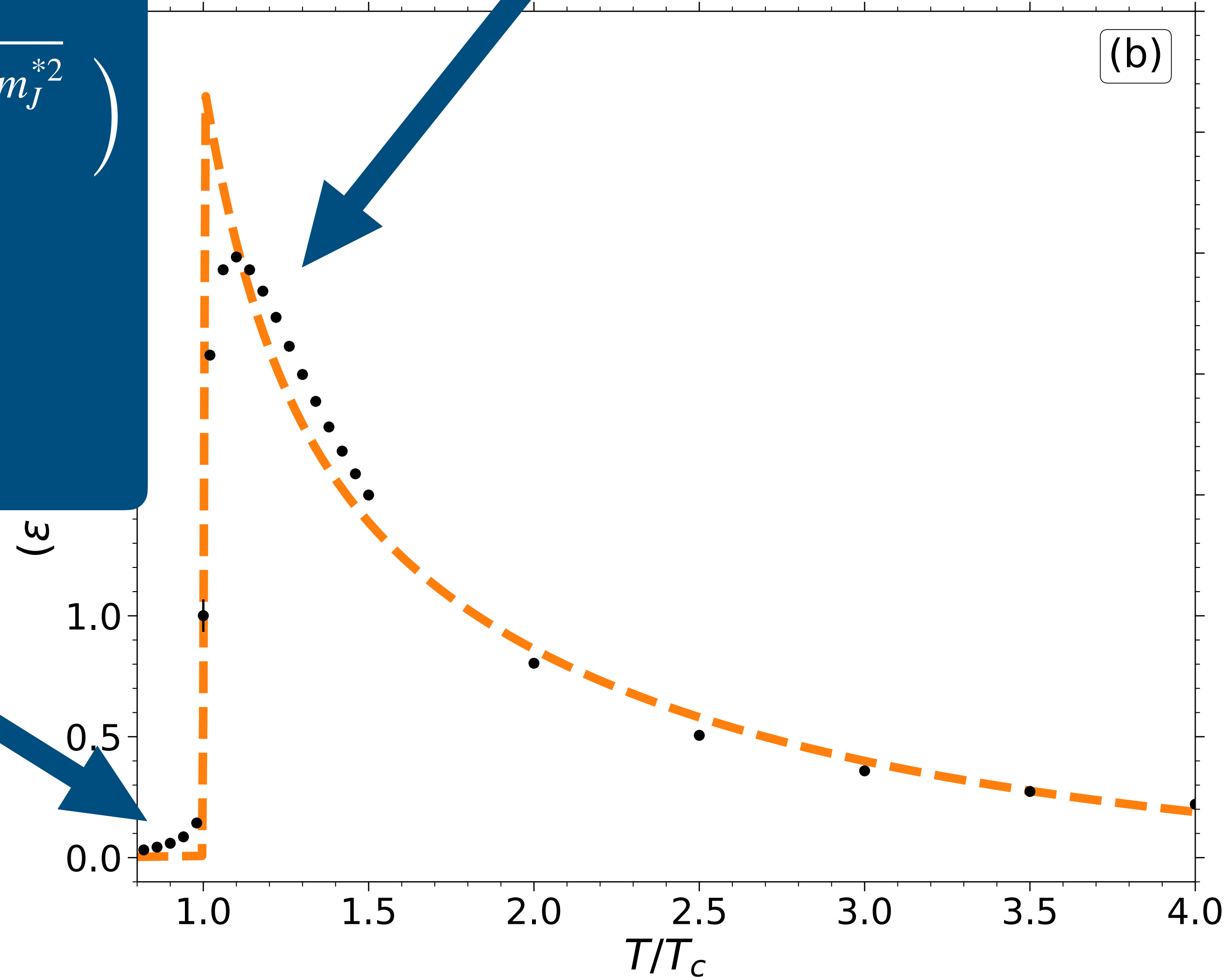
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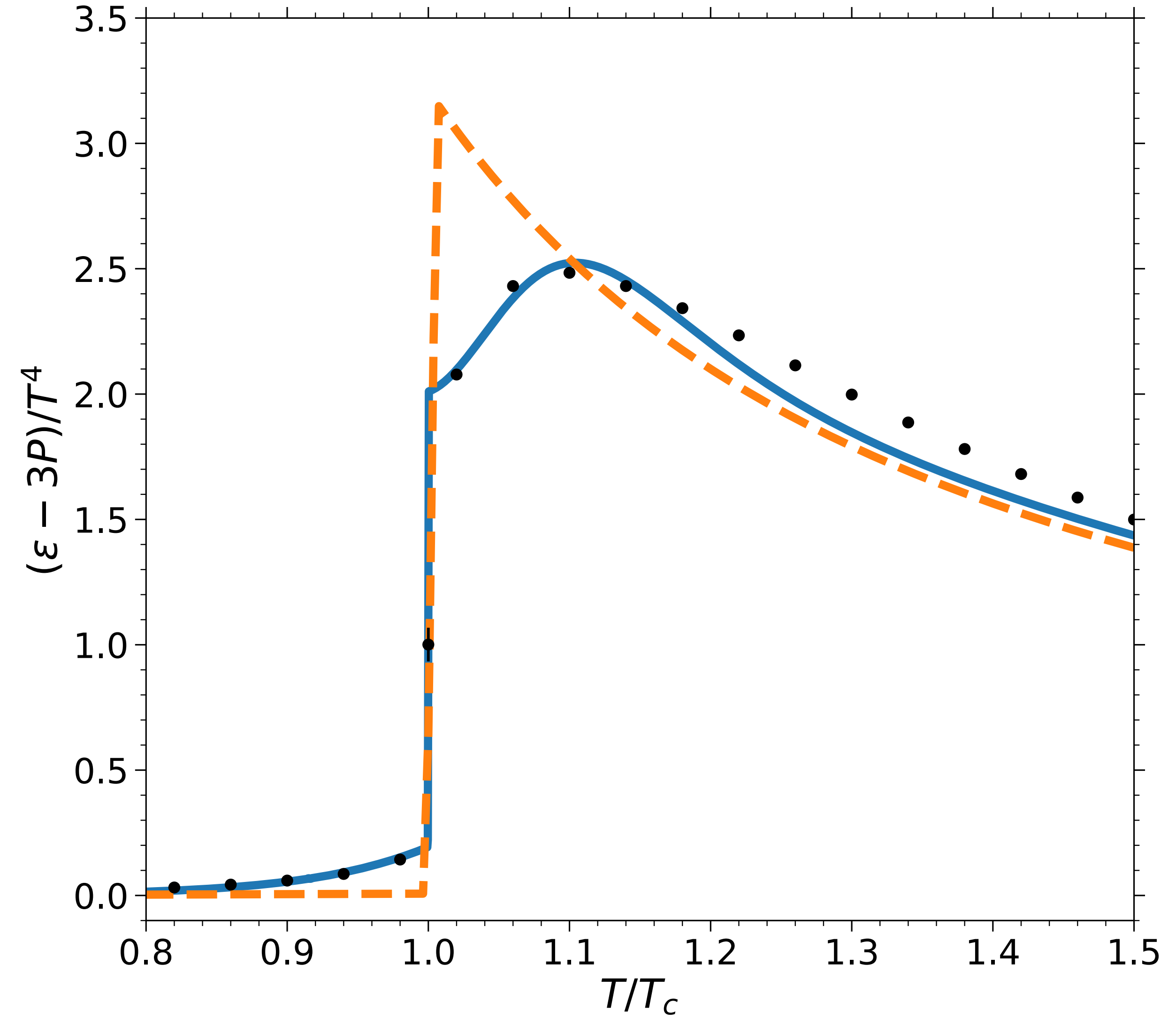
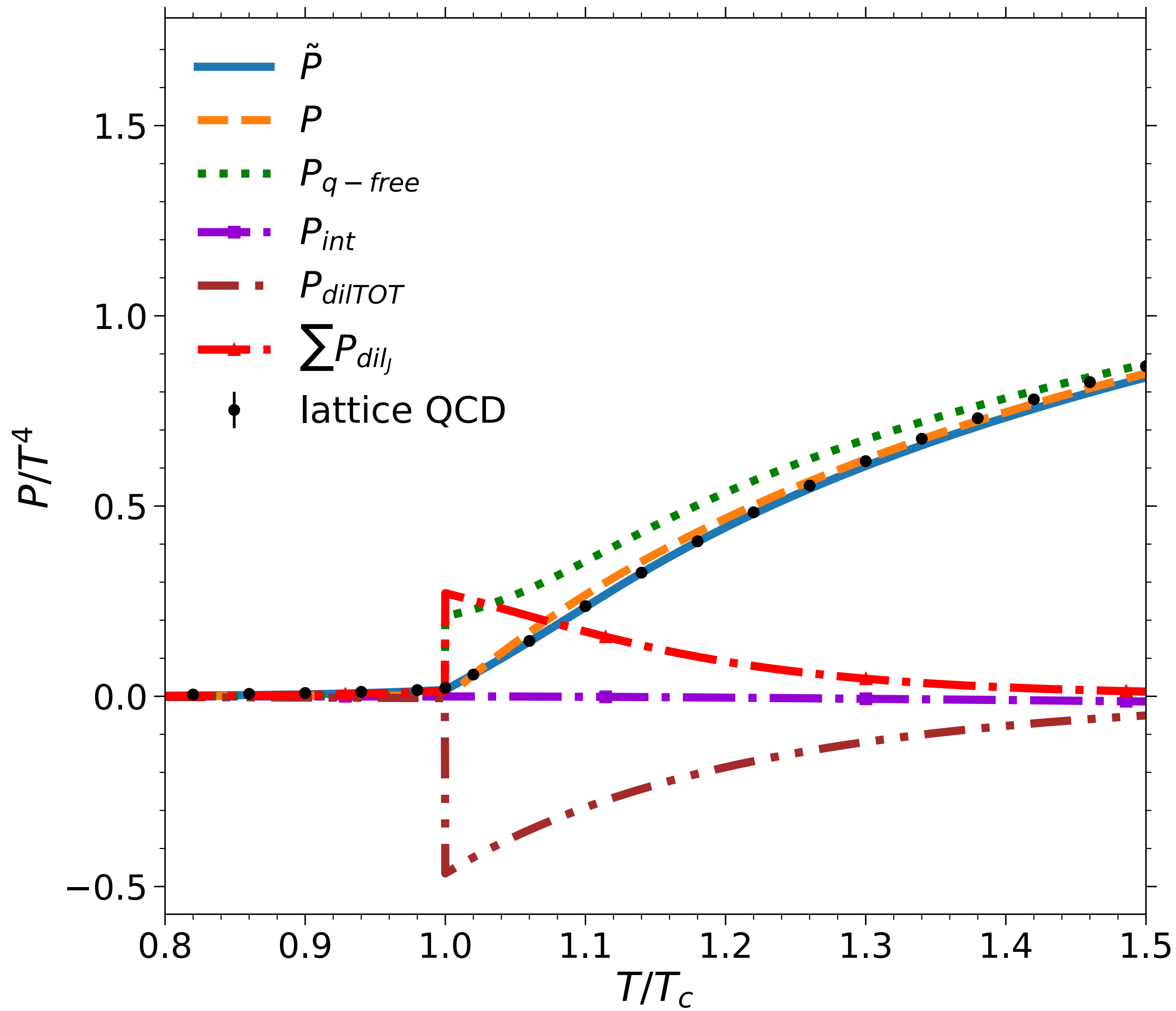
$$\frac{\tilde{\Omega}(\chi, T)}{V} = \frac{\Omega(\chi, T)}{V} - \sum_J^N P_{\text{dil}_J}$$

Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1 - \chi^2} = \frac{A}{1 - \bar{\chi}^2 - \langle \Delta_\chi^2 \rangle \cdot F(T)}$$



Pure gauge $SU(3)_c$ sector



Pure gauge $SU(3)_c$ sector

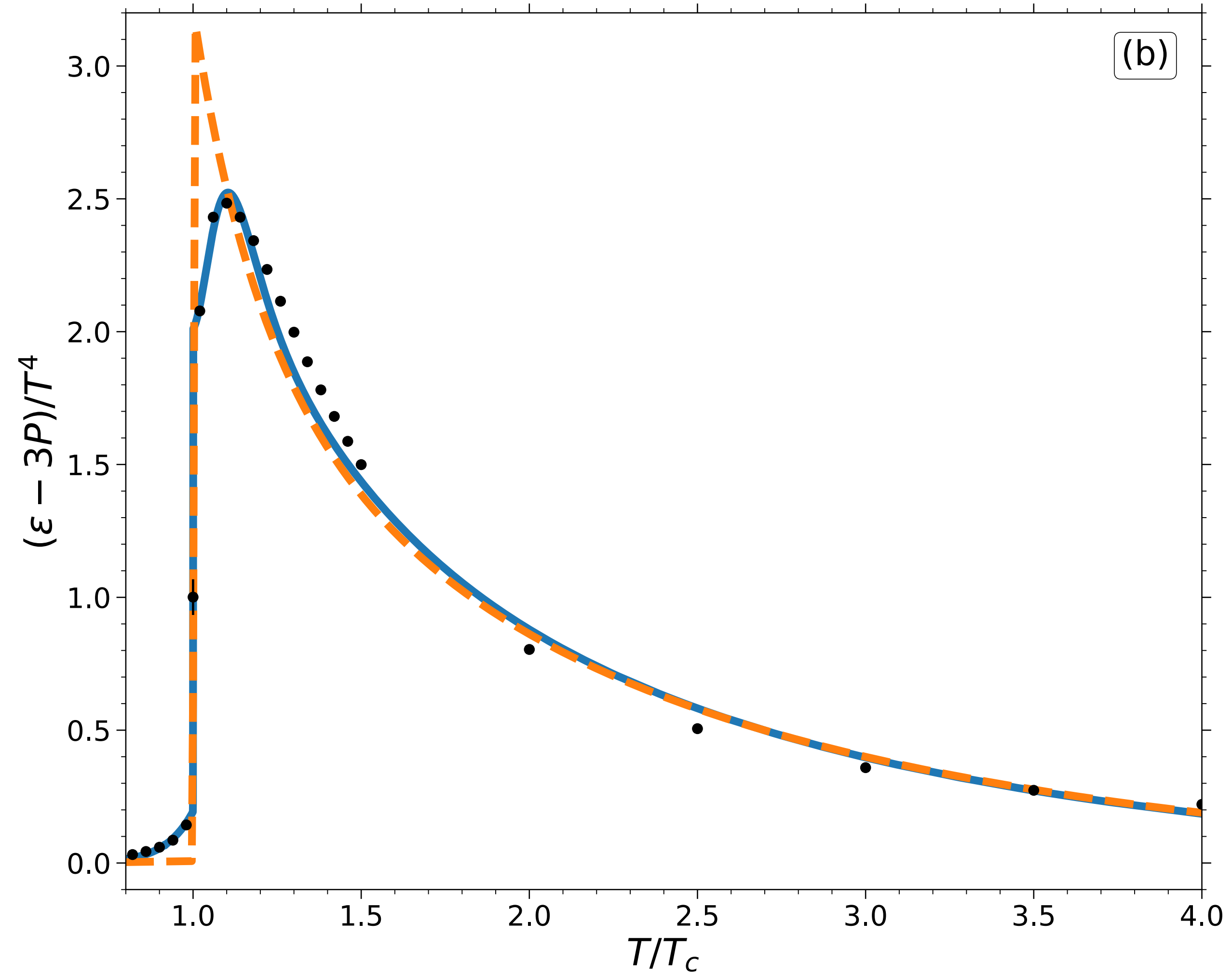
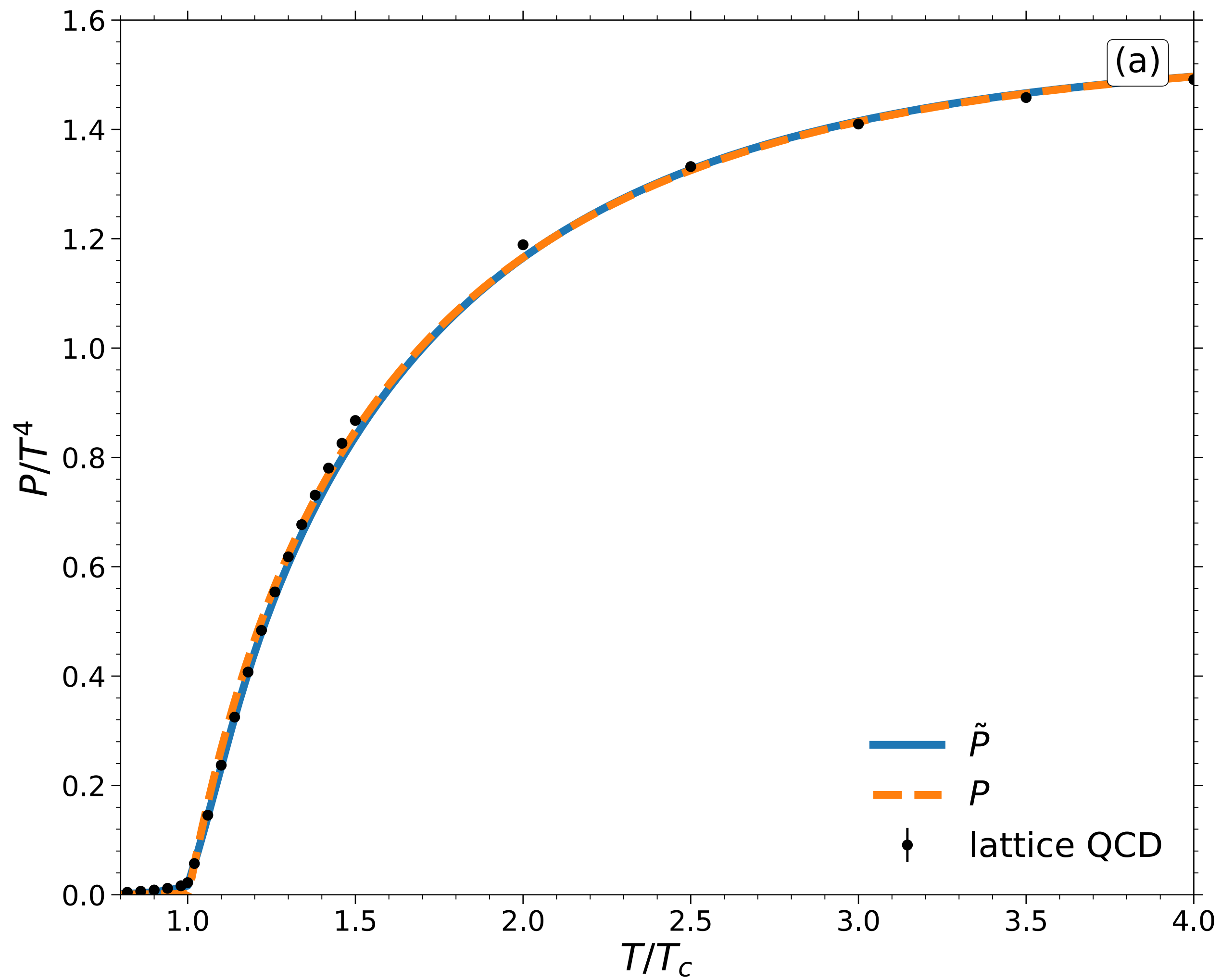


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3 Modeling finite-temperature EoSs

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 - **Meson and baryon sector**
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Meson and baryon sector at finite chemical potential

We consider the **meson** and **baryon** sector at **finite chemical potential** through the introduction of an effective Lagrangian that incorporates broken scale in addition to explicit broken chiral symmetry

Meson and baryon sector at finite chemical potential

We consider the **meson** and **baryon** sector at **finite chemical potential** through the introduction of an effective Lagrangian that incorporates broken scale in addition to explicit broken chiral symmetry

$$\begin{aligned} \tilde{\mathcal{L}} = & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} G_{\omega\phi} \phi^2 \omega_\mu \omega^\mu + \frac{1}{2} G_{b\phi} \phi^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu \\ & + \left[(G_4)^2 \omega_\mu \omega^\mu \right]^2 - \tilde{\mathcal{V}} + \bar{N} \left[\gamma^\mu \left(i \partial_\mu - g_\omega \omega_\mu - \frac{1}{2} g_\rho \mathbf{b}_\mu \cdot \boldsymbol{\tau} \right) - g \sqrt{\sigma^2 + \pi^2} \right] N \end{aligned}$$

$$\tilde{\mathcal{V}} = \mathcal{V}(\chi) - \frac{1}{2} B_0 \delta \chi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} + \frac{1}{2} B_0 \delta \chi^2 \left[\frac{\sigma^2 + \pi^2}{\sigma_0} - \frac{\chi^2}{2} \right] - \frac{1}{4} \epsilon'_1 \chi^2 \left[\frac{4\sigma}{\sigma_0} - 2 \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] - \frac{3}{4} \epsilon'_1.$$

Meson and baryon sector at finite chemical potential

ω field equation

$$4G_4 \omega_0^3 + m_\omega^2 \left(\bar{\chi}^2 + \langle \Delta_\chi^2 \rangle \right) \omega_0 - g_\omega \rho_B = 0$$

ρ field equation

$$m_\rho^2 \left(\bar{\chi}^2 + \langle \Delta_\chi^2 \rangle \right) b_0 - g_\rho \rho_3 = 0$$

σ field equation

$$\left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2 + \pi^2}} \right\rangle [\rho_S^p + \rho_S^n] - \left\langle \frac{B_0\sigma^2}{\sigma_0^4} \delta\chi^4 \frac{\sigma}{\sigma^2 + \pi^2} + \left(B_0\delta + \epsilon'_1 \right) \chi^2\sigma - \epsilon'_1 \frac{\chi^2}{\sigma_0} \right\rangle = 0$$

χ field equation

$$\left\langle 4B_0\chi^3 \ln \chi - 2B_0\delta\chi^3 \ln \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) + B_0\delta\chi \frac{\sigma^2 + \pi^2}{\sigma_0^2} - B_0\delta\chi^3 + \epsilon'_1\chi \frac{\sigma^2 + \pi^2}{\sigma_0} + \epsilon'_1\chi^3 \right\rangle \\ - 2\epsilon'_1\bar{\chi} \frac{\bar{\sigma}}{\sigma_0} - m_\omega^2 \bar{\chi} \left(\omega_0^2 + \langle \Delta\omega_\mu \Delta\omega^\mu \rangle \right) - m_\rho^2 \bar{\chi} \left(b_0^2 + \langle \Delta b_\mu \Delta b^\mu \rangle \right) = 0$$

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$$\langle A(\Phi_i + \Delta\Phi_i) \rangle = \int \prod_i dz_i P(z_i, \langle \Delta_i^2 \rangle) A(\Phi_i + z_i)$$

$$\Phi = (\phi, \sigma, \pi_0, \pi_+, \pi_-)$$

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$$n_i = \begin{cases} 1 & \text{for } \phi, \sigma, \pi_0, \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu, b_{0\mu}, b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

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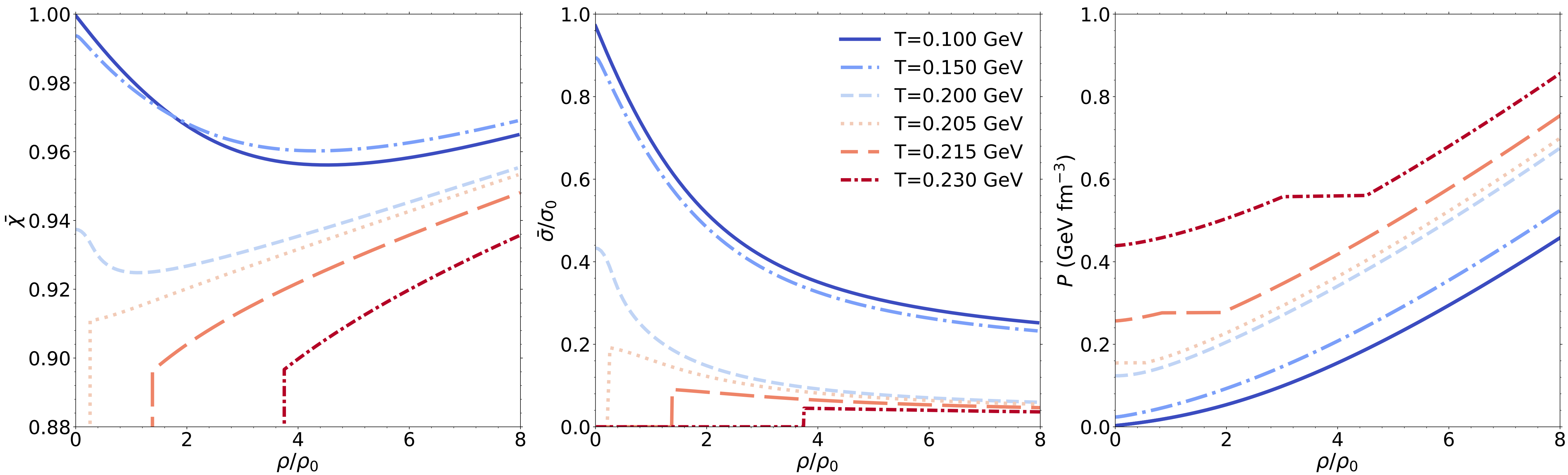
$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i^*} \frac{1}{e^{\beta(\epsilon_i^* - \mu_i^*)} - 1} dk$$

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$$\begin{aligned} m_\pi &= m_\pi(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle) \\ m_\sigma &= m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle) \\ m_\chi &= m_\chi(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle) \\ m_\rho &= m_\rho(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle) \\ m_N &= m_N(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle) \end{aligned}$$

Meson and baryon sector at finite chemical potential



Meson and baryon sector: phase diagram

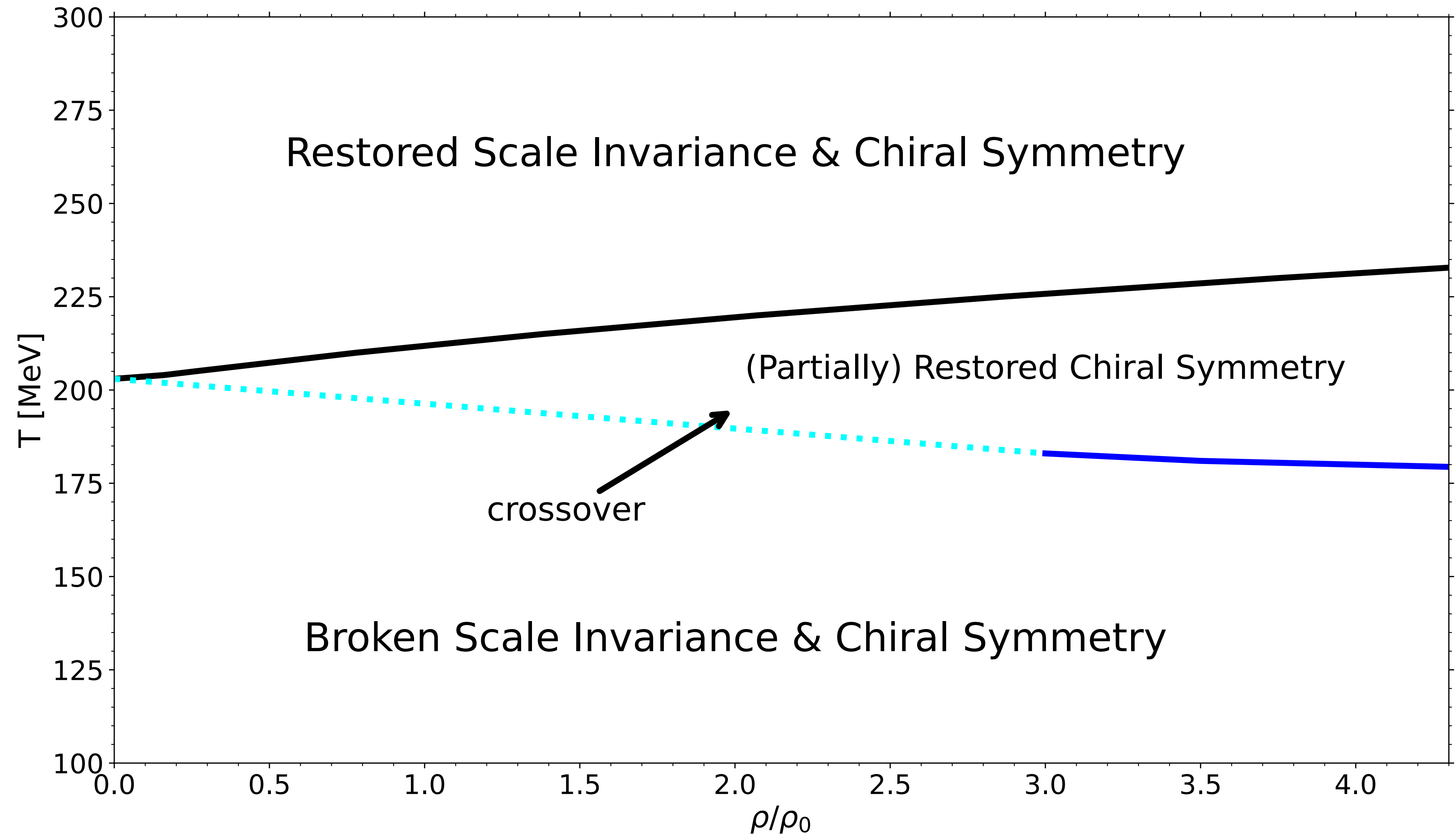


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- Outlooks

Summary

- Development of a numerical code for **RMF predictions** of the dense-matter EOS and **Bayesian analysis** applications to neutron stars
- The developed model **solves** the **stability problems** of the complex potential of the previous model in the literature
- Phenomenological approach **reproduces main EOS lattice QCD** results for pure gauge $SU(3)_c$ thermodynamics, consistently showing a **first-order phase transition** at 0.270 GeV.
- A model beyond mean field has been developed, including **mesonic** and **baryonic sectors**, enabling the exploration of a **wide range** of **temperatures** and **densities**, including **thermodynamic fluctuations**.

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Outlooks

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- Take into account new degrees of freedom:
 - $SU(3)_f$ **quarks (color-dielectric model)**
 - hadronic phase with **strange** meson and baryon

Outlooks

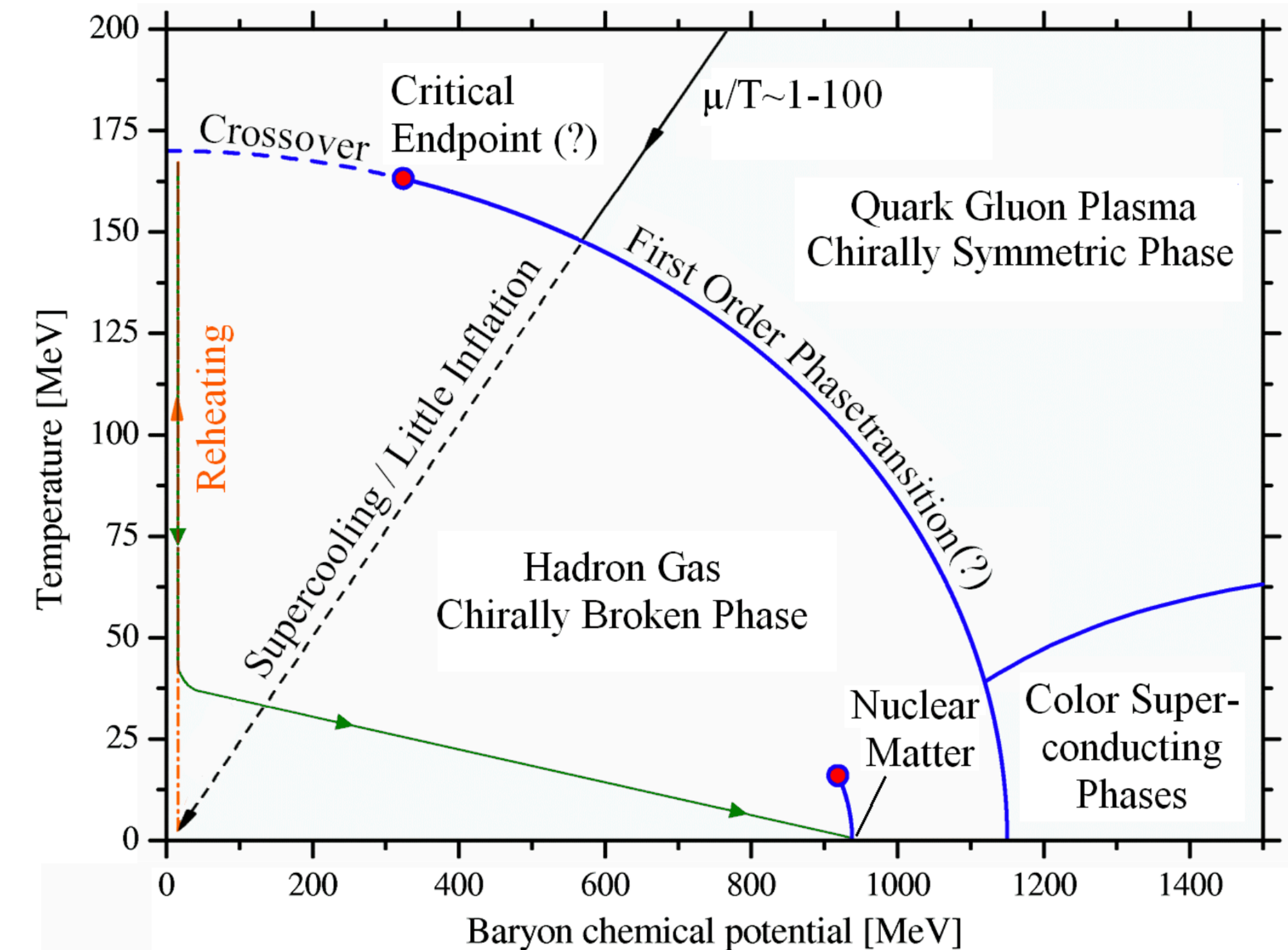
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Outlooks

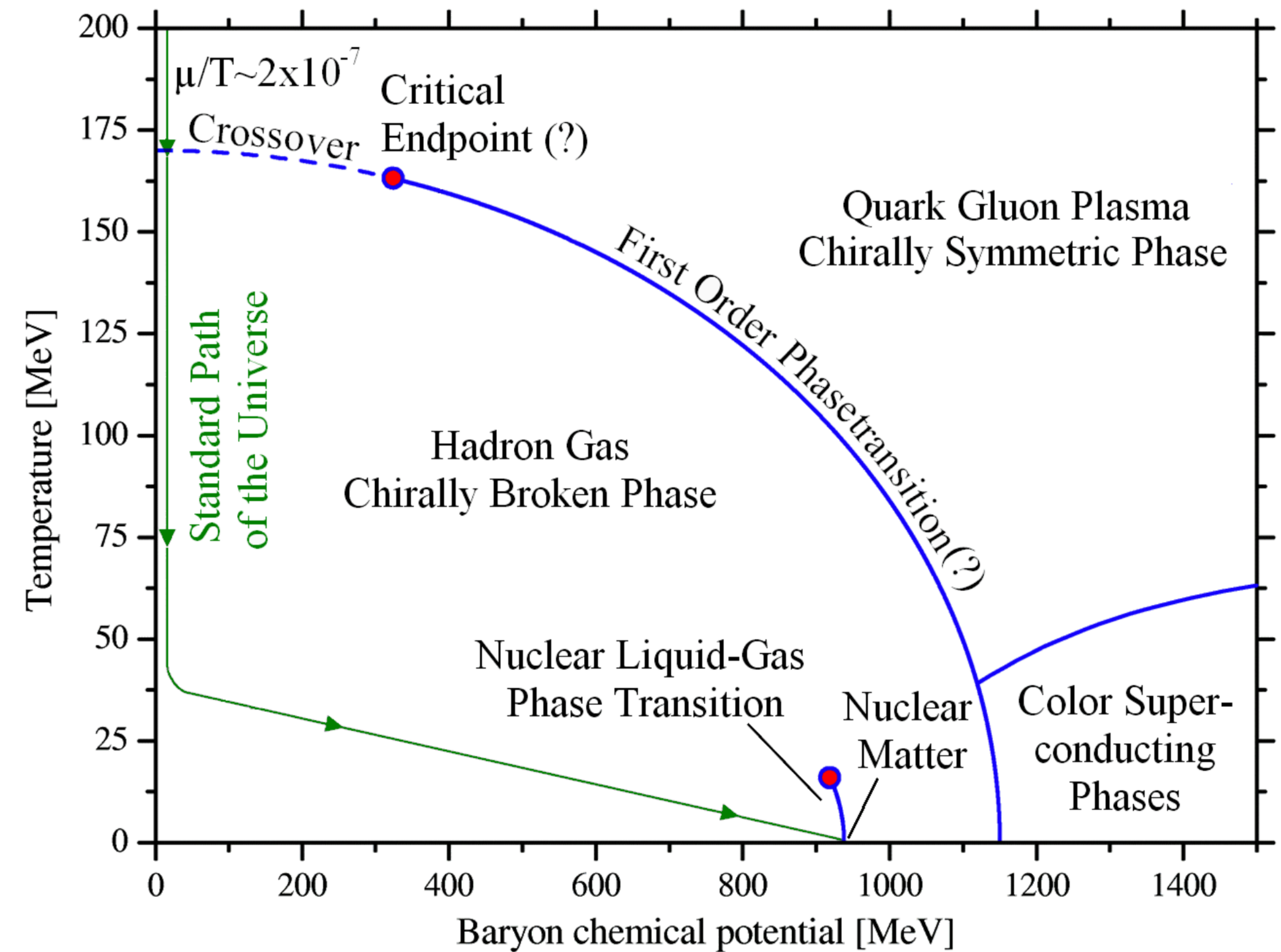
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 - **cosmological trajectories (little inflation scenario)** in the QCD epoch of the early universe
- Explore **double family scenario** through Bayesian analysis

Thank you

Backup slides

Equation of State for: cosmic trajectories

When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory



Equation of State for: cosmic trajectories

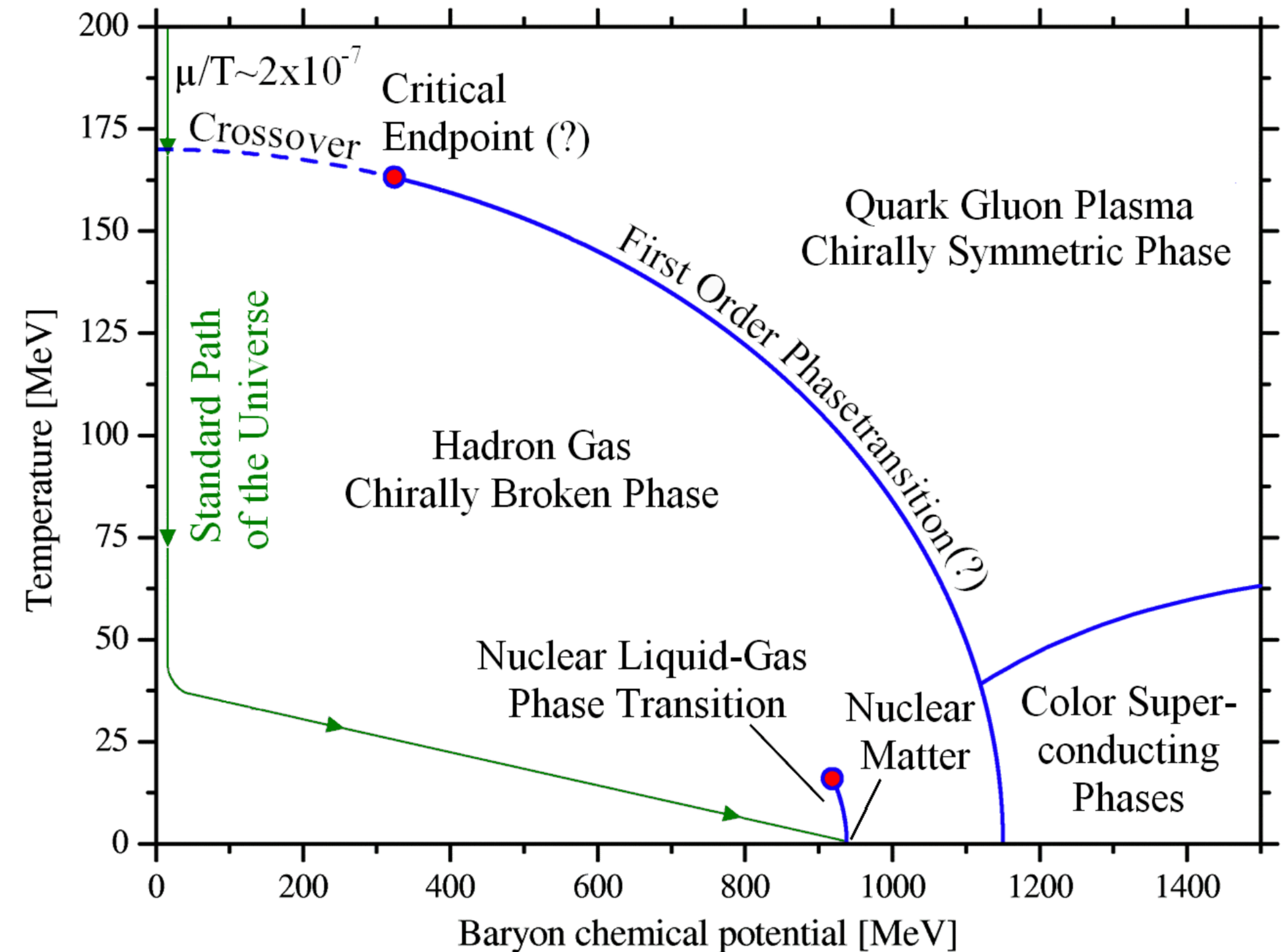
When the Universe expands and cools down, it follows a certain path in the QCD phase diagram, the so-called cosmic trajectory

Conservation laws

1. Lepton number: $l_\alpha s = n_\alpha + n_{\nu_\alpha}$, $\alpha = e, \mu, \tau$

2. Baryon number: $bs = \sum_i B_i n_i$

3. Electric charge: $qs = \sum_i Q_i n_i$



Equation of State for: cosmic trajectories

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2. Baryon number: $b s = \sum_i B_i n_i$ $b = 8.6 \times 10^{-11}$

$q = 0$ (charge neutrality)

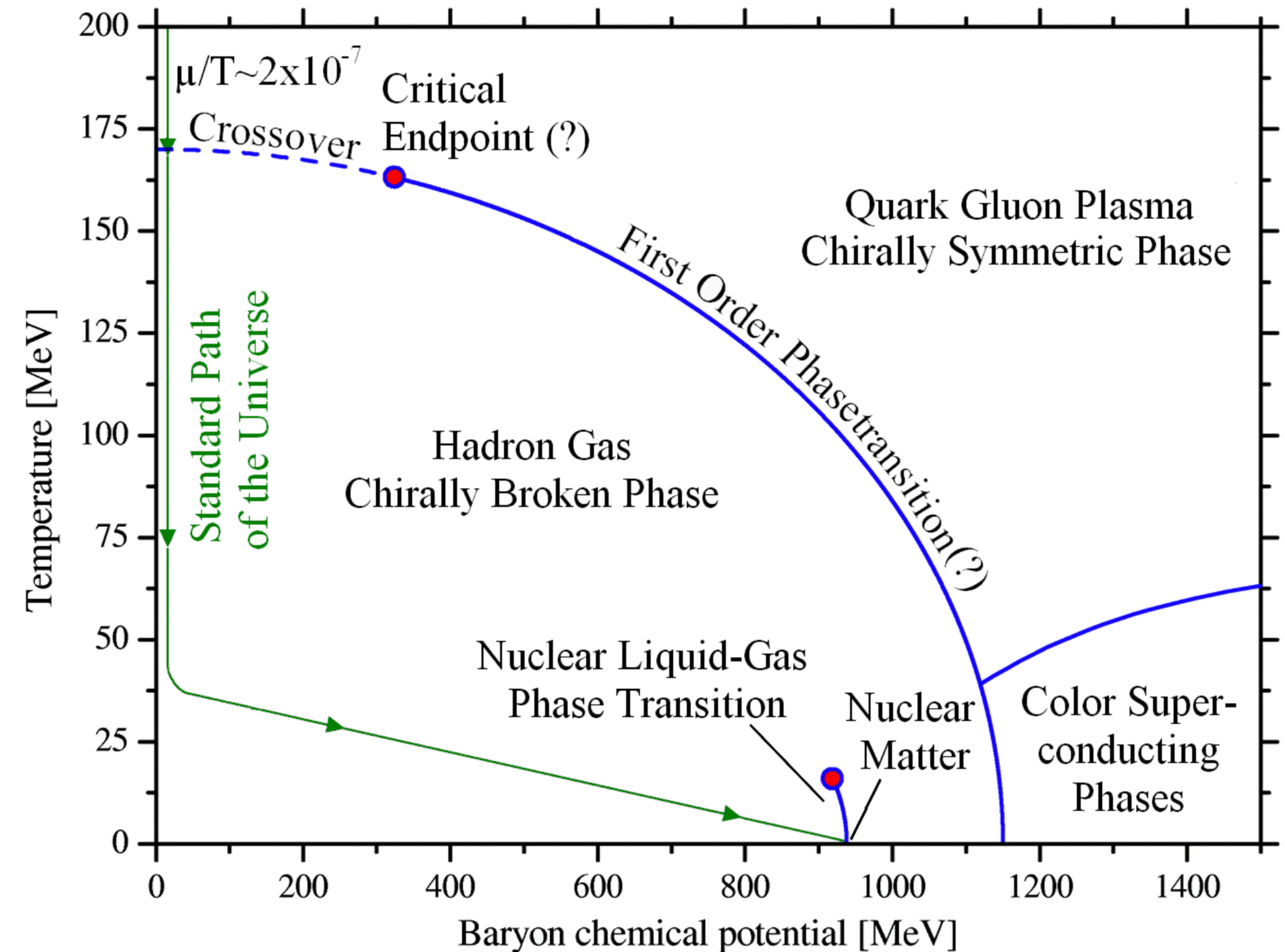
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5 conservation laws

→ 5 equations

→ 5 chemical potentials

$\mu_{L_\alpha}, \mu_B, \mu_Q$



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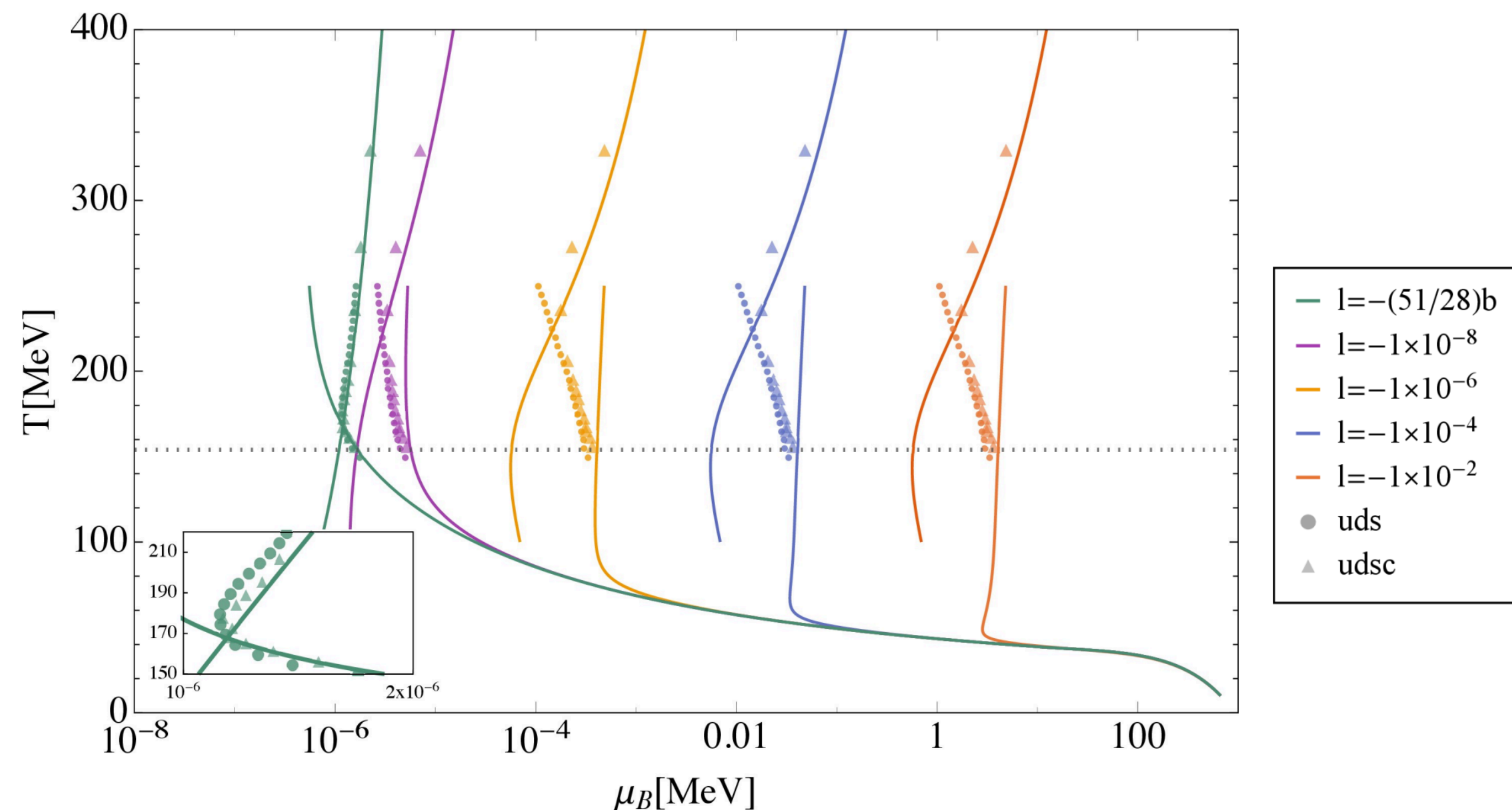
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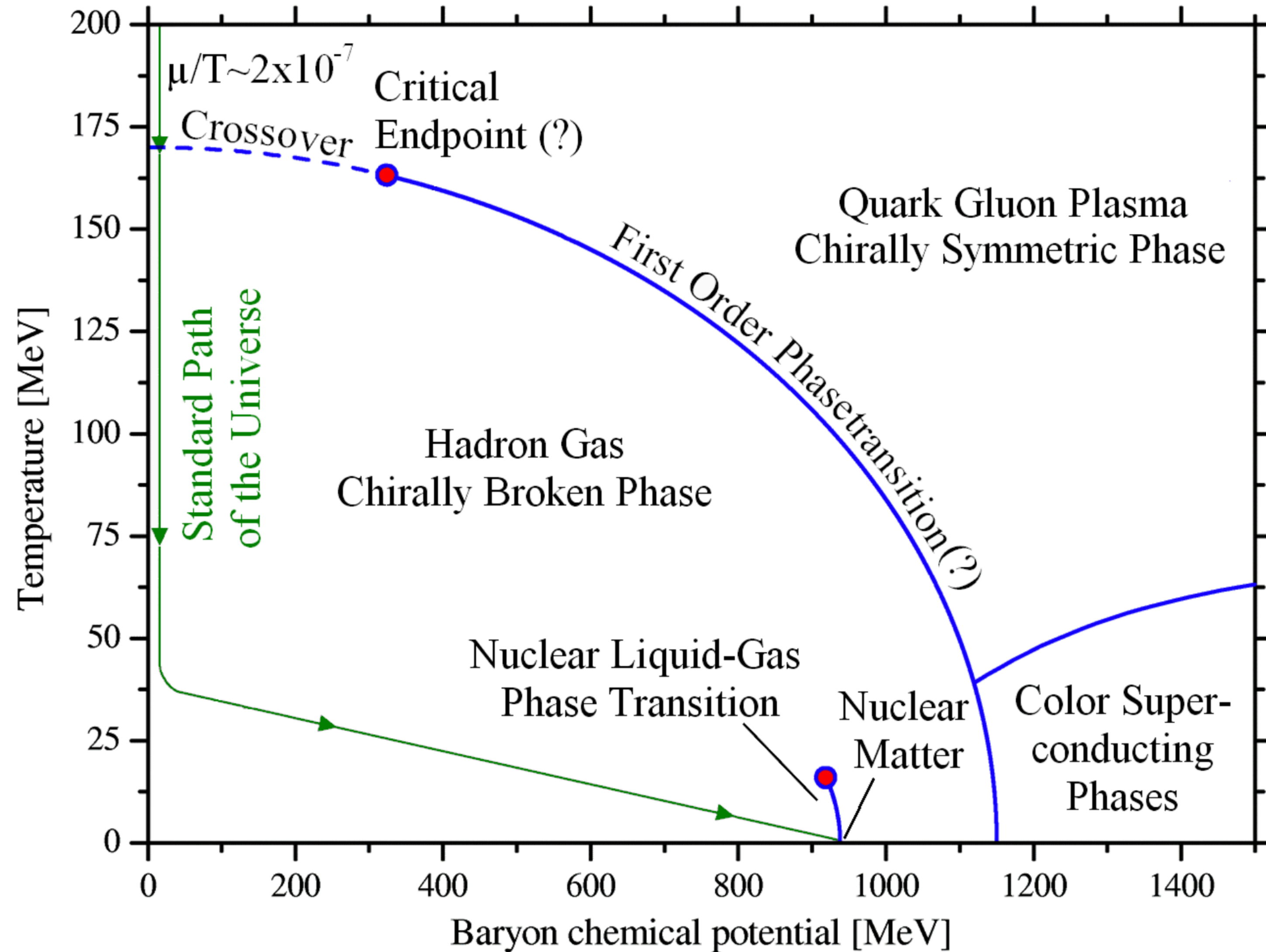
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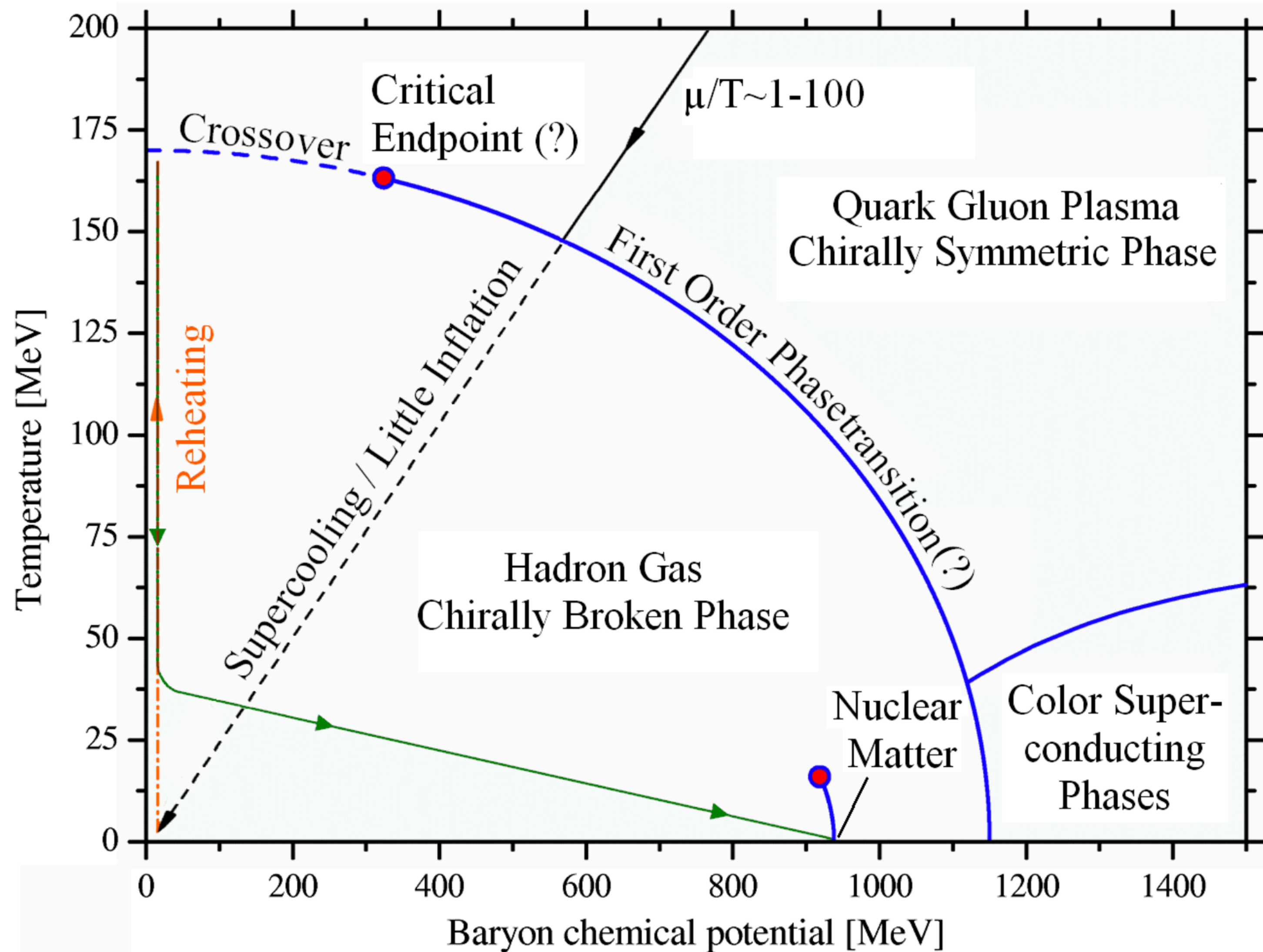


Little inflation scenario



- Negligible baryon density: $\mu_B \ll T$
- QCD phase transition: **crossover**

Little inflation scenario



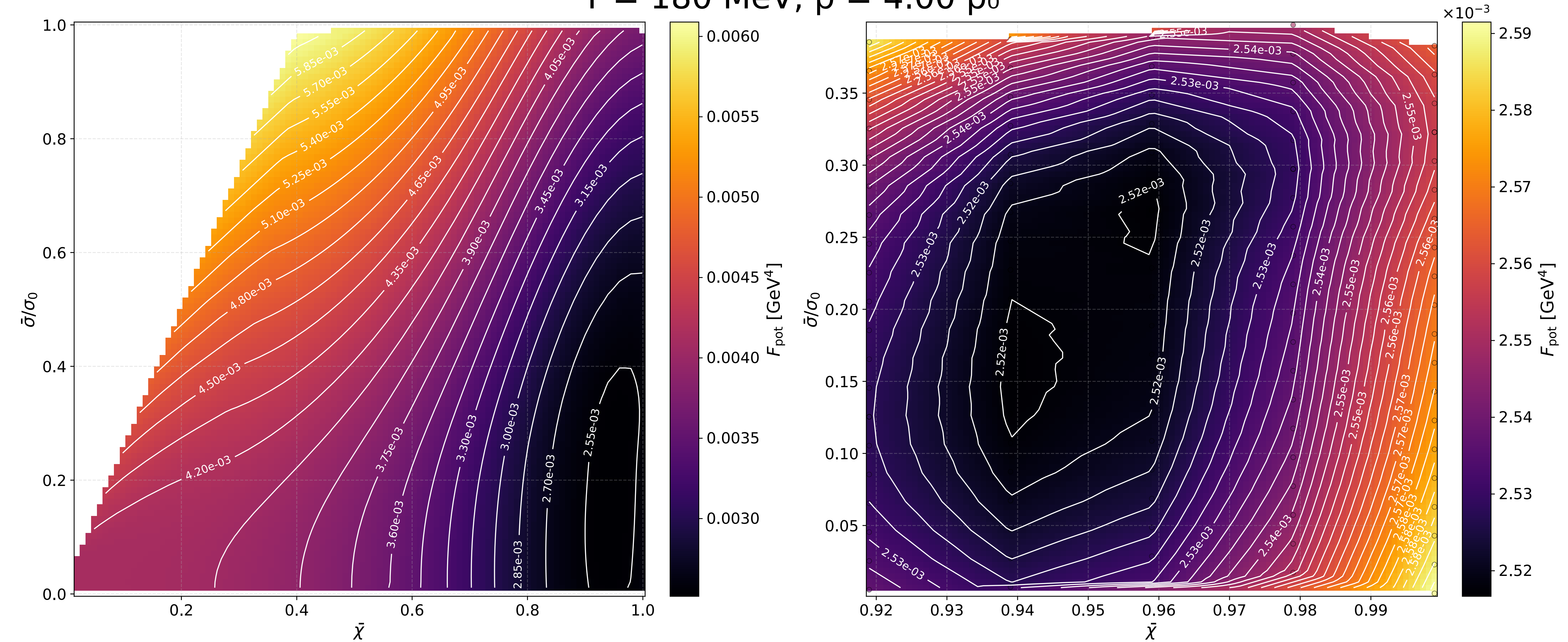
- Non vanishing μ_B : $\mu_B/T \sim O(1)$
- $\Omega(T, V, \mu_B)$ has two minima
- System sticks in **false vacuum**

⇒ **LITTLE INFLATION**

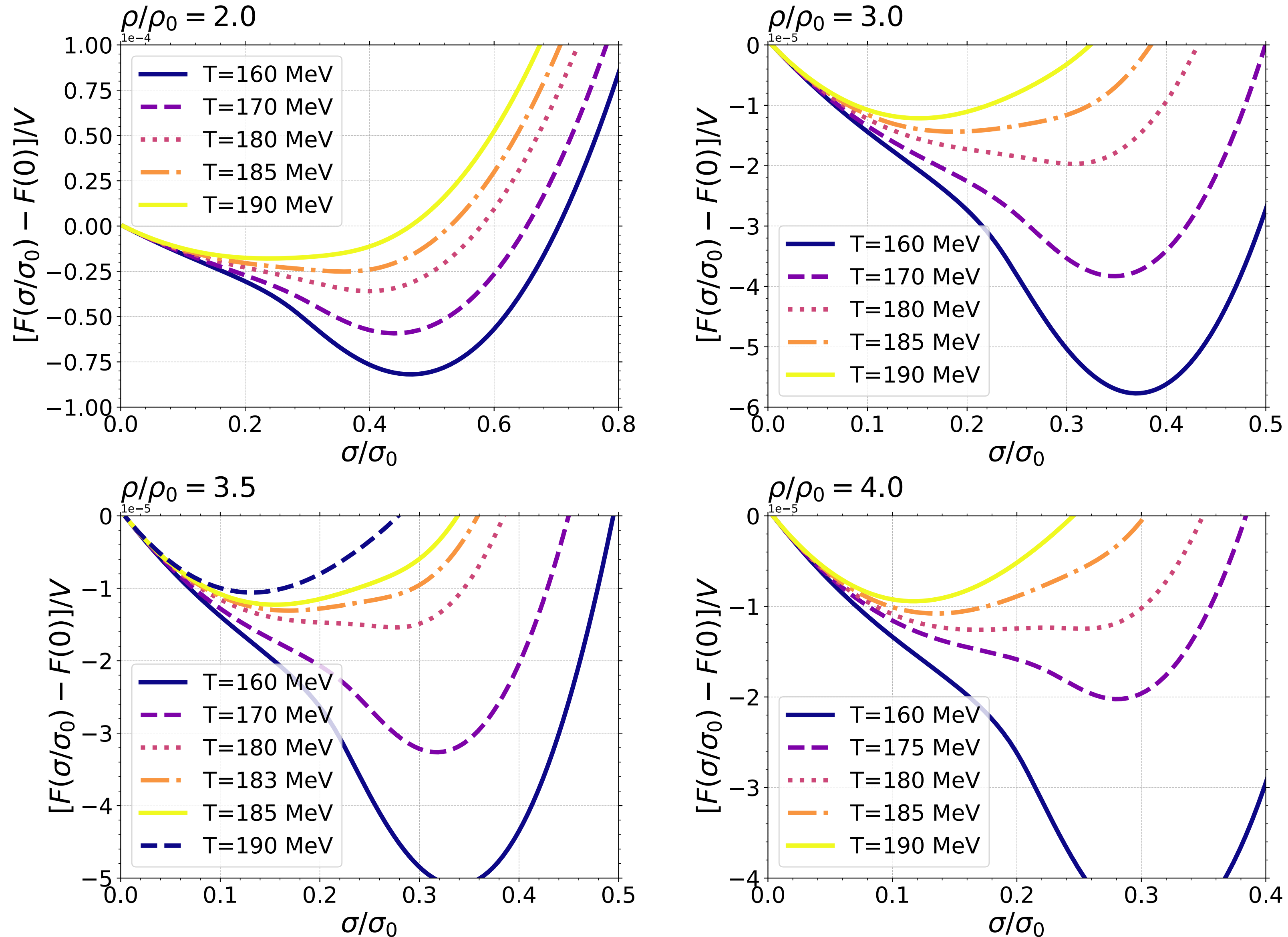
- **First order phase transition:** release of false vacuum energy causes **reheating**

Little inflation scenario

$T = 180 \text{ MeV}, \rho = 4.00 \rho_0$



Meson and baryon sector at finite chemical potential



Symmetries of the model

Scale anomaly

A theory is invariant under scale transformations if its action remains constant when the fields are transformed as follows under the scale operator $U(\lambda)$:

$$\phi(x) \rightarrow U^+ \phi(x) U(\lambda) = \lambda^\Delta \phi(\lambda x)$$

where Δ is the scale dimension of ϕ . The action remains unchanged *only if* no dimensional parameter is present: scale invariance is broken due to quantum corrections due to the appearance of a dimensional parameter Λ .

Chiral symmetry

Chiral symmetry is spontaneously broken in the QCD vacuum. The vacuum is non-trivial and not invariant under the chiral transformation $SU(N_f)_L \times SU(N_f)_R$, even in the chiral limit, so that the order parameter $\langle \bar{q}_L^i q_R^j \rangle$ acquires a non-zero expectation value. By increasing temperature and/or density, the quark condensate melts, restoring chiral symmetry.

Numerical details

4 field equations:

$\chi, \sigma, \omega, \rho$

2 conservation equation:

Total baryonic charge

Total electric charge

6 mass equations:

$$m_\chi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\sigma = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\pi = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\omega = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_\rho = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$m_N = m_\sigma(\chi, \sigma, \pi, \omega, \rho, \langle \Delta_i^2 \rangle)$$

$$\begin{cases} \vdots \\ \langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty dk \frac{k^2}{e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle)} \frac{1}{\exp \left[\beta \left(e_i^*(\bar{\phi}_j, \langle \Delta_j^2 \rangle) - \mu_i^* \right) \right] - 1} \\ \vdots \end{cases}$$

$$e_i^* = \sqrt{k^2 + m_i^{*2}}$$

Pure gauge $SU(3)_c$ sector

To improve accuracy with latticeQCD data:

- Inclusion **excited glueballs**:

$$m^2(n = 1, J^{PC}) = a(n + J) + b_{J^{PC}}$$

$$P_{\text{dil}_J} = - (2J + 1)T \int \frac{dk^3}{(2\pi)^3} \ln \left(1 - e^{-\beta \sqrt{k^2 + m_J^{*2}}} \right)$$

$$\frac{\tilde{\Omega}(\chi, T)}{V} = \frac{\Omega(\chi, T)}{V} - \sum_J^N P_{\text{dil}_J}$$

J^{PC}	m [GeV]	$b_{J^{PC}}$ [GeV ²]
0^{++}	1.647(25)	-2.78 ± 0.21
2^{++}	2.367(30)	-10.87 ± 0.57
0^{-+}	2.572(38)	1.12 ± 0.27
2^{-+}	3.11(5)	-6.79 ± 0.66
1^{+-}	2.955(37)	-2.25 ± 0.45

String tension term

$$\sigma(T) = \left(1 - \frac{1}{1 + e^{-(T-\alpha)/\delta}} \right)$$

- E. Trotti, S. Jafarzade, and F. Giacosa, *Eur.Phys.J.C* 83 (2023) 5, 390
- N. Cardoso and P. Bicudo, *Phys. Rev. D* 85, 077501 (2012).

Pure gauge $SU(3)_c$ sector

To improve accuracy with latticeQCD data:

- Inclusion excited glueballs;
- Modification **infrared cut-off**:

$$I = \frac{s}{T^3} - \frac{4P}{T^4} \Rightarrow s/T^3 \gg 4P/T^4$$

Modified infrared cut-off

$$\tilde{K}(\chi, T) = \frac{A}{1 - \chi^2} = \frac{A}{1 - \bar{\chi}^2 - \langle \Delta^2 \rangle \cdot F(T)}$$

Form factor

$$F(T) = \frac{\Theta(T - T_c)}{1 + \left(\frac{T - T_c}{\gamma}\right)^2}$$

Meson and baryon sector at finite chemical potential

The **thermodynamical potential** is

$$\begin{aligned} \frac{\tilde{\Omega}}{V} = & \langle \tilde{\mathcal{V}} \rangle - \frac{1}{2} m_\omega^2 \chi^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \chi^2 b_0^2 - \frac{G_4}{4} \omega_0^4 - \frac{1}{2} m_\sigma^{*2} \langle \Delta_\sigma^2 \rangle - \frac{1}{2} m_\pi^{*2} \langle \boldsymbol{\pi}^2 \rangle - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle \\ & + \frac{T}{2\pi^2} \int dk k^2 [\ln(1 - e^{-\beta\epsilon_\sigma}) + 3 \ln(1 - e^{-\beta\epsilon_\omega})] + \frac{T}{2\pi^2} \int dk k^2 \left[\sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\pi^* - \mu_\pi^{*a})}) + 3 \sum_{a=1,3} \ln(1 - e^{-\beta(\epsilon_\rho^* - \mu_\rho^{*a})}) \right] \\ & - \frac{T}{\pi^2} \sum_{i=p,n} \int dk_i k_i^2 \left[\ln(1 + e^{-\beta(E_i^* - \mu_i^*)}) + \ln(1 + e^{-\beta(E_i^* + \mu_i^*)}) \right] \end{aligned}$$

$$\tilde{\mathcal{V}} = \mathcal{V}(\chi) - \frac{1}{2} B_0 \delta \chi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2} + \frac{1}{2} B_0 \delta \chi^2 \left[\frac{\sigma^2 + \pi^2}{\sigma_0} - \frac{\chi^2}{2} \right] - \frac{1}{4} \epsilon'_1 \chi^2 \left[\frac{4\sigma}{\sigma_0} - 2 \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) - \chi^2 \right] - \frac{3}{4} \epsilon'_1.$$

Meson and baryon sector: thermal fluctuations

$$\langle \Delta_i^2 \rangle = \frac{n_i}{2\pi^2} \int_0^\infty \frac{k^2}{\epsilon_i^*} \frac{1}{e^{\beta(\epsilon_i^* - \mu_i^*)} - 1} dk$$

$$n_i = \begin{cases} 1 & \text{for } \phi, \sigma, \pi_0, \pi_+, \text{ and } \pi_- \text{ mesons} \\ -3 & \text{for } \omega_\mu, b_{0\mu}, b_{+\mu}, \text{ and } b_{-\mu} \text{ mesons} \end{cases}$$

$$P_\sigma(z) = \sqrt{\frac{1}{2\pi\langle \Delta_\sigma^2 \rangle}} \exp\left(-\frac{z^2}{2\langle \Delta_\sigma^2 \rangle}\right),$$

$$P_\pi(y) = \sqrt{\frac{2}{\pi}} \left(\frac{3}{\langle \Delta_\pi^2 \rangle}\right)^{(3/2)} \exp\left(-\frac{3y^2}{2\langle \Delta_\pi^2 \rangle}\right),$$

$$\langle O(\bar{\sigma} + \Delta_\sigma, \pi^2) \rangle = \int_{-\infty}^\infty dz P_\sigma(z) \int_0^\infty dy y^2 P_\pi(y) O(\bar{\sigma} + z, y^2) .$$

The thermodynamical potential in the mean field approximation

$$\frac{\Omega(\chi, T)}{V} = \langle \mathcal{V}(\chi) \rangle - P_{\text{dil}}(\chi, T) - P_{\text{q-free}}(\chi, T) - P_{\text{int}}(\chi, T) + \frac{B_0}{4} - \frac{1}{2} m_\chi^{*2} \langle \Delta_\chi^2 \rangle$$

Where

$$P_{\text{dil}}(\chi, T) = \frac{1}{6\pi} \int_0^\infty dk \frac{k^4}{\sqrt{k^2 + m_\chi^{*2}}} \frac{1}{e^{\beta\sqrt{k^2 + m_\chi^{*2}}} - 1}$$

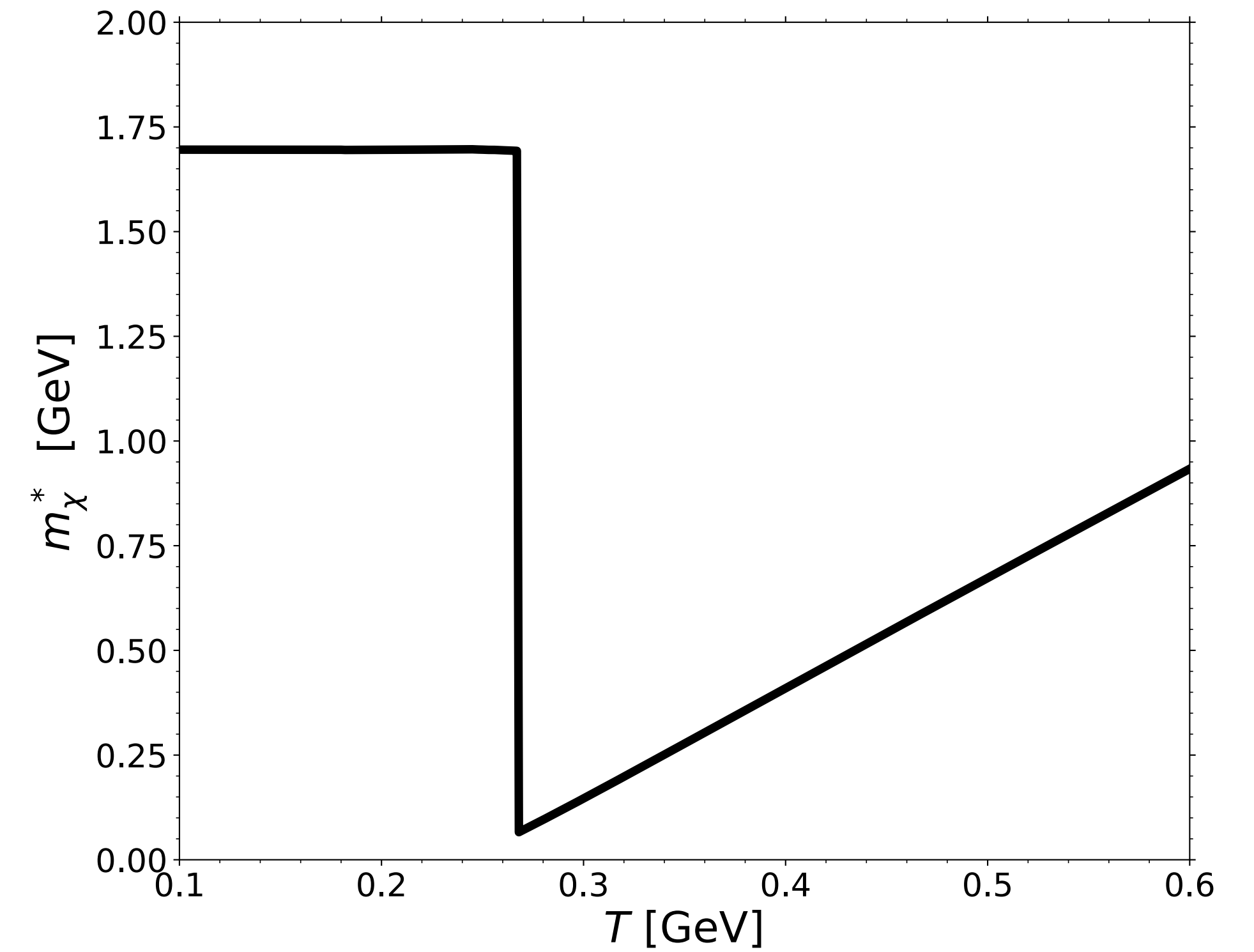
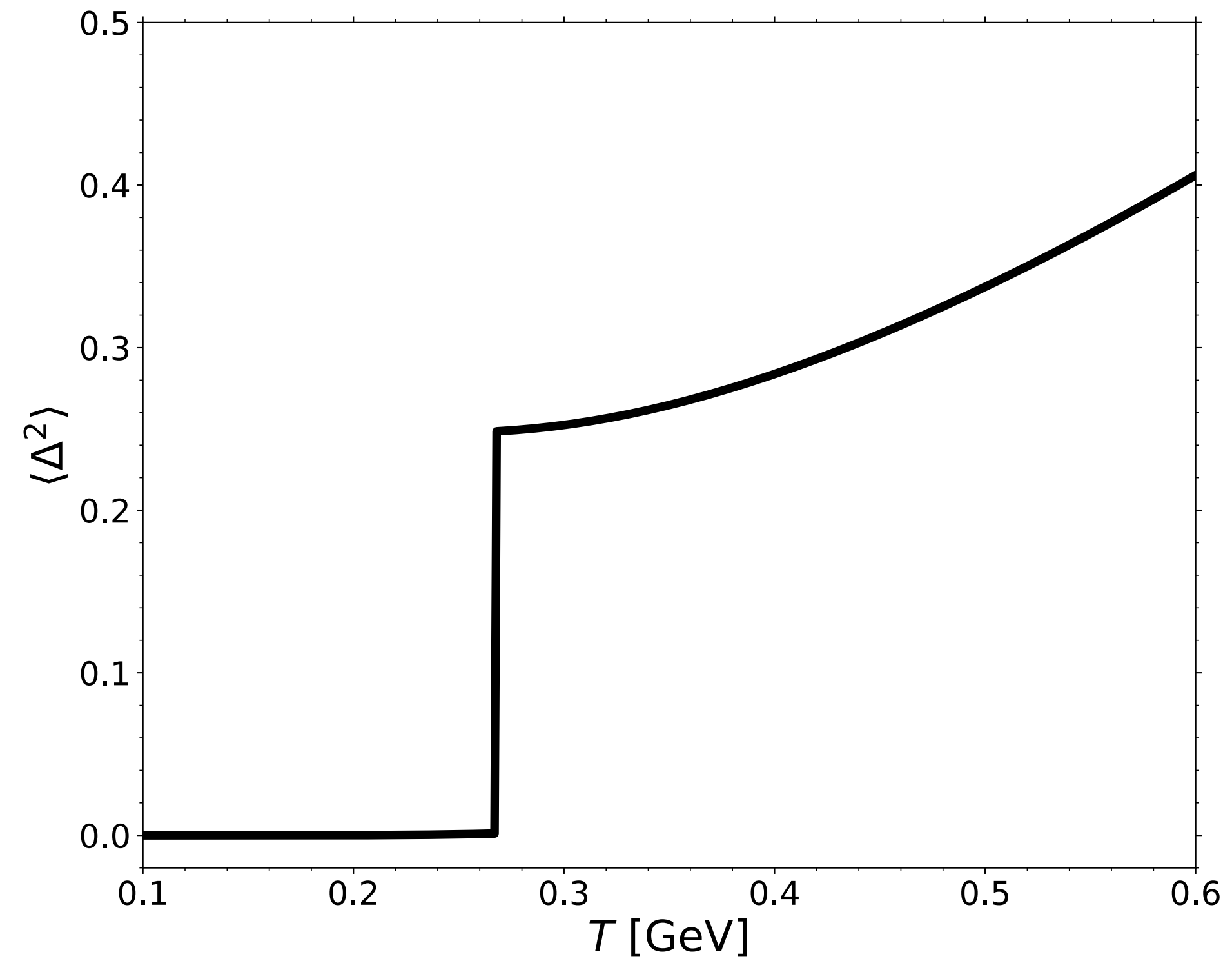
$$P_{\text{q-free}}(\chi, T) = -2(N_c^2 - 1)T \int \frac{d^3k}{(2\pi)^3} \ln(1 - e^{-k/T}) \Theta(k - K(\chi))$$

$$K(\chi) = \frac{A}{(1 - \chi)}$$

$$g^2(T) = \frac{48\pi^2}{11N_c \ln\left(\frac{T^2 + S^2}{\Lambda^2}\right)}$$

$$P_{\text{int}}(\chi, T) = g^2 N_c (N_c^2 - 1) \left\{ (-3) \left[\int \frac{d^3k}{(2\pi)^3} \frac{1}{k} N_B\left(\frac{k}{T}\right) \Theta(k - K(\chi)) \right]^2 + \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{1}{k_1 k_2} N_B\left(\frac{k_1}{T}\right) N_B\left(\frac{k_2}{T}\right) \right. \\ \left. \times \Theta(k_1 - K(\chi)) \Theta(k_2 - K(\chi)) \times \left[\frac{9}{4} \Theta(|\mathbf{k}_1 + \mathbf{k}_2| - K(\chi)) - \frac{1}{4} \Theta(|\mathbf{k}_1 - \mathbf{k}_2| - K(\chi)) \right] \right\}$$

Pure gauge $SU(3)_c$ sector



MCMC and Bayesian analysis

- If the set of parameters predicts at least one of the NEP to be out of the range given, then the prior probability is $p_{\text{prior}} = 0$

$$\log p_{\text{MCMC}} = -\frac{1}{2} \sum_{i=1}^N \left(\frac{E(\text{model}, n_i) - E(\chi\text{EFT}, n_i)}{\Delta E(\chi\text{EFT}, n_i)} \right)$$

i runs over the densities n_i , and where $E(\chi\text{EFT}, n_i)$ and $\Delta E(\chi\text{EFT}, n_i)$ represent the centroids and uncertainties of the χEFT .

- In case the RMF model is inside the χEFT band, the probability becomes constant: if $\log p_{\text{MCMC}} \geq -N/2$, then $\log p_{\text{MCMC}} = -N/2$.

NEPs	E_{sat}	n_{sat}	K_{sat}	$E_{\text{sym},2}$	L_{sym}	$m_{D,\text{sat}}^*$
unif.dist.	$[-16.4, -15.2]$	$[0.145, 0.165]$	$[180, 280]$	$[28, 36]$	$[30, 90]$	$[0.45, 1.05]$

RMF predictions for the dense-matter EOS and application to NSs

$$\begin{aligned}\mathcal{L} = & \sum_N \bar{\psi}_N (i\gamma_\mu \partial^\mu - m_N + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu \\ & - \frac{1}{2} g_\rho(n_b) \gamma_\mu \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu) \psi_N + \frac{1}{2} (\partial_\mu \sigma \partial^\mu - m_\sigma^2 \sigma^2) \\ & - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} \\ & - \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{3} b m_N (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4\end{aligned}$$

$$g_\rho(n_b) = g_\rho(n_{\text{sat}}) \exp \left[-a_\rho \left(\frac{n_b}{n_{\text{sat}}} - 1 \right) \right]$$

The density dependence of g_ρ introduces a rearrangement contribution Σ_r to the nucleon self-energies:

$$\Sigma_r = -\frac{1}{2} a_\rho g_\rho(n_b) \frac{n_p - n_n}{n_{\text{sat}}} \rho_{03}.$$

Color-dielectric model

$\mathcal{L}$ describes a system of interacting quarks, pions, sigmas and a scalar-isoscalar chiral singlet field χ , whose potential $U(\chi)$ has an absolute minimum for $\chi = 0$. Thus for the single nucleon problem, the effective quark mass $-g\sigma/\chi$ diverges outside the nucleon.

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{g}{\chi}\bar{\psi}(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})\psi + \frac{1}{2}(\partial_\mu\chi)^2 - U(\chi) + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - W(\sigma, \vec{\pi})$$

$$U(\chi) = \frac{1}{2}M^2\chi^2$$

$$W(\sigma, \vec{\pi}) = k \cdot (\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2$$

Strange quark matter

