Axion Bounds from Supernovae (and their interplay with colliders)

Ludovico Vittorio (Sapienza University of Rome & INFN, Sezione di Roma)

Light Dark Sectors at LNF (LDS@LNF) Workshop – November the 28th 2025

Mainly (but not only) based on works with M.Cavan-Piton, D.Guadagnoli et al.
[PRL '24 (2401.10979), JHEP '25 (2411.04170), JHEP '25 (2503.17490)]

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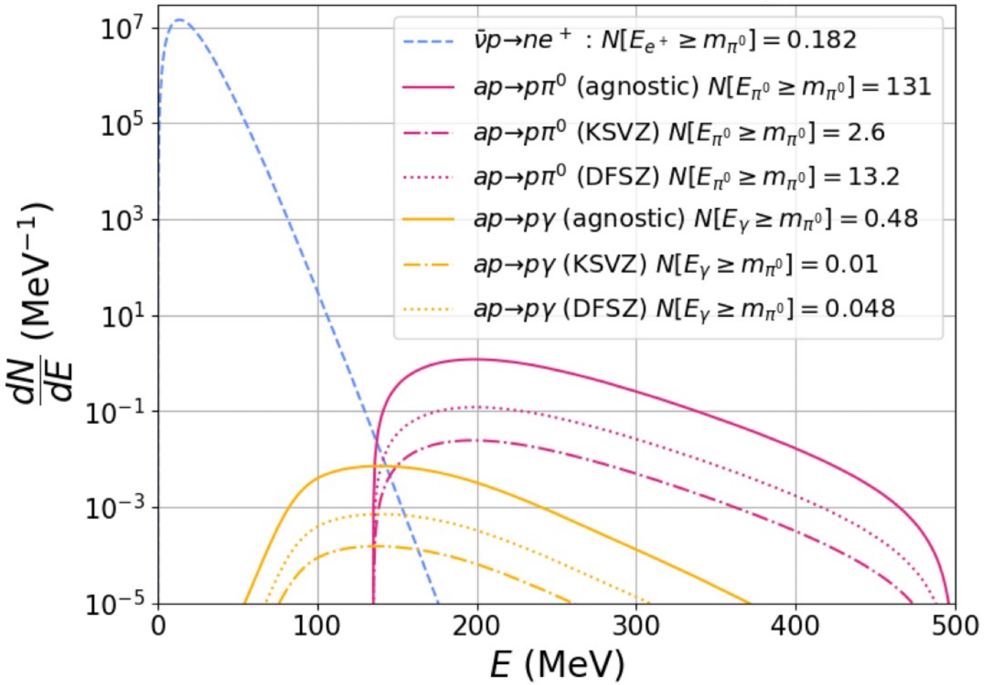

The QCD-axion

Axions are among the best motivated, most sought-after particles beyond the SM :

- *Elegant solution to the strong CP problem:*

$$\delta\mathcal{L}_{\rm QCD} = \theta\frac{g_s^2}{32\pi^2}G\tilde{G} \qquad |\theta| \lesssim 10^{-10}$$

θ is promoted to a dynamical field

$$\theta \rightarrow \frac{a}{f_a} \quad \textit{with} \quad \langle a \rangle = 0$$

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$$m_a = 5.691(51)\,\mu\text{eV}\,\frac{10^{12}\ \text{GeV}}{f_a}$$

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- *Excellent DM candidate*

J. Preskill, M. B. Wise and F. Wilczek, PRL 1983
L. F. Abbott and P. Sikivie, PLB 1983

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Di Luzio, Giannotti, Nardi, Visinielli, Phys.Rept. 870 (2020) 1 [2003.01100]

$$\mathcal{L}_a^{\rm int} \supset \frac{1}{4} g_{a\gamma} a F\tilde{F} + g_{af} \frac{\partial_\mu a}{2m_f} \bar{f}\gamma^\mu\gamma_5 f - \frac{i}{2} g_d\, a\, \bar{n}\sigma_{\mu\nu}\gamma_5 n F^{\mu\nu}$$

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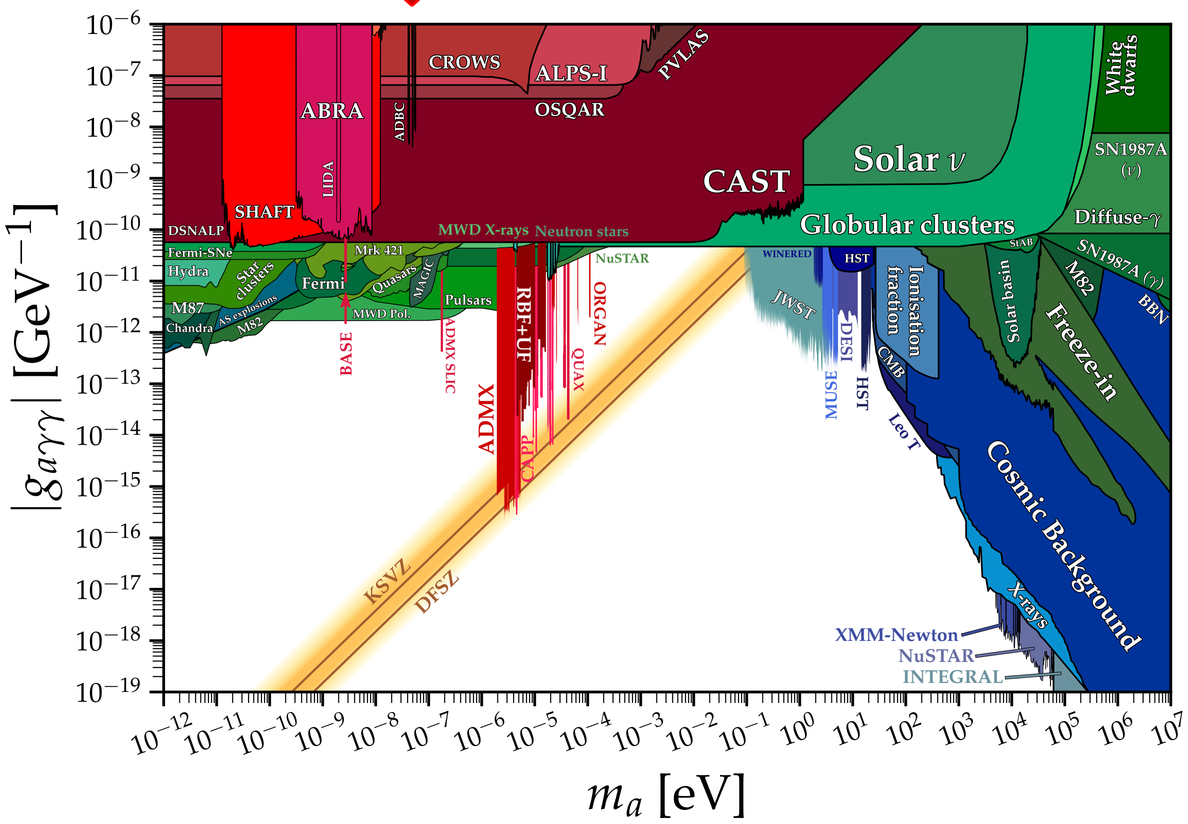

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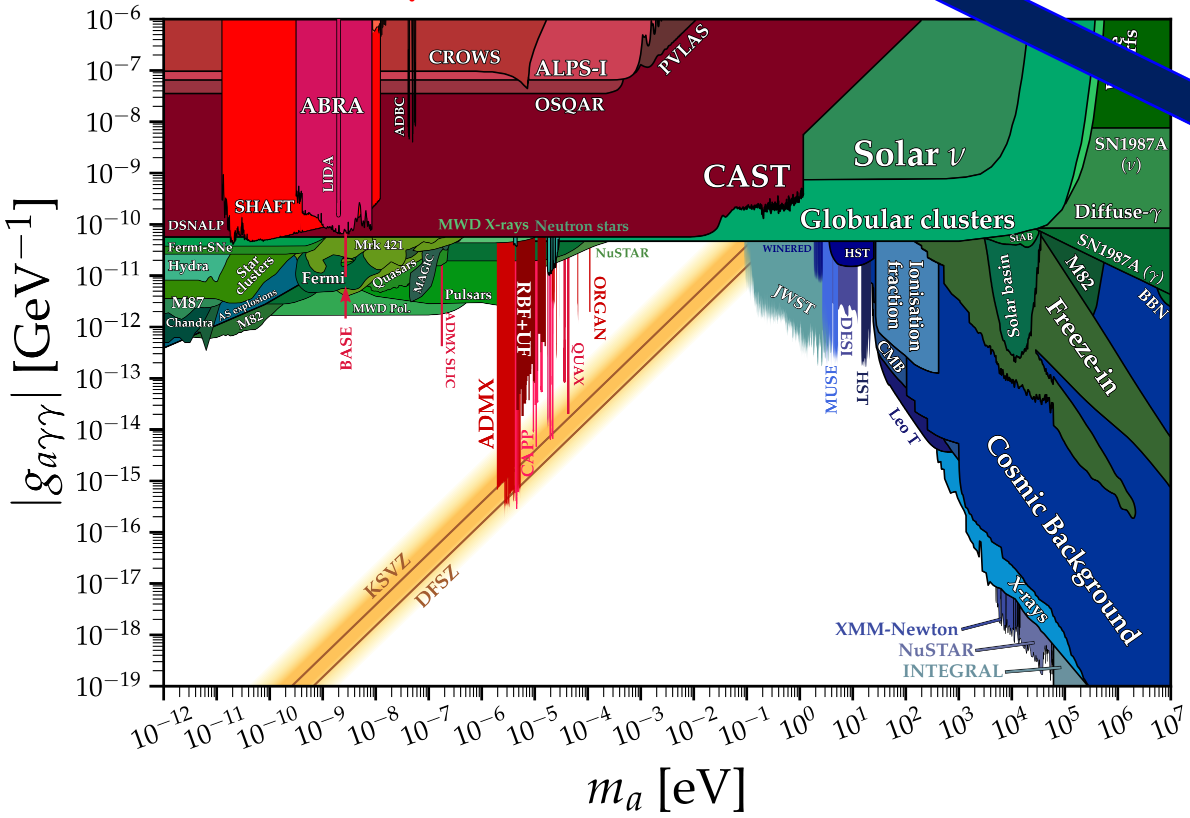
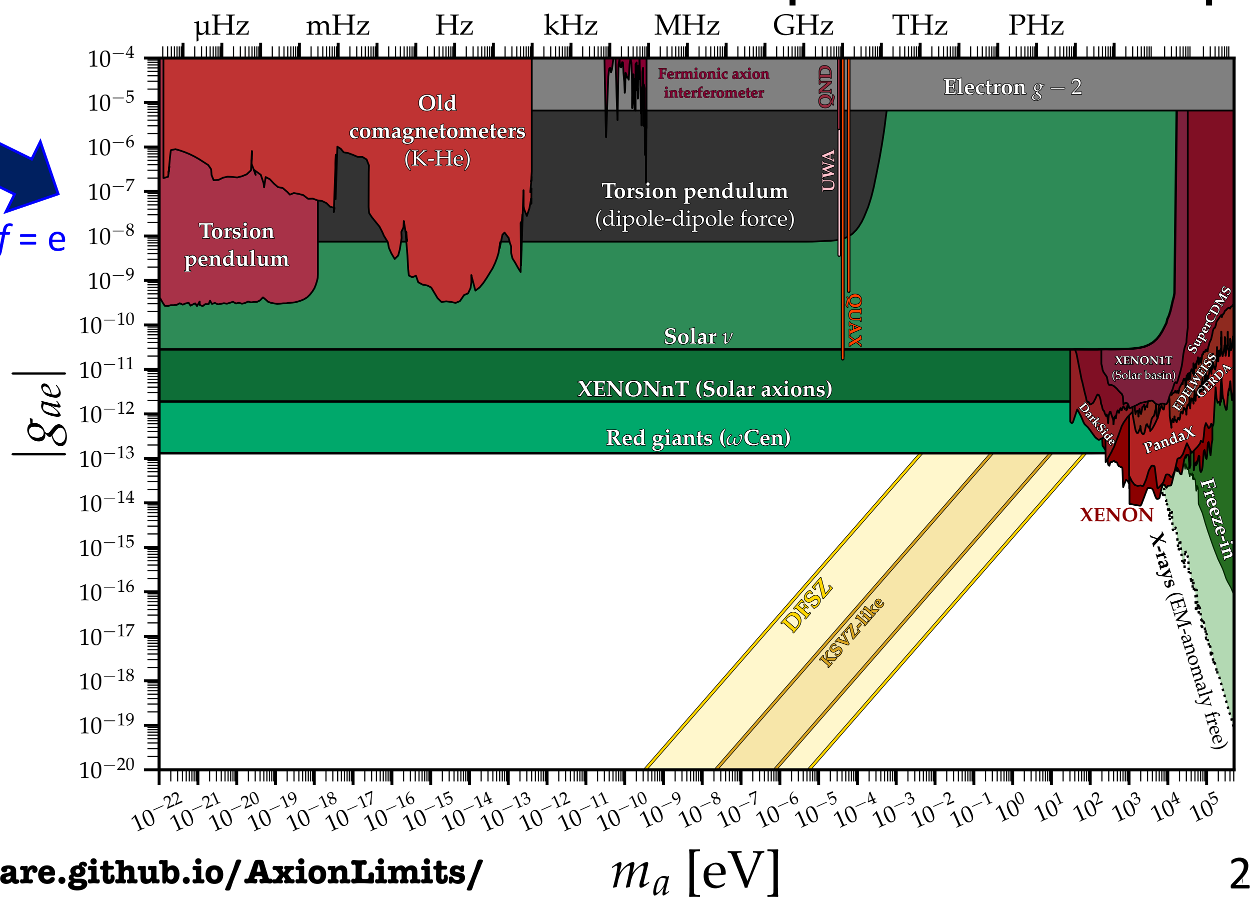


for f = e



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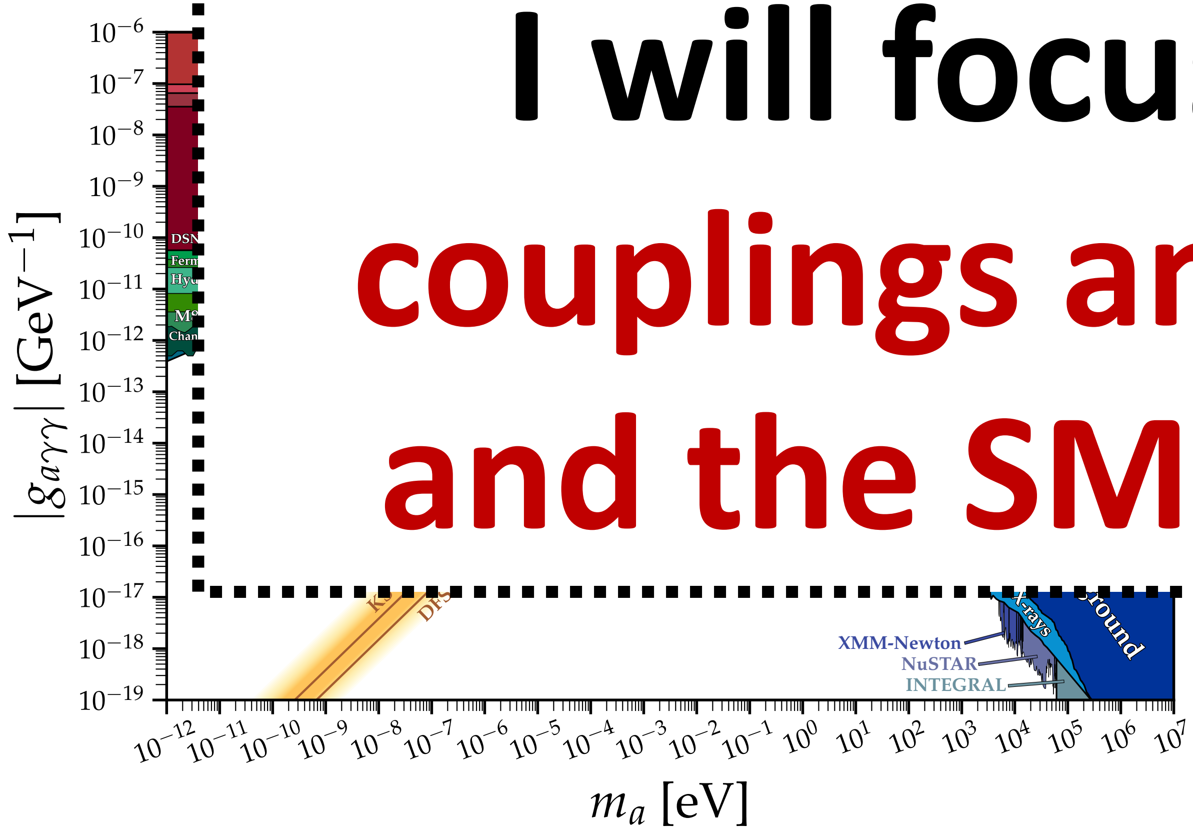
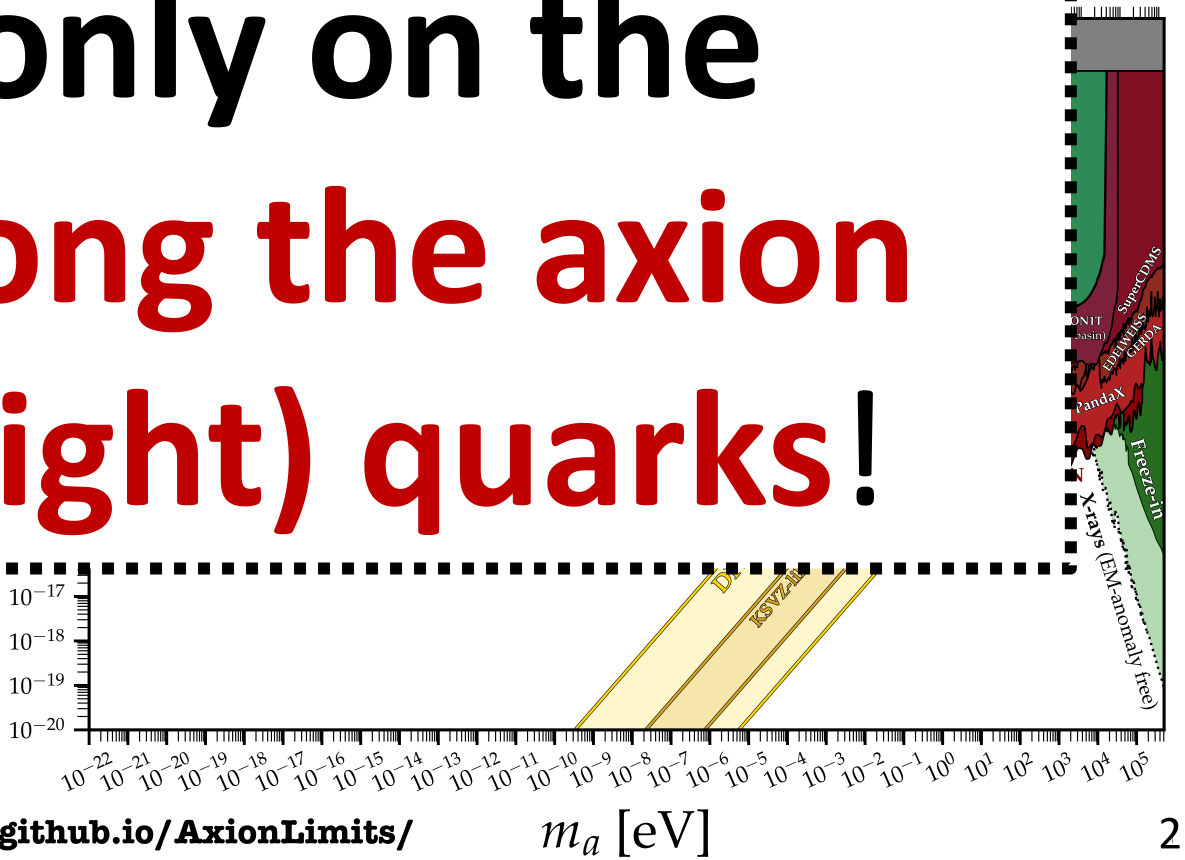
$\mathcal{L}_a^{\text{in}}$

For the purposes of this talk:

I will focus only on the couplings among the axion and the SM (light) quarks!





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$$q = (u, d, s)^T$$

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- $x^b_{R,L}(k_{R,L})$: axion-hadron couplings parametrized in terms of the fundamental k-couplings
- U: meson-octet field
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axion-hadron couplings parametrized in terms of the fundamental k-couplings

meson-octet field

baryon-octet field

In short: ChPT augmented with an axion!

Axion-quark couplings

In short: the axion-hadron dynamics can be parametrized in terms of the fundamental *k*-couplings (i.e. axion-quark couplings). In general, we have ten parameters to deal with:

$$(k_{\mathrm{V,A}})_{11,22,33,23,32}$$

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Seven parameters left !!

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In order to obtain interesting contraints on QCD axion's couplings with light generations of quarks, we are going to analyze two different probes in what follows:

- Exotic sources of cooling of core-collapse SuperNovae (SNe)

Constraints from axion emission ***Constraints from axion absorption @ Cherenkov exps.***

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- Three-body Kaon decays into final states w/ an axion → $K_S \to \pi\pi a,\ K_S \to \mu\mu a$

1. Axion bounds from Supernovae

Core-collapse supernovae

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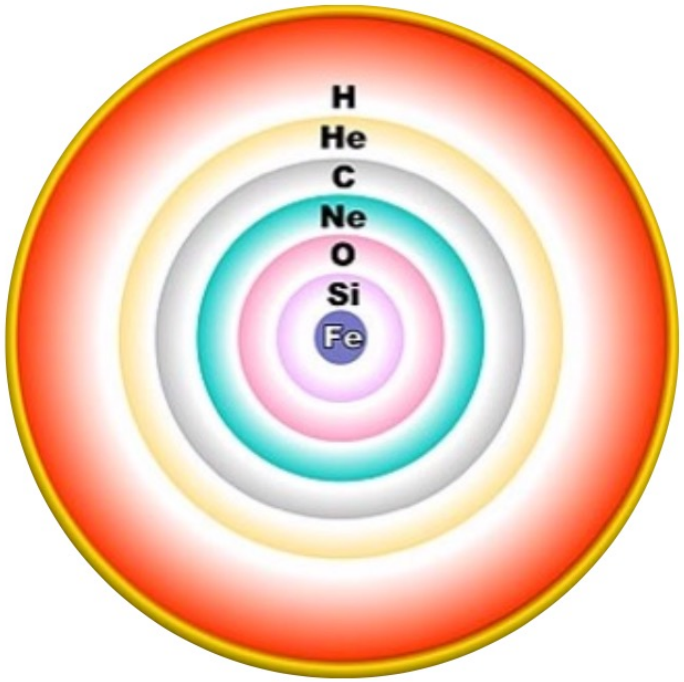

Sketch of the "onion-like" layers of a quite massive star

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The medium starts to become instable due to two reactions:

- **Photodissociation of iron** $\gamma + {}^{56}\mathrm{Fe} \rightarrow 13\alpha + 4\mathrm{n}$
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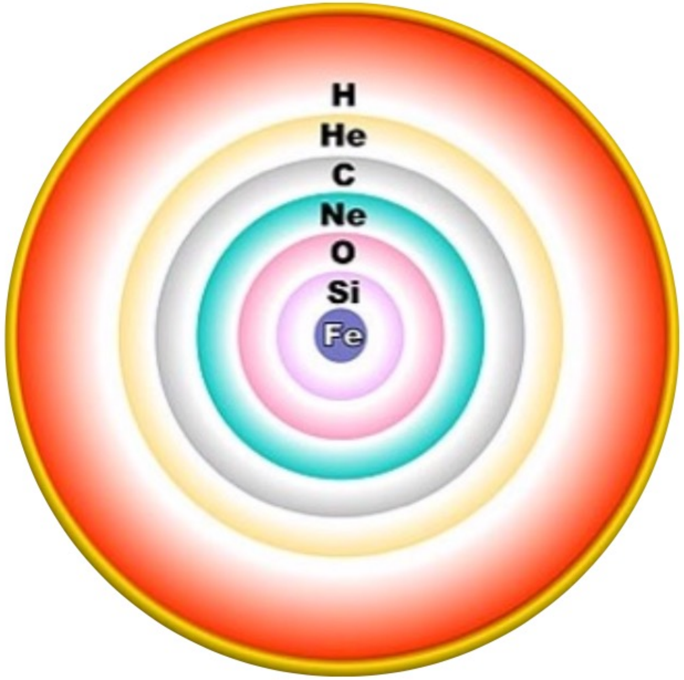

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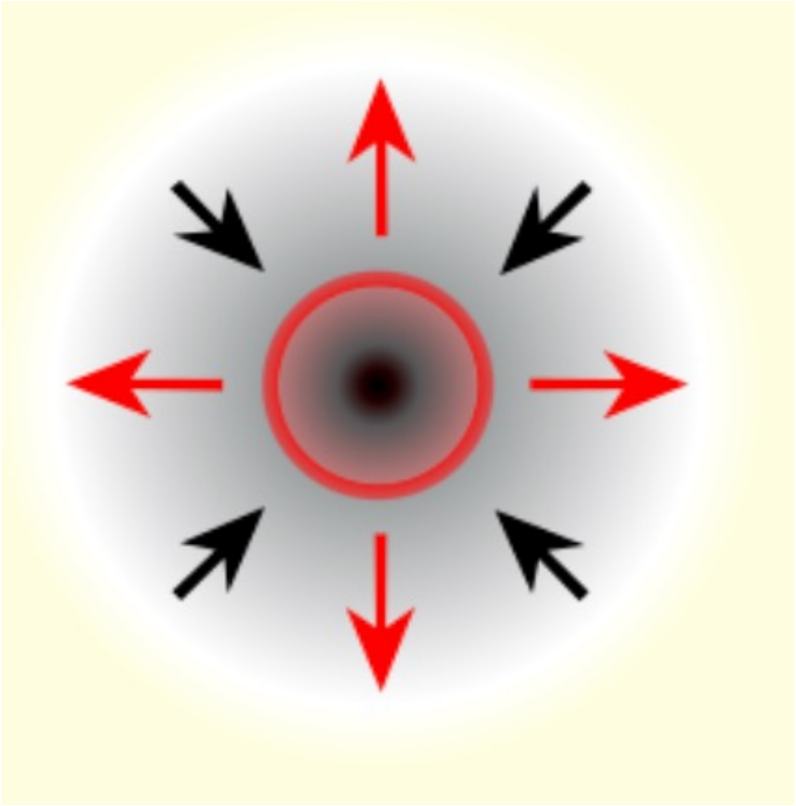

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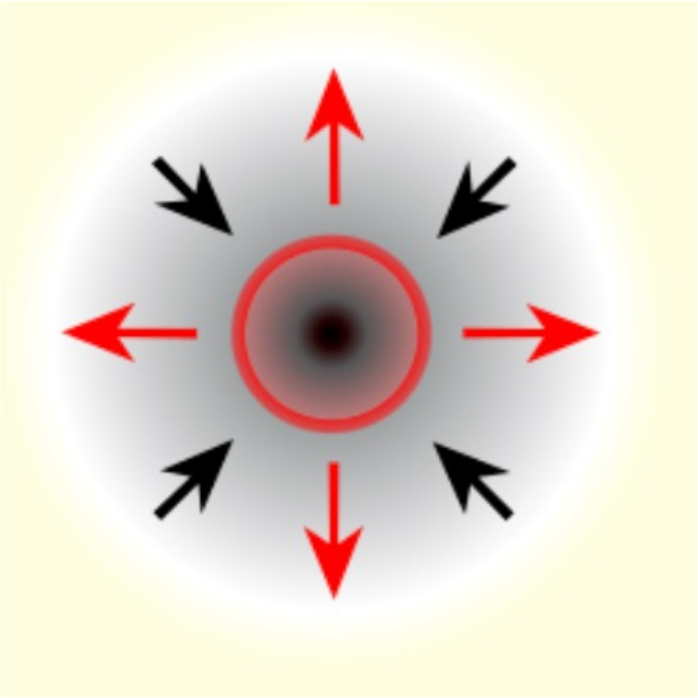
Bounce of infalling matter and formation of a shock-wave

Core-collapse supernovae

This shock wave moves outward, depositing energy and thus dissociating the nuclei of the medium as it passes!

This effects induces a sudden decrease of the coherent neutrino cross-sections and, thus, to a break-out of the neutrino luminosity.



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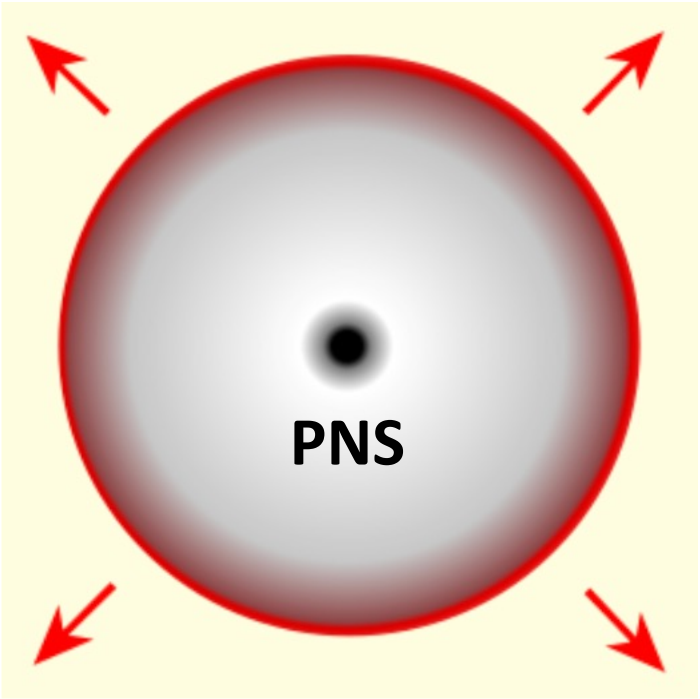

The shock reaches the edge of the iron core **after ~1s: a Proto Neutron Star (PNS) has formed, and about half of its binding energy, $E_b \sim 10^{53}$ erg, has already been emitted**.

In the following **~10s**, most of the remaining binding energy is radiated from the neutrino sphere: **PNS neutrino cooling**

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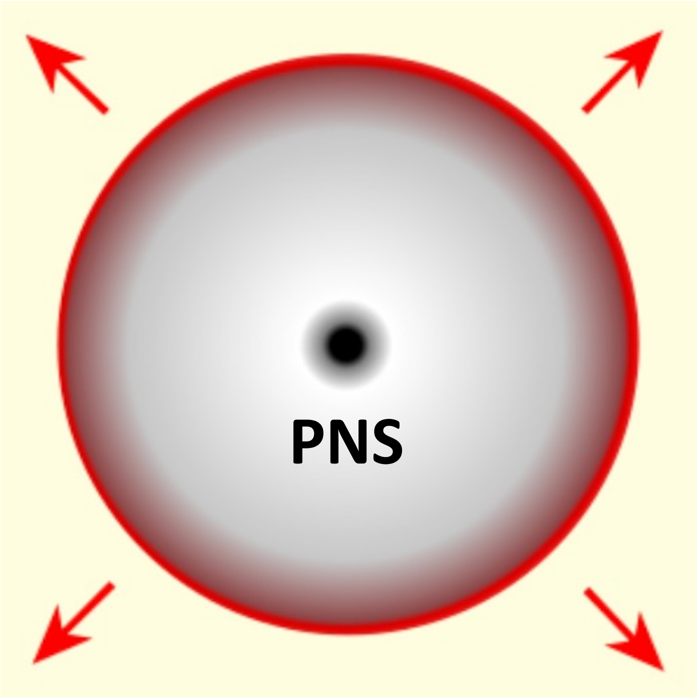

This picture is confirmed by the analysis of the data from the neutrino observations of SN1987A @ IMB, K-II, SNO

… and what about exotic sources of cooling ?

Following the argument in Raffelt, Phys.Rept. 198 (1990) 1-113

Let us assume that some BSM particle may be present within the SN: in particular, this BSM particle should be light enough to be thermally produced in the SN core and more weakly interacting than neutrinos.

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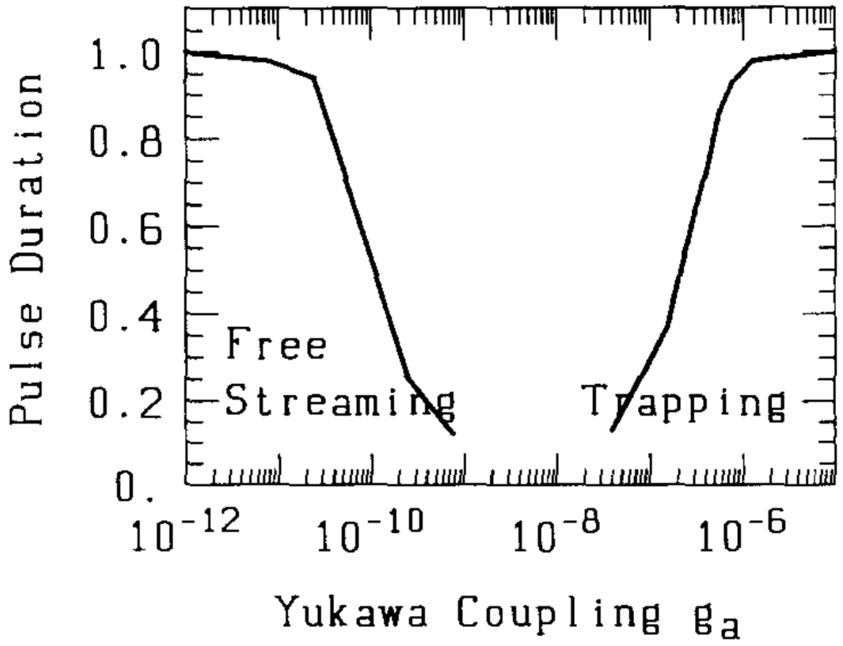
Novel forms of energy loss will mostly compete with neutrino cooling after the first burst, and will mostly shorten the "cooling tail" of the signal. Therefore, the main observable to constrain particle parameters is the **duration of the neutrino signal!**



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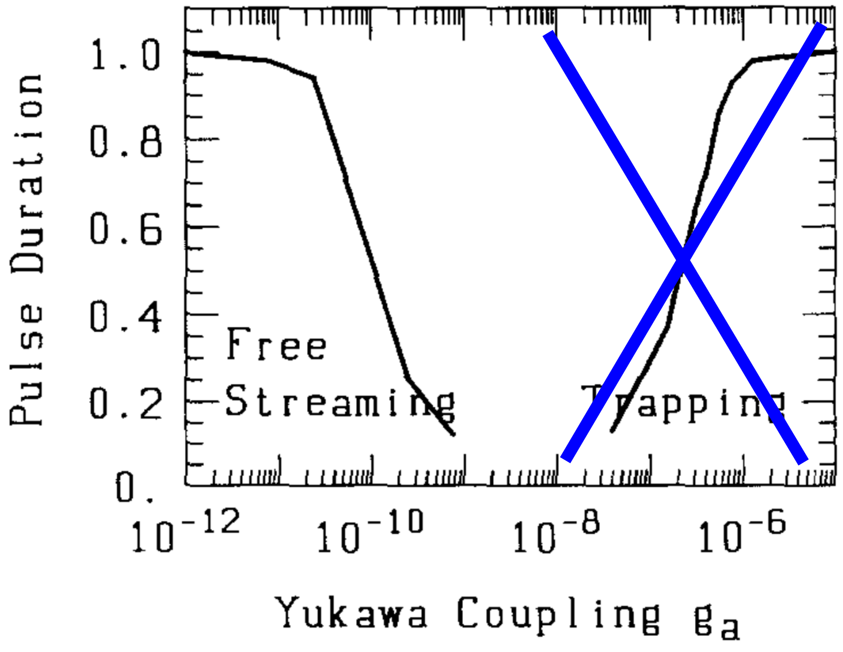
Trapping regime is excluded:

In case of strong couplings the ALP flux would have produced a signal in Kamiokande II.

But: no excess in the background of K-II around SN 1987A event!

Lella et al, PRD '24 [2306.01048]

Carenza et al., PRC '24 [2306.17055]



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Emission of axions from SNe

The neutrino burst associated to SN1987A strongly constrains exotic sources of cooling:
by defining the emissivity Q_i as the power radiated in the particle *i* per unit volume we have

$$Q_a < Q_\nu$$

Raffelt, Phys.Rept. 198 (1990) 1-113

Emission of axions from SNe

The neutrino burst associated to SN1987A strongly constrains exotic sources of cooling:
by defining the emissivity Q_i as the power radiated in the particle *i* per unit volume we have

$$Q_a < Q_\nu$$

Raffelt, Phys.Rept. 198 (1990) 1-113

For what concerns Q_ν, it is difficult to go beyond a crude estimate :

$$Q_\nu \sim L_\nu \frac{\rho}{M} \sim 3 \times 10^{33}\ \mathrm{erg.s^{-1}.cm^{-3}}$$

where $\rho \sim \rho_{core} \sim 3 - 8 \times 10^{14}\,\mathrm{g/cm^3},\ M \sim M_\odot.$

Emission of axions from SNe

***Q_a* is calculable as :**

$$Q_a = \int \begin{bmatrix} elem.\ of \\ phase\ space \end{bmatrix} \times E_a \times \begin{vmatrix} matrix\ elem.\ of \\ axion - producing \\ process \end{vmatrix}^2 \times \begin{bmatrix} ext.\ states' \\ distr.\ function \end{bmatrix}$$

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This computation can be carried out within assumptions :

- Distribution functions not obvious away from ideal-gas assumption
- Inclusion of in-medium effects: use of effective quantities as in mean-field approach

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The comparion between the neutrino emissivity and the axion emissivity leads to bounds on axion coupling with matter

Thermodynamics

For what concerns **modelling of the SN volume affected by axion emission** : assuming weak equilibrium for the interactions that involve the strange quark, and neglecting the presence of muons, the state of matter is characterised by three thermodynamic parameters :

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$$T = \{30, 40\}\ \mathrm{MeV};\ n_B = \{1, 1.5\}\, n_{\mathrm{sat}},\ \ n_{\mathrm{sat}} = 1.6 \times 10^{38}\,\mathrm{cm}^{-3}$$

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On top of the above thermodynamic-parameter choices, **we also consider two equation-of-state (EoS) models consistently containing the full baryonic octet:**

- DD2Y [Marquez et al. PRC '17 (1706.02913)] **- SFHoY [Fortin et al. PASA '18 (1711.09427)]**

This choice is guided by the following arguments among the others:

i) these models specifically predict different amounts of strangeness inside hot and dense matter;

ii) they are compatible with astro- and nuclear-physics constraints and they are publicly available;

Relevant processes for axion production

Most established bounds obtained from **nucleon axion-strahlung** :

$$N_1 N_2 \to N_3 N_4 a$$

See the discussion in Caputo, Raffelt, PoS COSMICWISPers (2024) 041 [2401.13728]

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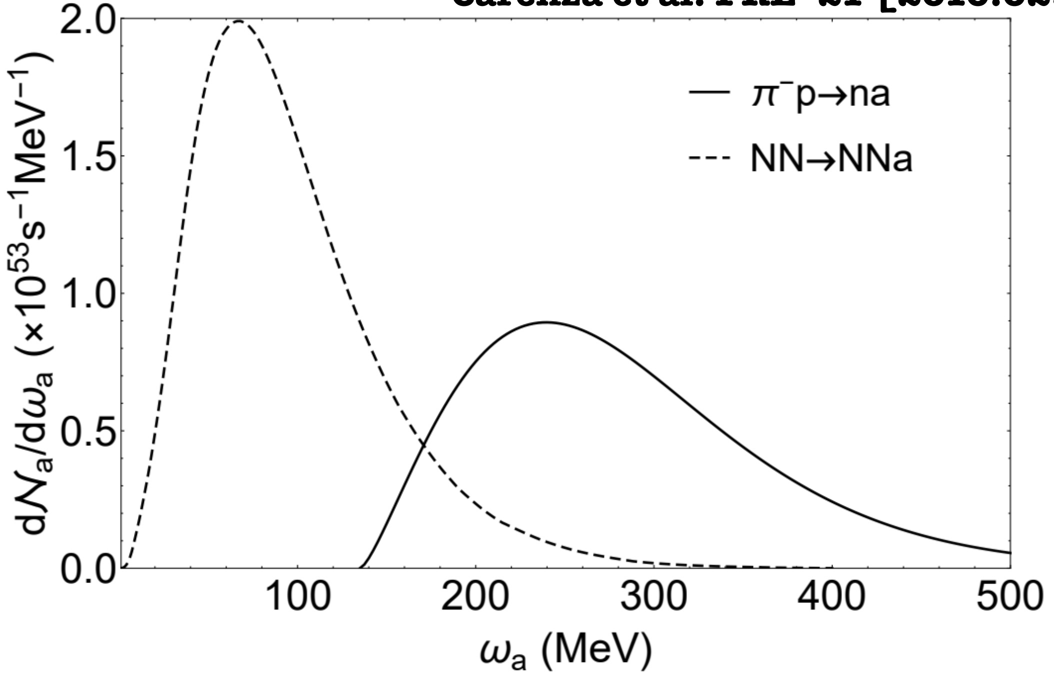
Carenza et al. PRL '21 [2010.02943]



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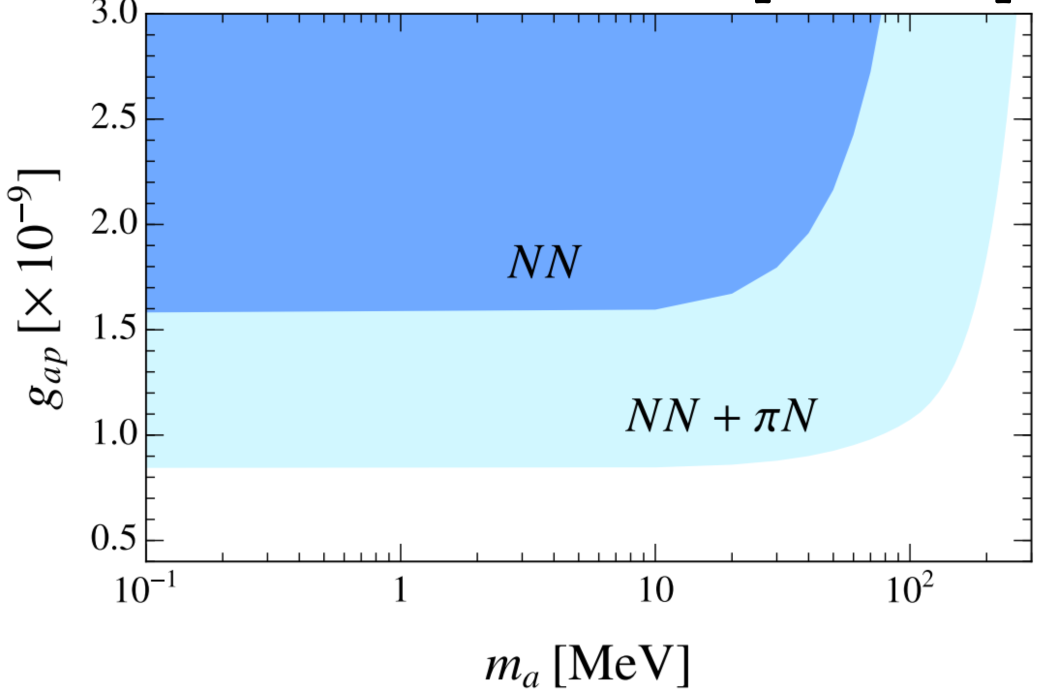

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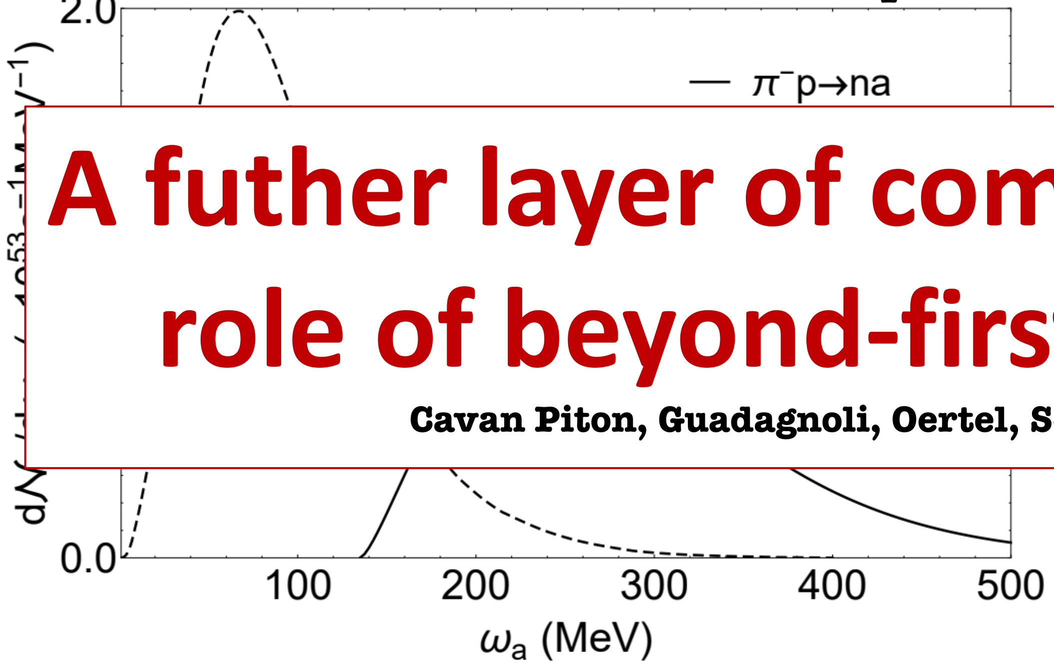

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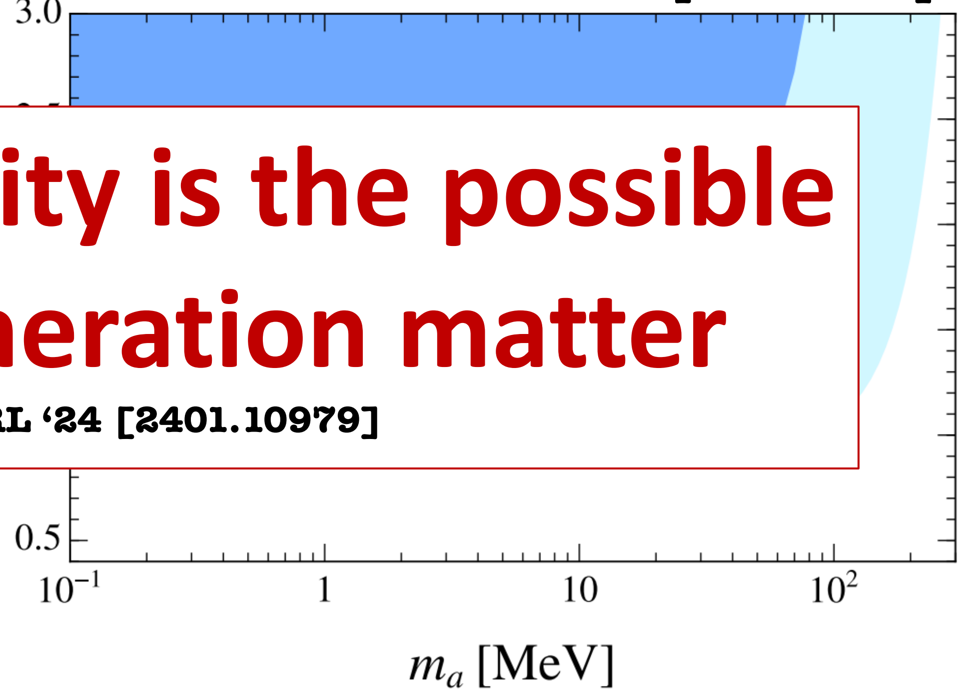

A futher layer of complexity is the possible role of beyond-first-generation matter

Cavan Piton, Guadagnoli, Oertel, Seong, LV, PRL '24 [2401.10979]

What we do in short

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Two key observations :

1. ***Q_a* is by construction positive definite**
2. **Even if the fractions of B_i, B_f and M are «small», the large number of processes yields a relevant constraint**

Results of axion emission with strange matter

Cavan Piton, Guadagnoli, Oertel, Seong, LV, PRL '24 [2401.10979]

Noting that:

- Q_a is less stringent than $K^+ \to \pi^+ a$ to constrain $|(k_V)_{23}|$
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We are left with four parameters: $(k_A)_{11,22,33}$ and $|(k_A)_{23}|$

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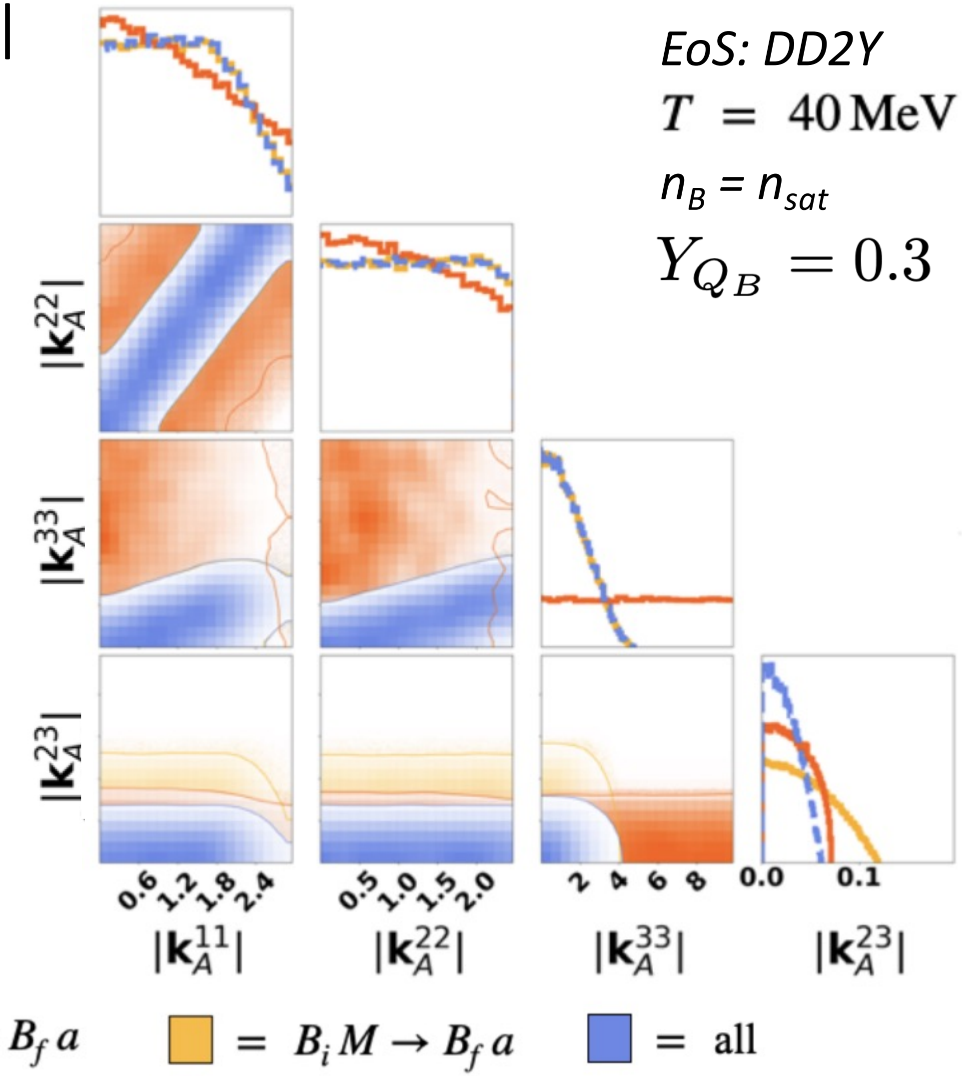

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MAIN FINDINGS :

(a) Strong correlations among fl.-univ. couplings (more pronounced for higher T)

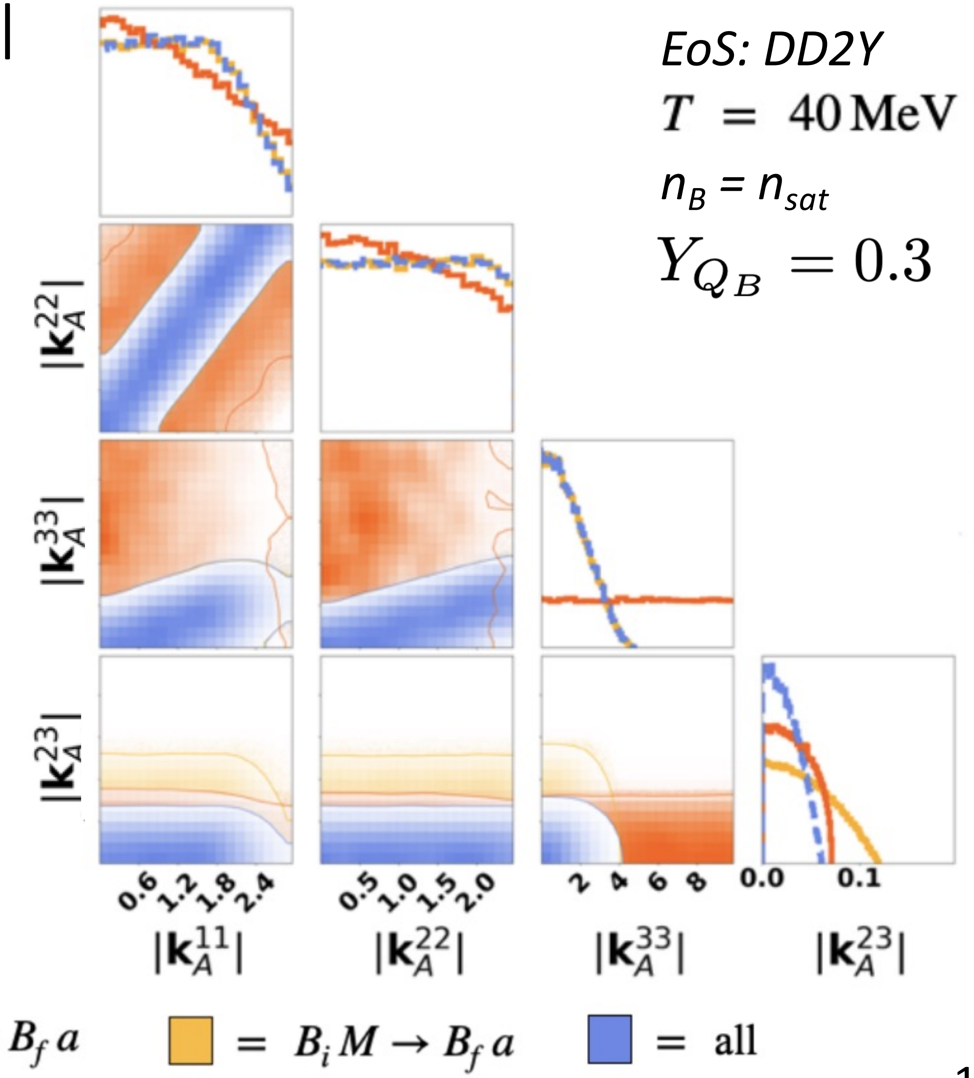
(b) Novel contraint on $|(k_A)_{23}|$, of $O(10^{-1}\text{-}10^{-2})$ (depending on EoS, T value …)

EoS: DD2Y

$T = 40\,\text{MeV}$

$n_B = n_{sat}$

$Y_{Q_B} = 0.3$



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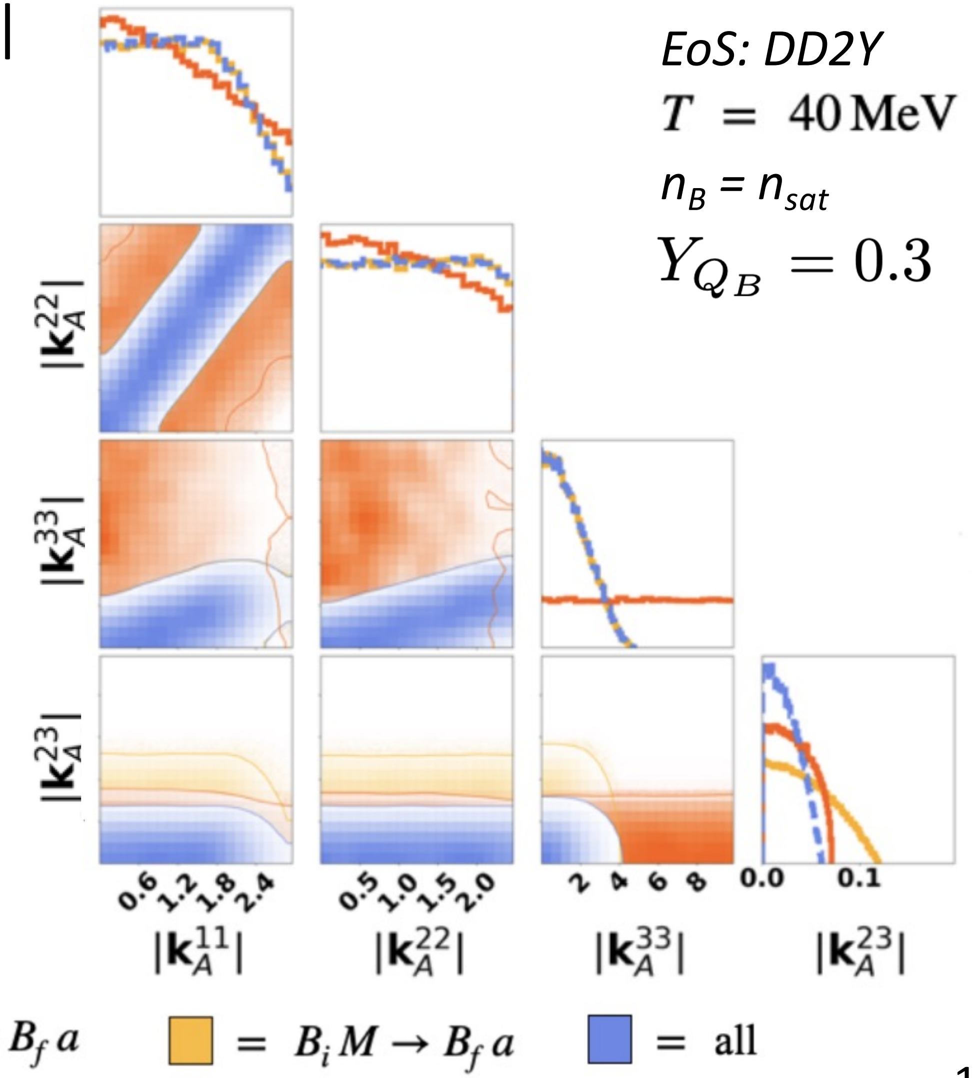

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Stronger than flavour bounds! *But …*

Other constrains on FV couplings:
Axion bounds from 3-body K decays

What about rare decays at colliders ?

Clear advantage : no intrinsic sources of uncertainty in the bounds on axion's couplings we will obtain!

Martin Camalich et al., PRD '20 [2002.04623]
(interesting updates in 2503.17323)

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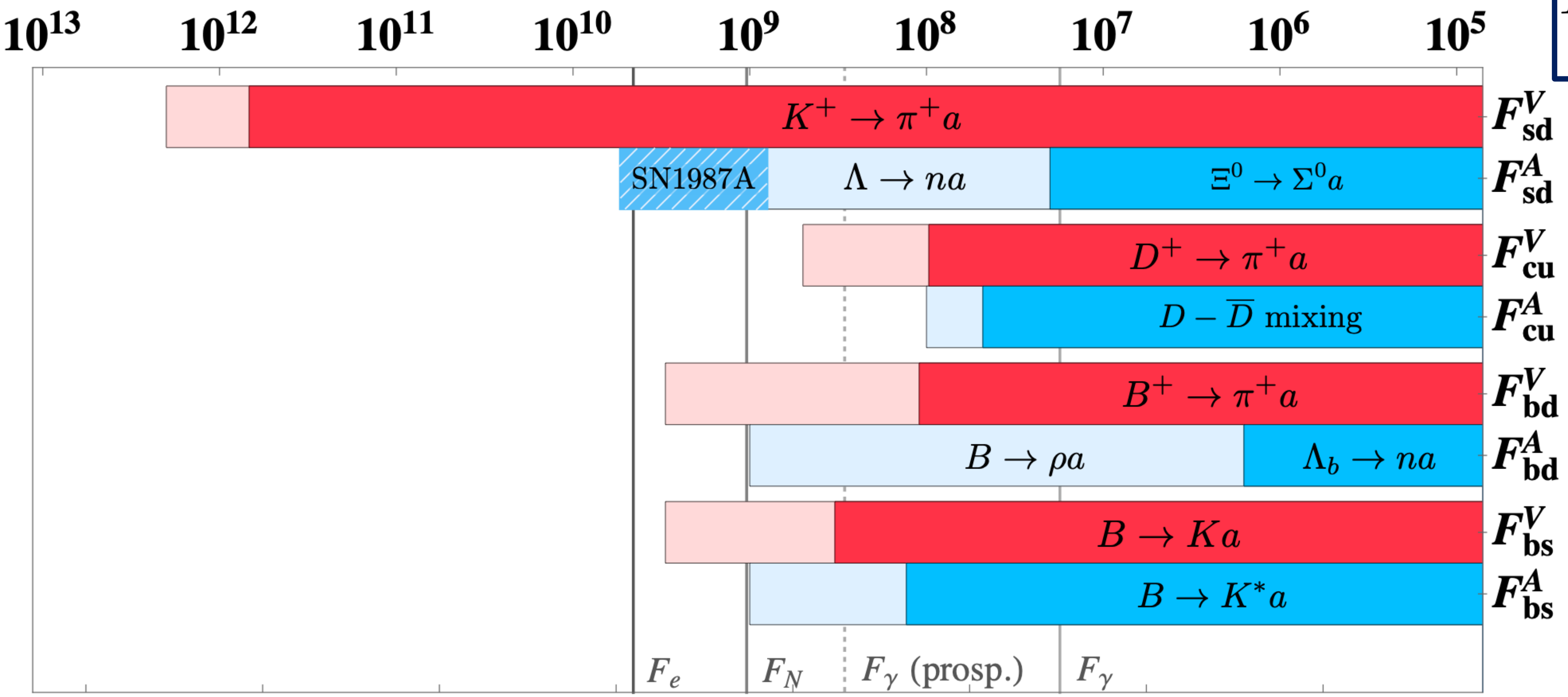

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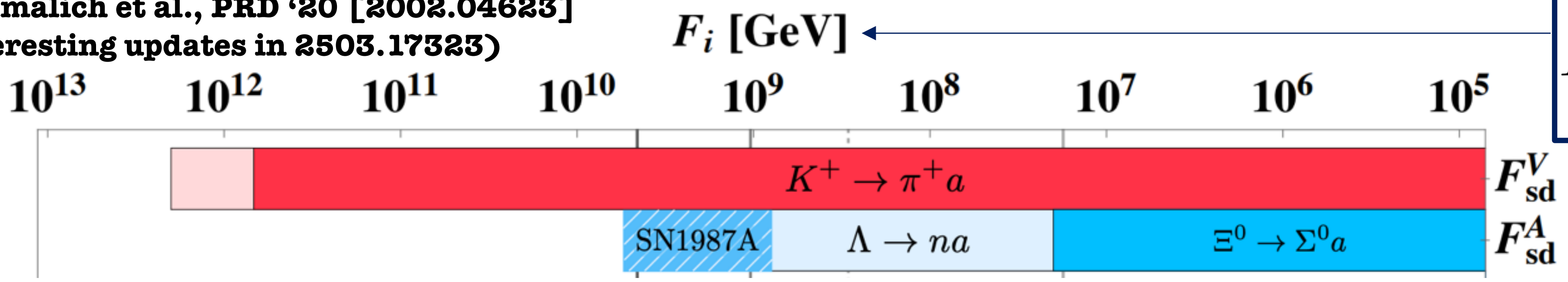

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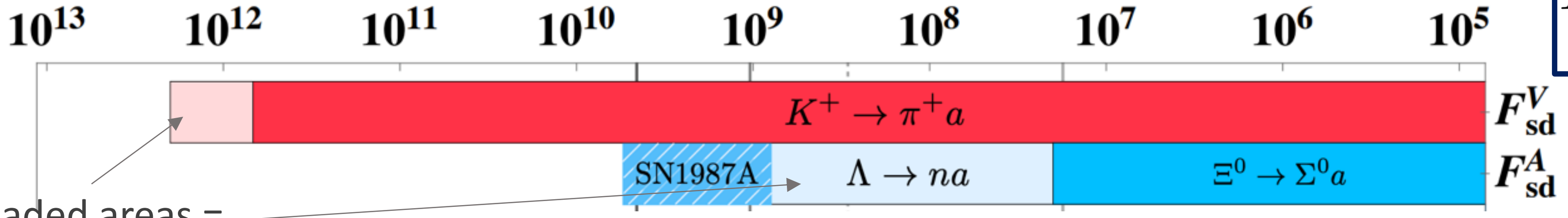

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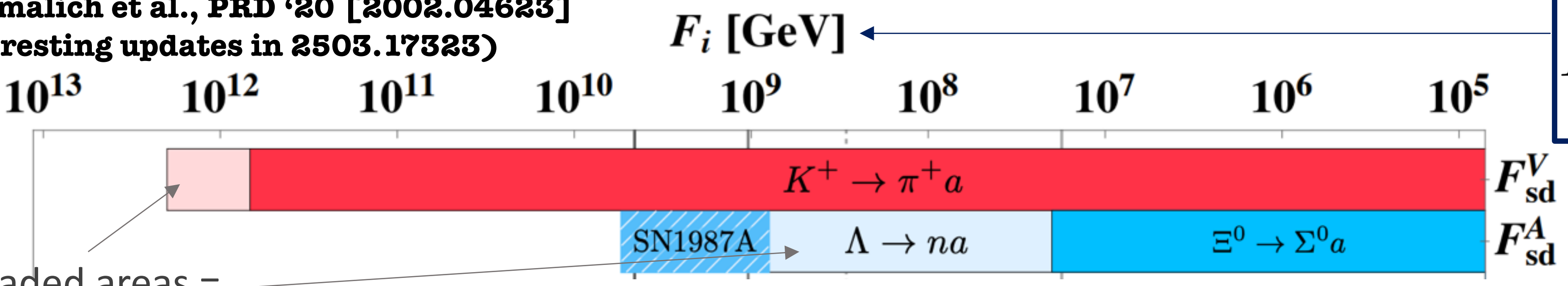

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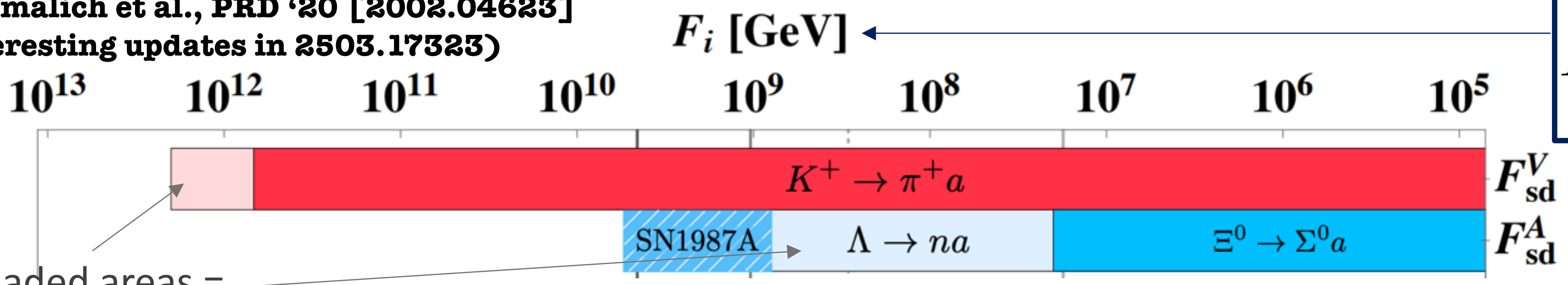

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... can we still obtain a bound on the axial-vector $(k_A)_{23}$ coupling from K decays ?

Three-body kaon decays into final states w/ an axion

Yes: K → $\pi\pi$a (and also K → $\mu\mu$a) transitions constrain only $(k_A)_{23}$ by parity conservation

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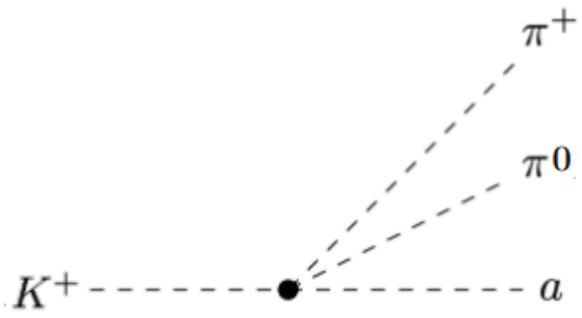

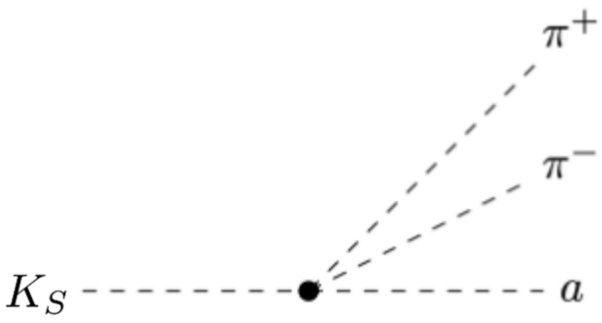

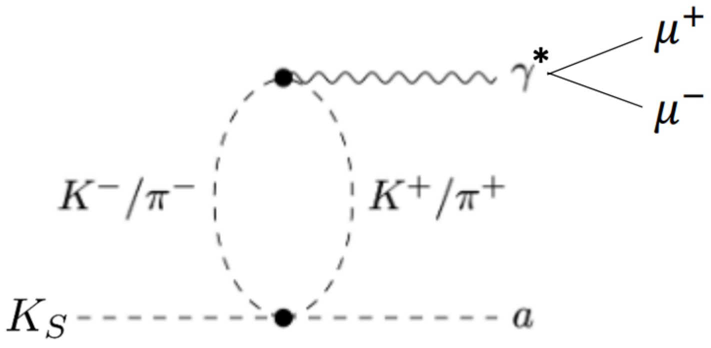

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(Both) relevant for LHCb

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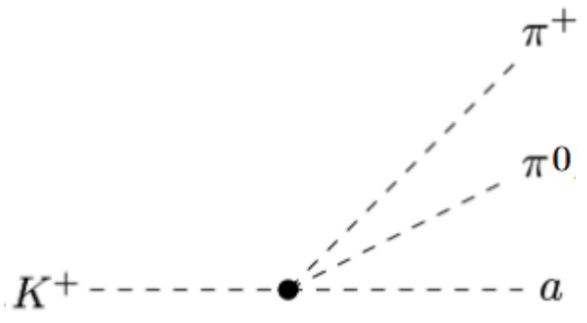

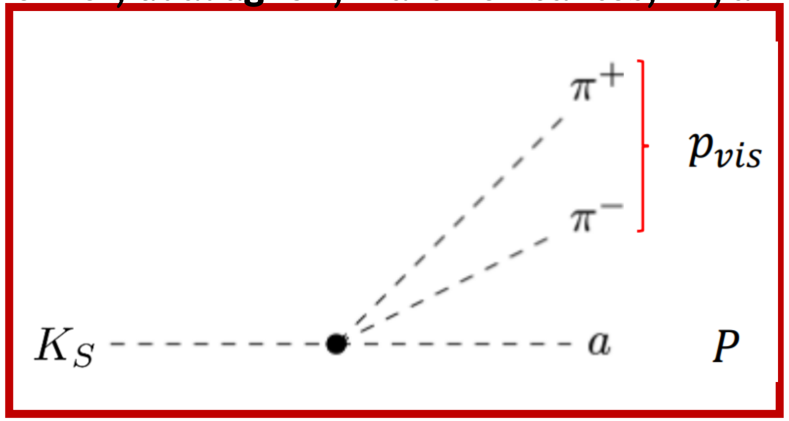

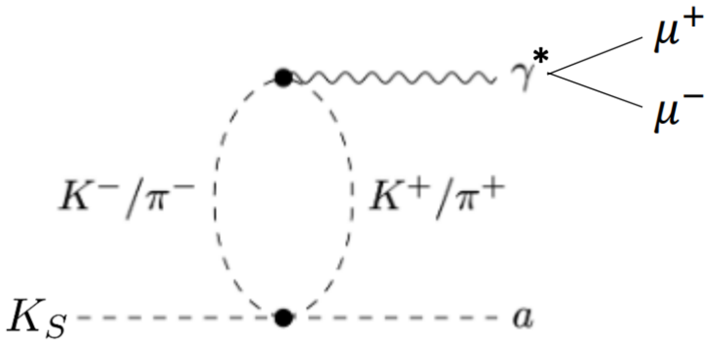

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In an actual exp. search, the **signal** is a reconstructed hh momentum (p_{vis}) + a missing momentum (P). By momentum conservation

$$(p_{vis} + P)^2 = \mathrm{m}_\mathrm{K}^2$$

Pending an actual search, we identify the dominant backgrounds to infer the signals' sensitivities

Three-body kaon decays into final states w/ an axion

Cavan Piton, Iohner, Guadagnoli, Martinez-Santos, LV, JHEP '25 (2411.04170)

We use the **sensitivity S_{eff}** to estimate of the actual BR-limit achievable through a dedicated search, where

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Since the QCD-axion can be considered (for the purposes of this discussion) as massless, one can identify the photon-like backgrounds and, then, plug them into the above formula.

Focusing e.g. on $K^+ \to \pi^+\pi^0 a$: the backgrounds are

$$\mathcal{B}(K^+ \to \pi^+\pi^0\gamma) = (6.0 \pm 0.4) \times 10^{-6}\,, \qquad \mathcal{B}(K^+ \to \mu^+\pi^0\nu) = (3.352 \pm 0.034) \times 10^{-2}$$

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$(k_A)_{23}$ **is here!**

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One can derive the **theoretical BR from** the expression (obtained in Chiral Perturbation Theory + axion):

$$\frac{d^2\Gamma(K^+ \to \pi^+\pi^0 a)}{dsdt} = 3m_K \frac{\omega_\pi^2}{\eta_\pi}\left(\frac{|(k_A)_{23}|}{f_a}\right)^2$$

$$\omega_\pi \equiv m_a^2 + m_K^2 + 2m_\pi^2 - s - 2t$$

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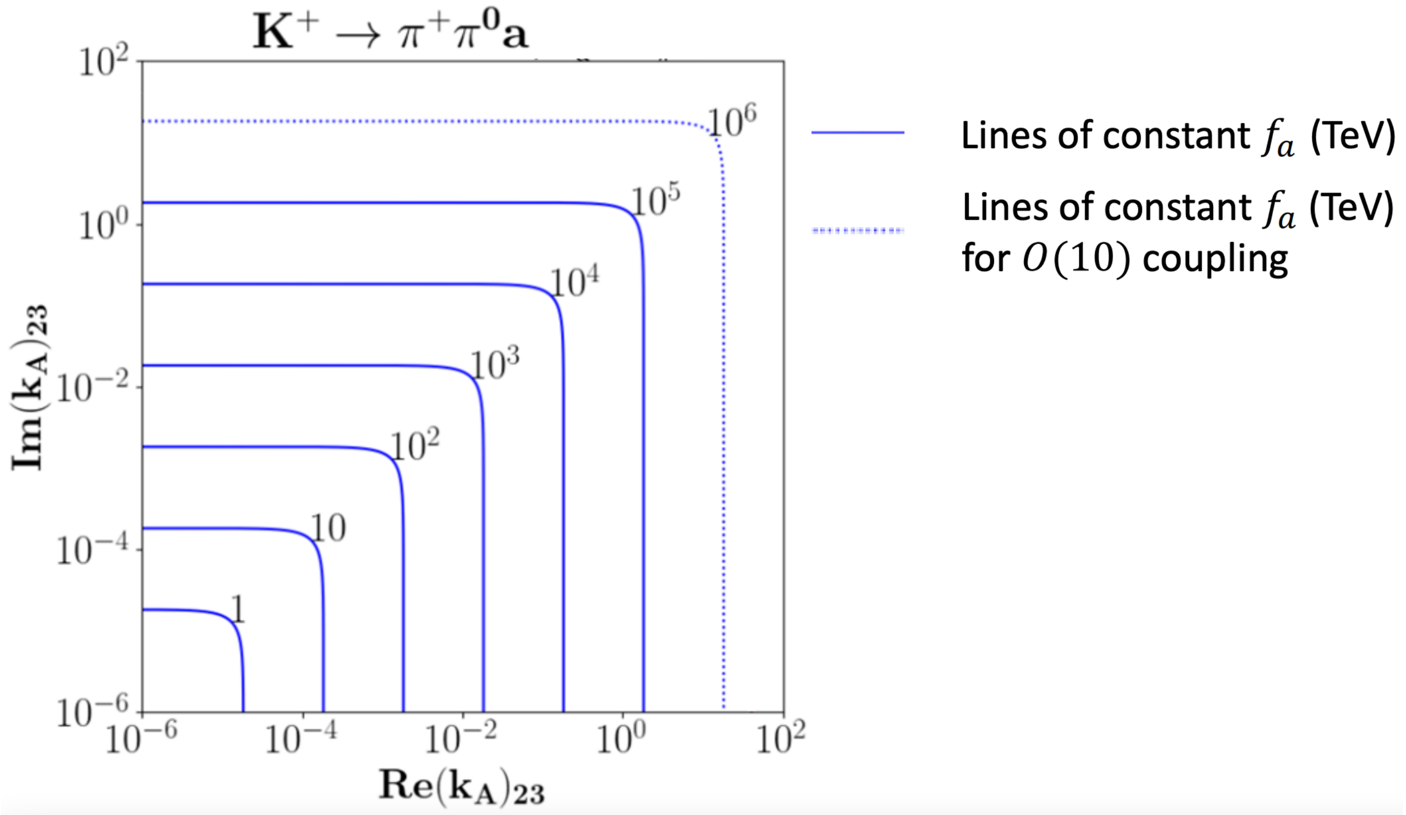
Exploiting the fact that $S_{\text{eff}} \approx 5.0 \times 10^{-7}$ one can obtain **bounds on $(k_A)_{23}$** :



Three-body kaon decays into final states w/ an axion

Cavan Piton, Iohner, Guadagnoli, Martinez-Santos, LV, JHEP '25 (2411.04170)

One can derive the **theoretical BR from** the expression (obtained in Chiral Perturbation Theory + axion):

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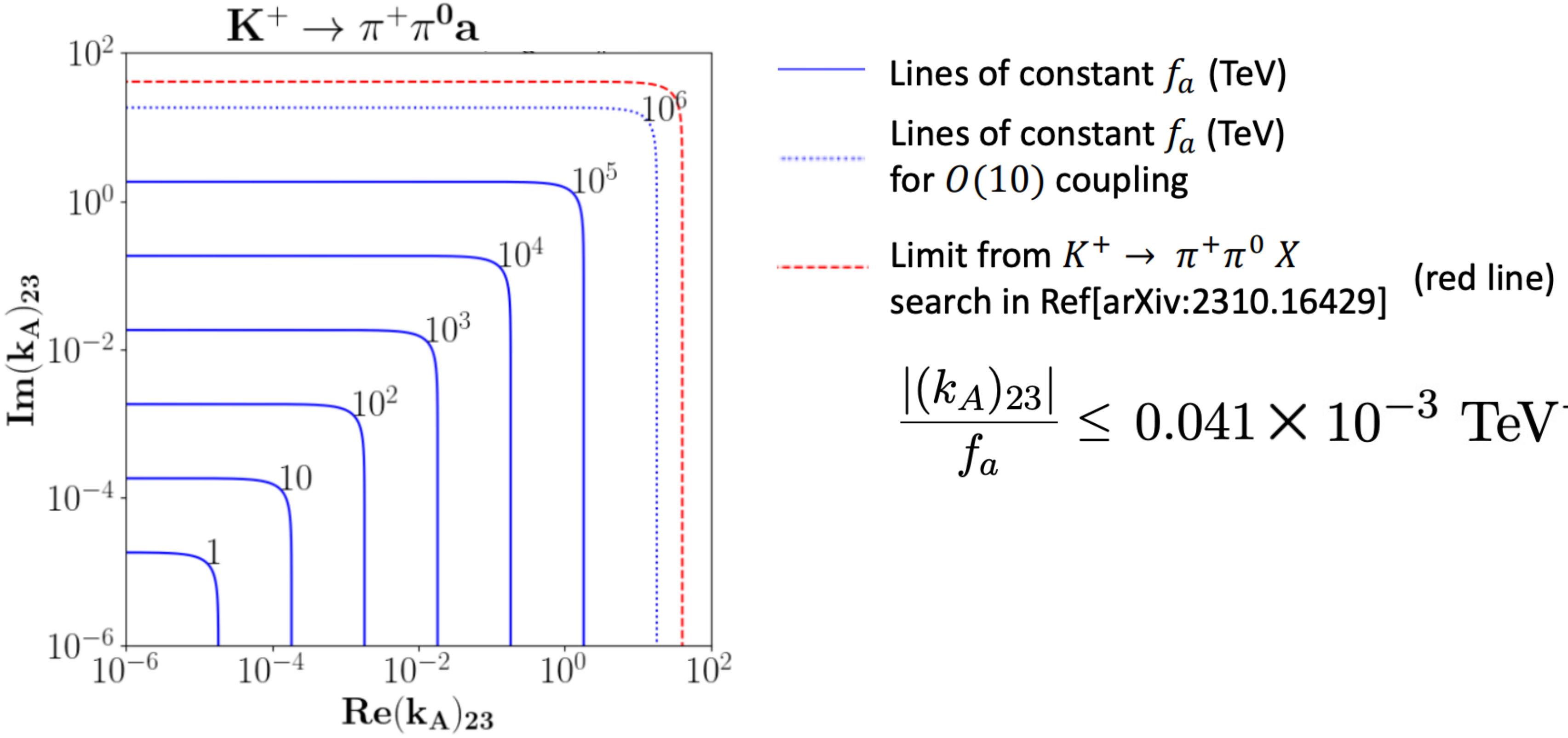

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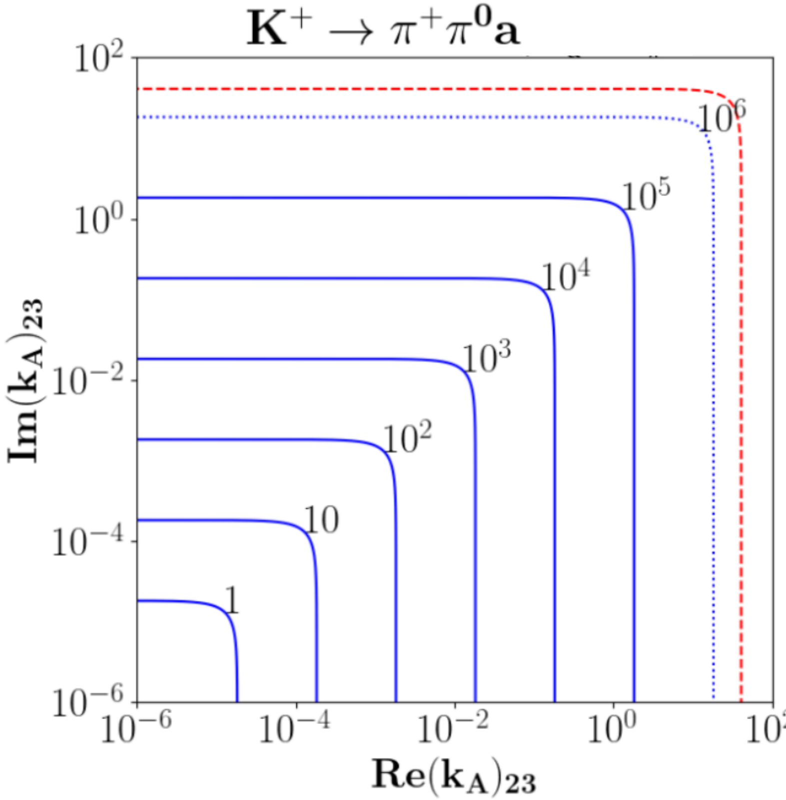

Lines of constant f_a (TeV)

Lines of constant f_a (TeV) for $O(10)$ coupling

Limit from $K^+ \to \pi^+\pi^0 X$ search in Ref[arXiv:2310.16429] (red line)

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for $f_a = 10^6$ TeV ➡ $|(k_A)_{23}| \leq 41$

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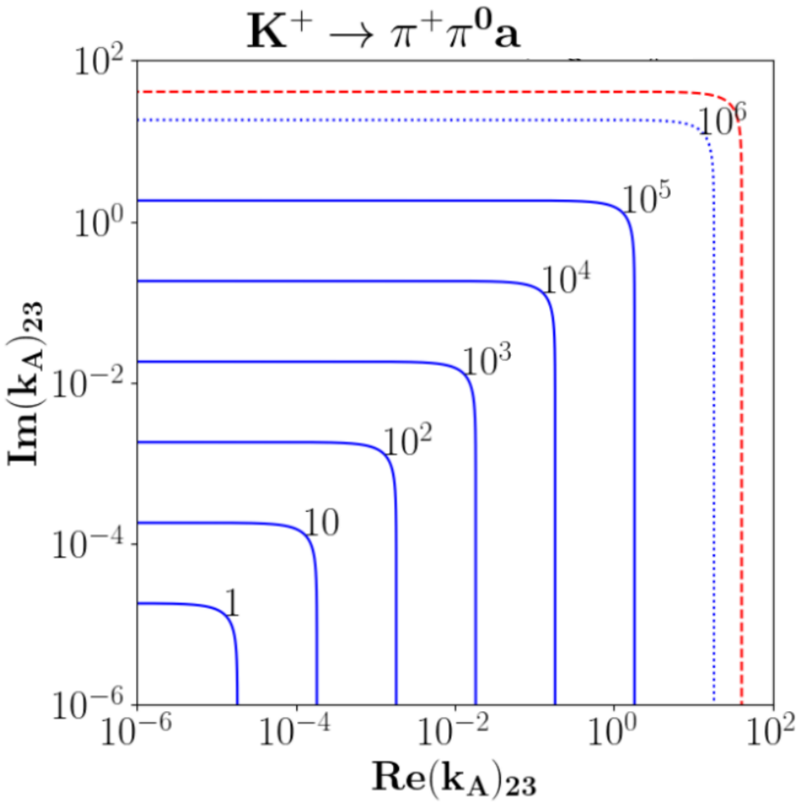

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Strong validation of the results obtained through the sensitivity study !

Three-body kaon decays into final states w/ an axion

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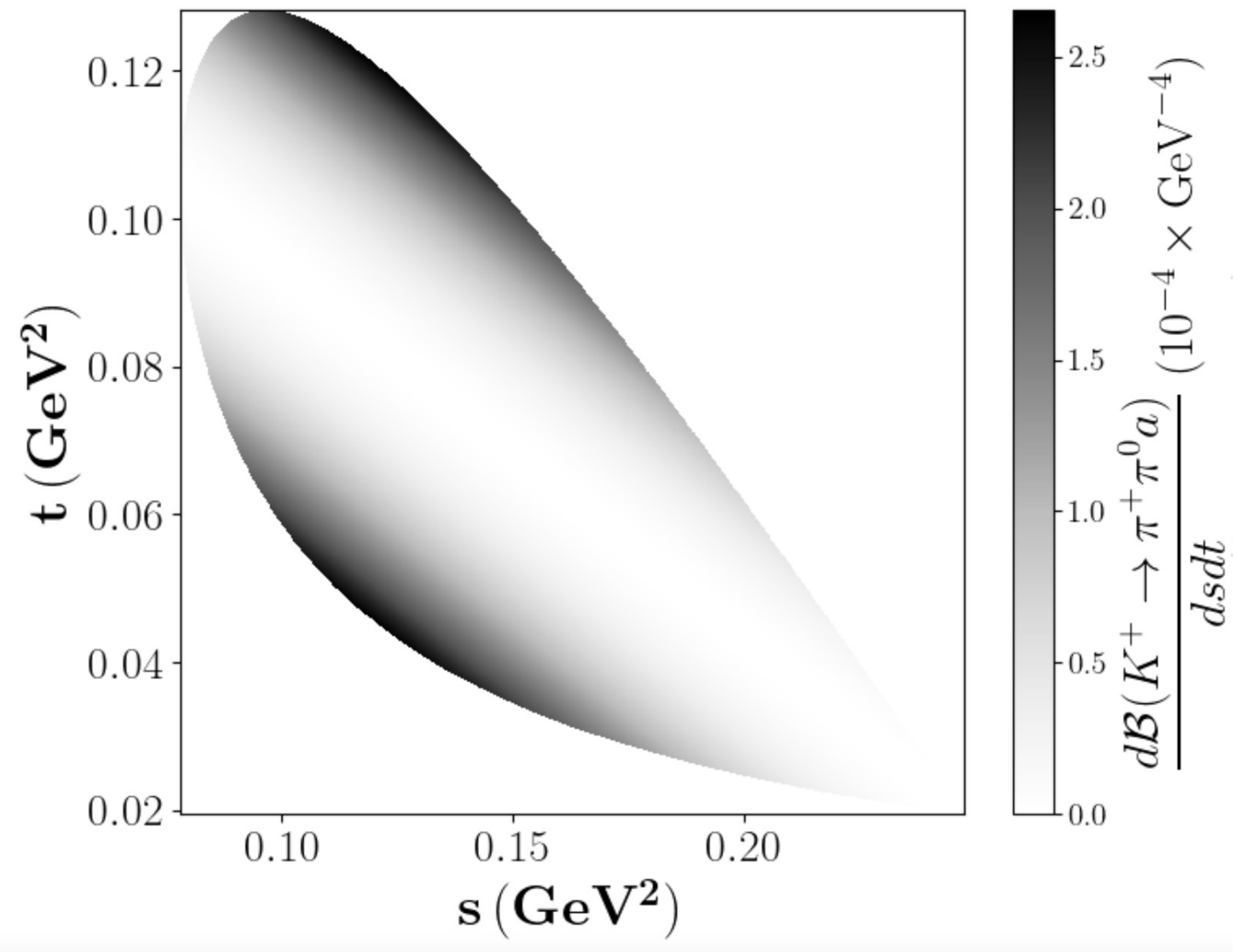
Novel proposal: let us exploit the differential decay widths (ddw) as well :



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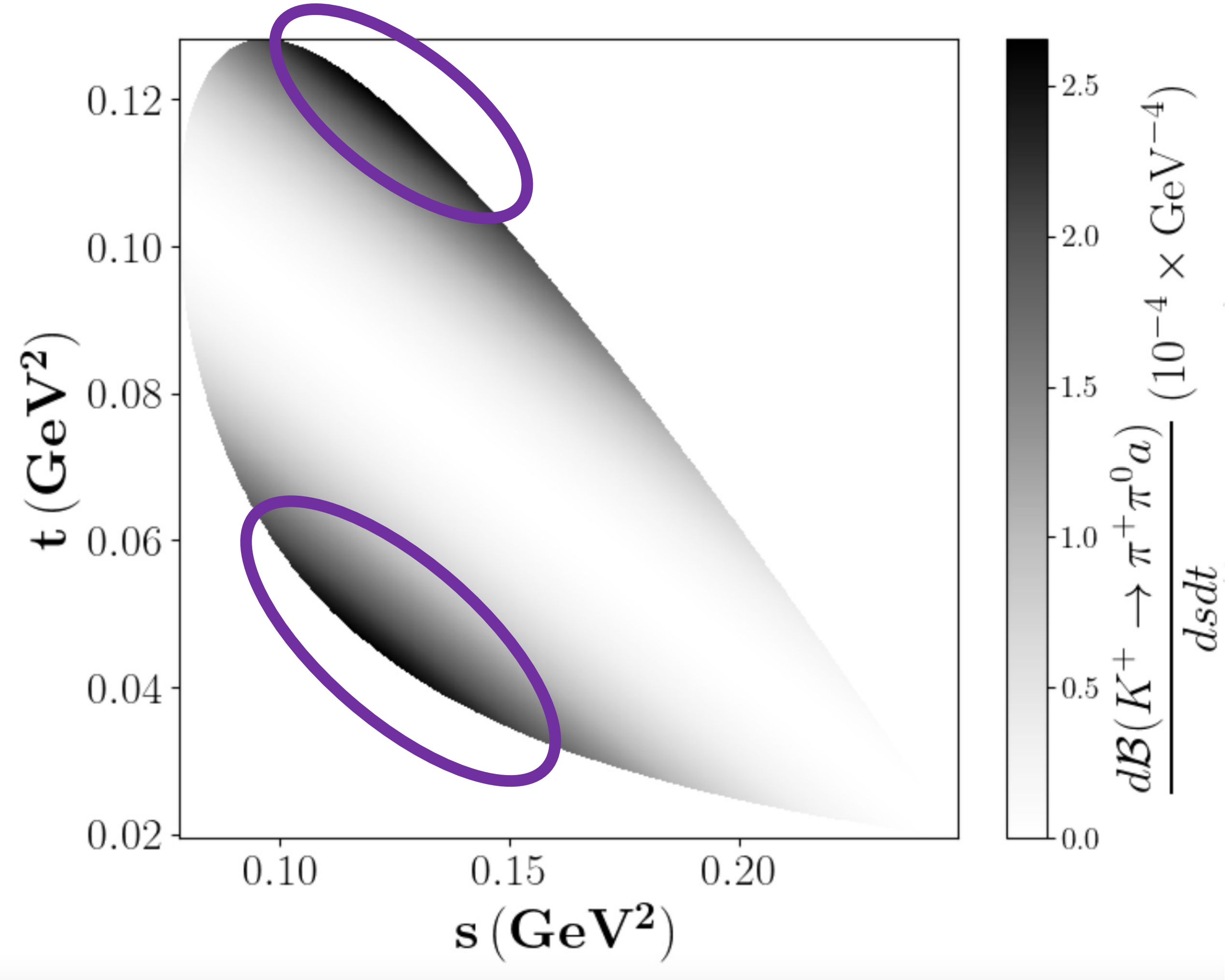


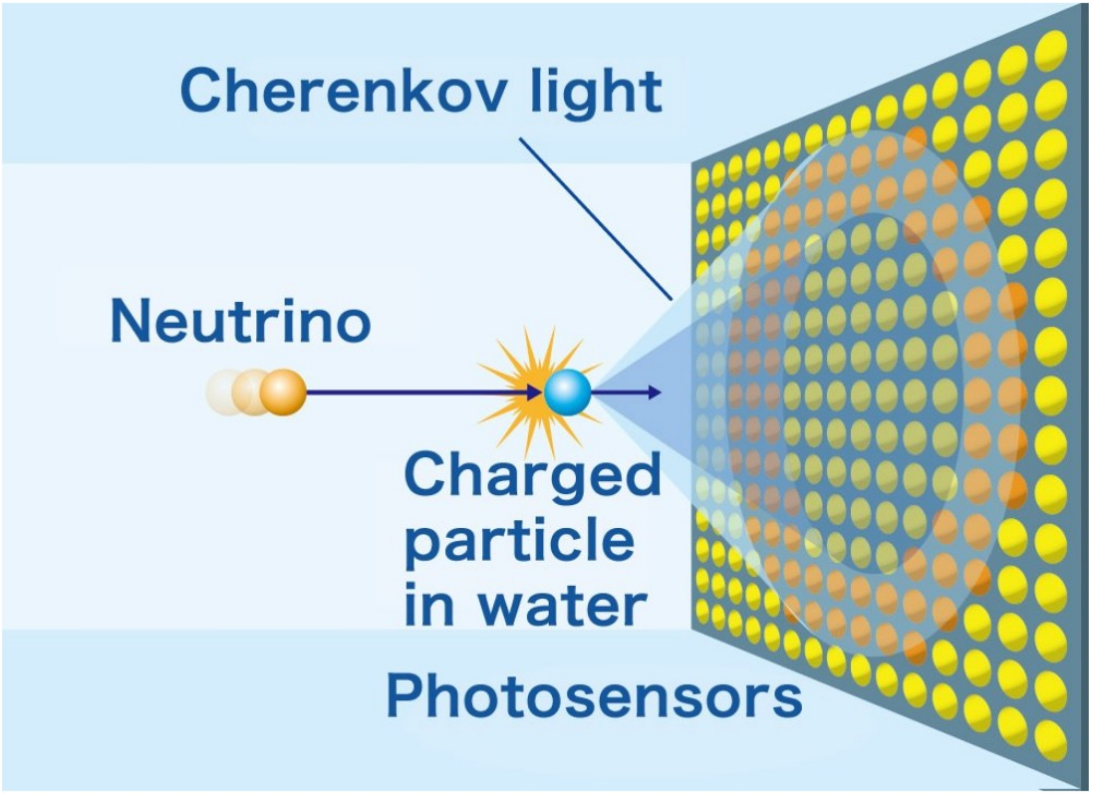
Prospects for future studies:
since there are some regions in which the theoretical ddw is enhanced w.r.t. others, dedicated measurements in these regions would give **stronger bounds on $(k_A)_{23}$**

Other constrains
on FU couplings (w/out s):

Axion bounds
from Cherenkov exps.

Water Cherenkov experiments: neutrino vs. axion fluxes

- A primary role in the physics potential of **Water Cherenkov experiments** (e.g. HyperKamiokande, see **1805.04163**) is played by **neutrino astrophysics**. One of the main goals of these exps. is, indeed, the **detection of neutrino fluxes from astrophysical sources, such as supernovae**



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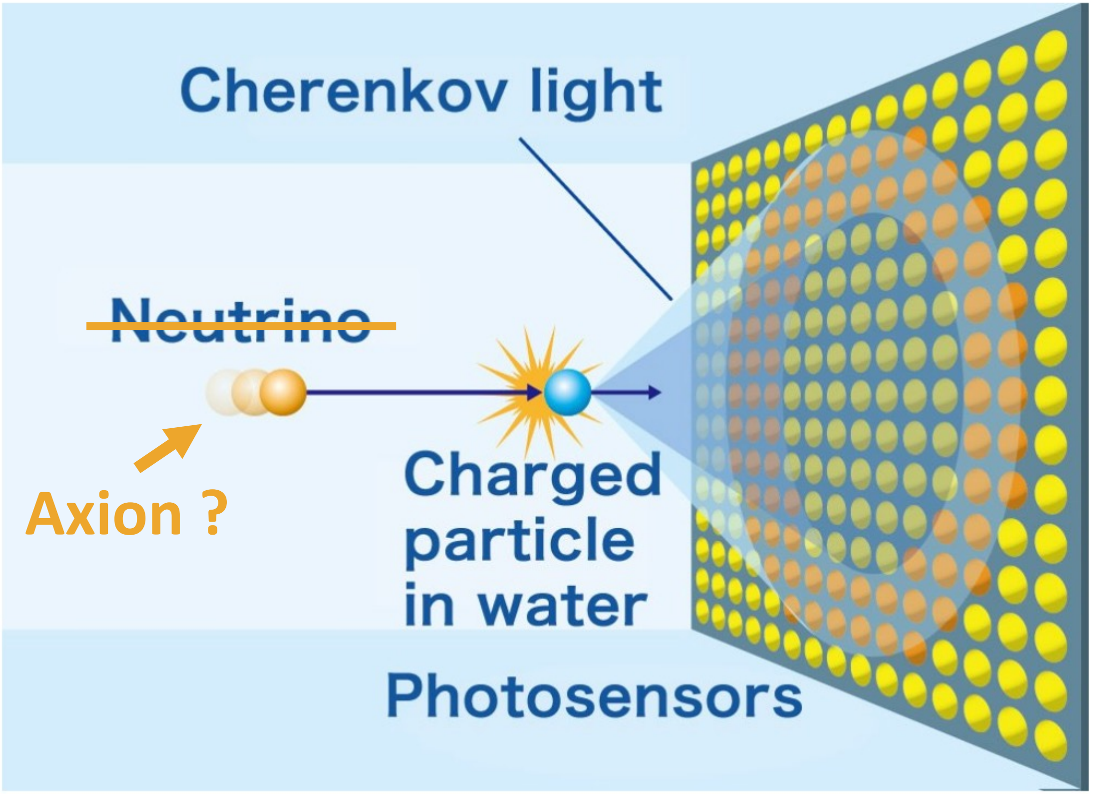

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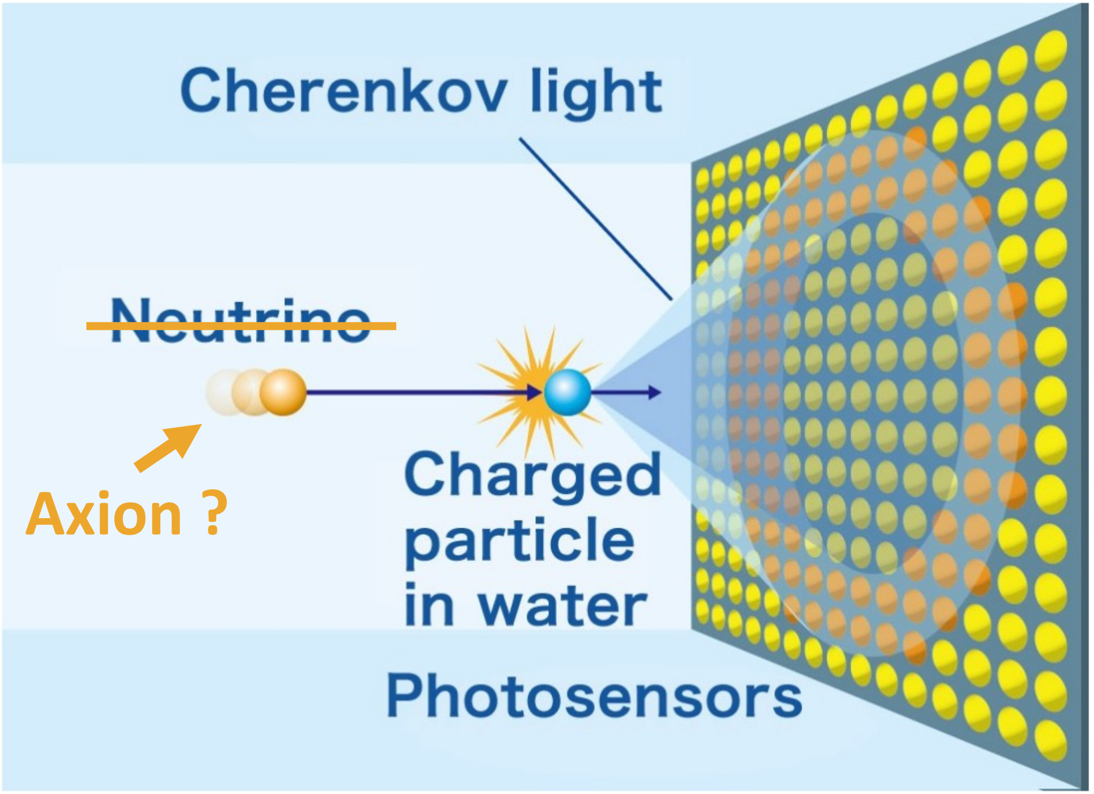

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Several studies on this subject:

- Lucente et al, PRD '22 [2203.15812]
- Engel et al, PRL 65 (1990) 960-963
- Carenza et al, PRC '24 [2306.17055]
- Li and Zhang, PRD '22 [2208.02696]
- Chakraborty et al, PRD '24 [2403.12169]
- Arias-Aragón et al, PRD '25 [2411.19327]
- Alonso-González et al, PRD '25 [2412.09595, 2412.19890]
- …

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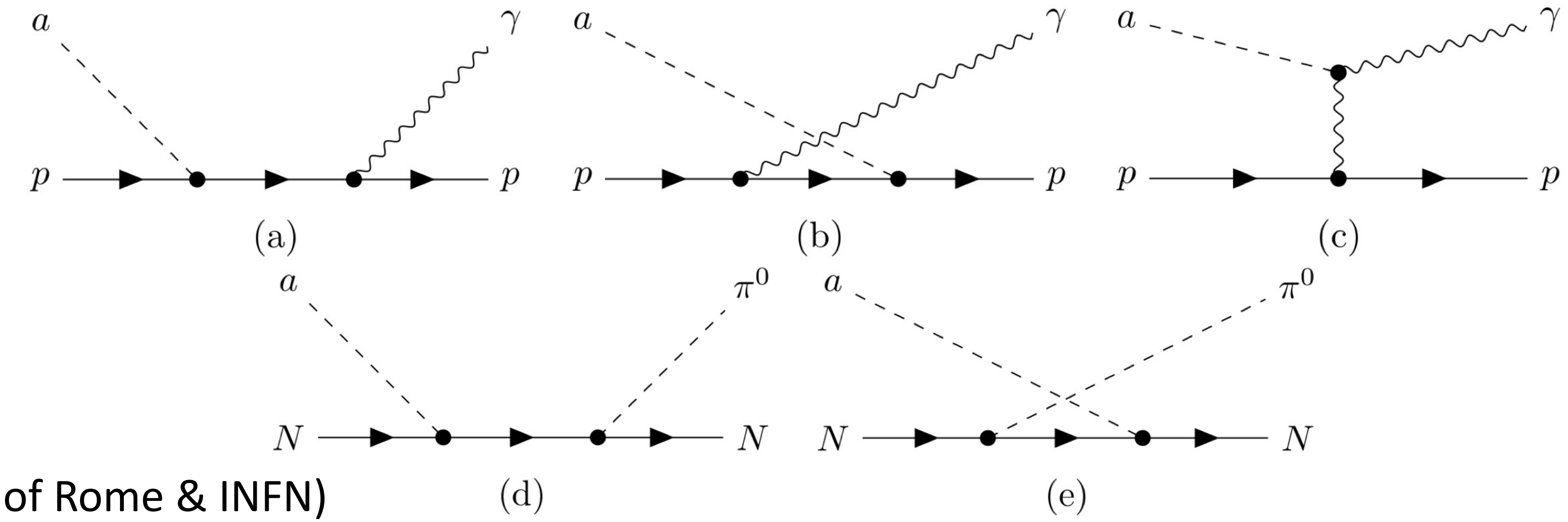

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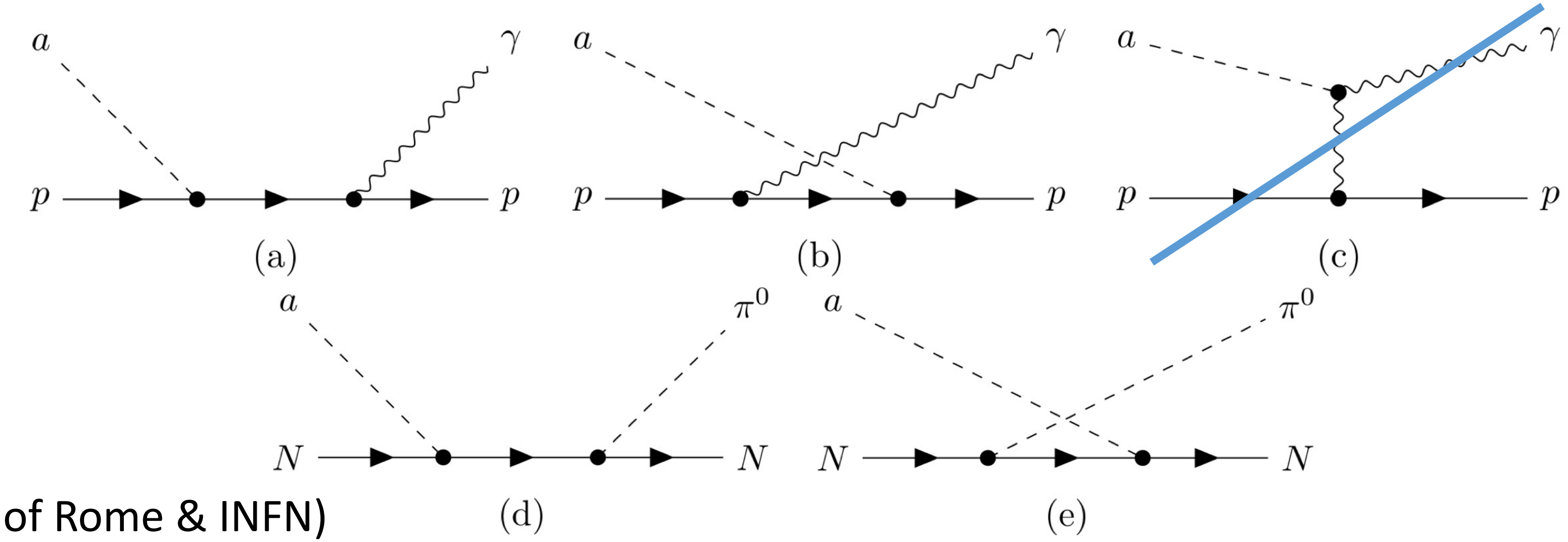

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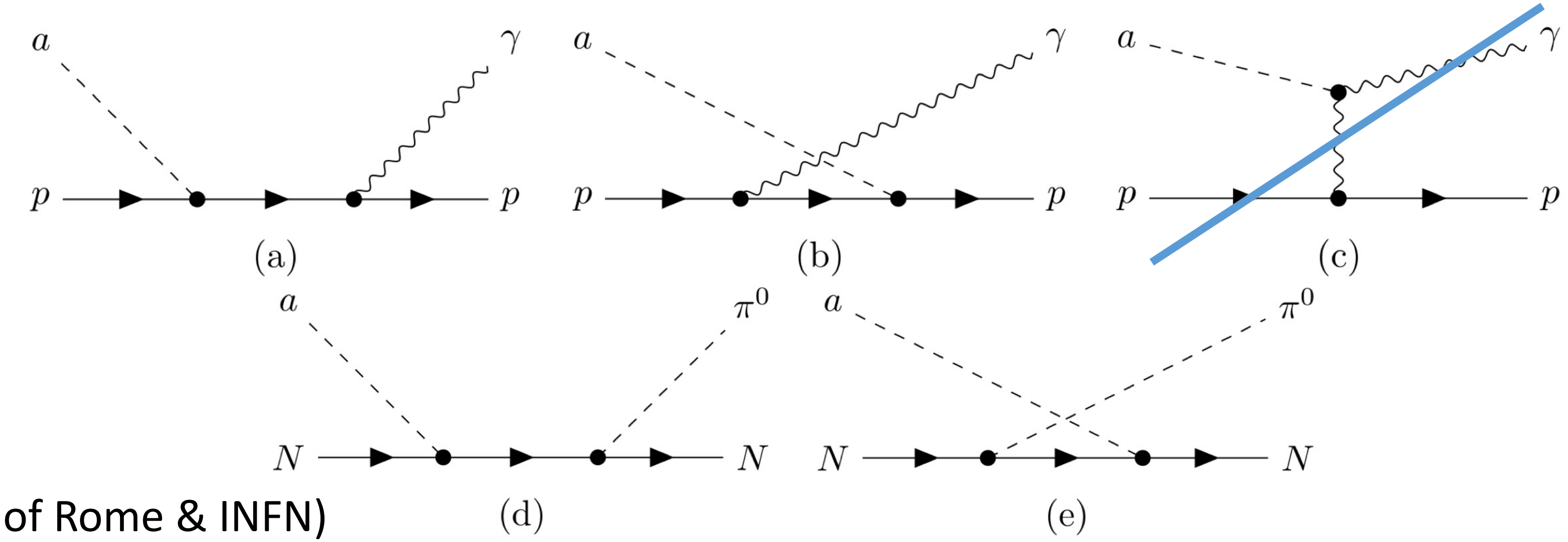
Backgrounds:

$$\bar{\nu}\,p \rightarrow n\,e^+$$
$$\nu\,n \rightarrow p\,e^-$$



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Possible estimate

$$\frac{dN_a}{dE_a} = \kappa\,\frac{d\dot{n}_a}{dE_a}, \text{ w/ } \frac{d\dot{n}_a}{dE_a} = \int (2\pi)^4\delta^4(p)\,|\mathcal{M}|^2\,\mathcal{F}_i\,\mathcal{F}_M\,(1-\mathcal{F}_f)\,\frac{d\Omega_a E_a}{2(2\pi)^3}\prod_m^{\{i,f,M\}}\frac{d^3\boldsymbol{p}_m}{(2\pi)^3 2E_m}$$

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$$C_{ann}|_{\mathrm{KSVZ}} = 0.012(28) \ , \qquad C_{app}|_{\mathrm{KSVZ}} = -0.452(28)$$

2. DFSZ [Dine, Fischler, Srednicki, PLB 104 (1981) 199; Zhitnitski, SJNP 31 (1980) 260]:

$$\begin{aligned} C_{ann}|_{\mathrm{DFSZ}} &= -0.123(30) + 0.406(15)\sin^2(\beta) \\ C_{app}|_{\mathrm{DFSZ}} &= -0.169(30) - 0.430(15)\sin^2(\beta) \end{aligned}$$

$\tan\beta$ is the ratio between the vacuum expectation values of the up- and down-sector Higgs doublets

Disclaimers (before showing the results ☺)

- **Three benchmark scenarios for the axion-nucleon couplings :**

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3. «Agnostic» : C_{aNN} solely required to comply with existing data :

- Neutron Star cooling:

Buschmann et al, PRL '22 [2111.09892]

$$C_{aNN}\frac{m_N}{f_a} \lesssim 10^{-9}$$

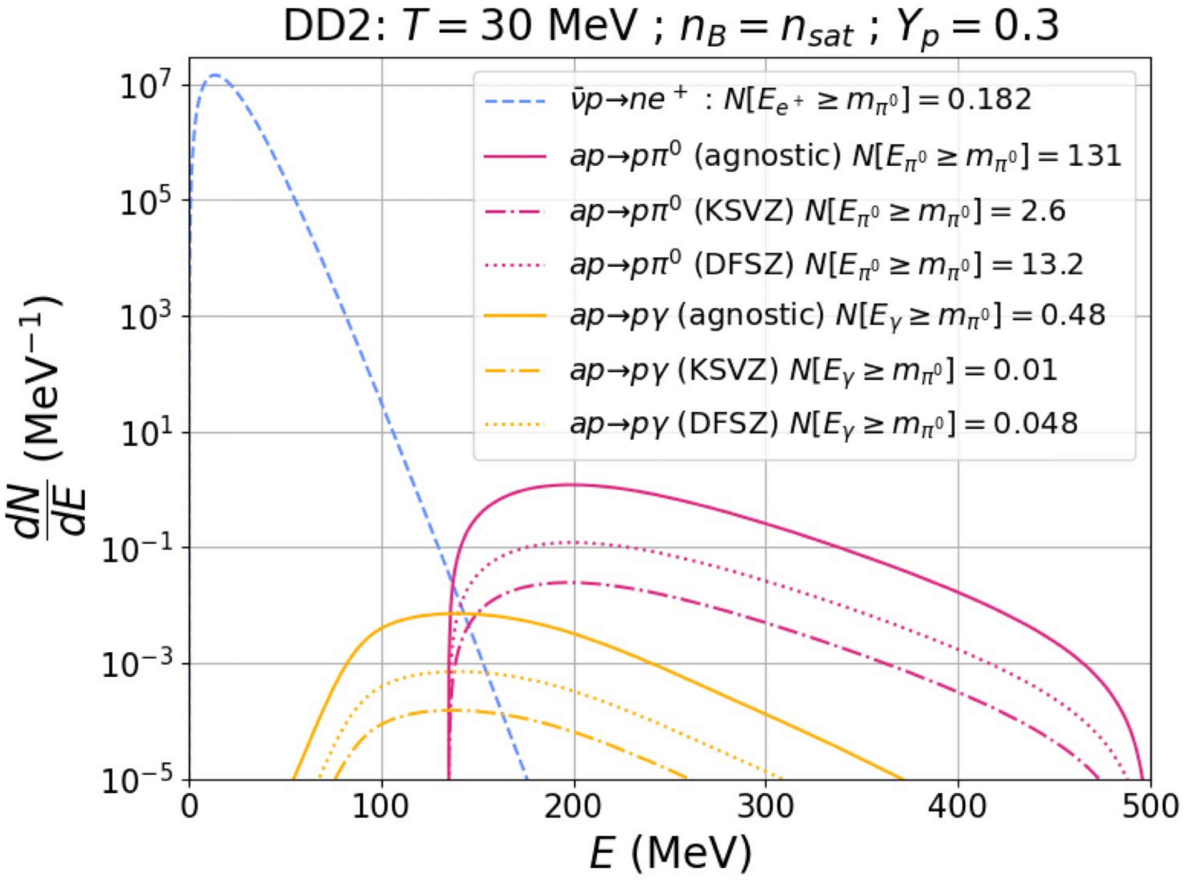
- SN cooling:

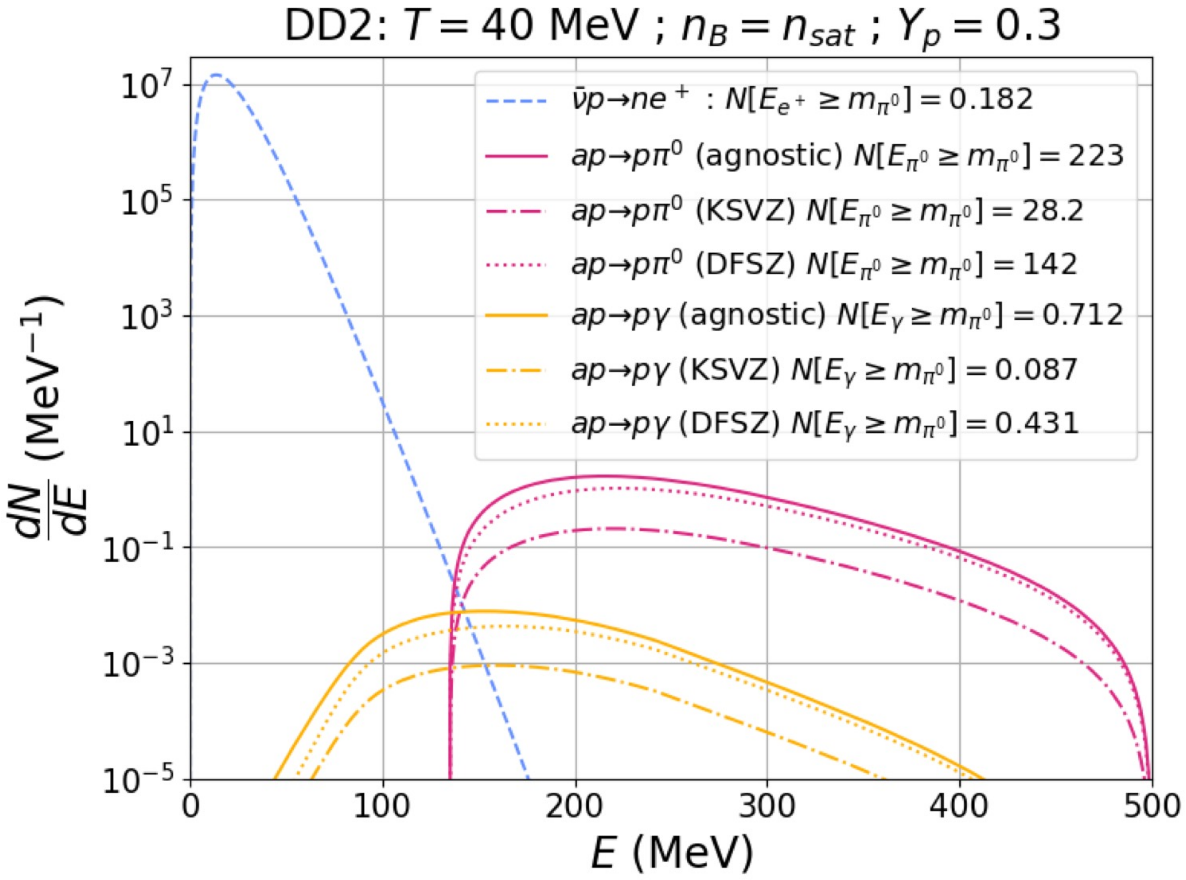
$$L_a \lesssim L_\nu \sim 3\times 10^{52}\mathrm{erg/s}$$

G. Raffelt, Phys. Rept. 198 ('90)

Cherenkov-light spectra

Cavan Piton, Guadagnoli, Iohner, Fernandez-Mendez, LV, JHEP '25 (2503.17490)





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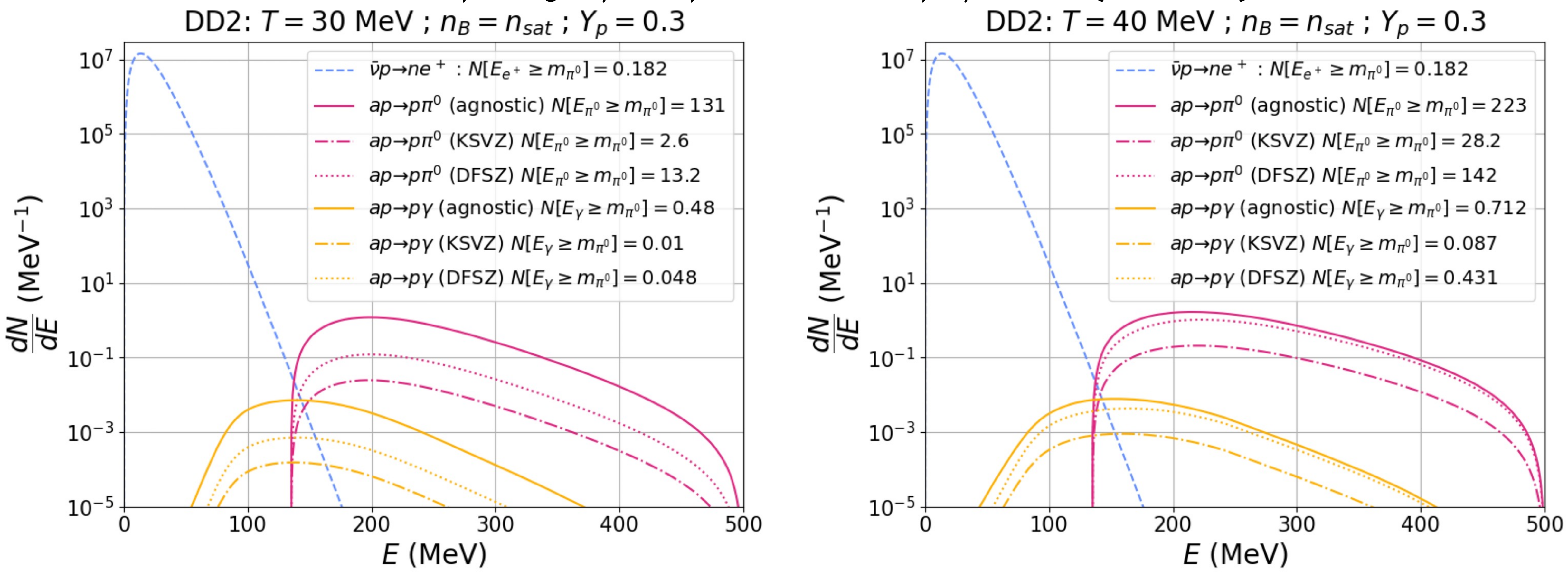
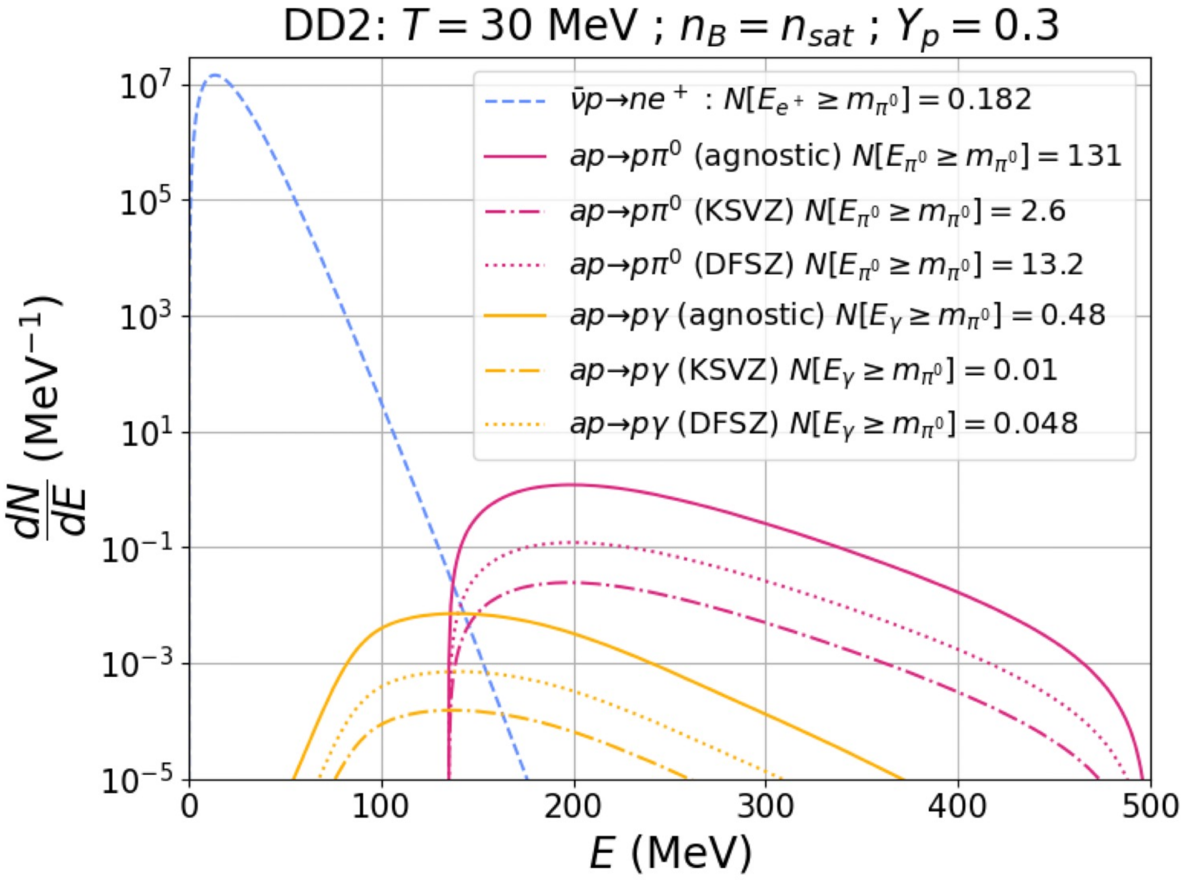
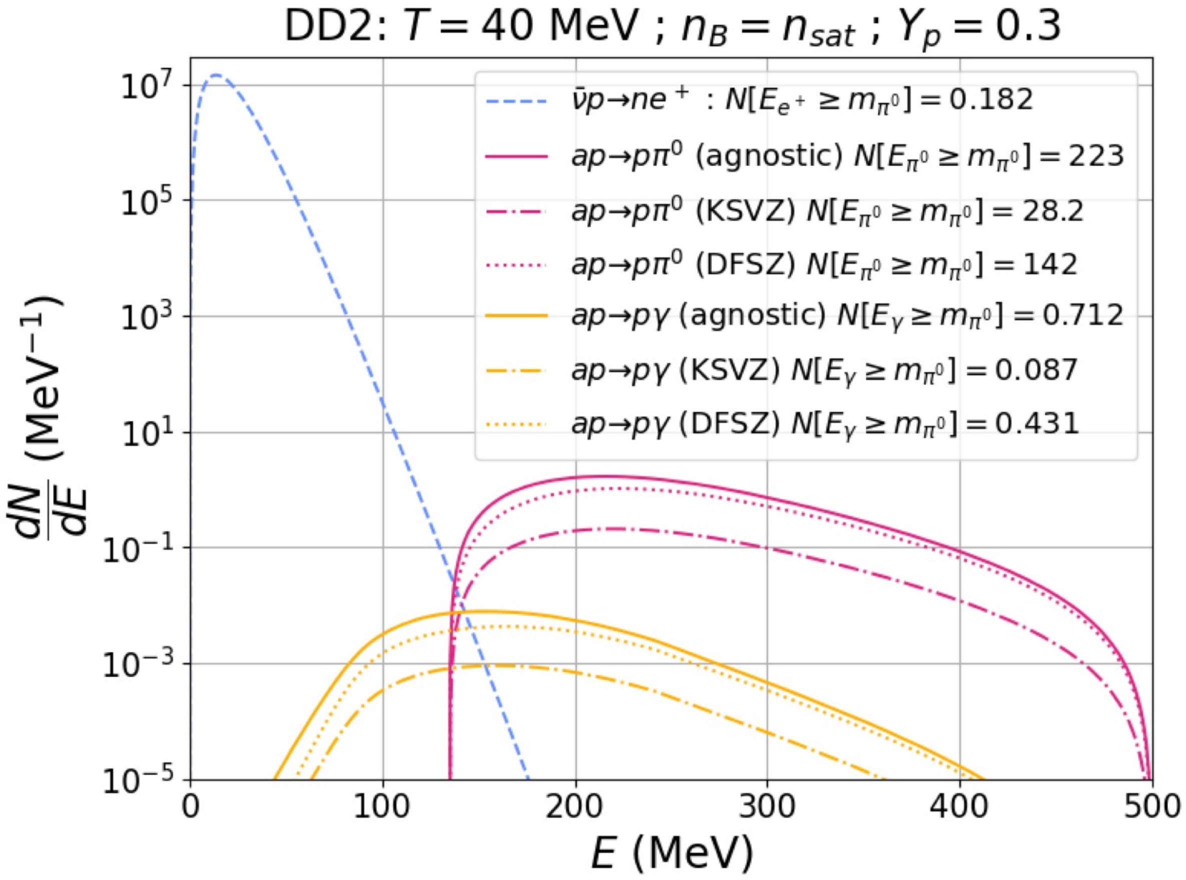


- **The Cherenkov-light spectra induced by neutrino vs. axion absorption peak at well separated energies**

Cherenkov-light spectra

Cavan Piton, Guadagnoli, Iohner, Fernandez-Mendez, LV, JHEP '25 (2503.17490)





- **The Cherenkov-light spectra induced by neutrino vs. axion absorption peak at well separated energies**
- **This feature is particularly evident for the process a p → p π^0**

Couplings dependence of expected number of event

Expected # of signal events

$$\left\langle N_{\pi^0}^{(R)} \right\rangle \equiv \lambda_{\pi^0}(C_{app}, C_{ann}) = \int_R dE_{\pi^0} \frac{dN_{\pi^0}^{(a)}(C_{app}, C_{ann})}{dE_{\pi^0}}$$

{ R ϵ [m_π, +infinity) }

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{ R ∈ $[m_\pi, +\text{infinity})$ }

The probability of observing any given number N_{π^0} of events obeys a **Poisson distribution** with λ_{π^0}

→ the **observed number of events** can thus be estimated as $\lambda_{\pi^0} \pm \sqrt{\lambda_{\pi^0}}$

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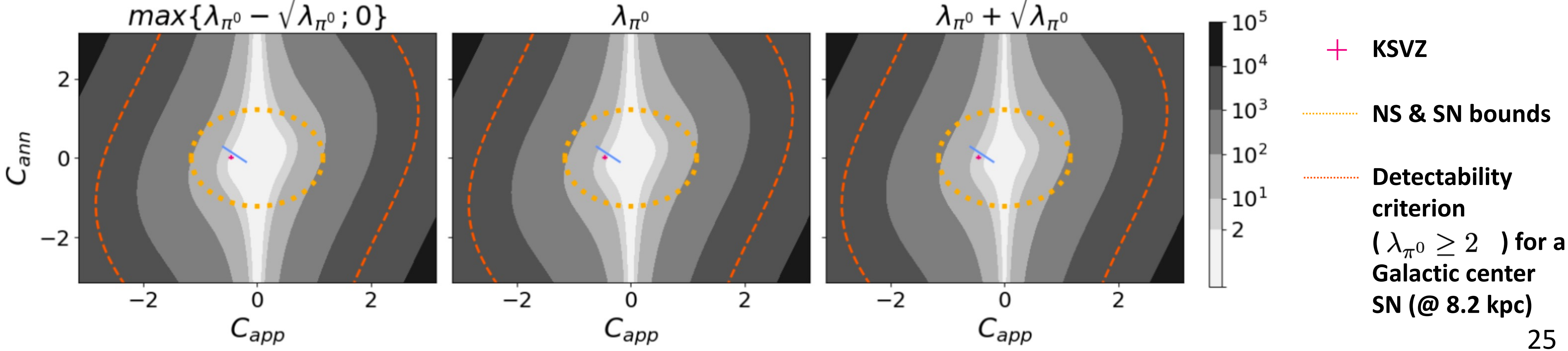
➡ the **observed number of events** can thus be estimated as $\lambda_{\pi^0} \pm \sqrt{\lambda_{\pi^0}}$

***EXAMPLE*: # of event from a SN candidate at 0.2 kpc (e.g. Betelgeuse)**

DD2: $T = 30$ MeV ; $n_B = n_{sat}$; $Y_p = 0.3$

Cavan Piton, Guadagnoli, Iohner, Fernandez-Mendez, LV, JHEP '25 (2503.17490)



Conclusions

Multiple probes of the couplings of the QCD axion with the SM (light) generations of quarks can be exploited to obtain bounds and have been discussed in this talk :

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1. **Axion emission from SuperNovae: they probe not only interactions with ordinary matter (i.e. protons and neutrons), but also beyond 1st-generation ones.**

→ ***Improved understanding of the sources is, at present, crucial to go beyond O(1) answers !***

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2. **Rare kaon decays at colliders: no intrinsic sources of uncertainties in this case. Our sensitivity study showsa that dedicated searches @ LHCb and other exps. will be crucial for future studies**

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Conclusions

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   → ***Improved analyses of the differential decay widths (Dalitz plots) can be also implemented!***

3. **Axion absorption @ Cherenkov exps. : π^0 production seems more promising than γ one (higher peak, and at higher E).**

   → ***Theoretical and experimental improvements are essential to reveal potential o.o.m. enhancement***

GRAZIE PER LA VOSTRA ATTENZIONE !

BACK-UP SLIDES

Axion effective Lagrangian

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G} + \frac{1}{4}g^0_{a\gamma}aF\tilde{F} + \boxed{\frac{\partial_\mu a}{2f_a}\bar{q}c^0_q\gamma^\mu\gamma_5 q} - \bar{q}_L M_q q_R + \text{h.c.}\,.$$

KSVZ models:

$$C_u = \Delta C_u = \mathcal{O}(10^{-2}),$$

$$C_d = \Delta C_d = \mathcal{O}(10^{-2}),$$

Kim, PRL 1979
Shifman, Vainshtein, and Zakharov, NPB 1980

DFSZ models:

$$C_u = 2\cos^2\beta + \Delta C_u,$$

$$C_d = 2\sin^2\beta + \Delta C_d$$

Dine, Fischler, and Srednicki, PLB 981
Zhitnitsky, SJNP 1980

$$\left\{ \tan\beta \in [0.25, 170] \right\}$$

KSVZ (following 2003.01100)

SM field content +

- A vector-like fermion $\mathcal{Q} \sim (3, 1, 0)$
- A SM-singlet complex scalar $\Phi \sim (1, 1, 0)$

The Lagrangian is

$$\mathcal{L}_{\mathrm{KSVZ}} = |\partial_\mu \Phi|^2 + \bar{\mathcal{Q}} i \not{D} \mathcal{Q} - \left(y_{\mathcal{Q}} \bar{\mathcal{Q}}_L \mathcal{Q}_R \Phi + \mathrm{h.c.} \right) - V(\Phi)$$

U(1)$_{\mathbf{PQ}}$ symmetry: $\Phi \to e^{i\alpha} \Phi \,, \qquad \mathcal{Q}_L \to e^{i\alpha/2} \mathcal{Q}_L \,, \qquad \mathcal{Q}_R \to e^{-i\alpha/2} \mathcal{Q}_R \,.$

$$V(\Phi) = \lambda_\Phi \left(|\Phi|^2 - \frac{v_a^2}{2} \right)^2$$

➡ **U(1)$_{\mathbf{PQ}}$ is spontaneously broken:**

$$\boxed{\Phi = \frac{1}{\sqrt{2}} (v_a + \varrho_a) e^{ia/v_a}}$$

Thus, the term responsible for generating the $aG\tilde{G}$ operator is

$$\mathcal{L}_{\mathrm{KSVZ}} \supset -m_{\mathcal{Q}} \bar{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_a} + \mathrm{h.c.} \,, \qquad \left\{ m_{\mathcal{Q}} = y_{\mathcal{Q}} v_a / \sqrt{2}. \right\}$$

KSVZ (following 2003.01100)

$$\mathcal{L}_{\rm KSVZ} \supset -m_{\mathcal{Q}} \bar{\mathcal{Q}}_L \mathcal{Q}_R e^{ia/v_a} + \text{h.c.}\,,$$

By performing a field-dependent axial transformation

$$\mathcal{Q}_L \to e^{i\frac{a}{2v_a}} \mathcal{Q}_L \text{ and } \mathcal{Q}_R \to e^{-i\frac{a}{2v_a}} \mathcal{Q}_R$$

Q becomes independent of *a* !! We can integrate it out. Moreover, since this transfomration is anomalous under QCD :

$$\delta\mathcal{L}_{\rm KSVZ} = \frac{g_s^2}{32\pi^2} \frac{a}{v_a} G\tilde{G}$$

No coupling to photons, no coupling to fermions!

DFSZ (following 2003.01100)

The field content of the DFSZ model includes:

- Two Higgs doublets $H_u \sim (1,2,-\frac{1}{2})$ and $H_d \sim (1,2,+\frac{1}{2})$
- A SM-singlet complex scalar $\Phi \sim (1,1,0)$

Potential V:

$$V(H_u, H_d, \Phi) = \tilde{V}_{\text{moduli}}(|H_u|, |H_d|, |\Phi|, |H_u H_d|) + \boxed{\lambda H_u H_d \Phi^{\dagger 2}} + \text{h.c.}$$

Responsible for the symm. breaking :

$$U(1)_{H_u} \times U(1)_{H_d} \times U(1)_\Phi \to U(1)_Y \times U(1)_{\text{PQ}}$$

By eploiting the assumption of universality of the PQ charges :

$$\mathcal{L}^Y_{\text{DFSZ}-\text{I}} = -Y_U \bar{q}_L u_R H_u - Y_D \bar{q}_L d_R H_d - Y_E \bar{\ell}_L e_R H_d + \text{h.c.}\,.$$

All the scalar fields pick up a VEV :

$$H_u \supset \frac{v_u}{\sqrt{2}} e^{i\frac{a_u}{v_u}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad H_d \supset \frac{v_d}{\sqrt{2}} e^{i\frac{a_d}{v_d}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \Phi \supset \frac{v_\Phi}{\sqrt{2}} e^{i\frac{a_\Phi}{v_\Phi}}$$

DFSZ (following 2003.01100)

To *define* the axion, we introduce the current

$$J^{\rm PQ}_\mu = -\mathcal{X}_\Phi \Phi^\dagger i\overleftrightarrow{\partial}_\mu \Phi - \mathcal{X}_{H_u} H_u^\dagger i\overleftrightarrow{\partial}_\mu H_u - \mathcal{X}_{H_d} H_d^\dagger i\overleftrightarrow{\partial}_\mu H_d + \dots \quad \supset \quad J^{\rm PQ}_\mu|_a = \sum_{i=\Phi,u,d} \mathcal{X}_i v_i \partial_\mu a_i$$

so that

$$\boxed{a = \frac{1}{v_a}\sum_i \mathcal{X}_i v_i a_i\,, \qquad v_a^2 = \sum_i \mathcal{X}_i^2 v_i^2}$$

In this way

$$J^{\rm PQ}_\mu|_a = v_a \partial_\mu a \text{ and } \langle 0|J^{\rm PQ}_\mu|_a|a\rangle = i v_a p_\mu \qquad \text{(Goldstone theorem)}$$

For what concerns the PQ charges:

$$\mathcal{X}_\Phi = 1\,, \qquad \mathcal{X}_{H_u} = 2\cos^2\beta\,, \qquad \mathcal{X}_{H_d} = 2\sin^2\beta \qquad \left\{\begin{array}{l} v_u/v = \sin\beta \\ v_d/v = \cos\beta \end{array}\right\}$$

⇓

$$v_a^2 = v_\Phi^2 + v^2(\sin 2\beta)^2 \quad \Rightarrow \quad v_a \simeq v_\Phi$$

DFSZ (following 2003.01100)

In this case, the **axion couples with** everything:

- **Gluons and photons:**

$$\delta\mathcal{L}_{\mathrm{DFSZ-I}} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{\alpha}{8\pi}\left(\frac{E}{N}\right)\frac{a}{f_a}F\tilde{F}$$

- **SM fermions:**

$$\delta(\bar{u}i\partial\!\!\!/ u) = \mathcal{X}_{H_u}\frac{\partial_\mu a}{2v_a}\,\bar{u}\gamma^\mu\gamma_5 u = \left(\frac{1}{3}\cos^2\beta\right)\frac{\partial_\mu a}{2f_a}\bar{u}\gamma^\mu\gamma_5 u\,,$$

$$\delta(\bar{d}i\partial\!\!\!/ d) = \mathcal{X}_{H_d}\frac{\partial_\mu a}{2v_a}\,\bar{d}\gamma^\mu\gamma_5 d = \left(\frac{1}{3}\sin^2\beta\right)\frac{\partial_\mu a}{2f_a}\bar{d}\gamma^\mu\gamma_5 d\,,$$

$$\delta(\bar{e}i\partial\!\!\!/ e) = \mathcal{X}_{H_d}\frac{\partial_\mu a}{2v_a}\,\bar{e}\gamma^\mu\gamma_5 e = \left(\frac{1}{3}\sin^2\beta\right)\frac{\partial_\mu a}{2f_a}\bar{e}\gamma^\mu\gamma_5 e\,,$$

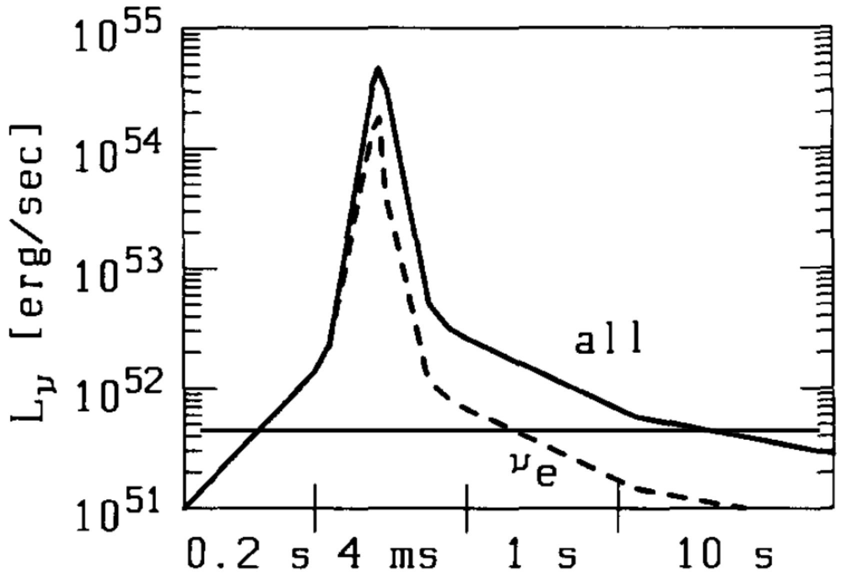
To generate flavour-violating (FV) couplings: relaxation of PQ universality ! Let us assume that quarks with the same EM charge but of different generations couple to different Higgs doublets, to which we assign the same but different PQ charges :

$$\mathcal{L}_{12}^{Y_U} = -(Y_U)_{11}\,\bar{q}_{1L}u_{1R}H_1 - (Y_U)_{22}\,\bar{q}_{2L}u_{2R}H_2 - (Y_U)_{12}\,\bar{q}_{1L}u_{2R}H_1 + \ldots\,.$$

Axion emission

After the formation of the iron core (no longer production of energy by nuclear burning) :

1. Photodissociation of iron + electron capture ⟶ almost free-fall collapse
2. As the core becomes hotter and denser, neutrinos become trapped
3. infall is halted only when the medium reaches nuclear densities ⟶ «bounce»
4. Dissociation of the nuclei of the medium due to the passage of the shock wave
5. When it reaches the neutrino-sphere, sudden decrease of the neutrino cross-sections and break-out of neutrino lumin.



Raffelt, Phys.Rept. 198 (1990) 1-113

Fig. 10.1. Schematic view of the neutrino luminosity expected from a type II supernova (adapted from Cooperstein in ref. [159]). There are several breaks of scale in the horizontal axis, separating the following periods: first ~0.2 s, infall from core density, $\sim 10^{10}$ g cm^{-3}, to maximum scrunch, $\sim 10^{15}$ g cm^{-3}; next ~0.004 s, from bounce to shock breakout at the neutrino sphere; further ~1 s until shock reaches edge of iron core; in the following ~10 s, most of the remaining binding energy is radiated from the neutrino sphere.

Axion emission

Table 10.1

Neutrino burst from SN 1987A in the IMB detector [166]. The event time is relative to the first event which occurred on 23 February 1987, 7:35:41.374 (UT), with an uncertainty of ±0.05 s. The angle is the polar angle with respect to the direction away from the SN. The energy is the measured energy of the electron or positron. If the events were due to $\bar{\nu} + p \to n + e^+$ on free protons, $E_{\bar{\nu}}$ was typically ~2 MeV larger than the measured e^- energy

Event	Time [s]	Angle [deg]	Energy [MeV]
1	0.00	80 ± 10	38 ± 7
2	0.41	44 ± 15	37 ± 7
3	0.65	56 ± 20	28 ± 6
4	1.14	65 ± 20	39 ± 7
5	1.56	33 ± 15	36 ± 9
6	2.68	52 ± 10	36 ± 6
7	5.01	42 ± 20	19 ± 5
8	5.58	104 ± 20	22 ± 5

Table 10.2

Neutrino burst from SN 1987A in the Kamiokande-II detector [168]. The event time is relative to the first event which occurred on 23 February 1987, 7:35:35 (UT), with an uncertainty of ±1:00 min. The angle is the polar angle with respect to the direction away from the SN. The energy is the measured energy of the electron or positron

Event	Time [s]	Angle [deg]	Energy [MeV]
1	0.00	18 ± 18	20.0 ± 2.9
2	0.11	40 ± 27	13.5 ± 3.2
3	0.30	108 ± 32	7.5 ± 2.0
4	0.32	70 ± 30	9.2 ± 2.7
5	0.51	135 ± 23	12.8 ± 2.9
6	0.69	68 ± 77	6.3 ± 1.7
7	1.54	32 ± 16	35.4 ± 8.0
8	1.73	30 ± 18	21.0 ± 4.2
9	1.92	38 ± 22	19.8 ± 3.2
10	9.22	122 ± 30	8.6 ± 2.7
11	10.43	49 ± 26	13.0 ± 2.6
12	12.44	91 ± 39	8.9 ± 1.9

Successful exponential cooling model to fit the data !

Axion emission

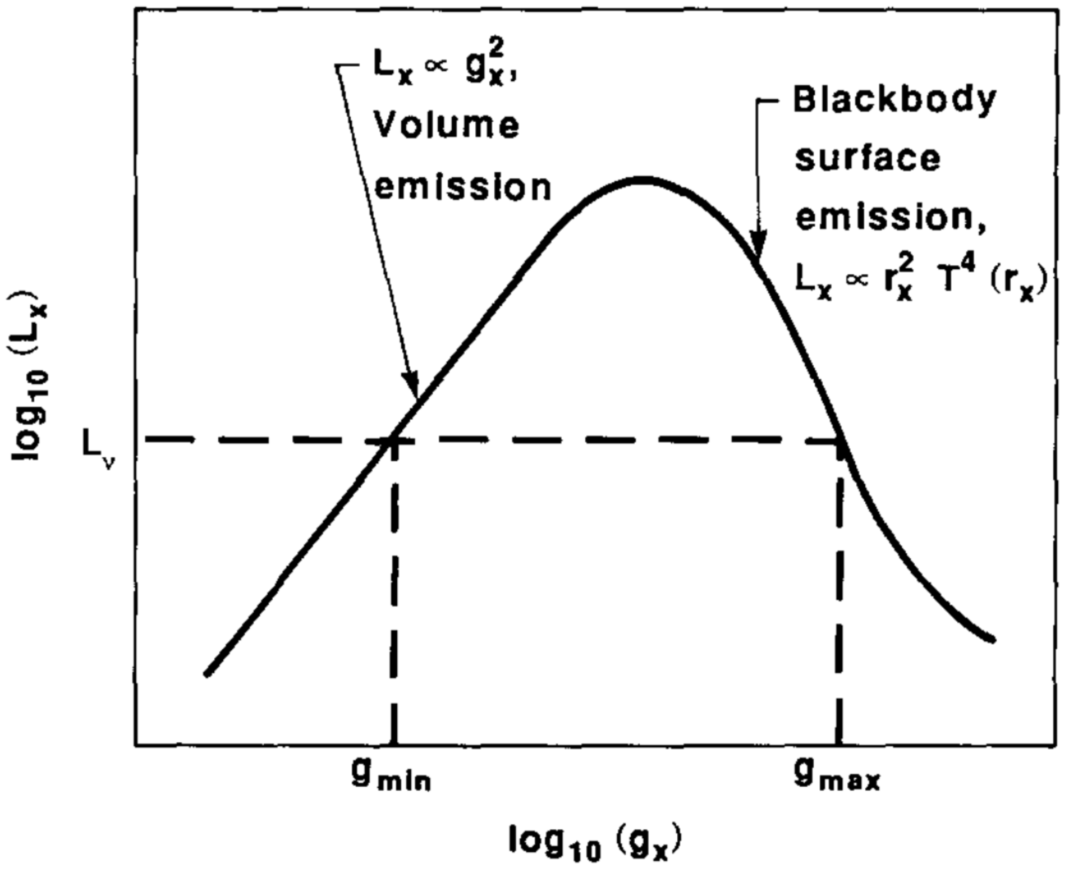
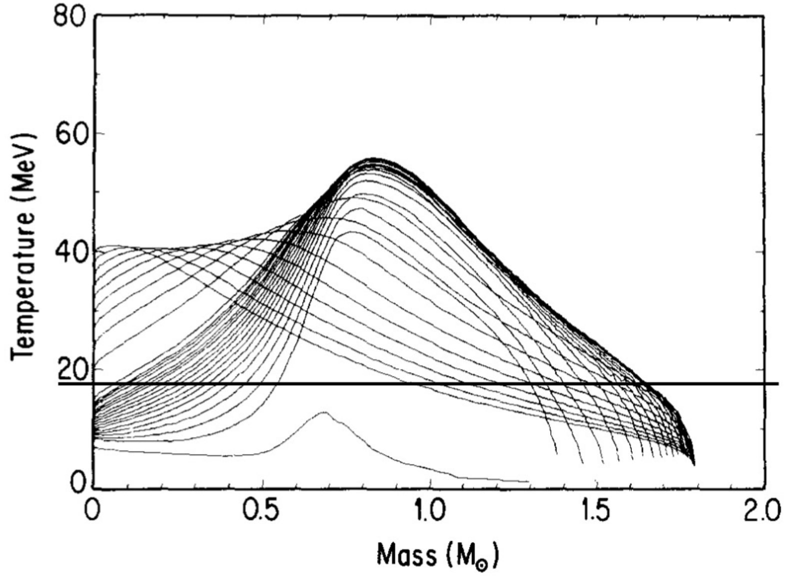


Fig. 10.4. Schematic dependence of the "exotic" luminosity, L_x, on the coupling strength of the new particles, g_x, which could be, for example, the Yukawa coupling of axions to nucleons or a "right-handed Fermi constant". In the range $g_{min} < g_x < g_{max}$, the novel energy loss would exceed the neutrino luminosity, L_ν. (Taken from Raffelt and Seckel [92].)

Raffelt, Phys.Rept. 198 (1990) 1-113

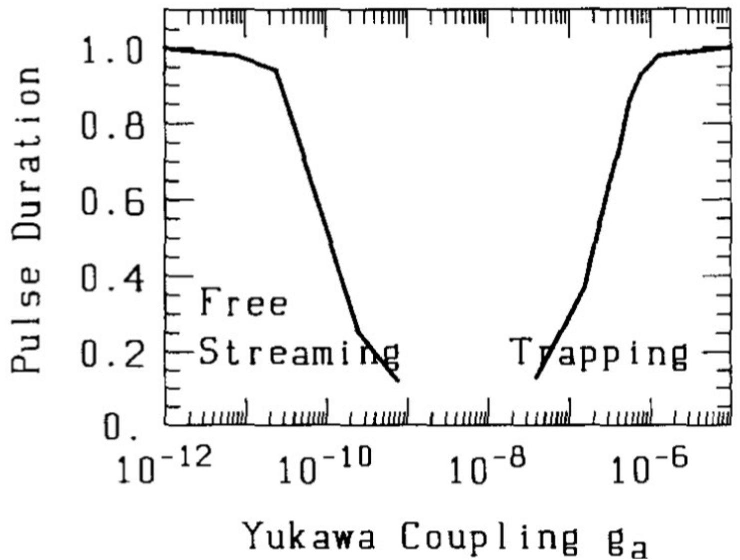
Axion emission

The total amount of energy emitted in neutrinos is relatively insensitive to the X coupling strength !



The emission of X-particles, however, will typically be dominated by the inner core where the densities are highest**. This part of the core, however, is at first at relatively low temperatures.**

Thus, the emission of X-particles will start slowly as energy diffuses into the inner core, and thus will be important mostly during the exponential cooling phase after the first neutrino burst ⟶ it would shorten the «cooling tail»



Axion emission

Hence, the X-particle luminosity is expected not to exceed the neutrino luminosity. Since

$$L_\nu \sim 3 \times 10^{52}\ \mathrm{erg/s}$$

then

$$\epsilon_X \lesssim 10^{19}\ \mathrm{erg\ g^{-1}\ s^{-1}}$$

(where $\epsilon_X = Q_X / \rho$)

Thermodynamics

The meson octet is treated as an ideal Bose gas with chemical potentials obtained from the EoS model.

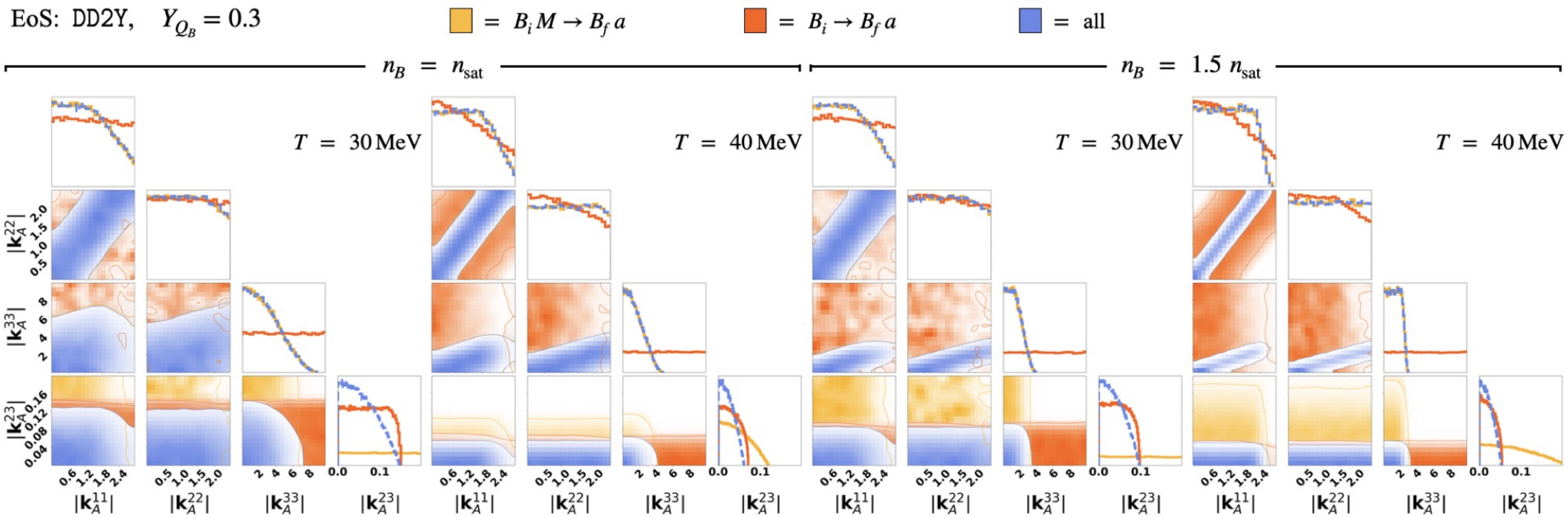
As written, the emissivity treats all particles involved as ideal gases. However, in-medium effects in hot and dense matter are expected to have a strong impact. Such effects may be captured at the mean-field level by shifting effective potentials, masses and energies of all particles entering the EoS with their “effective” counterparts:

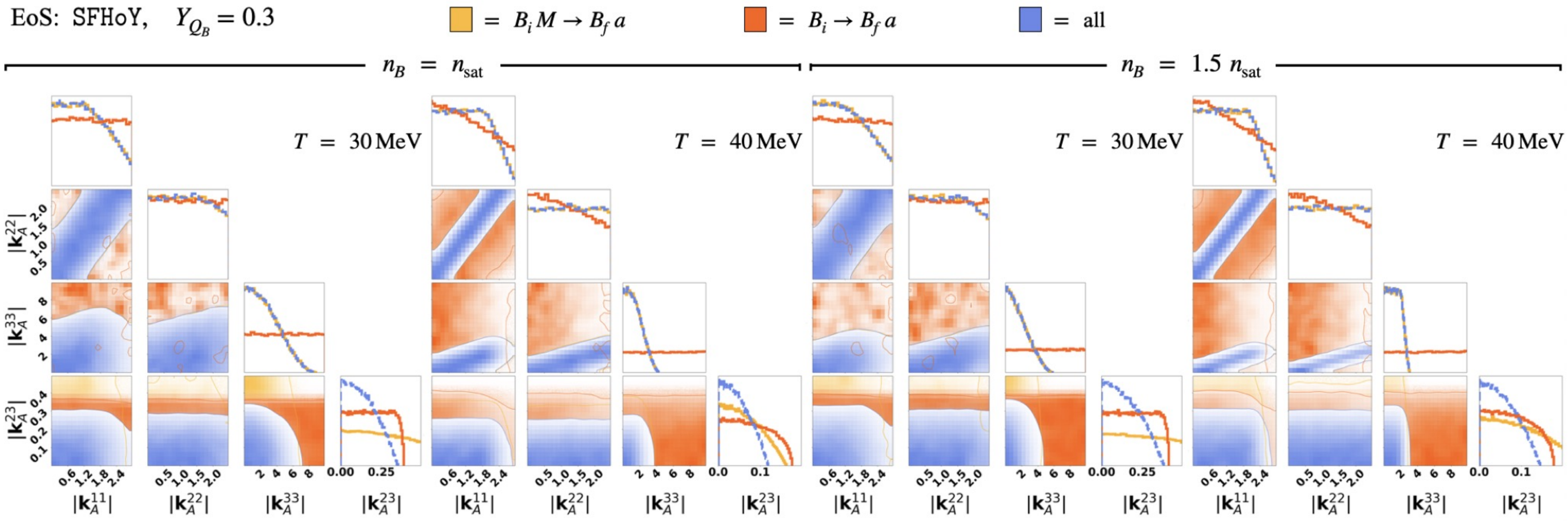
$$m_j \to m_j^* \ , \quad E_j \to E_j^* \ , \quad \mu_j \to \mu_j^* = \mu_j - U_j \ ,$$

The r.h.s.’s are calculated within each of our considered thermodynamic conditions. In particular:

1. **the effective chemical potential contains the mean-field in- teraction potential U_j, which also enters the energy- conservation equation and is determined self-consistently;**
2. **this treatment of mean-field effects also allows to correctly recover the particles’ number densities by integrat- ing their distribution functions over phase space.**

Formally the system can be treated as a free gas, with additional self-consistent equations determining the effective quantities and potential terms for energy and pressure (the distribution function still has the form of a free gas)


EoS: DD2Y, $Y_{Q_B} = 0.3$
= $B_i M \to B_f a$
= $B_i \to B_f a$
= all
$n_B = n_{\rm sat}$
$n_B = 1.5\, n_{\rm sat}$
$T = 30\,\mathrm{MeV}$
$T = 40\,\mathrm{MeV}$
$|\mathbf{k}_A^{11}|$
$|\mathbf{k}_A^{22}|$
$|\mathbf{k}_A^{33}|$
$|\mathbf{k}_A^{23}|$


EoS: SFHoY, $Y_{Q_B} = 0.3$
= $B_i M \to B_f a$
= $B_i \to B_f a$
= all
$n_B = n_{\rm sat}$
$n_B = 1.5\, n_{\rm sat}$
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$T = 40\,\mathrm{MeV}$
$|\mathbf{k}_A^{11}|$
$|\mathbf{k}_A^{22}|$
$|\mathbf{k}_A^{33}|$
$|\mathbf{k}_A^{23}|$

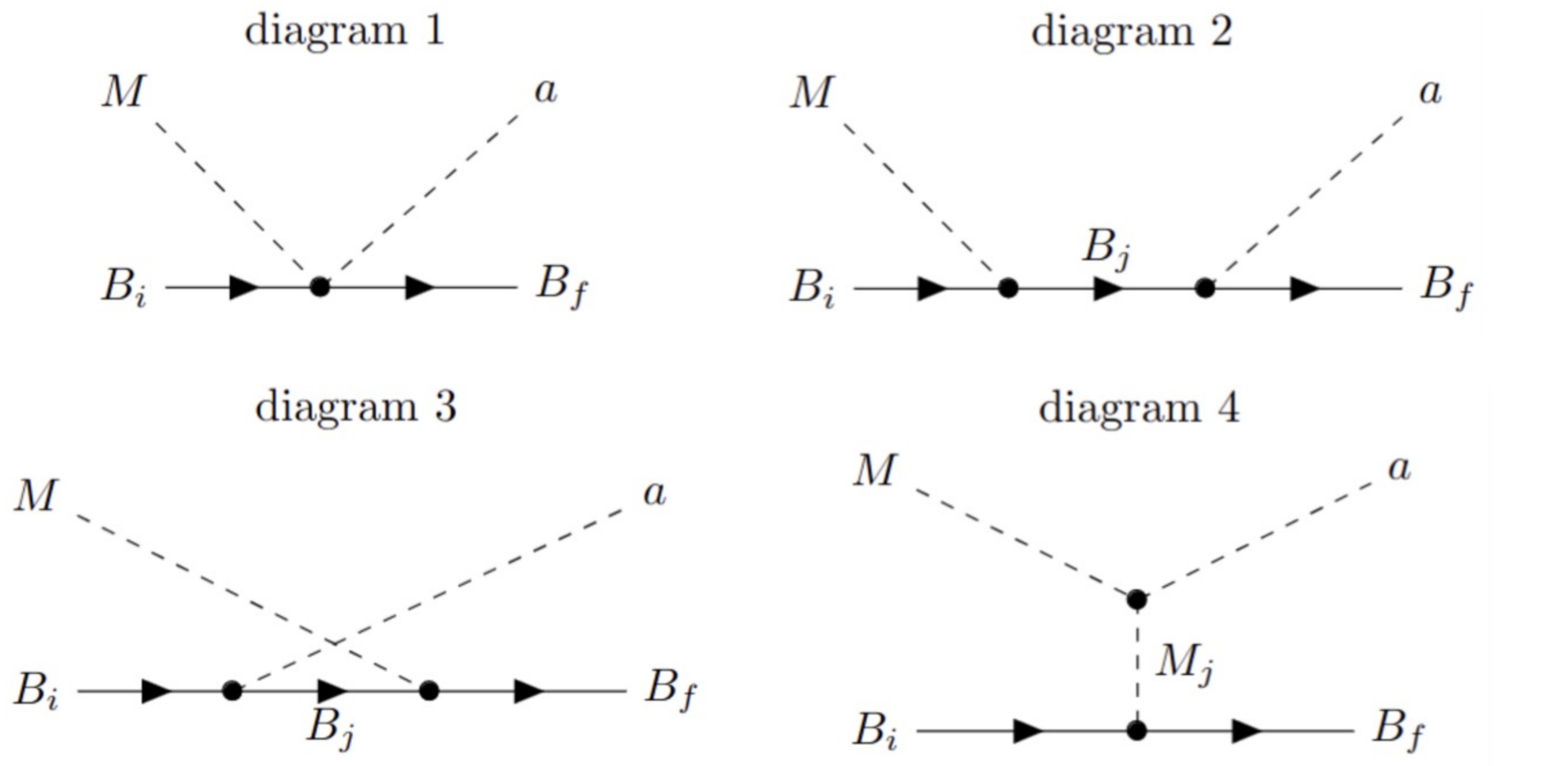


FIG. 1: The diagrams contributing to $B_i\,M \to B_f\,a$, with $B_{i,f}$ initial- or final-state octet baryons, M octet mesons, and a the axion.

Georgi, Kaplan, Randall, PLB 1986

Starting point :

$$\mathcal{L}^a = \tfrac{1}{2}(\partial^\mu a)(\partial_\mu a)$$

$$+ (\partial^\mu a/f)\left(x_\varphi \varphi^\dagger i\overleftrightarrow{D}_\mu \varphi + \sum_\psi \bar{\psi}_L \gamma_\mu X_\psi \psi_L\right)$$

$$- (a/f)[(g_3^2/32\pi^2)G\tilde{G} + C_{aWW}(g_2^2/32\pi^2)W\tilde{W}$$

$$+ C_{aYY}(g_1^2/32\pi^2)Y\tilde{Y}] \ .$$

Georgi, Kaplan, Randall, PLB 1986

At 1 GeV :

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu a)(\partial^\mu a)$$
$$- \bar{q}_R \exp(iaQ_A/f) M \exp(iaQ_A/f) q_L + \text{h.c.}$$
$$+ (\partial_\mu a/f)\bar{q}\gamma^\mu[(X_V + Q_V) + (X_A + Q_A)\gamma_5]q$$
$$- (a/f)[(e^2/32\pi^2)C_{a\gamma\gamma}F\tilde{F}] + \text{leptonic terms.}$$

To be matched with

$$\mathcal{L} = \tfrac{1}{4}f_\pi^2\,\text{tr}(D^\mu\Sigma D_\mu\Sigma^\dagger) + \tfrac{1}{2}f_\pi^2\mu\,\text{tr}(M\Sigma^\dagger) + \text{h.c.},$$
$$D^\mu\Sigma = \partial^\mu\Sigma + ieA^\mu[Q, \Sigma]$$

matrix $\Sigma = \exp(2i\Pi_a T_a/f_\pi)$, where the Π_a are the pseudoscalar meson fields, the T_a are the Gell-Mann matrices and $f_\pi \simeq 93$ MeV. The Σ field transforms as $\Sigma \to L\Sigma R^\dagger$ under $SU(3)_L \times SU(3)_R$.

Georgi, Kaplan, Randall, PLB 1986

the vector and axial vector currents are

$$j^{\mu}_{Va} = \mathrm{i}\tfrac{1}{2} f_{\pi}^{2}\, \mathrm{tr}[T_a(\Sigma D^{\mu}\Sigma^{\dagger} + \Sigma^{\dagger} D^{\mu}\Sigma)] \;,$$

$$j^{\mu}_{Aa} = \mathrm{i}\tfrac{1}{2} f_{\pi}^{2}\, \mathrm{tr}[T_a(\Sigma D^{\mu}\Sigma^{\dagger} - \Sigma^{\dagger} D^{\mu}\Sigma)] \;.$$

$$\begin{aligned} &2(\partial_{\mu}a/f) \sum_{a=1} \{ j^{\mu}_{Va}\, \mathrm{tr}[T_a(X_V + Q_V)] \\ &\quad + j^{\mu}_{Aa}\, \mathrm{tr}[T_a(X_A + Q_A)] \} \\ &\quad + \tfrac{1}{2} f_{\pi}^{2} \mu [\mathrm{i}(a/f)\, \mathrm{tr}(\{M, Q_A\}\Sigma) \qquad (12) \\ &\quad - \tfrac{1}{2}(a/f)^2\, \mathrm{tr}(\{\{M, Q_A\}, Q_A\}\Sigma) + O[(a/f)^3] + \mathrm{h.c.}] \,. \end{aligned}$$

Free streaming

$$Q_a = \int E_a \frac{d^3 p_a}{(2\pi)^3} \Gamma_a = \int dE_a \frac{|p_a| E_a^2}{2\pi^2} e^{E_a/T} \Gamma_a \tag{1}$$

Moreover, we know that

$$Q_a = \int dE_a E_a \frac{d\dot{n}_a}{dE_a} \tag{2}$$

Thus,

$$\Gamma_a = 2\pi^2 \frac{e^{E_a/T}}{E_a|p_a|} \frac{d\dot{n}_a}{dE_a} = \frac{2\pi^2}{V\Delta t} \frac{e^{-E_a/T}}{E_a|p_a|} \frac{dN_a}{dE_a} \tag{3}$$

The mean free path is defined as $l_a = 1/\Gamma_a$, which leads to

$$l_a > R = \left(\frac{3V}{4\pi}\right)^{1/3} \Leftrightarrow \frac{2\pi^2}{V\Delta t} \frac{e^{-E_a/T}}{E_a|p_a|} \frac{dN_a}{dE_a} < \left(\frac{4\pi}{3V}\right)^{1/3} \tag{4}$$

$$\Leftrightarrow \frac{e^{-E_a/T}}{E_a|p_a|} \frac{dN_a}{dE_a} < \left(\frac{4\pi}{3V}\right)^{1/3} \frac{V\Delta t}{2\pi^2} = 1.2 \times 10^{55}\ \text{MeV}^{-3} \tag{5}$$

The function $\frac{e^{-E_a/T}}{E_a|p_a|}$ is decreasing with energy and increasing with temperature. Since the support of the axionic emission spectrum starts at approximately $E_a \simeq 50$ MeV and $T \in \{30; 40\}$ MeV, we have $\frac{e^{-E_a/T}}{E_a|p_a|} \lesssim 10^{-4}\ \text{MeV}^{-2}$. Therefore, to remain in the free-streaming regime, it is sufficient that

$$\frac{dN_a}{dE_a} < 1.2 \times 10^{59}\ \text{MeV}^{-1} \tag{6}$$

Analogous formulation for mesons – Bauer et al PRL '21

At 1 GeV :

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} + c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial^\mu a}{f} \left(\bar{q}_L \boldsymbol{k}_Q \gamma_\mu q_L + \bar{q}_R \boldsymbol{k}_q \gamma_\mu q_R + \dots \right). \quad (1)$$

Chiral rotation of the quarks :

$$q(x) \to \exp\left[-i \left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5 \right) c_{GG} \frac{a(x)}{f} \right] q(x)$$

δ and κ are hermitian matrices, which describe exactly the same physics !

Analogous formulation for mesons – Bauer et al PRL '21

To perform computations :

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\text{Tr}\left[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4}\,B_0\,\text{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\right] + \frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\,,$$

Where

$$\hat{\boldsymbol{m}}_q(a) = \exp\left(-2i\boldsymbol{\kappa}_q c_{GG}\,\frac{a}{f}\right)\boldsymbol{m}_q$$

$$i\boldsymbol{D}_\mu\boldsymbol{\Sigma} = i\partial_\mu\boldsymbol{\Sigma} + e\,A_\mu\,[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(\hat{\boldsymbol{k}}_Q\,\boldsymbol{\Sigma} - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\right)$$

Covariant derivative à la Gasser- Leutwyler [NPB '85]

$$\mathcal{J}^{b\mu}_{L,R} = \underbrace{(\mathcal{J}^{(U)}_{L,R})^{b\mu}}_{\text{purely mesonic}} + \underbrace{(\mathcal{J}^{(B)}_{L,R})^{b\mu}}_{\text{baryonic-plus-n mesons part}}$$

$$(\mathcal{J}^{(U)}_{L})^{\mu b} = +i\frac{F_0^2}{2}\,\mathrm{Tr}\left[\frac{\lambda^b}{2}(D^\mu U)^\dagger U\right]\,, \qquad (\mathcal{J}^{(U)}_{R})^{\mu b} = -i\frac{F_0^2}{2}\,\mathrm{Tr}\left[\frac{\lambda^b}{2}U(D^\mu U)^\dagger\right]$$

$$(\mathcal{J}^{(B)}_{L,R})^{b\mu} = (\mathcal{J}^{(B,K_1)}_{L,R})^{\mu b} + (\mathcal{J}^{(B,K_2)}_{L,R})^{\mu b} + (\mathcal{J}^{(B,D)}_{L,R})^{\mu b} + (\mathcal{J}^{(B,F)}_{L,R})^{\mu b}\,.$$

$$(\mathcal{J}^{(B,K_1)}_{L,R})^{\mu b} = \frac{1}{4}\,\mathrm{Tr}\Big(\bar{B}\gamma^\mu\left[\lambda^b \mp \frac{f^{\lambda\phi b}}{2F_0}, B\right]\Big)\,,$$

$$(\mathcal{J}^{(B,K_2)}_{L})^{\mu b} = -\frac{1}{8}\,\mathrm{Tr}\left(\bar{B}\gamma^\mu\left[\lambda^b - \frac{f^{\lambda\phi b}}{2F_0} - u\left(\lambda^b - \frac{f^{\lambda\phi b}}{2F_0}\right)u^\dagger, B\right]\right)\,,$$

$$(\mathcal{J}^{(B,K_2)}_{R})^{\mu b} = -\frac{1}{8}\,\mathrm{Tr}\left(\bar{B}\gamma^\mu\left[\lambda^b + \frac{f^{\lambda\phi b}}{2F_0} - u^\dagger\left(\lambda^b + \frac{f^{\lambda\phi b}}{2F_0}\right)u, B\right]\right)\,,$$

$$(\mathcal{J}^{(B,D)}_{L})^{\mu b} = +\frac{D}{8}\,\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5\left\{\lambda^b - \frac{f^{\lambda\phi b}}{2F_0} + u\left(\lambda^b - \frac{f^{\lambda\phi b}}{2F_0}\right)u^\dagger, B\right\}\right)$$

$$(\mathcal{J}^{(B,D)}_{R})^{\mu b} = -\frac{D}{8}\,\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5\left\{\lambda^b + \frac{f^{\lambda\phi b}}{2F_0} + u^\dagger\left(\lambda^b + \frac{f^{\lambda\phi b}}{2F_0}\right)u, B\right\}\right)$$

We adhere to the notation in Ref. [61], in particular we define the mesonic field $U = \exp(i\phi/F_0)$, with covariant derivative $D_\mu U = \partial_\mu U - i\frac{\partial_\mu a}{f_a}\left(\hat{\boldsymbol{k}}_R U - U\hat{\boldsymbol{k}}_L\right) - ieA_\mu [Q, U]$ and $Q = \mathrm{diag}(Q_u, Q_d, Q_s)$. We normalize the 3-flavour pion-field matrix $\phi = \phi^a \lambda^a$ (where λ^a are the Gell-Mann matrices), so that e.g. the (1,1) entry is $\pi_0 + \eta_0/\sqrt{3}$, and $F_0 \approx 93$ MeV. We assume the chiral-symmetry transformation to be $U \to RUL^\dagger$. Also, we use $u = \sqrt{U}$, and the shortcut notation $f^{XYc} = f^{abc}X^aY^b$, with f the $SU(3)$ structure constants, and a, b, c SU(3) indices in the adjoint representation. Again, we normalize the hyperon field $B = B^a\lambda^a$ as in [61], such that e.g. the (1,1) entry is $\Sigma^0/\sqrt{2} + \Lambda/\sqrt{6}$ and we take $D = 0.81$ and $F = 0.44$ [62].

Class	**Processes**
$N\,\pi \to N\,a$	$n\,\pi^0 \to n\,a$ ‡ †, $n\,\pi^+ \to p\,a$, $p\,\pi^- \to n\,a$, $p\,\pi^0 \to p\,a$ ‡ †,
$N\,\pi \to \Lambda\,a$	$n\,\pi^0 \to \Lambda\,a$, $p\,\pi^- \to \Lambda\,a$,
$N\,\pi \to \Sigma\,a$	$n\,\pi^- \to \Sigma^-\,a$, $n\,\pi^0 \to \Sigma^0\,a$, $n\,\pi^+ \to \Sigma^+\,a$ ‡ †, $p\,\pi^- \to \Sigma^0\,a$, $p\,\pi^0 \to \Sigma^+\,a$,
$N\,K \to N\,a$	$n\,K^0 \to n\,a$, $n\,\bar{K}^0 \to n\,a$, $n\,K^+ \to p\,a$, $p\,K^- \to n\,a$, $p\,K^0 \to p\,a$, $p\,\bar{K}^0 \to p\,a$,
$N\,K \to \Lambda\,a$	$n\,\bar{K}^0 \to \Lambda\,a$, $p\,K^- \to \Lambda\,a$,
$N\,K \to \Sigma\,a$	$n\,K^- \to \Sigma^-\,a$, $n\,\bar{K}^0 \to \Sigma^0\,a$, $p\,K^- \to \Sigma^0\,a$, $p\,\bar{K}^0 \to \Sigma^+\,a$,
$N\,K \to \Xi\,a$	$n\,K^- \to \Xi^-\,a$ ‡ †, $n\,\bar{K}^0 \to \Xi^0\,a$ ‡ †, $p\,K^- \to \Xi^0\,a$ ‡ †,
$N\,\eta \to N\,a$	$n\,\eta \to n\,a$ ‡ †, $p\,\eta \to p\,a$ ‡ †,
$N\,\eta \to \Lambda\,a$	$n\,\eta \to \Lambda\,a$,
$N\,\eta \to \Sigma\,a$	$n\,\eta \to \Sigma^0\,a$, $p\,\eta \to \Sigma^+\,a$,

TABLE II: $B_i\,M \to B_f\,a$ processes, with $B_i = N$.

$\Lambda\,\pi \to N\,a$	$\Lambda\,\pi^0 \to n\,a$, $\Lambda\,\pi^+ \to p\,a$,
$\Lambda\,\pi \to \Lambda\,a$	$\Lambda\,\pi^0 \to \Lambda\,a$ ‡ †,
$\Lambda\,\pi \to \Sigma\,a$	$\Lambda\,\pi^- \to \Sigma^-\,a$, $\Lambda\,\pi^0 \to \Sigma^0\,a$ ‡ †, $\Lambda\,\pi^+ \to \Sigma^+\,a$,
$\Lambda\,\pi \to \Xi\,a$	$\Lambda\,\pi^- \to \Xi^-\,a$, $\Lambda\,\pi^0 \to \Xi^0\,a$,
$\Lambda\,K \to N\,a$	$\Lambda\,K^0 \to n\,a$, $\Lambda\,K^+ \to p\,a$,
$\Lambda\,K \to \Lambda\,a$	$\Lambda\,K^0 \to \Lambda\,a$, $\Lambda\,\bar{K}^0 \to \Lambda\,a$,
$\Lambda\,K \to \Sigma\,a$	$\Lambda\,K^- \to \Sigma^-\,a$, $\Lambda\,K^0 \to \Sigma^0\,a$, $\Lambda\,\bar{K}^0 \to \Sigma^0\,a$, $\Lambda\,K^+ \to \Sigma^+\,a$,
$\Lambda\,K \to \Xi\,a$	$\Lambda\,K^- \to \Xi^-\,a$, $\Lambda\,\bar{K}^0 \to \Xi^0\,a$,
$\Lambda\,\eta \to N\,a$	$\Lambda\,\eta \to n\,a$,
$\Lambda\,\eta \to \Lambda\,a$	$\Lambda\,\eta \to \Lambda\,a$ ‡ †,
$\Lambda\,\eta \to \Sigma\,a$	$\Lambda\,\eta \to \Sigma^0\,a$ ‡ †,
$\Lambda\,\eta \to \Xi\,a$	$\Lambda\,\eta \to \Xi^0\,a$,

TABLE III: $B_i\,M \to B_f\,a$ processes, with $B_i = \Lambda$.

Class	Processes
$\Sigma\,\pi \to N\,a$	$\Sigma^-\,\pi^+ \to n\,a$, $\Sigma^0\,\pi^0 \to n\,a$, $\Sigma^0\,\pi^+ \to p\,a$, $\Sigma^+\,\pi^- \to n\,a\ ^{\ddagger\,\dagger}$, $\Sigma^+\,\pi^0 \to p\,a$,
$\Sigma\,\pi \to \Lambda\,a$	$\Sigma^-\,\pi^+ \to \Lambda\,a$, $\Sigma^0\,\pi^0 \to \Lambda\,a\ ^{\ddagger\,\dagger}$, $\Sigma^+\,\pi^- \to \Lambda\,a$,
$\Sigma\,\pi \to \Sigma\,a$	$\Sigma^-\,\pi^0 \to \Sigma^-\,a\ ^{\ddagger\,\dagger}$, $\Sigma^-\,\pi^+ \to \Sigma^0\,a$, $\Sigma^0\,\pi^- \to \Sigma^-\,a$, $\Sigma^0\,\pi^0 \to \Sigma^0\,a\ ^{\ddagger\,\dagger}$, $\Sigma^0\,\pi^+ \to \Sigma^+\,a$, $\Sigma^+\,\pi^- \to \Sigma^0\,a$, $\Sigma^+\,\pi^0 \to \Sigma^+\,a\ ^{\ddagger\,\dagger}$,
$\Sigma\,\pi \to \Xi\,a$	$\Sigma^-\,\pi^0 \to \Xi^-\,a$, $\Sigma^-\,\pi^+ \to \Xi^0\,a\ ^{\ddagger\,\dagger}$, $\Sigma^0\,\pi^- \to \Xi^-\,a$, $\Sigma^0\,\pi^0 \to \Xi^0\,a$, $\Sigma^+\,\pi^- \to \Xi^0\,a$,
$\Sigma\,K \to N\,a$	$\Sigma^-\,K^+ \to n\,a$, $\Sigma^0\,K^0 \to n\,a$, $\Sigma^0\,K^+ \to p\,a$, $\Sigma^+\,K^0 \to p\,a$,
$\Sigma\,K \to \Lambda\,a$	$\Sigma^-\,K^+ \to \Lambda\,a$, $\Sigma^0\,K^0 \to \Lambda\,a$, $\Sigma^0\,\bar{K}^0 \to \Lambda\,a$, $\Sigma^+\,K^- \to \Lambda\,a$,
$\Sigma\,K \to \Sigma\,a$	$\Sigma^-\,K^0 \to \Sigma^-\,a$, $\Sigma^-\,\bar{K}^0 \to \Sigma^-\,a$, $\Sigma^-\,K^+ \to \Sigma^0\,a$, $\Sigma^0\,K^- \to \Sigma^-\,a$, $\Sigma^0\,K^0 \to \Sigma^0\,a$, $\Sigma^0\,\bar{K}^0 \to \Sigma^0\,a$, $\Sigma^0\,K^+ \to \Sigma^+\,a$, $\Sigma^+\,K^- \to \Sigma^0\,a$, $\Sigma^+\,K^0 \to \Sigma^+\,a$, $\Sigma^+\,\bar{K}^0 \to \Sigma^+\,a$,
$\Sigma\,K \to \Xi\,a$	$\Sigma^-\,\bar{K}^0 \to \Xi^-\,a$, $\Sigma^0\,K^- \to \Xi^-\,a$, $\Sigma^0\,\bar{K}^0 \to \Xi^0\,a$, $\Sigma^+\,K^- \to \Xi^0\,a$,
$\Sigma\,\eta \to N\,a$	$\Sigma^0\,\eta \to n\,a$, $\Sigma^+\,\eta \to p\,a$,
$\Sigma\,\eta \to \Lambda\,a$	$\Sigma^0\,\eta \to \Lambda\,a\ ^{\ddagger\,\dagger}$,
$\Sigma\,\eta \to \Sigma\,a$	$\Sigma^-\,\eta \to \Sigma^-\,a\ ^{\ddagger\,\dagger}$, $\Sigma^0\,\eta \to \Sigma^0\,a\ ^{\ddagger\,\dagger}$, $\Sigma^+\,\eta \to \Sigma^+\,a\ ^{\ddagger\,\dagger}$,
$\Sigma\,\eta \to \Xi\,a$	$\Sigma^-\,\eta \to \Xi^-\,a$, $\Sigma^0\,\eta \to \Xi^0\,a$,

TABLE IV: $B_i\,M \to B_f\,a$ processes, with $B_i = \Sigma$.

$\Xi\,\pi \to \Lambda\,a$	$\Xi^-\,\pi^+ \to \Lambda\,a$, $\Xi^0\,\pi^0 \to \Lambda\,a$,
$\Xi\,\pi \to \Sigma\,a$	$\Xi^-\,\pi^0 \to \Sigma^-\,a$, $\Xi^-\,\pi^+ \to \Sigma^0\,a$, $\Xi^0\,\pi^- \to \Sigma^-\,a\ ^{\ddagger\,\dagger}$, $\Xi^0\,\pi^0 \to \Sigma^0\,a$, $\Xi^0\,\pi^+ \to \Sigma^+\,a$,
$\Xi\,\pi \to \Xi\,a$	$\Xi^-\,\pi^0 \to \Xi^-\,a\ ^{\ddagger\,\dagger}$, $\Xi^-\,\pi^+ \to \Xi^0\,a$, $\Xi^0\,\pi^- \to \Xi^-\,a$, $\Xi^0\,\pi^0 \to \Xi^0\,a\ ^{\ddagger\,\dagger}$,
$\Xi\,K \to N\,a$	$\Xi^-\,K^+ \to n\,a\ ^{\ddagger\,\dagger}$, $\Xi^0\,K^0 \to n\,a\ ^{\ddagger\,\dagger}$, $\Xi^0\,K^+ \to p\,a\ ^{\ddagger\,\dagger}$,
$\Xi\,K \to \Lambda\,a$	$\Xi^-\,K^+ \to \Lambda\,a$, $\Xi^0\,K^0 \to \Lambda\,a$,
$\Xi\,K \to \Sigma\,a$	$\Xi^-\,K^0 \to \Sigma^-\,a$, $\Xi^-\,K^+ \to \Sigma^0\,a$, $\Xi^0\,K^0 \to \Sigma^0\,a$, $\Xi^0\,K^+ \to \Sigma^+\,a$,
$\Xi\,K \to \Xi\,a$	$\Xi^-\,K^0 \to \Xi^-\,a$, $\Xi^-\,\bar{K}^0 \to \Xi^-\,a\ ^{\perp}$, $\Xi^-\,K^+ \to \Xi^0\,a$, $\Xi^0\,K^- \to \Xi^-\,a$, $\Xi^0\,K^0 \to \Xi^0\,a$, $\Xi^0\,\bar{K}^0 \to \Xi^0\,a$,
$\Xi\,\eta \to \Lambda\,a$	$\Xi^0\,\eta \to \Lambda\,a$,
$\Xi\,\eta \to \Sigma\,a$	$\Xi^-\,\eta \to \Sigma^-\,a$, $\Xi^0\,\eta \to \Sigma^0\,a$,
$\Xi\,\eta \to \Xi\,a$	$\Xi^-\,\eta \to \Xi^-\,a\ ^{\ddagger\,\dagger}$, $\Xi^0\,\eta \to \Xi^0\,a\ ^{\ddagger\,\dagger}$,

TABLE V: $B_i\,M \to B_f\,a$ processes, with $B_i = \Xi$.

TABLE VI: Particle fractions in SN matter for the different thermodynamic conditions considered, as specified in the first two columns, and $Y_{Q_B} = 0.3$. Only particle fractions $> 10^{-5}$ are listed.

	T (MeV)	Y_e	Y_n	Y_p	Y_Λ	Y_{Σ^-}	Y_{Σ^0}	Y_{Σ^+}	Y_{Ξ^-}	Y_{Ξ^0}	Y_{π^-}	Y_{π^0}	Y_{π^+}
DD2Y $n_B = n_{\text{sat}}$	30	0.298	0.696	0.300	0.004	2×10^{-4}	5×10^{-5}	1×10^{-5}	3×10^{-5}	1×10^{-5}	0.002	2×10^{-4}	2×10^{-5}
DD2Y $n_B = n_{\text{sat}}$	40	0.295	0.680	0.302	0.014	0.002	6×10^{-4}	2×10^{-4}	5×10^{-4}	2×10^{-4}	0.006	0.001	2×10^{-4}
SFHoY $n_B = n_{\text{sat}}$	30	0.298	0.697	0.300	0.003	1×10^{-4}	4×10^{-5}	1×10^{-5}	2×10^{-5}		0.002	2×10^{-4}	2×10^{-5}
SFHoY $n_B = n_{\text{sat}}$	40	0.294	0.686	0.301	0.011	0.001	5×10^{-4}	2×10^{-4}	3×10^{-4}	1×10^{-4}	0.006	0.001	2×10^{-4}
DD2Y $n_B = 1.5\, n_{\text{sat}}$	30	0.299	0.690	0.301	0.009	5×10^{-4}	8×10^{-5}	1×10^{-5}	1×10^{-4}	3×10^{-5}	0.001	1×10^{-4}	1×10^{-5}
DD2Y $n_B = 1.5\, n_{\text{sat}}$	40	0.296	0.667	0.304	0.023	0.003	8×10^{-4}	2×10^{-4}	0.001	3×10^{-4}	0.004	8×10^{-4}	1×10^{-4}
SFHoY $n_B = 1.5\, n_{\text{sat}}$	30	0.299	0.697	0.300	0.003	8×10^{-5}	2×10^{-5}		2×10^{-5}		0.001	1×10^{-4}	1×10^{-5}
SFHoY $n_B = 1.5\, n_{\text{sat}}$	40	0.295	0.686	0.301	0.011	9×10^{-4}	3×10^{-4}	1×10^{-4}	4×10^{-4}	1×10^{-4}	0.005	8×10^{-4}	1×10^{-4}

Three kaon decays into final states w/ an axion

$$\frac{d^2\Gamma(K_S \to \pi^+\pi^- a)}{dsdt} = 3m_K \frac{\omega_\pi^2}{\eta_\pi}\left(\frac{\mathrm{Re}(k_A)_{23}}{f_a}\right)^2 + m_K \frac{\rho^2}{3\eta_\pi}\left(\frac{\mathrm{Im}(k_A)_{23}}{f_a}\right)^2 ,$$

$$\frac{d^2\Gamma(K^+ \to \pi^+\pi^0 a)}{dsdt} = 3m_K \frac{\omega_\pi^2}{\eta_\pi}\left(\frac{|(k_A)_{23}|}{f_a}\right)^2 ,$$

$$\frac{d^2\Gamma(K_S \to \ell^+\ell^- a)}{dsdt} = 6m_K \frac{\alpha_{\mathrm{em}}^2 |\hat{I}^{(a)}|^2 \omega_\ell}{\eta_\ell}\left(\frac{\mathrm{Re}(k_A)_{23}}{f_a}\right)^2 .$$

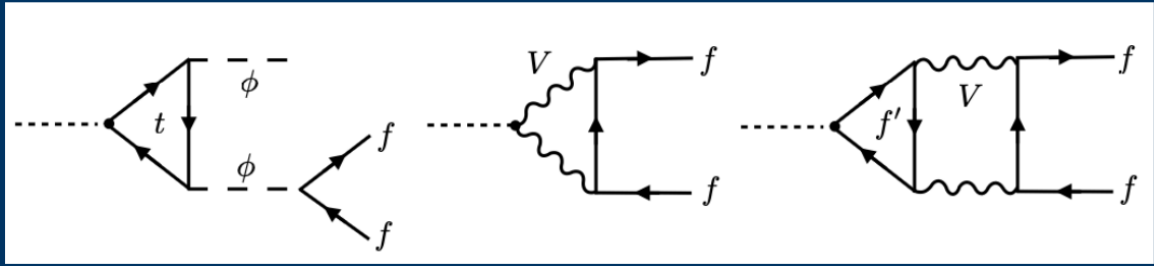
in units of $10^{-3}\,\mathrm{TeV}^{-1}$

	$m_a \ll m_\pi$			$m_a = m_\pi$		
Channel	$\frac{\lvert\mathrm{Re}[(k_A)_{23}]\rvert}{f_a}$	$\frac{\lvert\mathrm{Im}[(k_A)_{23}]\rvert}{f_a}$	$\frac{\lvert(k_A)_{23}\rvert}{f_a}$	$\frac{\lvert\mathrm{Re}[(k_A)_{23}]\rvert}{f_a}$	$\frac{\lvert\mathrm{Im}[(k_A)_{23}]\rvert}{f_a}$	$\frac{\lvert(k_A)_{23}\rvert}{f_a}$
$K_S \to \mu^+\mu^- a$	48	—	—	5.7	—	—
$K_S \to \pi^+\pi^- a$	3.0	1.0	—	3.7	0.57	—
$K^+ \to \pi^+\pi^0 a$	—	—	0.018	—	—	2.6

New Physics - axions

But axions have flavor violating couplings even if the UV theory is flavor diagonal : EW loops

$$\mathcal{L} = \frac{\partial_\mu a}{2f_a} C^A_{FF} \bar{\psi}\gamma^\mu\gamma_5\psi + c_{BB}\frac{\alpha_1}{8\pi f_a} a B_{\mu\nu}\tilde{B}^{\mu\nu} + c_{WW}\frac{\alpha_2}{8\pi f_a} a W^A_{\mu\nu}\tilde{W}^{\mu\nu,A} + c_{GG}\frac{\alpha_s}{8\pi f_a} a G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$

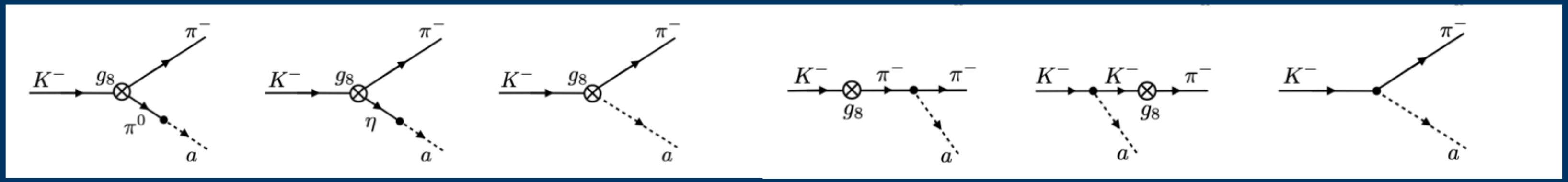


$$C^V_{ds}(\mu_c) = C^V_{ds}(\Lambda_{\rm UV}) + \sum_{F=t,c} \frac{y_F^2 V^*_{Fd} V_{Fs} C^A_{FF}}{16\pi^2}\left(\log\frac{\Lambda_{\rm UV}}{\mu_F} - f_F(x_F)\right) - \frac{g_2^4 N_2}{256\pi^4} V^*_{td} V_{ts} f_W(x_t),$$

$$N_2 = c_{WW}$$

New Physics - axions

But axions have flavor violating couplings even if the UV theory is flavor diagonal : QCD



$$
\begin{aligned}
i\mathcal{A}(K^- \to \pi^- a) &= \frac{N_8}{4f_a}\Big[8\,N_3\, m^2_{K-\pi}\xi_a + \left(4\,C^A_{ss} + 6\xi_a C^A_{uu+dd-2ss}\right) m_a^2 \\
&\quad + C^A_{2uu+dd+ss}\, m^2_{K-\pi-a} + C^V_{dd-ss}\, m^2_{K+\pi-a}\Big] - \frac{m^2_{K-\pi}}{2f_a} C^V_{ds}\,, \\
-i\sqrt{2}\mathcal{A}(\bar{K}^0 \to \pi^0 a) &= \frac{N_8}{4f_a}\Big[\left(8\,N_3\xi_a + C^A_{3dd+ss}\right) m^2_{K-\pi} + \left(C^A_{2uu-dd-ss} - 2\xi_a C^A_{uu+dd-2ss}\right) m_a^2 \\
&\quad - 2C^A_{uu-dd} m_a^2 \frac{m^2_{K-a}}{m^2_{\pi-a}} + C^V_{dd-ss} m^2_{K+\pi-a}\Big] - \frac{m^2_{K-\pi}}{2f_a} C^V_{ds},
\end{aligned}
$$

For K_L decays the CP conserving part is suppressed

$$K_L = \frac{(1+\epsilon)\,K^0 + (1-\epsilon)\,\bar{K}^0}{\sqrt{2(1+|\epsilon|^2)}}$$

$$|\epsilon| \approx 2 \times 10^{-3}$$

$$N_3 = c_{GG}$$

M. Bauer, presentation @ BFA 2025 Workshop

2503.05865

$$\mathcal{L} = \mathcal{L}_1 \mathcal{L}_2 \mathcal{L}_3$$

Poisson distribution for the total number of events

Poissonian constraint term for the number of bkg events

Multinomial piece for a shape analysis of the invisible inv. mass (the two components are distributed as a function of the invisible invariant mass squared)

$$\mathcal{L}_2 = \prod_{j=1}^{n_{\rm obs}} \left[\frac{n_b}{n_{\rm tot}} g_b(m_j^2) + \frac{n_a}{n_{\rm tot}} g_a(m_j^2) \right]$$

2503.05865

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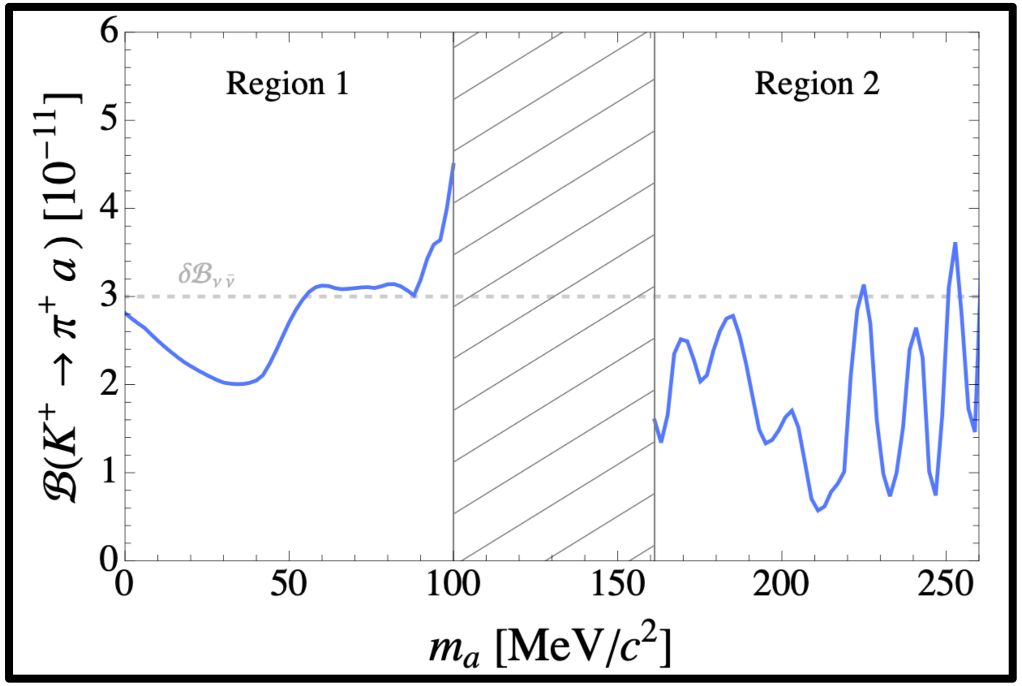
Poissonian constraint term for the number of bkg events



$$C_{aNN} \equiv C_{aNN}^{(D)}\ D + C_{aNN}^{(F)}\ F + C_{aNN}^{(S)}\ S$$

$$C_{ann}^{(D)} = \frac{2}{3} c_{GG} \frac{2-z}{1+z} + \frac{2(k_A)_{11} - (k_A)_{22} - (k_A)_{33}}{3} ,$$

$$C_{ann}^{(F)} = -2 c_{GG} \frac{z}{1+z} - (k_A)_{22} + (k_A)_{33} ,$$

$$C_{app}^{(D)} = -\frac{2}{3} c_{GG} \frac{1-2z}{1+z} - \frac{(k_A)_{11} - 2(k_A)_{22} + (k_A)_{33}}{3} ,$$

$$C_{app}^{(F)} = -2 \frac{c_{GG}}{1+z} - (k_A)_{11} + (k_A)_{33} ,$$

$$C_{app}^{(S)} = C_{ann}^{(S)} = \frac{2}{3} c_{GG} + \frac{(k_A)_{11} + (k_A)_{22} + (k_A)_{33}}{3} .$$

$$D = -\Delta_u/2 + \Delta_d - \Delta_s/2 = -0.813(43)$$

$$F = -\Delta_u/2 + \Delta_s/2 = -0.441(26) ,$$

$$S = \Delta_u + \Delta_d + \Delta_s = 0.405(62) ,$$

$$C_{ann} = 0.012(28) - 0.406(34)\ (k_A)_{11} + 0.848(26)\ (k_A)_{22} - 0.035(17)\ (k_A)_{33} ,$$

$$C_{app} = -0.452(28) + 0.848(34)\ (k_A)_{11} - 0.406(24)\ (k_A)_{22} - 0.035(17)\ (k_A)_{33}$$

Detector response

We estimate reco performance & signal detection at HK

→ PYTHIA 8 simulation of π^0 decays

\+

atmospheric-ν MCs

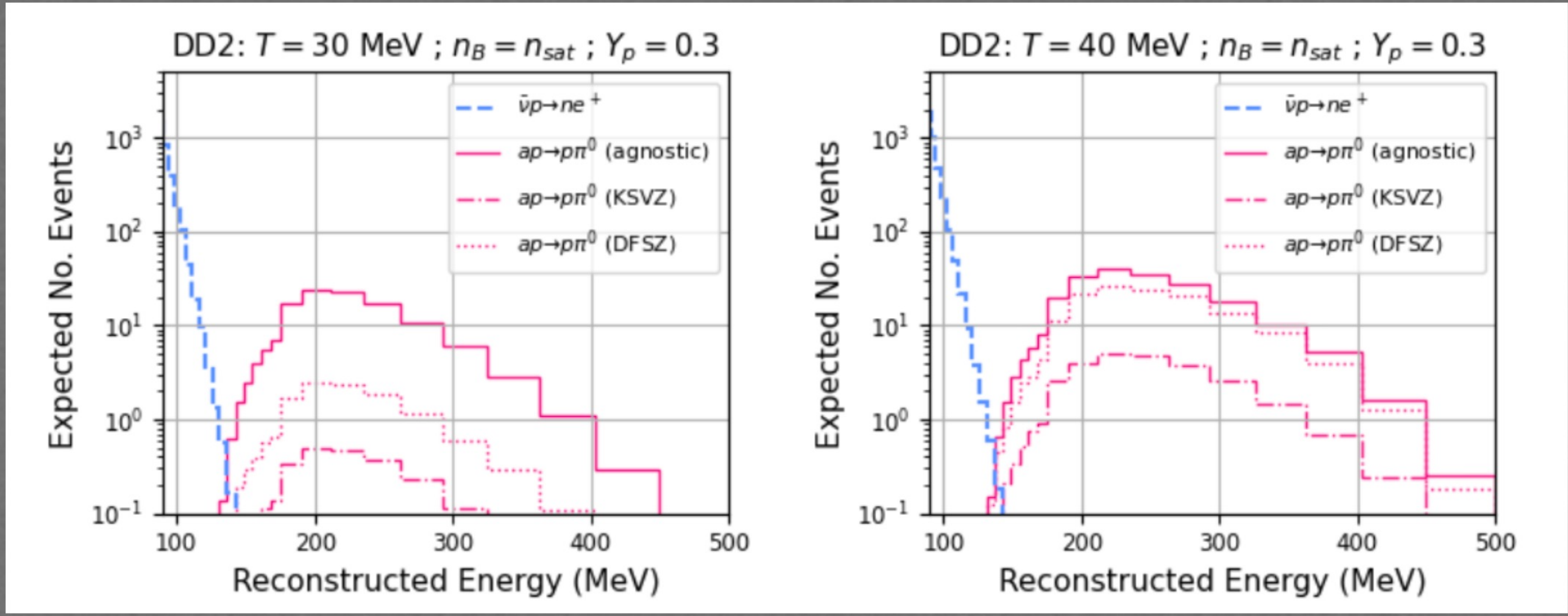
→ 5 topologies

Selection	Reconstructed as π^0 $T = 30$ MeV	Reconstructed as π^0 $T = 40$ MeV
$E_{\text{reco}} > 150$ MeV	0.974	0.985
$E_{\text{reco}} > 150$ MeV, 2-ring π^0-like	0.664	0.577
$E_{\text{reco}} > 150$ MeV, 1-ring π^0-like	0.148	0.204
$E_{\text{reco}} > 150$ MeV, 1-ring e-like	0.090	0.141
$E_{\text{reco}} > 150$ MeV, 1-ring μ-like	0.009	0.013
$E_{\text{reco}} > 150$ MeV, 2-ring other	0.064	0.050

- tiny $\bar{\nu}$-induced fractions in all categories

D. Guadagnoli's presentation @ workshop "The meV Mass Axion Frontier: Challenges and Opportunities"

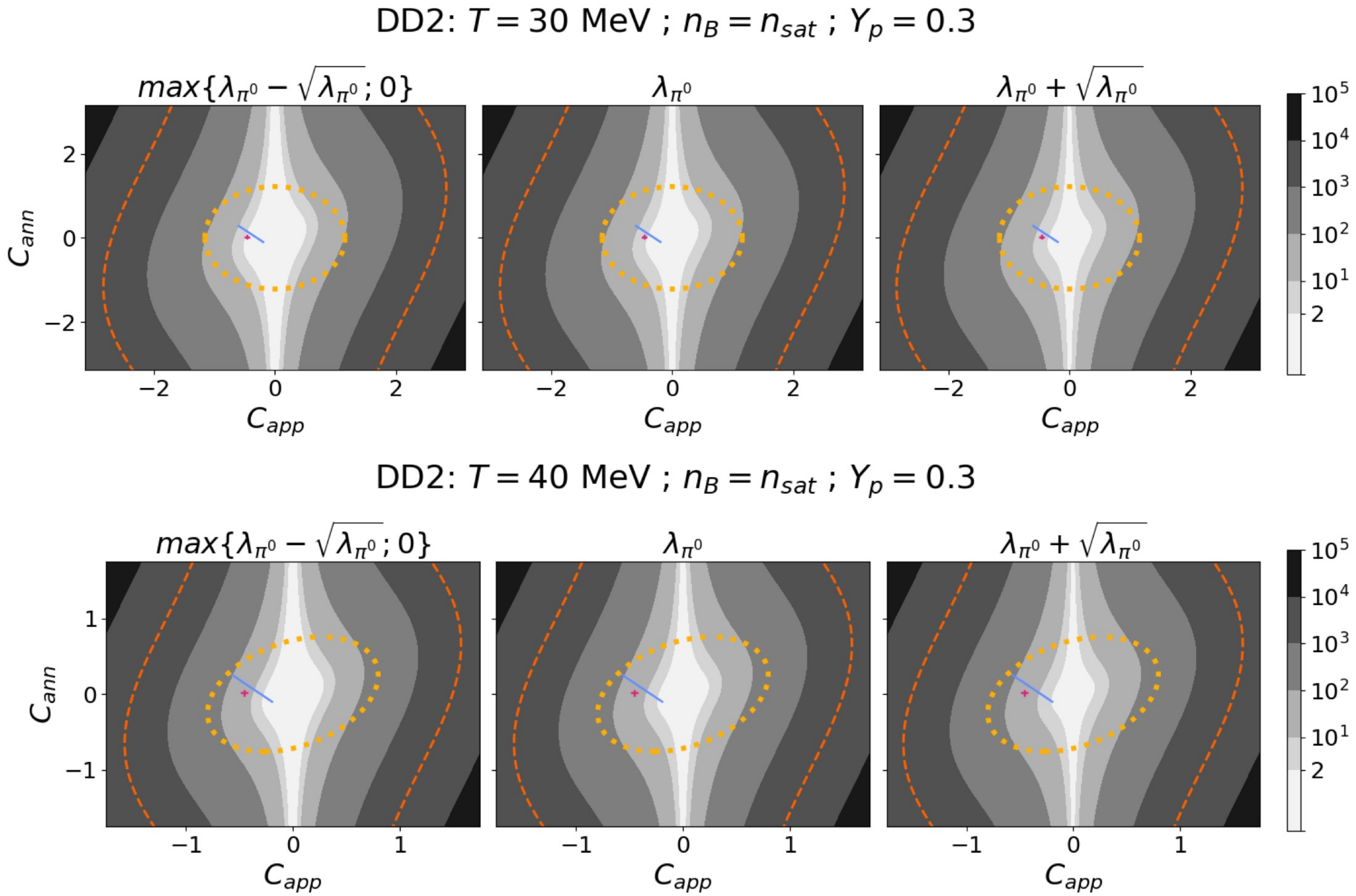
Detector-response corrected spectra



(injected spectra are those in previous figure)

= Main features remain unchanged as expected:

- peak positions
- relative peak heights



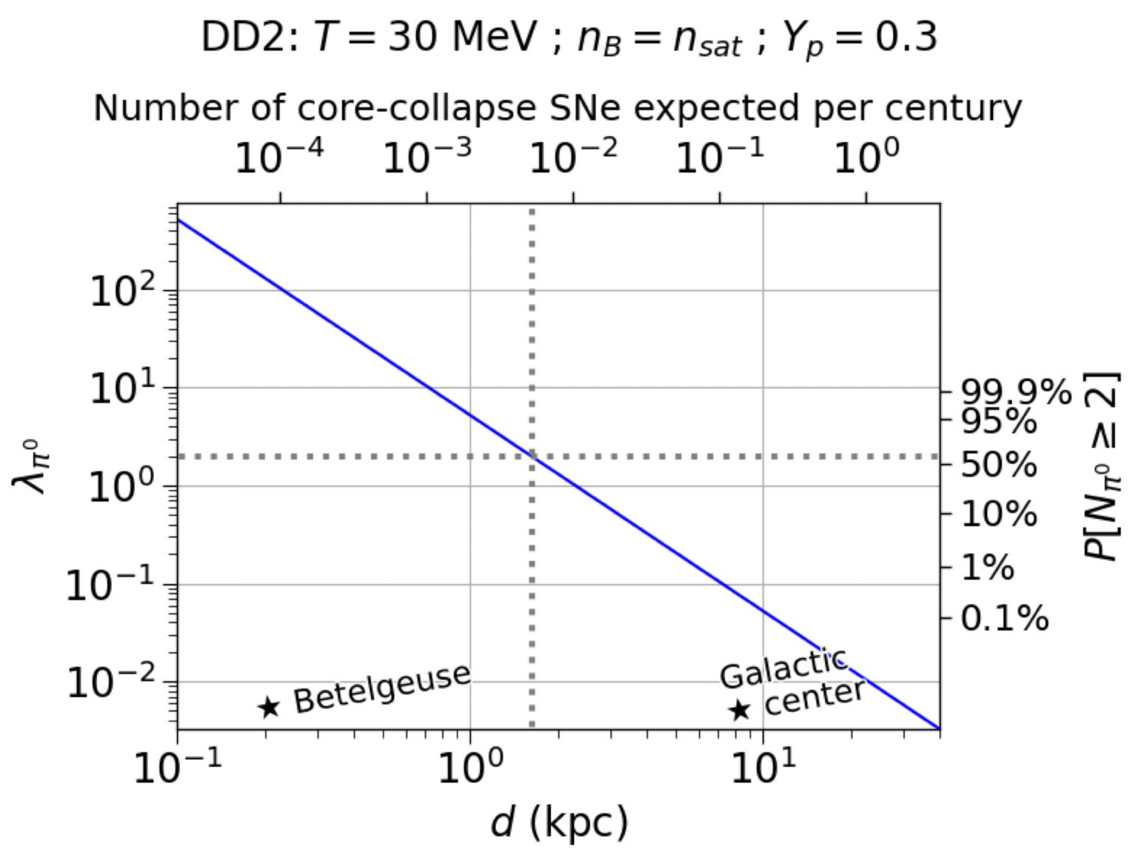
DD2: $T = 30$ MeV ; $n_B = n_{sat}$; $Y_p = 0.3$
$max\{\lambda_{\pi^0} - \sqrt{\lambda_{\pi^0}}; 0\}$
λ_{π^0}
$\lambda_{\pi^0} + \sqrt{\lambda_{\pi^0}}$
C_{ann}
C_{app}
10^5
10^4
10^3
10^2
10^1
2
DD2: $T = 40$ MeV ; $n_B = n_{sat}$; $Y_p = 0.3$
$max\{\lambda_{\pi^0} - \sqrt{\lambda_{\pi^0}}; 0\}$
λ_{π^0}
$\lambda_{\pi^0} + \sqrt{\lambda_{\pi^0}}$
C_{ann}
C_{app}
10^5
10^4
10^3
10^2
10^1
2

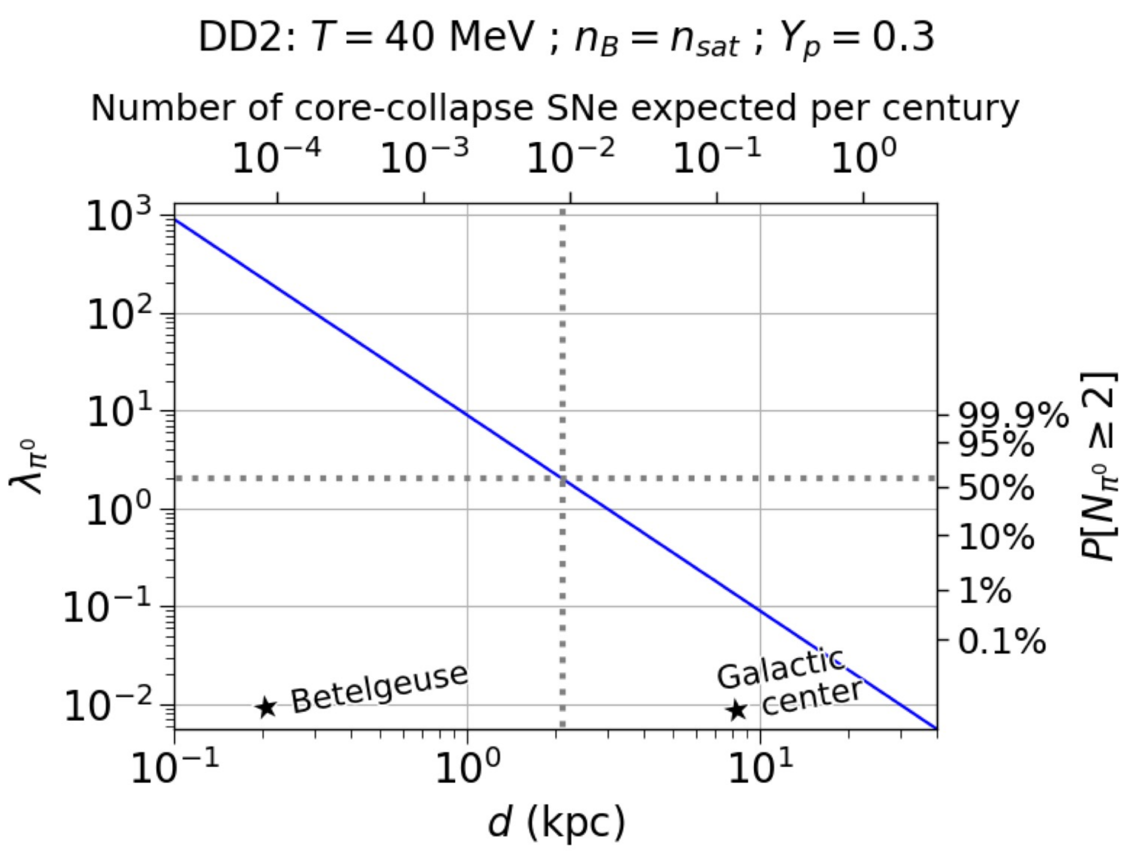
		DD2		SFHo	
T [MeV]	n_B	$\lvert C_{app}\rvert$	$\lvert C_{ann}\rvert$	$\lvert C_{app}\rvert$	$\lvert C_{ann}\rvert$
30	n_{sat}	2.1	2.0	1.8	1.7
30	$1.5\,n_{\text{sat}}$	2.0	1.9	1.5	1.5
40	n_{sat}	0.80	0.76	0.67	0.64
40	$1.5\,n_{\text{sat}}$	0.76	0.74	0.59	0.56

Table 2: Constraints on the absolute values of the axion couplings to nucleons $|C_{app}|$, $|C_{ann}|$ for $f_a = 10^9$ GeV for different EOS and for different thermodynamic conditions inside the SN core.

In turn, NS cooling data [32] imply the constraints are $|C_{app}| < 1.16$ and $|C_{ann}| < 1.22$ for $f_a = 10^9$ GeV.

Number of absorption events scales ad 1/d²: the larger *d*, the larger the number of SN candidates / time!





To evaluate the upper x-axis:

$$\dot{N}(d) = \left(\frac{d}{1\,\mathrm{kpc}}\right)^2 \times (2.0 \cdot 10^{-5}\mathrm{yr}^{-1})$$

Crude estimate! ☺

Tammann et al., Astrophys. J. Suppl. 92 (1994) 487