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Gauge Flexibility in PM based EOB models

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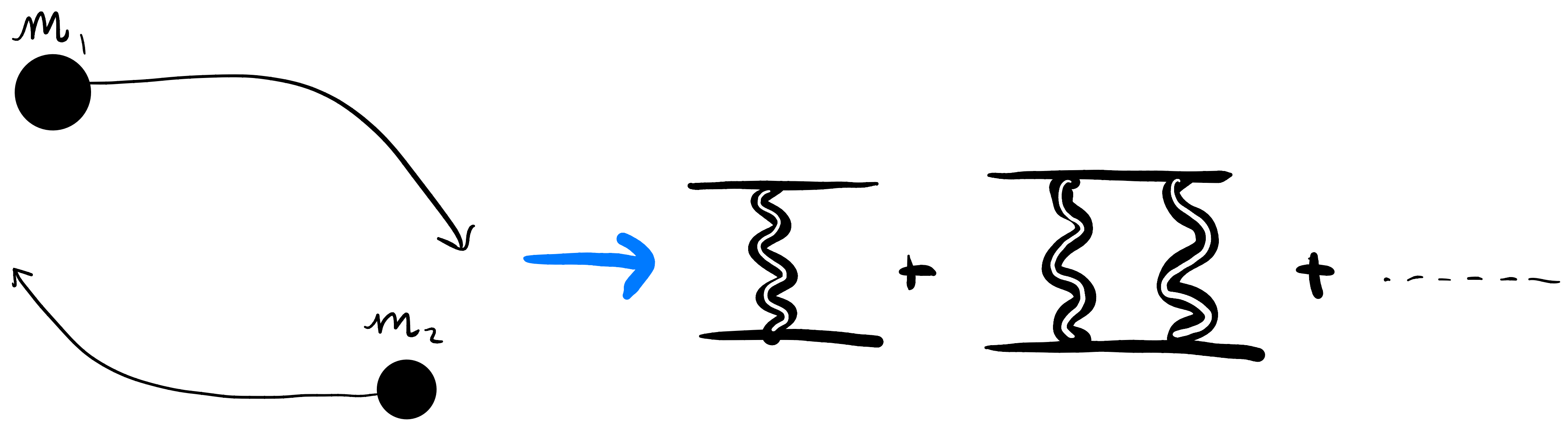
Lightning Introduction

Post-Minkowskian Theory

- Recent times have heralded many new results for PM 3PM Bern et. al. [2019], too many to list!
- Non-Spinning Radiative 5PM-1SF calculation has been completed, pushing the boundaries of current analytic information Driesse et. al. [2024]
- Spinning sector: All orders in spin results to 2PM Vines [2017], Aoude et. al. [2023] & quartic-in-spin at 3PM Akpinar et. al. [2025]. Higher orders currently restricted to spin-orbit terms.

$$\frac{GM}{rc^2} \ll 1$$



All orders in v

A key output is the scattering angle:

$$\theta^{n\text{PM}} = \sum_i 2\frac{\theta^{(i)}_{\text{PM}}}{p_\phi^i}$$

Lightning Introduction

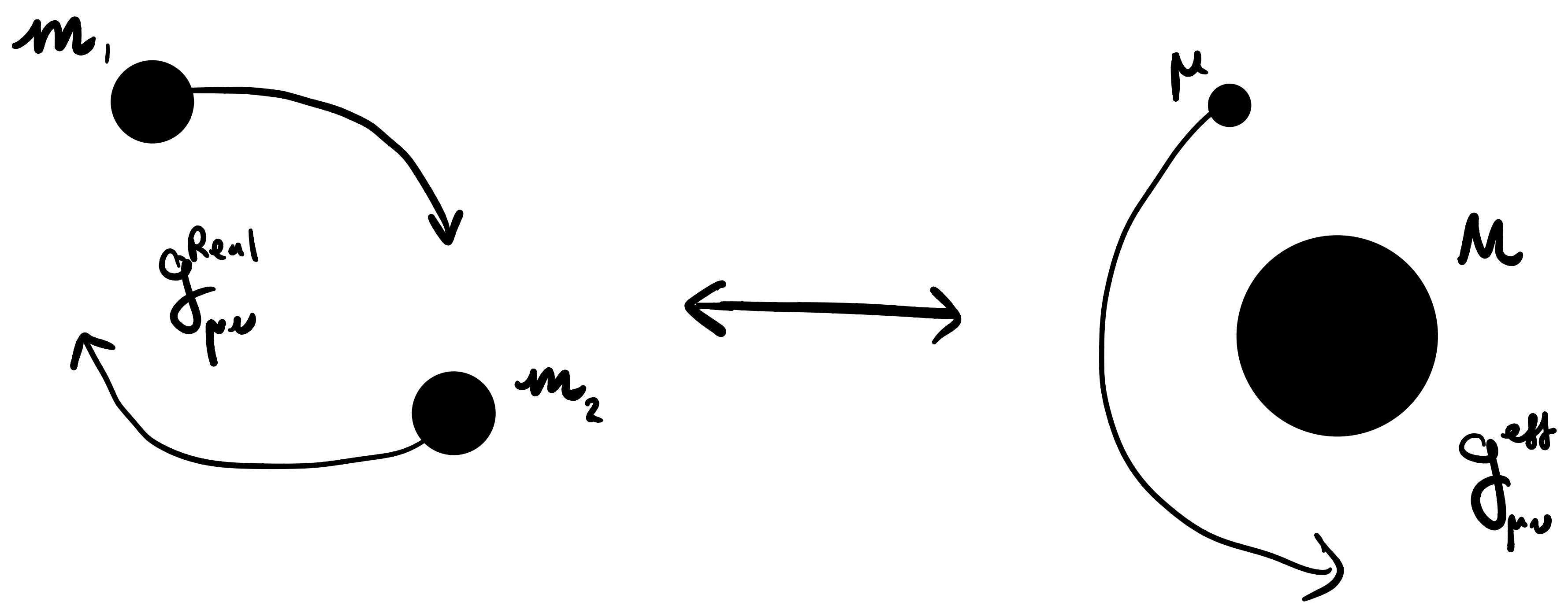
EOB-PM

- A key component of EOB is the mass-shell condition governing the dynamics Buonanno, Damour [1999]

$$g_{\text{eff}}^{\mu\nu} p_\mu p_\nu + \mu^2 + Q = 0$$

- This mass-shell condition involves many choices:

   1. Coordinate gauge: r vs $\bar{\text{r}}$
   2. "EOB Gauge": How is the mass-shell condition constructed
   3. Resummation Scheme - more on this later



EOB Gauges

- Introduce undetermined PM coefficients, which are determined by *matching to PM angles*

Post-Schwarzschild Damour [2018]	Post-Schwarzschild * Antonelli et. al. [2019]
$Q = \sum_i \frac{q_i}{r^i}$ $g^{\mu\nu}_{\text{Schw}} p_\mu p_\nu + \mu^2 + Q = 0$ PM deformations enter the Q term	No Q term: PM deformation enter the metric $g^{\mu\nu}_{\text{eff}} p_\mu p_\nu + \mu^2 = 0$ $A(r,\gamma,\nu) = A(r) + \sum_i \frac{\alpha_i}{r^i}$

Scattering angle -> Matching -> Coefficients

$$\theta^{n\text{PM}}_{\text{EOB}} = \theta^{n\text{PM}}$$

$$\theta^{\text{nPM}}_{\text{EOB}} = \pi - 2\ \text{PF} \int_{\text{r}_{\min}}^{\infty} \text{E}_\text{n} \left[\frac{\partial \text{p}_\text{r}}{\partial \text{p}_\phi} \right]$$

Resummation Scheme

- To test we focus on ***scattering trajectories.*** Solve the mass-shell condition for the radial momentum:

$$p_r^2 = (\gamma^2 - 1) - \frac{p_\phi^2}{r^2} + w(r, p_\phi, \gamma, \nu)$$

PM Expanded: w_{eob}

Damour, Rettegno [2022]; Rettegno + [2023]

$$w(r, p_\phi, \gamma, \nu) = \sum_i \frac{w_i(\gamma, \nu)}{r^i}$$

Resummed potential model:

$$w(r, p_\phi, \gamma, \nu) = p_r^2 - (\gamma^2 - 1) + \frac{p_\phi^2}{r^2}$$

Retains exact dependence on metric potentials - Used in SEOB-PM model

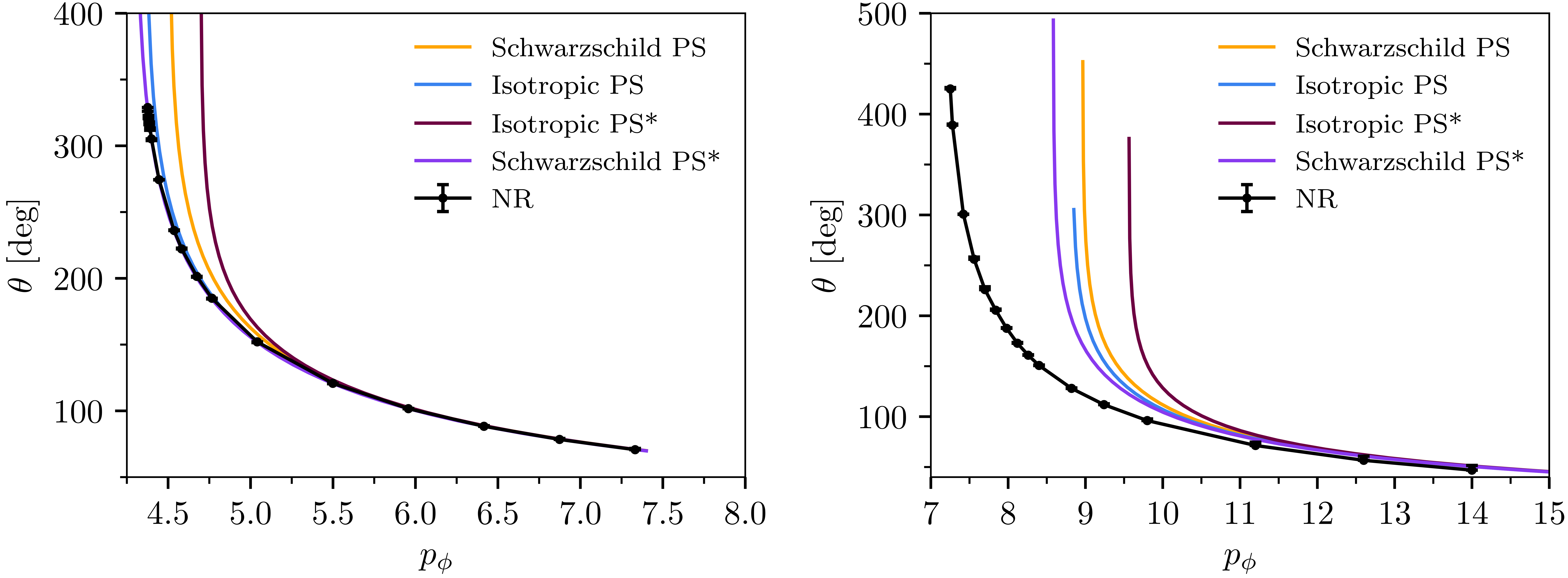
Buonanno, Jakobsen, Mogull [2024]

These definitions also depend on the choice of *radial coordinate*

Key comparison: The "gauge invariant" scattering angle:

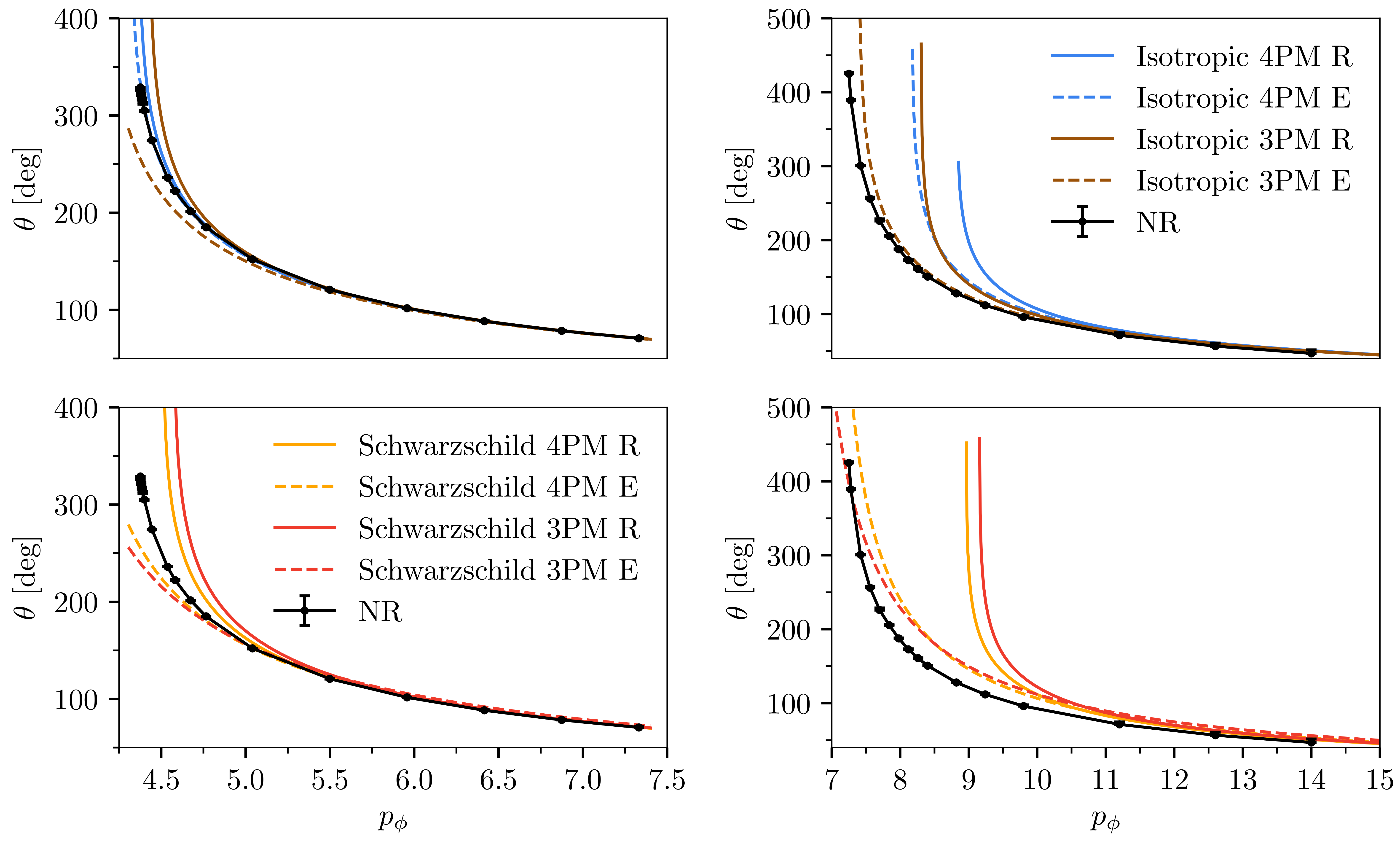
$$\theta^{\text{EOB-PM}} + \pi = -2\int_{r_{\min}}^{\infty} \frac{\partial p_r}{\partial p_\phi}$$

Key results: Non-Spinning Sector



"Resummed Potential" model: Varying EOB & Coordinate Gauge

Key Results: Non-Spinning Sector



- This shows the impact of ***resummation scheme***:
  - E - PM expanded w-potential
  - R - resummed potential model

EOB Gauge: PS

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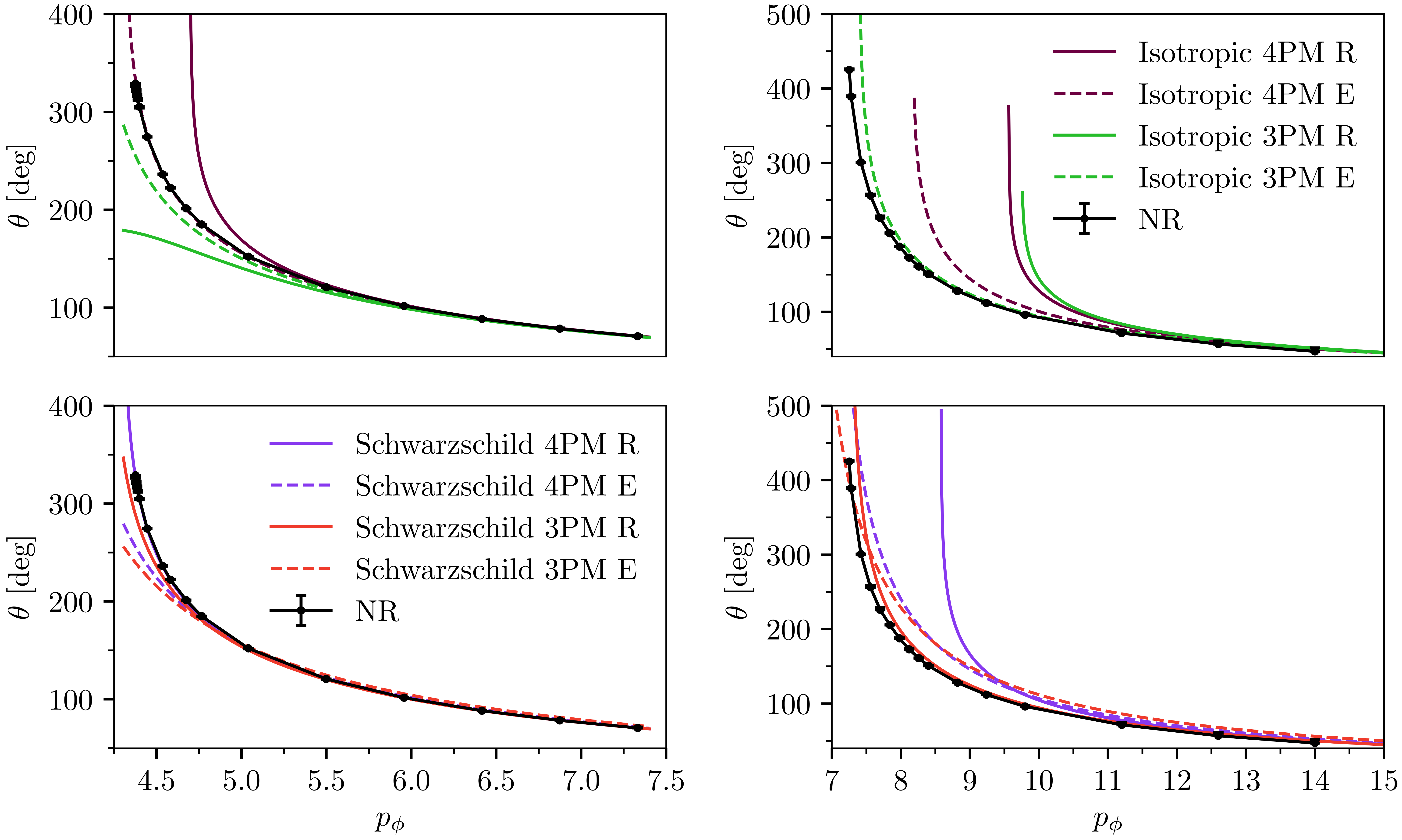
  EOB Gauge: PS*



Including Spins

The Centrifugal Radius

- Currently, spin information is included either by a dual PM/Spin expansion of the radial potential (w_{eob}) or through a gyro-gravitomagnetic terms (odd-spin corrections) and in the A - potential

Hamiltonian based approach (SEOB-PM):

$$g_{a_\pm} = r^2 \sum_{n\geq 2} \frac{\Delta g^{(n)}_{a_\pm}}{r^n}$$

$$\Delta g^{(n)}_{a_+} = \sum_{s=0}^{\lfloor \frac{n-2}{2} \rfloor} \sum_{i=0}^{s} \alpha^{(n)}_{(2(s-i)+1,2i)} \chi_+^{2(s-i)} \chi_-^{2i},$$

$$\Delta g^{(n)}_{a_-} = \sum_{s=0}^{\lfloor \frac{n-2}{2} \rfloor} \sum_{i=0}^{s} \alpha^{(n)}_{(2(s-i),2i+1)} \chi_+^{2(s-i)} \chi_-^{2i}.$$

$$A_{\text{eff}}(r,\gamma,a_\pm) = \frac{1 - \frac{2}{r} + \frac{\chi_+^2}{r^2} + \sum_n \frac{\Delta A^{(n)}}{r^n}}{1 + \frac{\chi_+^2}{r^2}\left(1+\frac{2}{r}\right)},$$

$$\Delta A^{(n)} = \sum_{s=0}^{\lfloor \frac{n-1}{2} \rfloor} \sum_{i=0}^{2s} \alpha_{(2s-i,i)} \delta^{\sigma(i)} \chi_+^{2s-i} \chi_-^{i},$$

$$H_{\text{eff}} = \frac{p_\phi (g_{\chi_+}\chi_+ + g_{\chi_-}\delta\chi_-)}{r^3 + a_+^2(r+2)} + \sqrt{A_{\text{eff}}\left(1 + \frac{p_\phi^2}{r^2} + (1 + B^{\text{Kerr}}_{np})p_r^2 + B^{\text{Kerr}}_{npa}\frac{p_\phi^2 \chi_+^2}{r^2}\right)}$$

w-potential approach:

$$w_n(p_\phi,\gamma,\nu,\chi_\pm) = w^{(n)}_{(0,0)}(\gamma,\nu) + w^{(n)}_{\text{SO}}(p_\phi,\gamma,\nu,\chi_\pm) + w^{(n)}_{\text{SS}}(p_\phi,\gamma,\nu,\chi_\pm).$$

Including Spins

The Centrifugal Radius

- We incorporated information into the *centrifugal radius* Damour, Nagar [2014] :

$$r_c^2 = r^2 + a^2 + \frac{2Ma^2}{r} + \delta a$$

Even in spin deformation go here

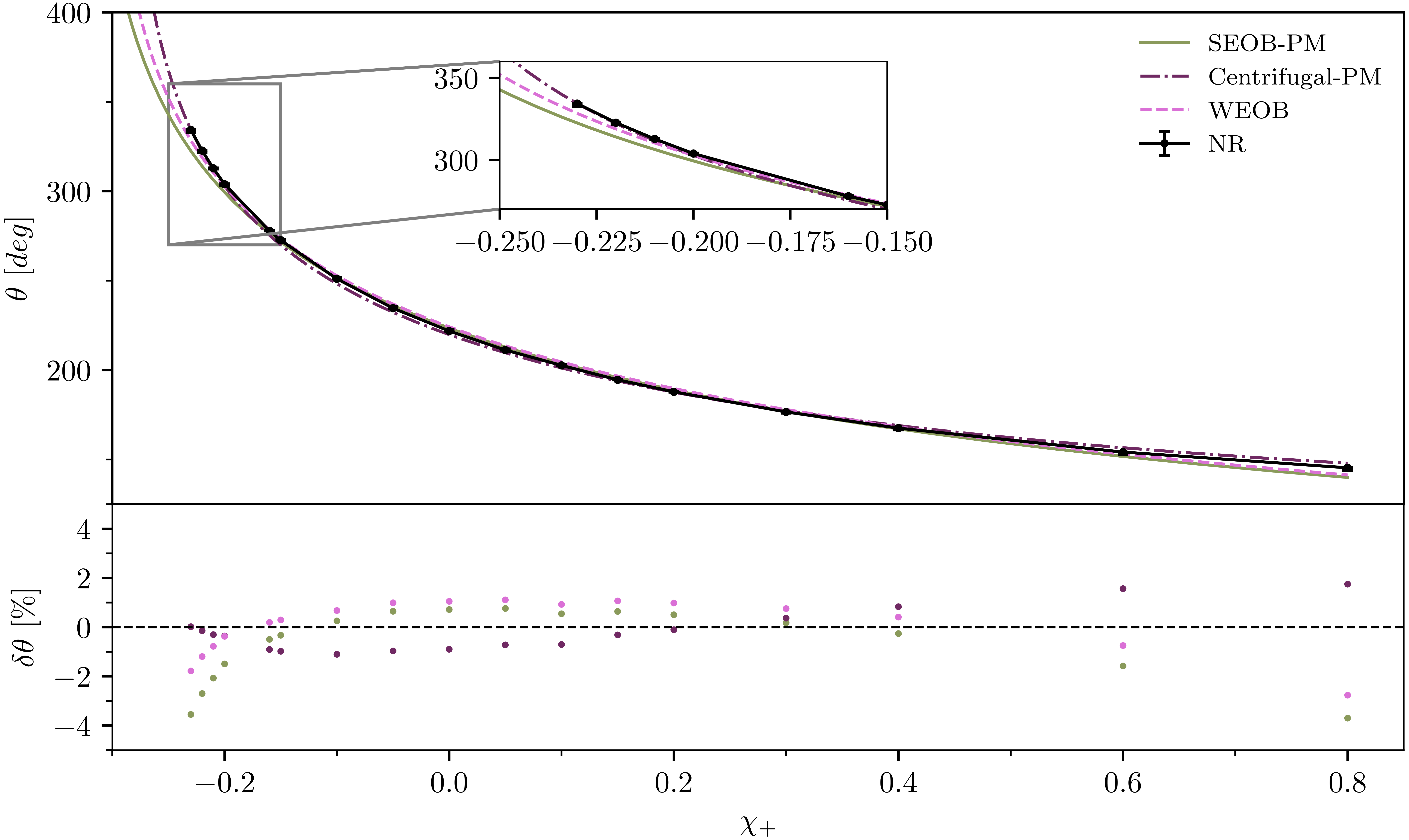
$$A_{\text{eff}} = \left(1 - \frac{2}{r_c} + \Delta A\right) \frac{1 + \frac{2}{r_c}}{1 + \frac{2}{r}}$$

Non-spinning deformations only

Odd in spin deformations in gyro-gravitomagnetic factors as before

Centrifugal Radius

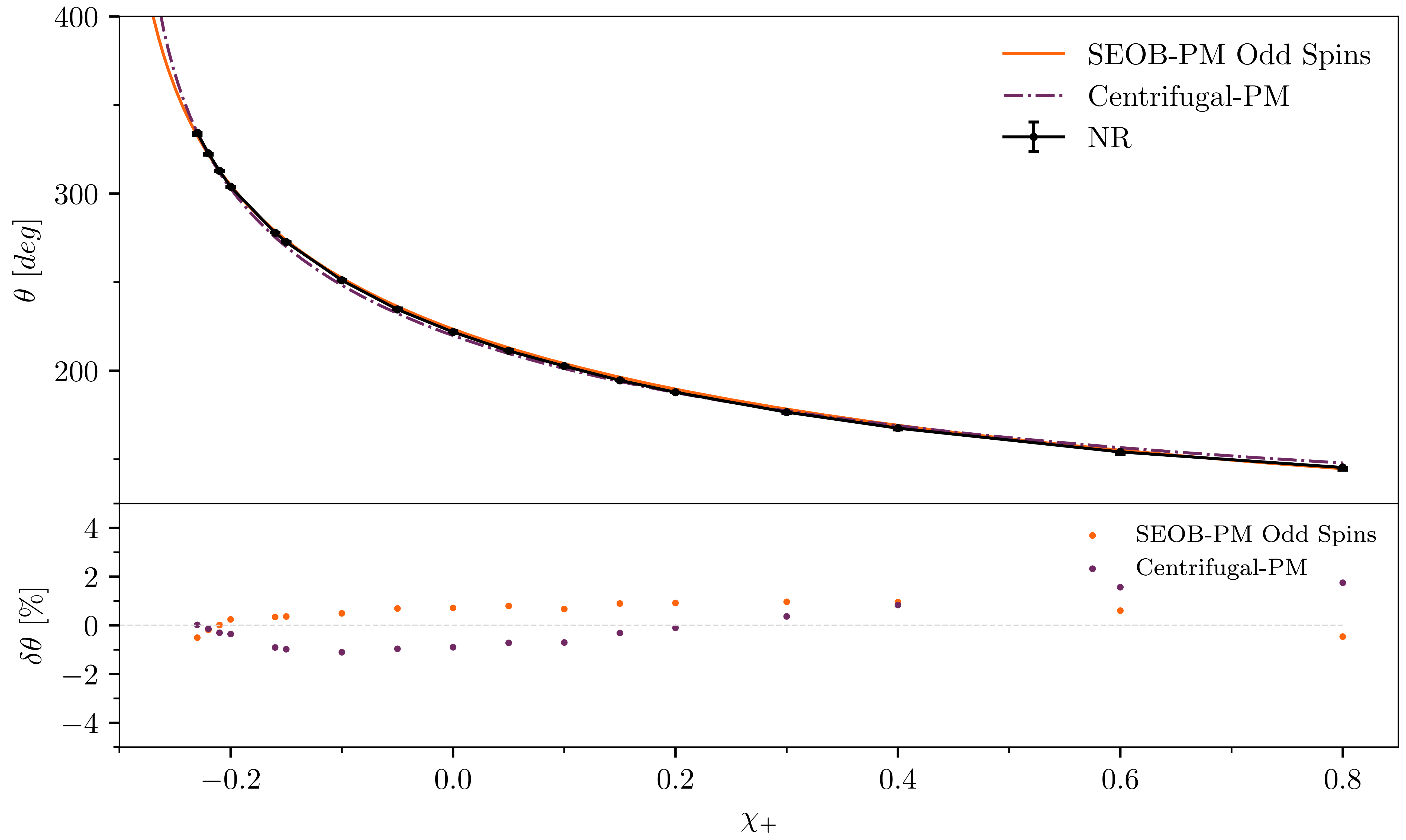
Performance



Including Spins

A Surprise...

- What if we don't include any spin corrections in the A-potential?
- This ad-hoc prescription simply switches off all of the even-in-spin information, leaving only the gyro-gravitomagnetic factors in play...



Key Takeaways

- Gauge choices can make or break an EOB-PM model - take care!
- For resummed models, there are pairs of gauge choices:
  (PS, Isotropic) & (PS*, Schwarzschild)
- There is also interaction between gauge choice and resummation scheme - Isotropic gauge works with a PM expanded w-potential whereas Schwarzschild requires resummation.
- In the spinning sector improvements can be made by using the *centrifugal radius to incorporate even in spin information.*
- Simply eliminating even in spin information is even better - there must be more to the story...