Spin precession across timescales

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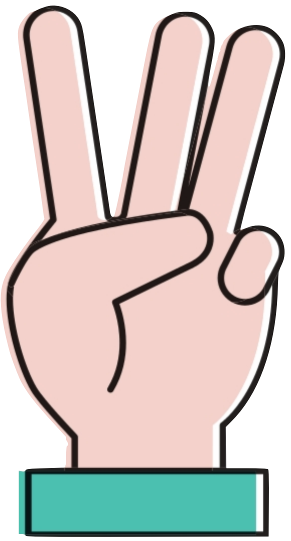
Black-hole binary spin precession

1. A sweet relativistic effect full of surprises
   Waveform modulations, frame dragging,
   resonances, instabilities, superkicks

2. A treasure trove for black-hole astrophysics
   Formation channels, supernova kicks, mass transfer, tides,
   core-envelope interactions, accretion disk physics

Three things today…



1. Precession-averaged PN dynamics

DG+ 2015, arXiv:1506.03492
DG+ 2023, arXiv:2304.04801

2. Up-down instability

DG+ 2015, arXiv:1506.09116
Mould, **DG** 2020, arXiv:2003.02281

3. What precession parameter

DG+ 2021, arXiv:2011.11948

(Eccentricity in the next talk by Giulia)

Fumagalli, **DG** 2023, arXiv:2310.16893

A tale of three timescales

1. **Orbital** motion → $t_{\rm orb} \propto (r/r_g)^{3/2}$ Kepler's third law
2. Spin & orbital-plane **precession** → $t_{\rm pre} \propto (r/r_g)^{5/2}$ Apostolatos et al 1994
3. GW emission and **inspiral** → $t_{\rm RR} \propto (r/r_g)^{4}$ Quadrupole formula Peters & Matthews 1963

if (Post-)Newtonian $r \gg r_g = GM/c^2$: timescale hierarchy

Orbit **<<** Precession **<<** Inspiral

BH binary multi-timescale analysis: **DG**+ 2015, 2023

1. Solve the dynamics (hopefully analytically) on the shorter time
2. Quasi-adiabatic evolution ("average") on the longer time

Two-spin PN dynamics

Orbit << Precession << Inspiral

- Three momenta, **9** components $\mathbf{L}, \mathbf{S_1}, \mathbf{S_2}$
- Spin magnitudes are const. Frame (3 constraints)

Spin precession is a **4D** problem: $r,\ \theta_1,\ \theta_2,\ \Delta\Phi$

- r and also $J = |\mathbf{L} + \mathbf{S_1} + \mathbf{S_2}|$ vary on t_{RR}
- Effective spin is constant (at least) at 2PN Racine 2008

Spin precession is (actually) a **1D** problem! **DG**+ 2015

$$\delta\chi = \frac{\chi_1 \hat{\mathbf{S}}_1 - q\chi_2 \hat{\mathbf{S}}_2}{1+q} \cdot \hat{\mathbf{L}}$$

…and it's analytically tractable! **DG**+ 2023

These 1D solutions are now at the heart of the "twisting up" procedure in some of the modern precessing GW waveforms

Averaging the average

Orbit << Precession << Inspiral

Now GW emission

- Quasi-adiabatic approach
- Only r and J vary on t_{RR}

Usual **orbit** average

$$\langle X \rangle_{\mathrm{orb}} = \frac{\int d\psi \; X \; dt/d\psi}{\int d\psi \; dt/d\psi}$$

Some parameters for the dynamics (say Kepler's true anomaly)

Orbital period

Now a **precession** average

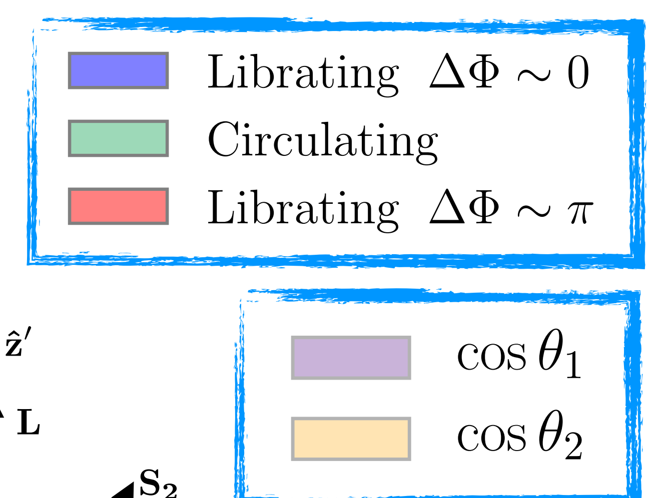
$$\langle X \rangle_{\mathrm{pre}} = \frac{\int d\delta\chi \, \langle X \rangle_{\mathrm{orb}} \, dt/d\delta\chi}{\int d\delta\chi dt/d\delta\chi}$$

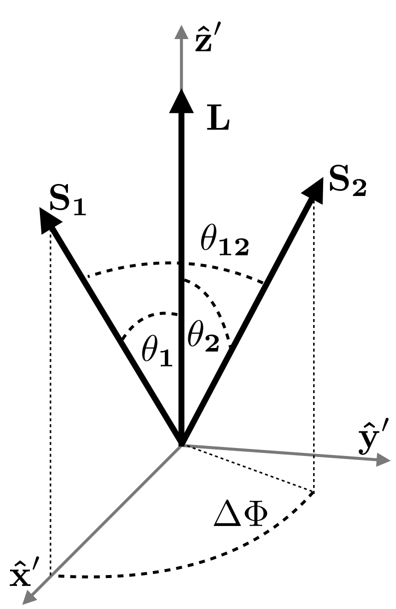
Dynamics is now parametrized by $\delta\chi$

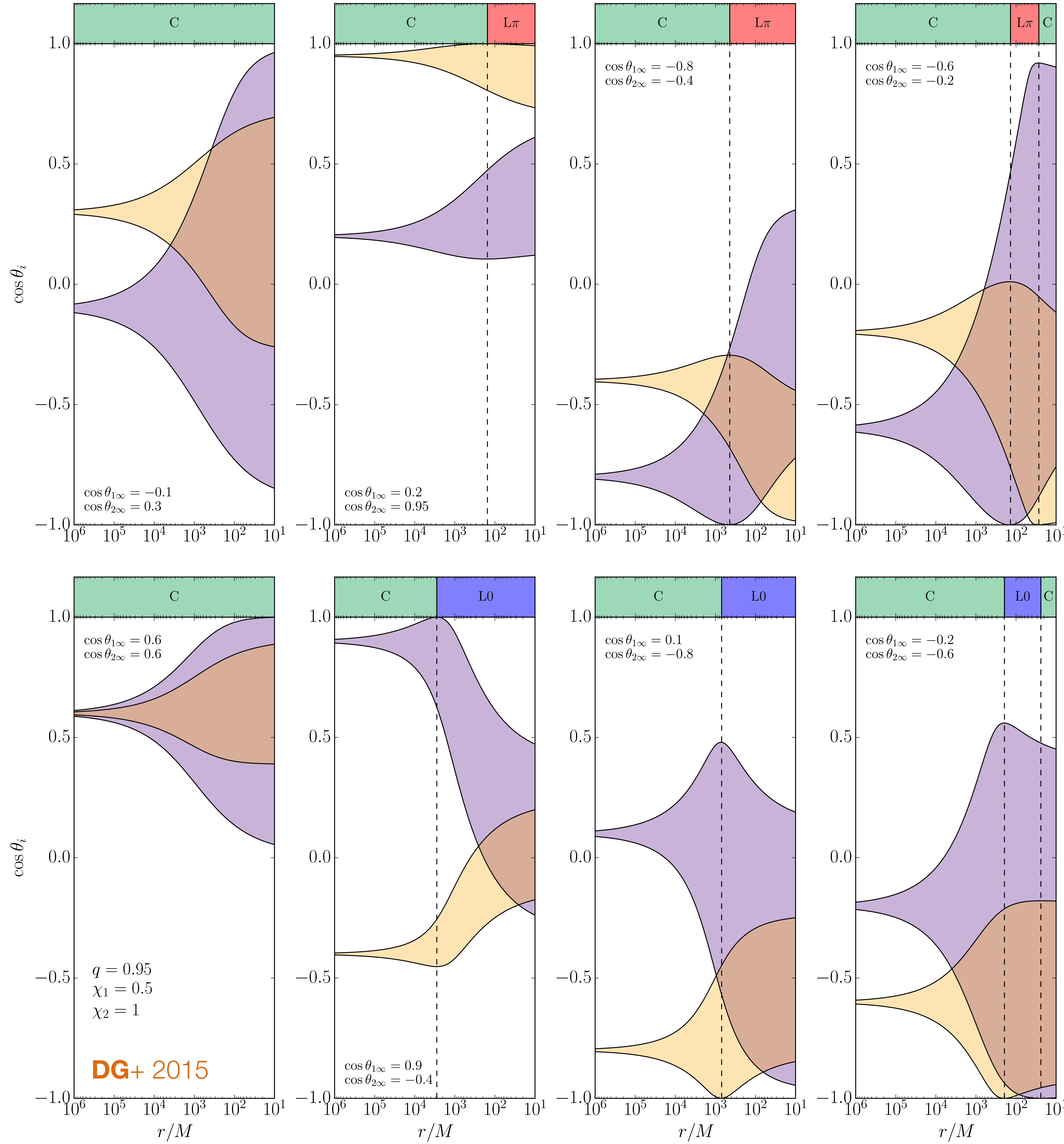
Precession period

Precesion-averaged evolutions

- Only way evolve from infinitely large separations
- Takes seconds!
- Now LVK reports spins at infinity with this formalism

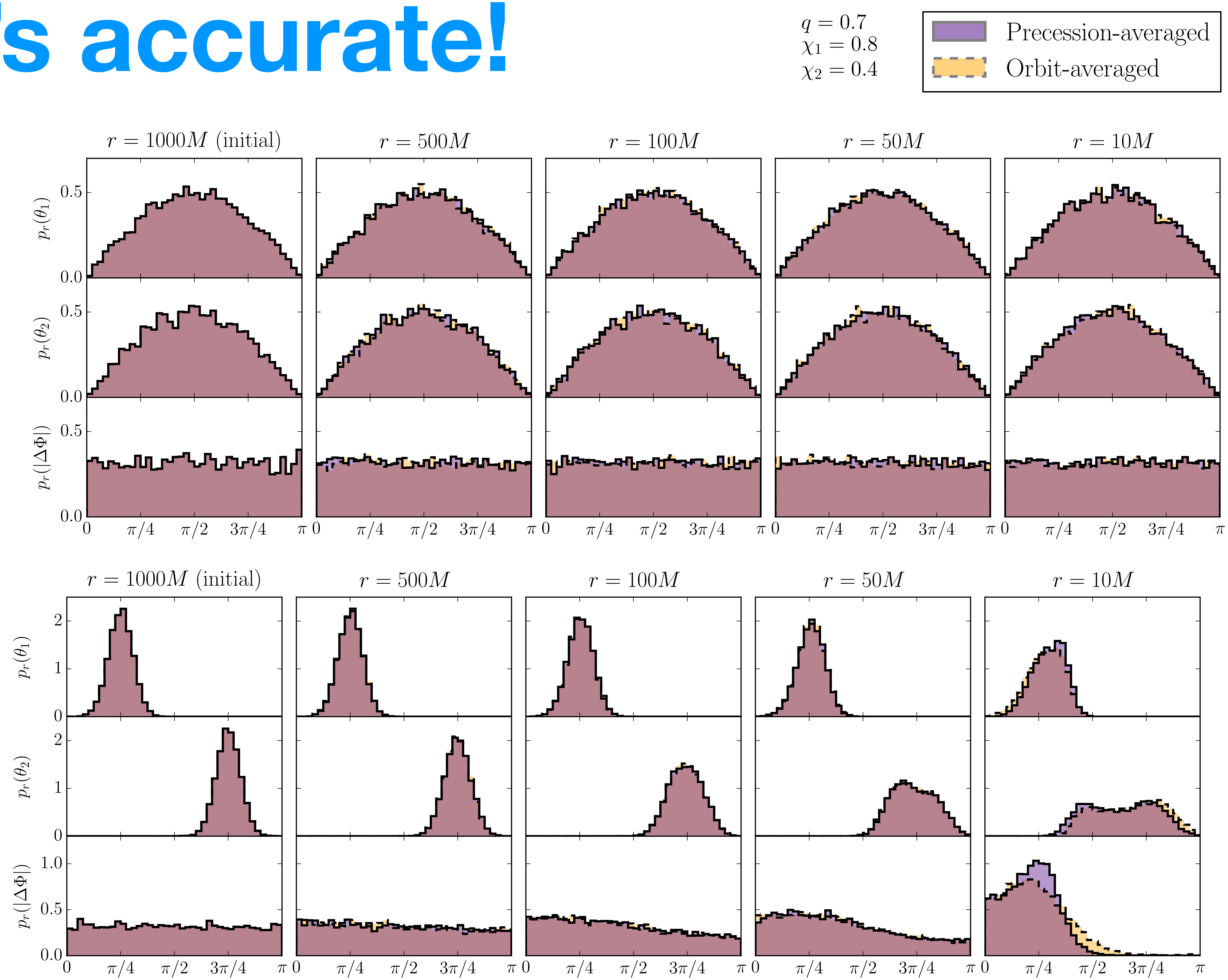






(Updates on this business in the very recent: **DG,** Foroni+ 2025)

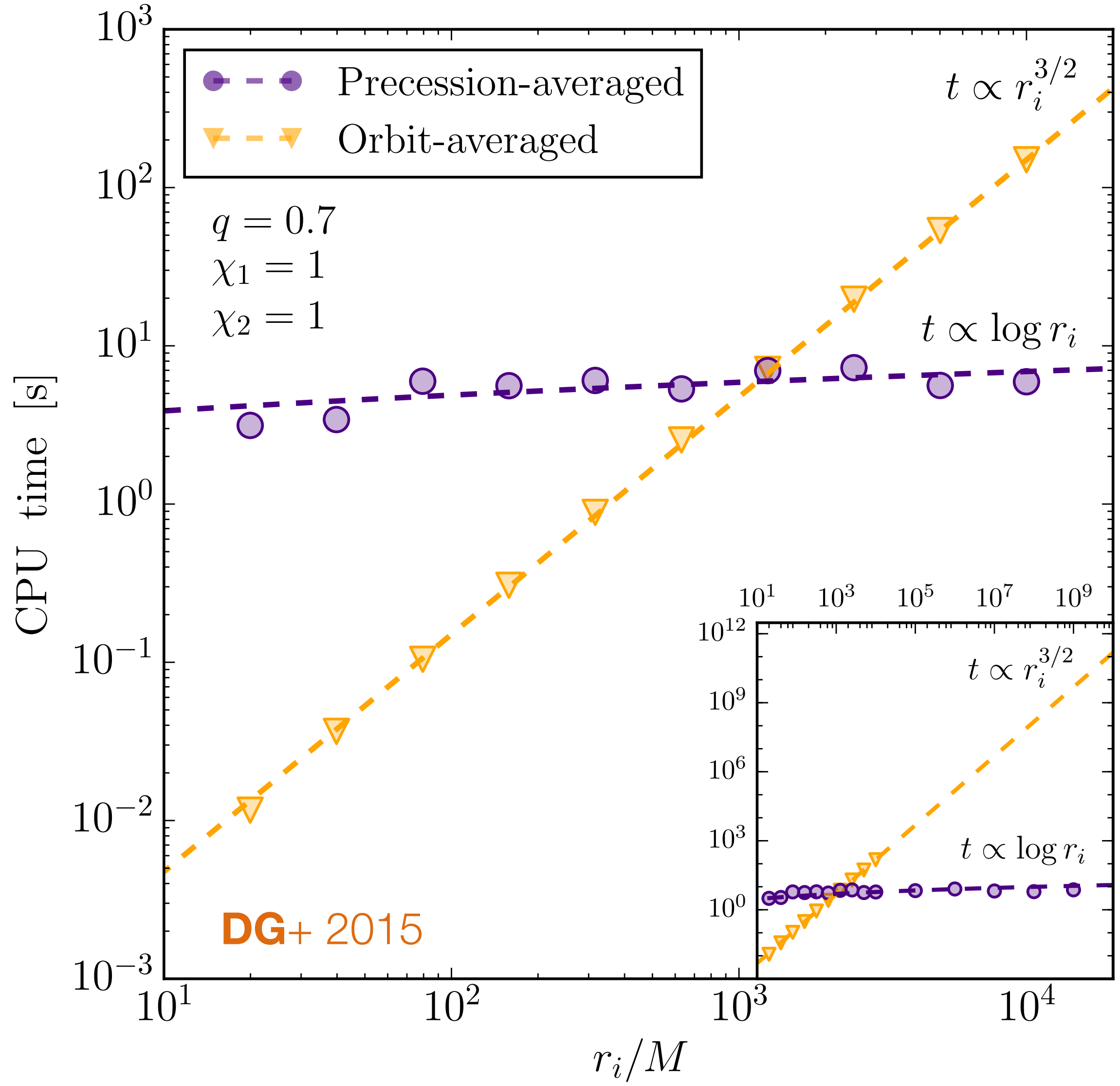
It's accurate!

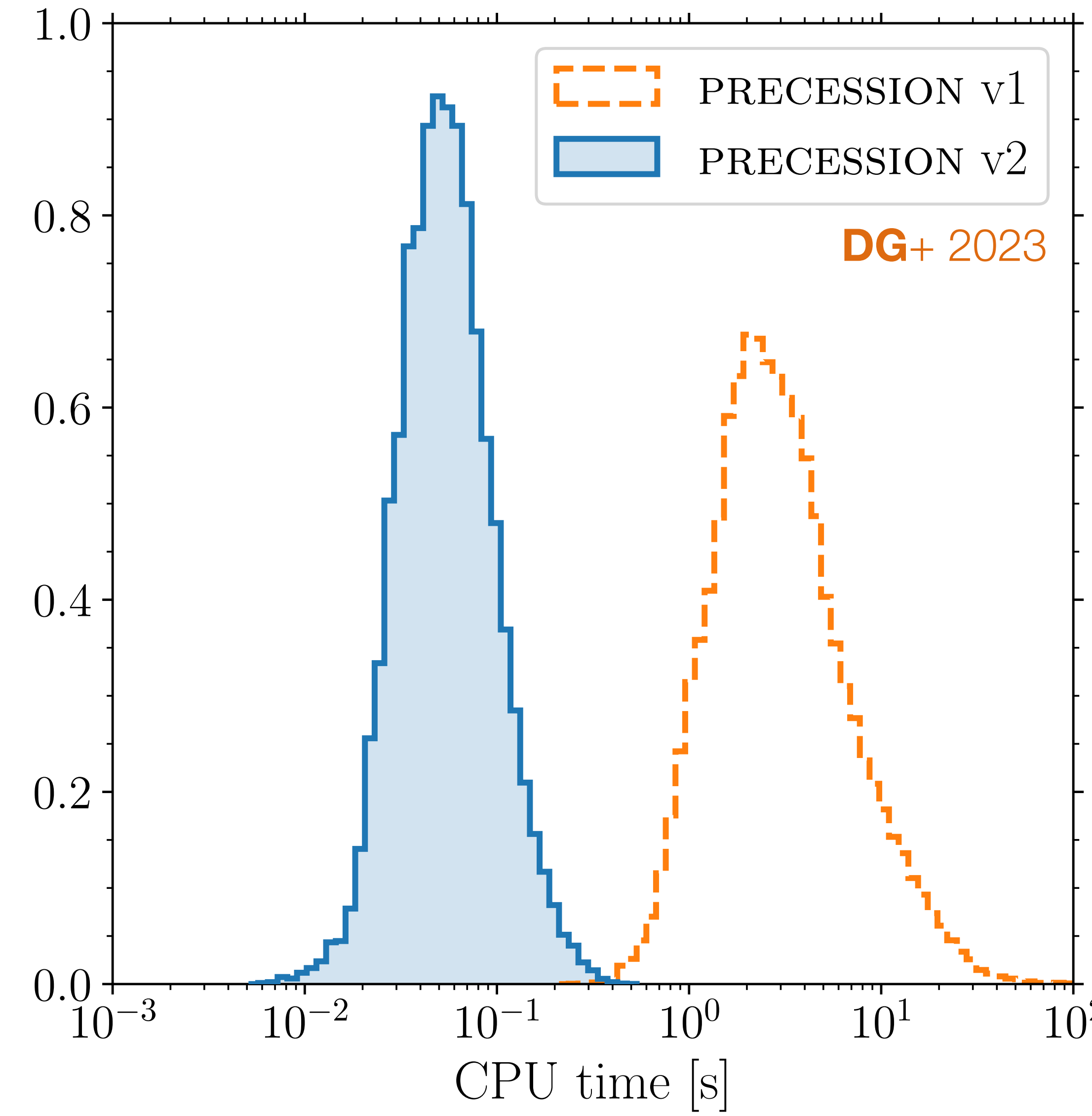


* = We might have a different definition of "accuracy"…

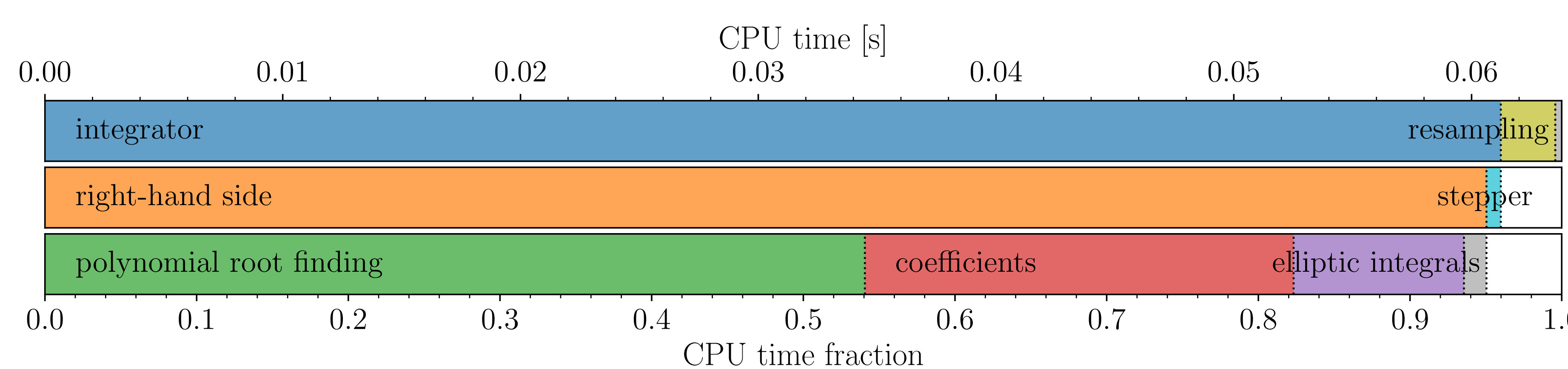
It’s fast!

Incredibly favourable scaling with the separation
(can actually integrate all the way to/from infinity!)





Current perfomance is O(0.1s) on a laptop, but one can do better



Try this at home

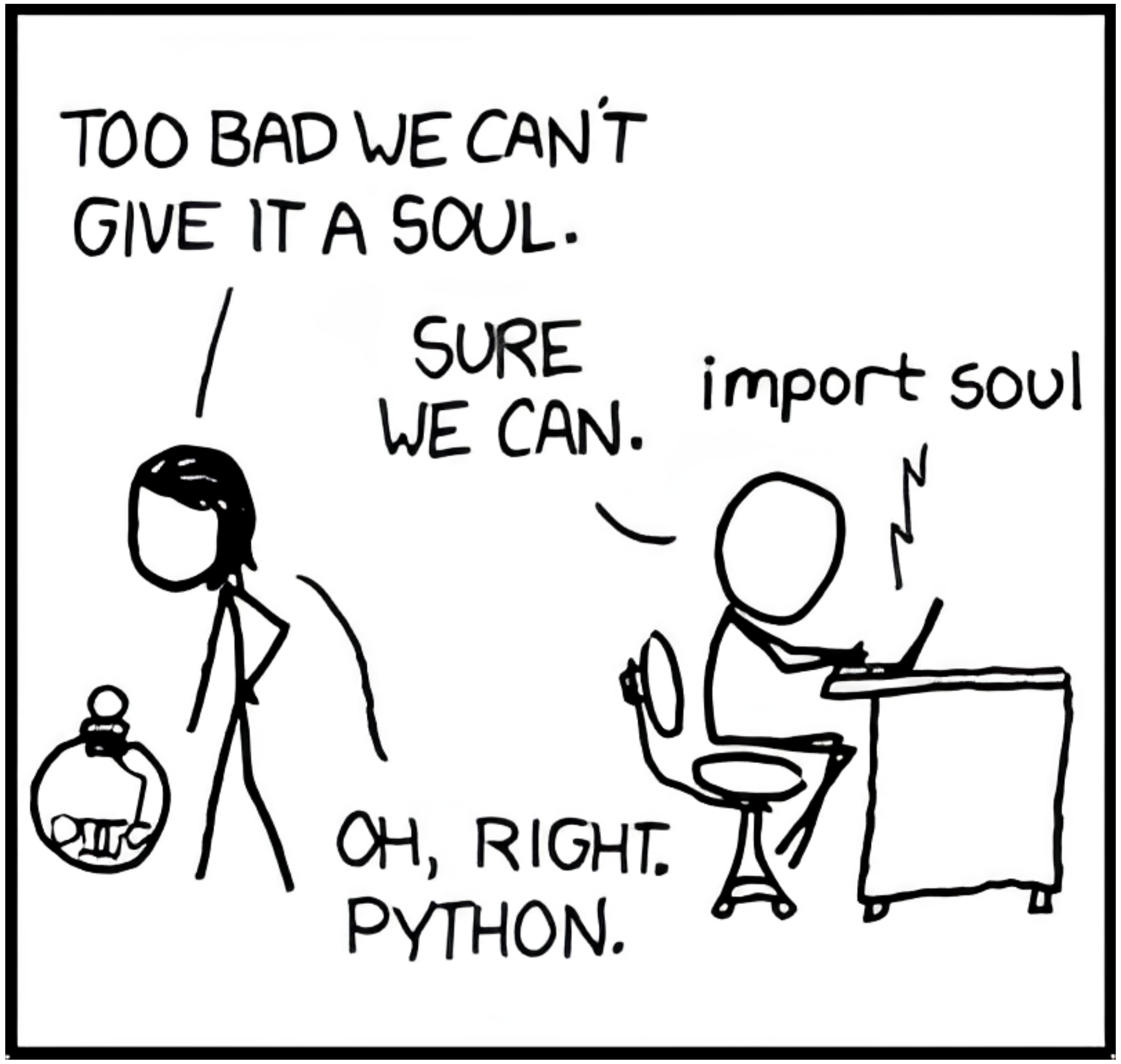
precession: open-source python module

DG+ 2015, **DG**+ 2023, Fumagalli **DG**+ 2023

```
pip install precession
>>> import precession
```

Features

1. Precessional dynamics
2. Orbit-averaged inspirals
3. Precession-averaged inspirals
4. Remnant predictions
5. API documentation
6. Tests and tutorial

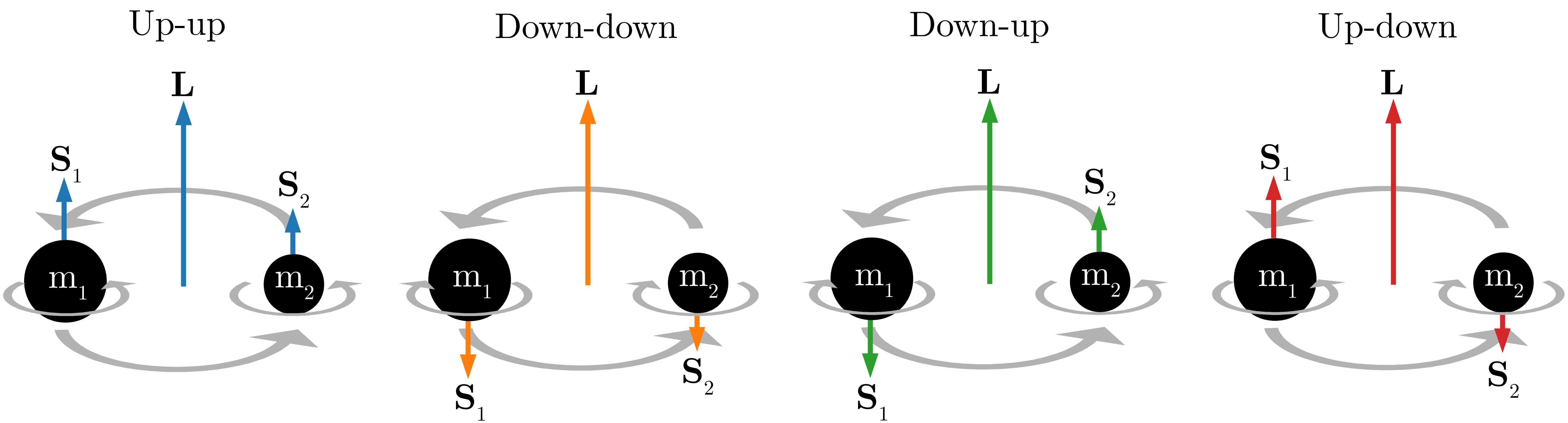


Used in >100 papers to date,
by many people

github.com/dgerosa/precession

Aligned configurations

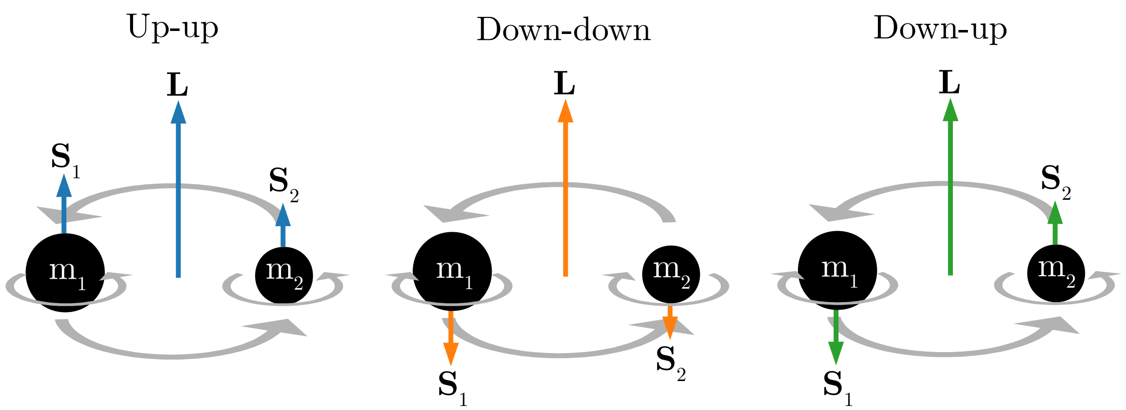
- Astrophysical processes drive binary here (maybe)
- Used as testbed for waveforms developing



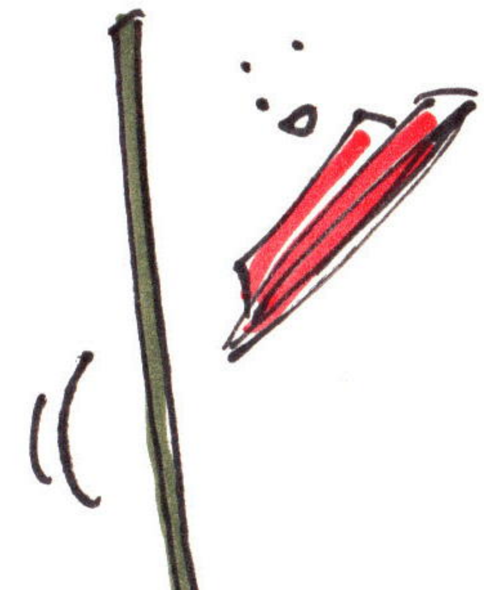
All these configurations are solutions
of the PN equations at any separation…

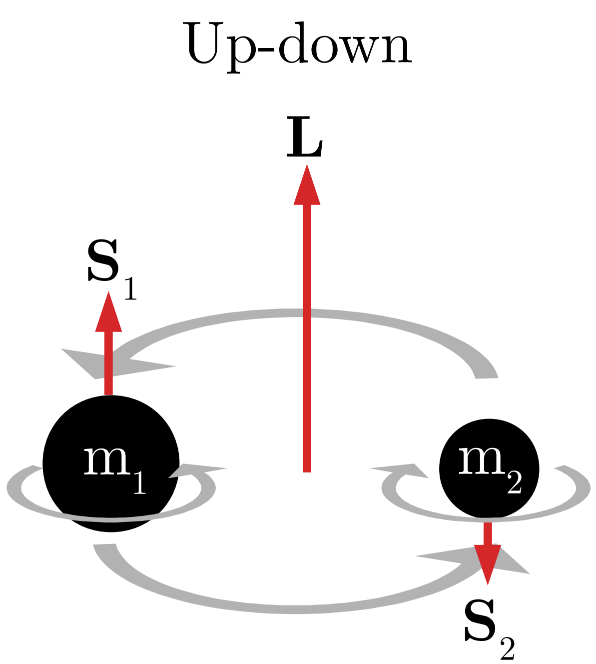
are they stable?

New precessional instability!



Stable at all frequencies





Up-down binaries can be unstable!

Instability kicks in at separations

$$r < r_{\rm ud+} \qquad r_{\rm ud+} = \frac{(\sqrt{\chi_1} + \sqrt{q\chi_2})^4}{(1-q)^2} M$$

- First found using the effective potentials **DG+** 2015b
- Then with explicit perturbation theory (still PN) Mould **DG** 2020, Lousto Healy 2016
- And then even with full numerical relativity! Varma Mould **DG** 2021
- Injections into LVK noise De Renzis **DG** 2022

Observational predictions

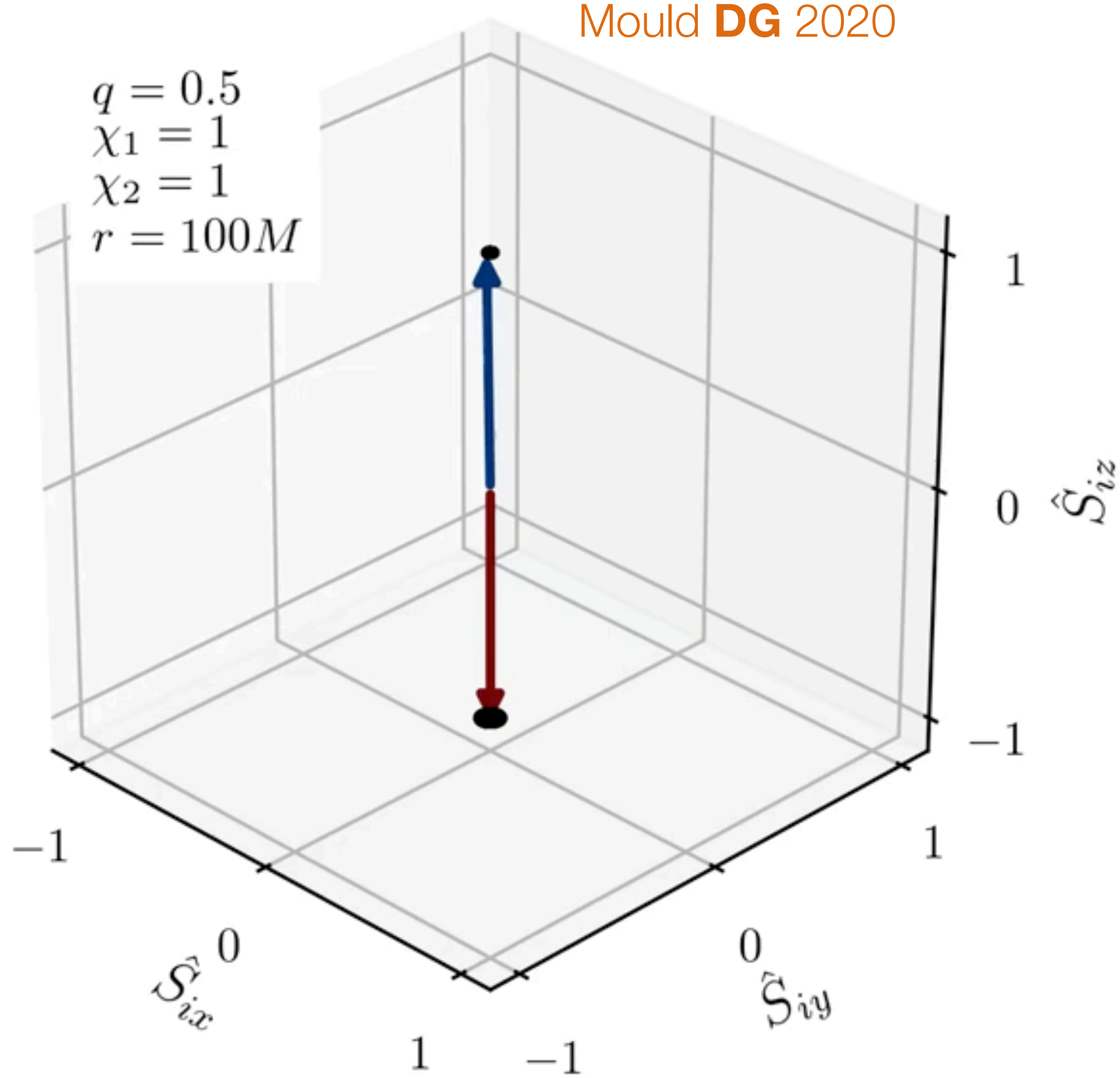
- Binary likely underwent the instability before entering the LIGO band
- Mechanisms to deplete the aligned region
- Well defined endpoint

$$\hat{\mathbf{S}}_1 = \hat{\mathbf{S}}_2$$

$$\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{L}} = \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{L}} = \frac{\chi_1 - q\chi_2}{\chi_1 + q\chi_2}$$

- But how about binaries that start precessing while being observed?!

Mould **DG** 2020



What's up with χ_p ?

precession

radiation reaction

$$\frac{d\mathbf{L}}{dt} = \frac{d\hat{\mathbf{L}}}{dt}L + \frac{dL}{dt}\hat{\mathbf{L}} = (\mathbf{\Omega}_L \times \hat{\mathbf{L}})L + \frac{dL}{dt}\hat{\mathbf{L}}$$

2PN spin precession equations:

$$\mathbf{\Omega}_L = \Omega_1\chi_1\hat{\mathbf{S}}_1 + \Omega_2\chi_2\hat{\mathbf{S}}_2$$

$$\Omega_1 = \frac{M^2}{2r^3(1+q)^2}\left[4+3q-\frac{3q\chi_\mathrm{eff}}{(1+q)}\frac{M^2}{L}\right]$$

$$\Omega_2 = \frac{qM^2}{2r^3(1+q)^2}\left[4q+3-\frac{3q\chi_\mathrm{eff}}{(1+q)}\frac{M^2}{L}\right]$$

$$\left|\frac{d\hat{\mathbf{L}}}{dt}\right|^2 = (\Omega_1\chi_1\sin\theta_1)^2 + (\Omega_2\chi_2\sin\theta_2)^2 + 2\Omega_1\Omega_2\chi_1\chi_2\sin\theta_1\sin\theta_2\cos\Delta\Phi$$

Schmidt+ 2015 Extremize $\cos\Delta\Phi = \pm 1$, take an aritmethic mean, and normalize by Ω_1

"Heuristic" $$\chi_\mathrm{p} \equiv \frac{1}{2\Omega_1}\left(\left|\frac{d\hat{\mathbf{L}}}{dt}\right|_+ + \left|\frac{d\hat{\mathbf{L}}}{dt}\right|_-\right) = \max\left(\chi_1\sin\theta_1, q\frac{4q+3}{4+3q}\chi_2\sin\theta_2\right)$$

But:

- Often $\cos\Delta\Phi = \pm 1$ is not allowed geometrically
- The three angles $\theta_1, \theta_2, \Delta\Phi$ vary on the same timescale!

DG+ 2021, De Renzis **DG**+ 2022

Two strategies to fix this

The common χ_{p} expression is a partial average over the precession timescale

Two things one can do:

1. Retain all variations on the precessional timescale. "Generalized" χ_{p}

$$\chi_{\mathrm{p}} \equiv \left|\frac{d\hat{\mathbf{L}}}{dt}\right| \frac{1}{\Omega_1} = \left[(\chi_1 \sin\theta_1)^2 + \left(q \frac{4q+3}{4+3q} \chi_2 \sin\theta_2\right)^2 + 2q \frac{4q+3}{4+3q} \chi_1 \chi_2 \sin\theta_1 \sin\theta_2 \cos\Delta\Phi \right]^{1/2}$$

same terms as before

this is new!

2. Precession-average them all "Averaged"

$$\langle \chi_{\mathrm{p}} \rangle = \frac{\displaystyle\int \chi_{\mathrm{p}}(\psi) \left(\frac{d\psi}{dt}\right)^{-1} d\psi}{\displaystyle\int \left(\frac{d\psi}{dt}\right)^{-1} d\psi}$$

A better precession parameter

DG+ 2021

Replace $\chi_\mathrm{p} = \max\left(\chi_1 \sin\theta_1, q\frac{4q+3}{4+3q}\chi_2 \sin\theta_2\right) \in [0,1]$

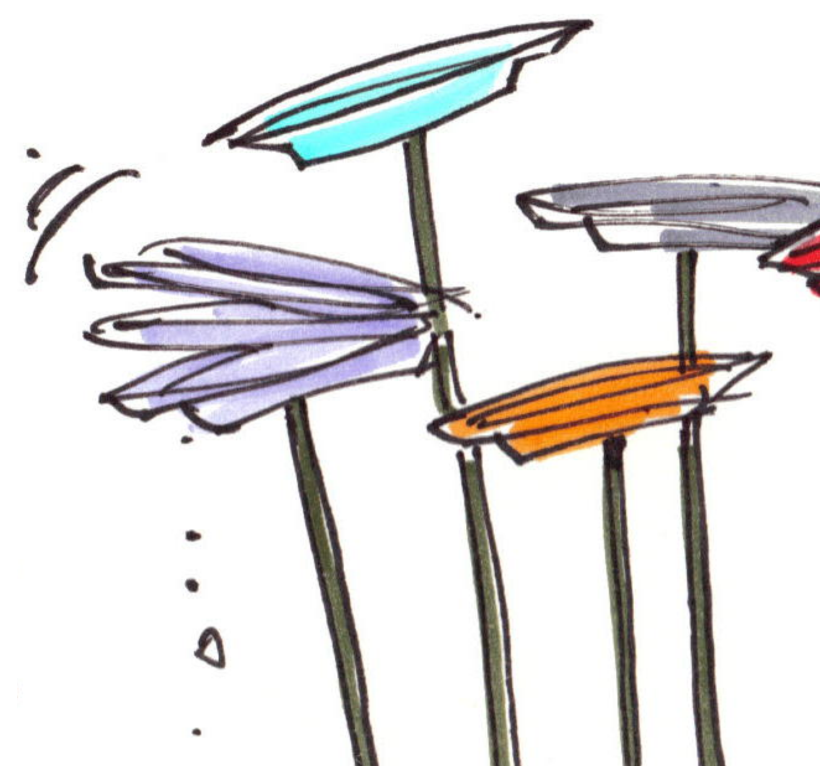
with $\chi_\mathrm{p} = \left[(\chi_1 \sin\theta_1)^2 + \left(q\frac{4q+3}{4+3q}\chi_2\sin\theta_2\right)^2 + 2q\frac{4q+3}{4+3q}\chi_1\chi_2\sin\theta_1\sin\theta_2\cos\Delta\Phi\right]^{1/2} \in [0,2]$

and average consistently $\langle\chi_\mathrm{p}\rangle$

The $\chi_\mathrm{p} \in [1,2]$ region is exclusive to binaries with two precessing spins

$\chi_\mathrm{p} > 0$ Evidence for **spin precession**

$\chi_\mathrm{p} > 1$ Evidence for **two-spin effects**



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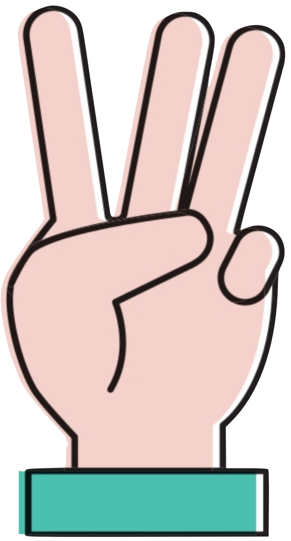

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More spin things!

Spins and astrophysics Tidal interactions, supernova kicks, mass transfer accretion **DG**+ 2013, 2018 and infintitely many more

Spins and waveforms Our solutions now somewhere in PhenomX
Chatziioannou+ 2017, Khan+ 2019, Pratten + 2023

Spins and population stats Need to go back to past time infinity! Now LVK GWTC4 catalog quoting spins back in time Mould, **DG**+ 2022

Spins and eccentricity Extremely interesting interplay (notion of "effective circular binary") Fumagalli, **DG**+ 2023, 2024

See Giulia's talk…

Spins and three body Clustering in the orbital plane
e.g. Antonini+ 2017

Spins and tides What happens with neutron stars?
LaHaye+ 2022

Spin and kicks Big kicks only come from spins!
Rodrigues+ 2007, Campanelli+ 2007, **DG**+ 2018

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