seit 1558

Real modes and null memory contributions in effective-one-body models

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EOB@Work25: 10 years of gravitational wave detections

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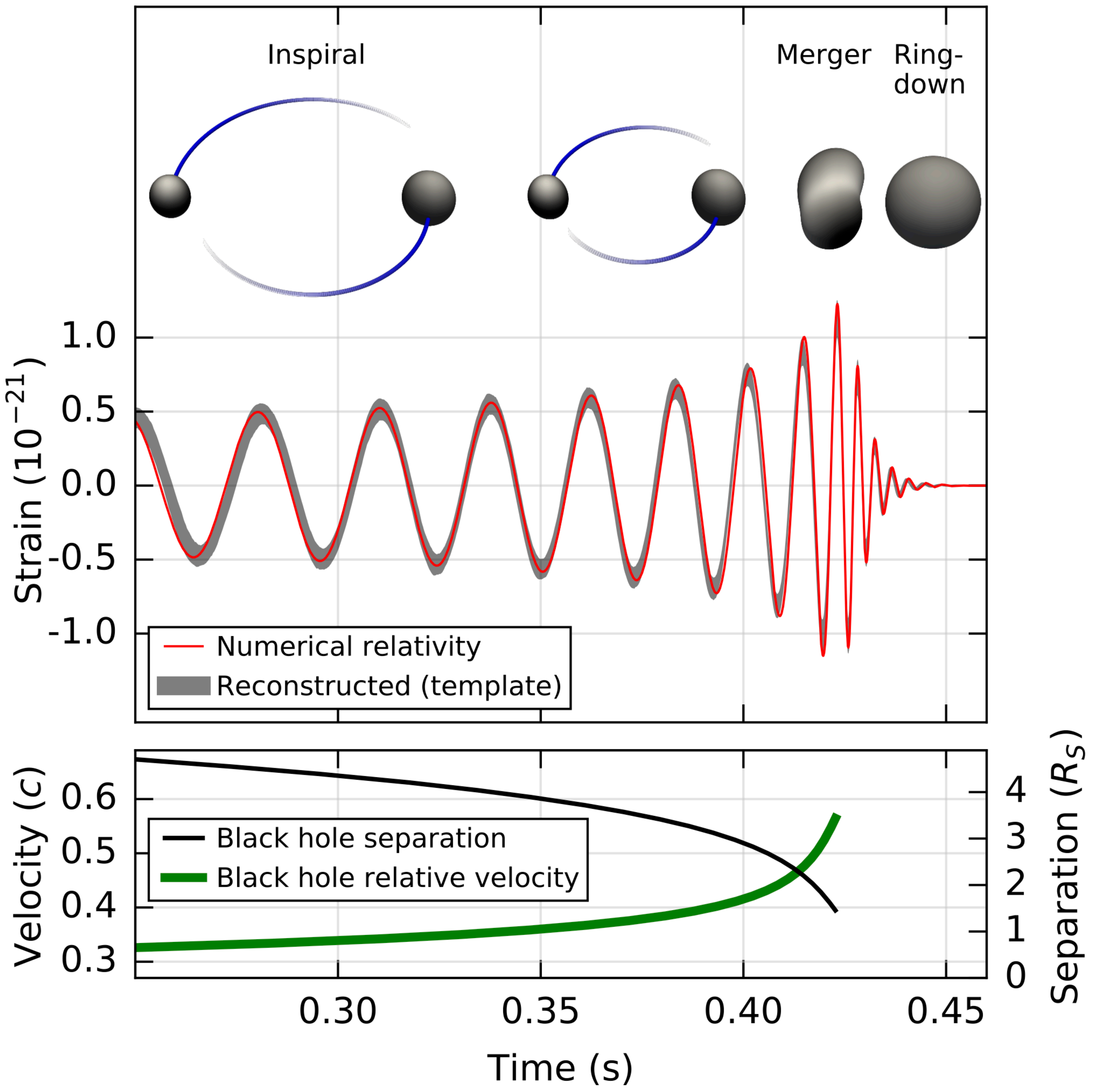
Complete EOBNR waveform

- “Pure” EOB approach describes inspiral+plunge

- EOB models completed with NR:
  - corrections to improve the late-inspiral/plunge
  - **ringdown model** to complete the EOB waveform

- Smooth connection between **EOB waveform** and ringdown model ensured by **NQC** (Next-to-Quasi-Circular) corrections

$$h_{\ell m} = \theta(t - t^{\rm match}_{\ell m}) h^{\rm inspl}_{\ell m} \hat{h}^{\rm NQC}_{\ell m} + \theta(t^{\rm match}_{\ell m} - t) h^{\rm ringdown}_{\ell m}$$



Ringdown modeling

- Modern ringdown models built in two steps [1]:
  1. fit the post-peak waveform with some phenomenological ansatz (**primary fits**)
  2. repeat for a sufficient large number of waveform and fit the found coefficients on the parameter space (**global fits**)

$$h_{\ell m} = \theta(t - t_{\ell m}^{\rm match}) h_{\ell m}^{\rm inspl} \hat{h}_{\ell m}^{\rm NQC} + \theta(t_{\ell m}^{\rm match} - t) h_{\ell m}^{\rm ringdown}$$

[1] Damour,Nagar:1406.0401

Ringdown: primary fits

$$h_{\ell m} = \theta(t - t^{\rm match}_{\ell m}) h^{\rm inspl}_{\ell m} \hat{h}^{\rm NQC}_{\ell m} + \theta(t^{\rm match}_{\ell m} - t) h^{\rm ringdown}_{\ell m}$$

- QNM-based phenomenological model based on [1]:
  - **factorize the dominant QNM contribution** for each multipole

$$\bar{h}_{\ell m}(\tau) = e^{\sigma^{+}_{\ell 1}\tau + i\phi^{\rm peak}_{\ell m}} h^{\rm rng}_{\ell m}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}}$$

  - fit the rescaled signal with **phenomenological ansatz**
  - some coefficients constrained by **continuity conditions using NR quantities**

$$A_{\bar{h}}(\tau) = \left(\frac{c_1^A}{1 + e^{-c_2^A \tau + c_3^A}} + c_4^A \right)^{\frac{1}{c_5^A}}$$

$$\phi_{\bar{h}}(\tau) = -c_1^{\phi} \ln\left(\frac{1 + c_3^{\phi} e^{-c_2^{\phi}\tau} + c_4^{\phi} e^{-2c_2^{\phi}\tau}}{1 + c_3^{\phi} + c_4^{\phi}} \right)$$

$$c_1^A = \frac{c_5^A \alpha_1}{c_2^A} (A_{\rm peak})^{c_5^A} e^{-c_3^A} (1 + e^{c_3^A})^2 \qquad c_5^A = -\frac{\ddot{A}_{\rm peak}}{A_{\rm peak}\alpha_1^2} + \frac{c_2^A}{\alpha_1} \frac{e^{c_3^A} - 1}{1 + e^{c_3^A}}$$

$$c_4^A = (A_{\rm peak})^{c_5^A} - \frac{c_1^A}{1 + e^{c_3^A}} \qquad c_1^{\phi} = \frac{1 + c_3^{\phi} + c_4^{\phi}}{c_2^{\phi}(c_3^{\phi} + 2c_4^{\phi})} \Delta\omega_{\rm peak}$$

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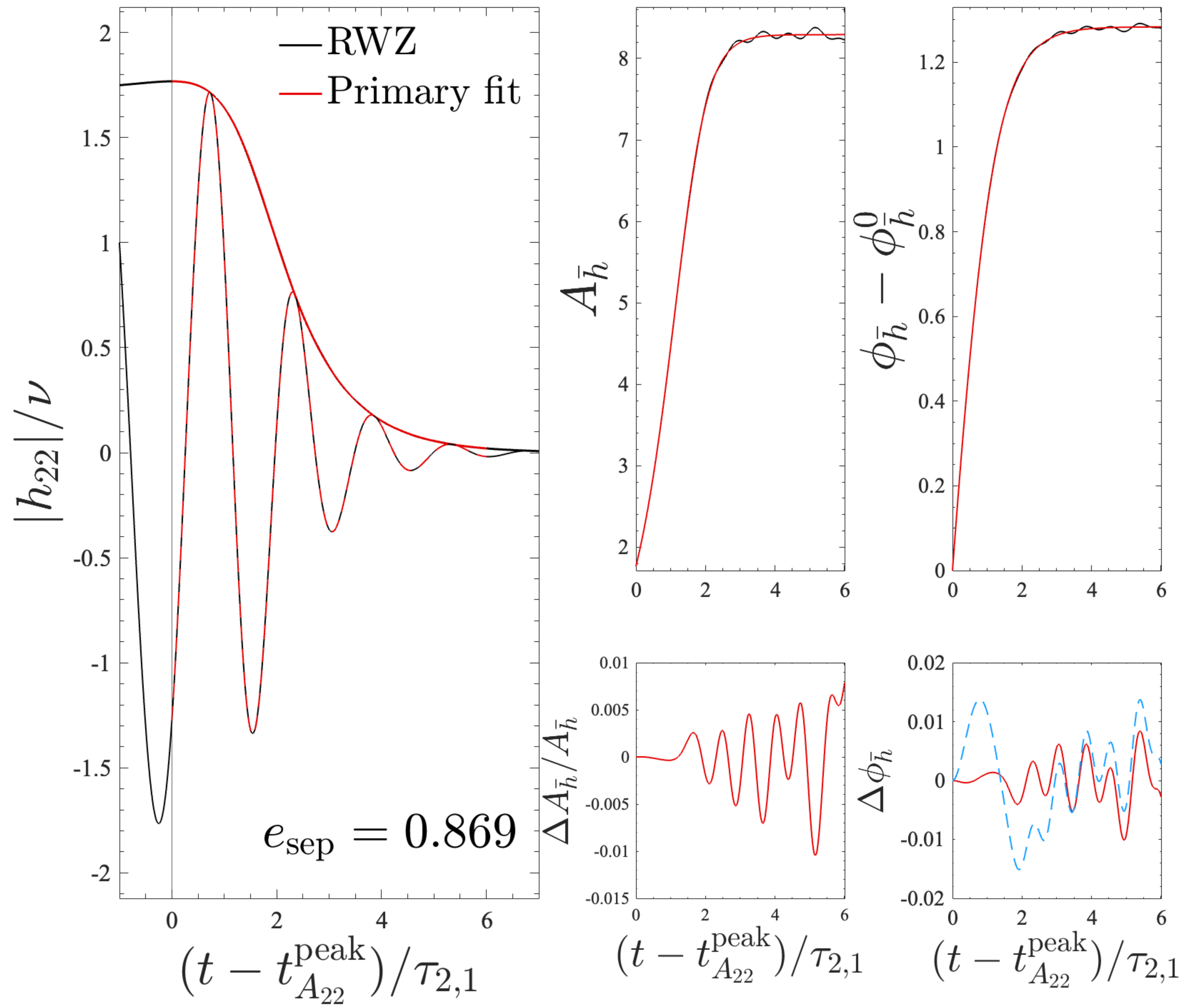
- Example of application in the test-mass case:
  - highly eccentric configuration
  - numerical waveform computed with `RWZHyp` [2,3]
  - For HMs, fits can be started at $A^{\rm peak}_{22}$ rather than at $A^{\rm peak}_{\ell m}$ [4]



[1] Damour,Nagar:1406.0401, [2] Bernuzzi,Nagar:1003.0597, [3] Bernuzzi+:1012.2456, [4] Cotesta+:1803.10701

Albanesi+:2305.19336

Matching and NQC

$$h_{\ell m} = \theta(t - t_{\ell m}^{\rm match}) h_{\ell m}^{\rm inspl} \hat{h}_{\ell m}^{\rm NQC} + \theta(t_{\ell m}^{\rm match} - t) h_{\ell m}^{\rm ringdown}$$

- Global fits across parameter space → **ringdown model done**

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- Matching to inspiral-plunge waveform by ensuring continuity with NQC corrections

$$\hat{h}^{\rm NQC}_{\ell m} = \left(1 + \sum_{i=1}^{3} a_i^{\ell m} n_i\right) \exp\left(i \sum_{j=1}^{3} b_j^{\ell m} n_{j+3}\right)$$

$$n_1 = \frac{p_{r_*}^2}{r^2\Omega^2}, \quad n_2 = \frac{\ddot{r}}{r\Omega^2}, \quad \dots$$

- Coefficients determined solving a linear system at $t^{\rm NQC}_{\ell m}$

$$A^{\rm EOB}_{\ell m}(t^{\rm NQC}_{\ell m}) = A^{\rm rng}_{\ell m}(t^{\rm NQC}_{\ell m}) \qquad \omega^{\rm EOB}_{\ell m}(t^{\rm NQC}_{\ell m}) = \omega^{\rm rng}_{\ell m}(t^{\rm NQC}_{\ell m})$$

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→ $\left\{a_i^{\ell m}, b_j^{\ell m}\right\}$

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→ $\left\{a_i^{\ell m}, b_j^{\ell m}\right\}$

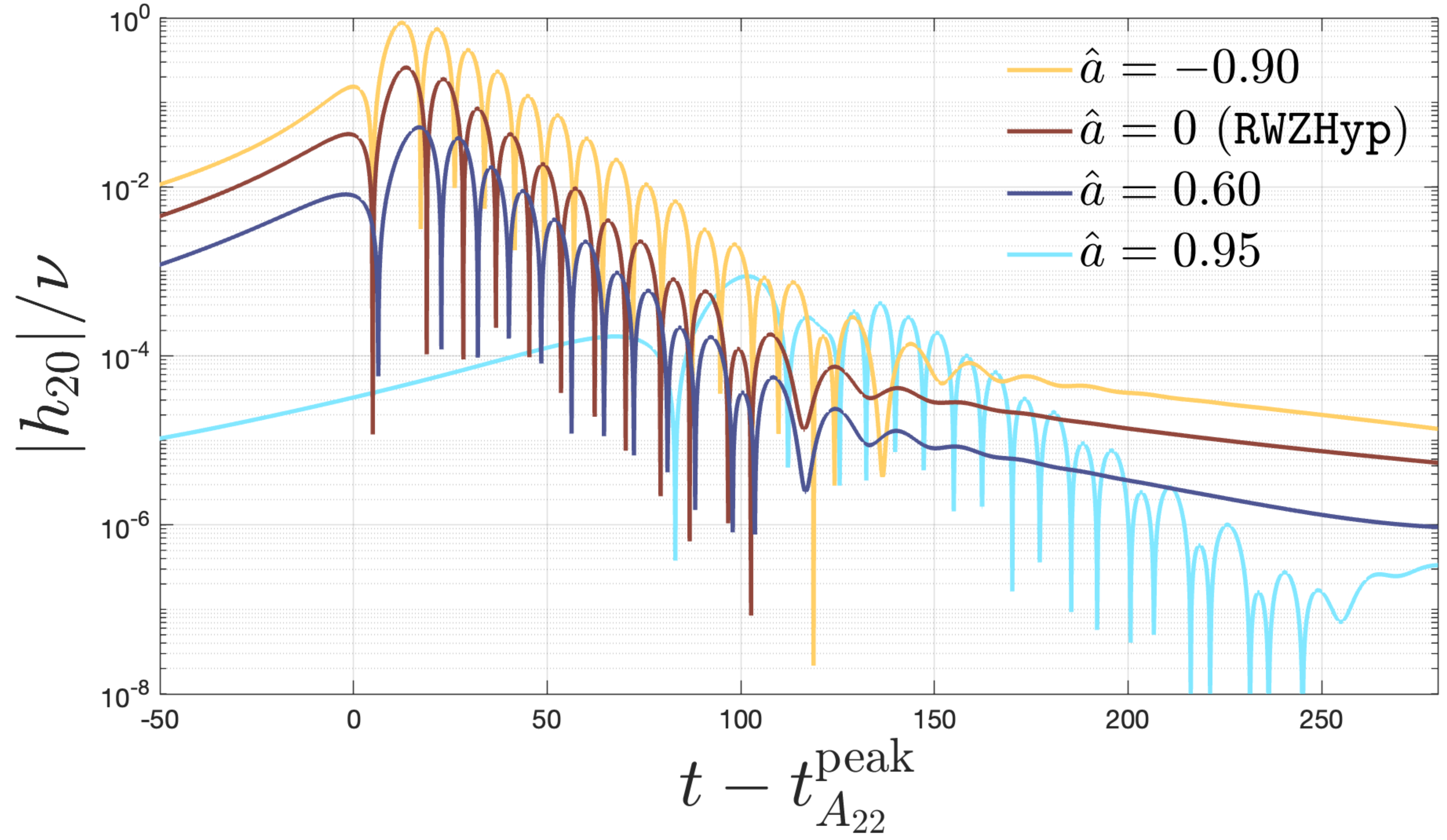
NQC acting on an extended time interval, but coefficients determined at a specific time:

→ having simple amplitude and phase behaviors helps the NQC to work properly

Ringdown for $m = 0$ modes

- $m = 0$ modes usually neglected, but enhanced by radial motion
  $\rightarrow$ relevant for generic orbits or quasi-circular plunges
- Ringdown model relies on complex ansatz and thus includes only $m > 0$ modes
- Advantages of complex templates:
  $\rightarrow$ phase and amplitude are well defined and monotonic before merger
  $\rightarrow$ “easy” to match with the inspiral/plunge waveform using NQC
- Extension to real $m = 0$ (real) with real (non-monotonic) templates would give issues with plunge/merger matching **:(**

Teukolsky waves for plunges in Kerr obtained with Teukode (Harms+:1406.5983)



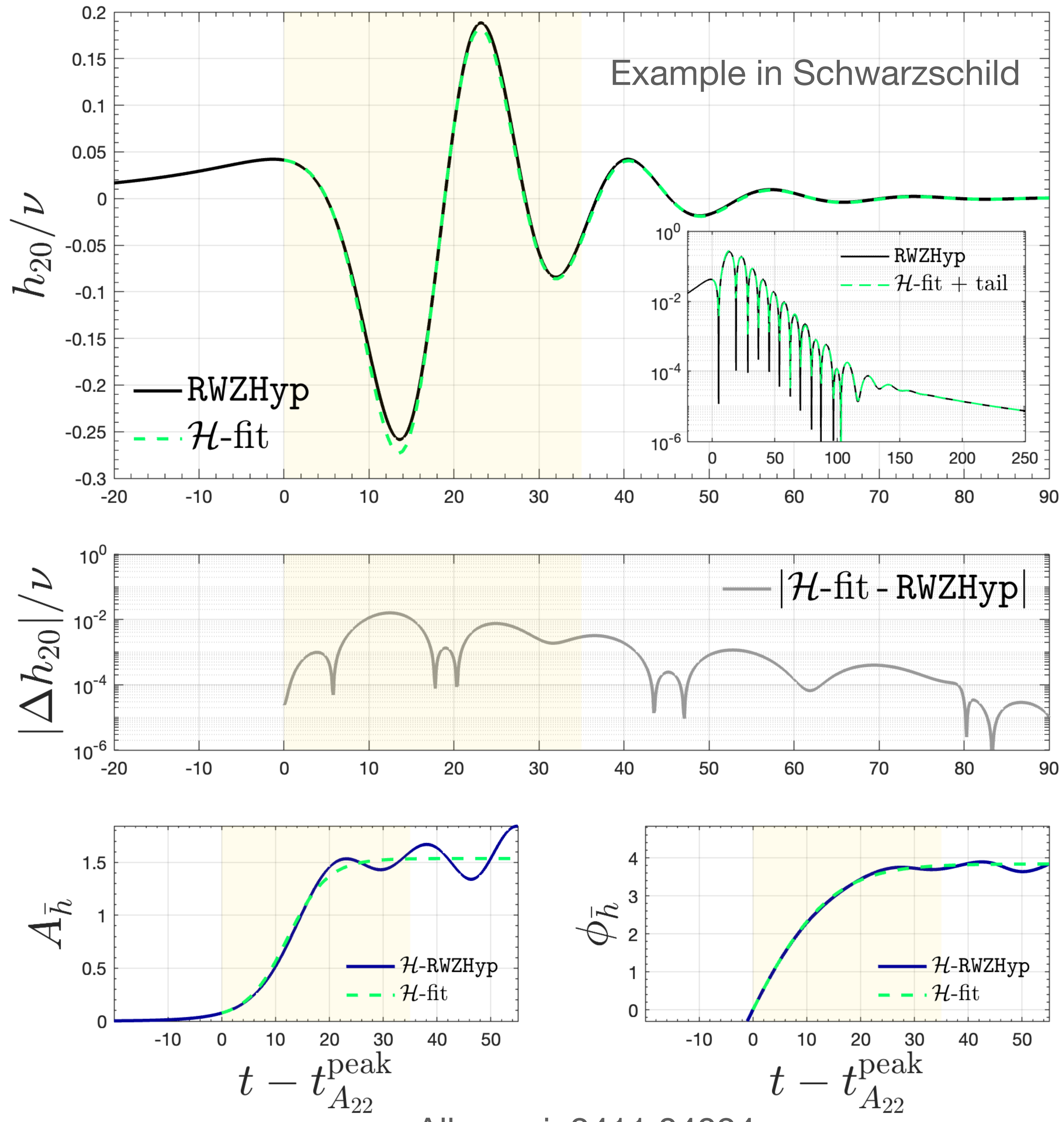
Ringdown for $m = 0$ modes: basic idea

- New approach proposed
  1. Complexify oscillatory part of $m = 0$ modes via **Hilbert transform** $\mathscr{H}[u]$

$$\mathscr{F}\left[\mathscr{H}[u]\right](\omega) = -i\,\mathrm{sgn}(\omega)\mathscr{F}[u](\omega),$$

analytic representation: $u_{\mathscr{H}}(t) = u - i\mathscr{H}[u](t)$

  2. Apply usual machinery **:)**
     A. primary fits with complex ansatz
     B. global fits
     C. match with (complexified) inspiral via standard NQC*: $h_{20}^{\rm inspiral}/\nu \propto r\ddot{r} + \dot{r}^2$ (to complexify before merger)
     D. consider only the physical part of the signal (i.e. the real part)

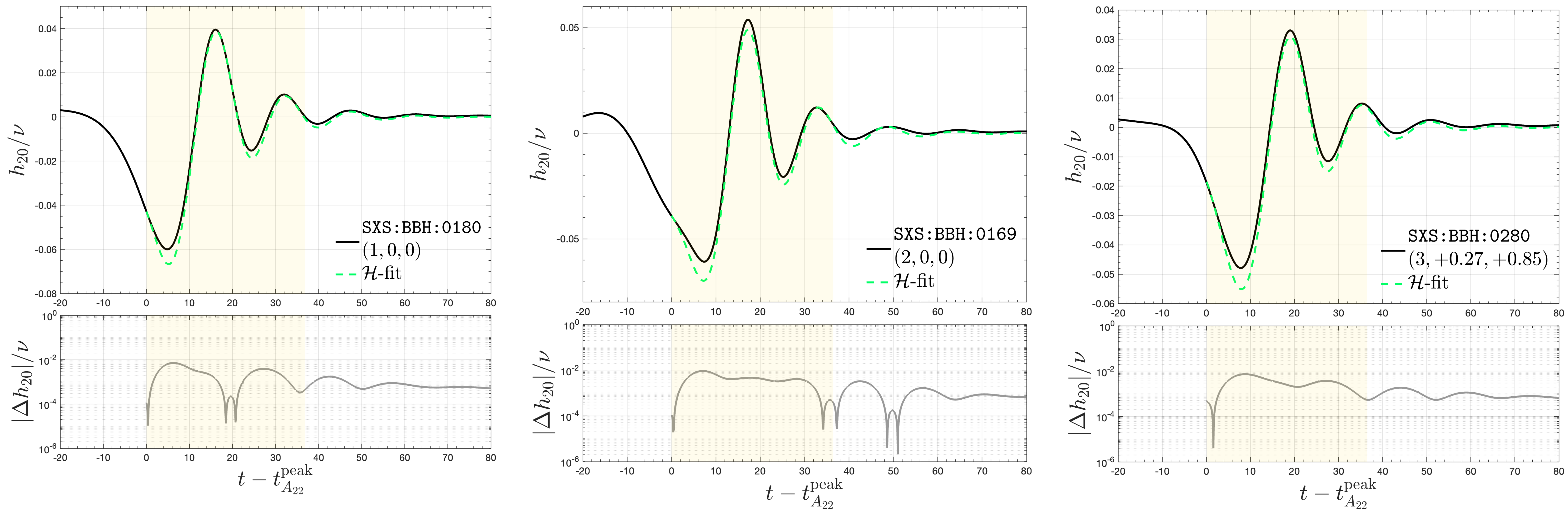


Albanesi: 2411.04024

* Matching the (2,0) at the peak of A_{22}

Ringdown for $m = 0$ modes: comparable mass

- Extension to comparable mass case: primary Hilbert-fits for a large number of NR-surrogate waveforms. Tried both direct-SXS (extrapolated from finite radius) and waves from **NRHybSur3dq8** [1], chosen the second to build the model



- **Global fits** performed over ν and the effective spin $\tilde{a}_0$:

$$f_{2D}(\tilde{a}_0, \nu) = b_0 + b_1\tilde{a}_0 + b_2\tilde{a}_0^2 + \left(c_1 + d_{11}\tilde{a}_0 + d_{21}\tilde{a}_0^2\right)\nu + \left(c_2 + d_{12}\tilde{a}_0 + d_{22}\tilde{a}_0^2\right)\nu^2$$

[1] Varma+:1812.07865

Ringdown for $m = 0$ modes: HMs

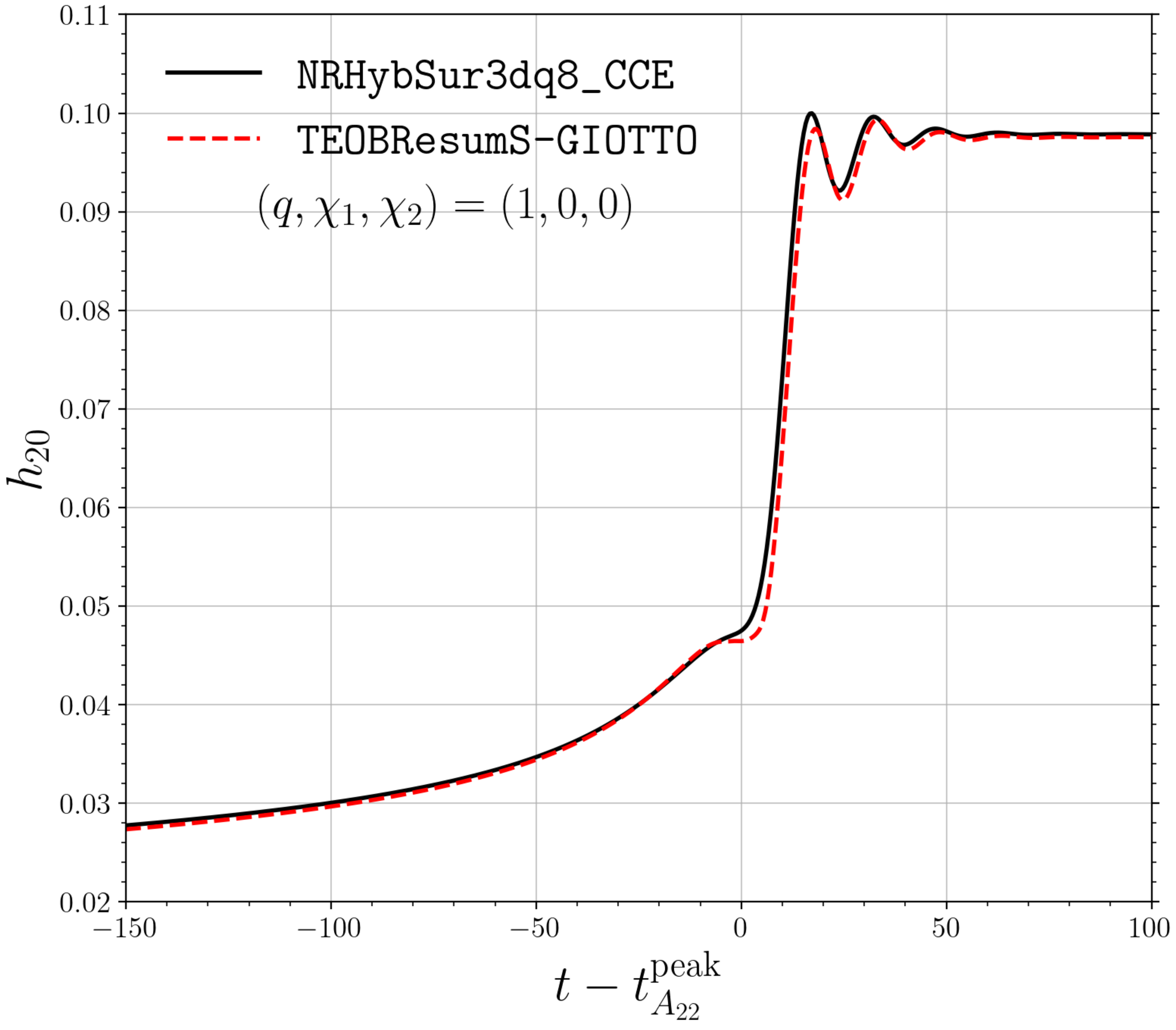
- Higher modes for a QC plunge in Schwarzschild

Ringdown for $m = 0$ modes: null memory

- Once that the oscillatory part is modeled over the parameter space, we can add null memory [1] by means of BMS balance laws [2]
- The displacement is given by the integral of the energy flux over the past history of the binary. In terms of the most relevant multipoles [3]:

$$h_{20}^{\rm memo}(t) = \frac{1}{7}\sqrt{\frac{5}{6\pi}}\int_{t_0}^{t} |\dot{h}_{22}|^2 dt - \frac{1}{14}\sqrt{\frac{5}{6\pi}}\int_{t_0}^{t} |\dot{h}_{21}|^2 dt + \\ + \frac{5}{2\sqrt{42\pi}}\int_{t_0}^{t}\left(\dot{h}_{22}^{\rm Re}\dot{h}_{32}^{\rm Re} + \dot{h}_{22}^{\rm Im}\dot{h}_{32}^{\rm Im}\right) - \frac{2}{11}\sqrt{\frac{2}{15\pi}}\int_{t_0}^{t} |\dot{h}_{44}|^2 dt$$

- Formally $t_0 \to -\infty$; in practice t_0 is the initial time of the EOB evolution
- To enforce $h_{20}^{\rm memo} \to 0$ for $t_0 \to -\infty$, we determine a shift using the 3.5 PN formula [4,5,6] (which has the "correct" low-frequency limit)

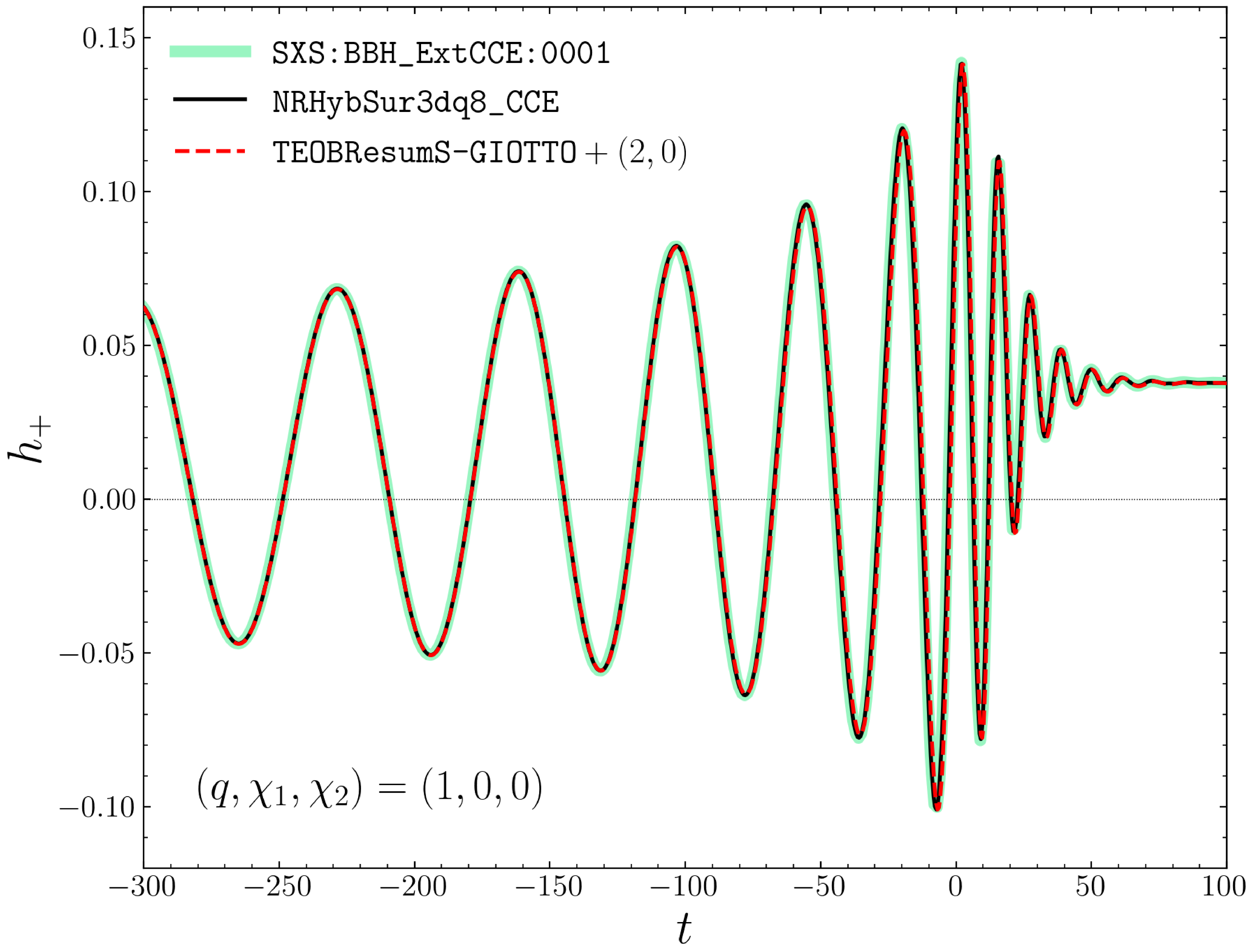


$$h_{20} = h_{20}^{\rm osc} + h_{20}^{\rm memo}$$

[1] Christodoulou-1991, [2] Mitman+:2011.01309, [3] Rosselló-Sastre+:2405.17302, [4] Favata:0812.0069, [5] Cunningham+:2410.23950

Ringdown for $m = 0$ modes: generic mass ratio

- Complete model available for spin-aligned quasi-circular binaries based on **TEOBResumS-GIOTTO** [1] with generic mass ratios. Will be public at some point.
- Quadrupolar waveform for the equal mass nonspinning case



[1] Nagar+:2304.09662

Conclusions

- Extended ringdown model to $m = 0$ modes using the analytical signal built with a Hilbert transform
- Easy to incorporate in EOB models using the complexified inspiral-plunge waveform and applying NQC as usual
- Built model for the (2,0) mode for QC spin-aligned BBH, including null memory effects
- Extendable to HMs and different orbital dynamics

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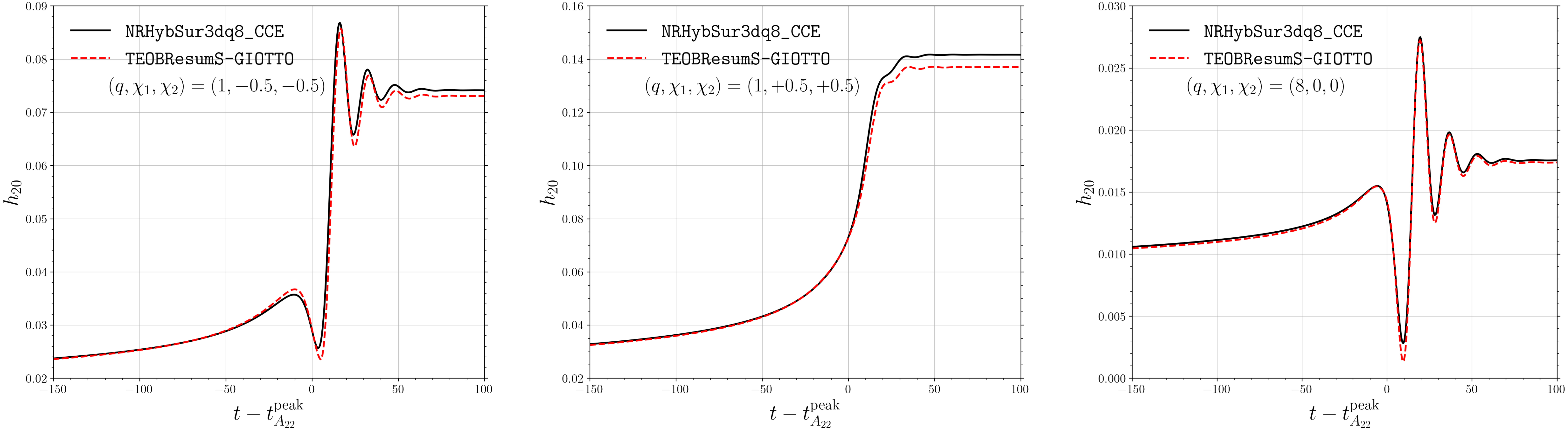
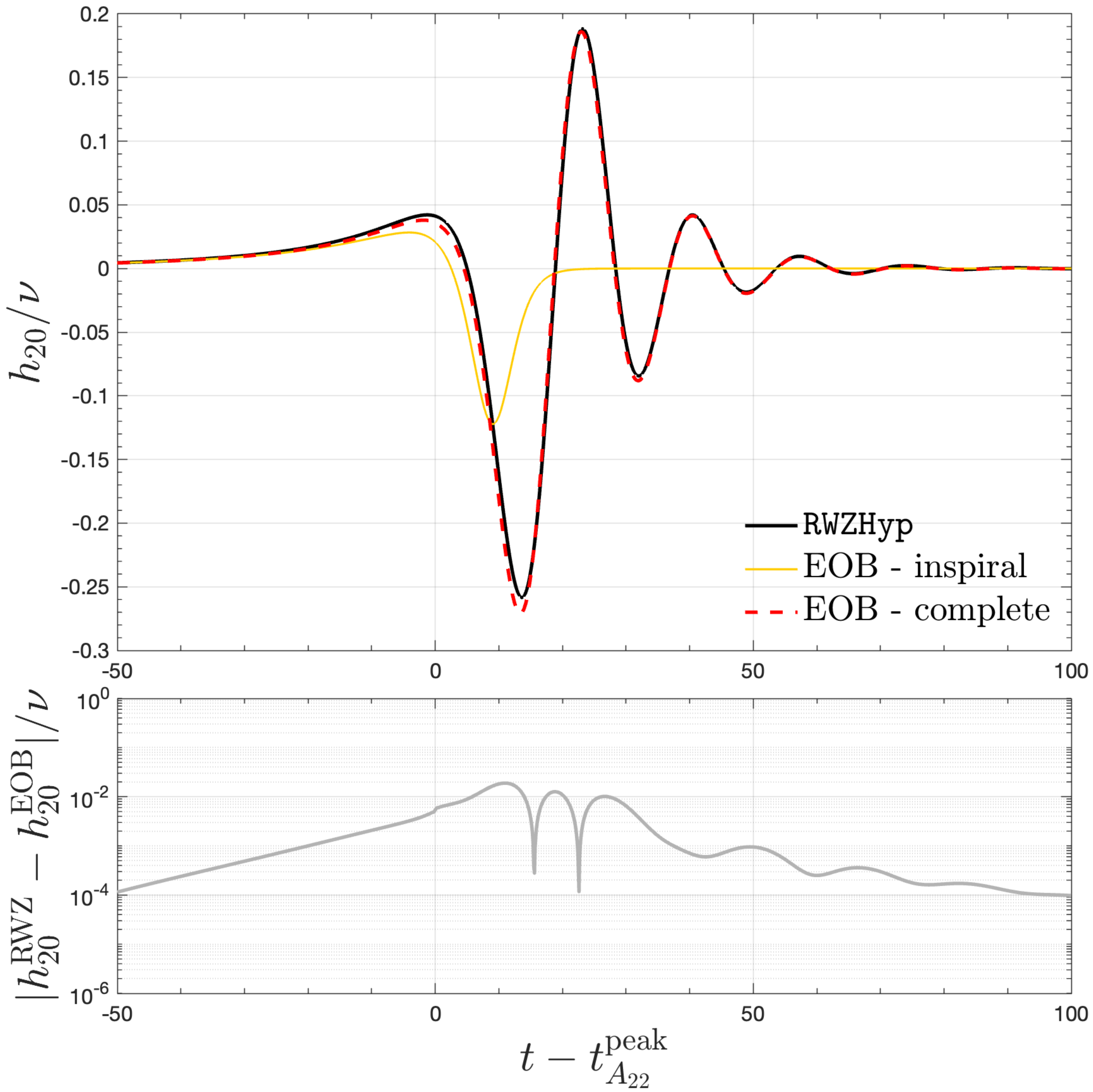
Thank you for the attention!

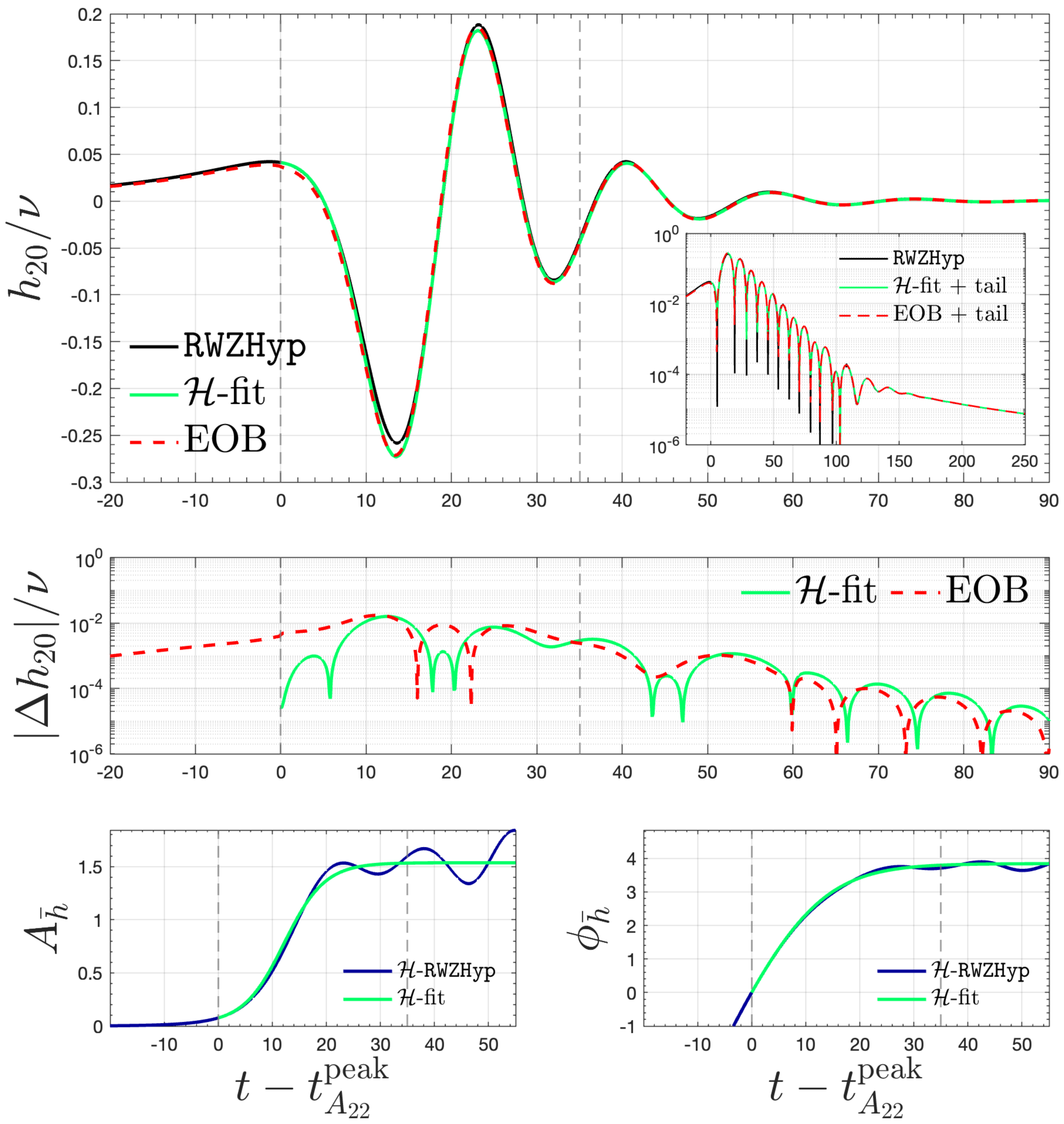
Additional comparisons



Matching



RWZHyp
EOB - inspiral
EOB - complete
h_{20}/ν
$|h_{20}^{\rm RWZ} - h_{20}^{\rm EOB}|/\nu$
$t - t_{A_{22}}^{\rm peak}$



RWZHyp
$\mathcal{H}$-fit
EOB
RWZHyp
$\mathcal{H}$-fit + tail
EOB + tail
h_{20}/ν
$\mathcal{H}$-fit
EOB
$|\Delta h_{20}|/\nu$
$A_{\bar{h}}$
$\mathcal{H}$-RWZHyp
$\mathcal{H}$-fit
$t - t_{A_{22}}^{\rm peak}$
$\phi_{\bar{h}}$
$\mathcal{H}$-RWZHyp
$\mathcal{H}$-fit
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