Descrizione *ab-initio* di sistemi a pochi nucleoni

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Introduction: *ab-initio* ... ?

$\boxed{A\text{-nucleon system}}$ $\leftrightarrow$ $\boxed{H = T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots}$

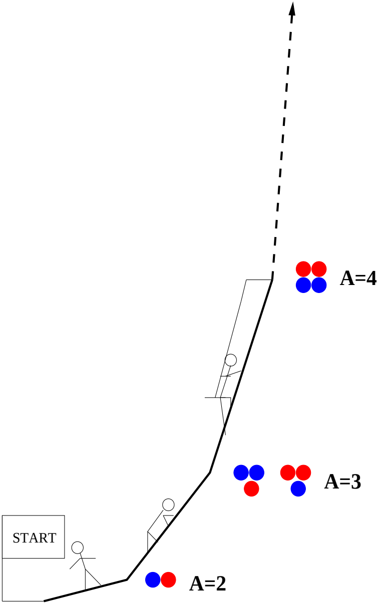
$\boxed{\text{Observable X}}$

Ab-initio method and *Ab-initio* results[1]

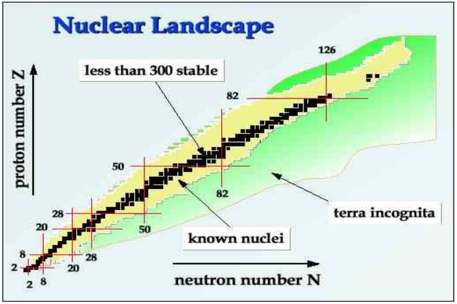
- *Ab-initio* method $\rightarrow$ obtain X by solving the relevant quantum many-body equations, without any uncontrolled approximation
- controlled approximations are allowed (expansion on a certain basis) $\rightarrow$ converged results = *ab-initio* results
- comparison of *ab-initio* results obtained with different *ab-initio* methods $\rightarrow$ benchmark calculations
- comparison of *ab-initio* results with data $\rightarrow$ test of H

[1] W. Leidemann and G. Orlandini, Progr. Part. Nucl. Phys. in press

Few-nucleon systems



Few-nucleon systems $\leftrightarrow$ *ab-initio* methods
$A \leq 12 \rightarrow A \leq 4$



- ideal "laboratory" to test H
- nuclear reactions for astrophysics

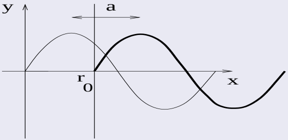
Few-nucleon observables (I)

Bound states: ^{3}H, ^{3}He, ^{4}He

- Binding energy: $B = \sum_i^A m_i - M$
- Mass/charge radius: $< r^2 >= \frac{\int d\tau \langle \Psi | r^2 | \Psi \rangle}{\int d\tau \langle \Psi | \Psi \rangle}$
- Asymptotic normalization constants: $\Psi_A \longrightarrow [\mathrm{ANC}] \times \{\Psi_{C_1} \Psi_{C_2} W(r)\}$
- Other quantities (unmeasurable): $< T > < V > P_{S,P,D,\cdots}$

Elastic scattering: $A_1 + A_2 \rightarrow A_1 + A_2, \quad A_1 + A_2 \leq 4$

- total/differential cross section: $\sigma / \frac{d\sigma}{d\Omega}$
- scattering length a: $a = -\lim_{k\to 0} \frac{\tan \delta_0(k)}{k} \leftrightarrow \lim_{k\to 0} \sigma = 4\pi a^2$



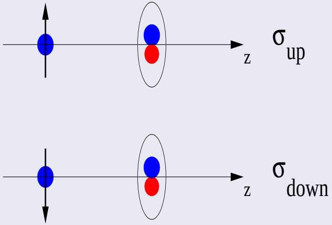
$$
\begin{aligned}
-\frac{\hbar^2}{2m} u''(r) &= E\, u(r) \\
u(r) &= A \sin(kr + \delta_0) \quad r > r_0 \\
u(r_0) = 0 &\Rightarrow \delta_0 = -k\, r_0 \Rightarrow a \equiv r_0
\end{aligned}
$$

$a \leftrightarrow V \Rightarrow a$ is spin-dependent

Few-nucleon observables (II)

Polarized elastic scattering: $\vec{A}_1 + A_2 \longrightarrow A_1 + A_2$

- polarization observables: vector analyzing power A_y



$$A_y \propto \frac{\sigma_{up} - \sigma_{down}}{\sigma_{up} + \sigma_{down}}$$

tensor polarizations $T_{11}, T_{20}, T_{21}, T_{22}$: $\vec{N} \leftrightarrow \vec{d}$

Electro-weak observables: H + electro-weak nuclear current

- electron-scattering: magnetic moments, charge and magnetic radii, form factors
- reactions observables: reaction rates, cross sections, polarization observables

Ab-initio methods

Bound-states

$A \leq 4$ "Faddeev" methods: F/FY - AGS

Variational methods: GEM - SVM - RGM - Hyperspherical Harmonics (HH) Method

"Effective interaction" methods: NCSM - EIHH - CC

Monte Carlo methods: VMC - GFMC

$A > 4$ NCSM, VMC/GFMC, CC, EIHH

......

Scattering-states: "direct calculation"

$A \leq 4$ F/FY, AGS, HH

$A > 4$ NCSM/RGM, CC

Scattering-states: "bound-state formulation"

$A = 3, 4, 6, 7$ Lorentz Integral Transform (LIT)

The Hyperspherical Harmonics (HH) method

Bound states

$$\Psi^{JJ_z} = \sum_{\mu} c_\mu \Psi_\mu$$

- $\Psi_\mu \rightarrow$ known functions (spin-isospin HH functions)
- Rayleigh-Ritz var. principle: $\delta_c \langle \Psi^{JJ_z} | H - E | \Psi^{JJ_z} \rangle = 0$
  $\Rightarrow$ Solve for E and c_μ

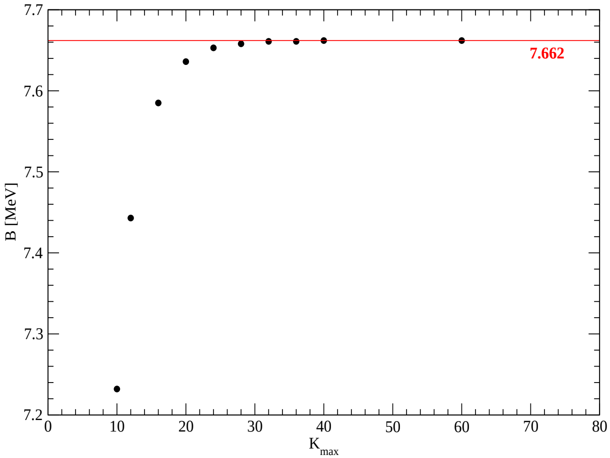
Scattering states

$$\Psi_{LSJ} = \Psi^{LSJ}_{core} + \Psi^{LSJ}_{asym}$$

- $\Psi^{LSJ}_{core} = \sum_\mu c_\mu \Psi_\mu$
- $\Psi^{LSJ}_{asym} \propto \Omega^R_{LS} + \sum_{L'S'} R_{LL',SS'} \Omega^I_{L'S'}$
- Kohn var. principle: $[R_{LL',SS'}] = R_{LL',SS'} - \langle \Psi_{L'S'J} | H - E | \Psi_{LSJ} \rangle$
  $\Rightarrow R_{LL',SS'} \rightarrow$ phase-shifts and mixing angles $\rightarrow$ scatt. length

Convergence of the method

B(^{3}H) with first 3 spin-isospin channel – N3LO



$A = 3, 4$ bound states

	^{3}H				^{3}He			
Potential	B (MeV)	r_p (fm)	$< V >$ (MeV)	P_D (%)	B (MeV)	r_p (fm)	$< V >$ (MeV)	P_D (%)
AV18	7.624	1.653	–54.351	8.510	6.925	1.872	–52.610	8.467
AV18/UIX	8.479	1.582	–59.754	9.301	7.750	1.771	–57.961	9.248
N3LO	7.854	1.655	–42.409	6.312	7.128	1.855	–40.917	6.313
N3LO/N2LO	8.474	1.611	–44.956	6.815	7.733	1.794	–43.478	6.818
Exp.	8.482	1.60	–	–	7.718	1.77	–	–

	^{4}He				
Potential	B (MeV)	r_p (fm)	$< V >$ (MeV)	P_D (%)	D_2^{dd} (fm^2)
AV18	24.21	1.514	–122.05	13.74	-0.115
AV18/UIX	28.46	1.430	–141.76	16.03	-0.113
N3LO	25.38	1.518	–94.62	10.74	
N3LO/N2LO	28.36	1.476	–103.29	10.79	
Exp.	28.30	1.47	–	–	-0.3$\pm$0.1 -0.19$\pm$0.04 -0.20$\pm$0.05

A. Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)

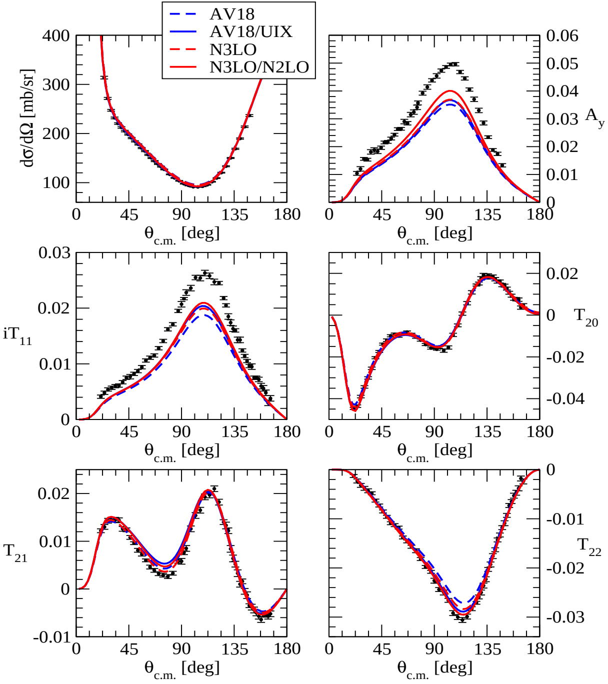
M. Viviani *et al.*, Phys. Rev. C **71**, 024006 (2005)

$A = 3, 4$ scattering lengths

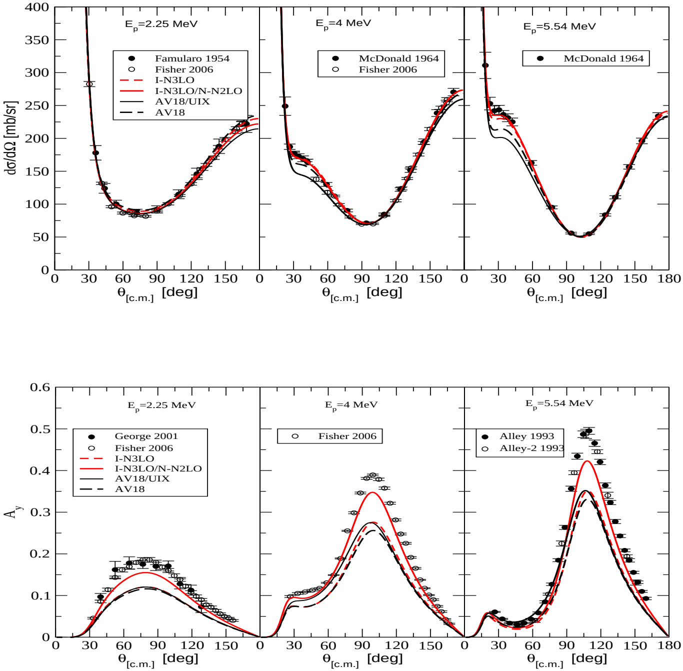
	AV18	AV18/UIX	N3LO	N3LO/N2LO	Exp.
${}^2a_{n-d}$ (fm)	1.248	0.590	1.100	0.675	0.645± 0.010
${}^4a_{n-d}$ (fm)	6.346	6.343	6.342	6.342	6.35±0.02
${}^2a_{p-d}$ (fm)	1.134	-0.089	0.876	0.072	–
${}^4a_{p-d}$ (fm)	13.662	13.662	13.646	13.647	–
${}^1a_{n-{}^3\mathrm{H}}$ (fm)	4.29	4.10	4.20	3.99	4.98±0.29
					4.45±0.10
${}^3a_{n-{}^3\mathrm{H}}$ (fm)	3.73	3.61	3.67	3.54	3.13±0.11
					3.32±0.02
${}^1a_{p-{}^3\mathrm{He}}$ (fm)	12.9	11.5	11.5	11.0	10.8±2.6
${}^3a_{p-{}^3\mathrm{He}}$ (fm)	10.0	9.1	9.2	8.6	8.1±0.5
					10.2±1.5

A. Kievsky *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101 (2008)

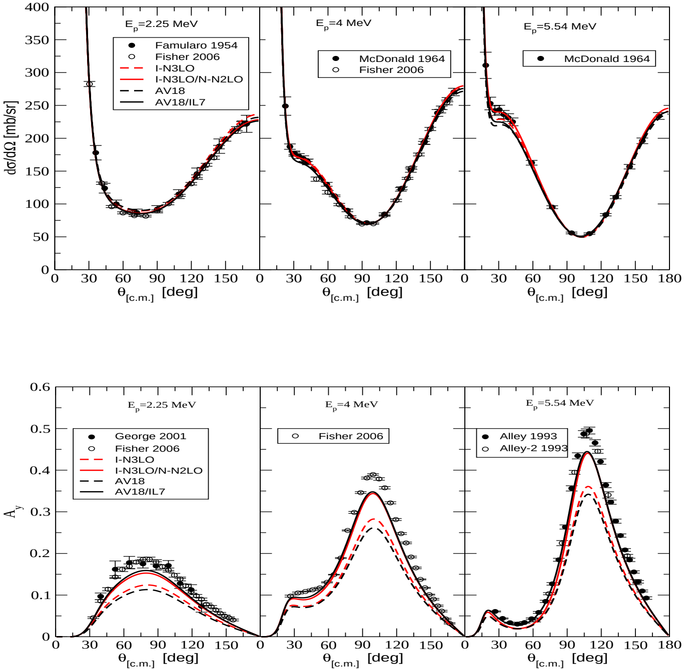
$p - d$ elastic scattering @ 2 MeV



$p - {}^3\mathrm{He}$ elastic scattering (AV18-AV18/UIX vs. N3LO-N3LO/N2LO)



$p - {}^3\mathrm{He}$ elastic scattering (UIX $\rightarrow$ IL7)



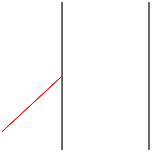
Electro-weak observables $\leftrightarrow$ electro-weak nuclear current

- Electromagnetic operators: $(\rho^\gamma, \mathbf{j}^\gamma)$
- Weak operators: $(\rho^{(V/A)}, \mathbf{j}^{(V/A)})$
- CVC $\Rightarrow (\rho^V/\mathbf{j}^V \rightarrow \rho^\gamma/\mathbf{j}^\gamma) \Rightarrow$ $\boxed{\text{4 operators}}$

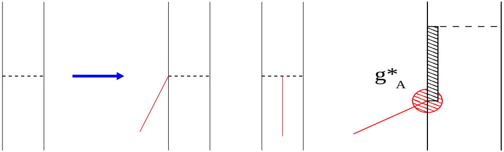
Current Conservation Relation (CCR) $\longrightarrow \mathbf{q}\cdot\mathbf{j}^\gamma \propto [\rho^\gamma, H]$

Realistic model

1b operators



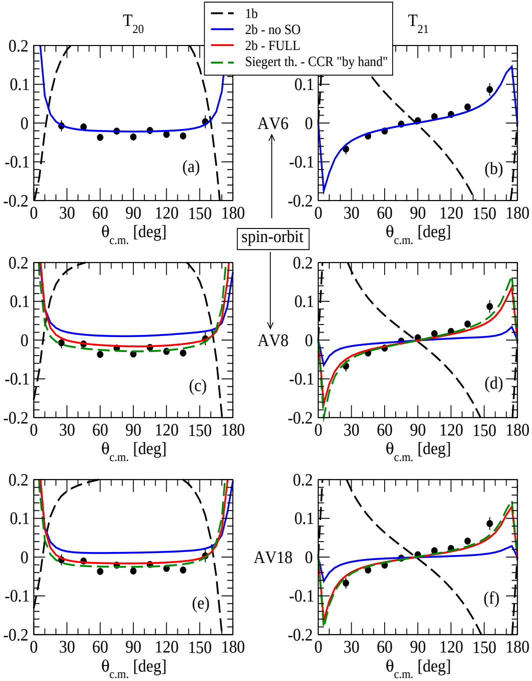
2b operators: Meson-exchange currents + $\mathbf{j}(\Delta)$



Interplay potential-current

CCR satisfied with AV18/UIX (L.E. Marcucci *et al.*, Phys. Rev. C **72**, 014001 (2005))

$\vec{d} + p \rightarrow {}^3\mathrm{He} + \gamma$
@ 2 MeV

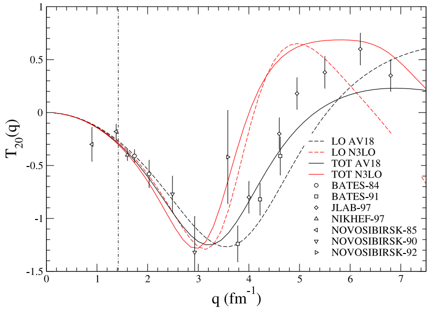


Some **PRELIMINARY** results with χEFT potential+current

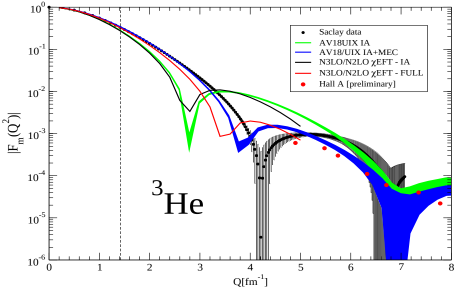
$A = 3$ m.m.

Nucleus	χEFT	phenom.	Exp.
^{3}H	2.9472	2.9525	2.9790
^{3}He	-2.0952	-2.1299	-2.1276

$e^- + \vec{d}$



$A = 3$ f.f.



Inclusive and exclusive electron-scattering

Inclusive: $A(e, e')$ $\Rightarrow$ only e' measured
Exclusive: final-state partially measured, e.g. $A(e, e'p)$

Inclusive

1. all final states to be included
2. Lorentz Integral Transform (LIT) method

$$\frac{d^2\sigma}{d\omega\, d\Omega_{e'}} = \alpha_{e'} \sum_{i=1}^{M} f_i(\omega, q, \theta_{e'})\, R_i(\omega, q)$$

$$R_i(\omega, q) = \sum\!\!\!\!\!\!\int_f dE_f |\langle \Psi(E_f)|\Theta_i|\Psi(E_0)\rangle|^2\, \delta(\omega + E_0 - E_f)$$

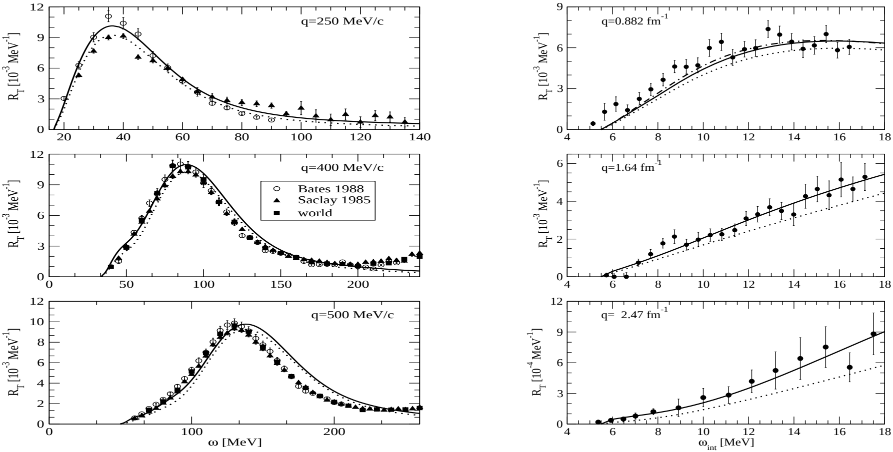
$$\mathrm{IT} \rightarrow L_i(\sigma; q) = \int dE\, R_i(E, q)\, K(E, \sigma) \leftrightarrow L_i(\sigma; q) = \langle \Psi(E_0)|K(H, \sigma)|\Psi(E_0)\rangle$$

$$\text{Lorentz kernel } \; K(E, \sigma) = \frac{1}{E - E_0 - \sigma_R - i\sigma_I}\, \frac{1}{E - E_0 - \sigma_R + i\sigma_I}$$

$$L_i(\sigma_R, \sigma_I; q) = \langle \tilde{\Psi}_i|\tilde{\Psi}_i\rangle \longrightarrow (H - E_0 - \sigma_R - i\sigma_I)\, \tilde{\Psi}_i = \Theta_i|\Psi(E_0)\rangle$$

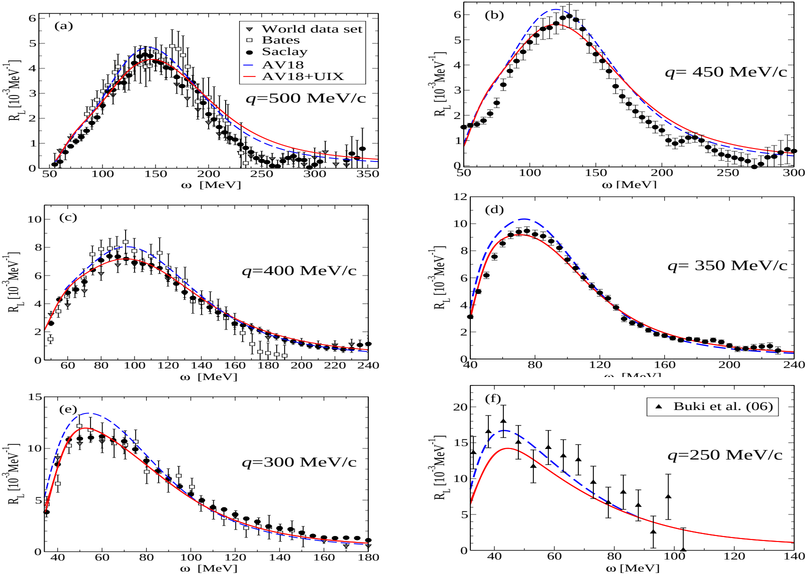
$$\longrightarrow \text{bound} - \text{state} - \text{like equation}$$

R_T for $^3\mathrm{He}(e,e')$



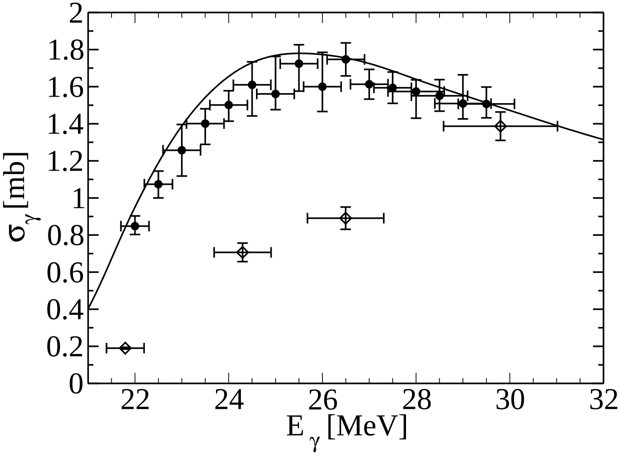
S. Della Monaca *et al.*, Phys. Rev. C **77**, 044007 (2008)

R_L for ^{4}He(e, e')



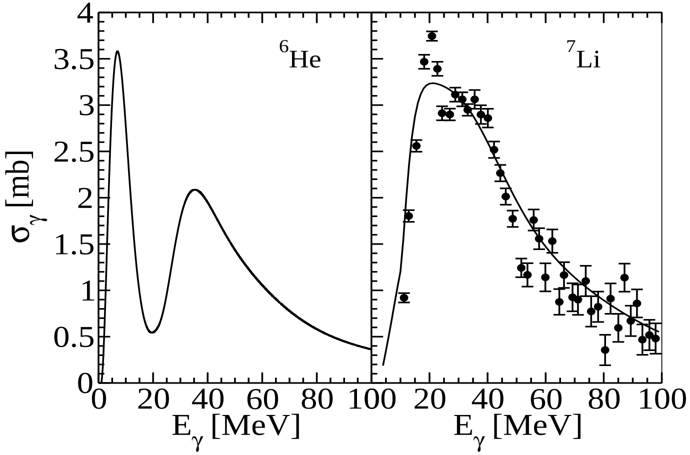
S. Bacca *et al.*, Phys. Rev. C **80**, 064001 (2009); *ibid.* Phys. Rev. Lett. **102**, 162501 (2009)

$^4He(\gamma, p)\,^3He$



S. Quaglioni *et al.*, Phys. Rev. C **69**, 044002 (2004)

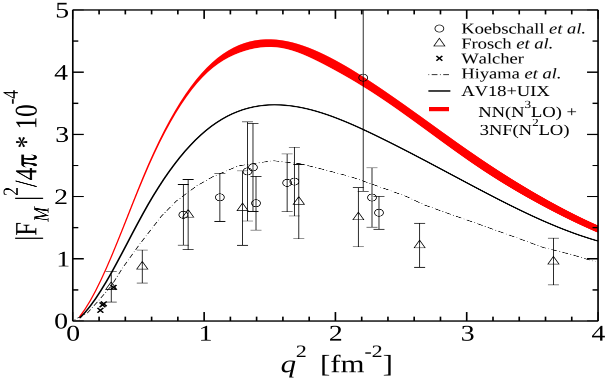
$\gamma + {}^6$He and $\gamma + {}^7$Li



S. Bacca *et al.*, Phys. Rev. C **69**, 057001 (2004); *ibid.*, Phys. Lett. B **603**, 159 (2004)

Isoscalar monopole resonance of ^{4}He

^{4}He*: O^+ narrow resonance ($E_R^{exp} = -8.20 \pm 0.05$ MeV)
$F_M \equiv$ transition f.f. for ^{4}He$(e, e')^4$He*



S. Bacca *et al.*, arXiv:1210.7255

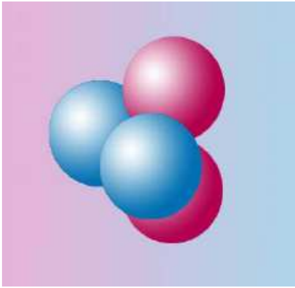
Summary

- *Ab-initio* methods: "exact" solution of the quantum many-body problem
- Few-nucleon systems $\leftrightarrow$ *ab-initio* methods: $A \leq 12$
- Ideal "laboratory" for testing potential+current models
- Nuclear reactions of astrophysical interest

Conclusions: to test $H \cdots$

~~Gli amici?~~
I nucleoni?
Meglio pochi

ma buoni!!



... o tanti ... ma sempre buoni!!