Teoria moderna delle forze nucleari

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work by:

Weinberg, Ordoñez, Van Kolck, Epelbaum, Gloeckle, Meissner, Entem, Machleidt,...

my contributions:

3NF at N4LO, nuclear e.m. 4-current, relativistic corrections, PV potential

collaborators:

A. Kievsky L. Marcucci M. Viviani (Pisa), S. Pastore (Argonne), R. Schiavilla (Jlab)

Recent reviews

- Epelbaum-Hammer-Meissner, Rev. Mod. Phys. 81 (2009) 1773
- Machleidt-Entem, Phys. Rept. 503 (2011) 1
- Lepage, "How to renormalize the Schroedinger equation, nucl-th/9706029

Outline

The problem of the nuclear interaction

- '30: Yukawa identifies the pion-exchange mechanism
- '50: meson field theories are mainly unsuccessful (lack chiral symmetry)
- '60: discovery of heavier mesons saves the situation. Meson exchange models
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however these approach suffer deficiencies:

- difficult assessment of theoretical uncertainty
- no further insight into consistent three-nucleon interaction
- hard to implement chiral and gauge symmetry
- lack of a clear contact with QCD

Effective theories and separation of scales

a probe of wavelength λ is insensible to details at short distances
$\longrightarrow$ replace the *true* short distance structure with a tower of *simpler* terms (cfr. multipole expansion)
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c, $d_{1,2}$ are LECs to be fixed from data
At a given order only a finite number of LECs $\Longrightarrow$ predictions

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
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a good compromise can be found within the *range of applicability* of the effective theory

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Explicit chiral symmetry violation $F_\pi^2 M_\pi^2 = (m_u + m_d)\langle 0|\bar{\psi}\psi|0\rangle + \ldots$

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In our case the effective Lagrangian must involve pions and nucleons through interpolating fields

$$U(x) = \exp\left(i\frac{\pi^a(x)\tau^a}{F_\pi}\right) \equiv u^2, \quad U \to U' = V_R U V_L^\dagger$$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \to N' = K(V_L, V_R, U)N, \quad V_R u = u' K$$

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nucleons can couple to pions only derivatively, via:

- the chiral connection Γ_μ appearing in the covariant derivative $D_\mu N$
- the "building block" $u_\mu = i u^\dagger \nabla_\mu U u^\dagger \to K u_\mu K^\dagger$

Chiral Lagrangians

A class of Lagrangians emerges, characterized by the number of derivatives and/or quark mass matrix $\mathcal{M} \sim O(p^2)$,

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$$\mathcal{L}_{\pi N} = \bar{N}(i\not{D} - M + \frac{1}{2}g_A \not{u}\gamma_5 + \ldots)N$$

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The above Lagrangians represents all the contributions with $\Delta = 0$

Disposing of large time-derivatives

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- heavy baryon formalism: integrate out the small field components
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- remove time derivatives from the Lagrangian using nucleon equations of motions (this amounts to redefine the interpolating fields). The lost Lorentz covariance of the ensuing Hamiltonian can be recovered at the end by imposing the Poincaré commutation relation, order by order in the chiral expansion

Chiral (power) counting

Count the power ν of low momenta in a generic Feynman diagram

- pion propagator $1/p^2 - m_\pi^2 \sim O(p^{-2})$
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$$\nu = 2 + 2L - \frac{E_N}{2} + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

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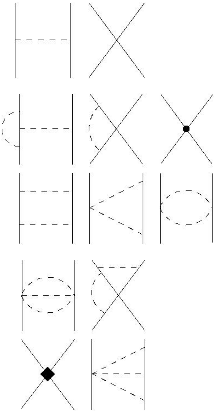
the chiral expansion is a loop expansion, since $\Delta_i \geq 0$

NN amplitude



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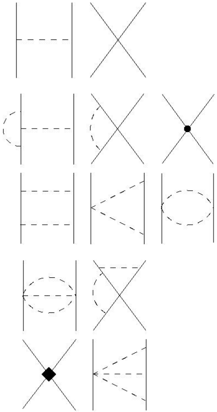

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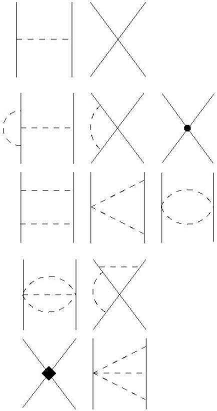

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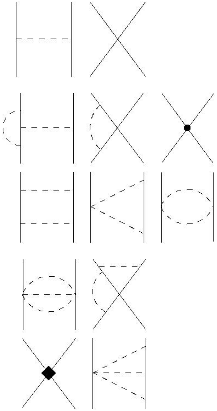

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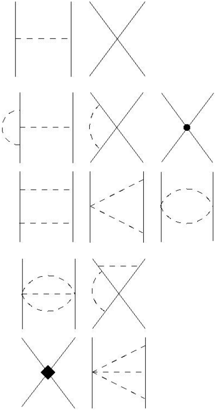

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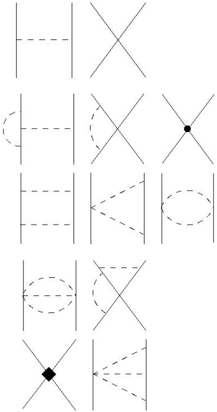
- at the order $\nu = 3$ we have 1-loop diagrams with subleading vertices
- at the order $\nu = 4$ (N3LO) we start to have 2-loop diagrams and $\Delta = 4$ contact terms (crucial for the $D - waves$)



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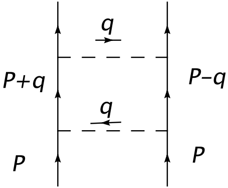


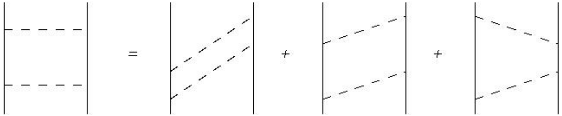
$$\sim \int d^4q \frac{1}{q^0 + i\epsilon} \frac{1}{q^0 - i\epsilon} \frac{P(q)}{(q^2 - M_\pi^2 + i\epsilon)^2}$$

"Pinch singularities"

This diagram is $O(p^0)$ instead of $O(p^2)$

Weinberg proposal

The origin of the problem is more manifest in time-ordered perturbation theory



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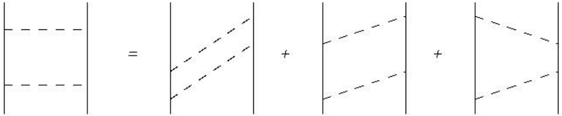

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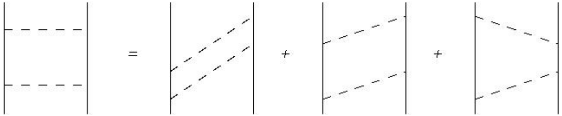

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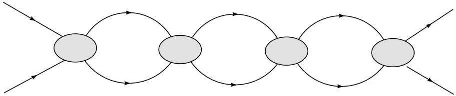

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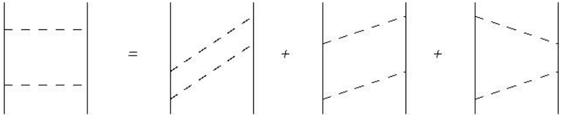

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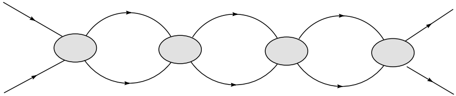

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- Apply the chiral counting only to the effective potential so as to avoid the pinch singularities

Accuracy of chiral potentials

Tables of χ^2/datum [Machleidt-Entem, Phys. Rep. 503 (2011) 1]

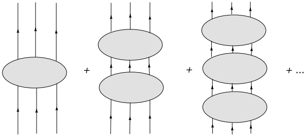
$T_{\rm lab}$ bin (MeV)	# of *np* data	*Idaho* N^{3}LO [68] (500–600)	*Juelich* N^{3}LO [171] (600/700–450/500)	Argonne V_{18} [174]
0–100	1058	1.0–1.1	1.0–1.1	0.95
100–190	501	1.1–1.2	1.3–1.8	1.10
190–290	843	1.2–1.4	2.8–20.0	1.11
0–290	2402	1.1–1.3	1.7–7.9	1.04

$T_{\rm lab}$ bin (MeV)	# of *pp* data	*Idaho* N^{3}LO [68] (500–600)	*Juelich* N^{3}LO [171] (600/700–450/500)	Argonne V_{18} [174]
0–100	795	1.0–1.7	1.0–3.8	1.0
100–190	411	1.5–1.9	3.5–11.6	1.3
190–290	851	1.9–2.7	4.3–44.4	1.8
0–290	2057	1.5–2.1	2.9–22.3	1.4

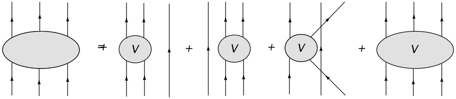
Up to N3LO there are 24 free LECs from contact operators, comparable with the number of parameters of phenomenological realistic potentials (35-40)

Three and more nucleons

Disconnected diagrams give the main contribution to the effective potential



where



Each disconnected piece is enhanced by an additional 4-momentum conserving δ-function

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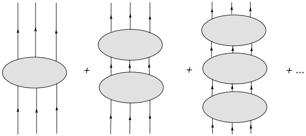

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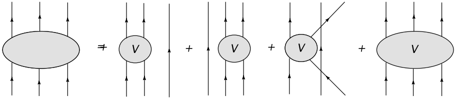


Each disconnected piece is enhanced by an additional 4-momentum conserving δ-function In an A-nucleon diagram, with C separately disconnected pieces

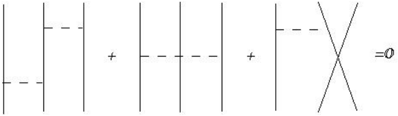
$$\nu = \sum_{i=1}^{C} [2 + 2L_i - A_i + (\sum \Delta)_i] - 4(C-1) = 4 - A - 2C + 2L + \sum_i \Delta_i$$

each participating nucleon decreases C by 1 and increases ν by 2

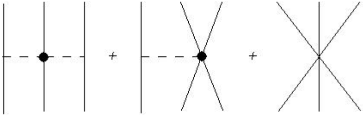
$\Longrightarrow$ hierarchy of nuclear forces

Three-nucleon interaction

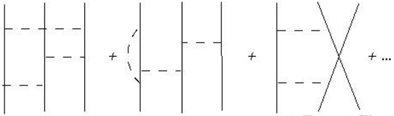
- in principle it starts to contribute at relative order $O(p^2)$ compared to the leading NN interaction, but this contribution vanishes



-
- the first non-vanishing contribution is therefore at $\delta\nu = 3$, or N2LO



-
- loops start at order N3LO, including pion rings (no free parameters)



Accuracy of three-nucleon interaction

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- this is not really surprising, given the small number of adjustable parameters: chiral TNI has just 2 free parameters up to N3LO
- since discrepancies arise at low energies, where interactions reduce to contact terms we have investigated the subleading $3N$ contact interaction [Girlanda et al. PRC84 (2011) 014001]

Imposing parity and time reversal we get a list of 146 operators

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$$\overleftrightarrow{\nabla}_1 \cdot \overleftrightarrow{\nabla}_2 [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3]$$
$$\overleftrightarrow{\nabla}_1 \cdot \vec{\sigma}_1 \, \overleftrightarrow{\nabla}_2 \cdot \vec{\sigma}_2 [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3]$$
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same as before with $\overleftrightarrow{\nabla} \to \overrightarrow{\nabla}$

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Constraints from Pauli principle and relativity

The anticommuting nature of the nucleon fields implies relationship among all possible operators.

In addition, we have to impose the requirements of Poincaré covariance

As a result, the subleading $3N$ effective Hamiltonian consists of

- fixed terms (relativistic corrections to the lower order terms)
- free terms, which have to commute with the lowest order boost operator $\mathbf{K}_0$

with the choice $N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3}\, b_s(\mathbf{p})\, \chi_s\, \mathrm{e}^{-ip\cdot x}$ $\mathbf{K}_0$ acts as

$$[\mathbf{K}_0\,,\; b_s(\mathbf{p})] = -i\, m\, \boldsymbol{\nabla}_{\mathbf{p}}\, b_s(\mathbf{p})$$

and only 10 independent combinations of the 14 operators can be found to commute with $\mathbf{K}_0$

Subleading contact potential

Choosing a momentum cutoff depending only on momentum transfers the coordinate space potential can be given a local form

$$
\begin{aligned}
V = \sum_{i\neq j\neq k} & (E_1 + E_2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
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Most terms are ordinary 2-body interactions between particles ij with a further dependence on the coordinate of particle k

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Spin-orbit terms suitable for the A_y puzzle [Kievsky PRC60 (1999) 034001]

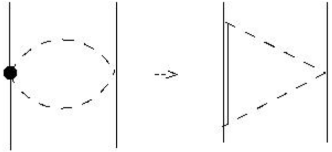
- work is in progress to determine the 10 LECs from data

Including the Δ

The proximity of the Δ resonance challenges the convergence properties of the effective theory

$$m_\pi \lesssim \Delta M = M_\Delta - M_N = 293\ \mathrm{MeV}$$

this reflects itself in unnatural values of some LECs
To improve the convergence one can include explicitly the Δ and treat $\Delta M \sim m_\pi \sim O(p)$



some contributions are promoted to lower orders and the LECs assume a more natural value

Integrating out the pion

Conversely, one can build the pionless theory, valid at $k \ll m_\pi$

$$\Lambda \sim m_\pi$$

due to the large scattering lengths, the contact interactions have to be resummed to all orders

$$\mathcal{A} = \frac{4\pi}{M} \frac{1}{1/a + ik} \left[1 + \frac{r_0/2}{1/a + ik} k^2 + \frac{(r_0/2)^2}{(1/a+ik)^2} + \ldots \right]$$

thus the expansion is in kr_0 but not limited to $ka << 1$ anymore
In the three-body sector the theory is usually formulated in terms of dimeron (auxiliary) fields
It encompasses universal phenomena, valid for systems with large scattering lengths $\longrightarrow$ Efimov physics, and allow to compute systematically corrections to the *unitary limit*, with controlled uncertainty

External currents

Electroweak currents are naturally implemented in the formalism, since they are the Noether currents of chiral symmetries [Park-Min-Rho, 1996]

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \\ D_\mu N &= \partial_\mu N + \Gamma_\mu N \\ \Gamma_\mu &= \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger(v_\mu + a_\mu)u - \frac{i}{2}u(v_\mu - a_\mu)u^\dagger \end{aligned}$$

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The transition operator is computed in the low-energy expansion, and maybe calculated inside realistic wave functions (hybrid approach)
With interactions and currents computed within the same scheme, electroweak nuclear observables may be worked out consistently, with controlled theoretical uncertainty

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\nabla_\mu U &= \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \\
D_\mu N &= \partial_\mu N + \Gamma_\mu N \\
\Gamma_\mu &= \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger (v_\mu + a_\mu) u - \frac{i}{2} u (v_\mu - a_\mu) u^\dagger
\end{aligned}
$$

The transition operator is computed in the low-energy expansion, and maybe calculated inside realistic wave functions (hybrid approach) With interactions and currents computed within the same scheme, electroweak nuclear observables may be worked out consistently, with controlled theoretical uncertainty This allows to construct currents consistent with the interactions, with controlled theoretical uncertainty

Conclusions and outlook

- The understanding of nuclear interaction, starting from Yukawa, has now come back to pion exchange, but within a systematic EFT framework constrained by chiral symmetry of QCD
- The EFT machinery can be viewed as a way to optimally implement symmetry constraints from the underlying theory. The LECs parametrize our ignorance on the dynamics
- This understanding is also quantitative: modern N3LO chiral potentials provide a very accurate description of *NN* data, comparable to "realistic" models
- The frontier is now the 3NF, where the guidance of power counting and symmetry is essential. We are confident to have identified the needed component of 3NF to become "realistic"
- Few-nucleon systems provide a unique laboratory to test and constrain the interaction and nuclear electroweak transition operators