Tensor Interaction and its effect on Spin-orbit Splitting of Shell Model States

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Abstract

After inclusion of tensor interaction in Skyrme Hartree Fock theory SKP set of parameters have been used to calculate the splitting of single particle shell model states of ^{208}Pb where there is abundant experimental data. For proton states, nuclei ^{207}Tl and ^{209}Bi have been considered for comparison whereas for neutron states nuclei ^{207}Pb and ^{209}Pb were considered. The level splittings of spin-orbit partners are reproduced quite admirably thus vindicating the importance of inclusion of tensor interaction. Neutron skin has also been calculated which may shed light on nuclear symmetry energy.

The genesis of magic numbers is known to be due to a strong spin-orbit interaction in nuclei [1-3]. However, the evolution of single particle energy levels with increasing N or Z forming islands of stability does not follow a simple geometrical rule. With the advent of new experimental facilities like radioactive ion beams it has been observed that new areas of magicity have developed [4-8] while conventional shell gaps have weakened thus necessitating a relook in the mean field type of nuclear structure calculations.

Conventional mean-field calculations [9-15] mainly deal with bulk properties of nuclei, viz., binding energy, rigidity modulus, charge density radius etc. From the point of view of nuclear theory, ^{208}Pb, the doubly magic nucleus, is one of the anchor points in the parameterizations of effective interactions for mean-field calculations. Moreover, there are several experimental results available for lead isotopes [16-20] which provide important information about single particle shell model states near the Fermi surface and systematic data of isotope shifts. Effect of tensor interaction in mean-field type calculations on the evolution of shell structure has recently drawn a lot of interest [21-26] due to its simplistic approach and wide applicability. In this paper we have incorporated the tensor interaction in Skyrme-Hartree-Fock theory to investigate the effect of tensor interaction on the splitting of spin-orbit partners of shell model states near the Fermi surface of ^{208}Pb. We have also calculated the neutron skin of ^{208}Pb given by $S = \sqrt{<R_n^2>} - \sqrt{<R_p^2>}$, which has a bearing on the evaluation of the symmetry energy for the isotope.

The spin-orbit potential in Skyrme Hartree-Fock theory with the inclusion of tensor component is given by

$$V_{s.o.}^{q} = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right) \quad (1)$$

where $J_{q(q')}(r)$ is the proton or neutron spin-orbit density defined as

$$J_{q(q')}(r) = \frac{1}{4\pi r^3}\sum_i v_i^2(2j+1)\left[j_i(j_i+1) - l_i(l_i+1) - \frac{3}{4}\right]R_i^2(r) \quad (2)$$

The tensor interaction is given by

$$V_T = \frac{T}{2}\left\{\left[(\boldsymbol{\sigma_1}.\boldsymbol{k}')(\boldsymbol{\sigma_2}.\boldsymbol{k}') - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2})k'^2\right]\delta(\boldsymbol{r_1}-\boldsymbol{r_2}) + \delta(\boldsymbol{r_1}-\boldsymbol{r_2})\left[(\boldsymbol{\sigma_1}.\boldsymbol{k})(\boldsymbol{\sigma_2}.\boldsymbol{k}) - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2})k^2\right]\right\} + U\{(\boldsymbol{\sigma_1}.\boldsymbol{k}')\delta(\boldsymbol{r_1}-\boldsymbol{r_2})(\boldsymbol{\sigma_1}.\boldsymbol{k}) - \frac{1}{3}(\boldsymbol{\sigma_1}.\boldsymbol{\sigma_2})\times[\boldsymbol{k}'.\delta(r_1-r_2)\boldsymbol{k}]\} \quad (3)$$

The coupling constants T and U denote the strength of the triplet-even and triplet-odd tensor interactions respectively.

In Eq.(1) $\alpha = \alpha_c + \alpha_T$ and $\beta = \beta_c + \beta_T$. The central exchange contributions are written in terms of the usual Skyrme parameters as

$$\alpha_c = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1x_1 + t_2x_2) \quad (4)$$

$$\beta_c = -\frac{1}{8}(t_1x_1 + t_2x_2) \quad (5)$$

The tensor contributions are expressed as

$$\alpha_T = \frac{5}{12}U, \;\; \beta_T = \frac{5}{24}(T + U), \quad (6)$$

In our calculation we have recast α as

$$\alpha = \alpha_c(1 + \alpha_T/\alpha_c) = S_f\,\alpha_c ,$$

where S_f is the scale factor = $1 + \alpha_T/\alpha_c$. Similarly for β we get

$$\beta = S_f'\beta_c , \text{ where } S_f' = (1 + \beta_T/\beta_c) .$$

We have used SKP set of parameters to calculate the splitting of single particle shell model states of ^{208}Pb. The reasons behind the use of SKP set are: i) it takes the $\mathbf{J}^2$ terms from the central force into account which is necessary for inclusion of the tensor force, ii) it reproduces the ground state properties and the single particle structure of ^{208}Pb quite nicely. From the inclusion of tensor interaction in the Skyrme energy density functional one expects that it will affect the spin-orbit splitting by altering the strength of the spin-orbit field in spin-unsaturated nuclei as expressed in Eq. (1). However, one must remember that the spin-orbit potential is readjusted through each pair of scale factors which are connected to the tensor coupling terms.

The success of the SKP parameters led us to use this set to evaluate the effect of inclusion of tensor forces in finding out the evolution of single particle states in different areas of nuclear chart. The optimal parameters of tensor interaction α_T and β_T were found for SKP forces by optimizing the reproduction of observed splitting of spin-orbit partners of single particle states around ^{208}Pb. In a two parameter search we have found the scale factors f = -2.4 and f' = -1.6 produces the best result. Because of the fact that the formation of shells depends crucially on the spin-orbit splitting of the single particle states we have chosen the scale factors such that it creates the shell gap in the right place. At the same time, locations of the individual states have not been compromised. In our calculation the value of α came out to be -80.18 and the value of β is 78.14.

In Table I we present our calculated single particle spectrum for ^{208}Pb for different parameter sets SKM, Z_σ, Sly4, SKX and SKP. Single particle behaviour of these states has been well established from detailed spectroscopic measurements. It is quite apparent from the table that SKP produces the best single particle (-hole) spectrum both for proton as well as neutron states of ^{208}Pb. Out of eighteen states studied only in the cases of proton $1g_{9/2}$ state and neutron $1h_{9/2}$ and $1i_{11/2}$ states we find some discrepancies of the order of 1 MeV, otherwise there is a very good agreement.

In Table II the calculated splitting of the spin-orbit partners of proton and neutron single particle states around ^{208}Pb have been presented along with their experimental values. For proton states nuclei ^{207}Tl and ^{209}Bi have been considered for comparison whereas for neutron states nuclei ^{207}Pb and ^{209}Pb were considered. The level splittings are reproduced quite admirably by SKP parameter set with the inclusion of tensor interaction thus vindicating the importance of inclusion of tensor interaction.

In Table III the calculated root mean square proton and neutron radii for ^{208}Pb is presented along with their experimental counterparts for comparison. Though for charge radius we have obtained a very good fit, for neutron the calculated rms radius is slightly smaller than the experimental value. As a result we have obtained a smaller skin thickness S.

In fig. 1 we have presented the charge and neutron distribution of ^{208}Pb. In contrast to a peak in the charge distribution near the centre, the neutron distribution shows a dip. In fig. 2 spin-density J_q (r) for both neutron and proton cases have been presented. As ^{208}Pb is spin-unsaturated in neutron and proton systems tensor interaction has an important role to play.

In this paper we have shown that inclusion of tensor interaction in the Skyrme Hartree Fock theory the shell gap at Z = 82 and N = 126 is better reproduced than the conventional mean-field type calculation.

TABLE I

Single particle levels in ^{208}Pb

nlj	$-E_{nlj}$ (MeV)					
	SKM	Z_σ	Sly4	SKX	SKP	EXPT
Proton						
$1g_{9/2}$	16.12	17.50	16.44	16.15	15.18	16.03
$1g_{7/2}$	12.32	13.34	14.67	11.36	11.43	11.51
$2d_{5/2}$	10.16	10.88	11.25	9.64	10.33	10.23
$2d_{3/2}$	8.28	8.87	7.07	7.54	8.80	8.38
$3s_{1/2}$	7.56	8.04	9.01	7.04	8.11	8.03
$1h_{11/2}$	8.42	9.58	8.11	9.95	8.72	9.37
$1h_{9/2}$	2.95	3.62	5.54	3.07	3.52	3.60
$2f_{7/2}$	1.81	2.13	2.32	2.47	3.21	2.91
Neutron						
$1h_{11/2}$	17.27	18.09	16.32	16.52	15.03	14.50
$1h_{9/2}$	11.73	12.09	14.09	9.46	9.84	11.28
$2f_{7/2}$	11.50	11.68	11.21	9.98	10.50	10.38
$2f_{5/2}$	8.53	8.40	9.81	6.67	8.11	7.95
$3p_{3/2}$	8.56	8.63	8.65	7.10	8.21	8.27
$3p_{1/2}$	7.40	7.33	8.10	5.81	7.32	7.38
$1i_{13/2}$	9.42	9.55	7.06	10.37	8.51	9.38
$1i_{11/2}$	2.07	1.83	3.82	1.02	1.84	3.15
$2g_{9/2}$	3.35	2.92	1.64	3.19	3.71	3.74
$2g_{7/2}$	0.27	1.03	0.04	0.95	0.71	1.45

TABLE II

Splitting of spin-orbit doublets

Protons	Energy in MeV					
	Sly5	Sly4	SKM*	SIII	SKP-T	EXP
Δ1h	6.43	6.22	5.94	5.20	4.64	5.56
Δ2d	1.95	1.89	2.37	1.63	1.55	1.33
Δ2f	2.69	2.61	2.57	2.32	2.17	1.93
Δ3p	1.05	1.02	0.97	0.87	0.79	0.84
Neutrons						
Δ1h	5.83	5.59	5.55	4.73	5.32	5.10
Δ1i	7.65	7.25	7.26	6.39	6.93	6.46
Δ2f	2.05	1.96	2.93	2.67	2.27	2.03
Δ2g	3.68	3.57	3.58	3.30	2.83	2.51
Δ3p	1.17	1.13	1.13	1.02	0.84	0.90
Δ3d	1.72	1.67	1.60	1.47	1.31	0.97

TABLE III

R.M.S. Proton and Neutron Radius of ^{208}Pb

R_c (Th.) (fm.)	R_c (Expt.) (fm.)	R_n (Th.) (fm.)	R_n (Expt.) (fm.)	$R_n - R_p$ (Th.) (fm.)	R_n–R_p (Expt.)[a)] (fm.)
5.497	5.501	5.554	5.653	0.11	0.195 $\pm$ 0.057

a) Ref. 27

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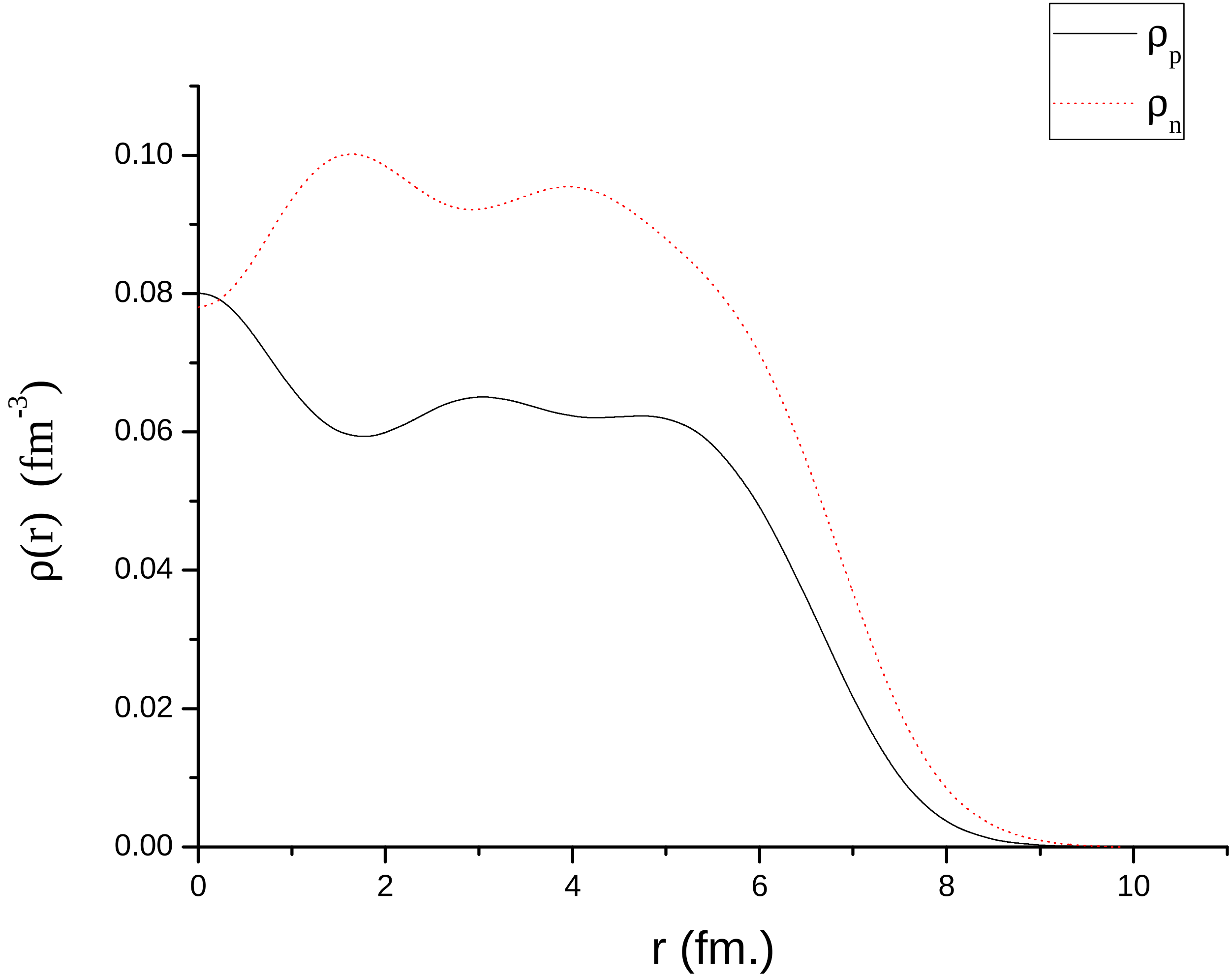
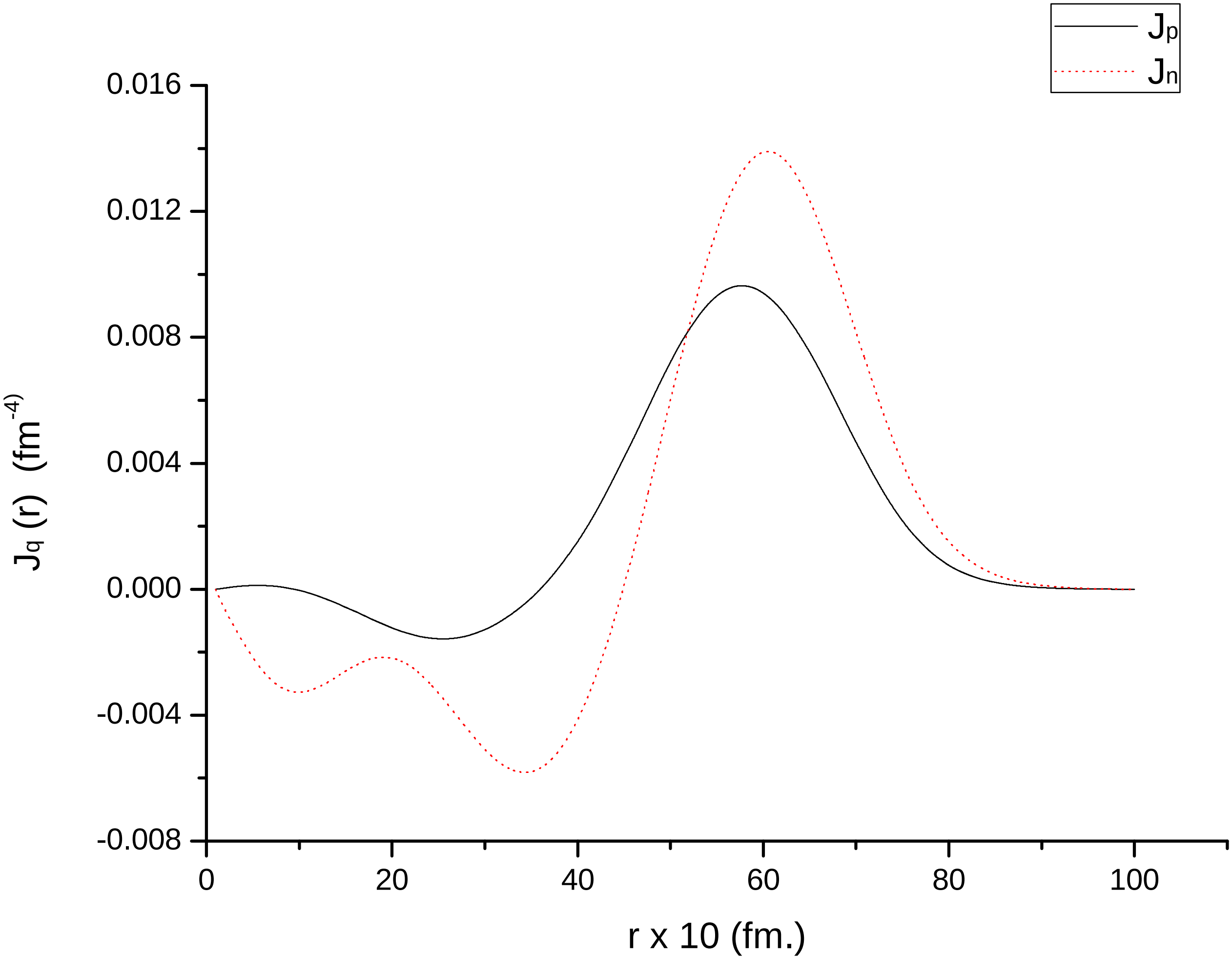
Figure Caption:

1. Charge and neutron distribution of ^{208}Pb.

2. Spin-density distributions of ^{208}Pb.



ρ(r) (fm⁻³)
r (fm.)
ρp
ρn
0.00
0.02
0.04
0.06
0.08
0.10
0
2
4
6
8
10



J_p
J_n
0.016
0.012
0.008
0.004
0.000
-0.004
-0.008
J_q (r) (fm^{-4})
0
20
40
60
80
100
r x 10 (fm.)