Symmetry energy and nucleon-nucleon cross sections

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Nucleus-nucleus cross section in the nuclear matter are practically unknown.

Typically, free nucleon-nucleon cross sections are used in simulations of nucleus-nucleus collisions, occasionally scaled down by some factor.

Density-dependence of nucleon-nucleon cross sections is approximated only empirically.

True in-medium nucleon-nucleon cross sections must depend on equation of state of the nuclear matter.

Project of Senior International Expert's Stay at Chinese Academy of Sciences (02/2012 - 04/2012):

Van der Waals equation of state in nuclear transport model

It is planned to implement phenomenological nuclear equation of state of van der Waals form to modify the parametrizations of the potential and reaction cross sections entering into the transport models such as the Boltzmann-Uehling-Uhlenbeck equation. Behavior of various observables will be examined, in particular their sensitivity to variation of parameters of the EoS, specifically the the symmetry energy term and the dependence on nuclear density and isospin asymmetry. In the comparison with available experimental data, it will be possible to strengthen constraints on the density-dependence of the symmetry energy.

What is so specific about Van der Waals-like equation of state ?

It allows to describe by simple model the liquid-gas phase transition in isospin-asymmetric nuclear matter, including the possible interplay of 1^{st} and 2^{nd} order phase transition with variation of the isospin asymmetry (M. Veselsky, IJMPE 17 (2008) 1883; arXiv.org:nucl-th/0703077)

It provides a parameter, called “proper volume”, which describes the volume per constituent particle of the non-ideal gas

Equation of state of asymmetric nuclear matter

The nuclear potential acting on neutron or proton can be in the Boltzmann-Uhling-Uhlenbeck equation (BUU) represented as

$$\frac{U}{A} = a\rho + b\rho^{\kappa} + 2a_s(\frac{\rho}{\rho_0})^{\gamma}\tau_z I \tag{1}$$

where $I = (\rho_n - \rho_p)/\rho$, τ_z assumes value 1 for neutron and -1 for proton and coefficients a and b represent properties of the symmetric nuclear matter while the last term describes the influence of the symmetry energy. Corresponding contribution into thermodynamical equation of state can be obtained as

$$\Delta p_{non-ideal} = -(\frac{dU}{dV})_T = a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s\rho_0(\frac{\rho}{\rho_0})^{1+\gamma}\tau_z I \tag{2}$$

The equation of state of asymmetric nuclear matter can be then written in this simple parametrization as

$$p = \rho T + a\rho^2 + b\kappa\rho^{1+\kappa} + 2\gamma a_s\rho_0(\frac{\rho}{\rho_0})^{1+\gamma}\tau_z I \tag{3}$$

is obtained from simple Weizsaecker formula, where a_s represents the coefficient of the symmetry energy term, assuming the $\rho^{2/3}$-dependence characteristic for the Fermi gas. The values of a_s typically range from 18 to 25 MeV. The above equation of state can reasonably describe properties of asymmetric nuclear matter at sub-saturation densities, such as multifragmentation and isospin-asymmetric liquid-gas phase transition.

Estimation of volume using the Van der Waals equation of state

When looking for relation of equation of state and emission rates one can consider the van der Waals equation of state. It is written as

$$(p + a'\rho^2)(V - Nb') = NT \tag{4}$$

or

$$(p + a'\rho^2)(1 - \rho b') = \rho T \tag{5}$$

where the parameter a' is related to attractive interaction among particles and b' represents the proper volume of the constituent particles. In geometrical picture the volume of the particle can be directly related to its cross section for interaction with particles. It is possible to formally transform the equation of state of asymmetric nuclear matter (and practically any equation of state of any form) into the van der Waals equation. Then one obtains coefficients

$$a' = -a \tag{6}$$

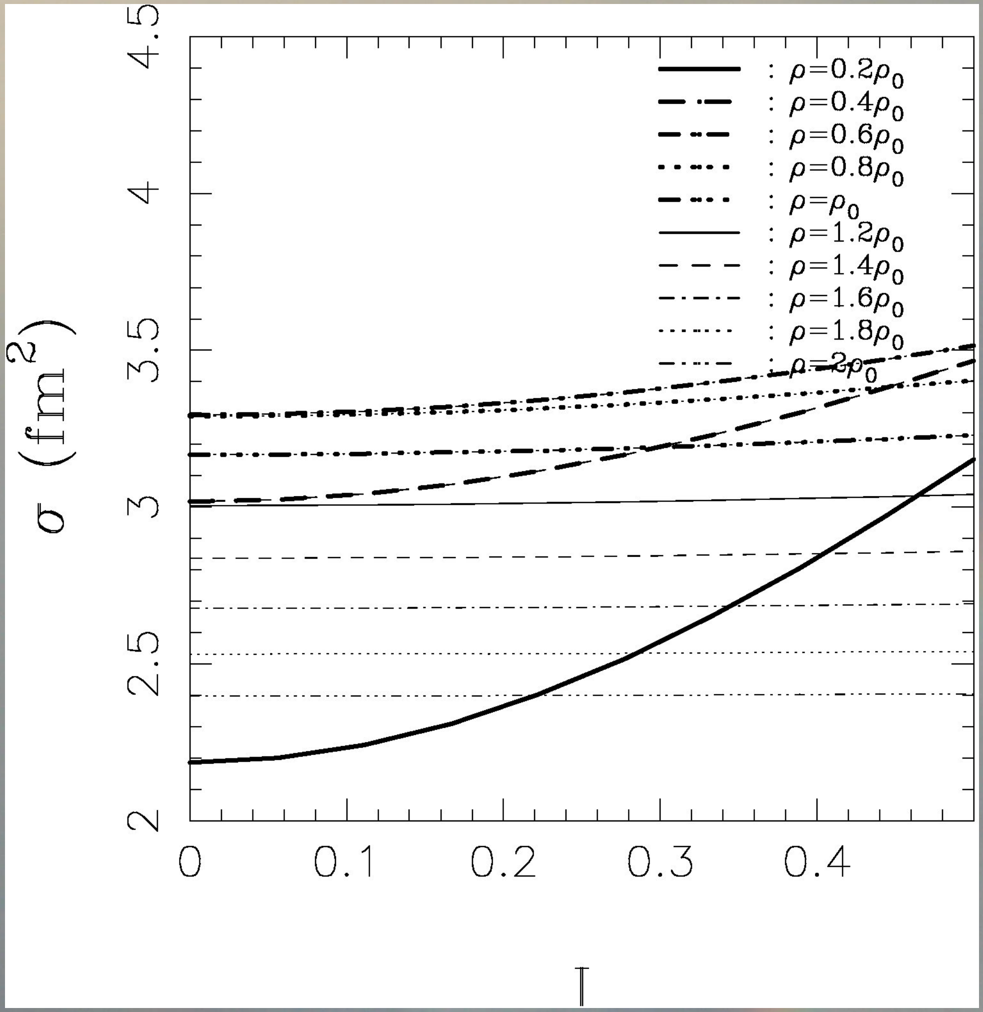
and

$$b' = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s(\frac{\rho}{\rho_0})^{\gamma}\tau_z I}{p - a\rho^2} = \frac{b\kappa\rho^{\kappa} + 2\gamma a_s(\frac{\rho}{\rho_0})^{\gamma}\tau_z I}{\rho T + b\kappa\rho^{1+\kappa} + 2\gamma\rho_0 a_s(\frac{\rho}{\rho_0})^{1+\gamma}\tau_z I} \tag{7}$$

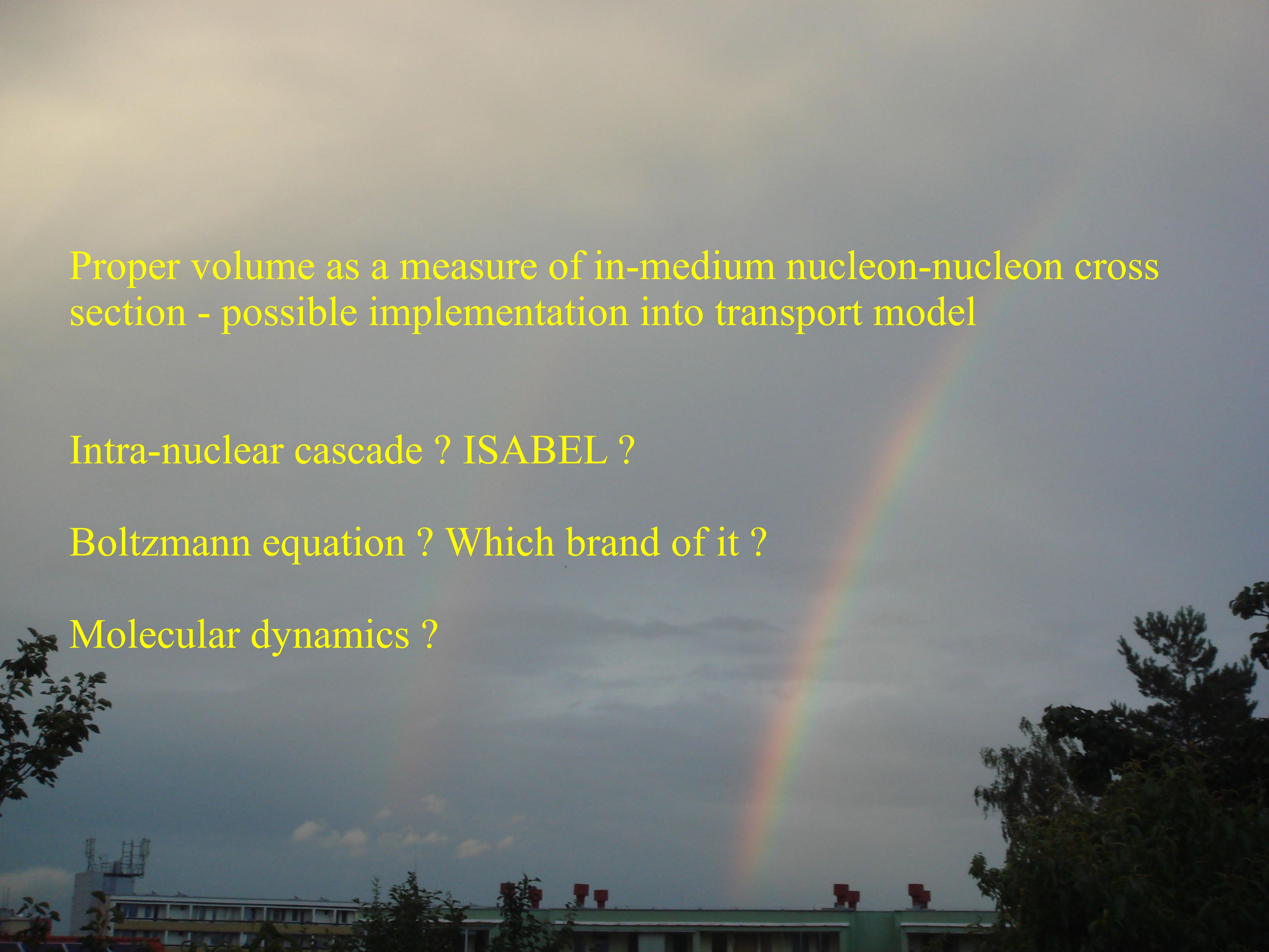
where the latter provides a measure of the proper volume of the constituent of the gas, nucleon in this case, as a measure of deviation from its behavior of the ideal gas. The proper volume of nucleon can be used to estimate its cross section within the nucleonic medium

$$\sigma = 1.209\ b'^{2/3} \tag{8}$$

which can be implemented into the collision term of the BUU equation.



Nucleon-nucleon cross section, corresponding to proper volume, estimated using the Van der Waals-like EoS (from M. Veselsky, IJMPE 17 (2008) 1883; arXiv.org:nucl-th/0703077).



Proper volume as a measure of in-medium nucleon-nucleon cross section - possible implementation into transport model

Intra-nuclear cascade ? ISABEL ?

Boltzmann equation ? Which brand of it ?

Molecular dynamics ?

Boltzmann equation

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{\mathbf{p}}{m} + \frac{\partial f}{\partial \mathbf{p}} \cdot \mathbf{F} = \left. \frac{\partial f}{\partial t} \right|_{\mathrm{coll}} .$$

No collision term – Liouville or Vlasov equation, explains e.g. Boltzmann distribution, behavior of plasma in electromagnetic field

Collision term – BGK form – describes simple transport phenomena – Ohm's law, thermodiffusion, thermo-electric phenomena, Fokker-Planck equation (deep-inelastic transfer)

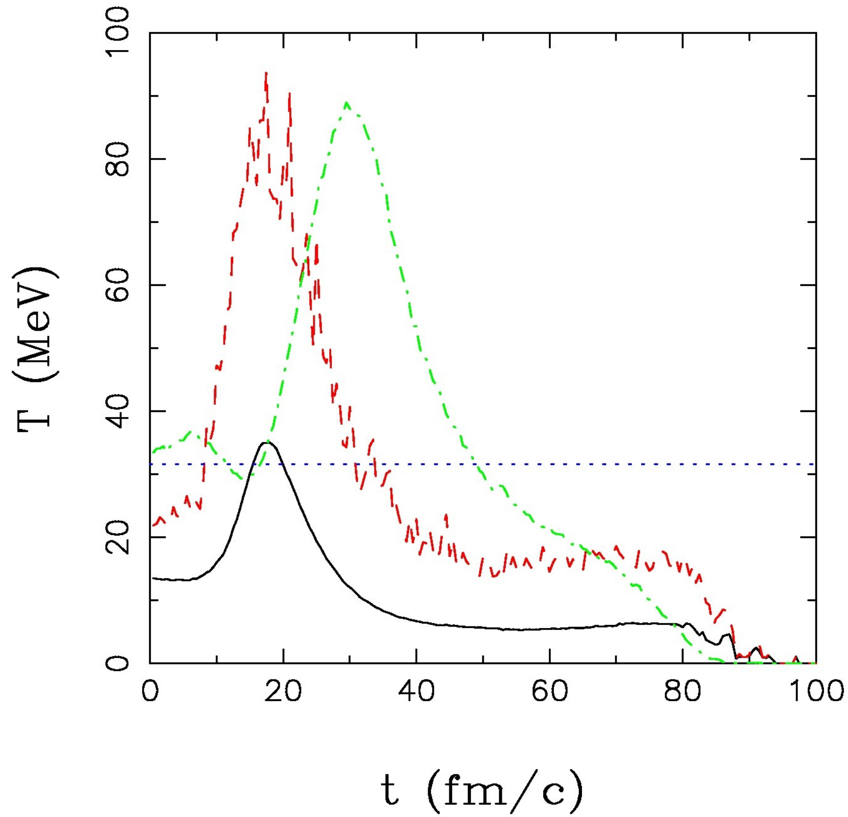
Collision term with pair collisions – Boltzmann-Uehling-Uhlenbeck, Vlasov-Uehling-Uhlenbeck, Landau-Vlasov – suitable for description of nucleus-nucleus collisions ? In-medium nucleon-nucleon cross sections ?

Implementation into the BUU equation

While BUU equation is formulated in terms of density, it does not consider temperature directly. Therefore temperature T must be estimated. It is possible to estimate temperature using the Maxwellian momentum distribution of nucleons

$$f(\vec{p}) = \frac{1}{(2\pi mT)^{3/2}} \mathrm{e}^{-\frac{p_x^2+p_y^2+p_z^2}{2mT}} \tag{9}$$

where m is the nucleon mass. Using this formula, local temperature can be estimated from momentum distribution in the c.m. frame by evaluating the momentum variance. This can be done for transverse momentum since it provides better measure of mutual thermalization of particles from the projectile and target, since it proceeds by distant elastic collisions generating the transverse momentum. More violent collisions would lead to emission of a given nucleon. This temperature estimate can be done without requiring stopping and formation of the source equilibrated in all three dimensions, better analogue would be the friction of two dilute gas clouds passing through each other.



$$T_{fireball}=(2.5(E/A)_{proj})^{1/2}$$

Figure 1: Evolution of average (solid line) and maximal (dashed line) temperature with the time. The total volume of the fireball is shown in arbitrary scale as dash-dotted line. The results were obtained using the BUU in reaction $^{48}Ca+^{48}Ca$ at 400 AMeV using the impact parameter 3 fm. Dotted line shows the fireball temperature estimate for a given beam energy obtained from the systematics of pre-equilibrium spectra [?].

Values of "transverse" temperature look reasonable

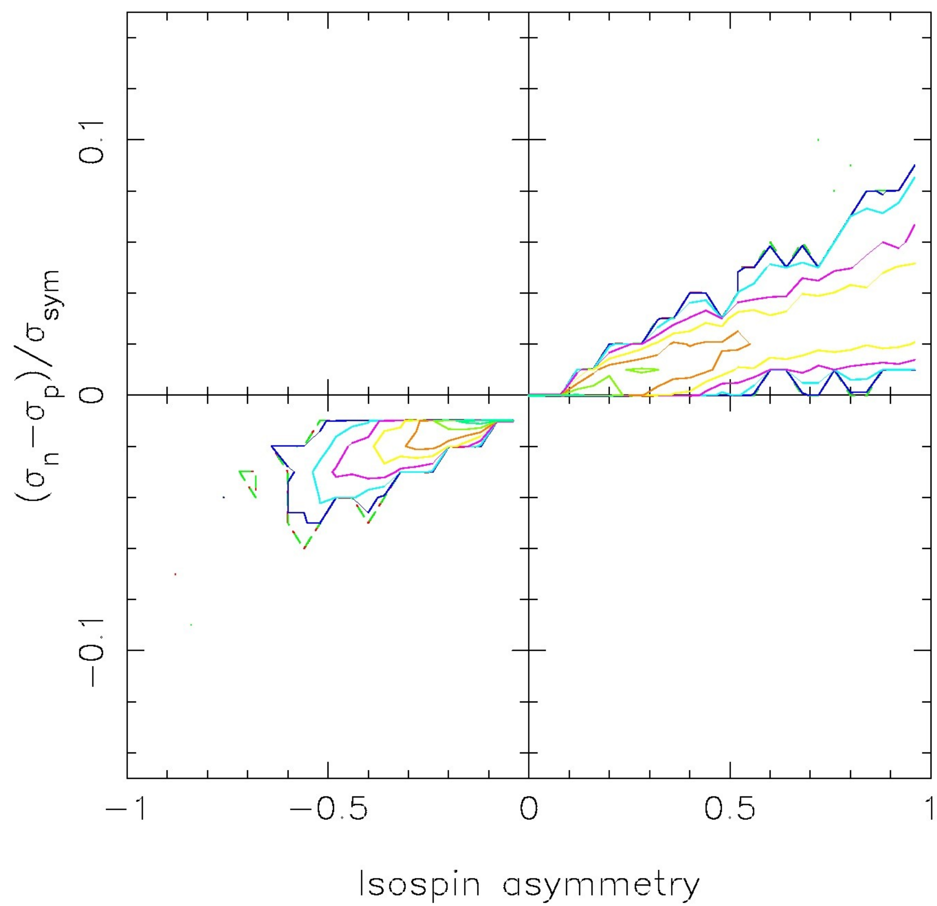


Figure 3: Relative difference of the isopin-dependent neutron-neutron and proton-proton cross sections as a function of isospin asymmetry of the volume cell. The results were obtained using the BUU in reaction ^{48}Ca+^{48}Ca at 400 AMeV using the impact parameter 3 fm.

Relative difference just few %, nevertheless sensitivity is there

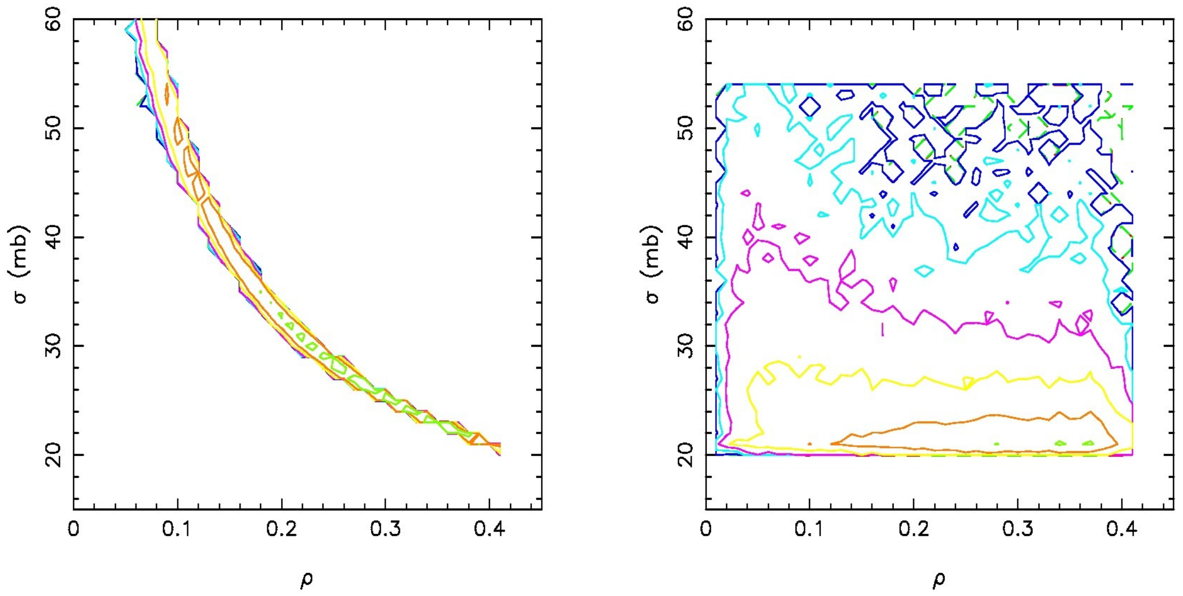


Figure 5: Comparison of the nucleon-nucleon cross sections in two variants of the BUU calculations. On the left panel are the isospin-dependent nucleon-nucleon cross sections, obtained as the proper volume of the Van der Waals form of the equation of state, as a function of density, while on the right panel are shown the corresponding nucleon-nucleon cross sections, obtained using standard energy dependent parametrization, used in BUU calculation. The results were obtained using the BUU in reaction ^{48}Ca+^{48}Ca at 400 AMeV using the impact parameter 3 fm.

Global $1/\rho^{2/3}$ dependence of nucleon-nucleon cross sections from EoS

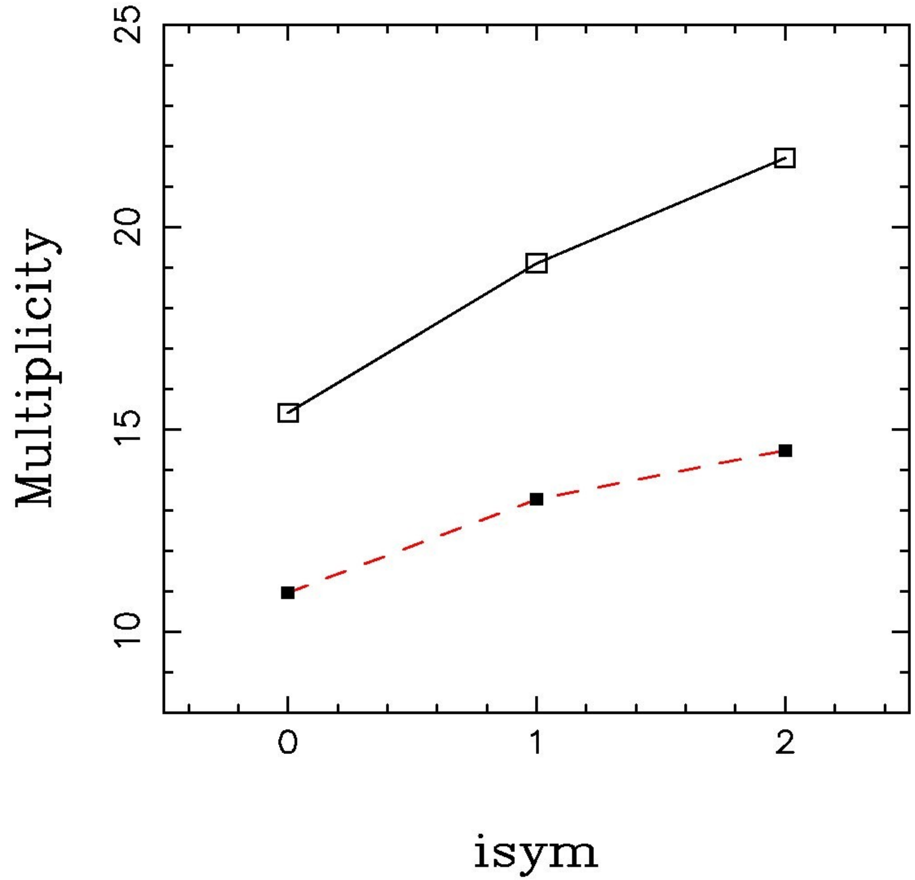
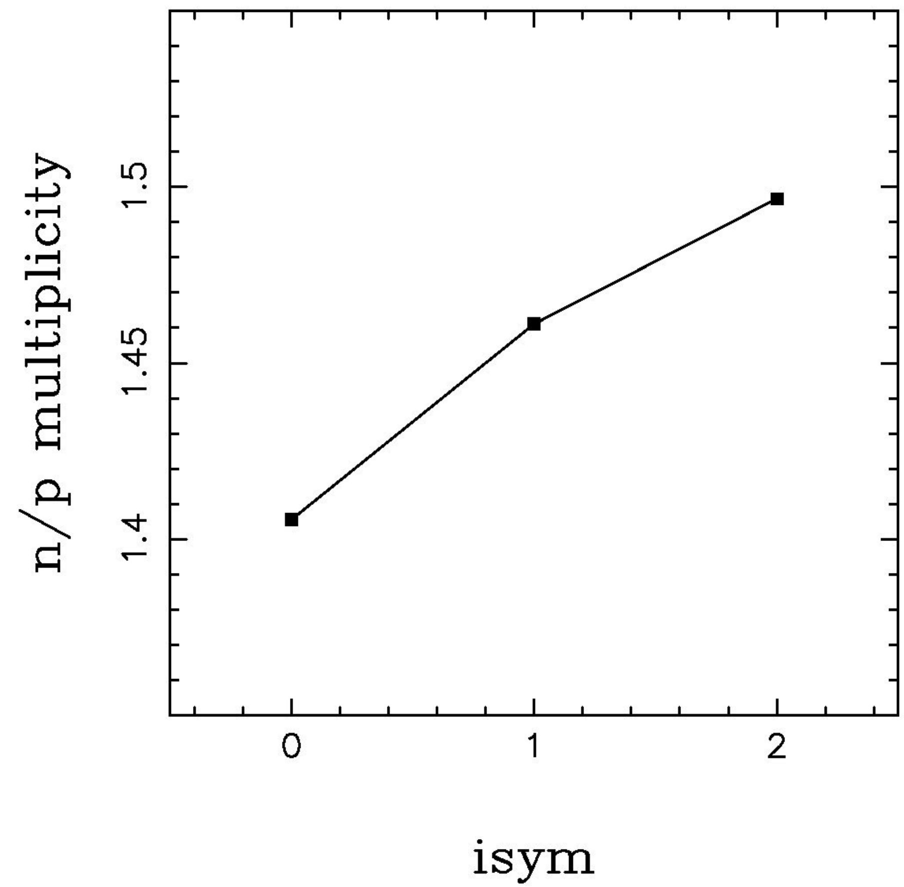


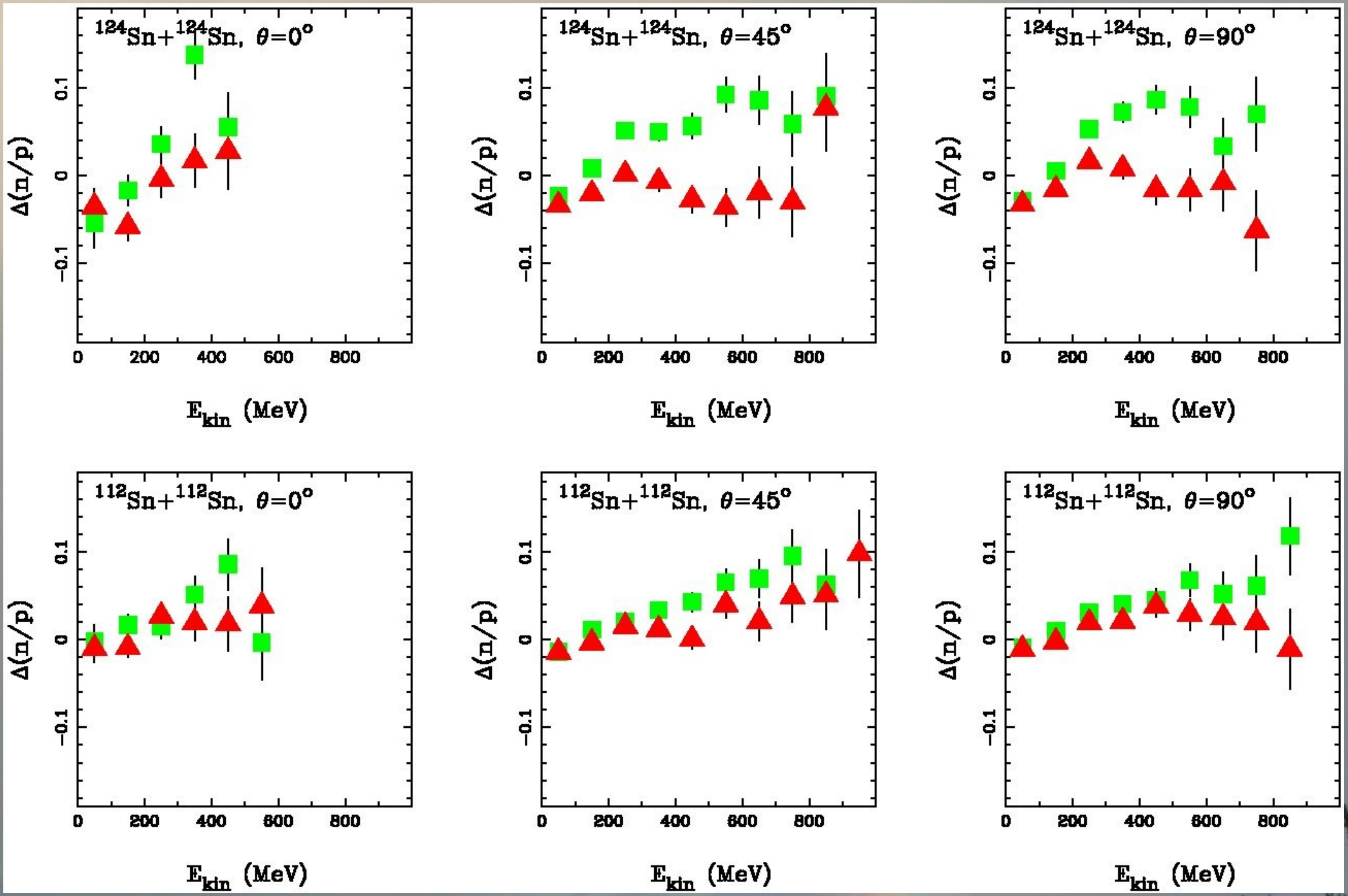
Figure 4: Evolution of the neutron and proton multiplicity from left to the right for standard BUU (isym=0), BUU with isospin-dependent potential (isym=1) and for BUU with both isospin-dependent potential and nucleon-nucleon cross sections (isym=2). The results were obtained using the BUU in reaction $^{48}Ca+^{48}Ca$ at 400 AMeV using the impact parameter 1 fm. Solid black line - total neutron multiplicity, Dashe line - total proton multiplicity,



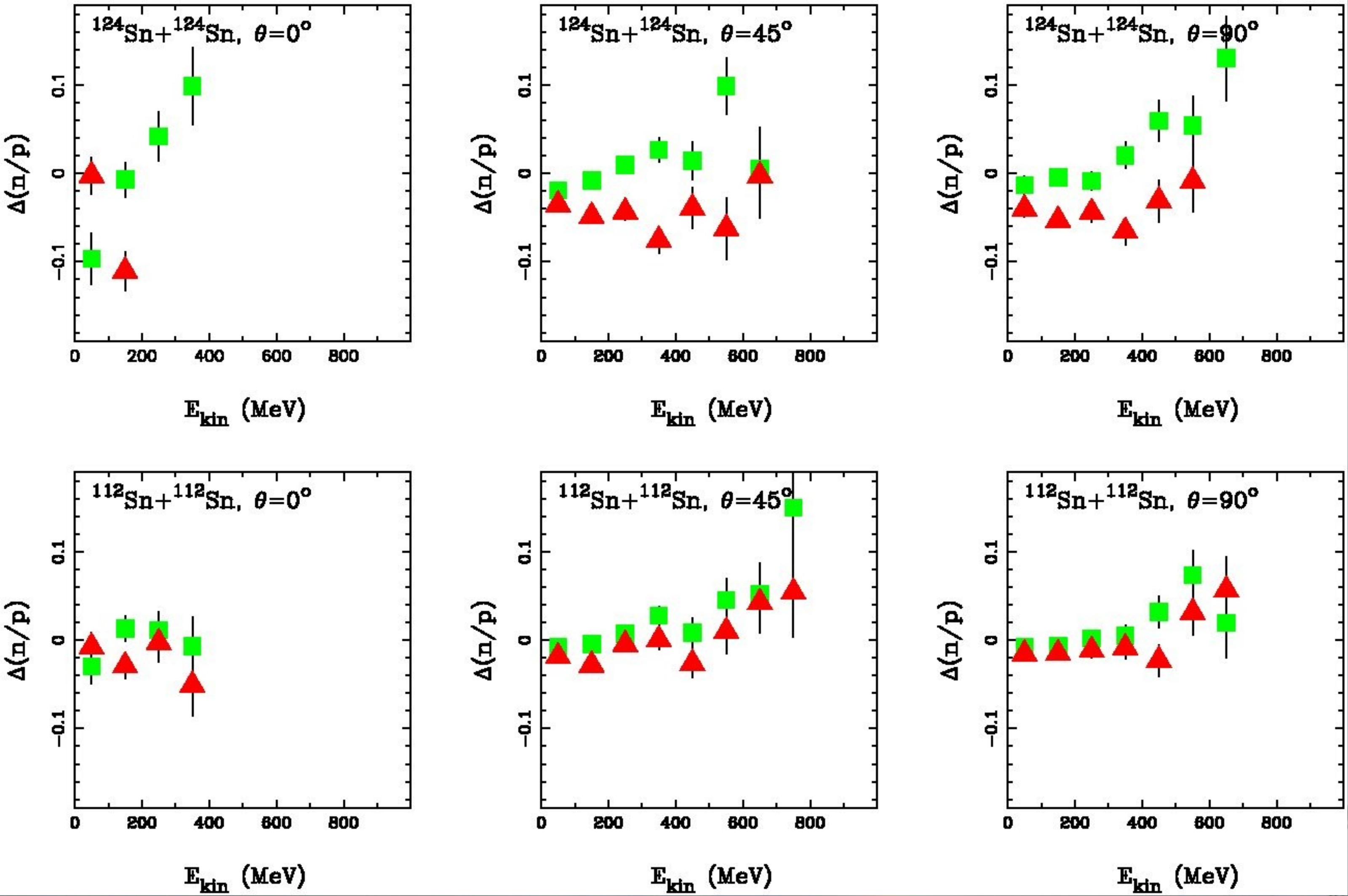
Soft EOS
Asystiff γ=2

Figure 2: Evolution of the n/p multiplicity ratio from left to the right for standard BUU (isym=0), BUU with isospin-dependent potential (isym=1) and for BUU with both isospin-dependent potential and nucleon-nucleon cross sections (isym=2). The results were obtained using the BUU in reaction $^{48}Ca+^{48}Ca$ at 400 AMeV using the impact parameter 1 fm.

For 40Ca+40Ca at 400 AMeV no sensitivity observed, as expected



Evolution of the difference of n/p multiplicity ratio between BUU calculation with both isospin-dependent mean-field and nucleon-nucleon cross sections and BUU calculation with isospin-dependent mean-field and free cross sections for three angular ranges. The results were obtained using the BUU in reactions 124Sn+124Sn and 112Sn+112Sn at 400 AMeV using the impact parameter 1 fm at the time 200 fm/c. Squares show the result stiff symmetry energy while triangles show results for soft symmetry energy. Soft nuclear equation of state is used.



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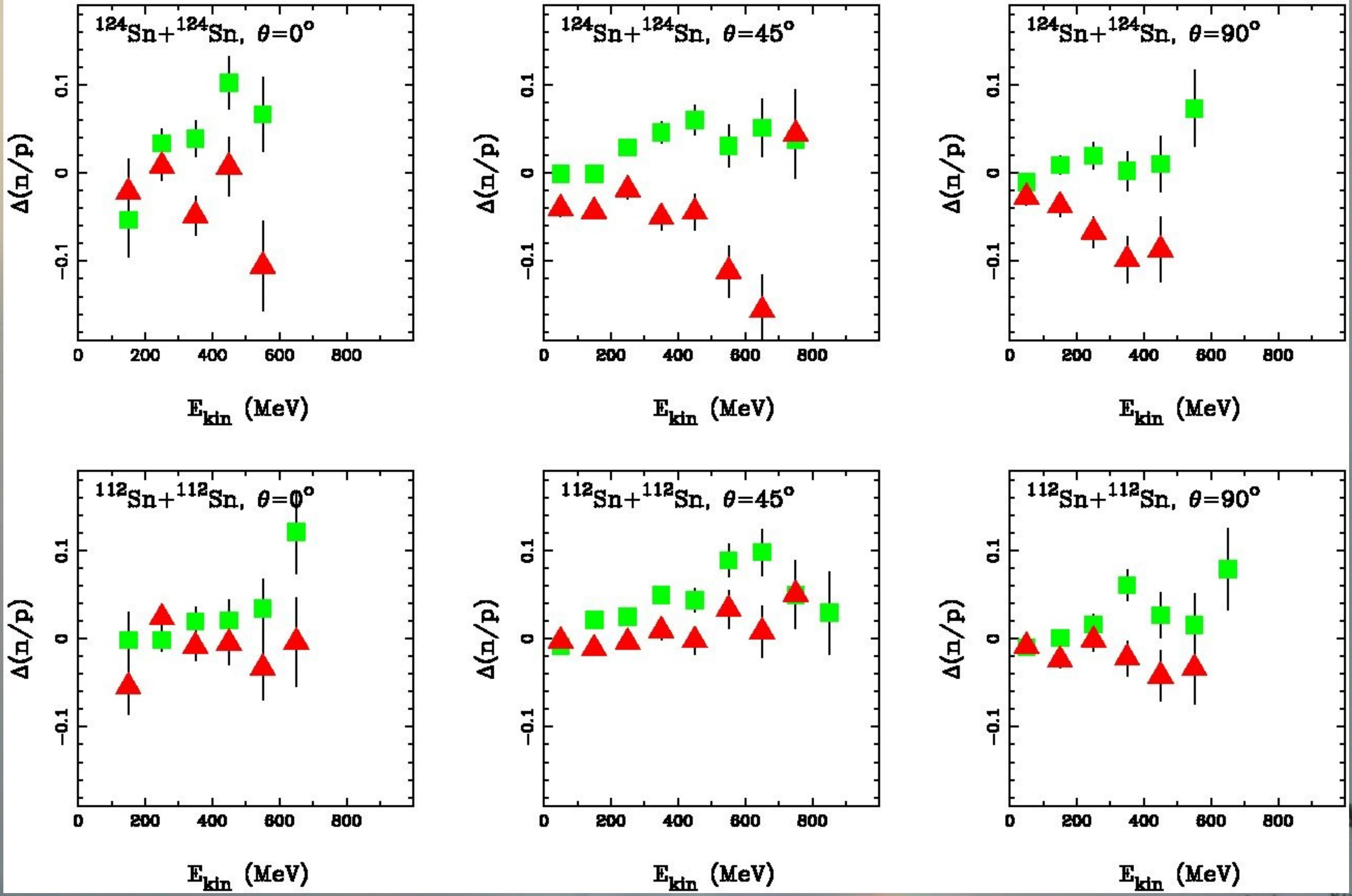
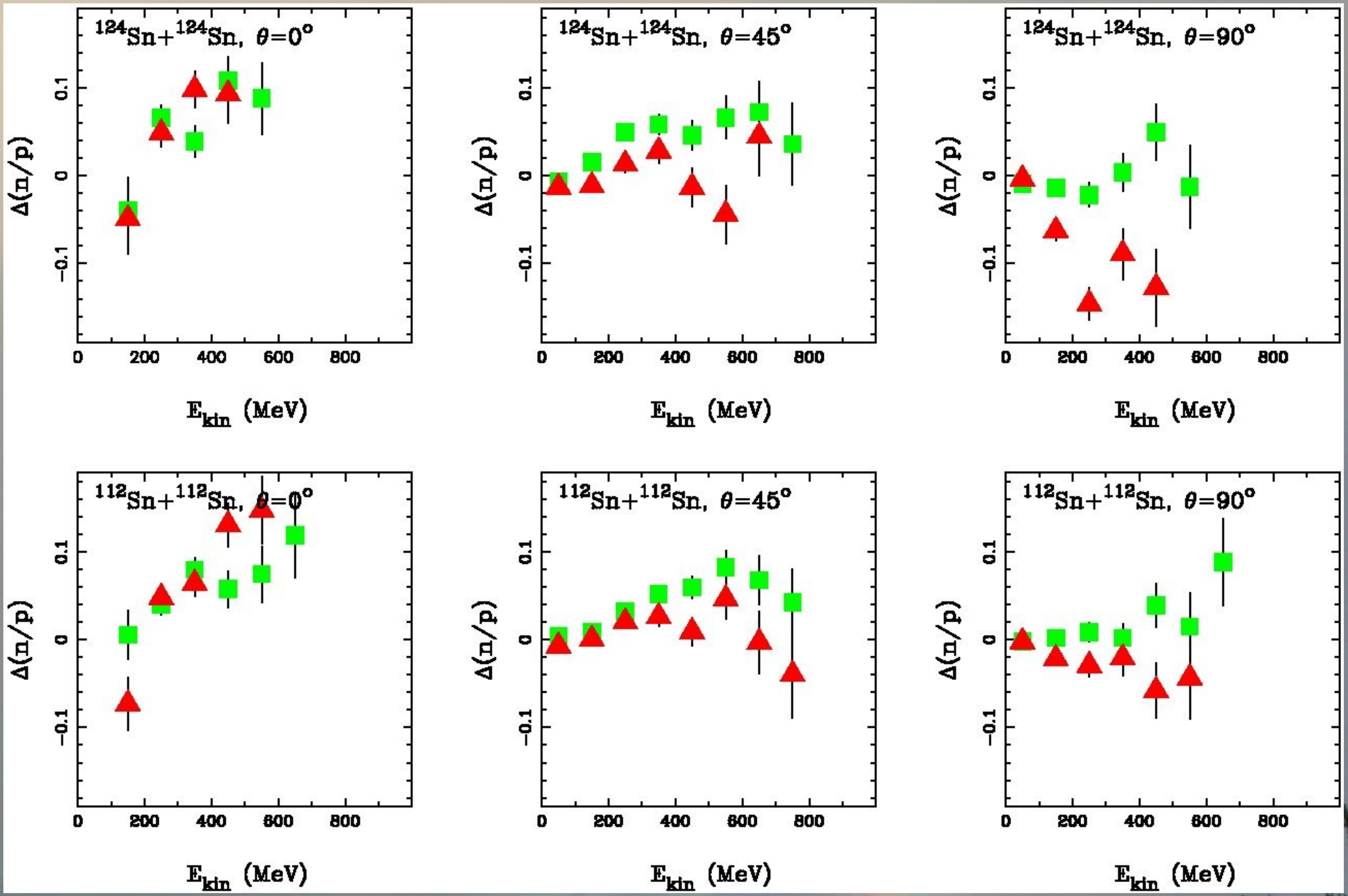


Evolution of the difference of n/p multiplicity ratio between BUU calculation with both isospin-dependent mean-field and nucleon-nucleon cross sections and BUU calculation with isospin-dependent mean-field and free cross sections for three angular ranges. The results were obtained using the BUU in reactions 124Sn+124Sn and 112Sn+112Sn at 400 AMeV using the impact parameter 6 fm at the time 200 fm/c. Squares show the result stiff symmetry energy while triangles show results for soft symmetry energy. Soft nuclear equation of state is used.



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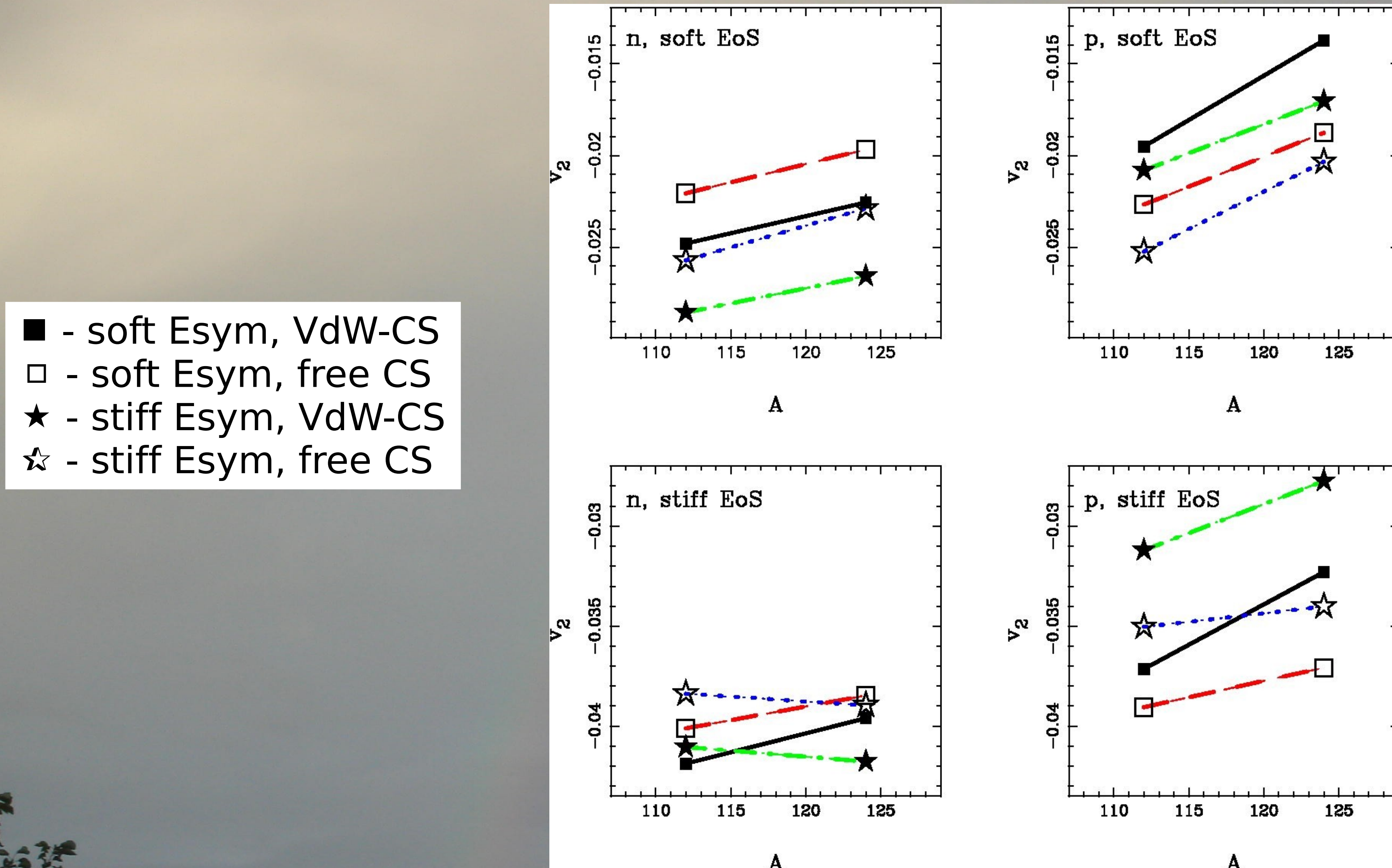
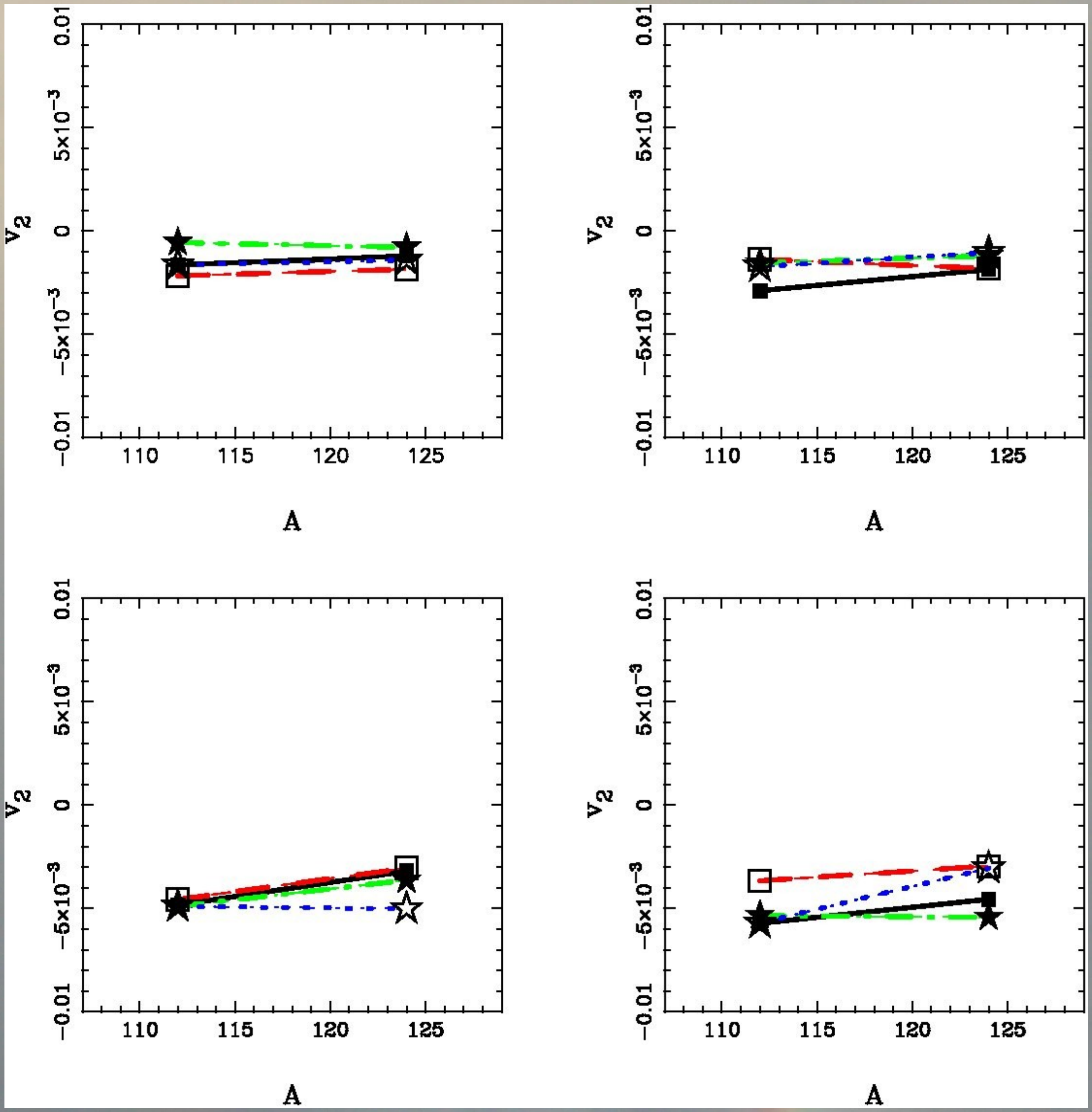


Elliptic flow of neutrons and protons in reactions 124Sn+124Sn and 112Sn+112Sn at 400 AMeV at the impact parameter 6 fm Solid and open squares show results of BUU calculation with both isospin-dependent mean-field and nucleon-nucleon cross sections and BUU calculation with isospin-dependent mean-field and free cross sections, respectively, with soft symmetry energy. Solid and open asterisks show analogous results with stiff symmetry energy.



Elliptic flow of neutrons and protons in reactions 124Sn+124Sn and 112Sn+112Sn at 400 AMeV at the impact parameter 1 fm Solid and open squares show results of BUU calculation with both isospin-dependent mean-field and nucleon-nucleon cross sections and BUU calculation with isospin-dependent mean-field and free cross sections, respectively, with soft symmetry energy. Solid and open asterisks show analogous results with stiff symmetry energy.

Symmetry energy dependence:

asystiff ESym leads to larger effect than asysoft ESym

for both density dependences of Esym the effect is larger with soft EoS than with stiff EoS

Sensitivity is higher for early stage (higher kinetic energies), where emission from compressed dense fireball occurs

Conclusions:

- N-N cross sections derived from the equation of state appear to work

- increased sensitivity to isospin dynamics

- increasing sensitivity with isospin asymmetry

- testing of available reaction data needed

- “self-consistent” testing of equations of state possible

- application to QMD ?

- possible far-reaching consequences for nuclear astrophysics (supernovae, neutron stars)

From nuclear physics to genomics: Determining sequence and time of mutations
M. Veselsky, to appear in the Journal of Computational Biology

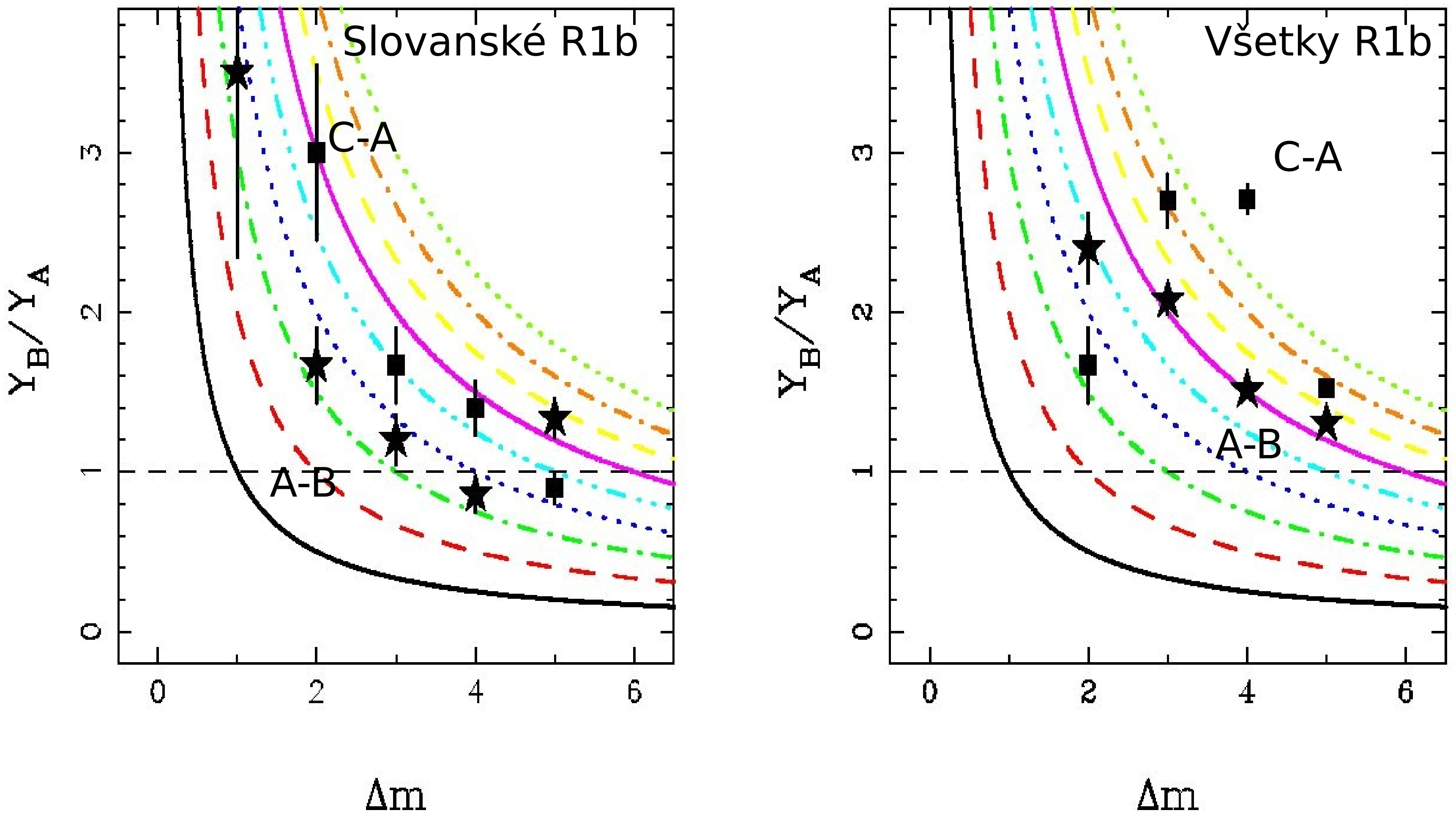


Figure 3: Left panel - Ratios of numbers of haplotypes with a given mutation distance Δm for pairs of haplotypes A–B (asterisks) and C–A (squares) found in the search restricted to Slavic countries, lines - calculated probability ratios P_m/P_{m+1} for $m = 0 - 8$, expressed as a function of the mean number of mutations. Right panel - As in the left panel except that the search is without geographical restrictions.