

# Symmetry Energy & Maximum Rotation of Neutron Stars



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## Motivation

$\Omega_{\text{Kepler}}$ : absolute upper limit on the rotational frequency of neutron stars

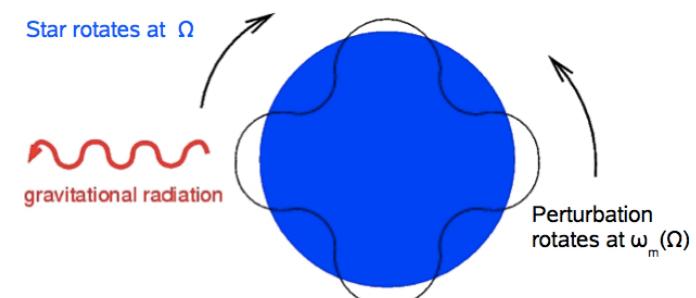
(matter ejected from equator for  $\Omega > \Omega_{\text{Kepler}}$ )

$$\Omega_{\text{Kepler}} \approx 7800 \sqrt{\left(\frac{M}{M_{\text{sun}}}\right) \left(\frac{10 \text{ km}}{R}\right)^3} \text{ s}^{-1}$$

But instabilities can prevent neutron stars from reaching  $\Omega_{\text{Kepler}}$   $\rightarrow$  more stringent limit in rotation

R-mode instability : toroidal mode of oscillation

- ✓ restoring force: Coriolis
  - ✓ emission of GW in hot & rapidly rotating NS (CFS mechanism)
    - GW makes the mode unstable
    - Dissipation (viscosity) stabilizes the mode
- $\tau_{\text{GW}}, \tau_{\text{diss}} \rightarrow$  Critical angular velocity  $\Omega_c$



$$A \propto A_0 e^{-i\omega_m(\Omega)t + im\phi - t/\tau(\Omega)}$$

In this talk ...

Study the role of the symmetry energy slope parameter  $L$  on the maximum rotational frequency of neutron stars by using both microscopic (BHF, APR & AFDMC) and phenomenological (Skyrme & RMF) approaches of the nuclear matter EoS

based on:



Phys. Rev. C 85, 045808 (2012)

# Microscopic approaches

## 💡 BHF with Av18 + UIX

- $\frac{E}{A}(\rho, \beta) = \frac{1}{A} \sum_{\tau} \sum_{k \leq k_{F_\tau}} \left( \frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2} \text{Re}[U_\tau(\vec{k})] \right)$

- ✓  $G(\omega) = V + V \frac{Q}{\omega - E - E' + i\eta} G(\omega)$
- ✓  $E_\tau(k) = \frac{\hbar^2 k^2}{2m_\tau} + \text{Re}[U_\tau(k)]$
- ✓  $U_\tau(k) = \sum_{\tau'} \sum_{k' \leq k_{F_{\tau'}}} \langle \vec{k} \vec{k}' | G(\omega = E_\tau(k) + E_{\tau'}(k')) | \vec{k} \vec{k}' \rangle_A$

## 💡 APR & AFDMC parametrized

- $\frac{E}{A}(\rho, \beta) = E_0 u \frac{u - 2 - s}{1 + us} + S_0 u^\gamma \beta^2, \quad u = \rho / \rho_0$

Infinite summation of two-hole line diagrams



Partial summation of pp ladder diagrams

$$\langle \dots \rangle = \langle \dots \rangle + \langle \dots \rangle + \langle \dots \rangle + \dots =$$

$$= \langle \dots \rangle + \langle \dots \rangle$$

- ✓ Pauli blocking
- ✓ Nucleon dressing

Heiselberg & Hjorth-Jensen Phys. Rep. 328, 237 (2000)

- $\frac{E}{A}(\rho, \beta) = E_0 + a(\rho - \rho_0)^2 + b(\rho - \rho_0)^3 e^{\gamma(\rho - \rho_0)} + C_s \left( \frac{\rho}{\rho_0} \right)^{\gamma_s} \beta^2$  Gandolfi et al., MNRAS 404, L35 (2010)

# Phenomenological approaches

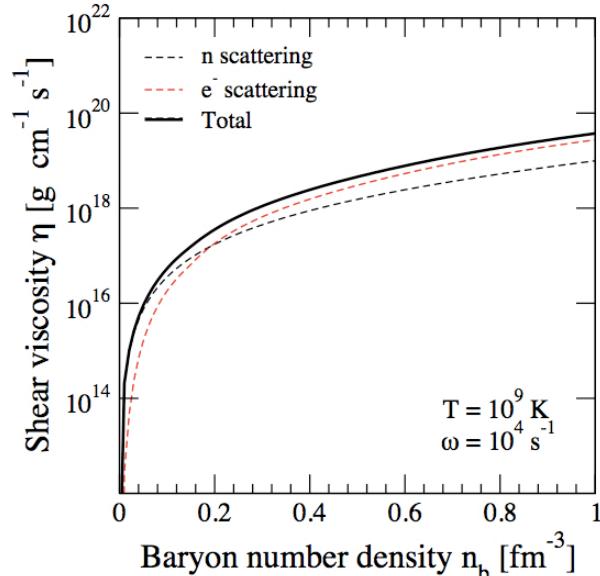
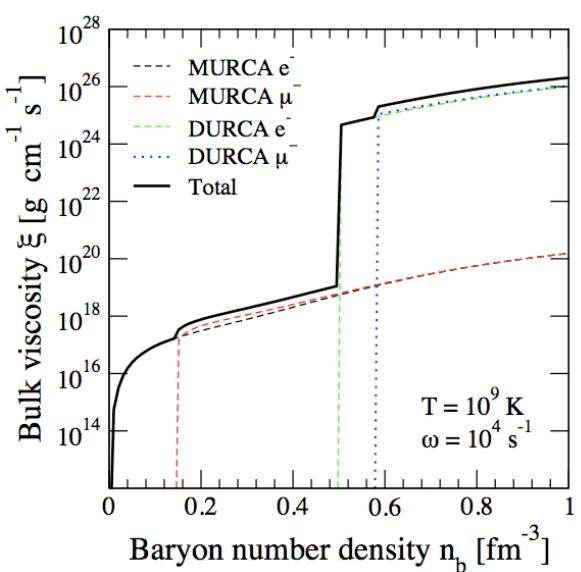
## 💡 Skyrme

- Lyon group  $SLy$ :  $SLy0$ - $SLy10$ ,  $Sly230a$
- $SkI$  family:  $SkI1$ - $SkI6$
- $Rs$ ,  $Gs$ ,  $SGI$ ,  $SkMP$ ,  $SkO$ ,  $skO'$ ,  $skT4-5$ ,  $SV$

## 💡 Relativistic mean field models

- Non-linear Walecka models (NLWM) with constant coupling constants:  $GM1$ ,  $GM3$ ,  $TM1$ ,  $NL3$ ,  $NL3-II$ ,  $NL-SH$
- Density dependent hadronic models (DDH) with density dependent coupling constants:  $DDME1$ ,  $DDME2$ ,  $TW99$ ,  $PK1$ ,  $PKIR$ ,  $PKDD$

# Bulk & shear viscosities



$$\xi = \xi_{MURCA} + \xi_{DURCA} = \sum_{nl} \frac{|\lambda_{nl}|}{\omega^2} \left| \frac{\partial P}{\partial X_l} \right| \frac{\partial \xi_l}{\partial n_b} + \sum_l \frac{|\lambda_l|}{\omega^2} \left| \frac{\partial P}{\partial X_l} \right| \frac{\partial \xi_l}{\partial n_b}$$

✓ Modified URCA



✓ Direct URCA



Haensel et al., AA 357, 1157 (2000); AA 372, 130 (2001)

$$\eta = \eta_n + \eta_e$$

✓ n scattering<sup>1</sup>

$$\eta_n = 2 \times 10^{18} \left( \frac{\rho}{10^{15} g cm^{-3}} \right)^{9/4} \left( \frac{T}{10^9 K} \right)^{-2}$$

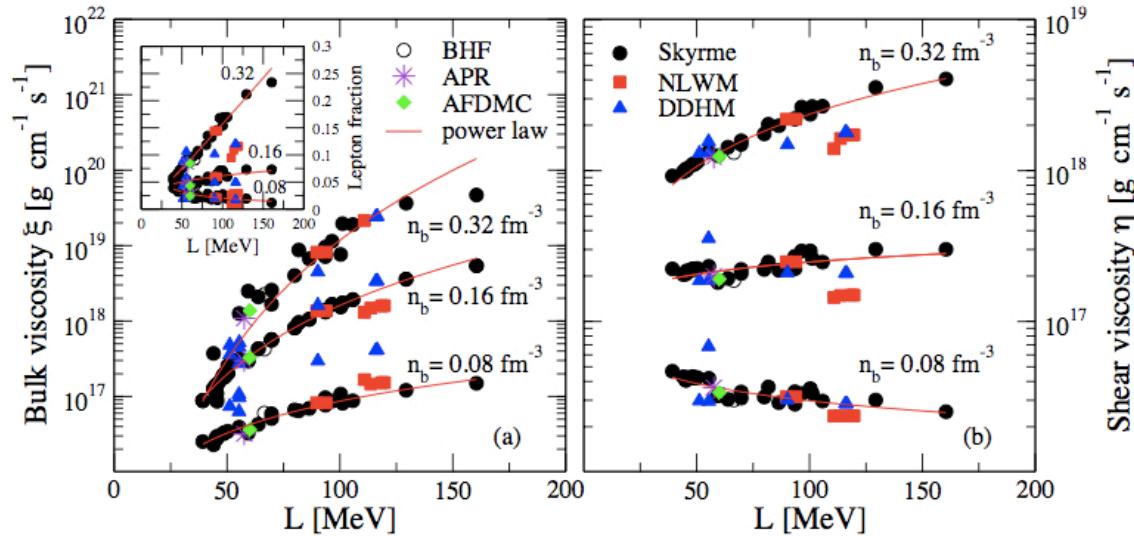
✓ e<sup>-</sup> scattering<sup>2</sup>

$$\eta_e = 4.26 \times 10^{-26} (x_p n_b)^{14/9} T^{-5/3}$$

<sup>1</sup> Cutler & Lindblom., ApJ 314, 234 (1987).

<sup>2</sup> Shternin & Yakovlev, PRD 78, 063006 (2008)

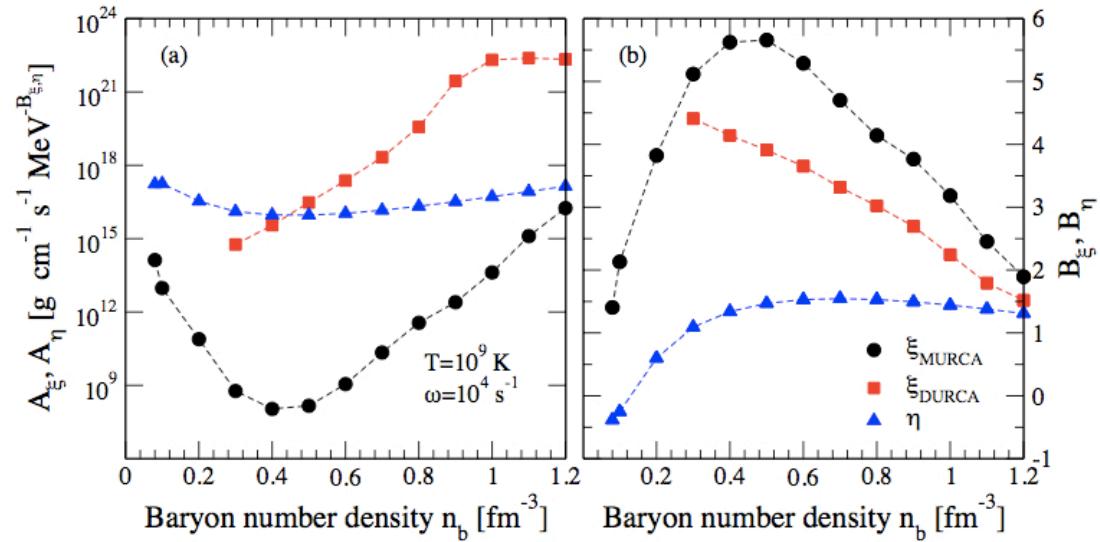
# $L$ dependence of $\xi$ and $\eta$



- ✓  $\xi$  increases with  $L$  for all densities
  - ✓  $\eta$  increases with  $L$  for  $n_b > n_0$  & decreases with  $L$  for  $n_b < n_0$
- consequence of lepton fraction dependence

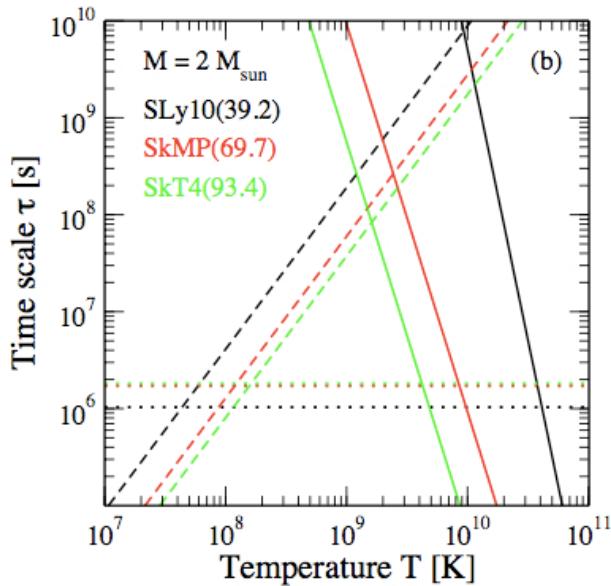
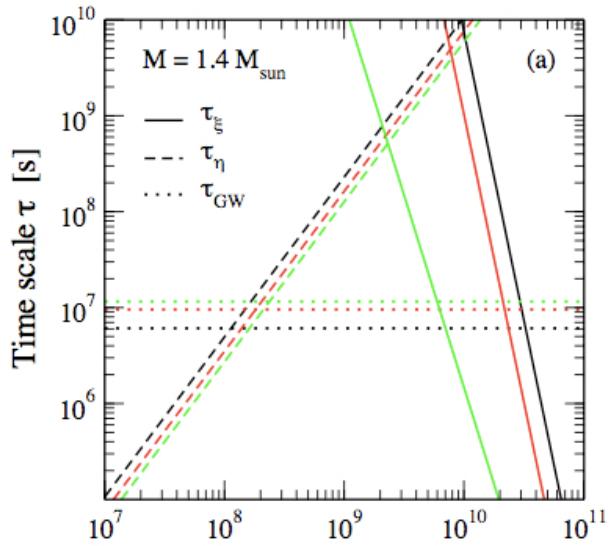
$L$  dependence described by simple power laws

$$\xi = A_\xi L^{B_\xi}, \eta = A_\eta L^{B_\eta}$$



# Dissipative time scales of r-modes

$$\frac{1}{\tau_i} = -\frac{1}{2E} \left( \frac{dE}{dt} \right)_i$$



- $\frac{1}{\tau_\xi} = \frac{4\pi}{690} \left( \frac{\Omega^2}{\pi G \rho} \right)^2 R^{2l-2} \left[ \int_0^R \rho r^{2l+2} dr \right]^{-1} \int_0^R \xi \left( \frac{r}{R} \right)^2 \left[ 1 + 0.86 \left( \frac{r}{R} \right)^2 \right] r^2 dr$
  - $\frac{1}{\tau_\eta} = (l-1)(2l+1) \left[ \int_0^R \rho r^{2l+2} dr \right]^{-1} \int_0^R \eta r^{2l} dr$
  - $\frac{1}{\tau_{\text{GW}}} = \frac{32\pi G \Omega^{2l+2}}{c^{2l+3}} \frac{(l-1)^{2l}}{[(2l+1)!!]^2} \left( \frac{l+2}{l+1} \right)^{2l+1} \int_0^R \rho r^{2l+2} dr$
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- ✓  $\tau_{\text{GW}}$  larger for models with larger L  
Larger L → stiffer EoS → less compact star →  $\tau_{\text{GW}}$  larger
- ✓  $\tau_\xi$  &  $\tau_\eta$  smaller for models with larger L  
 $\tau_\xi$  ( $\tau_\eta$ ) decrease with  $\xi$  ( $\eta$ ) but  $\xi(\eta)$  increase with L
- ✓  $\tau_{\text{GW}}$ ,  $\tau_\xi$  &  $\tau_\eta$  decrease when increasing M  
Given an EoS: the more massive the star the denser it is  
→  $\tau_{\text{GW}}$ ,  $\tau_\xi \sim (\rho/\xi) R^2$  &  $\tau_\eta \sim (\rho/\eta) R^2$  decrease

# Critical angular velocity

- time dependence of an r-mode  $\sim e^{-i\omega t - t/\tau}$

$$\frac{1}{\tau(\Omega, T)} = -\frac{1}{\tau_{GW}(\Omega)} + \frac{1}{\tau_\xi(\Omega, T)} + \frac{1}{\tau_\eta(T)}$$

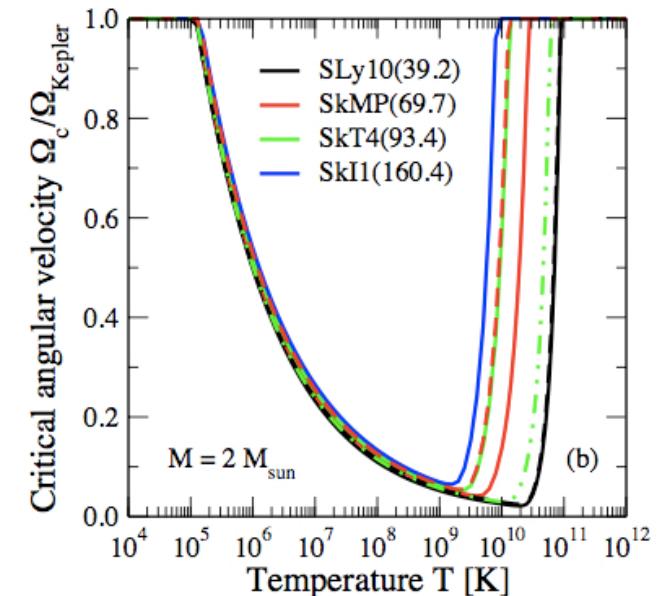
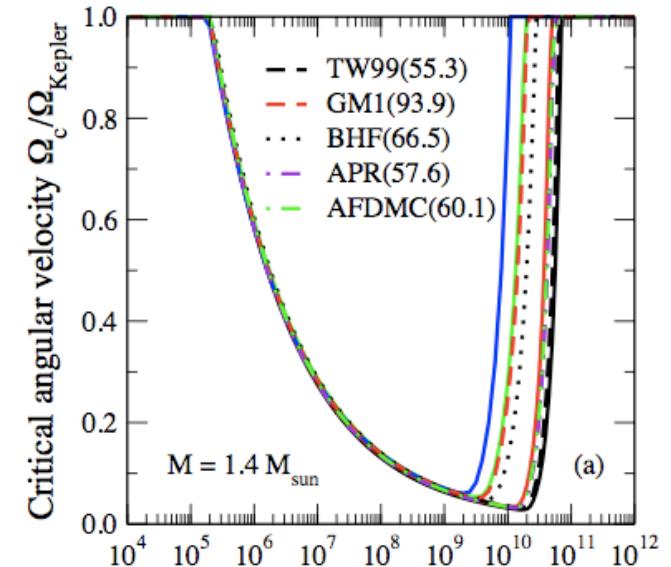
$\rightarrow \frac{1}{\tau(\Omega_c, T)} = 0$     r-mode instability region

$\Omega < \Omega_c$  stable

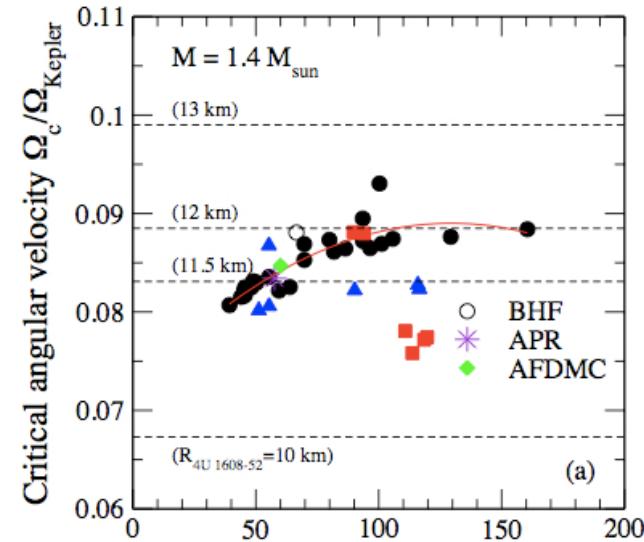
$\Omega > \Omega_c$  unstable

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- ✓ instability region smaller for models with larger L (increase of  $\xi$  &  $\eta$  with L)
- ✓ instability region larger for more massive star (time scales decrease when M increases)



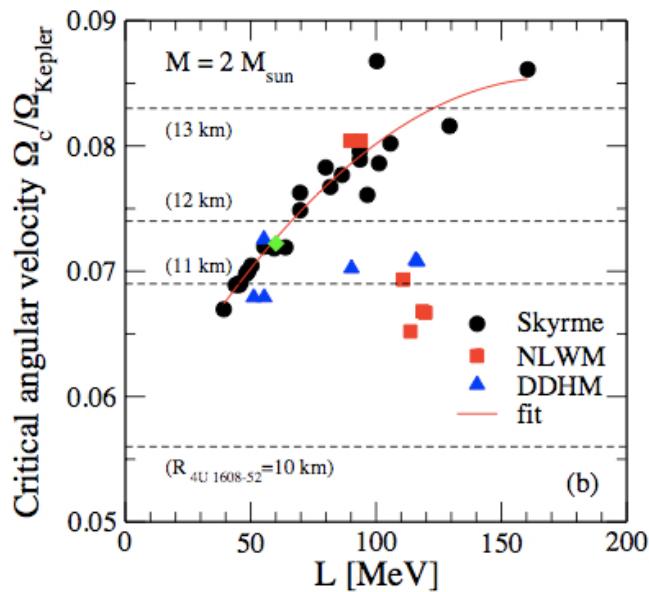
# Constraining L from LMXBs



- observational constraints from pulsar in LMXB 4U 1608-52

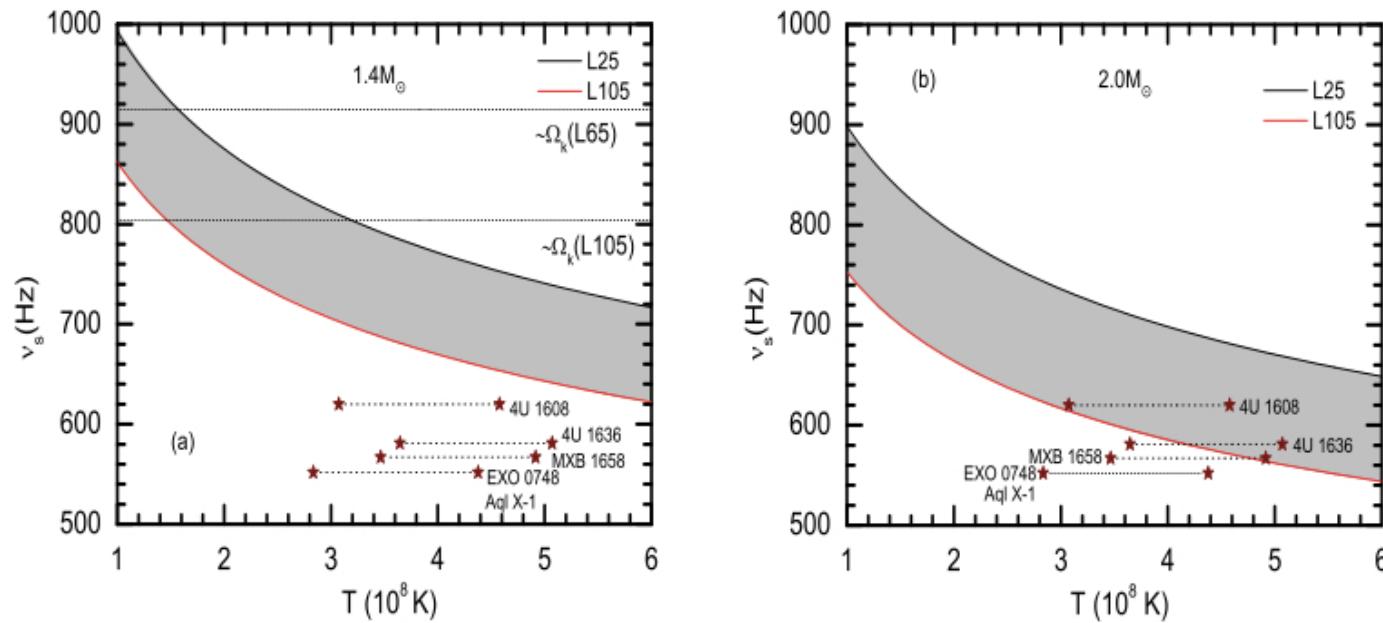
- ✓ estimated core temperature  $T \sim 4.55 \times 10^8$  K
- ✓ measured spin frequency 620 Hz

radius: 10, 11.5, 12 or 13 km       $\Omega_{\text{Kepler}} \approx 7800 \sqrt{\left(\frac{M}{M_{\text{sun}}}\right) \left(\frac{10\text{km}}{R}\right)^3} \text{s}^{-1}$   
mass: 1.4 or 2  $M_{\odot}$



- ✓ No constraint on L if
  - $R_{4U\ 1608-52} < 11$  km ( $\Omega_c > \Omega$ )
  - $R_{4U\ 1608-52} > 12(13)$  km &  $M=1.4(2)M_{\odot}$  ( $\Omega_c < \Omega$ )
- ✓  $L > 50$  MeV if (assuming 4U 1608-52 stable)
  - $R_{4U\ 1608-52}$  is 11.5-12(11.5-13) km &  $M=1.4(2)M_{\odot}$

This is in contrast with the recent work of Wen, Newton & Li where they obtain  $L < 60$  MeV (PRC 85, 025801 (2012))



However, they consider electron-electron scattering at the crust-core ( $\rho < \rho_c$ ) boundary as the main dissipation mechanism. Therefore, their calculation of is done in a region where  $\eta$  decreases with  $L$

## Summary & Conclusions

Study the role of the symmetry energy slope parameter L on the maximum rotational frequency of neutron stars by using both microscopic and phenomenological approaches of the nuclear matter EoS

- 💡 r-mode instability region smaller for models with larger L (increase of  $\xi$  &  $\eta$  with L).
- 💡 L dependence of  $\xi$  &  $\eta$  can be described by simple power laws  $\xi \approx A_\xi L^{B_\xi}$  &  $\eta \approx A_\eta L^{B_\eta}$ .
- 💡 Constraints on L from LMXB 4U 1608-52: L > 50 if 4U 1608-52 assumed stable with 11.5-12 (11.5-13) km and M=1.4(2)  $M_\odot$ . Otherwise no constraints.

