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Application of machine learning in the holographic QCD

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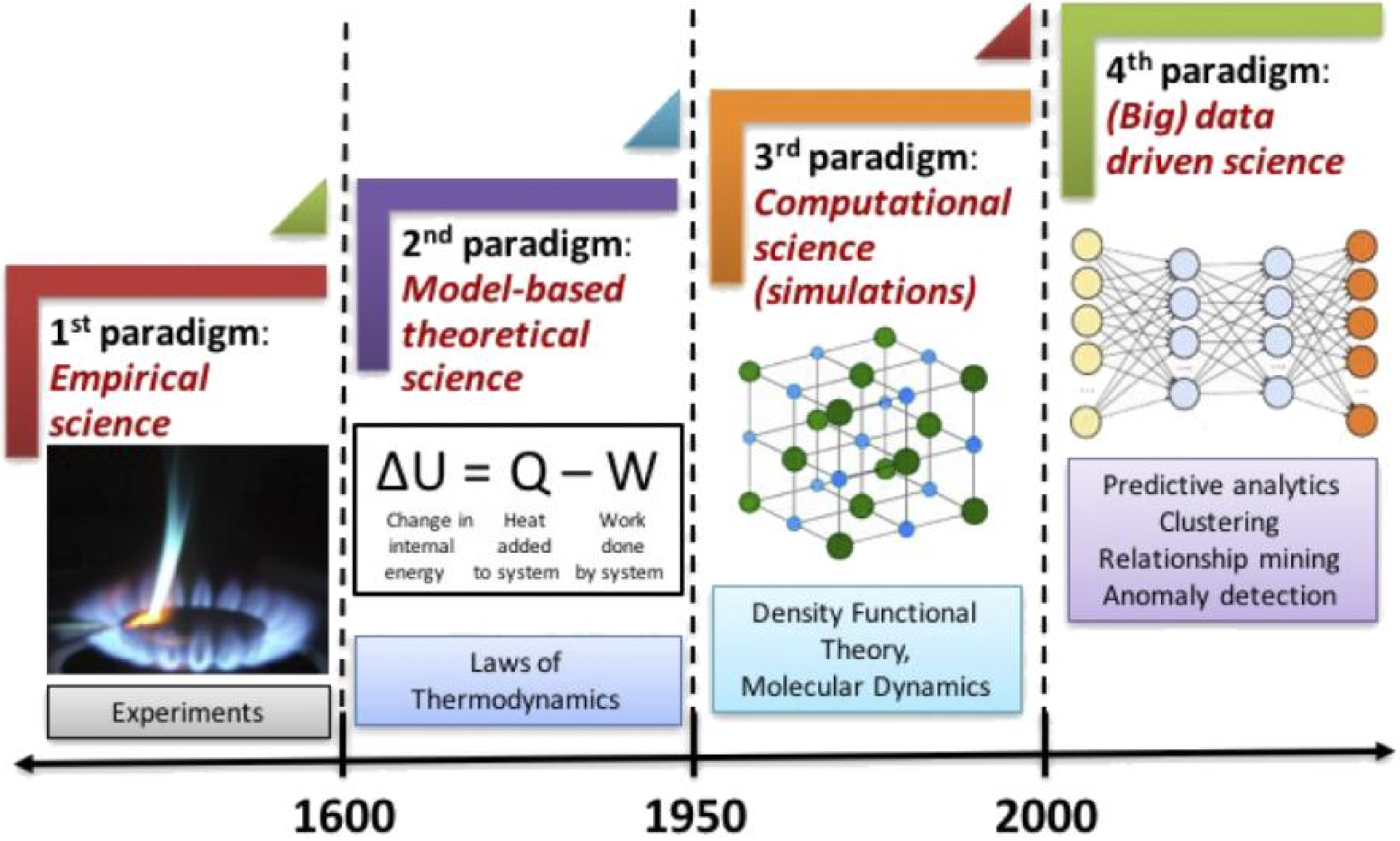
The Fourth Paradigm, Data-intensive Scientific Discovery

Tony Hey,

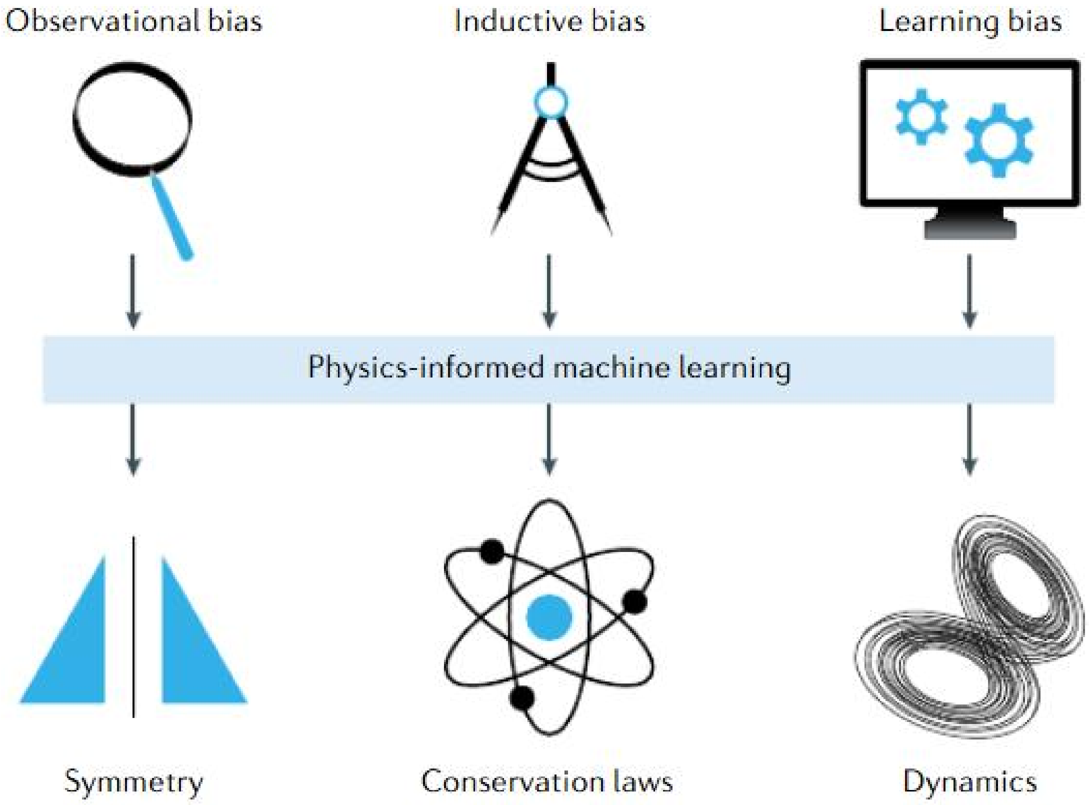
Communications in Computer and Information Science (CCIS), volume 317



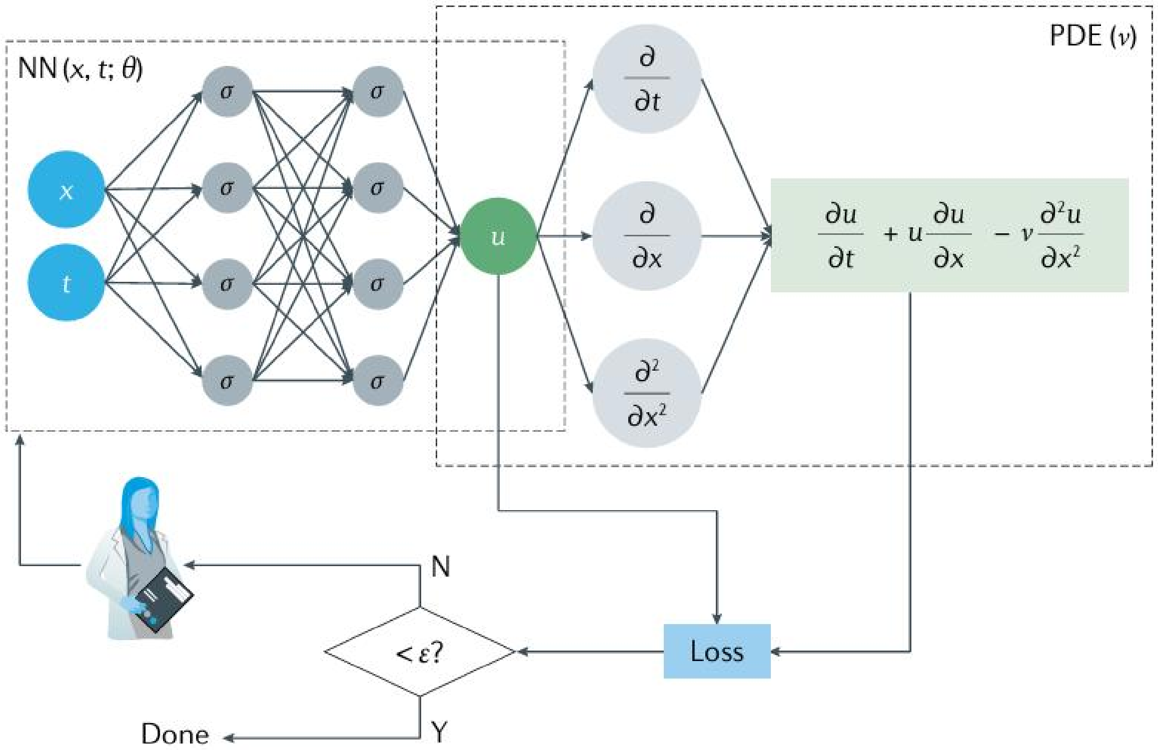
The four paradigms of science: empirical, theoretical, computational, and data-driven.

Physics Informed Machine Learning

George Em Karniadakis, Ioannis G. Kevrekidis, Sifan Wang and Liu Yang
Nature Reviews Physics volume 3, pages 422–440 (2021)



PINN



Automatically dertermining the model parameters

Standard model: more than 19 parameters.

Can we automatically dertermining the model parameters with machine learning?

Loss function: $f(x) = \frac{1}{N}\sum(y_{pre} - y_{\exp})\text{\^{}}2$

Gradient descent algorithm

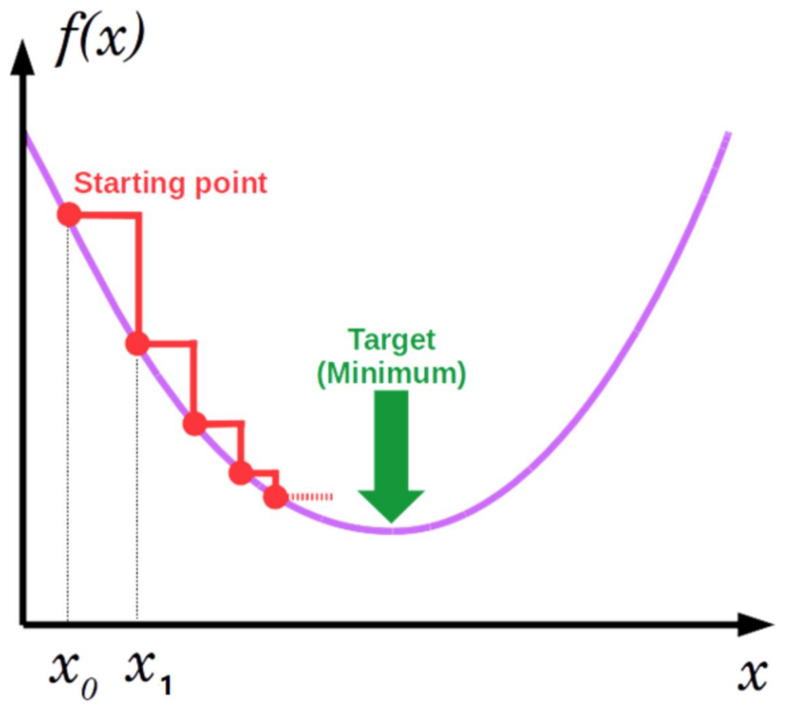
To minimize *f(x)*, parameters are updated as:

$$w_{new} = w - \eta \cdot \nabla f(x)$$

where:

η = learning rate (step size)

$\nabla f(x)$ = gradient at current point



Determining the parameters in the holographic model

***Xun Chen**, Mei Huang, Machine learning holographic black hole from lattice QCD equation of state, Phys.Rev.D* 109 (2024) 5, L051902.

***Xun Chen**, Mei Huang, Flavor dependent critical endpoint from holographic QCD through machine learning, JHEP 02 (2025) 123.*

Einstein-Maxwell-Dilaton model

$$S_b = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

$f(\phi)$ is the gauge kinetic function coupled with the Maxwell field. (derteming the dependent of chemical potential.)

$V(\phi)$ is the potential of the dilaton field. (Simulating the QCD properties at finite temperature.)

Ansatz of metric $$ds^2 = \frac{L^2 e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right]$$

$f(\phi)$ and $V(\phi)$ are unknow function, which need to be fixed.

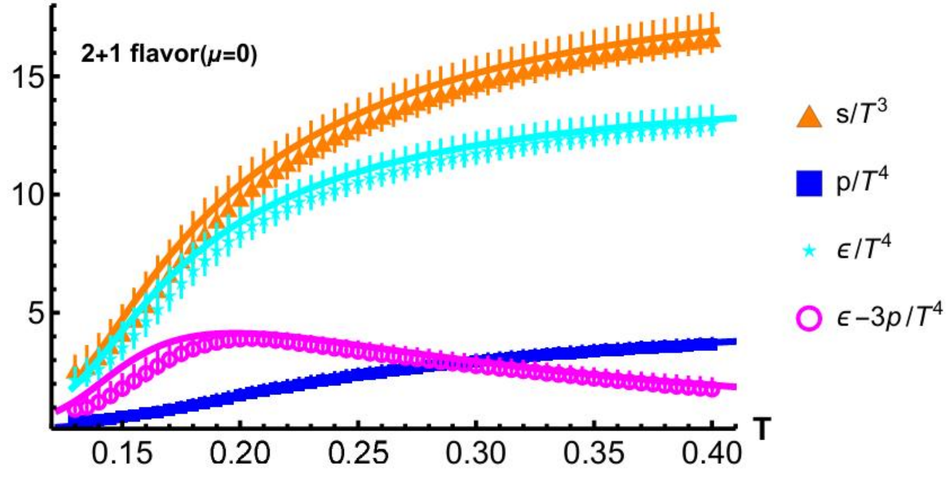
Equally, we assume the form of *A(z)* and *f(z):*

$$A(z) = d * \ln(az^2 + 1) + d * \ln(bz^4 + 1), \qquad f(z) = e^{cz^2 - A(z) + k}.$$

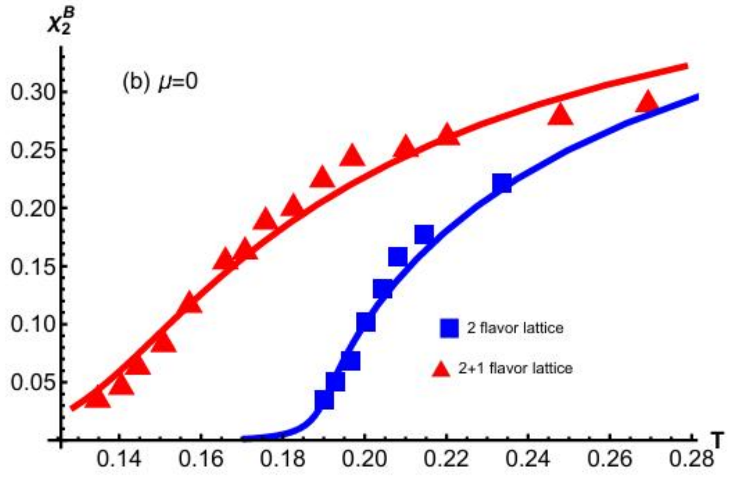
with undeterming parameters: *a,b,c,d,k*

entropy: $$s = \frac{e^{3A(z_h)}}{4G_5 z_h^3}.$$ Baryon number susceptibility $$\chi_2^B = \frac{1}{T^2}\frac{\partial\rho}{\partial\mu}.$$

QCD equation of state from lattice

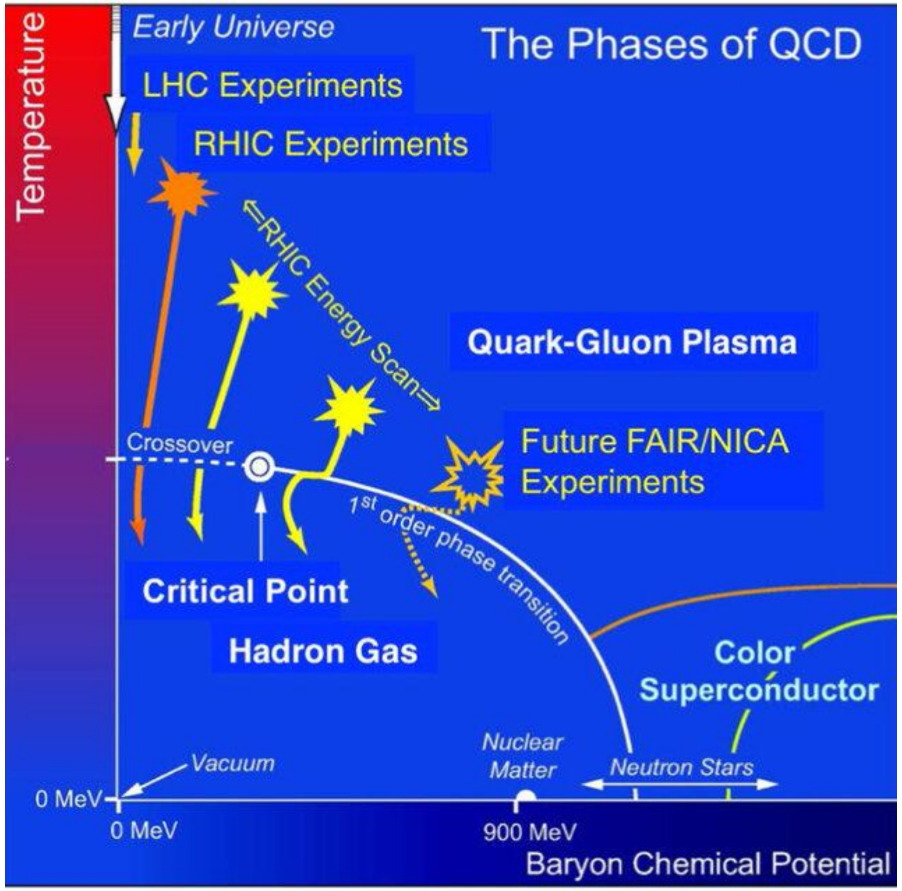


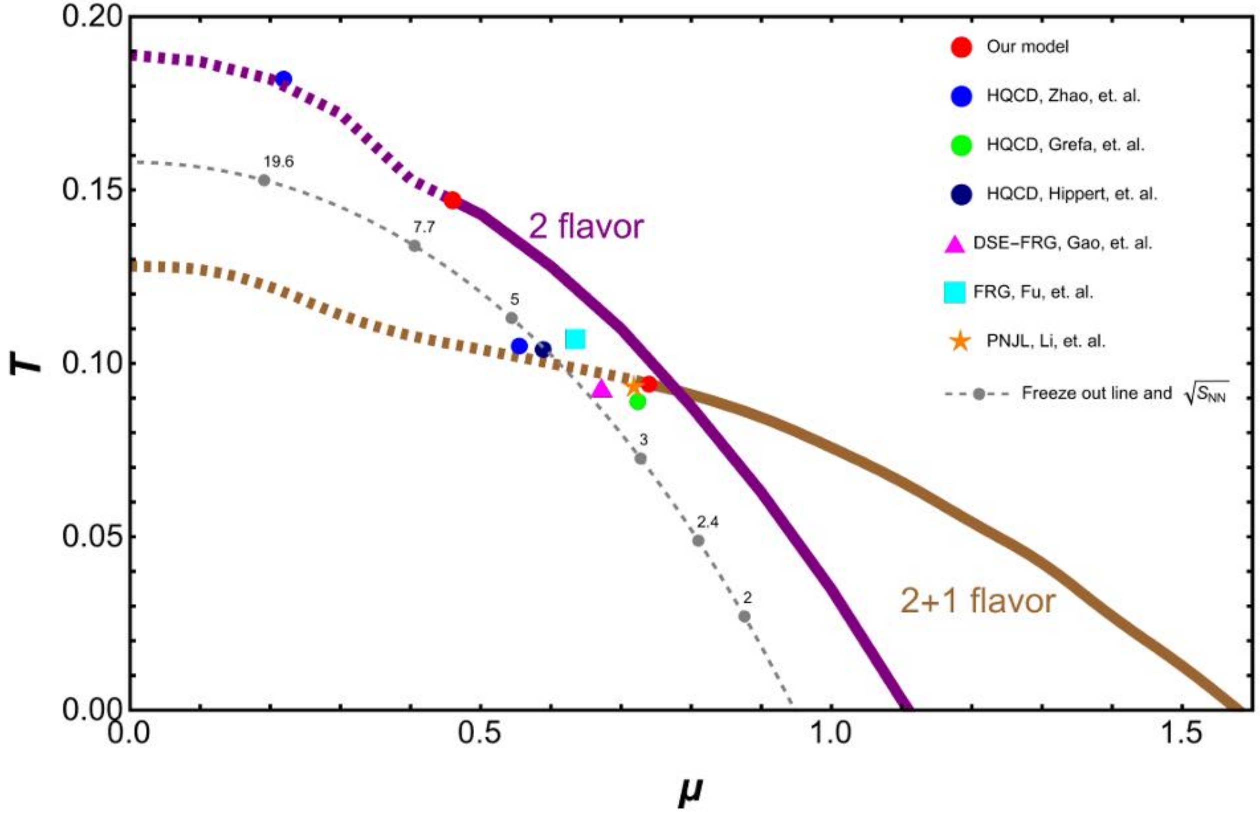
Baryon number susceptibility



	a	b	c	d	k	G_5	T_c
$N_f = 0$	0	0.072	0	-0.584	0	1.326	0.265
$N_f = 2$	0.067	0.023	-0.377	-0.382	0	0.885	0.189
$N_f = 2+1$	0.204	0.013	-0.264	-0.173	-0.824	0.400	0.128

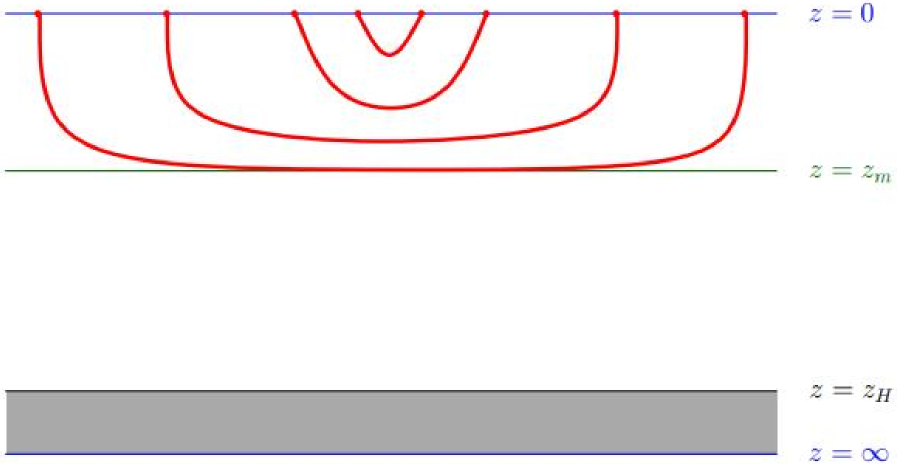
QCD phase diagram





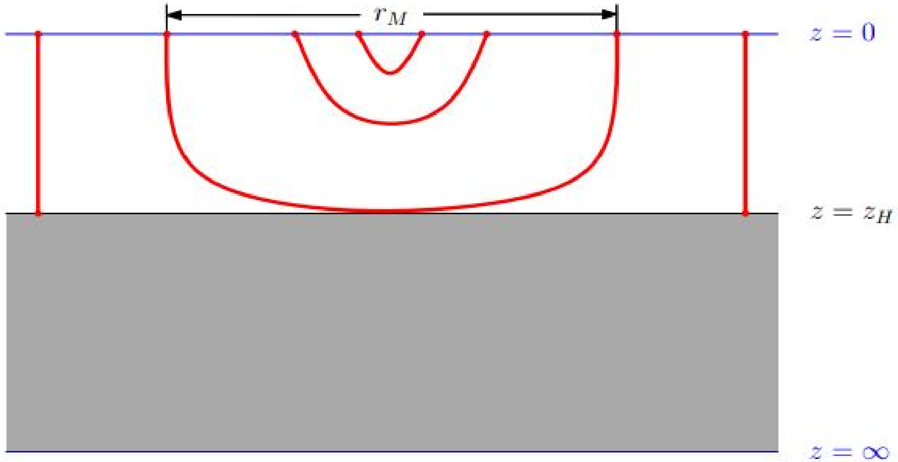
Heavy-quark potential

Pietro Colangelo, Floriana Giannuzzi, Stefano Nicotri, Holography, Heavy-Quark Free Energy, and the QCD Phase Diagram, Phys.Rev.D 83 (2011) 035015



Small black hole

(a)

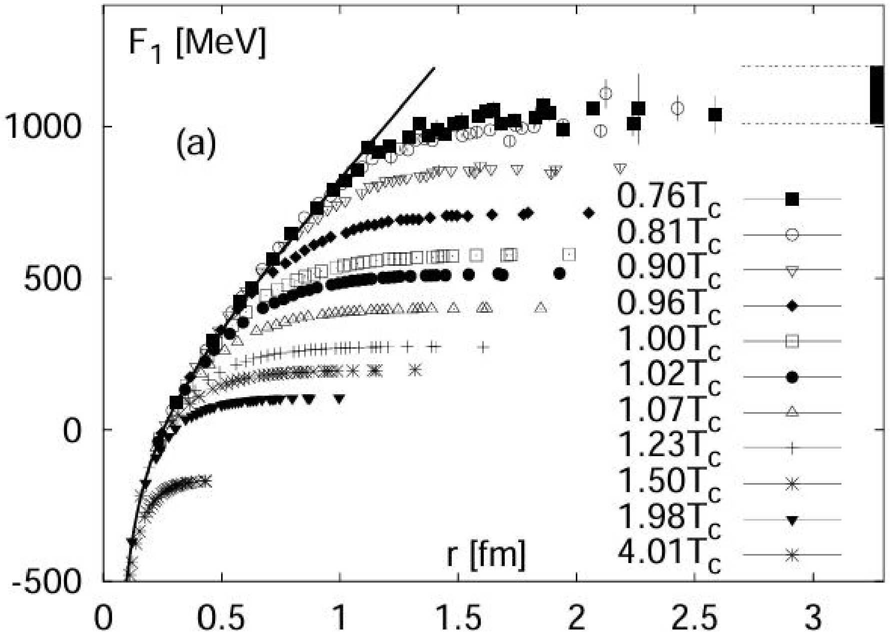


Large black hole

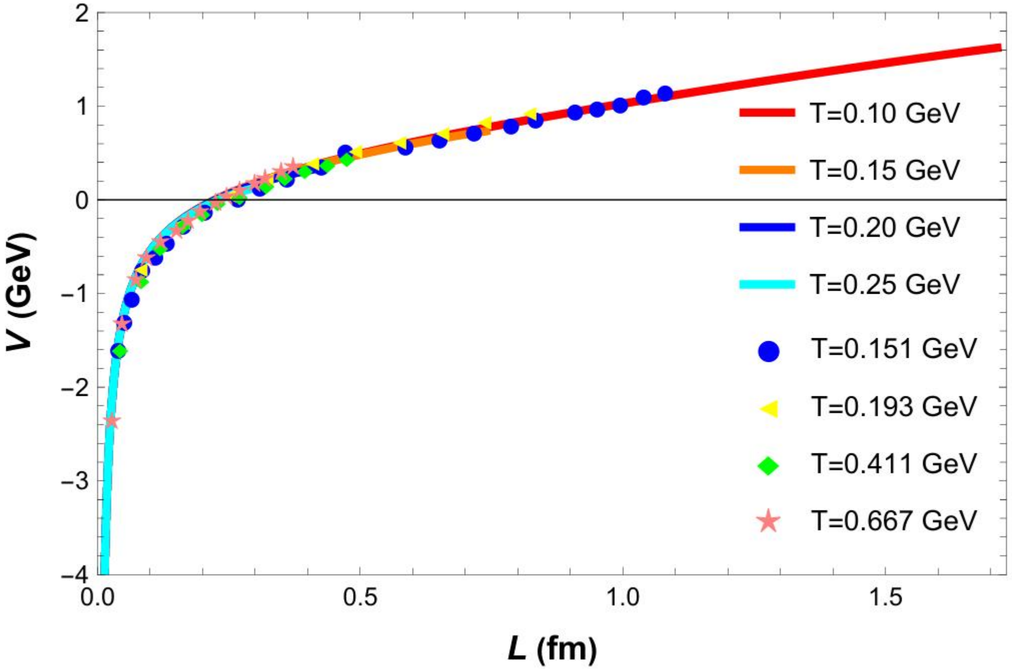
(b)

Heavy-quark potential

*Xi Guo, **Xun Chen**, et al, Potential energy of heavy quarkonium in flavor-dependent systems from a holographic model, Phys.Rev.D 110 (2024) 4, 046014*

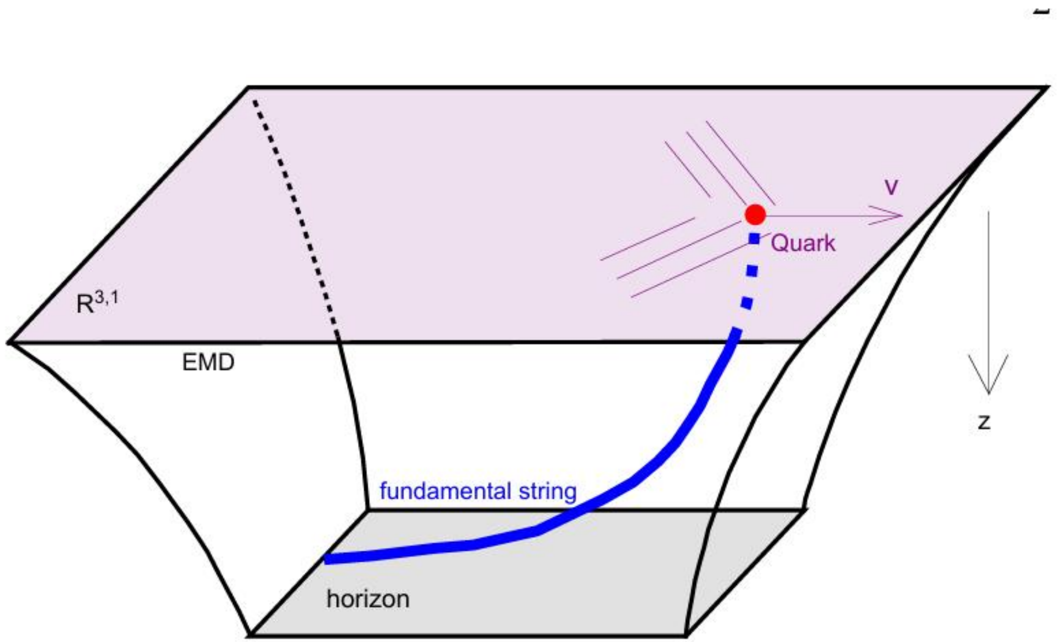


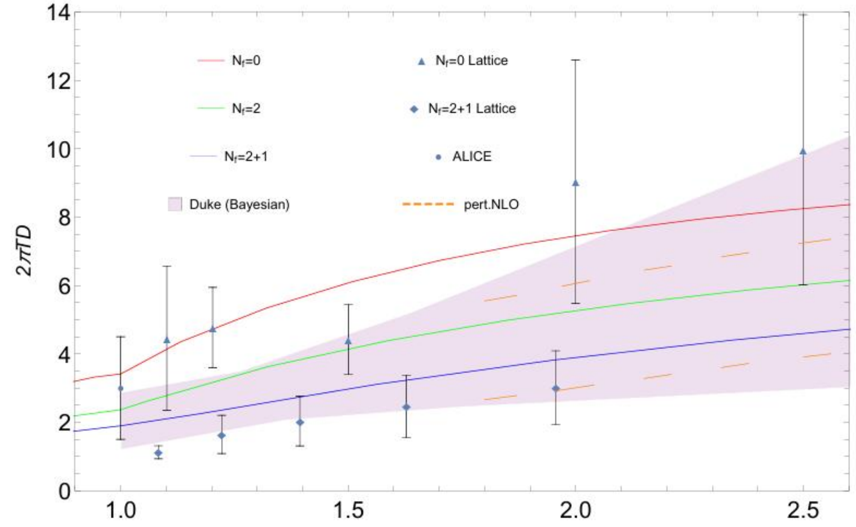
Phys.Rev. D 71 (2005) 114510
arXiv:hep-lat/0503017

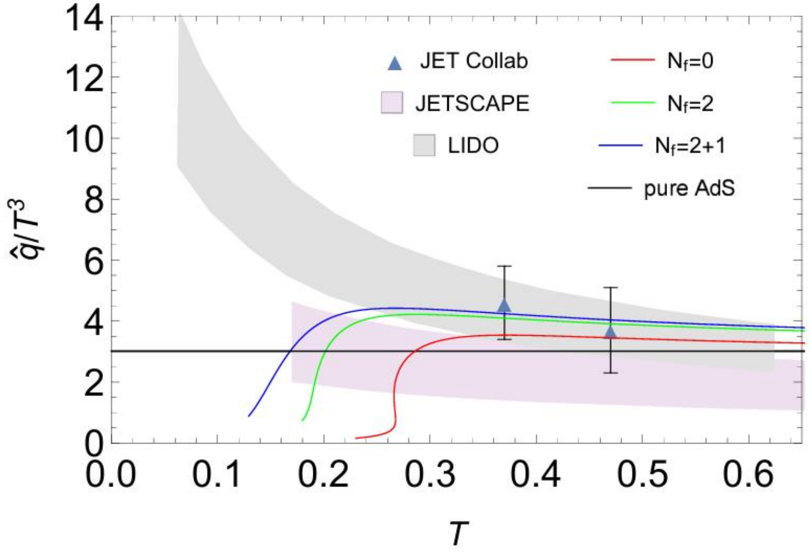


Phys.Rev.D 105 (2022) 5, 054513
arXiv: 2110.11659 [hep-lat]

Transport properties of QGP







*Bing Chen, **Xun Chen**, at al. Exploring transport properties of quark-gluon plasma in flavor-dependent systems with a holographic model, Phys.Rev.D 111 (2025) 8, 086033.*

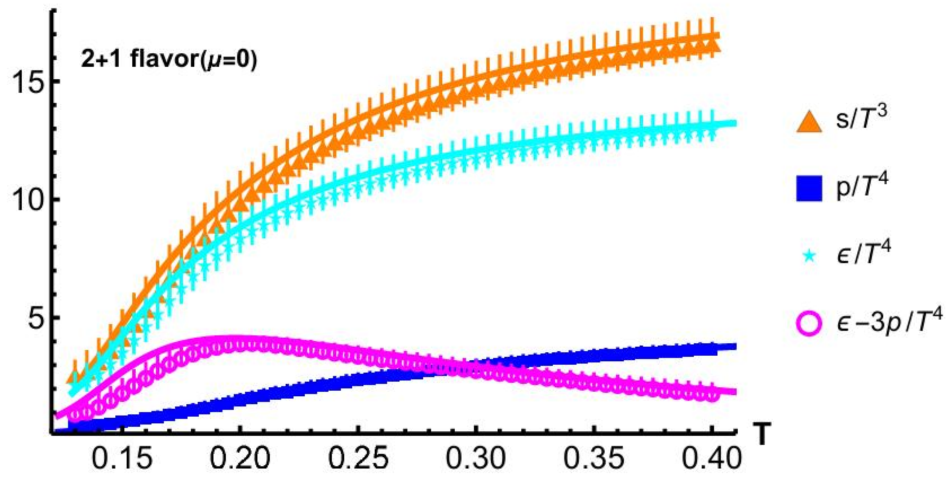
*Sheng Lin, Xuan Liu, **Xun Chen**, et al, Holographic Schwinger effect in flavor-dependent systems, Phys.Rev.D 111 (2025) 4, 046005*

*Fei Sun, **Xun Chen**, et al, High-precise determination of critical exponents in holographic QCD, arxiv: 2503.17642*

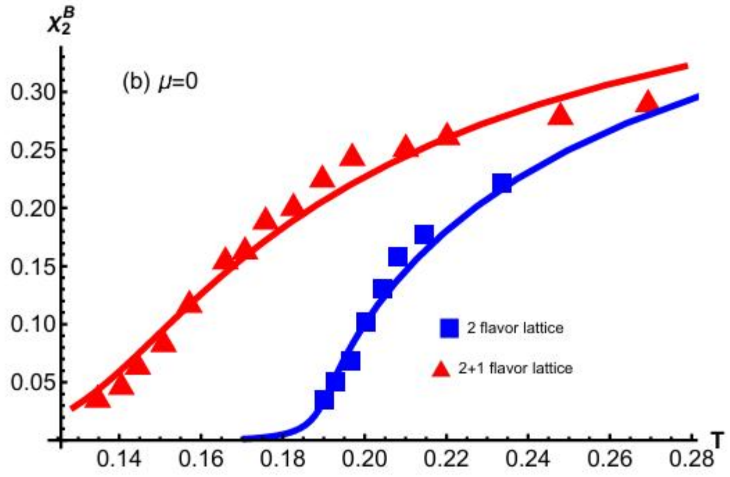
*Le Zhang, Lu Yin, Guo-Dong Zhou, Chao-Jie Fan, **Xun Chen**, Light quark energy loss in the flavor-dependent systems from holography, arXiv: 2504.04979 (accepted by PRD)*

In summary, to validate our model, we can calculate further physical quantities and compare them with experimental results or other models.

QCD equation of state from lattice



Baryon number susceptibility



Can we include the error bar in the holographic model?

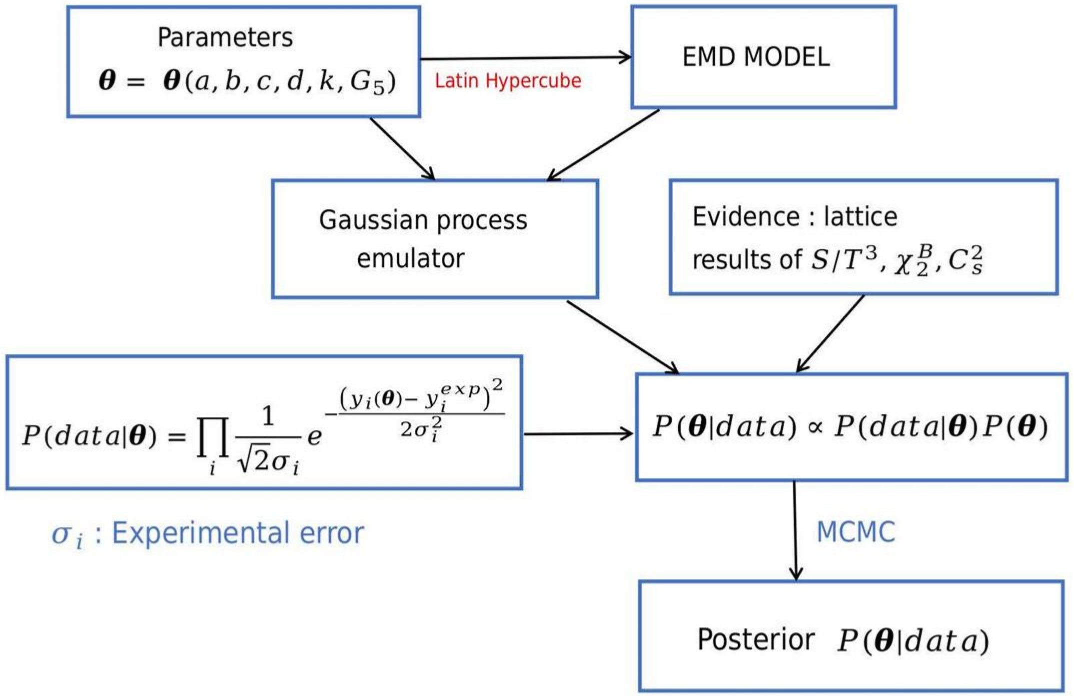
Sì

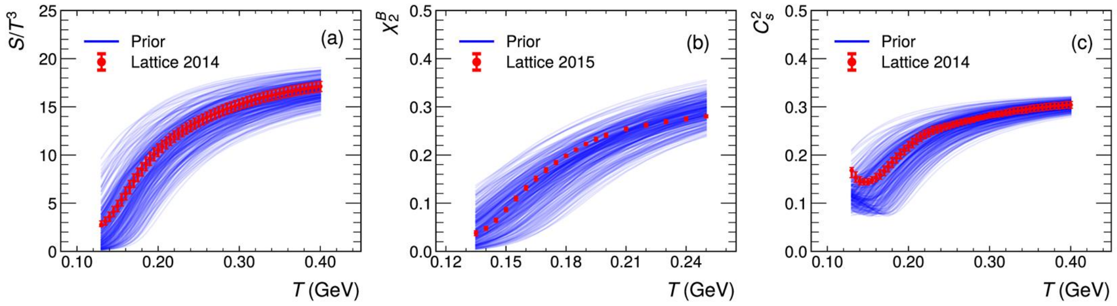
Yes

Shì(是)

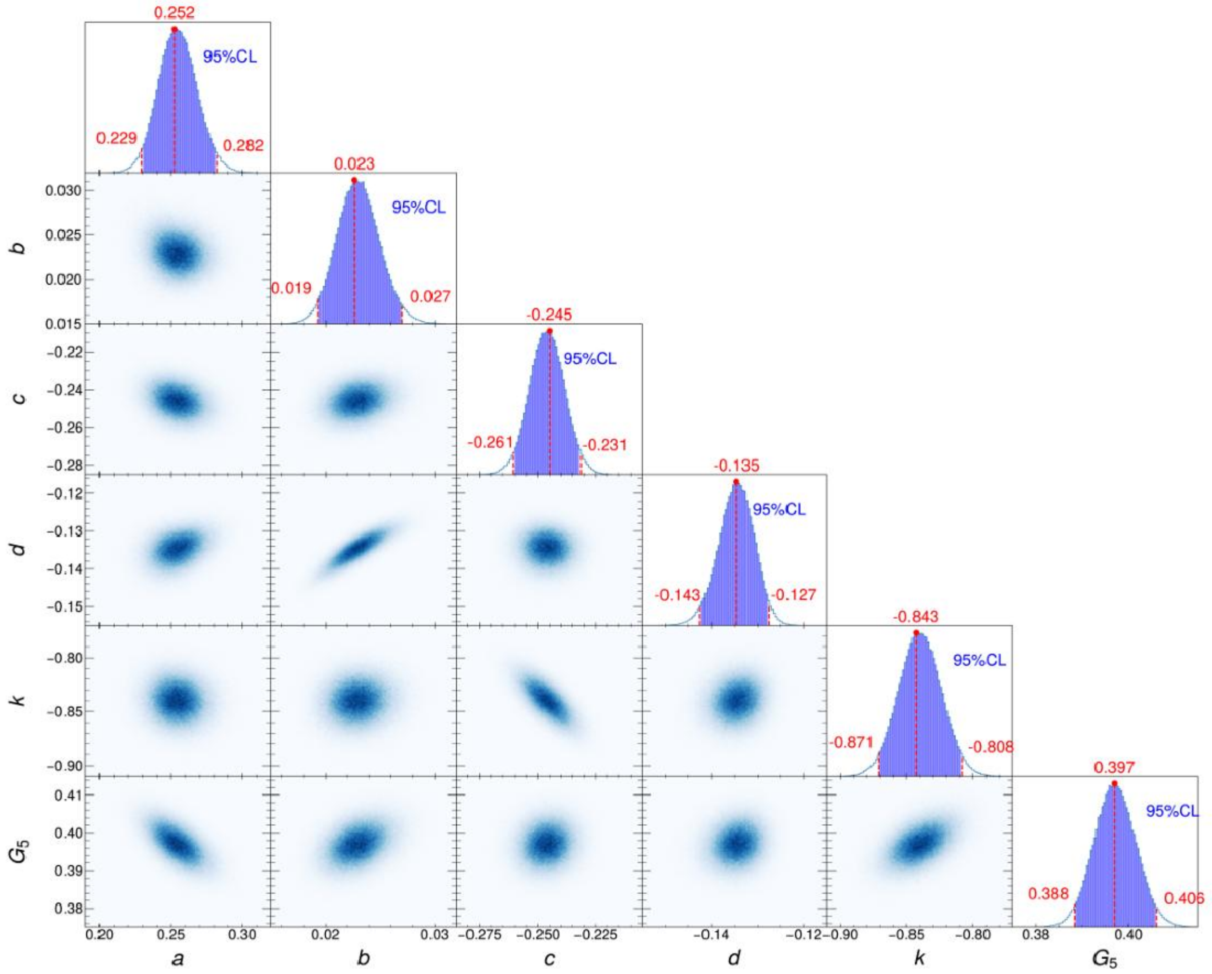
*Liqiang Zhu, **Xun Chen**, Kai Zhou, Hanzhong Zhang, Mei Huang,Bayesian Inference of the Critical Endpoint in 2+1-Flavor System from Holographic QCD, arXiv:2501.17763*

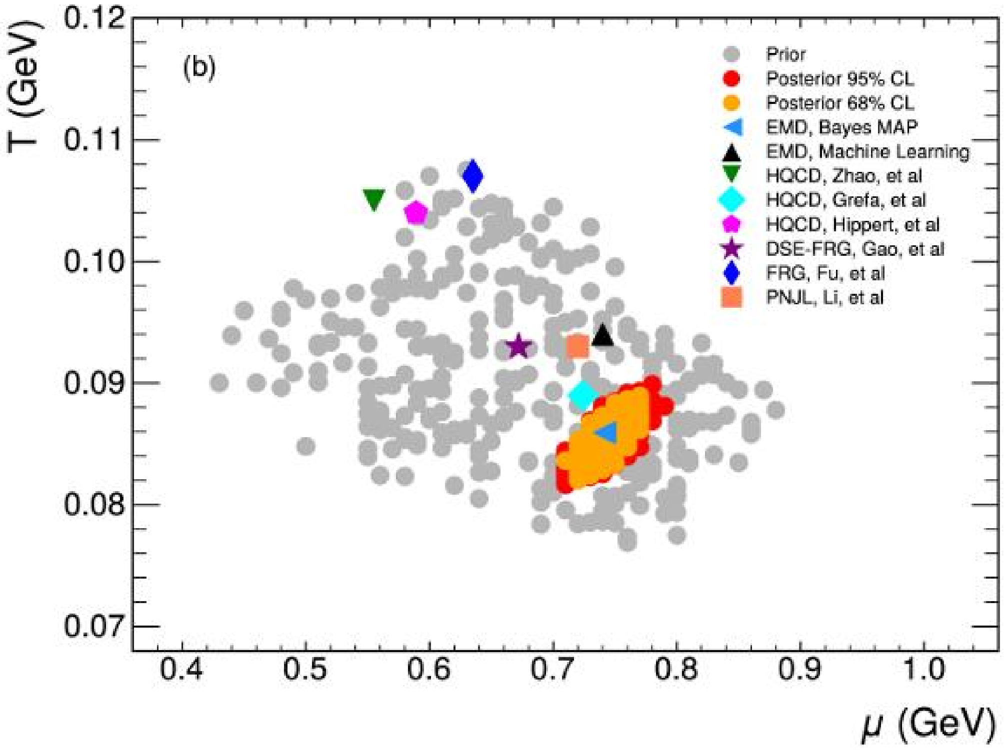
$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$





Prior		
Parameter	min	max
a	0.110	0.310
b	0.005	0.031
c	-0.280	-0.205
d	-0.240	-0.110
k	-0.910	-0.770
G_5	0.375	0.430


0.252
95%CL
0.229
0.282
0.023
0.019
0.027
-0.245
-0.261
-0.231
-0.135
-0.143
-0.127
-0.843
-0.871
-0.808
0.397
0.388
0.406
a
b
c
d
k
G_5

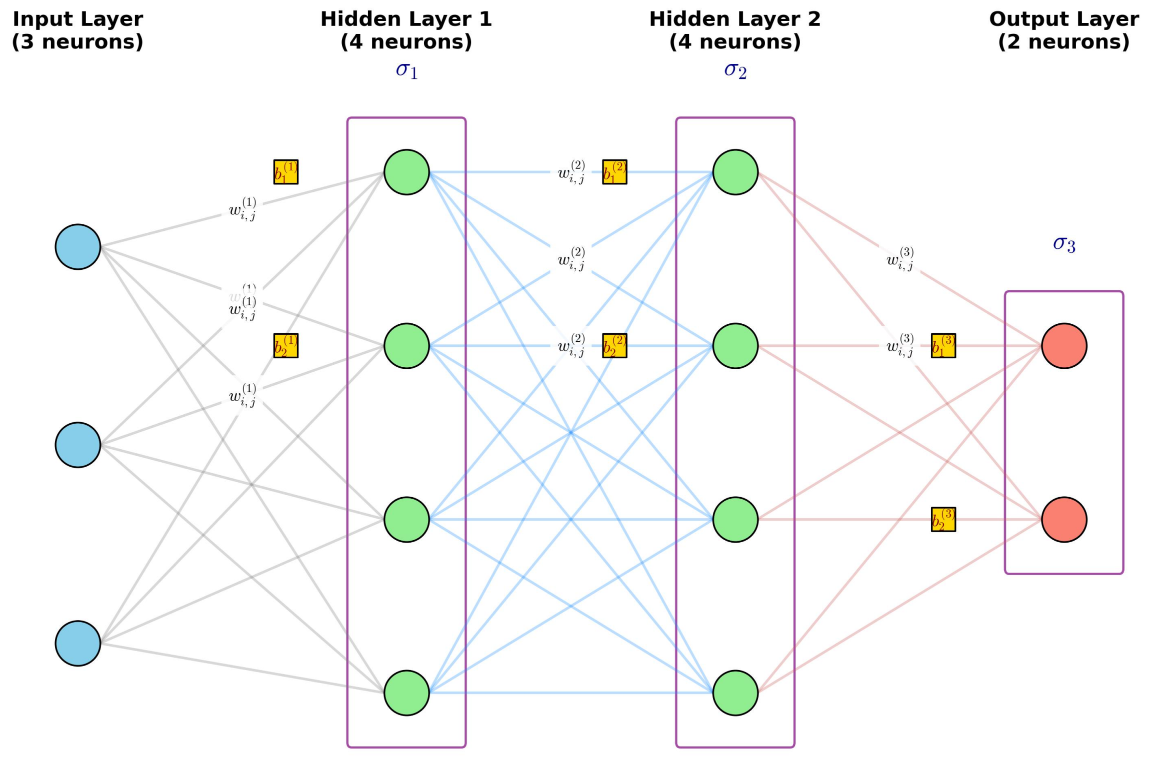

(b)
Prior
Posterior 95% CL
Posterior 68% CL
EMD, Bayes MAP
EMD, Machine Learning
HQCD, Zhao, et al
HQCD, Grefa, et al
HQCD, Hippert, et al
DSE-FRG, Gao, et al
FRG, Fu, et al
PNJL, Li, et al
T (GeV)
μ (GeV)

Universal Approximation Theorem

$$f(x) = \sum_{i=1}^{N} \alpha_i \, \sigma(w_i x + b_i).$$

σ: activation function
w:weight
b:bias

When N is sufficiently large, $f(x)$ can approximate any function arbitrarily closely.



Multilayer Perceptron (MLP) Architecture

Inverse problem

An inverse problem is the process of deducing unknown causes or system parameters from observed effects or data.

Miguel Angel Martin Contreras, Alfredo Vega and Saulo Diles, Heavy quarkonia spectroscopy at zero and finite temperature in bottom-up AdS/QCD

$$I_{\text{Vector } Q\bar{Q}} = -\frac{1}{4\,g_5^2}\int d^5x\,\sqrt{-g}\,e^{-\Phi(z)}\,g^{mp}\,g^{nr}\,F_{mn}\,F_{pr}$$

$$\Phi(z) = (\kappa\,z)^{2-\alpha} + M\,z + \tanh\left[\frac{1}{M\,z} - \frac{\kappa}{\sqrt{\Gamma}}\right]$$

$$-\phi_n''(z) + U(z)\,\phi_n(z) = M_n^2\,\phi_n(z) \qquad f_n^2 = \frac{1}{M_n^2\,g_5^2}\lim_{z\to 0} e^{-2\,\Phi(z)}\left|\frac{2\,\psi_n(z,q)}{z^2}\right|^2.$$

Charmonium States $I^G(J^{PC}) = 0^+(1^{--})$							
Parameters:		$\kappa = 1.8$ GeV, $M = 1.7$ GeV, $\sqrt{\Gamma} = 0.53$ GeV and $\alpha = 0.54$					
n	**State**	$M_{\mathbf{Exp}}$ **(MeV)**	$M_{\mathbf{Th}}$ **(MeV)**	$\%M$	$f_{\mathbf{Exp}}$ **(MeV)**	$f_{\mathbf{Th}}$ **(MeV)**	$\%f$
1	J/ψ	3096.916 ± 0.011	3140.1	1.42	416.16 ± 5.25	412.4	1.4
2	$\psi(2S)$	3686.109 ± 0.012	3656.9	0.9	296.08 ± 2.51	272.7	8.0
3	$\psi(4040)$	4039 ± 1	4055.7	0.4	187.13 ± 7.61	201.8	7.8
4	$\psi(4415)$	4421 ± 4	4376	0.9	160.78 ± 9.70	164.1	2.0
Nonlinear Regge Trajectory:				$M_n^2 = 8.097(0.39 + n)^{0.58}\text{GeV}^2$ with $R^2 = 0.999$			

Optimized Charmonium Mass Spectrum Predictions (Unit: MeV)
Ground state (n=1, J/ψ): 3096.9 (Experimental value: 3096.9)
First excited state (n=2, ψ(2S)): 3686.1 (Experimental value: 3686.1)
Second excited state (n=3, ψ(4040)): 4038.9 (Experimental value: 4039)
Third excited state (n=4, ψ(4415)): 4421.1 (Experimental value: 4421)

Optimized Charmonium Decay Constant Predictions (Unit: MeV)
Ground state (n=1, J/ψ): 416.1 (Experimental value: 416.16)
First excited state (n=2, ψ(2S)): 296.4 (Experimental value: 296.08)
Second excited state (n=3, ψ(4040)): 183.5 (Experimental value: 187.13)
Third excited state (n=4, ψ(4415)): 165.4 (Experimental value: 160.78)

Multi-Layer Perceptrons (MLP) and Kolmogorov-Arnold Network (KAN)

Ziming Liu, et al, arXiv: 2404.19756

Black box

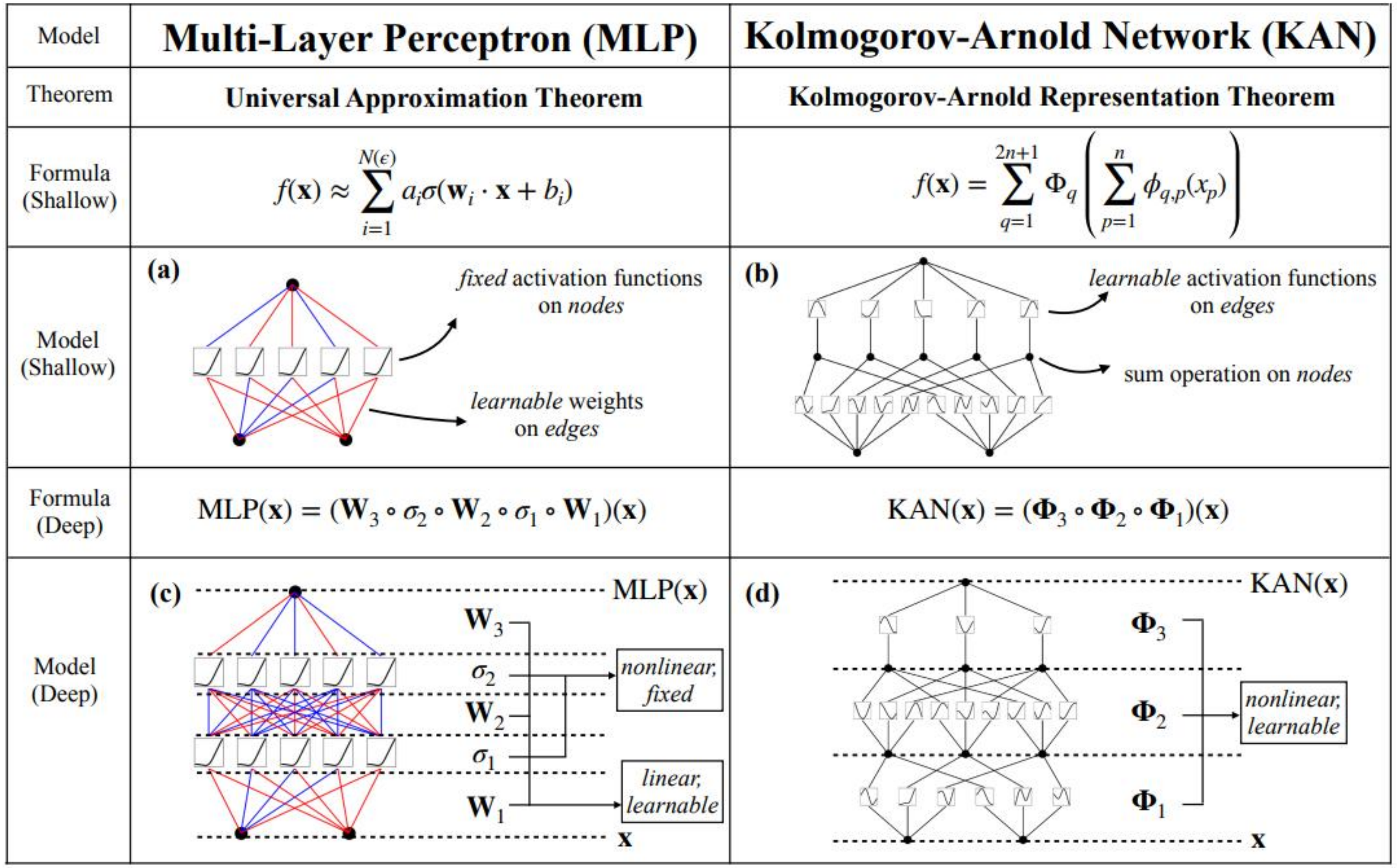


Figure 0.1: Multi-Layer Perceptrons (MLPs) vs. Kolmogorov-Arnold Networks (KANs)

Intrinsic Interpretability

Holographic heavy-quark potential

Oleg Andreev, Valentin I. Zakharov, On Heavy-Quark Free Energies, Entropies, Polyakov Loop, and AdS/QCD, JHEP 04 (2007) 100

Deformed AdS-RN black hole

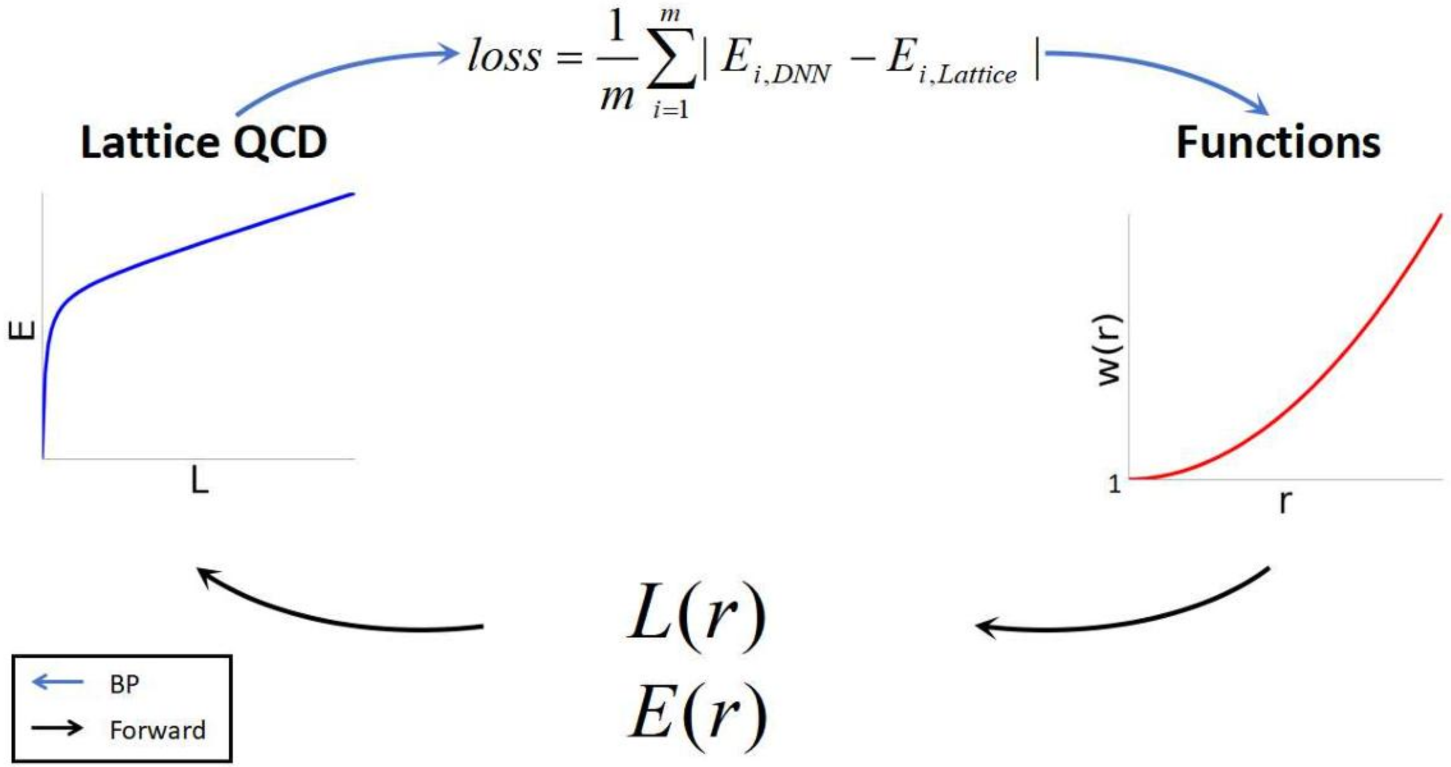
$$ds^2 = w(r)\frac{1}{r^2}\left[f(r)dt^2 + d\vec{x}^2 + f^{-1}(r)dr^2\right],$$

$$f(r) = 1 - \left(\frac{1}{r_h^4} + q^2 r_h^2\right) r^4 + q^2 r^6.$$

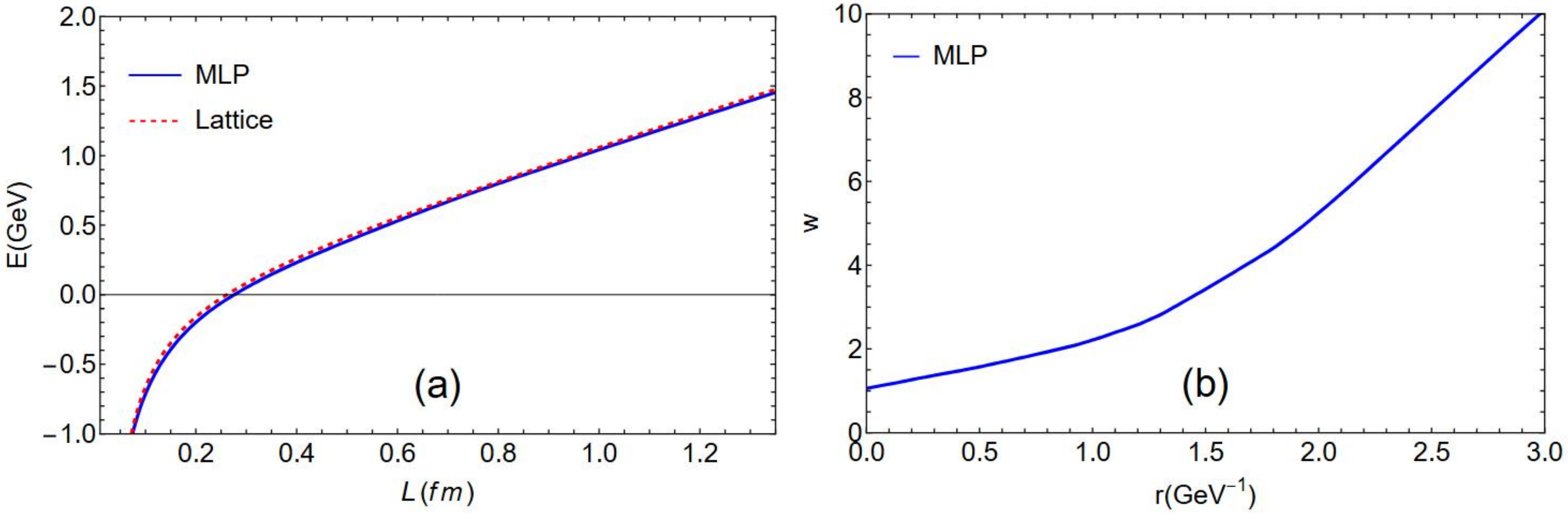
Deformed factor *w(r)* is added by hand in early holographic works

MLP training

*Ou-Yang Luo, **Xun Chen**, et al, Neural Network Modeling of Heavy-Quark Potential from Holography, arXiv: 2408.03784 (accepted by EPJC)*

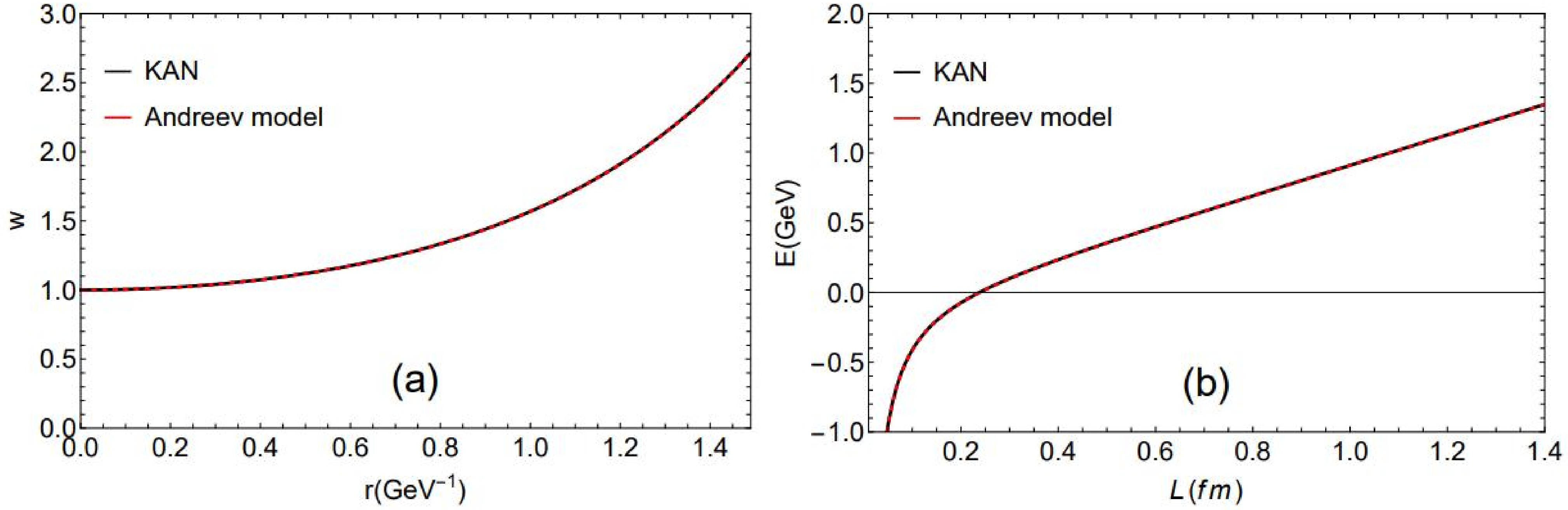


Construct the model from lattice QCD

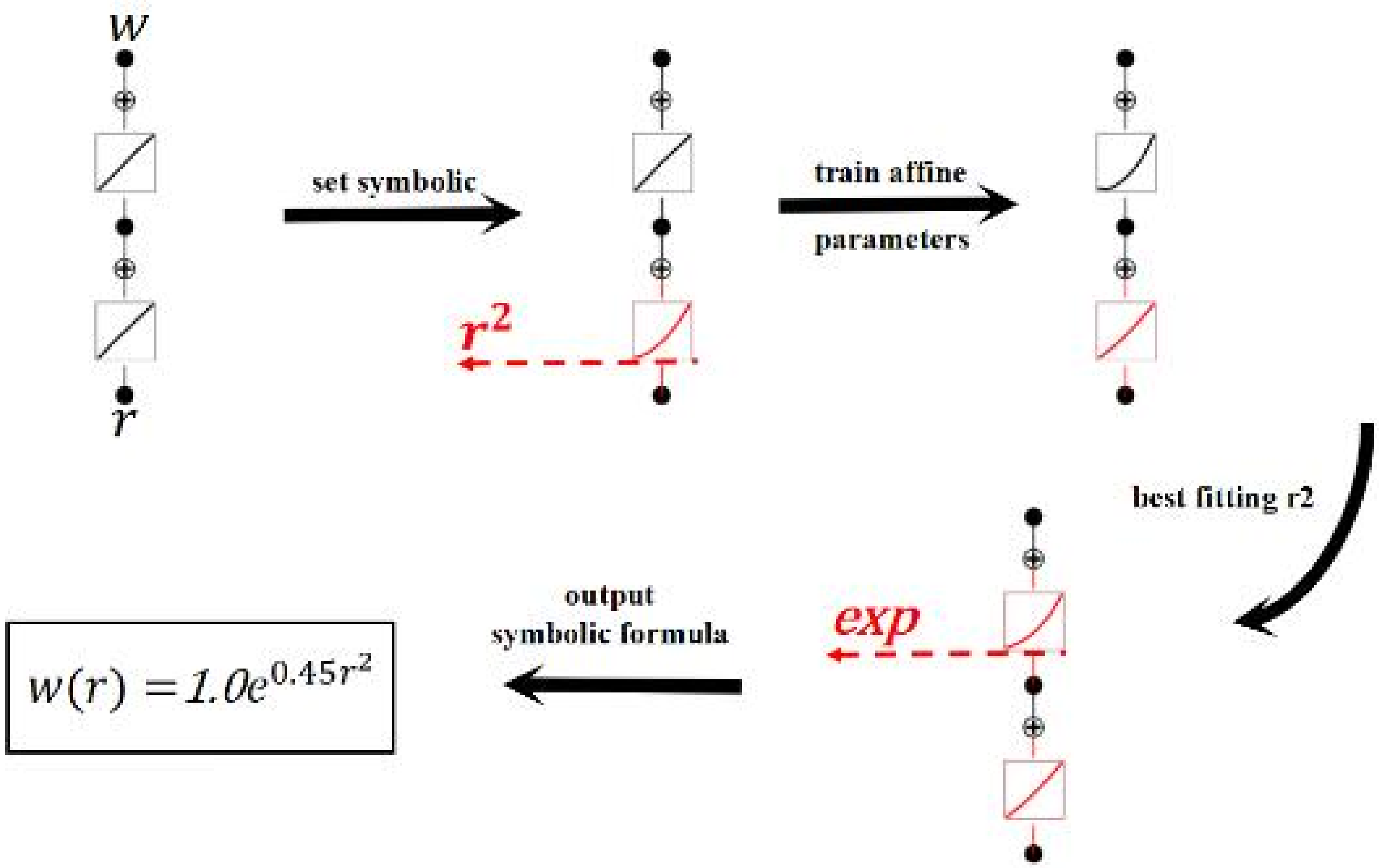


Only numerical solution of w(r)

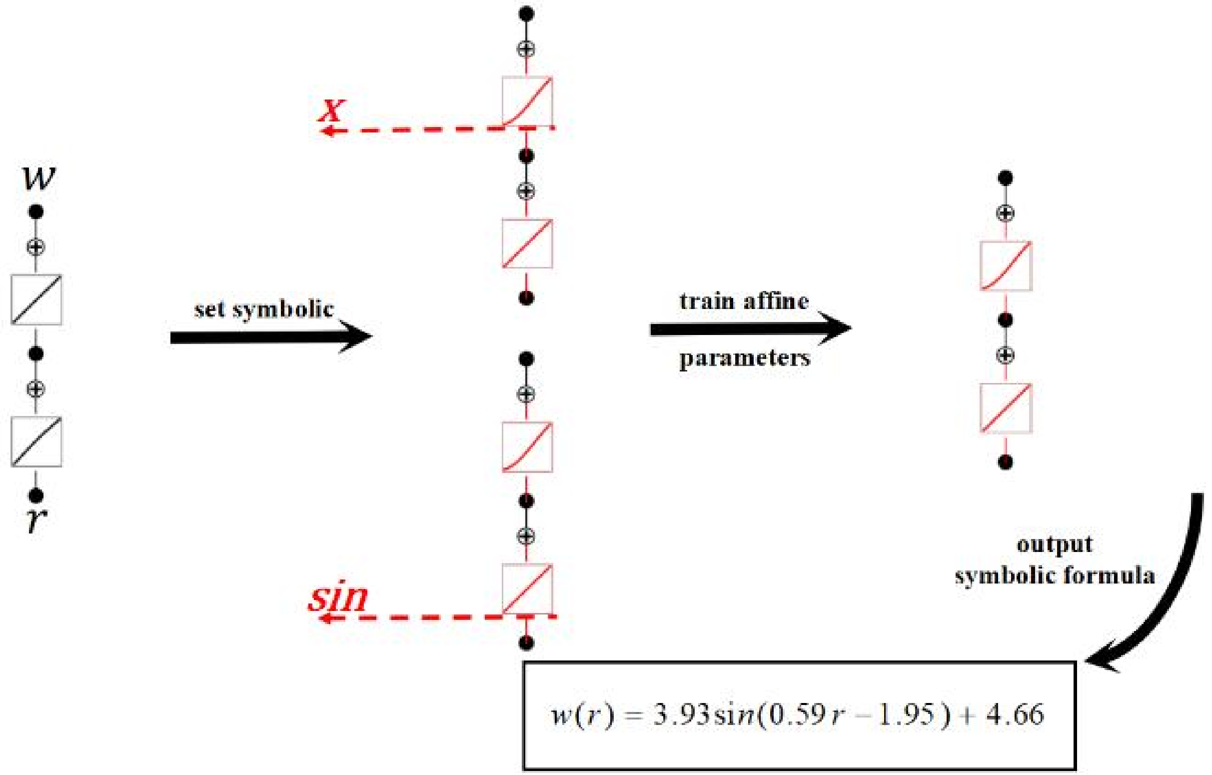
The KAN structure and its application to reproduce the Andreev-Zakharov model.

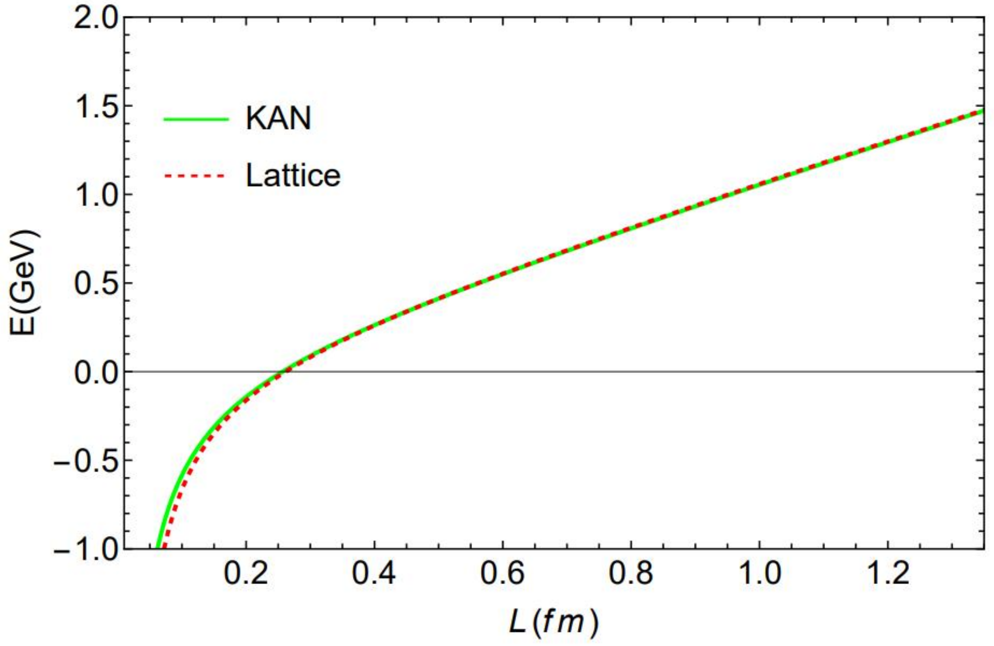


Reproduce the Andreev-Zakharov model

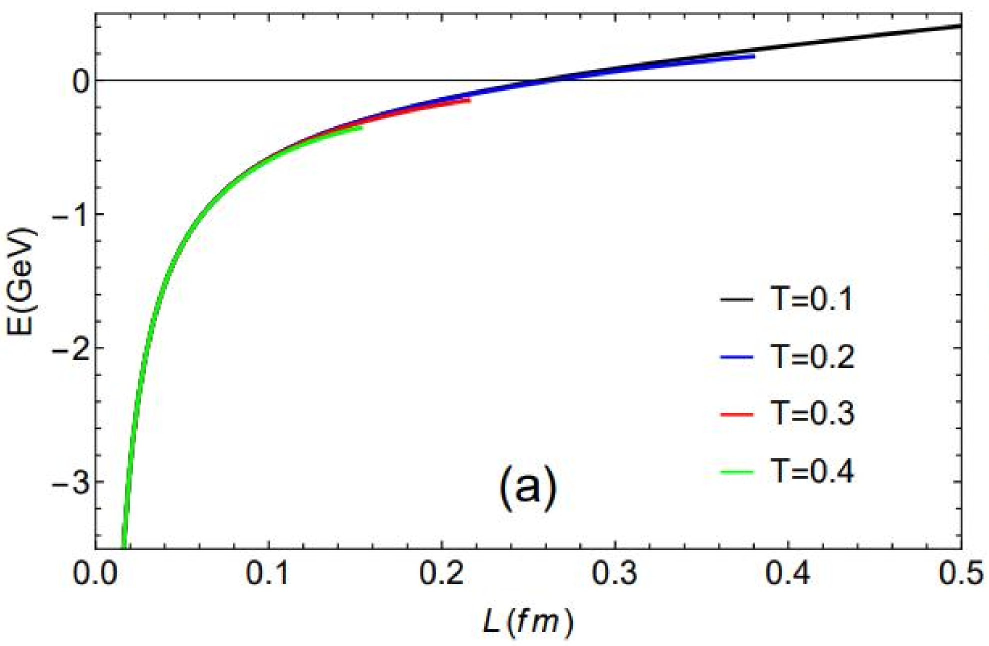


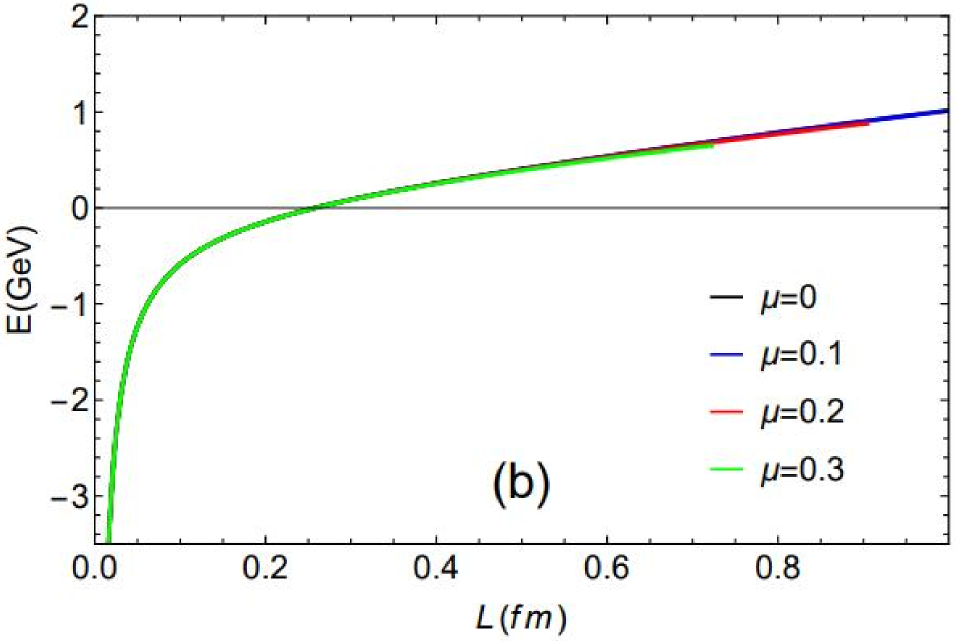
KAN architecture and results





Heavy-quark potential at finite temperature and chemical potential





Thank you for your time!