Bubble wall dynamics at the EW phase transition

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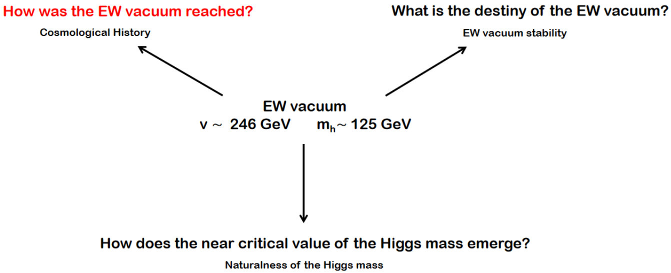
Based on C.B., A. Conaci, S. De Curtis, L. Delle Rose, *Electroweak Phase Transition and Bubble Wall Velocity in Local Thermal Equilibrium*, arXiv:2504.21213

Università degli Studi di Bari

Higgstory

Key ingredients in SM: EW vacuum ← Possibly least understood piece



Trivial vacuum → ? → EW vacuum

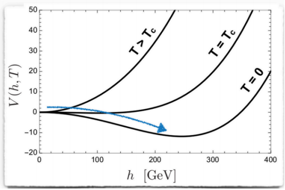
← T

EW Phase transition in the Standard Model

In the SM, EWPT is a crossover: the two phases are smoothly connected

Kajantie, Laine, Rummukainen, Shaposhnikov

- No barrier in the potential
- h rolls down to non-trivial minimum for $T < T_c$



No strong breaking of thermal equilibrium, no distinctive experimental signatures

Perturbative evaluation of the effective potential:

~~1st order transition~~

Unreliable

Barriers fully generated by fluctuations must be taken with caution

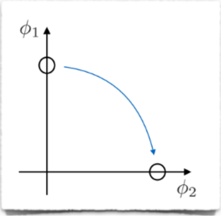
EWPhT and new physics

Multiple scalar fields

- Minima in several directions and barriers between them can be present at tree-level

$\Rightarrow$ New physics may provide a **first order** phase transition $\Leftarrow$

- h tunnels to non-trivial value at $T_n < T_c$
- Transition proceeds through **bubble nucleation**



The dynamics of the bubble wall can produce

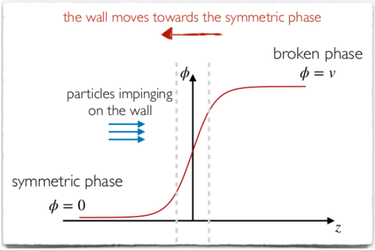
- A significant breaking of thermal equilibrium (baryogenesis)
- Bubble collisions & turbulences in the plasma (gravitational waves)

Signatures that in principle make theory of EWPhT falsifiable

Dynamics of the bubble wall

System: scalar field ϕ + plasma

ϕ representative for the fields driving the PhT



- Expansion of the bubble wall in the false vacuum drives the plasma out of equilibrium
- Plasma back-reacts: interactions between with the PhT front produce a friction
- Balance between outward pressure and friction: steady state regime with terminal velocity v_w

In the follwing we assume a planar wall and a steady-state regime

Runaway solutions with $v_w = 1$ do not exist

Gouttenoire, Jinno, Sala / Ai, Nagels, Vanvlasselaer / Azatov, Barni, Petrossian-Byrne / Ramsey-Musolf, Zhu / Ai, Carosi, Garbrecht, Tamarit, Vanvlasselaer

Set-up

Plasma described as a mixture of three components

1. Scalar fields participating in the transition
2. Background of weakly coupled species: $\sim$ local thermal equilibrium
3. Strongly coupled species: out-of-equilibrium contributions relevant

$$\text{For each particle species: } f(p,z) = \underbrace{f_v(p,z)}_{\text{LTE}} + \underbrace{\delta f(p,z)}_{\text{OOE}}$$

1. Scalar field EOMs

Moore, Prokopec

$$\Box\phi(z) - \partial_\phi V(\phi, T) = \frac{1}{\phi'(z)} \sum_i N_i \frac{dm^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_p} \delta f_i$$

2. EM conservation for background $\rightarrow$ space-dependent profiles

$$f_v = \frac{1}{e^{\beta(z)\gamma(z)(E - v_p(z)p_z)} \pm 1}$$

3. Boltzmann equation for out-of-equilibrium

$\mathcal{C}$ collision integral

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right)(f_v + \delta f) = -\mathcal{C}[f_v + \delta f]$$

Boltzmann equation and flow paths

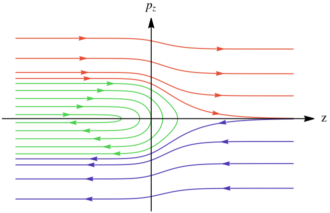
Along flow paths $p_\perp$ & $p_z^2 + m^2(z)$ are conserved De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

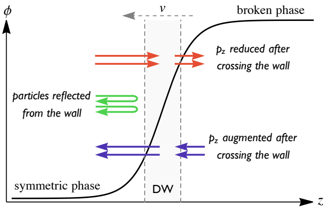
$$\mathcal{L} = \left(\frac{p_z}{E}\partial_z - \frac{(m^2(z))'}{2E}\partial_{p_z} \right) \quad \rightarrow \quad \frac{p_z}{E}\frac{d}{dz}$$

Trajectories in $(p_\perp, p_z, z)$ phase space in collisionless limit $C \rightarrow 0$

Three classes of solutions

$$m(z) = m_0(1 + \tanh(z/L))$$





Collision integral for $2 \rightarrow 2$ processes

$$\mathcal{C}[f] = \sum_i \frac{1}{4N_pE_p} \int \frac{d^3\mathbf{k}\, d^3\mathbf{p}'\, d^3\mathbf{k}'}{(2\pi)^5 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_i|^2 \delta^4(p + k - p' - k')\mathcal{P}[f]$$

$$\mathcal{P}[f] = f(p)f(k)(1 \pm f(p'))(1 \pm f(k')) - f(p')f(k')(1 \pm f(p))(1 \pm f(k))$$

Linearised Boltzmann equations

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2(z))'}{2E}\partial_{p_z}\right)\delta f + \overline{\mathcal{C}}[\delta f] = \frac{(m^2(z))'}{2E}\partial_{p_z} f_v$$

Higher order contributions subdominant

De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

Collision integral splits in two pieces

- Terms proportional to $\delta f(p)$ not integrated
- Terms with $\delta f(q)$ inside the integral $q = k, p', k'$

$\Rightarrow$ The equation can be put in the form $\left(\frac{d}{dz} - \frac{\mathcal{Q}}{p_z}\right)\delta f = \mathcal{S}$

- $\frac{\mathcal{Q}}{p_z} \leftrightarrow$ terms $\propto \delta f(p)$
- $\mathcal{S} \leftrightarrow$ terms with integrated δf: corrections by iteration

$$\delta f = \left[B(p_\perp, p_z^2 + m^2(z)) + \int_{\bar{z}}^{z} dz' e^{-\mathcal{W}(z')} S\right] \qquad \mathcal{W}(z) = \int^{z} dz' \frac{\mathcal{Q}}{p_z}$$

Out-of-equilibrium: an iterative approach

- Boltzmann: integro-differential equation hard to solve
- $\delta f \leftrightarrow v_p(z),\ T(z)$ depend on one another

Solutions using (truncated) moment expansion were used — Moore, Prokopec

But several shortcomings ...

Laurent, Cline / Dorsch, Huber, Konstandin / De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

Recently: iterative (numerical) approach to get full solution

De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

At each step:

1. Consider the Boltzmann equation in the form $\left(\frac{d}{dz} - \frac{\mathcal{Q}}{p_z}\right)\delta f = \mathcal{S}$ with $\mathcal{S}$ calculated from δf at previous step
2. Calculate $\mathcal{W}$ and determine δf
3. Calculate $\mathcal{S}$ to be used in the next step (+ profile corrections, see later)

First step: LTE solution

Hydrodynamics of the plasma

In LTE, hydrodynamics of the plasma fully determines bubble dynamics

LTE: drop $T^{\mu\nu}_{out}$

Equations for the plasma are obtained from conservation of the EMT

$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{pl} + T^{\mu\nu}_{out}$, $\qquad T^{\mu\nu}_{out} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \delta f$

$$T^{30} = w\gamma^2 v_p + T^{30}_{out} = c_1$$

$$T^{33} = \frac{(\partial_z\phi)^2}{2} - V(\phi, T) + w\gamma^2 v_p^2 + T^{33}_{out} = c_2$$

$w = T\partial_T V$ enthalpy, v_p plasma velocity, $\gamma = \left(\sqrt{1 - v_p^2}\right)^{-1}$

Constants $c_{1/2}$: boundaries for v_p and T $\qquad + =$ in front of the wall, $- =$ behind the wall

Combustion regimes

Two branches: $v_+ > v_-$, $v_+ < v_-$. $v_\pm$, $T_\pm$ depend on v_w: three classes

- Detonations, $v_w > v_J$:
  plasma at rest in front of the wall, followed by rarefaction wave

$$v_w = v_+, \; T_n = T_+$$

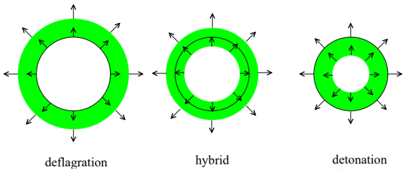
- Deflagrations, $v_w < c_s^-$:
  plasma at rest behind wall, shock wave in front of it

$$v_w = v_-, \; T_n = T_+^{SW}$$

- Hybrids, $c_s^- < v_w < v_J$:
  combination of both, shock front + rarefaction wave

$$v_- = c_s^-, \; T_n = T_+^{SW}$$

Espinosa, Konstandin, No, Servant



Scalar EOMs

Our interest is in 2-step PhTs driven by two fields h, s, $(0,0) \to (0,\bar{s}) \to (\bar{h},0)$

$$E_h \equiv -\partial_z^2 h + \frac{\partial V(h,s,T)}{\partial h} \left(+ \frac{F_h^{out}(z)}{h'} \right) = 0$$

$$E_s \equiv -\partial_z^2 s + \frac{\partial V(h,s,T)}{\partial s} \left(+ \frac{F_s^{out}(z)}{s'} \right) = 0$$

Approximate solution: tanh ansatz

$$h(z) = \frac{h_-}{2}\left(1 + \tanh\left(\frac{z}{L_h}\right)\right)$$

$$s(z) = \frac{s_+}{2}\left(1 - \tanh\left(\frac{z}{L_s} - \delta_s\right)\right)$$

h_- : $\partial_h V(h,0,T_-) = 0$, s_+ : $\partial_s V(0,s,T_+) = 0$

4 parameters to determine: T_-, δ_s, L_h, L_s

Constraints

Trade the differential equations for four constraints eqs

$$P_h = \int dz\, E_h h' = 0 \qquad G_h = \int dz\, E_h \left(2\frac{h}{h_-} - 1\right) h' = 0$$

$$P_s = \int dz\, E_s s' = 0 \qquad G_s = \int dz\, E_s \left(2\frac{s}{s_+} - 1\right) s' = 0$$

P pressure, G pressure gradients

In **LTE**

$P_h + P_s = \Delta V - \int dz (\partial_T V) T'$: non-trivial T-profile $\rightarrow$ LTE friction

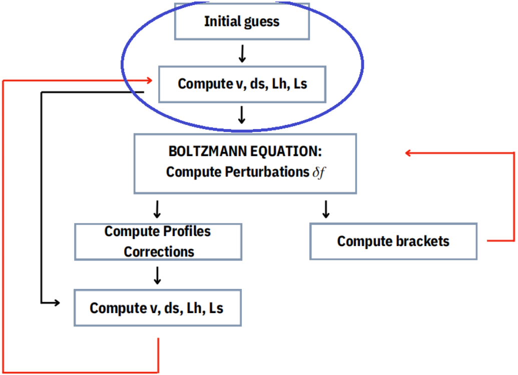
Analytic considerations suggest that:

-$P_h + P_s$ mainly depends on $T_- \rightarrow v_w$
-$P_h - P_s$ (mainly) determines δ_s
-$G_i = 2\phi_i^2/15L_i^2 + g_i(T_-, \delta_s, L_h/L_s) \qquad L_h/L_s \sim h_-/s_+$

Solving the eqs: numerical algorithm

As of now:

- In LTE (hydrodynamic + scalar) we explored the parameter space of models with 2-step PhT
- Inclusion of out-of-equilibrium done for benchmark points



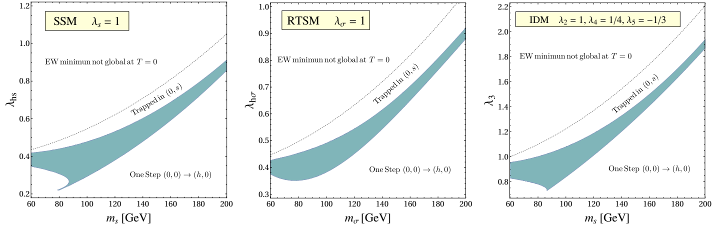
Scan on parameter space: initial guess = result on nearest neighbour

Models

We consider three models: singlet extension of the SM (SSM), triplet extension of the SM (RTSM), inert 2HDM (IDM), common tree-level potential

$$V_0(h,s) = \frac{\mu_h^2}{2}h^2 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_h}{4}h^4 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2 s^2$$

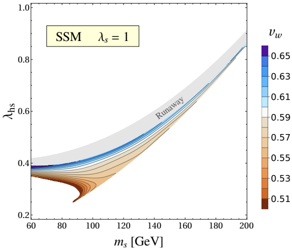
Preliminary: determine 2-step region w/ `CosmoTransitions` (slightly modified)

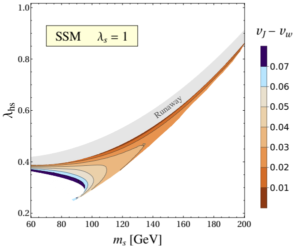


One can easily check that the barrier is mainly generated by Parwani-resummed tree-level potential

$$V_0^P(h,s,T) = V_0(h,s) + \frac{c_h T^2}{2}h^2 + \frac{c_s T^2}{2}s^2$$

Results: SSM, $\lambda_s = 1$

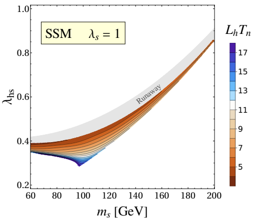


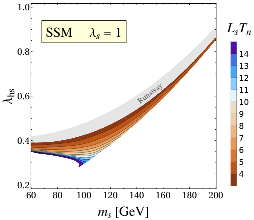


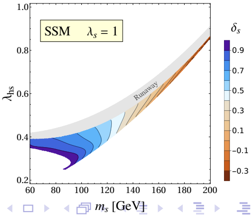
Only deflagrations in LTE

We complement this showing it with analytic study of P

Earlier suggestion in [Ai, Laurent, van de Vis, '23]

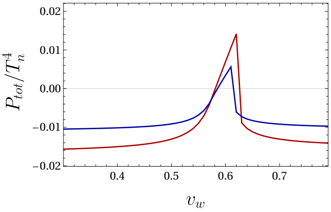


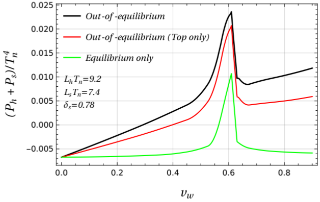




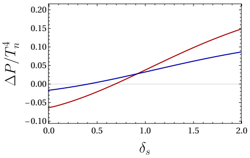
Constraints

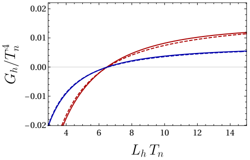
De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

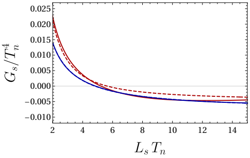




- Region around peak unphysical: second zero NOT a detonation
- Out-of-equilibrium: additional source of friction. **Strong impact !**

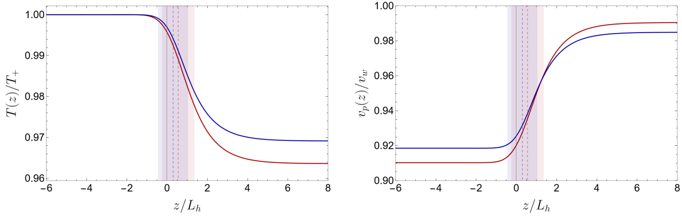






$L_h/L_s \sim h_-/s_+$ verified

Profiles



- Plasma profiles are nearly flat far from bubble front
- $\gamma(z)T(z) = cte$ due to entropy conservation in LTE   Ai, Garbrecht, Tamarit

$$P_{tot} = \Delta V - \int dz \frac{\partial V}{\partial T} T'(z)$$

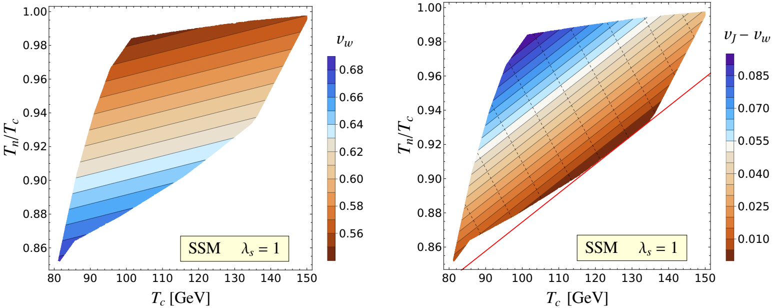
$\to$ $T(z)$ decreasing ensures $P_{tot} = 0$ has physical solution

$\uparrow$

Deflagrations

In detonations $T(z)$ decreases: **ONLY** OOE contributions can generate them

Results in terms of PhT parameters



v_w turns out to depend ($\sim$) linearly on T_c, T_n/T_c

- v_J too, with isolines nearly orthogonal
- $\Rightarrow$ upper bound on the amount of supercooling

Comparing the models

Model	$v_w(T_c/v, T_n/T_c)$	$(T_n/T_c)^{min}(T_c)$
SSM $\lambda_s = 1$	$1.60 + 0.15\,x - 1.14\,y$	$0.71 + 0.42\,x$
SSM $\lambda_s = 2$	$1.60 + 0.15\,x - 1.14\,y$	$0.71 + 0.42\,x$
RTSM $\lambda_\sigma = 1$	$1.60 + 0.13\,x - 1.12\,y$	$0.73 + 0.39\,x$
RTSM $\lambda_\sigma = 2$	$1.59 + 0.13\,x - 1.12\,y$	$0.72 + 0.40\,x$
IDM $\lambda_2 = 1/2$	$1.60 + 0.07\,x - 1.09\,y$	$0.75 + 0.34\,x$
IDM $\lambda_2 = 1$	$1.60 + 0.05\,x - 1.08\,y$	$0.76 + 0.32\,x$

- Weak dependence on the model and on the self-coupling of the additional state

Toward a model-independent characterization of v_w in terms of PhT parameters ... ?

Conclusions and outlook

Conclusions:

- First order EWPT: theoretically and experimentally compelling
- Strategy put forward to provide full solution of the (steady-state) wall dynamics
- In LTE: complete solution in the parameter space of BSM models
  1. Only deflagration solutions
  2. Linear fit of $v_w(T_c, T_n/T_c)$ to an excellent approximation
  3. Weak model dependence
- Out-of-equilibrium effects can have a significant impact

Outlook :

- Explore parameter space with out-of-equilibrium
- Inclusion of $1 \to 2$ and $2 \to 1$ processes in $\mathcal{C}$
- Possible release of the code
- Improve input: non-perturbative evaluation of the potential

Numerical algorithm: hydrodynamics + scalars

0. Calculate Jouguet velocity v_J and speed of sound c_s^- (Brent's method)

1. Starts with an initial guess for v_w, δ_s, L_h and L_s

2. Calculate the four constraints. Computation of the constraints requires $T(z)$: a function that computes it is then called. This function compares v_w to v_J, and classifies the tentative solution in terms of its combustion regime.

3a. Solve boundary conditions for detonations: $T_+ = T_n$ $(s_+ = s_n)$, $v_+ = v_w$. T_- and v_- found using the matching equations.

3b. Solve boundary conditions for deflagrations/hybrid: $T_+^{SW} = T_n$ and $v_- = v_w(c_s^-)$. Use fluid equations to determine T_+ (shooting method), then proceeds as for detonations.

4. Constraints are finally obtained to perform the numerical integration. The values of v_w, δ_s, L_h and L_s are progressively refined until the solution is found.