

Probing the structure of the $\chi_{c1}(3872)$ meson: heavy quark symmetries at work

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Based on: P.Colangelo, F. De Fazio, and G.Roselli

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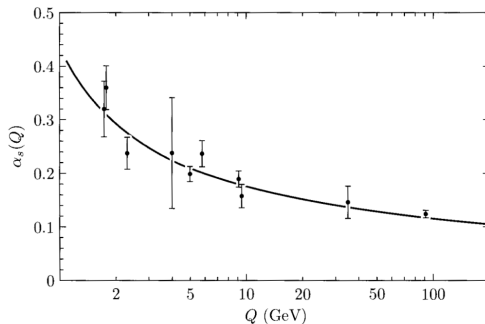
- 1 Features of strong interactions
- 2 Hadron spectroscopy: quark model classification and other possibilities
- 3 The case of $\chi_{c1}(3872)$ meson
- 4 Heavy Quark Effective theory (HQET)
- 5 Application to radiative decays of heavy quarkonia
- 6 Results and conclusions

- Difficulties in the application of perturbative methods.
- 1973: Non-Abelian gauge theories $\implies$ **asymptotic freedom**.
- Deep inelastic scattering $\implies$ evidences of almost free elementary constituents in the nucleon.
- 1973: **QCD** $\implies$ quarks.

Light quarks	Heavy quarks
up (u)	charm (c)
down (d)	bottom (b)
strange (s)	top (t)

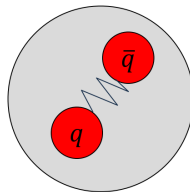
Strong Interactions

- Free quarks not observed $\implies$ all hadronic states are color singlets.
 $\implies$ confinement (non perturbative regime).
- Coupling constant is running:

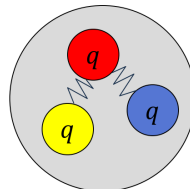


- Since 1964 hadronic states classified within the quark model:

- $q\bar{q}$ (mesons)

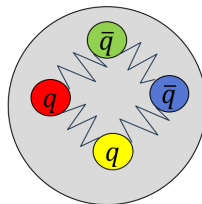


- qqq (baryons)

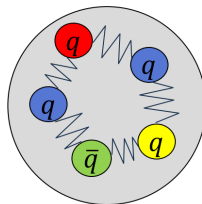


More complex structures, called **exotic** states, include:

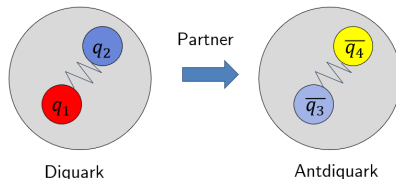
- $qq\bar{q}\bar{q}$ (tetraquarks)



- $qqq\bar{q}q$ (pentaquarks)

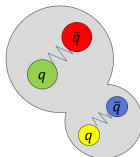


- **Tetraquarks and pentaquarks are compact multiquark states:**
 - Quarks form qq pairs (*diquarks*), which combine with antiquarks.

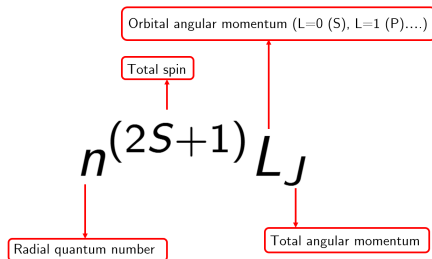


- **Molecular Picture:**

- Loosely bound systems of two color-neutral objects.

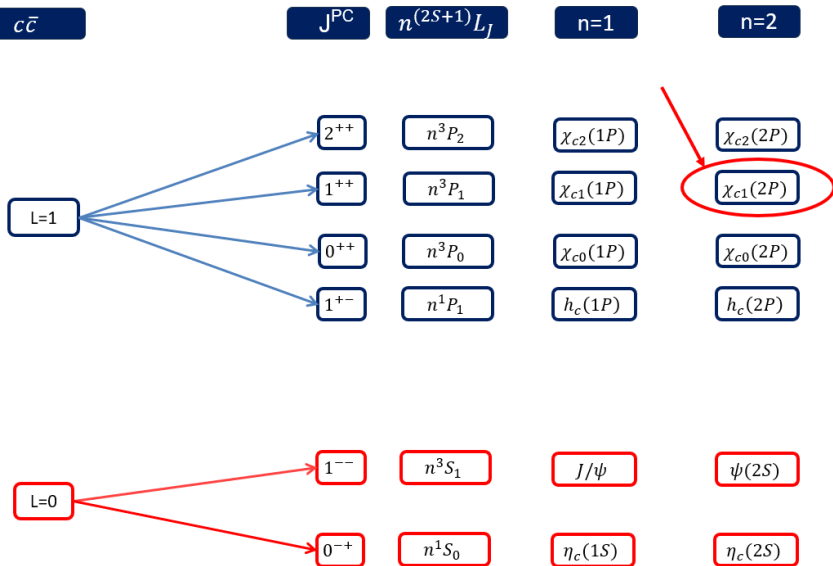


- Classification using the Spectroscopic notation:



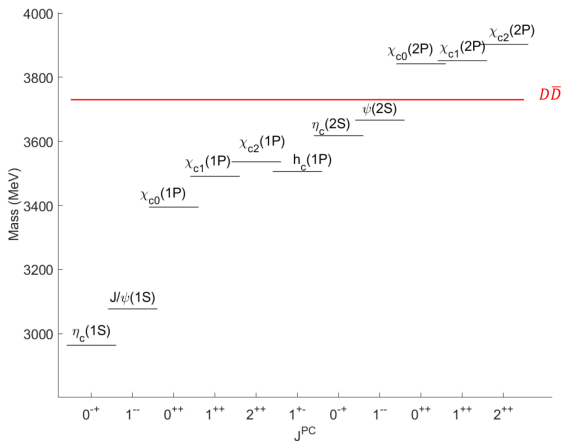
- For a meson $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \Rightarrow S = \begin{matrix} \nearrow 0 \\ \searrow 1 \end{matrix}$
- $P = (-1)^{L+1}$
- $C = (-1)^{L+S}$

Conventional states classification

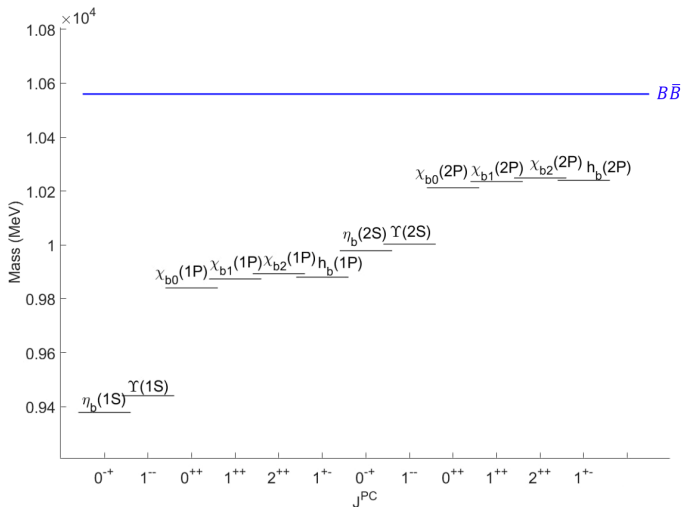


Charmonium States

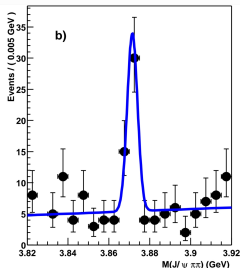
- Below the open-charm threshold, all states have been identified.
- Above this threshold, some conventional states remain unobserved, e.g., $2^1P_1 \rightarrow h_c(2P)$.



- Open-bottom threshold $M \approx 2m_B (= 10560 \text{ MeV})$

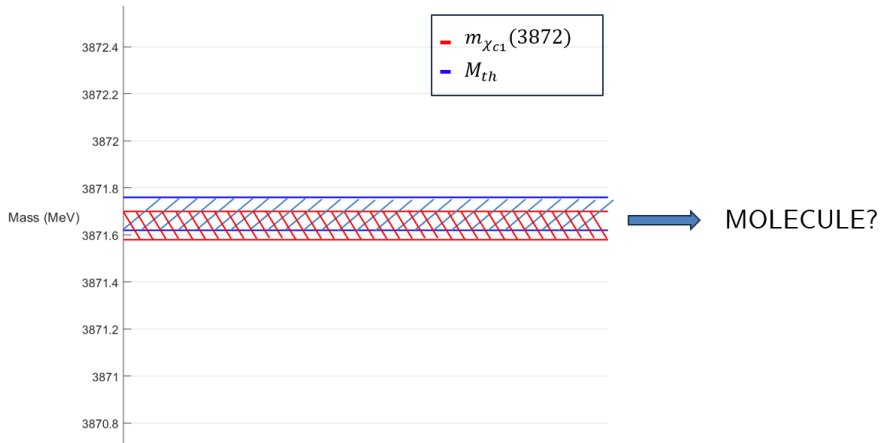


- Since 2003: New observations challenge the quark model.
- My work: the case of $\chi_{c1}(3872)$.
- Observed by the Belle collaboration in the decay $B \rightarrow K\pi^+\pi^-J/\psi \rightarrow$ narrow peak in the $\pi^+\pi^-J/\psi$ invariant mass distribution.



- Confirmed by BaBar, BESIII, LHCb, CDF, D0, ATLAS, CMS, ALICE
- Fixed $J^{PC} = 1^{++}$.
- No isospin partners found.

- Ordinary charmonium interpretation: most likely $\chi_{c1}(2P)$
- $m_{\chi_{c1}(3872)} = 3871.64 \pm 0.06$ MeV is very close to the $D^0 \bar{D}^{0*}$ threshold: $M_{th} = m(D^0) + m(\bar{D}^{0*}) = 3871.69 \pm 0.07$ MeV.



★ Isospin violation in decays: Non-conventional interpretation?

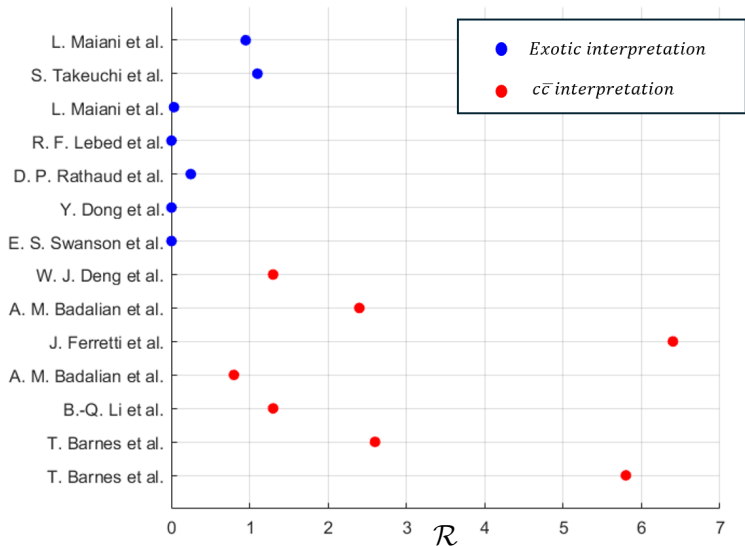
$$\frac{\mathcal{B}(\chi_{c1}(3872) \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(\chi_{c1}(3872) \rightarrow J/\psi \pi^+ \pi^-)} = \begin{cases} 1.0 \pm 0.4 \pm 0.3 & (\text{Belle}), \\ 0.8 \pm 0.3 \text{ } (1.7 \pm 1.3) & (\text{BaBar}), \\ 1.6^{+0.4}_{-0.3} \pm 0.2 & (\text{BESIII}). \end{cases}$$

- χ_{c1} as ordinary charmonium ($I = 0$):
 - Decay into ω conserves isospin ($I(\omega) = 0$).
 - Decay into ρ violates isospin ($I(\rho) = 1$).
 - Large phase space suppression of ω mode vs ρ mode.
 - LHCb collaboration analysis: ratio of isospin-violating to isospin-conserving $\chi_{c1}(3872)$ couplings: 0.29 ± 0.04 (six times larger than in isospin-violating decays of $\psi(2S)$).
- Molecular interpretation proposed: not well-defined isospin.
 - ★ Small production cross section disfavors the molecular model (being similar to the one expected for ordinary charmonium)

Isospin violation in $\chi_{c1}(3872)$ decays remains unresolved.

$$\mathcal{R} \equiv \frac{\Gamma(\chi_{c1}(3872) \rightarrow \psi(2S)\gamma)}{\Gamma(\chi_{c1}(3872) \rightarrow J/\psi\gamma)}$$

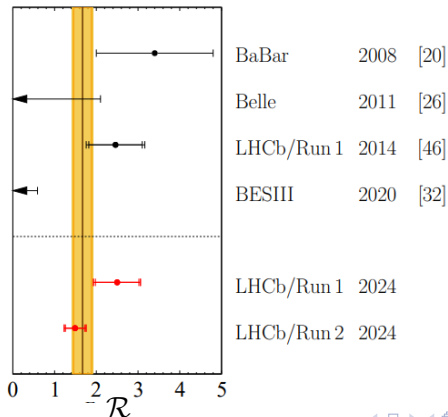
sensitive to the structure of $\chi_{c1}(3872)$



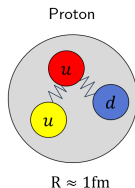
- Experimental results for the ratio $\mathcal{R}$:

LHCb Average 2024:

$$\mathcal{R} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04$$



- HQET: an effective theory of QCD valid in the large mass limit of heavy quarks (c, b).
- The hadron radius $R_{\text{had}} \approx 1 \text{ fm}$ corresponds to $\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$,



- For a heavy quark:

$$m_Q \gg \Lambda_{\text{QCD}} \implies \lambda_Q \ll R_{\text{had}}$$

- HQ acts like a static source of color field $\implies$ New symmetries arise.
- Remaining degrees of freedom have no sensitivity to HQ flavour or spin.

- QCD Lagrangian for a heavy quark:

$$\mathcal{L}_Q = \bar{Q}(i\not{D} - m_Q)Q,$$

- HQ nearly on-shell ($v^2 = 1$):

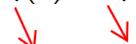
$$P_Q^\mu = m_Q v^\mu + k^\mu,$$

where $k \approx \mathcal{O}(\Lambda_{\text{QCD}})$.

- $Q(x)$ is redefined as:

$$Q(x) = e^{-im_Q v \cdot x} [h_v(x) + H_v(x)],$$

- $H_v(x)$ integrated out (EOM).


$$e^{im_Q v \cdot x} \frac{1 + \not{v}}{2} Q(x) \quad e^{im_Q v \cdot x} \frac{1 - \not{v}}{2} Q(x)$$

- The effective Lagrangian becomes:

$$\mathcal{L}_{\text{eff}} = \underbrace{\bar{h}_v i v \cdot D h_v}_{\mathcal{L}_{\text{HQET}}} + \underbrace{\frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v}_{\frac{\mathcal{L}_1}{2m_Q}} + \frac{1}{2m_Q} \underbrace{g \bar{h}_v \sigma_{\alpha\beta} G^{\alpha\beta} h_v}_{\text{CHROMOMAGNETIC OPERATOR}} + \mathcal{O}\left(\frac{1}{m_Q^2}\right),$$

KINETIC OPERATOR
 O_π Breaks flavour symmetry

CHROMOMAGNETIC OPERATOR
 O_G Breaks flavour symmetry and HQ spin symmetry

$\mathcal{L}_{\text{HQET}}$
 $\downarrow$
 No Dirac matrices $\Rightarrow$ HQ Spin invariance.
 No m_Q $\Rightarrow$ Flavour invariance.

- Due to these new symmetries, hadrons, which differ only for the orientation of SQ, can be collected in multiplets

- $\vec{J} = \vec{S} + \vec{L}$, for mesons: $S = \begin{matrix} \nearrow 0 \\ \searrow 1 \end{matrix}$

- e.g. with $L = 0$:

- $J = (0, 1) \implies$ Doublet: $J^P = (0^-, 1^-)$,
- Denoted as:

$$H_a = \frac{1 + \not{v}}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5] .$$

$\begin{matrix} \downarrow & \downarrow \\ 1^- & 0^- \\ \text{e.g. } D^* & D \end{matrix}$

Heavy Quarkonia Multiplets

- Identify states obtained rotating the spin of both HQs.
- The number of states in a multiplet depends on L.
- L=1 multiplet (P-wave):

$$J^\mu = \frac{1 + \not{v}}{2} \left\{ \overset{\chi_{Q2}}{H_2^{\mu\alpha}} \gamma_\alpha + \frac{1}{\sqrt{2}} \overset{\chi_{Q1}}{\epsilon^{\mu\alpha\beta\gamma}} v_\alpha \gamma_\beta H_{1\gamma} \right. \\ \left. + \frac{1}{\sqrt{3}} (\gamma^\mu - v^\mu) \underset{\chi_{Q0}}{H_0} + \underset{h_Q}{K_1^\mu} \gamma_5 \right\} \frac{1 - \not{v}}{2} ;$$

- L=0 multiplet (S-wave):

$$J = \frac{1 + \not{v}}{2} \left[\underset{\psi(\Upsilon)}{H_1^\mu} \gamma_\mu - \underset{\eta_Q}{H_0} \gamma_5 \right] \frac{1 - \not{v}}{2} .$$

Effective Lagrangians can be constructed in terms of the just presented multiplets:

- Radiative $nP \rightarrow mS$ transitions are described by:

$$\mathcal{L}_{nP \leftrightarrow mS} = \delta_Q^{nPmS} \text{Tr} [\bar{J}(m_S) J_\mu(n_P)] v_\nu F^{\mu\nu} + \text{h.c.} .$$

Hadron velocity

Electromagnetic field tensor:

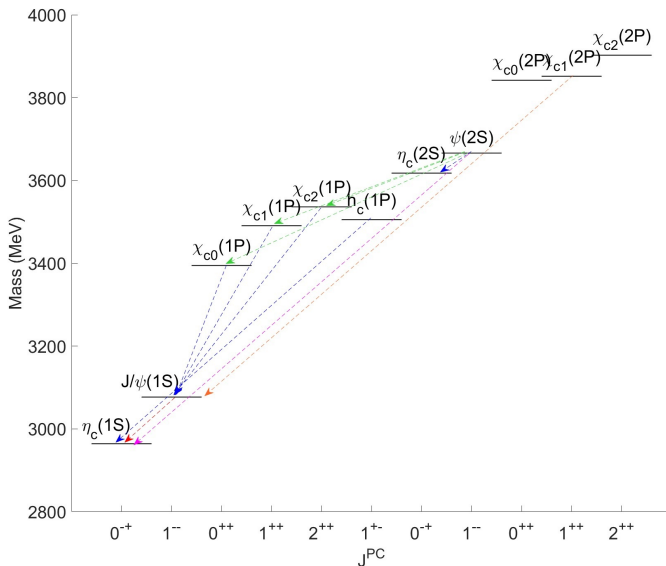
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

- Invariant under the symmetries:

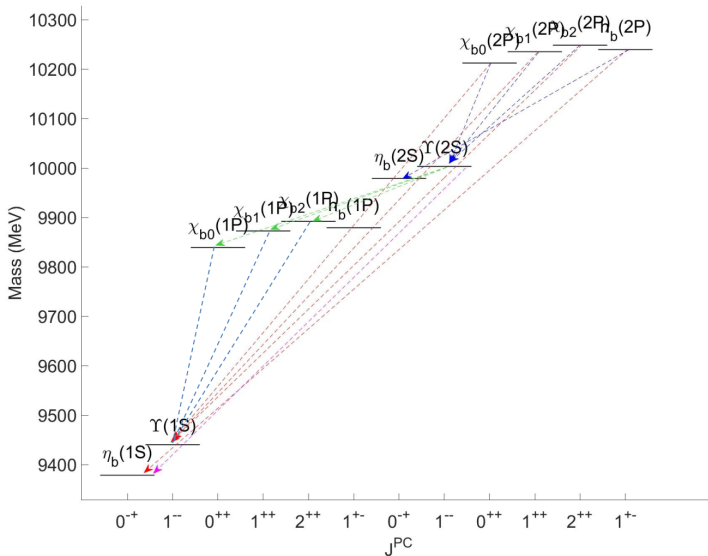
- C, P, T
- $SU(2)_{HQ}$

δ_Q^{nPmS} describes **all** the transitions among the members of the nP multiplet to the mS one.

Considered radiative decays



Considered radiative decays



I analyze the radiative decays:

Test HQ limit

- $(\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P), h_c(1P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \rightarrow (\eta_c(2S), \psi(2S)),$
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \rightarrow (\eta_c(2S), \psi(2S)).$

Coupling δ_Q^{nPmS} determined comparing HQ predictions to experimental data:

$$\begin{aligned}\Gamma(n^3P_J \rightarrow m^3S_1\gamma) &= \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}} \\ \Gamma(m^3S_1 \rightarrow n^3P_J\gamma) &= (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}} \\ \Gamma(n^1P_1 \rightarrow m^1S_0\gamma) &= \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}} \\ \Gamma(m^1S_0 \rightarrow n^1P_1\gamma) &= \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},\end{aligned}$$

Process	Branching ratio (Γ_i/Γ)
$J/\psi \rightarrow \gamma\eta_c(1S)$	$(1.41 \pm 0.14)\%$
$\chi_{c0}(1P) \rightarrow \gamma J/\psi$	$(1.41 \pm 0.09)\%$
$\chi_{c1}(1P) \rightarrow \gamma J/\psi$	$(34.3 \pm 1.3)\%$
$\chi_{c2}(1P) \rightarrow \gamma J/\psi$	$(19.5 \pm 0.8)\%$
$h_c(1P) \rightarrow \gamma\eta_c(1S)$	$(60 \pm 4)\%$
$\eta_c(2S) \rightarrow \gamma J/\psi$	$< 1.4\%$
$\psi(2S) \rightarrow \gamma\chi_{c0}(1P)$	$(9.77 \pm 0.23)\%$
$\psi(2S) \rightarrow \gamma\chi_{c1}(1P)$	$(9.75 \pm 0.27)\%$
$\psi(2S) \rightarrow \gamma\chi_{c2}(1P)$	$(9.36 \pm 0.23)\%$
$\psi(2S) \rightarrow \gamma\eta_c(1S)$	$(3.6 \pm 0.5) \times 10^{-3}$
$\psi(2S) \rightarrow \gamma\eta_c(2S)$	$(7 \pm 5) \times 10^{-4}$
$\chi_{c1}(3872) \rightarrow \gamma J/\psi$	$(7.8 \pm 2.9) \times 10^{-3}$

$$\Gamma(n^3P_J \rightarrow m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \rightarrow n^3P_J\gamma) = (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

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$$\delta_c^{1P1S} = (2.35 \pm 0.12) \times 10^{-1} \text{ GeV}^{-1}$$

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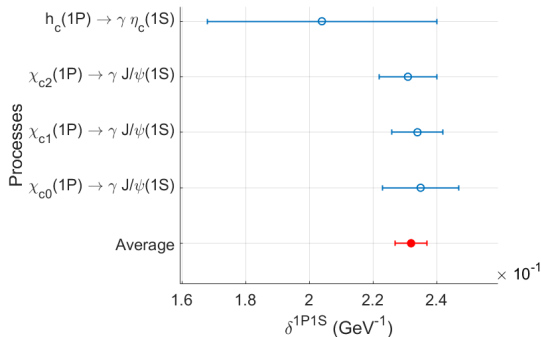
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$$\delta_c^{1P1S} = (2.00 \pm 0.40) \times 10^{-1} \text{ GeV}^{-1}$$



Average: $\delta_c^{1P1S} = (0.232 \pm 0.005) \text{ GeV}^{-1}$.

- Results fulfill the expectations based on HQ symmetries.

Goal: calculate ratio $\mathcal{R} \equiv \frac{\Gamma(\chi_{c1}(3872) \rightarrow \psi(2S)\gamma)}{\Gamma(\chi_{c1}(3872) \rightarrow J/\psi\gamma)}$

Identifying $\chi_{c1}(3872) \equiv \chi_{c1}(2P)$

I analyze the radiative decays:

- $(\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P), h_c(1P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{c0}(2P), \chi_{c1}(2P), \chi_{c2}(2P), h_c(2P)) \rightarrow (\eta_c(2S), \psi(2S)),$
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \rightarrow (\eta_c(1S), J/\psi),$
- $(\chi_{b0}(2P), \chi_{b1}(2P), \chi_{b2}(2P), h_b(2P)) \rightarrow (\eta_c(2S), \psi(2S)).$

Coupling δ_Q^{nPmS} determined comparing HQ predictions to experimental data:

- Predictions limited by incomplete knowledge of total widths for bottomonia states.
- Introducing:

$$\tilde{\delta}_b^{nPmS}(P_b) = \frac{\delta_b^{nPmS}}{[\Gamma_{\text{tot}}(P_b)]^{1/2}}, \quad P_b = \chi_{Q0}, \chi_{Q1}, \chi_{Q2}, h_Q$$

and

$$R_b^\delta(P_b) \equiv \frac{\delta_b^{2P2S}(P_b)}{\delta_b^{2P1S}(P_b)} = \frac{\tilde{\delta}_b^{2P2S}(P_b)}{\tilde{\delta}_b^{2P1S}(P_b)}$$

$$\Gamma(n^3P_J \rightarrow m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \rightarrow n^3P_J\gamma) = (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \rightarrow m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \rightarrow n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

Process	Branching ratio (Γ_i/Γ)
$\chi_{b0}(2P) \rightarrow \gamma\Upsilon(1S)$	$(3.8 \pm 1.7) \times 10^{-3}$
$\chi_{b0}(2P) \rightarrow \gamma\Upsilon(2S)$	$(1.38 \pm 0.30)\%$
$\chi_{b1}(2P) \rightarrow \gamma\Upsilon(1S)$	$(9.9 \pm 1.0)\%$
$\chi_{b1}(2P) \rightarrow \gamma\Upsilon(2S)$	$(18.1 \pm 1.9)\%$
$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \rightarrow \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \rightarrow \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b0}(2P)) = 13 \pm 4$$

$$\Gamma(n^3P_J \rightarrow m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \rightarrow n^3P_J\gamma) = (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \rightarrow m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

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$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \rightarrow \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \rightarrow \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b1}(2P)) = 8 \pm 1$$

$$\Gamma(n^3P_J \rightarrow m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \rightarrow n^3P_J\gamma) = (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \rightarrow m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \rightarrow n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

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$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(1S)$	$(6.6 \pm 0.8)\%$
$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \rightarrow \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \rightarrow \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(\chi_{b2}(2P)) = 7 \pm 1$$

Bottom production section

$$\Gamma(n^3P_J \rightarrow m^3S_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_1}}{M_{P_J}}$$

$$\Gamma(m^3S_1 \rightarrow n^3P_J\gamma) = (2J+1) \frac{(\delta_Q^{nPmS})^2}{9\pi} k_\gamma^3 \frac{M_{P_J}}{M_{S_1}}$$

$$\Gamma(n^1P_1 \rightarrow m^1S_0\gamma) = \frac{(\delta_Q^{nPmS})^2}{3\pi} k_\gamma^3 \frac{M_{S_0}}{M_{P_1}}$$

$$\Gamma(m^1S_0 \rightarrow n^1P_1\gamma) = \frac{(\delta_Q^{nPmS})^2}{\pi} k_\gamma^3 \frac{M_{P_1}}{M_{S_0}},$$

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$\chi_{b2}(2P) \rightarrow \gamma\Upsilon(2S)$	$(8.9 \pm 1.2)\%$
$h_b(2P) \rightarrow \gamma\eta_b(1S)$	$(22 \pm 5)\%$
$h_b(2P) \rightarrow \gamma\eta_b(2S)$	$(48 \pm 13)\%$

$$R_b^\delta(h_b(2P)) = 8 \pm 2$$

- Average result:

$$R_b^\delta = 9.0 \pm 0.7$$

Differences between $Q\bar{Q}$ and $Q\bar{q}$ symmetries:

- ✓ spin symmetry
- ✗ flavour symmetry $\implies \frac{1}{m_Q}$ suppression $\implies$ cancels in ratios
 $\implies R_c^\delta \approx R_b^\delta$.

- My Prediction for the ratio:

$$\mathcal{R}(\chi_{c1}(3872)) = \frac{\mathcal{B}(\chi_{c1}(3872) \rightarrow \psi(2S)\gamma)}{\mathcal{B}(\chi_{c1}(3872) \rightarrow J/\psi\gamma)} \propto (R_c^\delta)^2$$

$$\mathcal{R}(\chi_{c1}(3872)) = 1.7 \pm 0.3$$

Results



The predicted δ allows to find several other observables e.g.:

- Branching ratios:
 - $\mathcal{B}(\chi_{c2}(2P) \rightarrow J/\psi\gamma) = (3.1 \pm 1.5) 10^{-4}$
 - $\mathcal{B}(\chi_{c1}(3872) \rightarrow \psi(2S)\gamma) = (9.0 \pm 3.4) \times 10^{-3}$.
- The analogous $\mathcal{R}$ for $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$:
 - $\mathcal{R}(\chi_{c0}(2P)) = 1.5 \pm 0.2$
 - $\mathcal{R}(\chi_{c2}(2P)) = 2.9 \pm 0.4$

- Large HQ mass limit: HQ spin and HQ flavour symmetries.
- $\chi_{c1}(3872)$:
 - many observables, many puzzles, many interpretations.
- Under the assumption $\chi_{c1}(3872) \equiv \chi_{c1}(2P) \implies$ the prediction follows:

$$\mathcal{R} = \frac{\mathcal{B}(\chi_{c1}(3872) \rightarrow \psi(2S)\gamma)}{\mathcal{B}(\chi_{c1}(3872) \rightarrow J/\psi\gamma)} = 1.7 \pm 0.3$$

- Experimental LHCb 2024 average:

$$\mathcal{R} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04.$$

This agreement supports the assumed identification.

- Open questions:
 - Size of isospin-breaking in $\chi_{c1}(3872)$ decays.
 - Understanding production cross sections.
- Observation of missing conventional states preliminary and necessary step.
- Predictions for unobserved radiative decays to be compared to experimental data still to come → further test of the method and the results.

Thank You!

Further predictions are possible by constructing an $n\text{SmS}$ interaction Lagrangian:

- from the decay $J/\psi \rightarrow \eta_c(1S)\gamma$:
 $\delta_c^{1S1S} = 0.0485 \pm 0.0024 \text{ GeV}^{-1}$
- from the decay $\psi(2S) \rightarrow \eta_c(1S)\gamma$:
 $\delta_c^{2S1S} = (3.42 \pm 0.26) \times 10^{-3} \text{ GeV}^{-1}$
- from the decay $\psi(2S) \rightarrow \eta_c(2S)\gamma$:
 $\delta_c^{2S2S} = (0.66 \pm 0.24) \times 10^{-1} \text{ GeV}^{-1}$
- from the decay $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$:
 $\delta_b^{2S1S} = (4.50 \pm 0.05) \times 10^{-4} \text{ GeV}^{-1}$.

The large number of predictions confirms the important role of the effective theory approach.

- Degeneracy among spin multiplet members is broken by the chromomagnetic operator.
- Effective Lagrangian for spin symmetry breaking:

$$\mathcal{L}^{SpSB} = \frac{\lambda_J}{2m_Q} \frac{1}{\mathcal{N}} \text{Tr}[\bar{J}_{\mu_1 \dots \mu_n} \sigma_{\alpha\beta} J_{\nu_1 \dots \nu_n} \Phi^{\mu_1 \dots \mu_n \alpha\beta \nu_1 \dots \nu_n}]$$

with normalization $\mathcal{N} = \text{Tr}[\bar{J}_{\mu_1 \dots \mu_n} J^{\mu_1 \dots \mu_n}]$.

- Function Φ ensures invariance under P, C, T and satisfies antisymmetry of $\sigma_{\alpha\beta}$.
- For S-wave states, hyperfine splitting arises:

$$\mathcal{L}_S^{SpSB} = \frac{\lambda_S}{2m_Q} \frac{1}{\mathcal{N}} \text{Tr}[\bar{J} \sigma_{\alpha\beta} J \sigma^{\alpha\beta}]$$

with spin, parity, and charge quantum numbers $J^{PC} = (0^{-+}, 1^{--})$.

- Mass formula for S-wave states:

$$m_H = m_Q + \bar{\Lambda}_S + \frac{-\lambda_1 + d_H \lambda_S}{2m_Q}$$

- For P-wave states:

$$\mathcal{L}_P^{SpSB} = \frac{\lambda_P}{2m_Q} \frac{1}{\mathcal{N}} \text{Tr}[\bar{J}_\mu \sigma_{\alpha\beta} J_\nu \Phi^{\alpha\beta\mu\nu}]$$

- P-wave multiplet states $J^{PC} = (0^{++}, 1^{++}, 2^{++}, 1^{+-})$:

$$m_H = m_Q + \bar{\Lambda}_P + \frac{-\lambda_1}{2m_Q} + \frac{d_H^{(1)}\lambda_P^{(1)} + d_H^{(2)}\lambda_P^{(2)} + d_H^{(3)}\lambda_P^{(3)}}{2m_Q}$$

- Parameters for mass splitting:

$$\lambda_P^{(1)} = \frac{1}{24}(-m_0^2 + 3m_1^2 + m_2^2 - 3m_h^2), \quad \lambda_P^{(2)} = \frac{1}{24}(-2m_0^2 - 3m_1^2 + 5m_2^2),$$

$$\lambda_P^{(3)} = \frac{1}{24}(-2m_0^2 + 3m_1^2 - m_2^2)$$

where m_0, m_1, m_2, m_h are masses of states in the multiplet.

- Recent discoveries of exotic hadrons include:
 - $P_c(4380)^+$, $P_c(4440)^+$, $P_c(4457)^+$, and $P_c(4312)^+$, observed in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays.
 - $X(6900)$ in $J/\psi J/\psi$ final states.
 - $X_0(2900)$ and $X_1(2900)$ in $B^+ \rightarrow D^+ D^- K^+$.
 - $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ in $B^+ \rightarrow J/\psi \phi K^+$.
 - $T_{cc}(3875)^+$ in $D^0 D^0 \pi^+$.

Multiplets transform under the discrete symmetries C, P, and T:

$$J^{\mu_1 \dots \mu_k} \xrightarrow{P} \gamma^0 J_{\mu_1 \dots \mu_k} \gamma^0$$

$$J^{\mu_1 \dots \mu_k} \xrightarrow{C} \hat{C} (J^{\mu_1 \dots \mu_k})^T \hat{C}^\dagger (-1)^L$$

$$J^{\mu_1 \dots \mu_k} \xrightarrow{T} -\hat{T} J_{\mu_1 \dots \mu_k} \hat{T}^{-1}$$

$$\hat{C} = i\gamma^2\gamma^0 \text{ and } T = i\gamma^1\gamma^3.$$

The electromagnetic field strength tensor transforms as:

$$F^{\mu\nu} \xrightarrow{P} F_{\mu\nu}, \quad F^{\mu\nu} \xrightarrow{C} -(F^{\mu\nu}), \quad F^{\mu\nu} \xrightarrow{T} -F_{\mu\nu}.$$