Auto-calibration from response function

BULLKID-DM Collaboration Meeting
Ferrara, 1-2 July 2025

Davide Quaranta, 1st July 2025

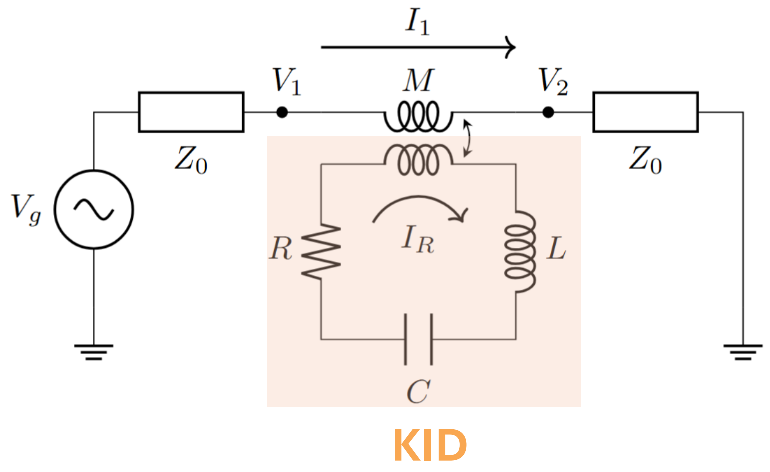
Basics of KID readout

- The key observable of a KID coupled to a transmission line is the **complex transmission function**

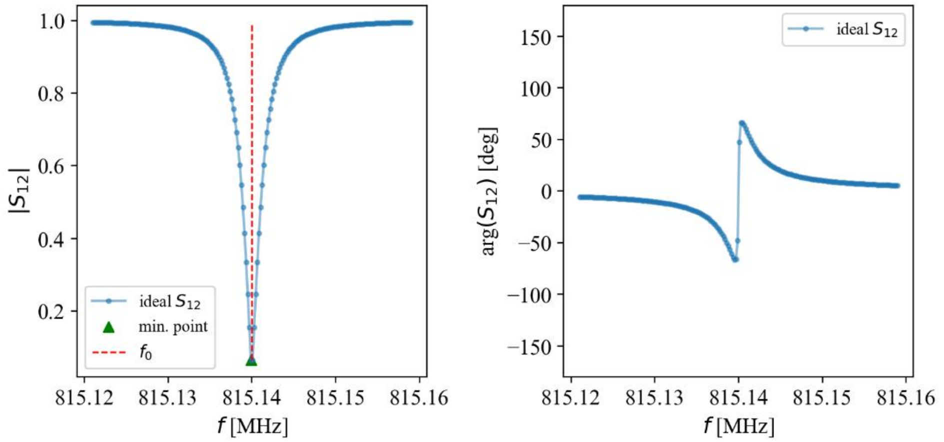
$$S_{21}(f) = \frac{V_2^-}{V_1^+} = 1 - \frac{Q}{Q_c}\frac{1}{1+2jQx}$$

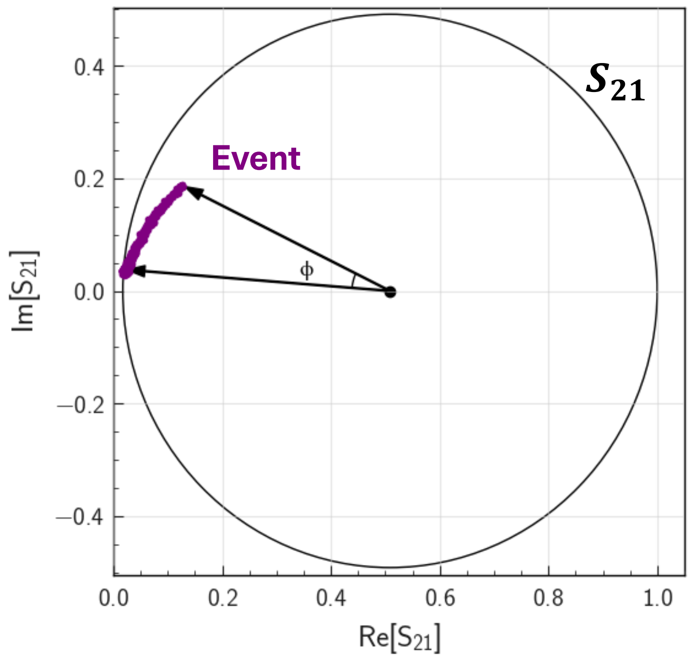
Fractional Frequency Shift

$$x = \frac{f - f_0}{f_0}$$



- As a function of x, S_{21} describes a **circle** in the complex plane:
  → ***VNA scan*** : f *variable,* f_0 *fixed*
  → ***Acquisition*** : f_0 *variable,* f *fixed*





BULLKID-DM KID Readout

- Event amplitude reconstructed using **calibrated phase**
  - → *Central angle from the working point*

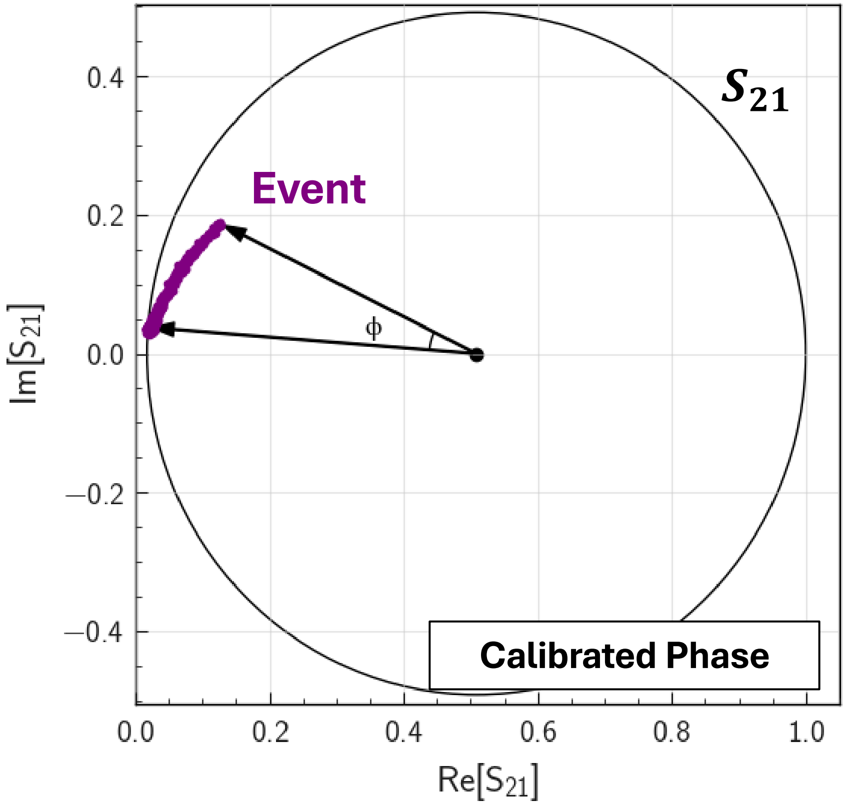
- Using the BCS theory, the phase responsivity is:

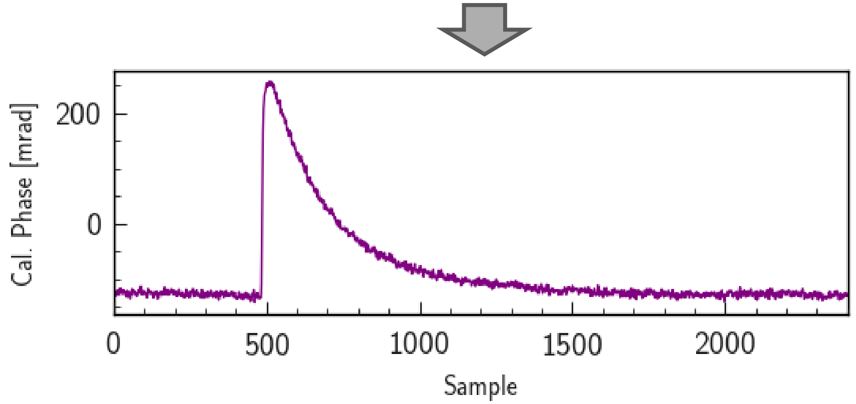
$$\frac{d\phi}{dE} = \boxed{\frac{\eta\alpha S_2(\omega_0, T)}{N_0 V \Delta^2}} \cdot \boxed{\frac{Q}{1+4y^2}} \qquad y = Qx$$

Material properties — Resonator Properties

- However, this quantity presents few problems:
  - → *Resonator-specific (Q dependance)*
  - → *Non-linearity (y dependance)*
  - → *Power-dependence (non-linearity of L_k)* → $y = y_0 \cdot z + \frac{a_0 z^3}{1+4y^2}$

L.J. Swenson et al, J. Appl. Phys. 113, 104501 (2013)
M. Vignati et al, arXiv:2102.09431





Key Idea
Use response function to build a variable which has equal responsivity among KIDs.

Fractional Frequency Shift

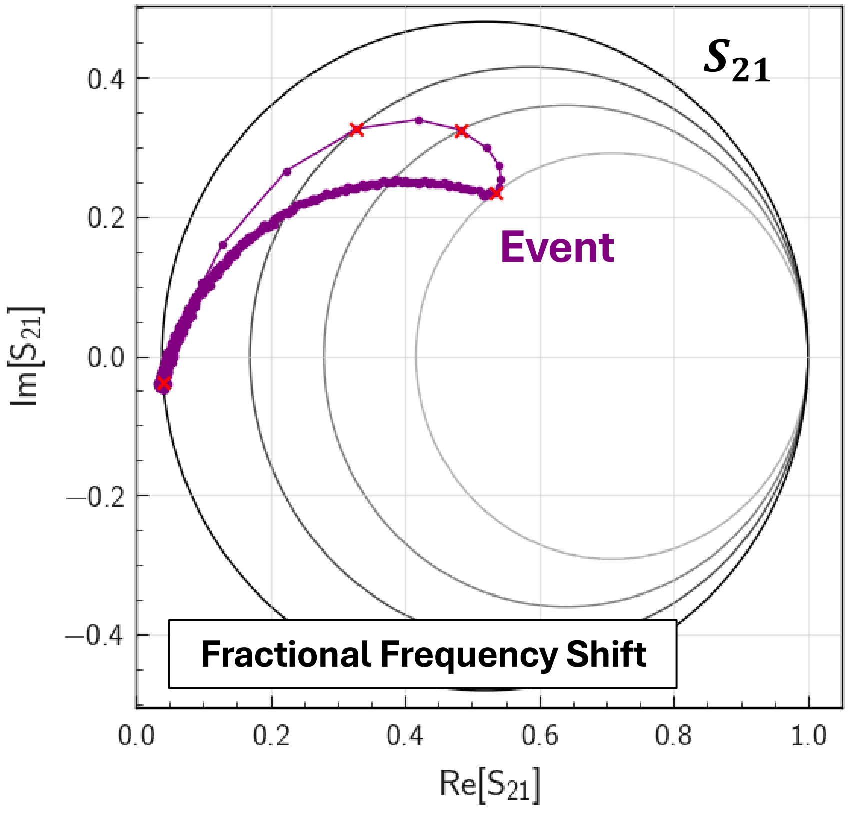
- The responsivity in **Fractional Frequency Shift** is given by:

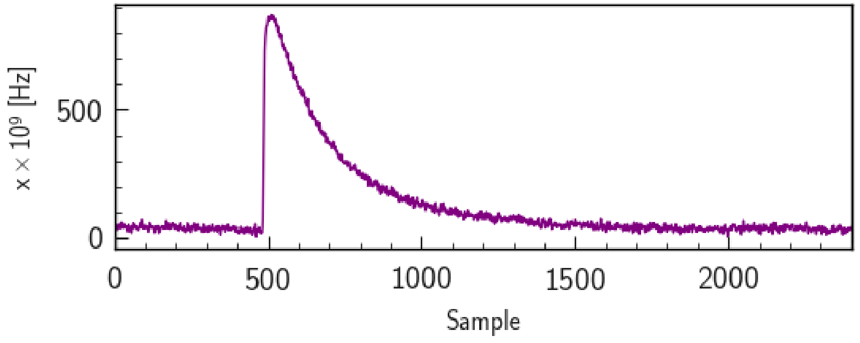
$$\frac{dx}{dE} = \eta \frac{\alpha S_2(\omega_0, T)}{4 N_0 V \Delta_0^2}$$

«No» resonator-specific dependence

How do we measure x?

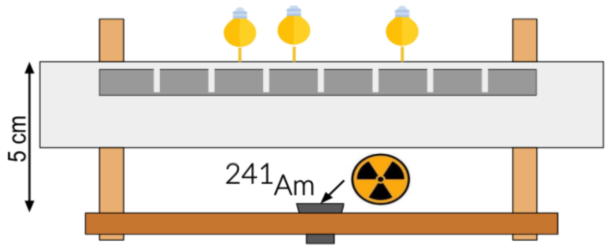
1. Re-compute Q at each sample $\quad Q = 2Q_c R \quad (R = \text{radius})$
2. Compute y from S21 $\quad y = \frac{Q}{2jQ_c(1 - S_{21})} + \frac{j}{2}$
3. Correct y for bias power $\quad y_0 = \frac{y}{z} - \frac{a_0 z^2}{1 + 4y^2}$
4. Compute x $\quad x = \frac{y_0}{Q}$

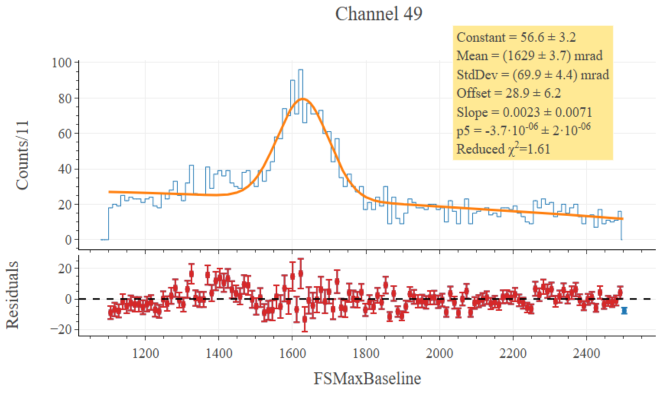


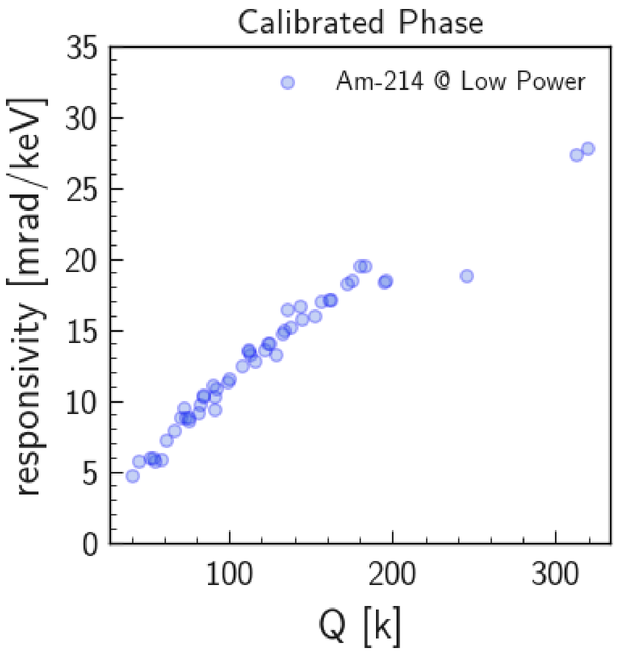


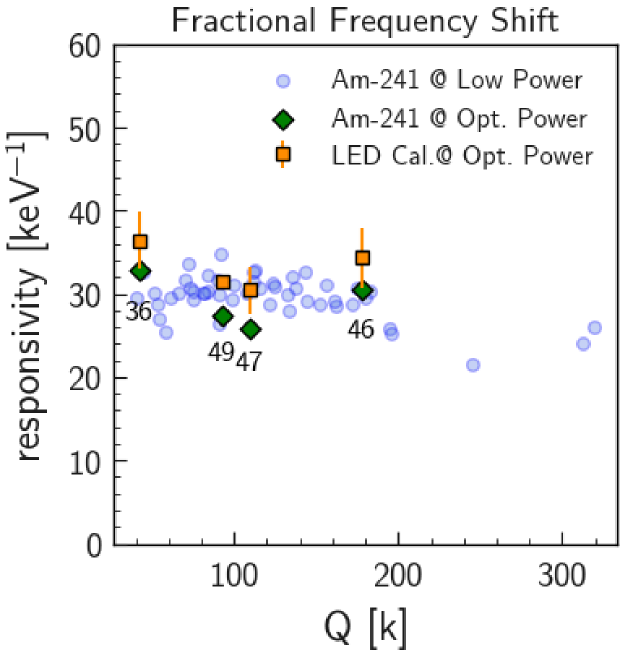
Whole Wafer Test with Am-241

- STACK-04 was illuminated with an **Am-241 source** placed just below internal Aluminum Pot
- 3 low-power runs to acquire whole wafer (<u>50 pixels</u>)
- Responsivity from gaussian fit of Am-241 **60 keV peak**
  - → *Thus no estimation of non-linearity*









- Double-checks done on 4 selected channels:
  - → ***Am-241 @ Optimum Power***
  - → ***Usual LED Calibration***

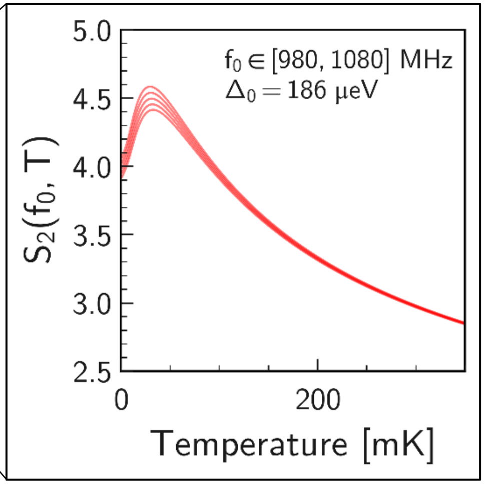
- <u>Efficient KID response equalization</u> was observed among all dice under test:
  - → *average responsivity* : **29.7 keV^{-1}**
  - → *better then* **10% agreement**

Effect of KID heating

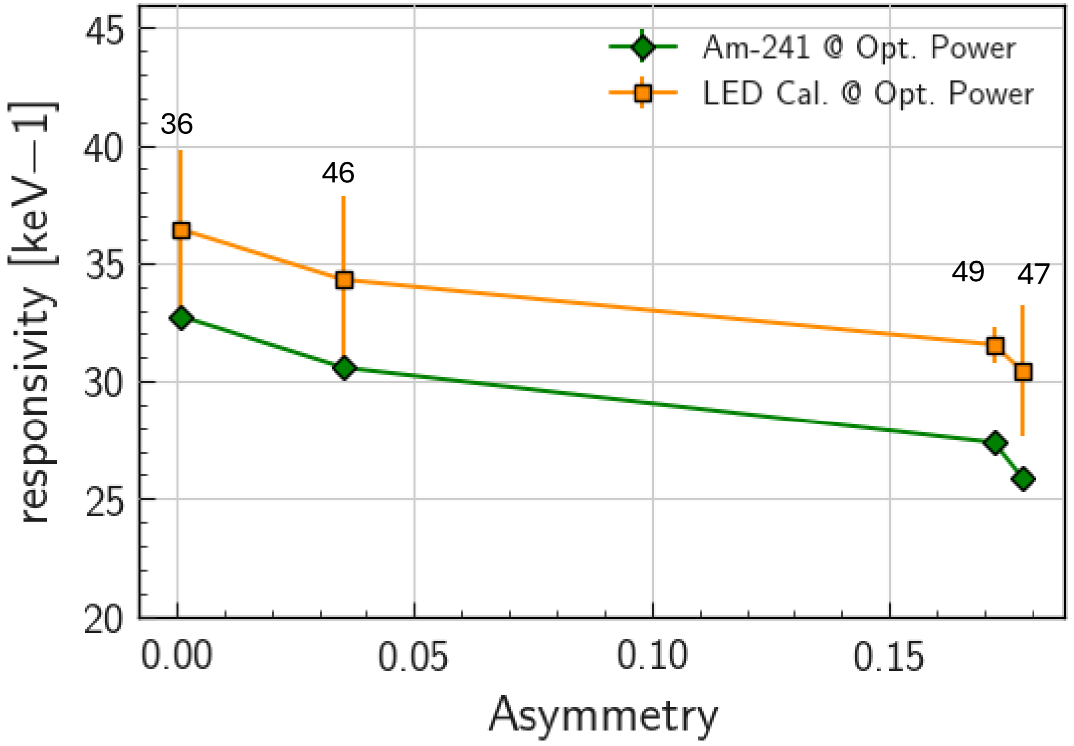
- Responsivity decreases with increasing asymmetry (a)
  - → *Observed on the 4 selected channels*
- Ascribed to Power-Induced Quasiparticles Creation
  - → *Readout wave energy breaks Cooper pairs*
  - → *Leads to increased quasiparticle density*
- Effect equivalent to a temperature increase
  - → *Effectively "heats" the KID*

  J.Gao et al, J Low Temp Phys (2008)
- $S_2(T)$ decreases with temperature
  - → *Responsivity diminishes*

$$\frac{dx}{dE} = \eta \frac{\alpha \boxed{S_2(\omega_0, T)}}{4 N_0 V \Delta_0^2}$$



$$a = \frac{2Q^2}{Q_c} \frac{P_g}{\omega_r E_*}$$



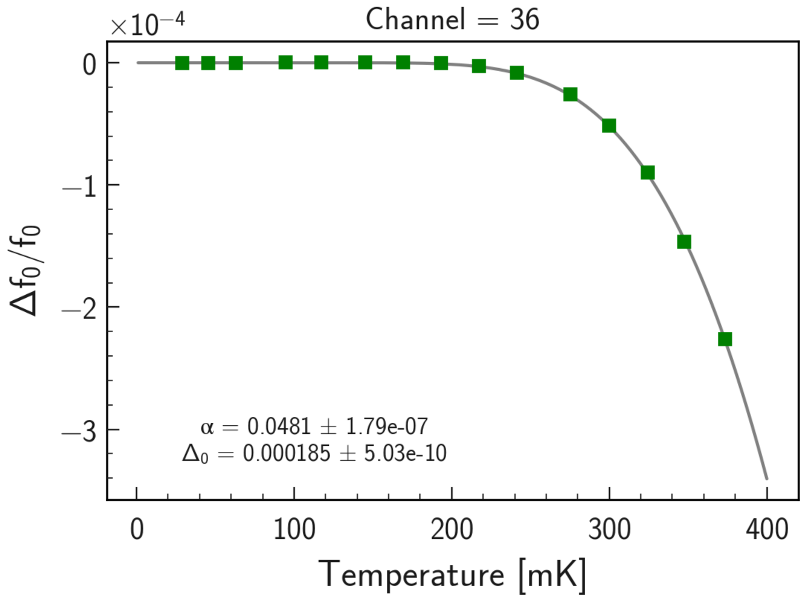
Effect of KID heating

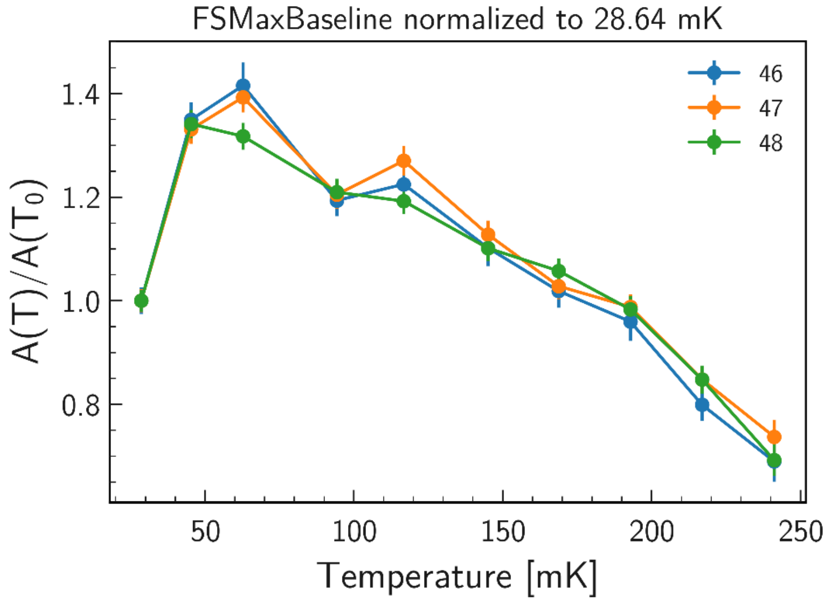
- We performed a temperature scan between 35 and 375 mK
  - → *At each T, measure f_0* shift
- LED pulse fired at each temperature
  - → *Intermediate energy, around 250 mrad*
- Compute amplitude ratio w.r.t base temperature
  - → *Obtain ratio of S_2*
- Express responsivities w.r.t base temperature

$$\frac{A_{LED}(T)}{A_{LED}(T_0)} = \frac{r(T)}{r(T_0)} = \frac{S_2(f_0,T)}{S_2(f_0,T_0)}$$

$$\Downarrow$$

$$r(T_0) = r(T)\left(\frac{A_{LED}(T)}{A_{LED}(T_0)}\right)^{-1}$$





Effect of KID heating

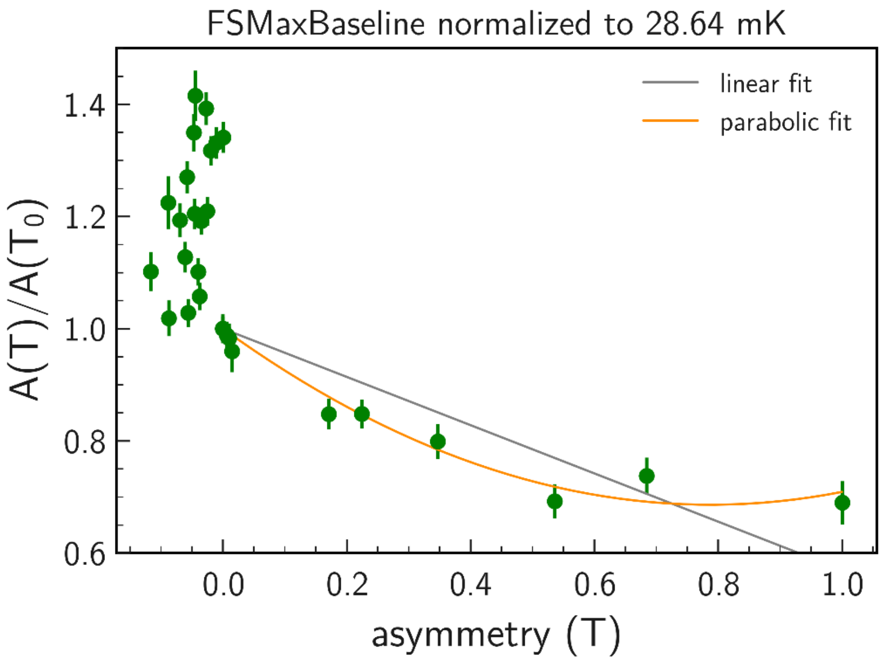
- At each T, compute the value of the "asymmetry"
  - → *To do this we use:*

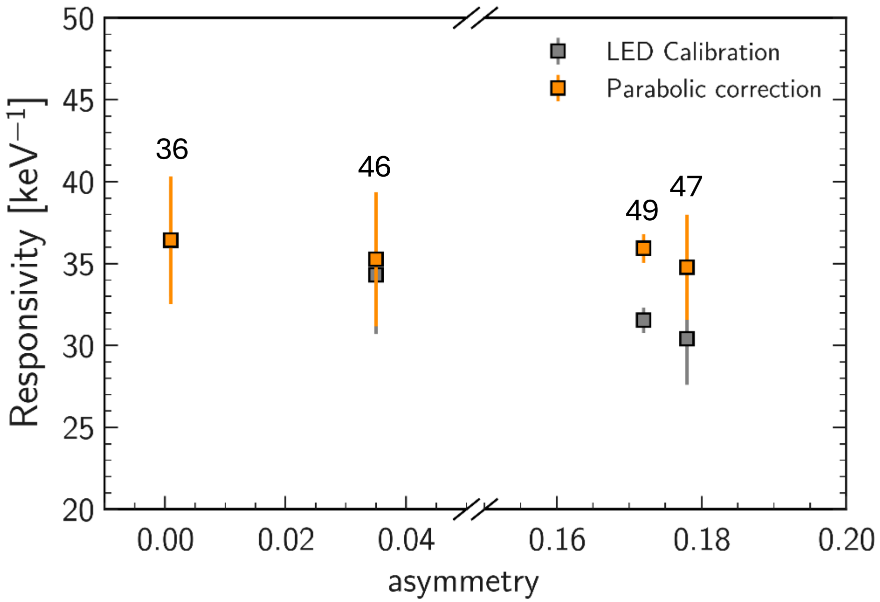
$$a(T) = -y_0|_{min} = -Q\frac{\Delta f_0}{f_0}(T)$$

- Fit the <u>amplitude ratio vs asymmetry plot</u>
  - → *simple parabola in absence of a model*

- Apply the correction to the previous responsivities
  - → *Given the asymmetry from the circle fit*

- The correction seems to go in the correct direction
  - → *Improvement of responsivity uniformity*





Conclusions

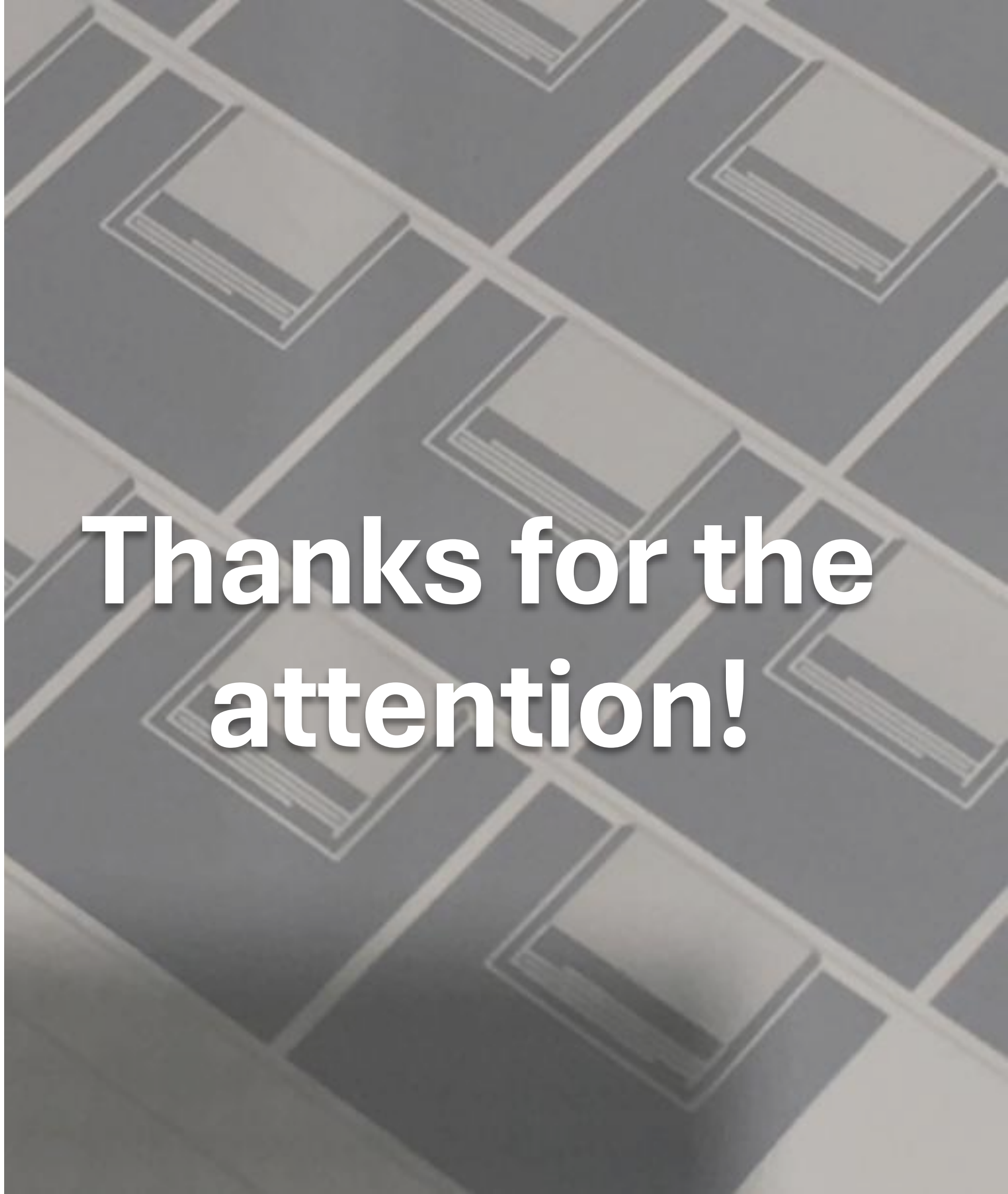
- We developed an algorithm to reconstruct the **Fractional Frequency Shift** of events in the BULLKID detector

- In contrast with Calibrated Phase, this variable allows to obtain **similar responsitivities** between different KIDs

- We demonstrated that the knowledge of the response function allows equalization of KID response down to **< 10% level**

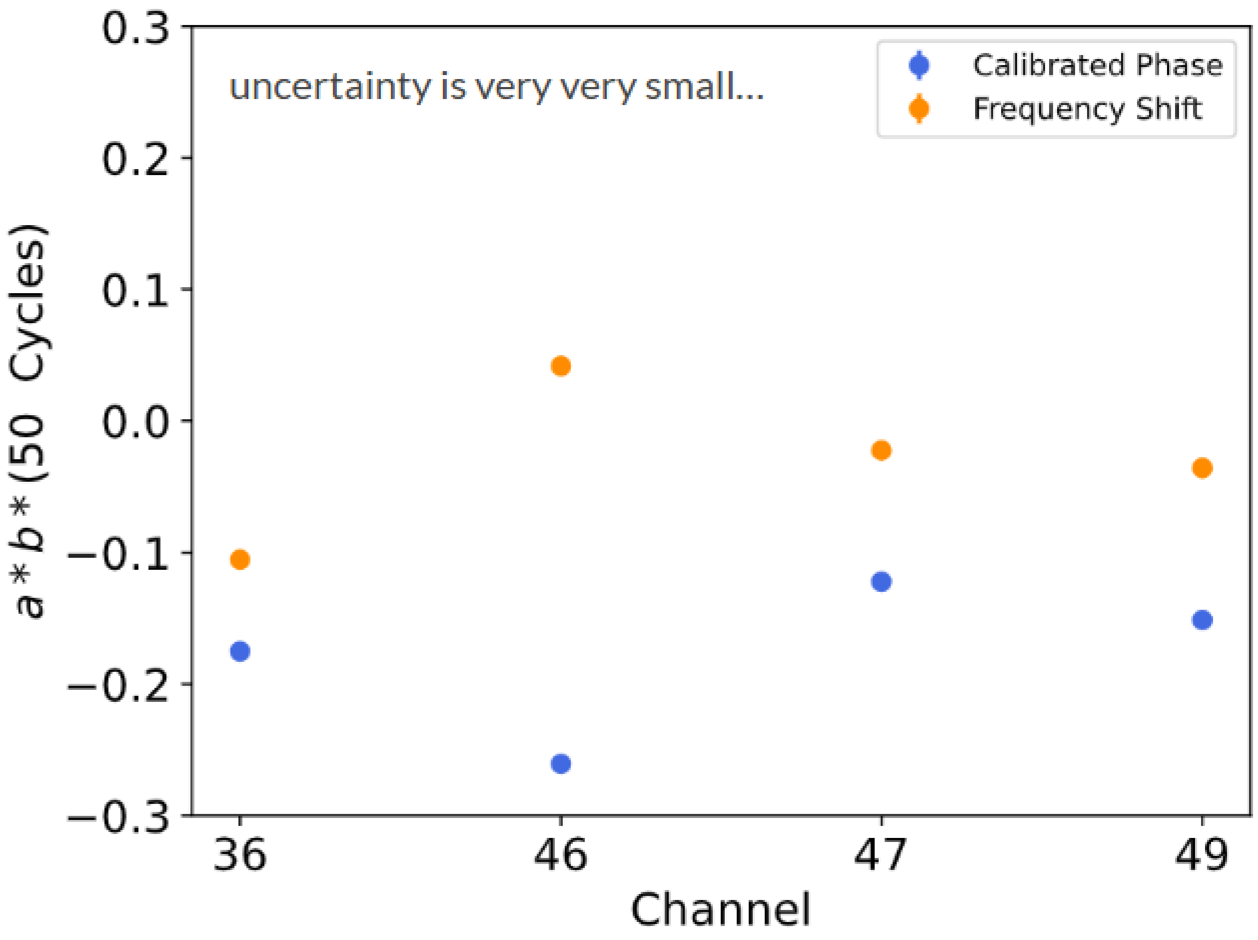
- Residual dependence of bias power has been accounted for exploiting the equivalence of temperature change and external pair breaking

- Any suggestion or comment on the presented procedure is greatly appreciated!



Backup Slides

Non-linearity



- To compare non-linearity of CP and FS:
  - Fit to : a*Cycles*(1+b*a*Cycles)
  - Compute b*a*Cycles @ 50 Cycles