The $VP\gamma^*$ form factor parametrizations
(arXiv: 1111.1291 + ...)

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April 17, 2012

Outline

Objectives

1. Introduce the modern status of the $VP\gamma^*$ transitions
   - $V \to P\gamma$
   - $V \to P\gamma^*$
2. Collect the different model predictions for the distribution shapes and compare them to data

   we will focus on $\omega \to \pi\gamma^*$

Definitions (form factor)

$VP\gamma^*$ transition amplitude

$$\mathcal{M}(V^{(\alpha)}\ P(k)\ \gamma^{*(\beta)}(q)) \propto e\ \mathcal{F}_{VP\gamma^*}(q^2)\ \varepsilon^{\mu\nu\alpha\beta} q_\mu k_\nu$$

$\varepsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol
q and k are 4-momenta of the photon and the pseudoscalar meson
q^2 is 4-momentum squared of the photon

transition form factor: $\mathcal{F}_{VP\gamma^*}(q^2)$

Radiative decays

Normalization at $q^2 = 0$

$$\Gamma(V \to P\gamma) \ = \ |\mathcal{F}_{VP\gamma^*}(0)|^2 \frac{e^2}{96\pi} \left(\frac{M_V^2 - m_P^2}{M_V} \right)^3$$

e.g., $\rho^0 \to \pi^0\gamma$, $\omega \to \pi^0\gamma$, $\phi \to \eta\gamma$

The radiative decays $V \to P\gamma$ are widely used as electromagnetic probes of the flavor contents of the mesons, and give important information on the isospin–breaking, G-parity–breaking and OZI–breaking in the light meson sector.

Radiative decays – some references

[O'Donnell, Rev.Mod.Phys. 53 (1981) 673]
[Landsberg, Phys.Rep. 128 (1985) 301]
[Kaiser, Meissner, Nucl.Phys. A519 (1990) 671]
[Prades, Z.Phys. C63 (1994) 491, EPJ C11 (1999) 571]
[Bramon, Grau, Pancheri, Phys.Lett. B344 (1995) 240]
[Hashimoto, Phys.Lett. B381 (1996) 465]
[Hashimoto, Phys.Rev. D54 (1996) 5611]
[Bramon, Escribano, Scadron, Phys.Lett. B503 (2001) 271]
[Napsuciale, Rodriguez, Alvarado-Anell, PR D67 (2003) 036007]
[Escribano, Frere, JHEP 0506 (2005) 029]
[Benayoun et al., Eur.Phys.J. C55 (2008) 199]
[Benayoun et al., Eur.Phys.J. C65 (2010) 211]
[Pham, Phys.Lett. B694 (2010) 129]
and many other papers . . .

Radiative decays – status

$\Gamma(V \to P\gamma)$ — Experiment

- The data are available for miscellaneous decay modes [PDG]

$\Gamma(V \to P\gamma)$ — Theory

- State of the art: simultaneous fit with a reasonable quality

The overall picture of $\Gamma(V \to P\gamma)$ is good

Yet another story is the $V \to P\gamma^*$ with the virtual photon.

Conversion decays

The decay line shape

$$\frac{d\,\Gamma(V \to P\mu^+\mu^-)}{d\,Q^2} = \frac{\alpha}{3\pi}\frac{\Gamma(V \to P\mu^+\mu^-)}{Q^2}\left(1+\frac{2m_\mu^2}{Q^2}\right)\sqrt{1-\frac{4m_\mu^2}{Q^2}}$$

$$\times\left(\left(1+\frac{Q^2}{M_V^2-m_P^2}\right)^2-\frac{4M_V^2Q^2}{(M_V^2-m_P)^2}\right)^{3/2}\left|F(Q^2)\right|^2$$

[L.G. Landsberg et al., Phys.Rep. 128 (1985) 301]

$M_{\mu^+\mu^-} \equiv M_{\gamma^*} \equiv \sqrt{Q^2}$

The conversion decays $V \to P\gamma^*(\to Pl^+l^-)$ are yet another and complimentary electromagnetic probes of mesons.

Conversion decays – some references

Models

HLS	[Hashimoto, Phys.Rev. D54 (1996) 5611]
ENJL	[Prades, Z.Phys. C63 (1994) 491, EPJ C11 (1999) 571]
EVMD	[Faessler et al., Phys.Rev., C61, 035206 (2000)]
DSE	[Maris and Tandy, Phys.Rev., C65, 045211 (2002)]
LCQM	[Qian and Ma, Phys.Rev, D78, 074002 (2008)]
EFT	[Terschlüsen and Leupold, Phys.Lett., B691, 191 (2010)]
EFT	[Terschlüsen and Leupold, Prog.Part.Nucl.Phys. 67 (2012) 401]
EFT	[Ivashyn, Prob.Atom.Sci.Tech. 2012N1 (2012) 179]

Conversion decays – Experiment

Form factors

Lepton-G (Serpukhov) $\omega \to \pi\mu^+\mu^-$

[R.Dzhelyadin et al., Phys.Lett., B102 (1981) 296]

NA60 (CERN) $\omega \to \pi\mu^+\mu^-$

[R.Arnaldi et al., Phys.Lett., B677 (2009) 260]

[G.Usai Nucl.Phys., A855 (2011) 189]

CMD-2 (Novosibirsk) $\omega \to \pi e^+e^-$

[Akhmetshin et al., Phys.Lett., B613 (2005) 29]

SND (Novosibirsk) $\phi \to \eta e^+e^-$

[Achasov et al., Phys.Lett., B504 (2001) 275]

SND (Novosibirsk) $\omega \to \pi e^+e^-$

[Achasov et al., JETP, 107 (2008) 61]

KLOE (Frascati) $\phi \to \eta e^+e^-$

[first results on the slope — **today** by J.Zdebik at QNP2012 in Paris]

Conversion decays – Experiment

Experimental prospects and plans

Novosibirsk

[Obrazovsky, Nucl.Phys.Proc.Suppl. 181-182 (2008) 243]

Frascati

[Amelino-Camelia et al., Eur.Phys.J. C68 (2010) 619]

Jülich

[A.Kupść, private communication]

. . . ?

"Traditional VMD"

$$F^{VMD}_{\omega\to\pi\gamma^*}(Q^2) \;=\; \frac{M_V^2}{M_V^2 - Q^2}$$

The *normalized* form factor is controlled by only one parameter:
$M_V = M_\rho = 775.5$ MeV.

An attempt to fit the NA60 data gives $M_V \approx 668$ MeV $\neq M_\rho$
[G.Usai [NA60 Collaboration]. Nucl. Phys. A, 855 (2011) 189]
This indicates that the data and the model are not compatible.

"Traditional VMD" $\equiv$ VMD 2
[nomenclature: O'Connell et al. Prog.Part.Nucl.Phys. 39 (1997) 201]

“Traditional VMD”

$$F^{VMD}_{\omega\to\pi\gamma^*}(Q^2) \;=\; \frac{M_V^2}{M_V^2 - Q^2}$$

The *normalized* form factor is controlled by only one parameter: $M_V = M_\rho = 775.5$ MeV.

The asymptotic limit is satisfied automatically
$F^{VMD}(Q^2 \to -\infty) = 0$

“Traditional VMD” $\equiv$ VMD 2

[nomenclature: O’Connell et al. Prog.Part.Nucl.Phys. 39 (1997) 201]

EFT: Terschlussen and Leupold

[Terschlüsen and Leupold, Phys.Lett., B691, 191 (2010)]
[Terschlüsen and Leupold, Prog.Part.Nucl.Phys. 67 (2012) 401]

$$F^{TL}_{\omega\to\pi\gamma^*}(Q^2) = (1 - g_{\omega\pi^0}) + g_{\omega\pi^0}\frac{M_\rho^2}{M_\rho^2 - Q^2}$$
$$= 1 + g_{\omega\pi^0}\frac{Q^2}{M_\rho^2 - Q^2}$$

$g_{\omega\pi^0} \approx 2.01$ from other processes

The asymptotic limit is violated due to chosen values of the constants
$F^{TL}(Q^2 \to -\infty) = 1 - g_{\omega\pi^0} \approx -1.01$

"Terschlussen, Leupold" $\equiv$ VMD 1

[nomenclature: O'Connell et al. Prog.Part.Nucl.Phys. 39 (1997) 201]

EFT: Eidelman et al.

The formulae of [Eidelman et al., Eur.Phys.J.C69, 103 (2010)] can be rewritten in the form

$$F^E_{\omega\to\pi\gamma^*}(Q^2) \;=\; (1-(1+\alpha)g_{\omega\pi^0}) + g_{\omega\pi^0}\left(\frac{M_\rho^2}{M_\rho^2-Q^2}+\alpha\frac{M_\phi^2}{M_\phi^2-Q^2}\right)$$

$g_{\omega\pi^0} \approx 1$
$\alpha \approx 0.001$ from the $\rho-\phi$ mixing data

The asymptotic limit in this approach is violated to a very small extent due to a “rough” G-parity breaking effect inclusion
$F^E(Q^2\to-\infty) = 1 - g_{\omega\pi^0} - \alpha g_{\omega\pi^0} \approx \alpha \approx -0.001$

“Eidelman et al.” $\approx$ VMD 1, $\approx$ VMD 2

EFT-fit, one octet

$$F^{EFT,\ fit\ 1}_{\omega\to\pi\gamma^*}(Q^2) \;=\; (1-g_{\omega\pi^0}) + g_{\omega\pi^0}\frac{M_\rho^2}{M_\rho^2-Q^2}$$

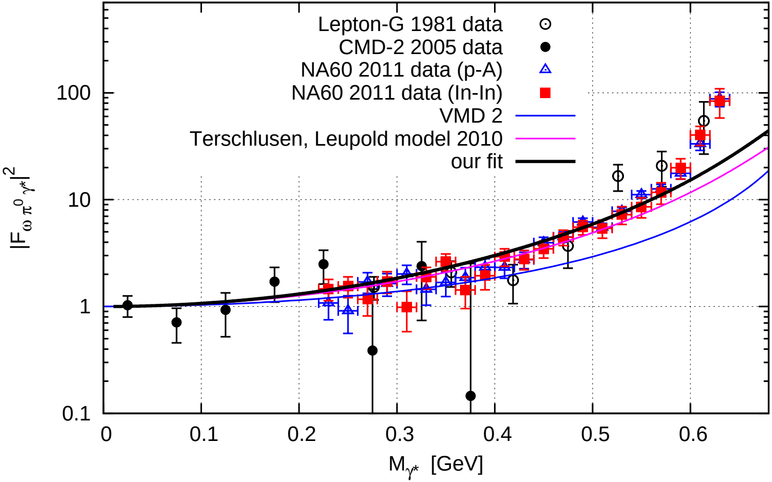
$g_{\omega\pi^0} = 1.705$ from fit to NA 60 data

[Ivashyn, Prob.Atom.Sci.Tech. 2012N1 (2012) 179]

The asymptotic limit is violated:
$F^{EFT,\ fit\ 1}(Q^2\to-\infty) = 1 - g_{\omega\pi^0} \approx -0.705$

"EFT-fit 1" $\equiv$ VMD 1

$\omega\pi^0$ form factor — one octet models



- unavoidable big discrepancy in the region of high M_{γ^*}
- failure of a “traditional” vector meson dominance (VMD 2)

Extended VMD (multiplicative)

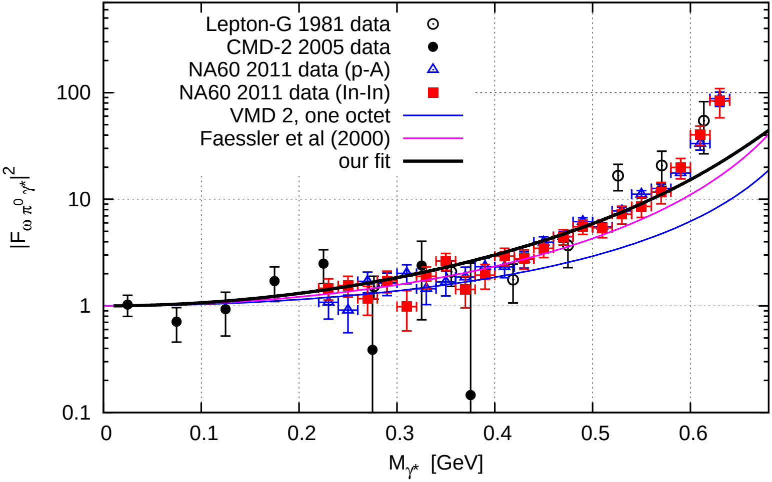
[Faessler et al., Phys.Rev., C61, 035206 (2000)]

$$F^F_{\omega\to\pi\gamma^*}(Q^2) = \frac{M_\rho^2 M_X^2}{(M_\rho^2 - Q^2)(M_X^2 - Q^2)}$$

$M_X = 1.2$ GeV
from fitting the slope of [Landsberg, Phys.Rep. 128 (1985) 301]

The asymptotic limit is automatically satisfied
$F^F(Q^2 \to -\infty) = 0$

$\omega\pi^0$ form factor — Extended VMD



- Much better than a one-octet VMD 2
- The agreement with data is still bad

Summary

- Status of $V \to P\gamma^*$ is not satisfactory
- Most of the $V \to P\gamma^*$ transition models give the form factor parametrizations equivalent to VMD

  Remark: there are two types of VMD

More models, new fits, form factor slope,…
$\Rightarrow$ at the “Transition form factors workshop” (Krakow)

- New precise data on $V \to P\gamma^*(\to P l^+ l^-)$ are welcome
- Any chance to measure $\rho \to \pi\gamma^*$, $\rho \to \eta\gamma^*$, $\omega \to \eta\gamma^*$?
- There are also complimentary processes to be improved: $\gamma^* \to VP$, $P \to V\gamma^*$, $\gamma^* \to PPP$ …

Spare slides

Odd intrinsic parity transitions

Particles

the low-lying well-established quark-antiquark states

V	$I^G(J^{PC}) = 1^+(1^{--})$	ρ^0,
	$I^G(J^{PC}) = 0^-(1^{--})$	ω, ϕ
P	$I^G(J^{PC}) = 1^-(0^{-+})$	π^0,
	$I^G(J^{PC}) = 0^+(0^{-+})$	η, η'
γ	$(J^{PC}) = (1^{--})$	

Odd intrinsic parity transitions

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P	$I^G(J^{PC}) = 1^-(0^{-+})$	π^0,
	$I^G(J^{PC}) = 0^+(0^{-+})$	η, η'
γ	$(J^{PC}) = (1^{--})$	

$VP\gamma^*$ transitions

- these transitions are *P*-odd and are the phenomenological reflections of the Adler-Bell-Jackiw axial anomaly of QCD

$V\gamma P$ or VVP contact term?

There is an old debate on the correct realization of the VMD ansatz in the odd-intrinsic parity sector

$V\gamma P \gg VVP$

[Borasoy and Nißler, Eur.Phys.J., A19, 367 (2004)]

$V\gamma P \ll VVP$

[Bramon, Grau, Pancheri, Phys.Lett. B344 (1995) 240]
[Hashimoto, Phys.Lett. B381 (1996) 465]
[Terschlüsen and Leupold, Phys.Lett., B691, 191 (2010)]
[Benayoun et al., Eur.Phys.J. C65 (2010) 211]
[ ]

$V\gamma P \approx VVP$

[Ruiz-Femenía, Pich and Portolés, JHEP 0307.003 (2003)]

Another motivation (1)

- poor knowledge of $V \to \gamma^* \mathcal{P}$ was one of the obstacles in study of light scalar mesons in $e^+e^- \to \pi^0\pi^0\gamma$, $\pi^0\eta\gamma$

	Dominant	Suppressed
in $\gamma^* \to (\cdots) \to \pi^0\pi^0\gamma$:		
1-vector	$(\rho^0\pi^0)$, $(\omega\pi^0)$	$(\phi\pi^0)$
2-vector	$(\omega \to \rho^0\pi^0)$, $(\rho^0 \to \omega\pi^0)$	$(\phi \to \rho^0\pi^0)$, $(\phi \to \omega\pi^0)$ $(\rho^0 \to \phi\pi^0)$
in $\gamma^* \to (\cdots) \to \pi^0\eta\gamma$:		
1-vector	$(\rho\pi^0)$, $(\omega\pi^0)$ $(\rho\eta)$, $(\omega\eta)$	$(\phi\pi^0)$ $(\phi\eta)$
2-vector	$(\rho \to \omega\pi^0)$, $(\omega \to \rho\pi^0)$ $(\rho \to \rho\eta)$, $(\omega \to \omega\eta)$	$(\rho \to \phi\pi^0)$, $(\phi \to \rho\pi^0)$ $(\phi \to \phi\eta)$, $(\phi \to \omega\eta)$

[Eidelman et al., Eur.Phys.J.C69, 103 (2010)]

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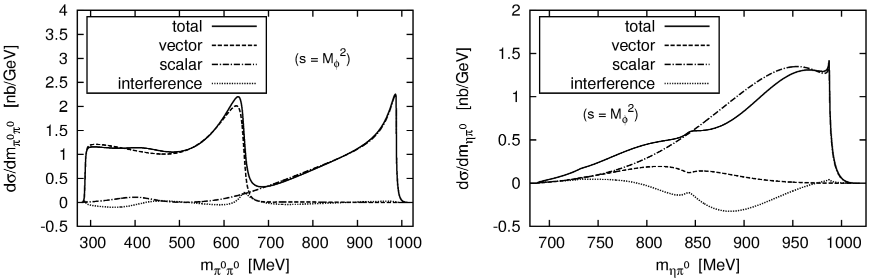
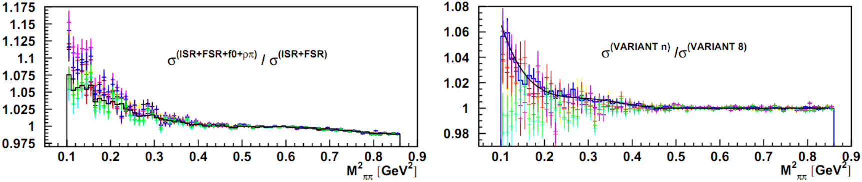


Differential cross section of the e^+e^- annihilation to $\pi^0\pi^0\gamma$ (left) and $\pi^0\eta\gamma$ (right) for $\sqrt{s} = M_\phi$

[Eidelman et al., Eur.Phys.J.C69, 103 (2010)]

Another motivation (2)

- $V \to \gamma^* \mathcal{P}$ and scalar meson decay contributions "contaminate" the pion form factor precision measurement when the radiative return method is used



(a) (b)

Figure 18: (a) Fractional contribution of the processes $e^+e^- \to \phi \to f_0\gamma \to \pi^+\pi^-\gamma$ and $e^+e^- \to \phi \to \rho^\pm\pi^\mp \to (\pi^\pm\gamma)\pi^\mp$ (estimated from MC simulation) to the signal. 10 different variants of the parameter sets have been used. The solid line histogram shows the variant with a set of parameters which gave the best fit result in [21] (variant 8). (b) Comparison between all the variants and the one with the best fit result. The solid histogram shows the effect of the variant with the largest deviation (variant 4 in [21]). Below 0.45 GeV^2, the solid histogram has been fitted with a fourth order polynomial.

[Picture is taken from KLOE analysis: http://www.lnf.infn.it/kloe/ppg]

Another motivation (3)

The $VV\mathcal{P}$ function of QCD is the basic object in calculation of the light-by-light contribution to the muon a.m.m.

$a_\mu^{LbL,\ had} = (93 \pm 34) \times 10^{-11}$ [F.Jegerlehner, book]

- the worst known object in the game
- not calculable, no data
- dominated by pion exchange ($\approx$ 95%)
- predictions of different approaches do not coincide
- difficult to estimate an error

Prospects for $\phi \to \eta e^+ e^-$ at Frascati

Eur. Phys. J. C (2010) 68: 619–681 647

4.2.3 Pseudoscalar form factors

Pseudoscalar production at the ϕ-factory (Fig. 11) associated to internal conversion of the photon into a lepton pair allows the measurement of the form factor $\mathcal{F}_P(q_1^2 = M_\phi^2, q_2^2 > 0)$ of the pseudoscalar P in the kinematical region of interest for the VMD model. The vector-meson-dominance assumption generally provides a good description of the photon coupling to hadrons but in the case of the transition form factor of the ω meson [264, 265] where standard VMD fails in predicting the strong rise of the coupling in the $(0.4 < M_{l^+l^-} < 0.6)$ GeV region. Recently, a model for implementing systematic corrections to the standard VMD calculations has been proposed [266] which correctly describes the $\omega \to \pi^0\mu^+\mu^-$ experimental results, and predicts deviation from standard VMD for the $\phi \to \eta e^+ e^-$ decay spectrum. The only existing data on the latter process come from the SND experiment at Novosibirsk which has measured the M_{ee} invariant mass distribution on the basis of 74 events [267]. The accuracy on the shape does not allow any conclusion on the models. The predictions can be tested with the analysis of KLOE and KLOE-2/step0 data, e.g., selecting the η sample by means of the reconstruction of $\eta \to \pi^+\pi^-\pi^0$ decays. This kind of procedure has been tested on a 90 pb^{-1} sample obtaining, as a figure of merit for the KLOE-2 reach, a clean sample with a selection efficiency of 15% which translates to 12,000 events/fb^{-1}.

[Amelino-Camelia et al., Eur.Phys.J. C68 (2010) 619]

Formalism: Lagrangian terms (RχT)

$$\mathcal{L}_{\gamma V} = -e f_V \partial^\mu B^\nu \left(\tilde{\rho}^0_{\mu\nu} + \frac{1}{3}\tilde{\omega}_{\mu\nu} - \frac{\sqrt{2}}{3}\tilde{\phi}_{\mu\nu}\right)$$

$$\tilde{V}_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\mathcal{L}_{VVP} = -\frac{4\sigma_V}{f_\pi}\epsilon_{\mu\nu\alpha\beta}\Big[\quad \pi^0 \partial^\mu \omega^\nu \partial^\alpha \rho^{0\beta} + \eta\big[(\partial^\mu \rho^{0\nu}\partial^\alpha \rho^{0\beta} + \partial^\mu \omega^\nu \partial^\alpha \omega^\beta)\frac{1}{2}C_q - \partial^\mu \phi^\nu \partial^\alpha \phi^\beta \frac{1}{\sqrt{2}}C_s\big]\Big]$$

$$\mathcal{L}_{V\gamma P} = -\frac{4\sqrt{2}e h_V}{3 f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \Big[\quad \big[\rho^{0\mu} + 3\omega^\mu\big]\partial^\nu \pi^0 + \big[(3\rho^{0\mu} + \omega^\mu)C_q + 2\phi^\mu C_s\big]\partial^\nu \eta\Big]$$

$\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor. We omit the η' meson terms.

Form factors: formulae

$$V_{\rho\pi\gamma^*}(Q^2) = \frac{4}{3f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\omega(Q^2)\right] = \frac{1}{C_q} V_{\omega\eta\gamma^*}(Q^2)$$

$$V_{\omega\pi\gamma^*}(Q^2) = \frac{4}{f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\rho(Q^2)\right] = \frac{1}{C_q} V_{\rho\eta\gamma^*}(Q^2)$$

$$V_{\phi\eta\gamma^*}(Q^2) = 2C_s \frac{4}{3f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\phi(Q^2)\right]$$

[Eidelman et al., Eur.Phys.J.C69, 103 (2010)]

$$D_V(Q^2) = [Q^2 - M_V^2 + i\sqrt{Q^2}\Gamma_{tot,V}(Q^2)]^{-1}$$

$f_{V_1} = 0.20173(86)$ from $\Gamma(\rho \to ee) = \frac{e^4 M_\rho f_V^2}{12\pi}$

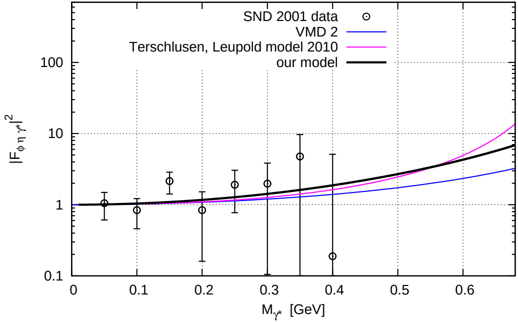
$h_{V_1} = 0.041(3)$ from $\Gamma(\rho \to \pi\gamma) = \frac{4\alpha M_\rho^3 h_V^2}{27 f_\pi^2}\left(1 - \frac{m_\pi^2}{M_\rho^2}\right)^3$

$f_\pi = 92.4$ MeV

the coefficients $C_q \approx 0.720$, $C_s \approx 0.471$ account for the η-η' mixing

normalization here: $\Gamma(V \to P\gamma) = \frac{\alpha M_V^3}{24 f_\pi^2}\left(1 - \frac{m_P^2}{M_V^2}\right)^3 |V(0)|^2$

$\phi \to \gamma^* \eta$ form factor (normalized)



- consistency with Novosibirsk data
- new precise data from KLOE will appear soon (2012 ?)
  $\Rightarrow$ important test of the models
  (does the vector meson dominance work here?)

$\phi\eta$ form factor — the light-cone quark model

WEN QIAN AND BO-QIANG MA PHYSICAL REVIEW D **78,** 074002 (2008)

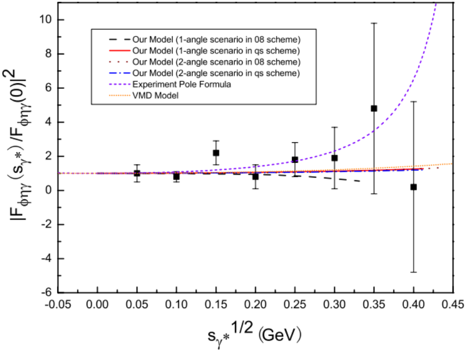


FIG. 7 (color online). The Q^2 behavior of the normalized form factor $F_{\phi\to\eta\gamma^*}(Q^2)/F_{\phi\to\eta\gamma^*}(0)$ using the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme compared with the experimental data [19] and the vector meson dominance model result in the timelike region.

$\omega\pi^0$ form factor — the light-cone quark model

WEN QIAN AND BO-QIANG MA PHYSICAL REVIEW D **78**, 074002 (2008)

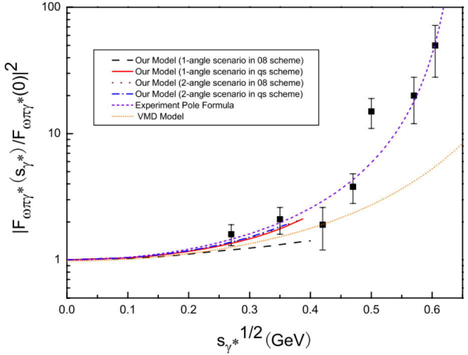


FIG. 6 (color online). The Q^2 behavior of the normalized form factor $F_{\omega\to\pi\gamma^*}(Q^2)/F_{\omega\to\pi\gamma^*}(0)$ using the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme compared with the experimental data [17,21] and the vector meson dominance model result in the timelike region.