VP: status and discussion

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. Introduction

.. HLMNT routine; status and comparison with other VP compilations; latest changes

... Narrow resonances: treatment and pitfalls

.... Higgs fit

Thanks to my collaborators Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura.

Introduction

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

— Corrections for data used as input for $g-2$: 'undressed' $\sigma^0_{\rm had}$

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

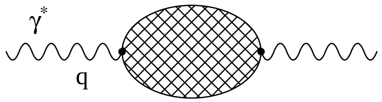
— Determination of α_s and quark masses from total hadronic cross section $R_{\rm had}$ at low energies and of resonance parameters.

— Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.

— $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.

$\longrightarrow$ Ingredient in MC generators for many processes.

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi \;=$ 

Full photon propagator $\sim\; 1 \;+\; \Pi \;+\; \Pi\cdot\Pi \;+\; \Pi\cdot\Pi\cdot\Pi \;+\; \dots$

$$\rightsquigarrow \qquad \alpha(q^2) \;=\; \frac{\alpha}{1-\mathrm{Re}\Pi(q^2)} \;=\; \alpha \,/\, \big(1-\Delta\alpha_{\mathrm{lep}}(q^2)-\Delta\alpha_{\mathrm{had}}(q^2)\big)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \;=\; -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}\,, \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1-\Pi|^2}$$

[$\to$ σ^0 requires ‘undressing’, e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$.
$\to$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

HLMNT routine; status and comparison

Features of the HLMNT VP code

— Latest routine VP_HLMNT_v2_1, version 2.1, available since 27 Jan. 2012

— Simple set of (standard) Fortran routines; completely standalone, no libs needed; all explanations in comment-headers

— Gives separately real and imaginary part ($\Delta\alpha(s)$ and $R(s)$)

— Tabulation/interpolation of hadronic part, for both space- and time-like region, including errors; input 'data' included in routine (rhad not needed)

— Leptonic part coded analytically; all special function included (partly with custom made expansions)

— top contribution in the same way

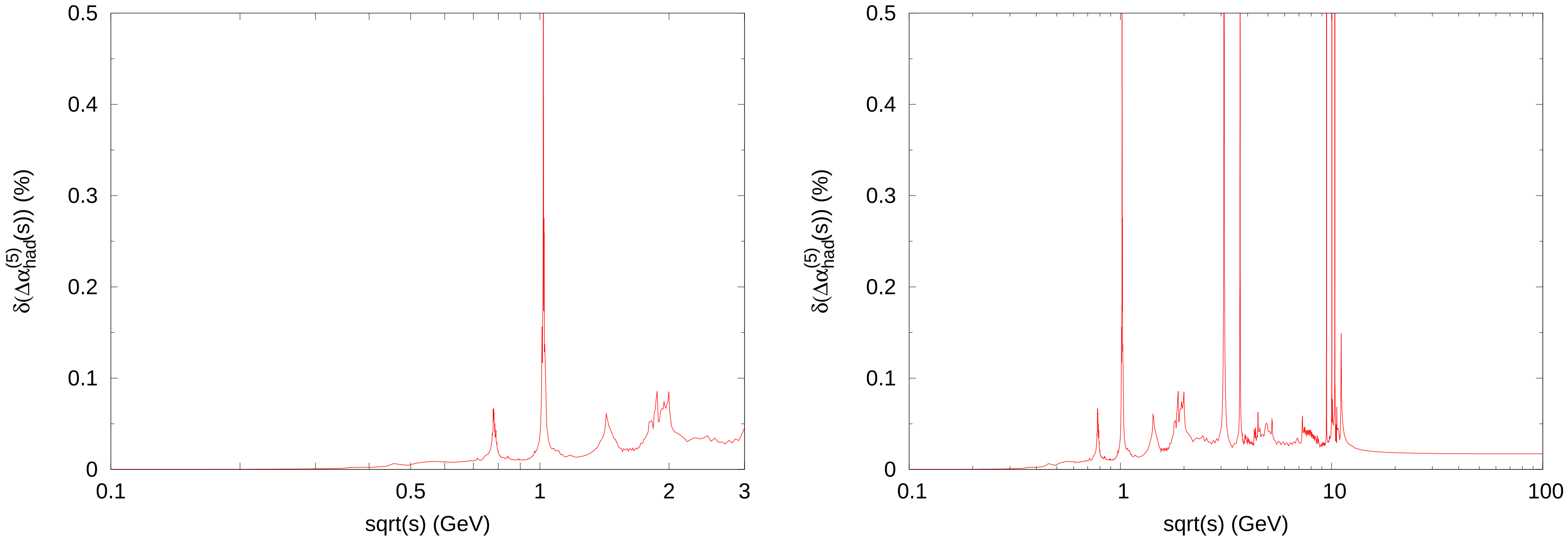
→ Flag to include or exclude narrow resonances $\boldsymbol{J/\psi}$, $\boldsymbol{\psi'}$, $\boldsymbol{\Upsilon(1-6\,S)}$

[But ϕ always included via integral over final state data (3π, KK).

Is this o.k. for users?]

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):

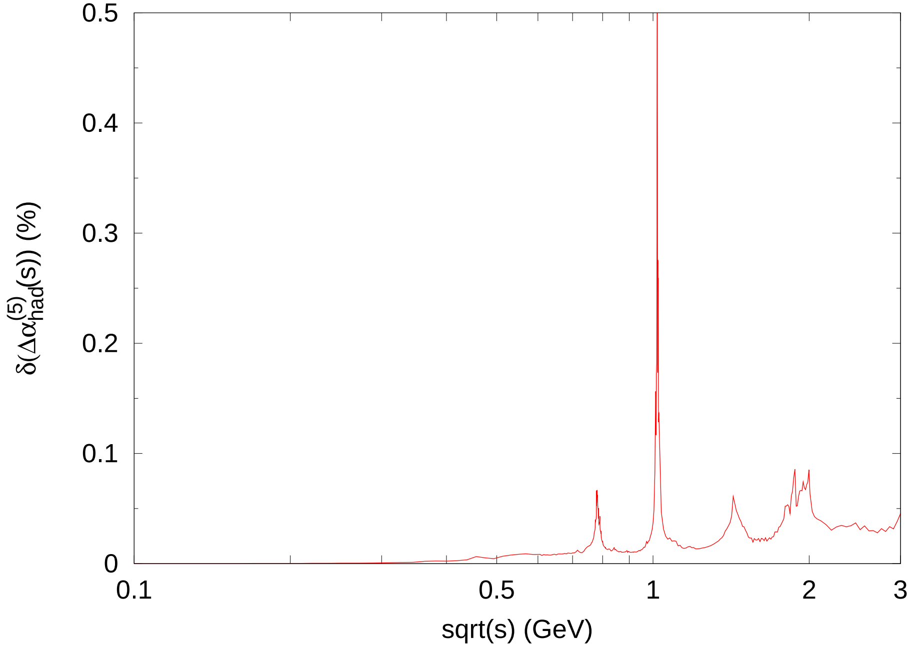


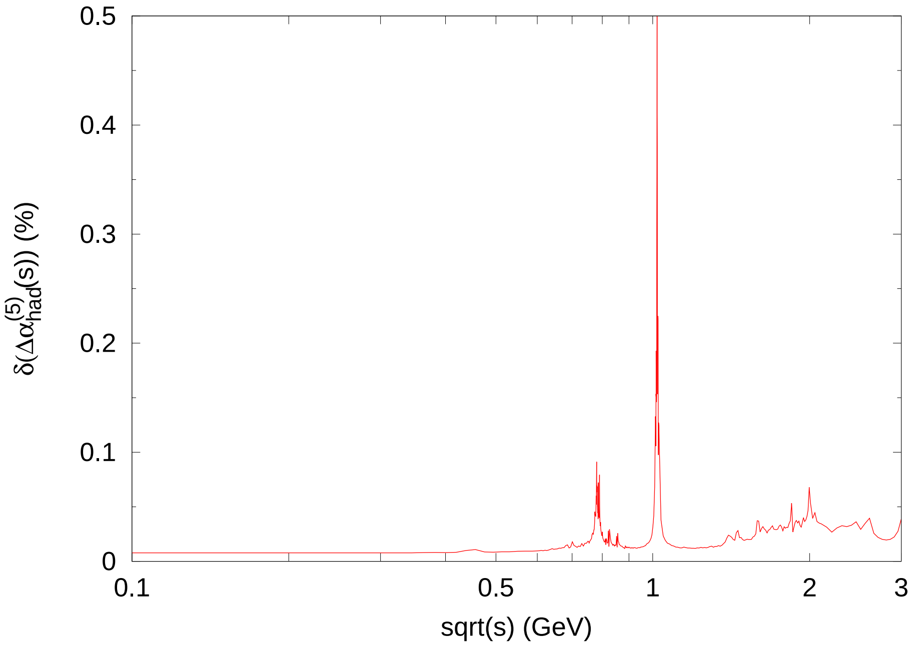
$\longrightarrow$ Below one per-mille (and typically $\sim 5\cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

Enough in the long term? Need for more work in resonance regions.

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

Error of VP in the timelike regime: old vs. **new** HLMNT '11 compilation):

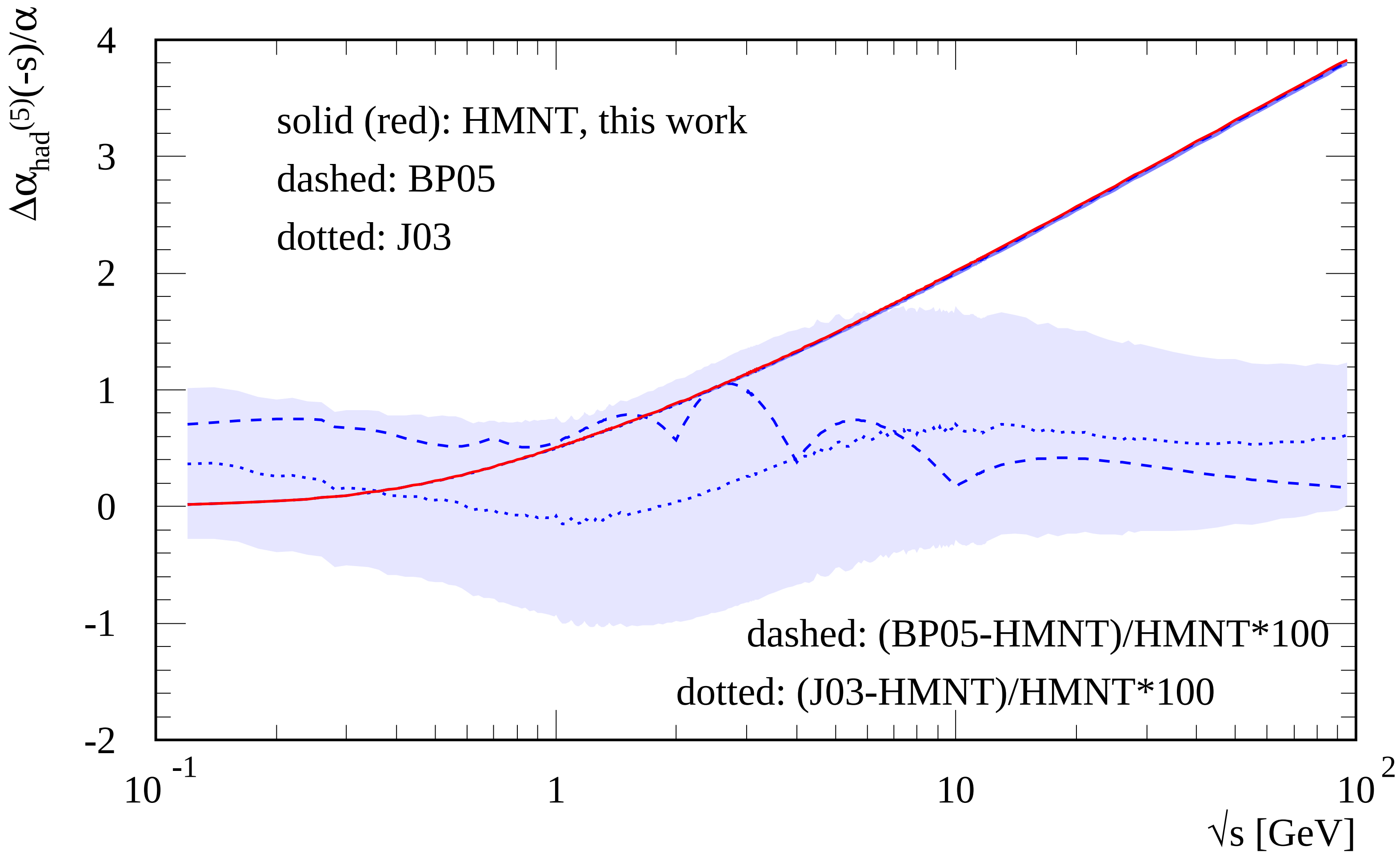




$\rightarrow$ region $1 < \sqrt{s} < 2$ GeV (and higher) improved,

ρ suffers from tension in 2π data (BaBar included).

- Comparison of Spacelike $\Delta\alpha^{(5)}_{\rm had}(-s)/\alpha$ (smooth $\alpha(q^2<0)$)

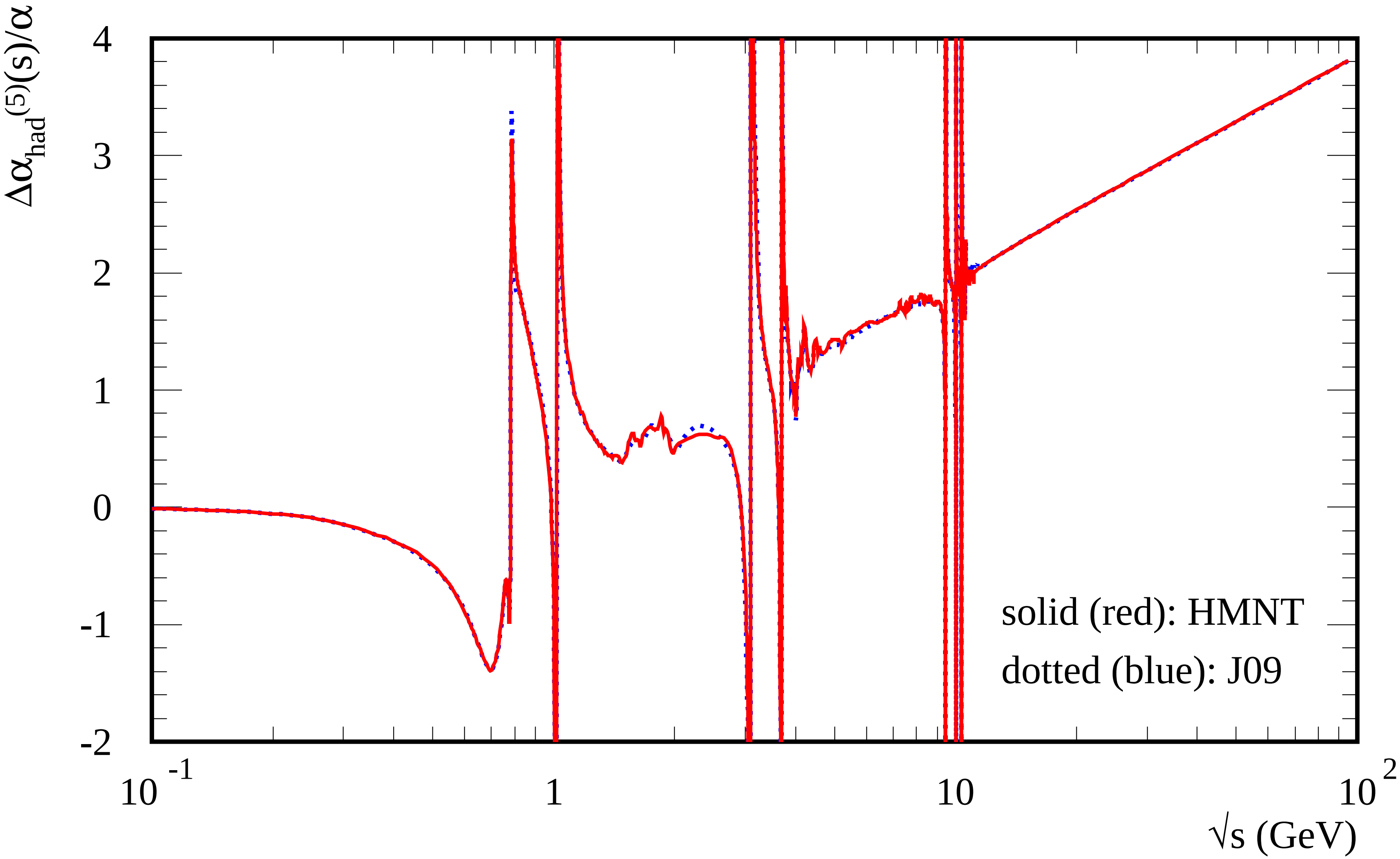


— Differences between parametrisations clearly visible but within error band (of HLMNT)

— Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.

— Encourage use of more accurate recent tabulations; $\boldsymbol{\Delta\alpha(M_Z^2)}$

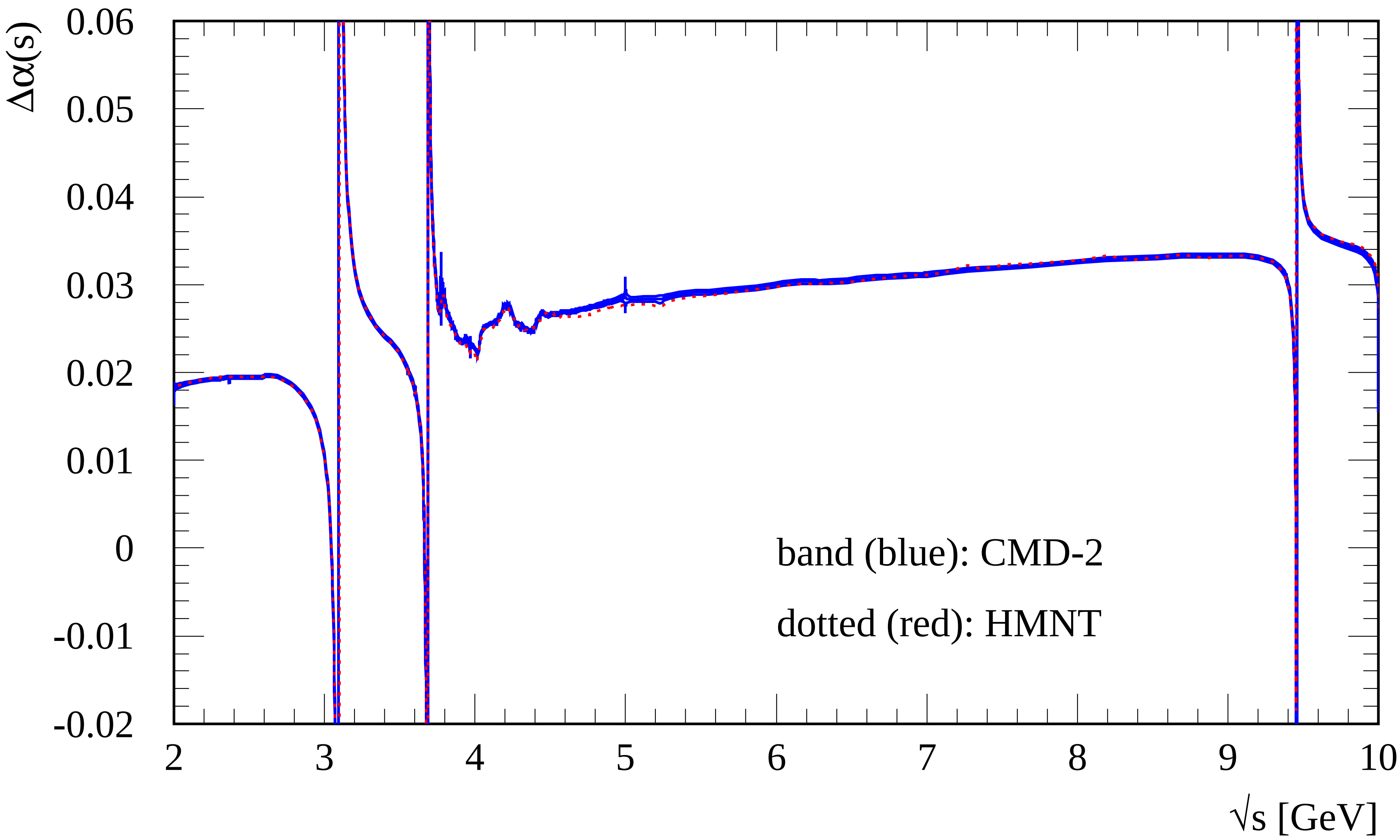
- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



$\longrightarrow$ with new version big differences (with 2003 version) gone

– smaller differences remain and reflect different choices, smoothing etc.

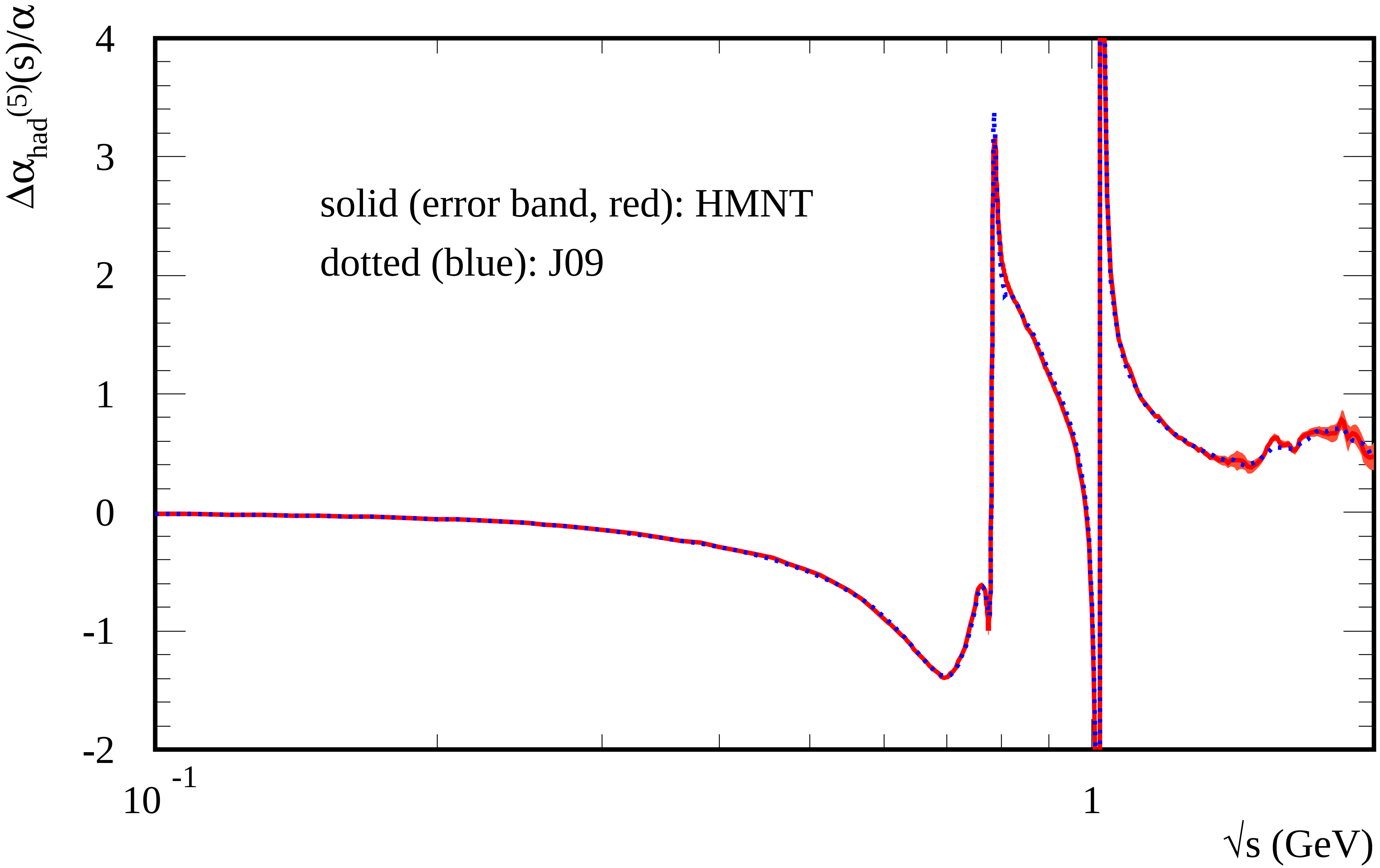
- HLMNT compared to Novosibirsk   Timelike, $\Delta\alpha(q^2)$



More comparison plots...

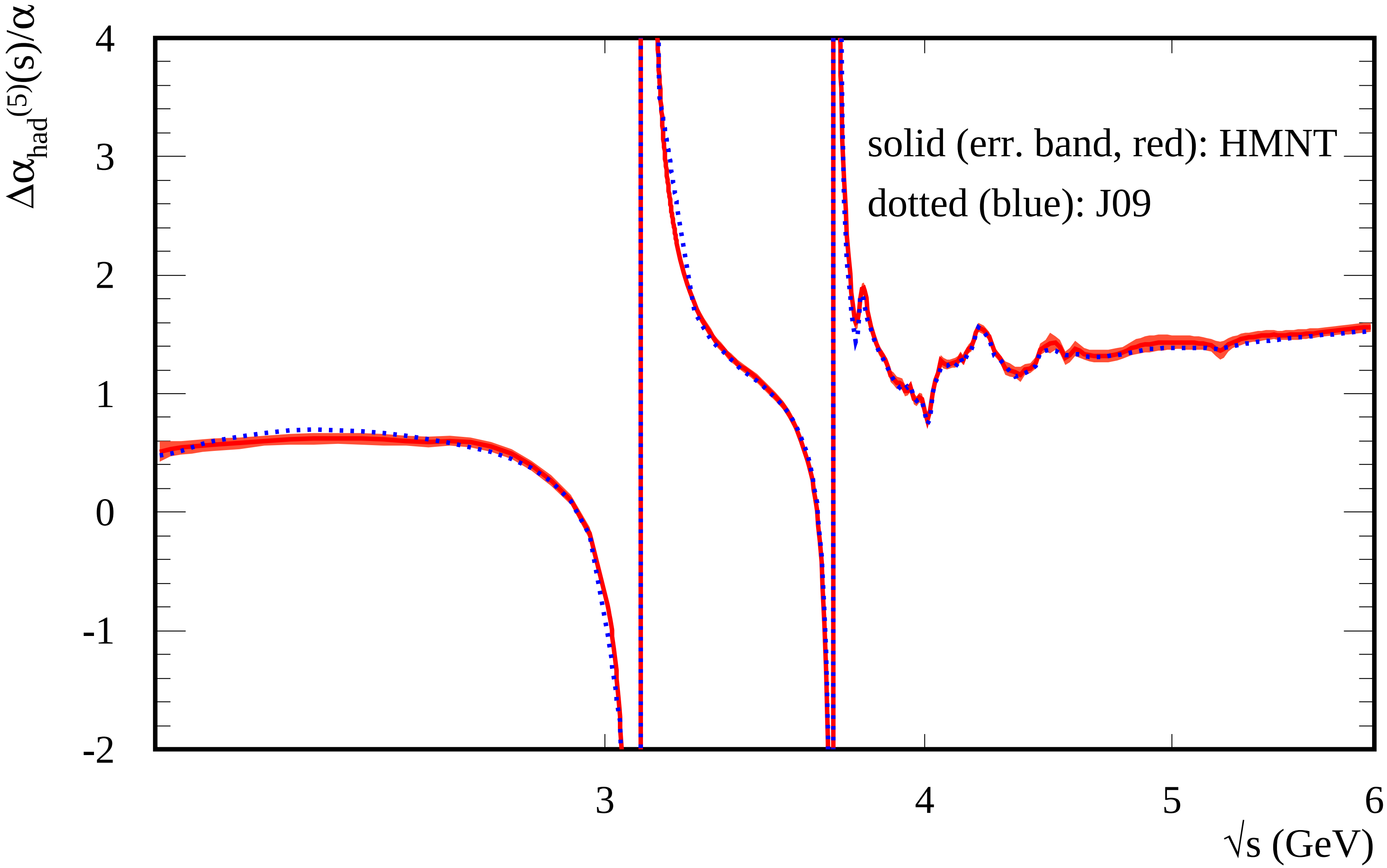
HLMNT compared to Fred Jegerlehner's new version: Detailed look

Low energies: ρ and ϕ



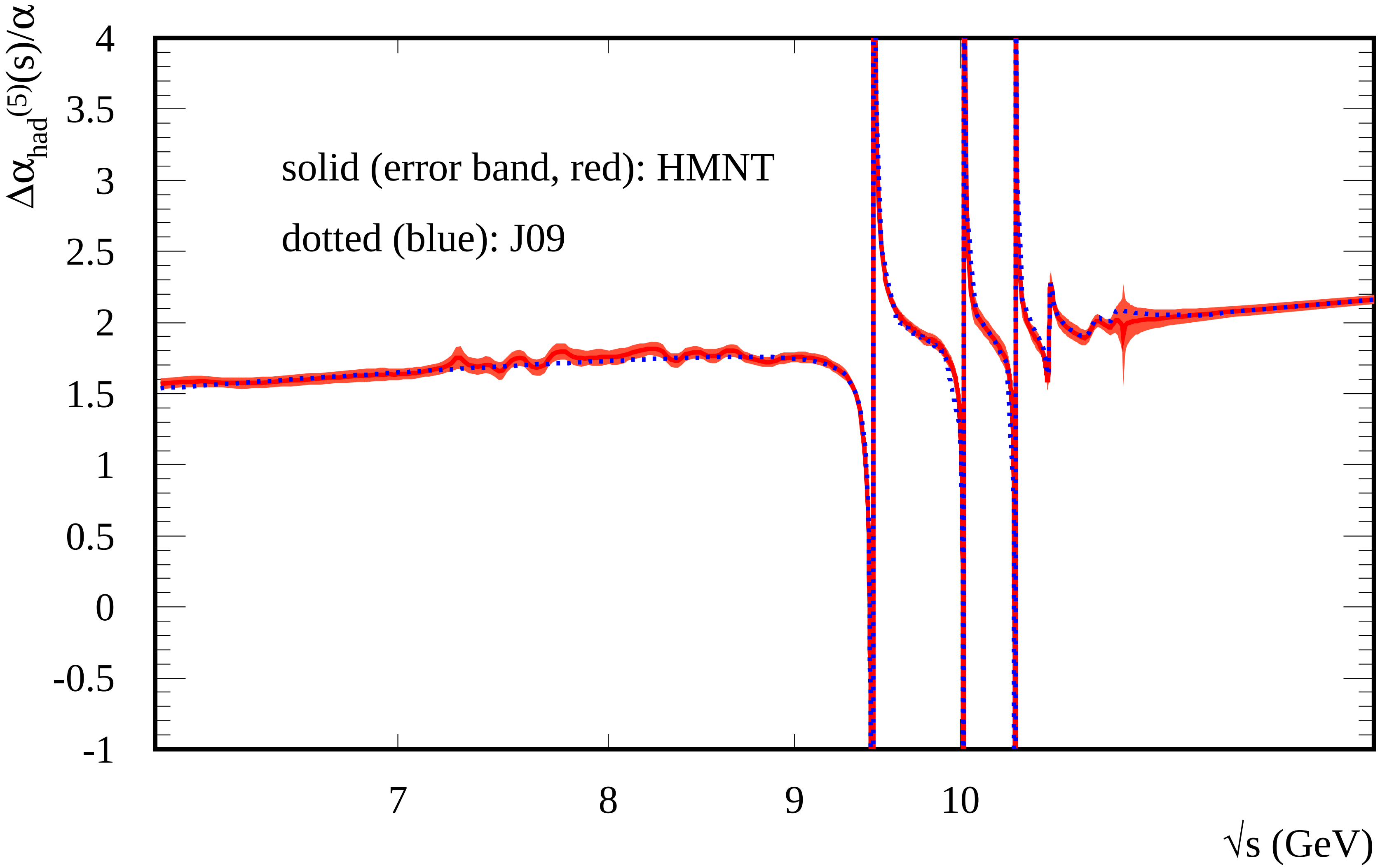
HLMNT compared to Fred Jegerlehner's new version: Detailed look

Medium energies: continuum and charm



HLMNT compared to Fred Jegerlehner's new version: Detailed look

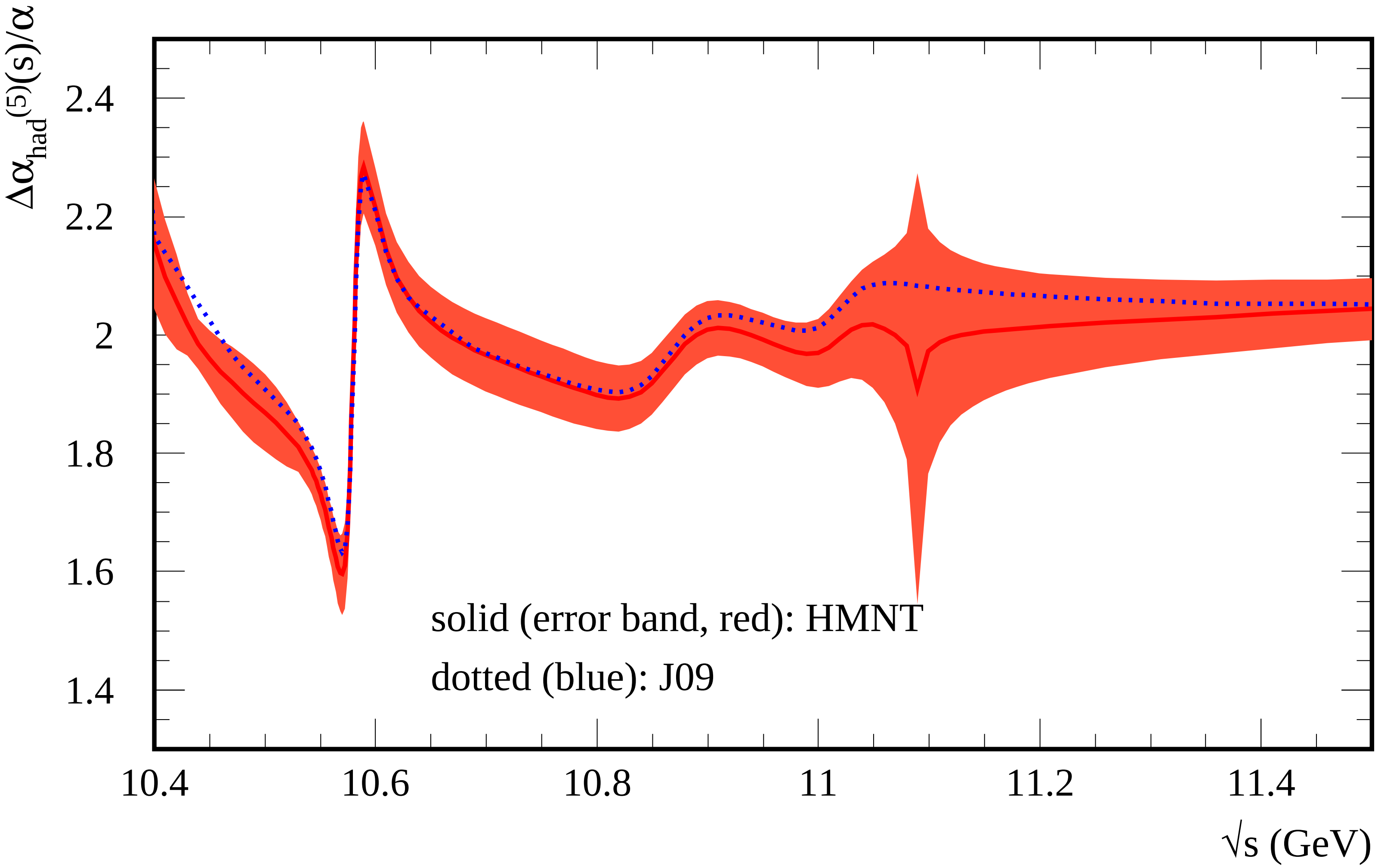
Higher energy continuum; bottom



HLMNT compared to Fred Jegerlehner's new version: Detailed look

old here

Details of higher $\Upsilon(4,5,6\,S)$ [10580, 10860, 11020] / open bottom region

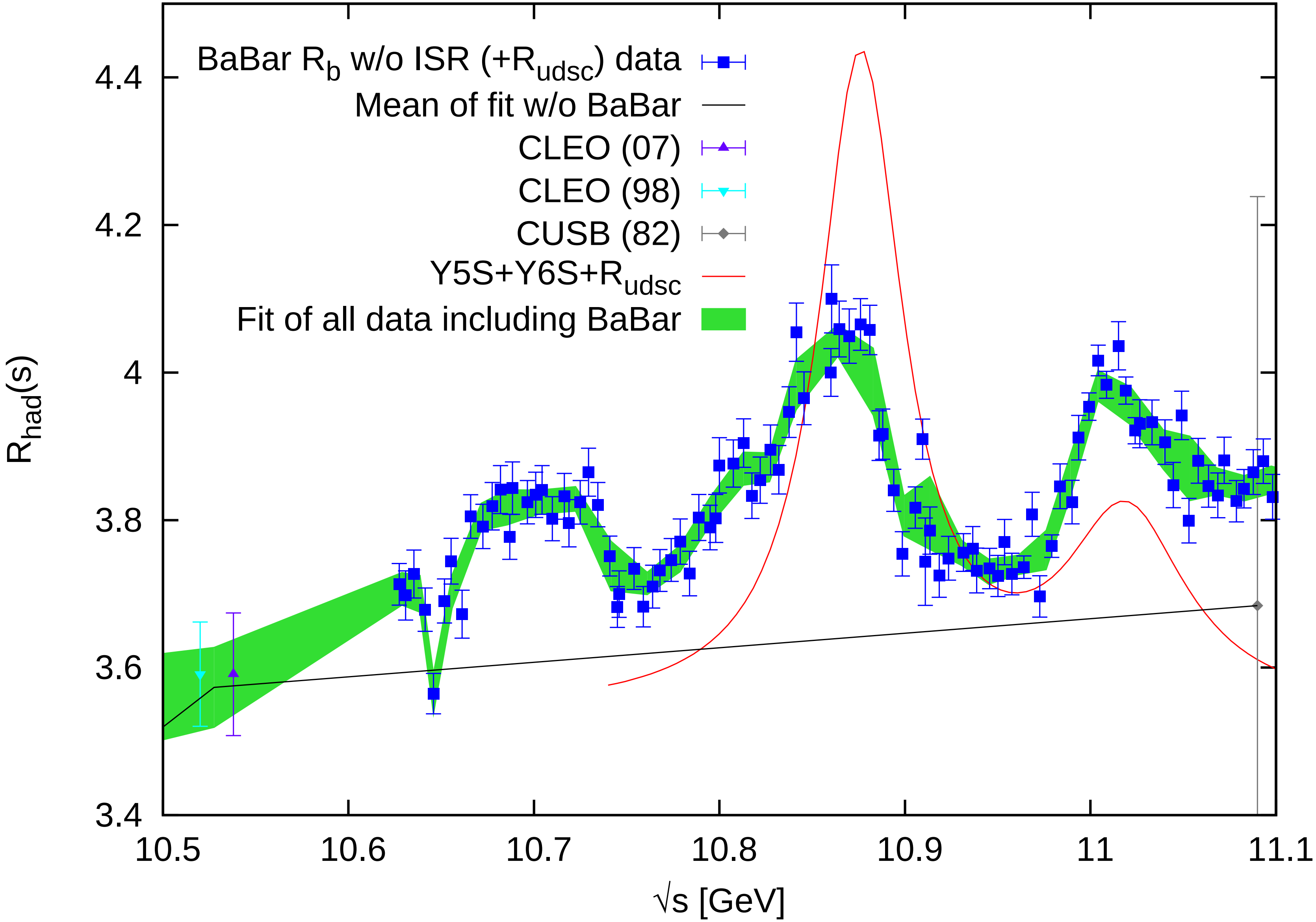


$\longrightarrow$ HLMNT still to include BaBar's $R_{b\bar{b}}$ data; ISR unfolding.. work in progress ✓

– expected to smooth and improve region above 11 GeV ✓

Latest changes, included in v2.1

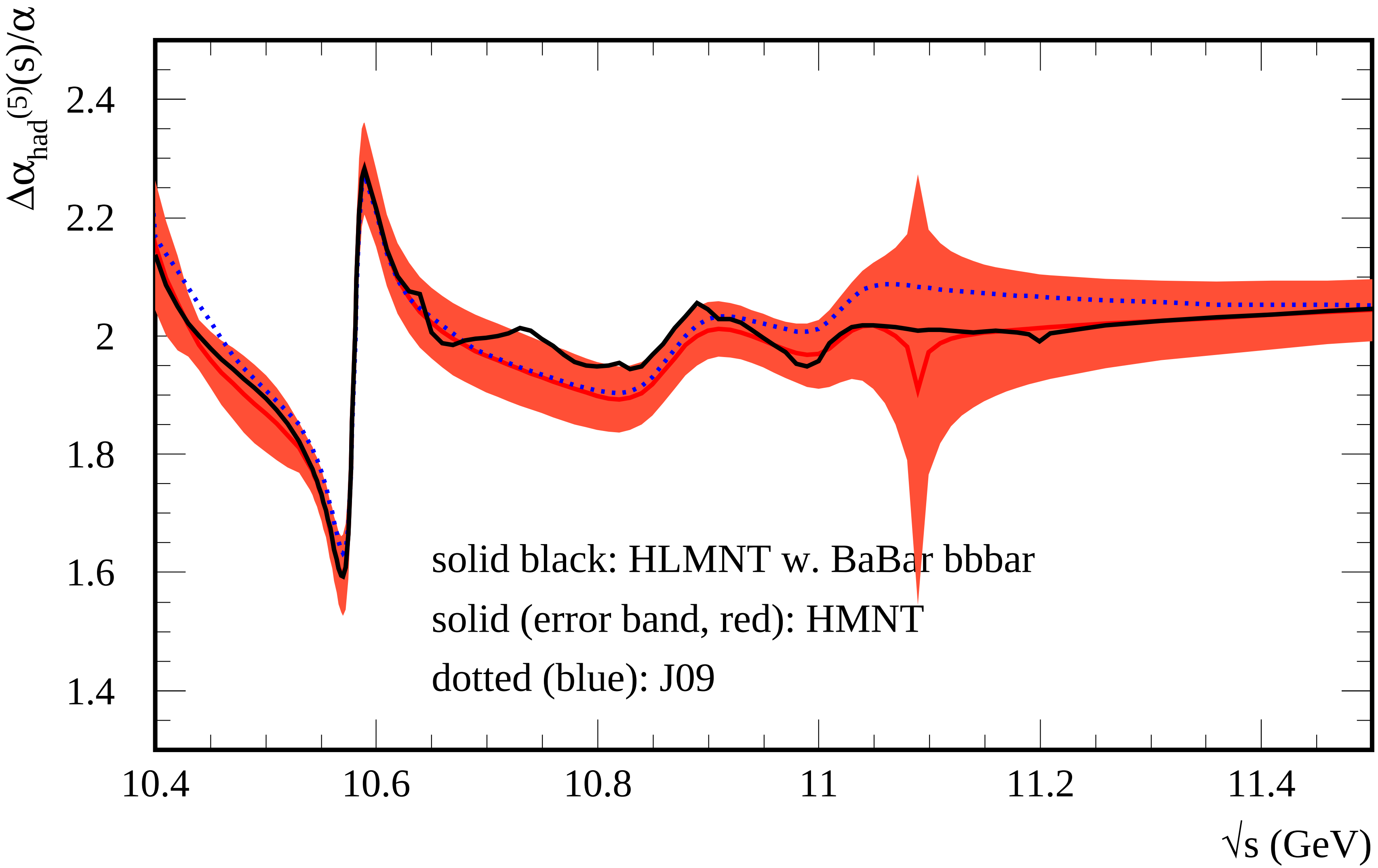
Inclusion of BaBar's $b\bar{b}$ after ISR deconvolution



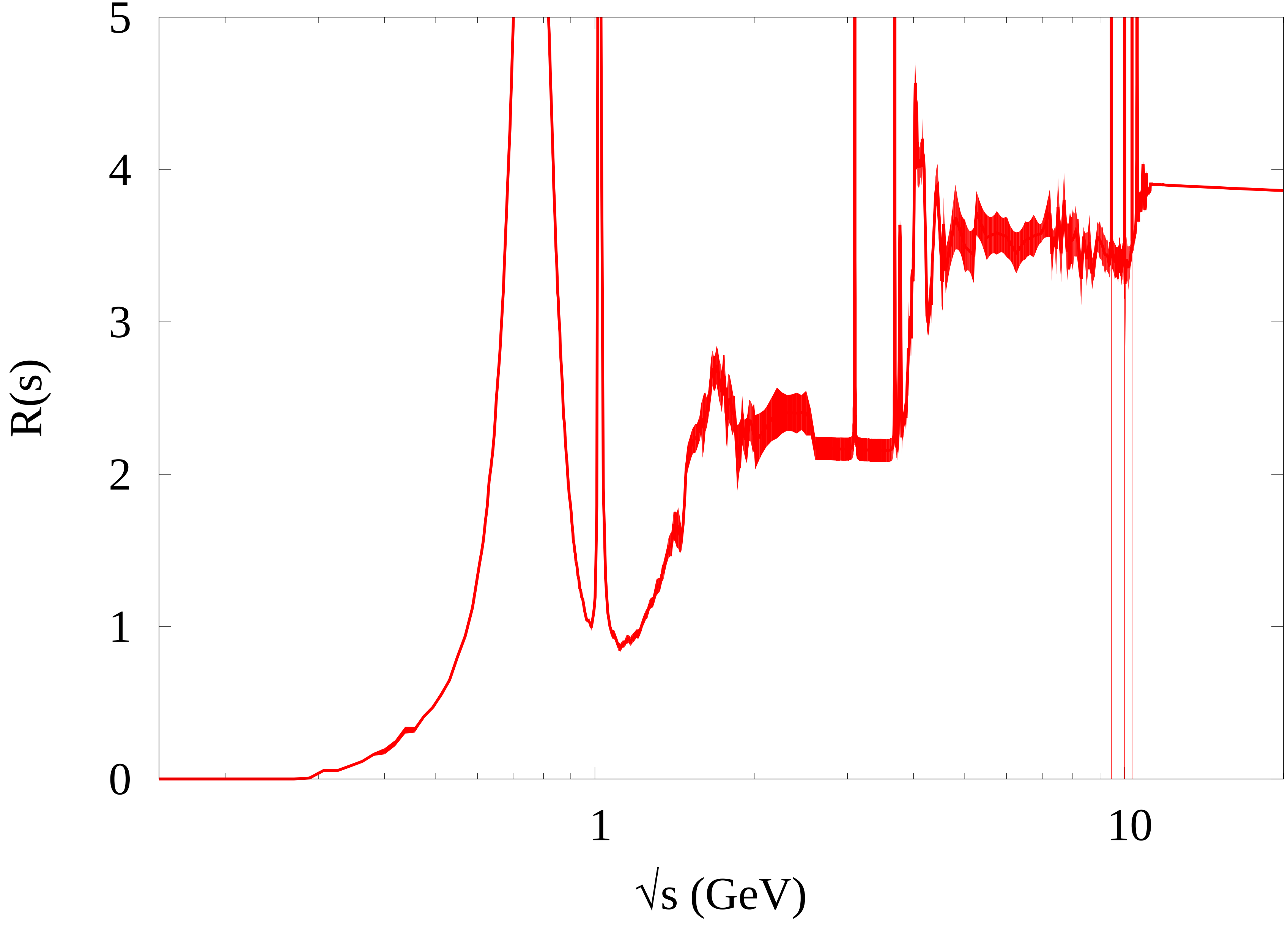
Inclusion of BaBar's $b\bar{b}$ after ISR deconvolution

new HLMNT routine v2.1

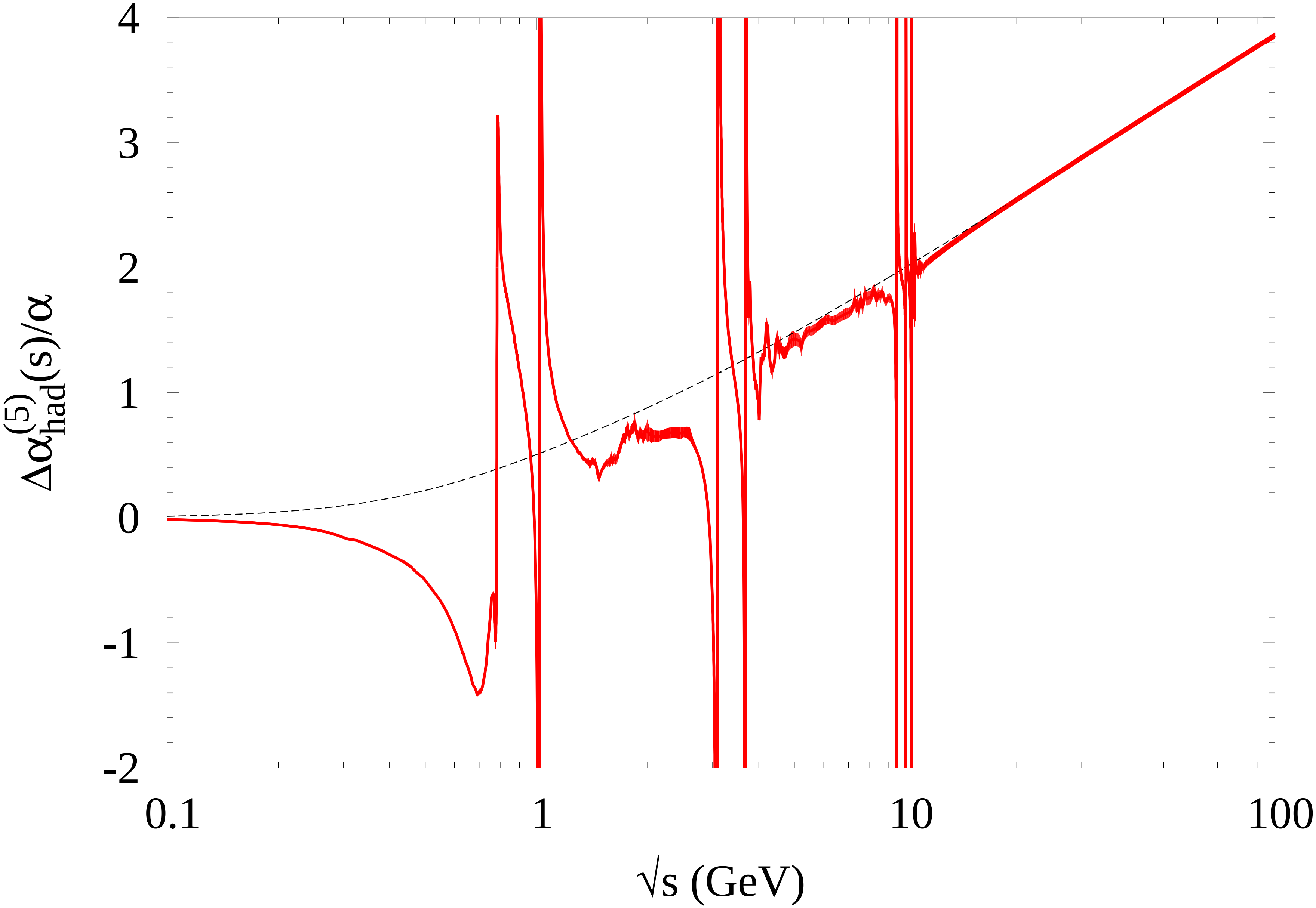
Higher energy continuum; bottom. No smoothing!



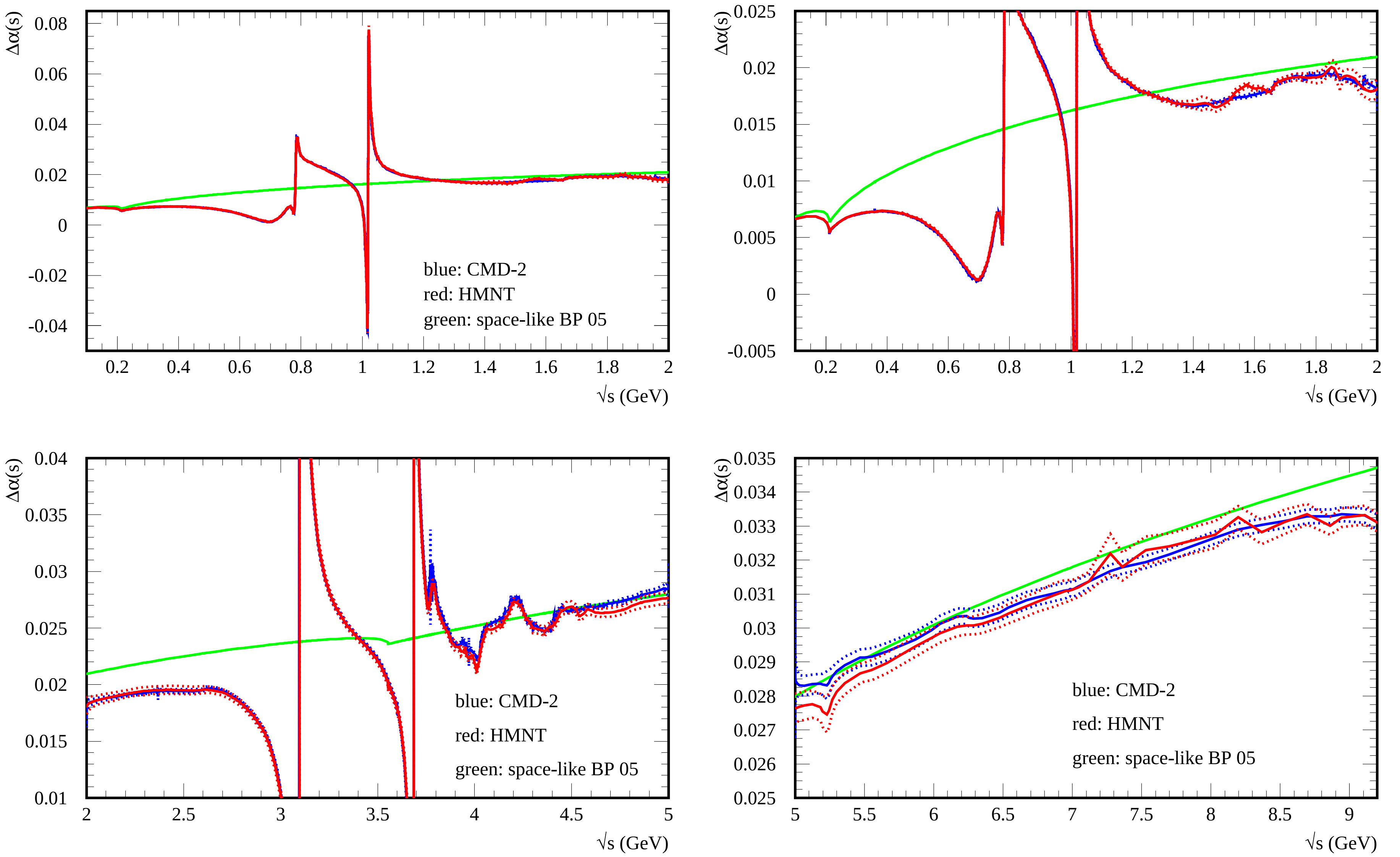
HLMNT '11: $\boldsymbol{R(s)}$



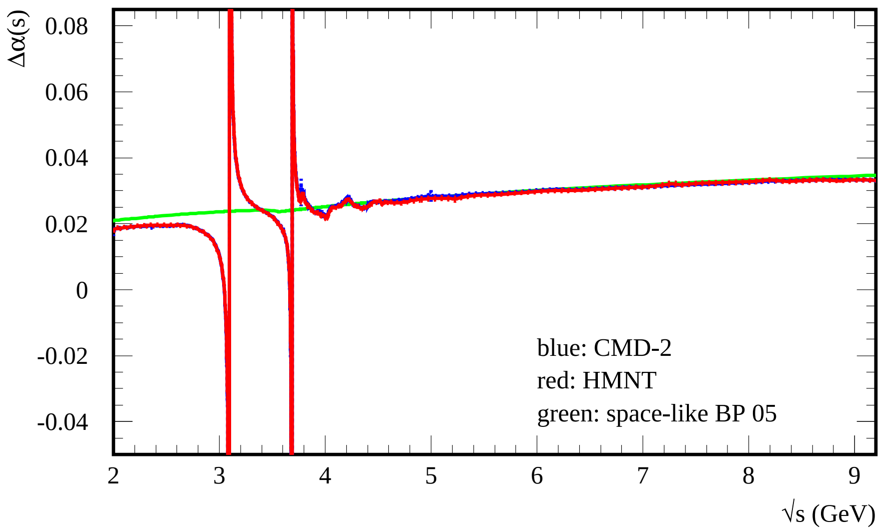
HLMNT '11: space- and time-like $\Delta\alpha^{(5)}_{\rm had}(s)/\alpha$

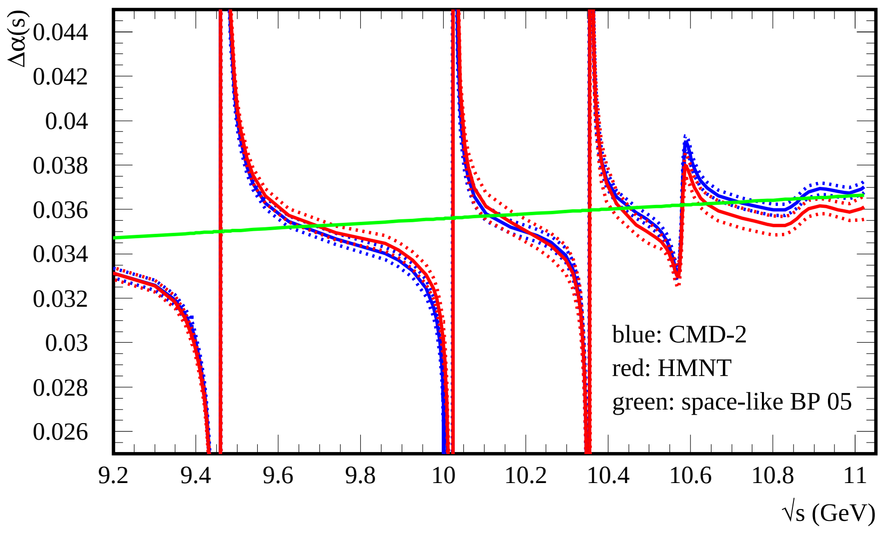


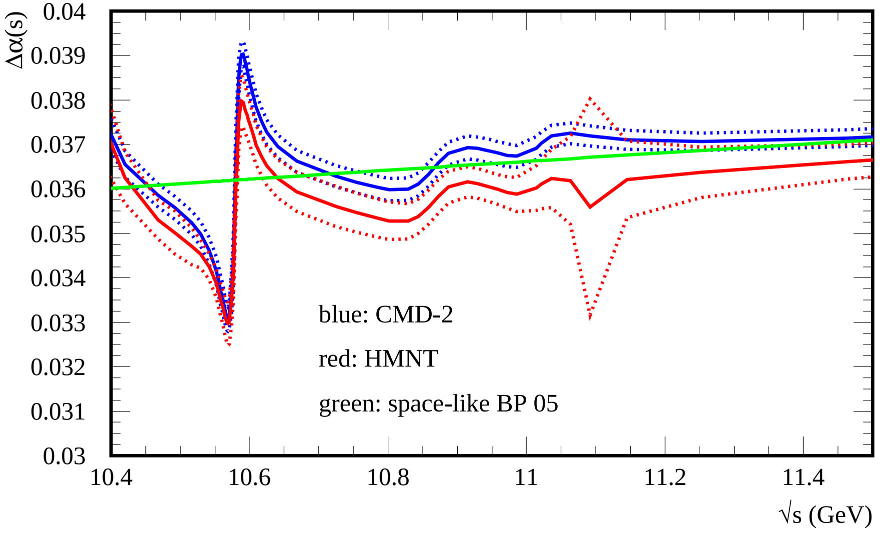
HLMNT compared to CMD-2's routine: Detailed looks



HLMNT compared to CMD-2's routine: three more zooms







Narrow Resonances: treatment and pitfalls

Note 1:

– For $\Delta\alpha$ or $g-2$, using NR or BW formulae with the dressed width Γ_{ee} for a resonance V is inconsistent and introduces sizeable effects (a few percent).

– Undressing via the smooth spacelike running $\alpha(-M_V)$ comes closer numerically but is not fully correct.

– Use undressing formula

$$\Gamma_{ee}^0 = \frac{\left[\alpha/\alpha_{\mathrm{no}\,V}(M_V^2)\right]^2}{1+3\alpha/(4\pi)}\,\Gamma_{ee}\,,$$

where 'no V' means that the resonance V is *excluded* from the running α.

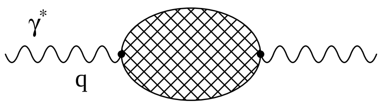
Note 2:

– Close to narrow resonance energies $|\Pi| \sim 1$ and the summation breaks down

↪ Need other formulation, e.g. Breit-Wigner resonance propagator interfering with γ:

$$\left(\frac{\alpha(s)}{s}\right)^2 \to \frac{1}{s^2}\left|\alpha_{\mathrm{no}\,V}(s) + \frac{3\Gamma_{ee}M_V}{s-M_V^2+i\Gamma M_V}\right|^2 .$$

The running QED coupling $\Delta\alpha(M_Z^2)$ ... and the Higgs mass



- Vacuum polarisation leads to the 'running' of α from $\alpha(q^2=0) = 1/137.035999084(51)$ to $\alpha(q^2=M_Z^2) \sim 1/129$
- $\alpha(q^2) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(q^2) - \Delta\alpha_{\rm had}(q^2)\right)$
- Hadronic uncertainties $\rightsquigarrow$ α the least well known EW param. of $\{G_\mu,\, M_Z\,,\, \alpha(M_Z^2)\}$!
- We find (HLMNT '11):

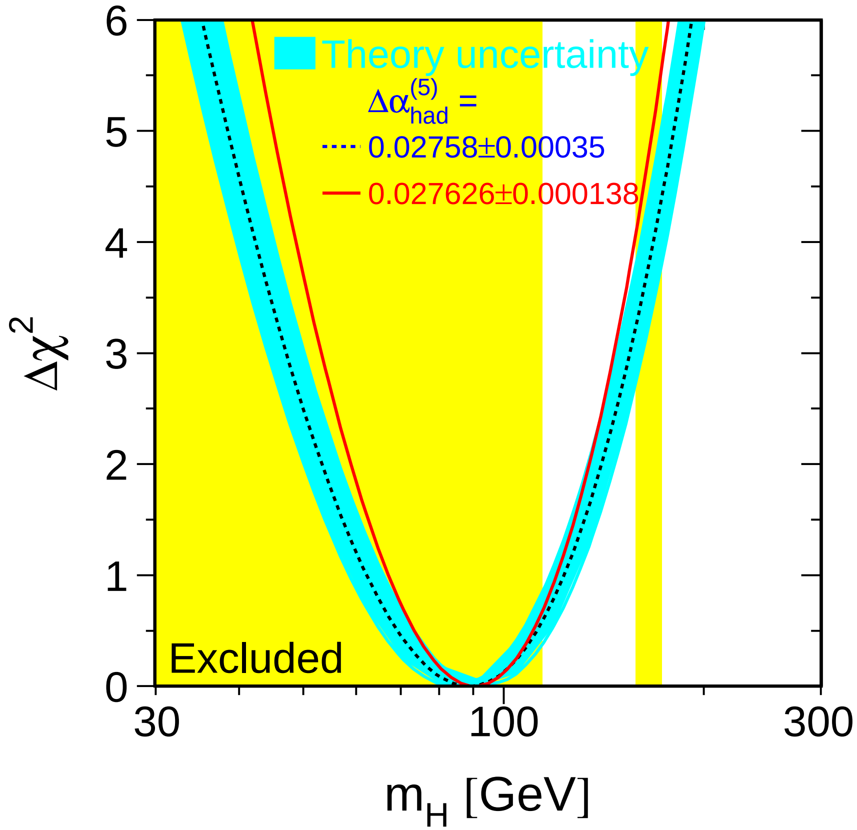
  $\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027626 \pm 0.000138}$

  i.e. $\boldsymbol{\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019}$
- evaluation from Davier *et al.* now very similar:

  $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02757 \pm 0.00010$

  [now more use of pQCD can only slightly improve error]

Fit of the SM Higgs mass: LEP EWWG



Fit and Fig. thanks to M. Grünewald

$\hookrightarrow$ $\boldsymbol{M_H = 91\,^{+30}_{-23}}$ GeV

vs. 'default' 89^{+35}_{-26} GeV

[w. $m_t = (173.3 \pm 1.1)$ GeV]

Olds:

Comparison of different compilations

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s = E^2) \;=\; \alpha \,/\, \Big(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\Big)$$

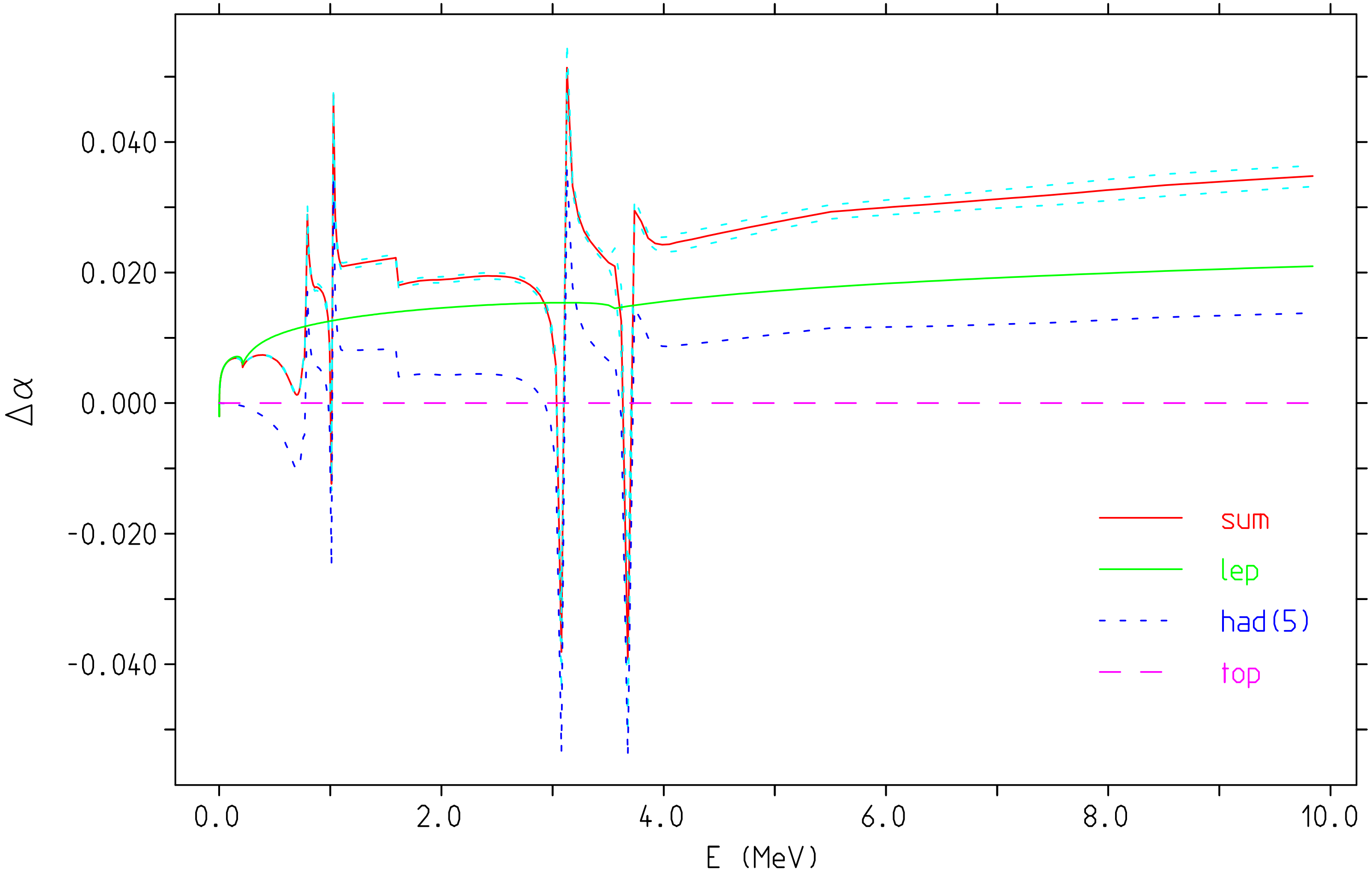
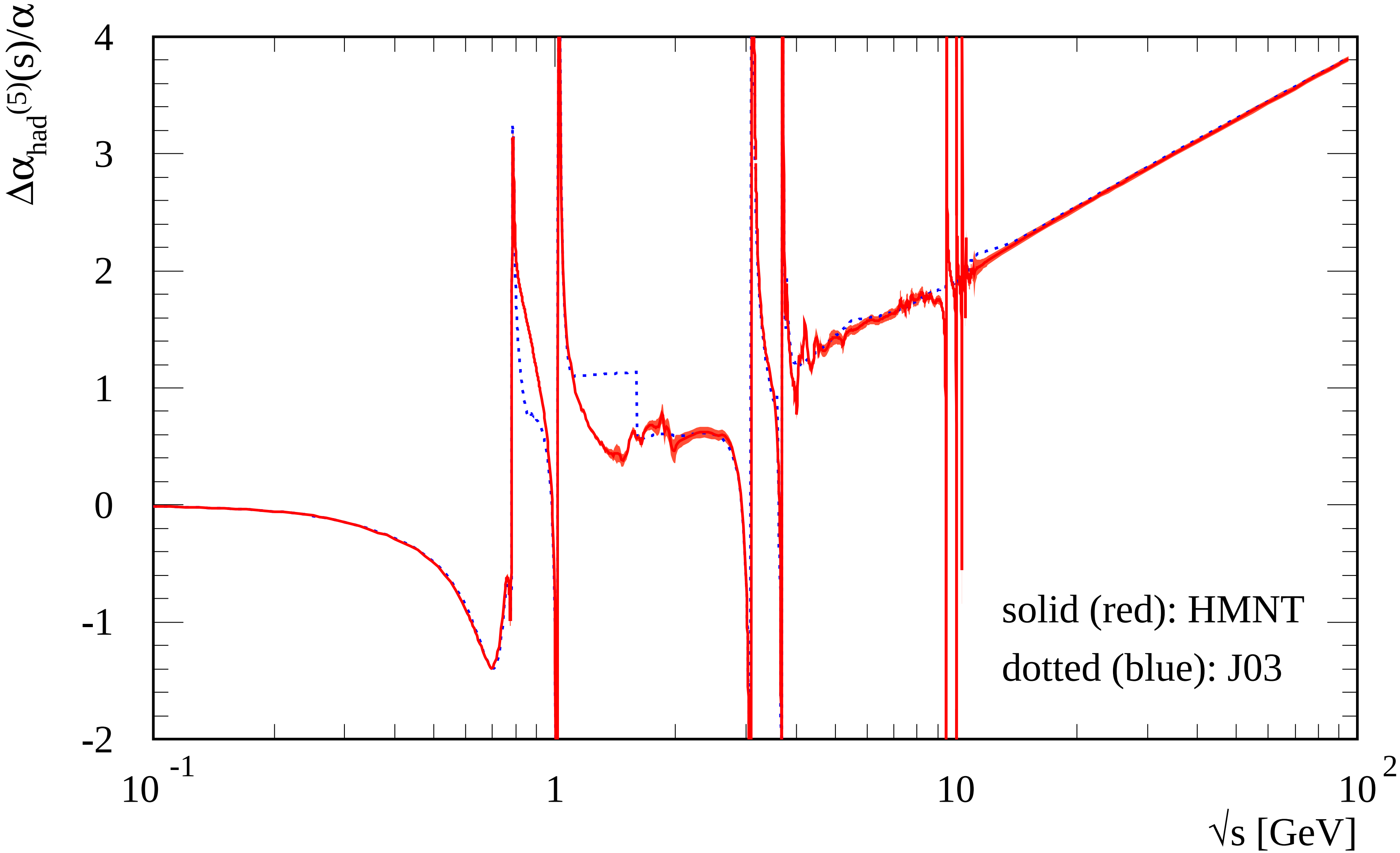


Figure from Fred Jegerlehner

Timelike $\alpha(s = q^2 > 0)$ follows resonance structure:



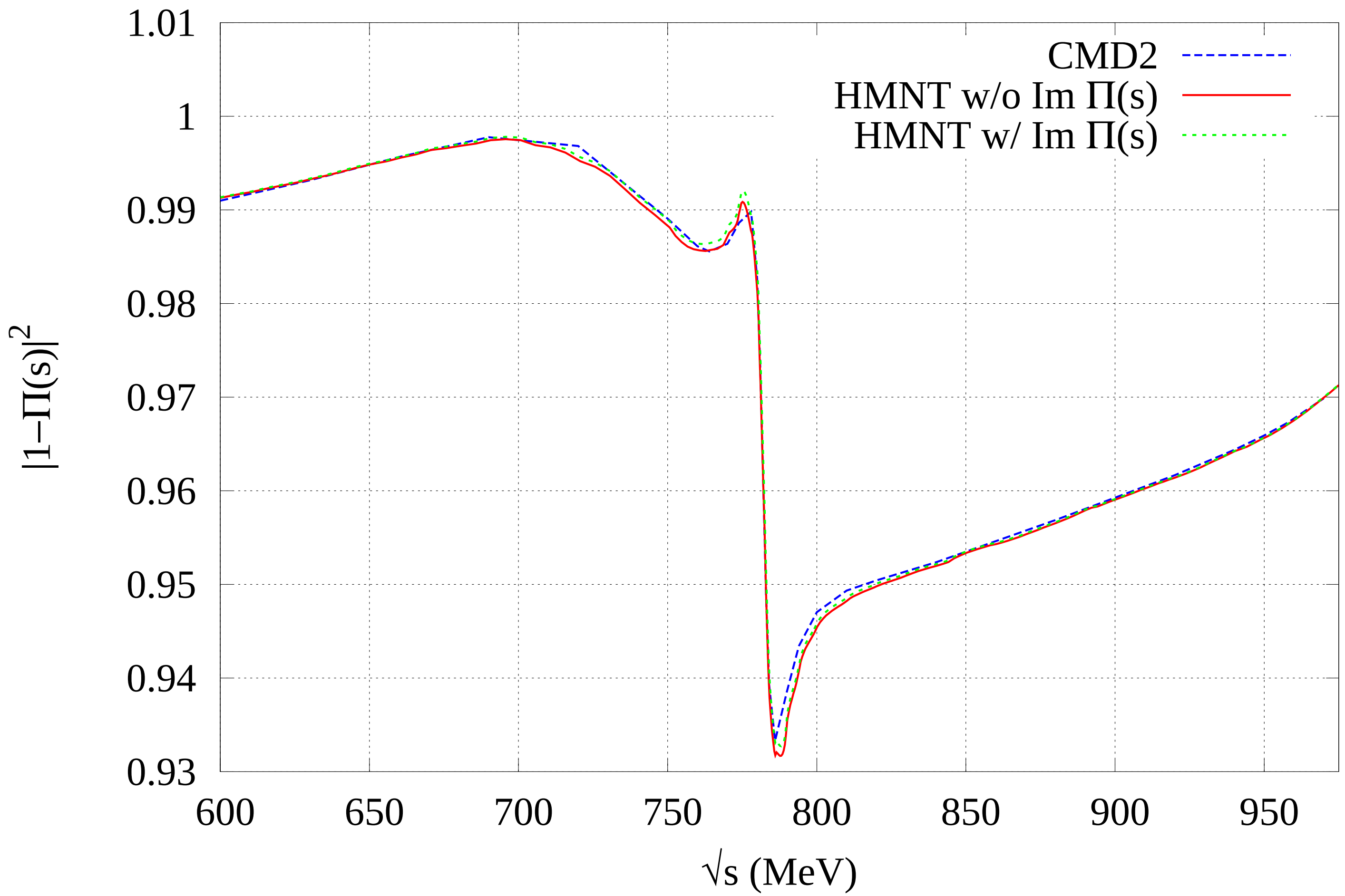
— Step below just a feature of unfortunate grid.

— Difference below 1 GeV not expected from data.

[Comparisons with other parametrisations confirm HMNT.]

- HMNT compared to Novosibirsk's parametrisation

Timelike $|1-\Pi(s)|^2 \sim (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!



→ Small but visible differences, as expected from independent compilations.