Problematic channels in exclusive R and $(g-2)_\mu/2$

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Outline

1. Missing channels
2. σ: Sum of intermediate channels vs. phase space
3. Interference
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5. Particular final states
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General

- How realistic is our accuracy of $a_\mu^{\mathrm{LO,had}}$?
  $a_\mu^{\mathrm{LO,had}} = (692.3 \pm 1.4 \pm 3.1 \pm 2.4) \cdot 10^{-10}$, M. Davier et al., 2011
- A very serious and complicated issue is experimental systematics; another important issue is averaging and summing; radiative corrections; there are also other problems with the way we determine exclusive R
- Missing channels, sum of intermediate channels vs. phase space, interference effects, isospin relations

Missing channels

- The final states studied by now are:
  $m(\pi^+\pi^-)n\pi^0,\ m = 1, 2, 3,\ n = 0, 1, 2$, but less than 7π and not more than $2\pi^0$,
  so $\pi^+\pi^-3\pi^0,\ \pi^+\pi^-4\pi^0$ have never been observed
- $K^+K^-,\ K_SK_L,\ K^+K^-\pi^0,\ K^\pm K_S\pi^\mp,\ K^+K^-\pi^+\pi^-,\ K^+K^-\pi^0\pi^0,\ 2(K^+K^-)$,
  so $\pi^+\pi^-3\pi^0,\ \pi^+\pi^-4\pi^0,\ K^0\bar{K}^0\pi\pi$ have never been observed
- Channels with η: $\eta\pi^+\pi^-,\ \eta\pi^+\pi^-\pi^0,\ \eta K^+K^-$,
  so η + more pions (kaon pairs) or $\eta\eta + X$ have never been observed
- Radiative decays $\pi^0\gamma,\ \eta\gamma,\ \pi^0\pi^0\gamma,\ \pi^0\eta\gamma$ below 1.4 GeV,
  but none of these or other channels above 1.4 GeV;
  SND recently measured $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ up to 1 GeV

Intermediate mechanisms

- Production of any final state proceeds basically via quasi-two-body intermediate mechanisms, e.g., $e^+e^- \to 2\pi^+2\pi^-$ can go via $e^+e^- \to a_1(1260)^+\pi^- \to \rho^0\pi^+\pi^- \to 2\pi^+2\pi^-$
- The situation with 4π is much richer: $a_1(1260)^+\pi^-,\ a_2(1320)^+\pi^-,\ \pi(1300)^+\pi^-,\ \rho^0 f_0,\ \rho^0 f_2(1270), \ldots,$ with subsequent decay of a resonance into $\rho\pi$ or 2π
- Each specific mechanism has its own angular and energy dependence, so the detection efficiency may be different between them and just phase space; the effect is smaller for ISR and grows when the solid angle is smaller
- The correct parameterization is $\frac{d\sigma}{d\Omega} \propto |\Sigma a_i|^2$
- Interference effects can be very large, even with narrow states, so the results should be presented as a set of complex numbers
- Multiple solutions!

Isospin relations

- For some of the missing channels, when there is no information on some of the charge combinations of the same final state, e.g., $\pi^+\pi^-4\pi^0$ from $2(\pi^+\pi^-\pi^0)$ and $3(\pi^+\pi^-)$
- The basic idea is to use Clebsch-Gordan (CG) coefficients from which, e.g., we learn that in K^{*0} decays $\Gamma(K^\pm\pi^\mp)/\Gamma(K^0\pi^0) = 2/1$
- Its application is limited because the CG idea assumes a pure isospin state. This is not true for multi-body final states, e.g., for $K\bar{K}\pi$ even with a single $K^*\bar{K}$ mechanism there is interference of two pure isospin amplitudes:
  $a(K^+K^-\pi^0) = a(K^{*+}[K^+\pi^0]K^-) + a(K^{*-}[K^-\pi^0]K^+)$,
  $a(K^0\bar{K}^0\pi^0) = a(K^{*0}[K^0\pi^0]\bar{K}^0) + a(\bar{K}^{*0}[\bar{K}^0\pi^0]K^0)$,
  $a(K^-K^0\pi^+) = a(K^{*0}[K^-\pi^+]K^0) + a(K^{*+}[K^0\pi^+]K^-)$,
- The situation is even more complicated when one adds $\phi\pi^0$
- Recent analysis of Davier et al., 2011 gives various relations for $K\bar{K}\pi$ and $K\bar{K}\pi\pi$ contributions to $a_\mu^{\rm LO,had}$, which are hardly correct (both interference with ϕ ignored and pure isospin states assumed)

Conclusions

- The contribution to $a_\mu^{\mathrm{LO,had}}$ from the range ($2m_\pi$-2) GeV is $\approx (640 \pm 4) \cdot 10^{-10}$
- Missing radiative channels ($\pi^0\gamma,\ \eta\gamma,\ \pi^0\pi^0\gamma,\ \ldots$ final states) can add 0.5 ± 0.5 to $a_\mu^{\mathrm{LO,had}}$
- Are there any exotic contributions like 7π, 8π, $\eta\eta n\pi$, $\eta' + X$?
- Various intermediate mechanisms with differing detection efficiency can change σ by 1.5 ± 1.5 (two 4π channels), technically long account of interference effects
- Contributions based on isospin relations can change σ by 2 ± 2 coming from $K\bar{K}n\pi,\ 6\pi$
- In total, 4 ± 4, i.e., a serious effect for the cross section and its systematic uncertainty, but how realistic?
- A lot of tedious additional work in experiment/theory!