$(g-2)_\mu$: Scope for improvement – role of various channels

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$(g-2)_\mu$: Hadronic contributions

▶ Hadronic Vacuum Polarisation from exp. $\sigma(e^+e^- \to \gamma^* \to hadrons(+\gamma))$ data

[or from $\tau \to \nu_\tau + hadrons$ spectral functions; isospin breaking...]

Use of dispersion integral (based on analyticity and unitarity):

$$a_\mu^{\text{had,VP LO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^{\infty} \mathrm{d}s\, \sigma_{\text{had}}^0(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s} \cdot (0.4 \ldots 1)$$

→ Kernel K ⇝ weighting towards smallest energies. σ_{had}^0 the undressed cross section

→ Similar approach with different kernel functions for NLO VP contributions $a_\mu^{\text{had,VP NLO}}$

▶ Hadronic Light-by-Light:

— No dispersion relation. *First Principles* calculations from lattice QCD are underway...

— Also first results based on Dyson-Schwinger eqs. by C. Fischer et al.

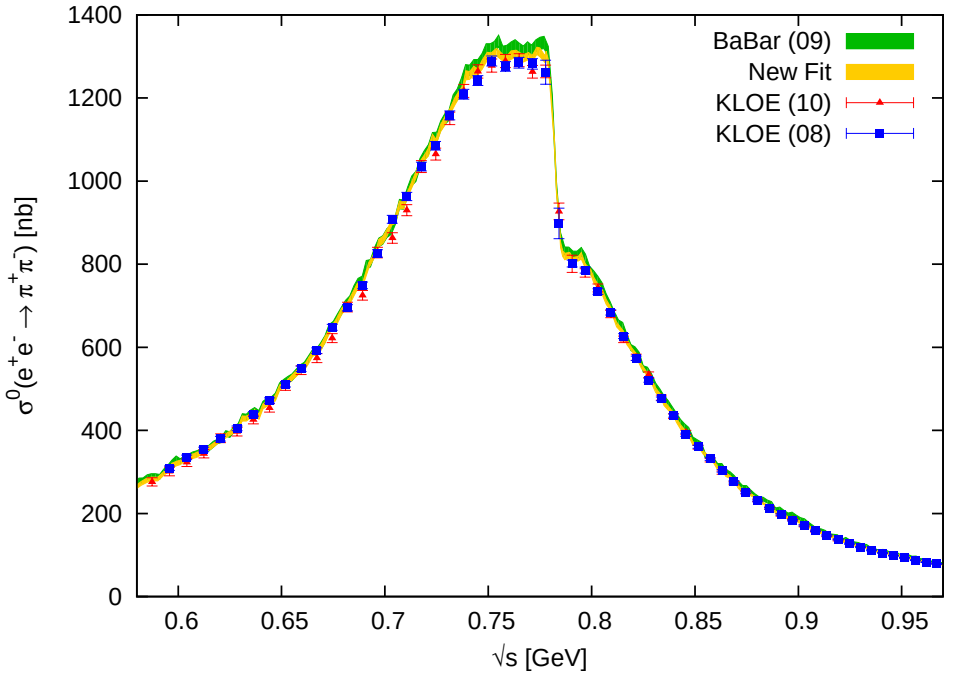
— 'Consensus' of different recent model calculations. HLMNT numbers below use compilation from J. Prades, E. de Rafael, A. Vainshtein: $\boldsymbol{a_\mu^{\text{L-by-L}} = (10.5 \pm 2.6) \cdot 10^{-10}}$

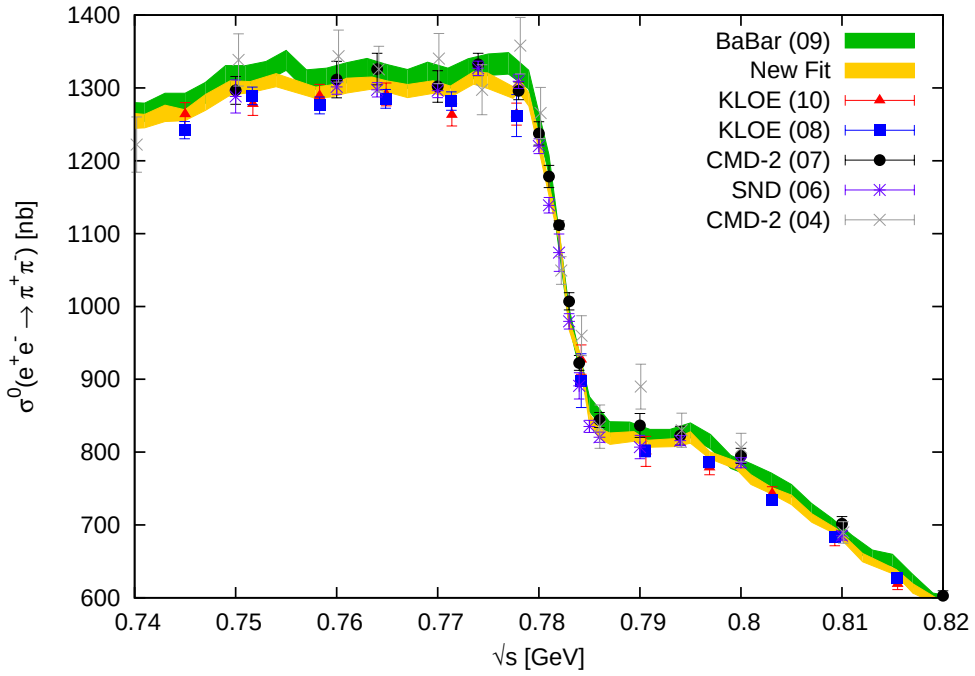
— Compatible result from F. Jegerlehner, A. Nyffeler: $a_\mu^{\text{L-by-L}} = (11.6 \pm 4.0) \cdot 10^{-10}$

Hadronic VP contributions

- For low energy $\sigma^0_{\rm had}(s)$, need to sum ~ 25 exclusive channels $[2\pi,\ 3\pi,\ KK,\ 4\pi,\ \ldots]$
- $\sqrt{s} \sim 1.4 - 2$ GeV: sum exclusive channels and/or use old inclusive data
- above ~ 2 GeV: inclusive data *or* use of perturbative QCD [+narrow resonances]

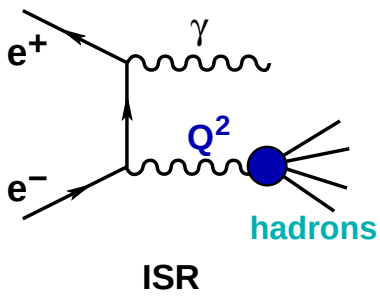
▶ The most important $\boldsymbol{2\pi}$ channel ($> 70\%$) HLMNT '11 use 879 data points; needed!?



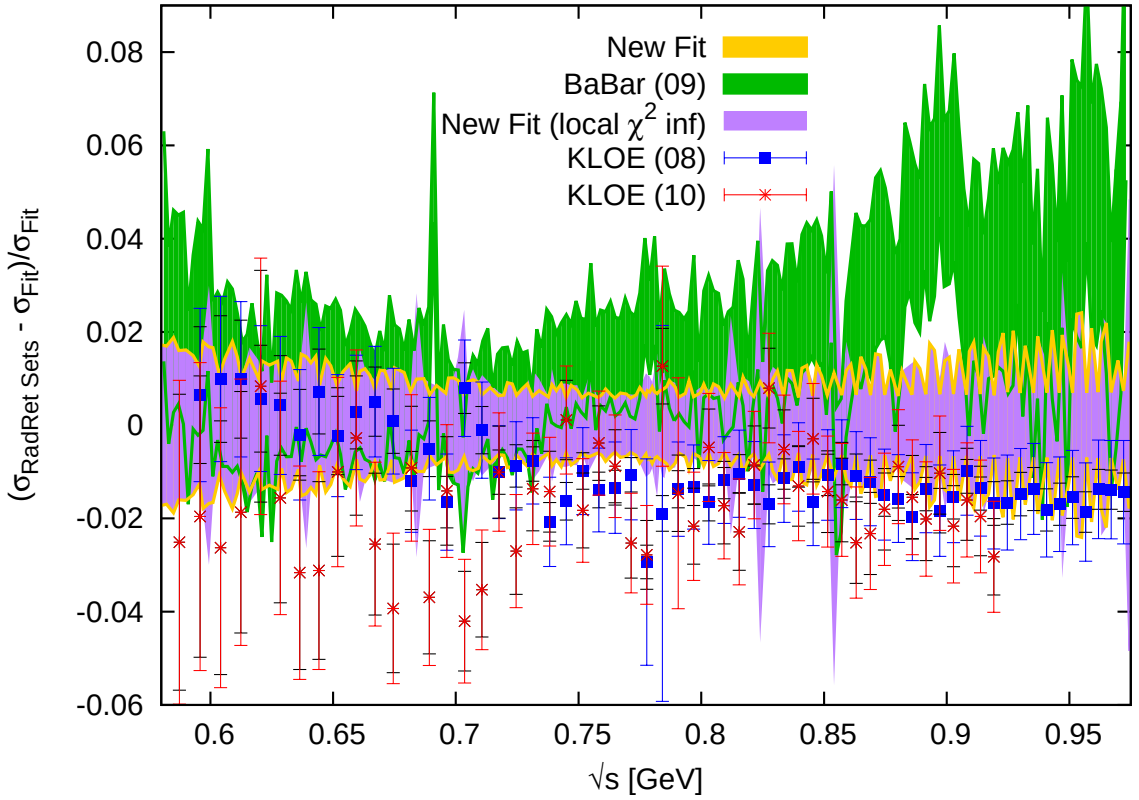


Overall, the data combination incl. 'Direct Scan' and 'Radiative Return' looks fine, but...

Radiative Return $\pi\pi(\gamma)$ data [KLOE 08/10 and BaBar 09] compared to combination of all



Normalised difference of cross sections [HLMNT '11]



→ Radiative Return (at fixed e^+e^- energy) a powerful method, *complementary to direct energy scan*

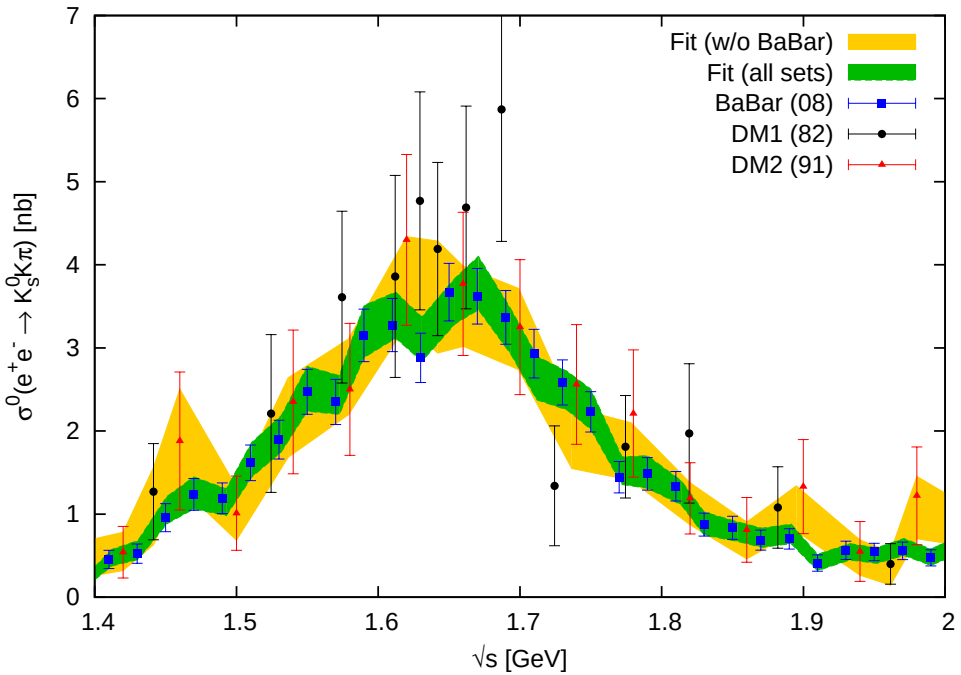
⇝ Differences in shape and BaBar high at medium and higher energies

⇝ limited gain in accuracy due to 'tension'; pull-up (mainly from BaBar)

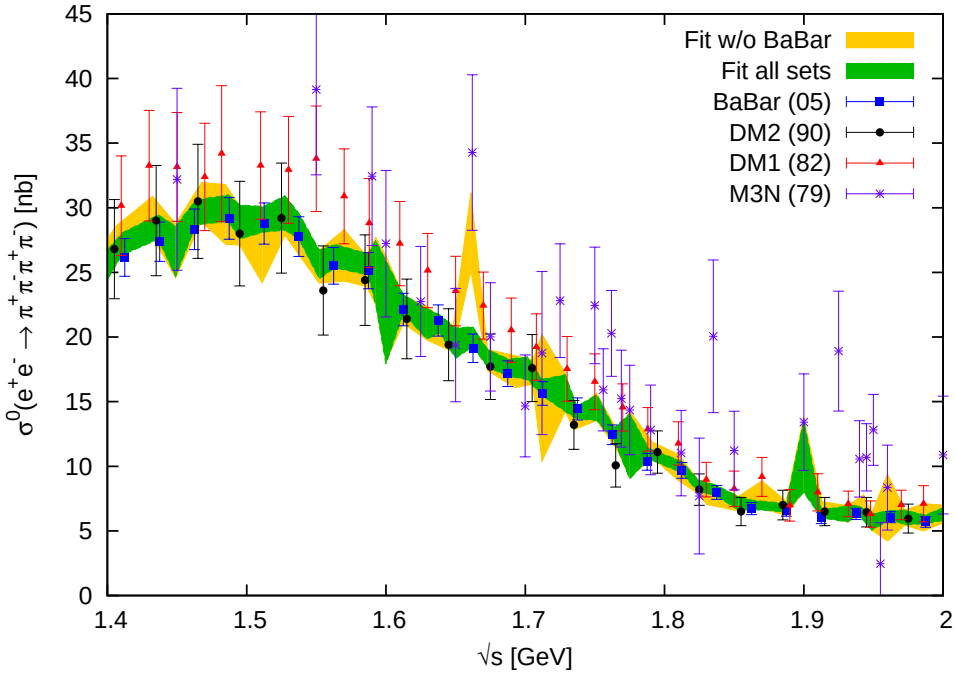
- Comb. of all data on same footing, before integration (purple band): still good $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.5$ of fit
- $a_\mu^{2\pi}(0.32 - 2\,\text{GeV}) = (\mathbf{504.2 \pm 3.0}) \cdot 10^{-10}$, $a_\mu^{2\pi,\text{w/outRad. Ret.}} = (\mathbf{498.7 \pm 3.3}) \cdot 10^{-10}$
  ⇝ Pull-up of a_μ from Rad. Ret. by $\sim \mathbf{5.5}$; and: comp. w. DHMZ: Their $a_\mu^{2\pi}$ is higher by about $\mathbf{2.1}$ units.
- Clarification/improvement with more, possibly even more precise data (from both scan and ISR) needed!

Region below 2 GeV: impact of recent BaBar Rad. Ret. analyses

$K^0_S K\pi$ channel



$2\pi^+2\pi^-$ channel

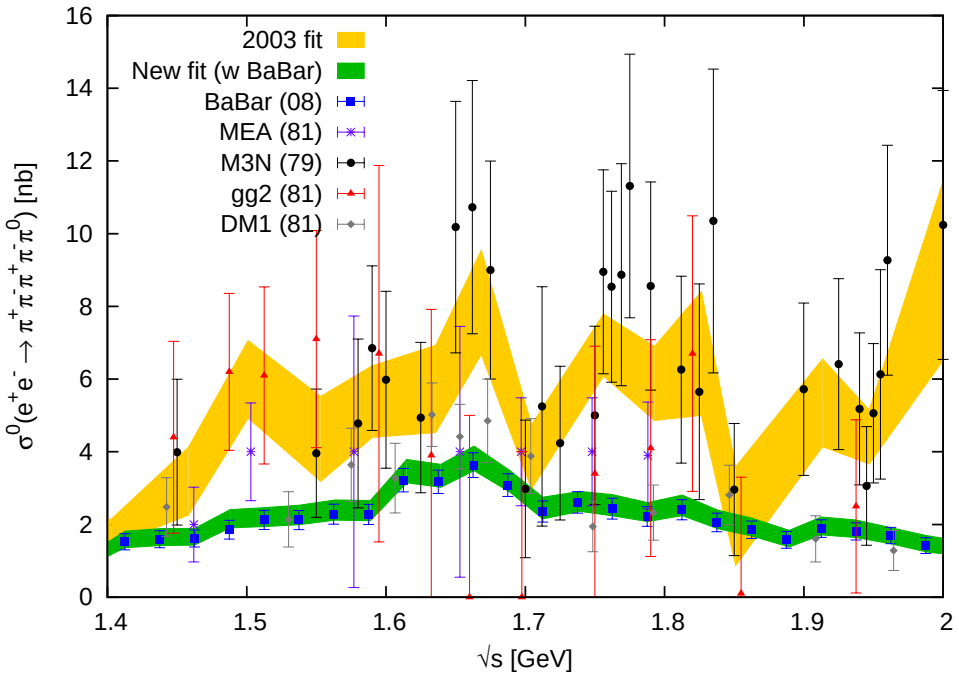


→ Big improvements over earlier data compilations in many channels.

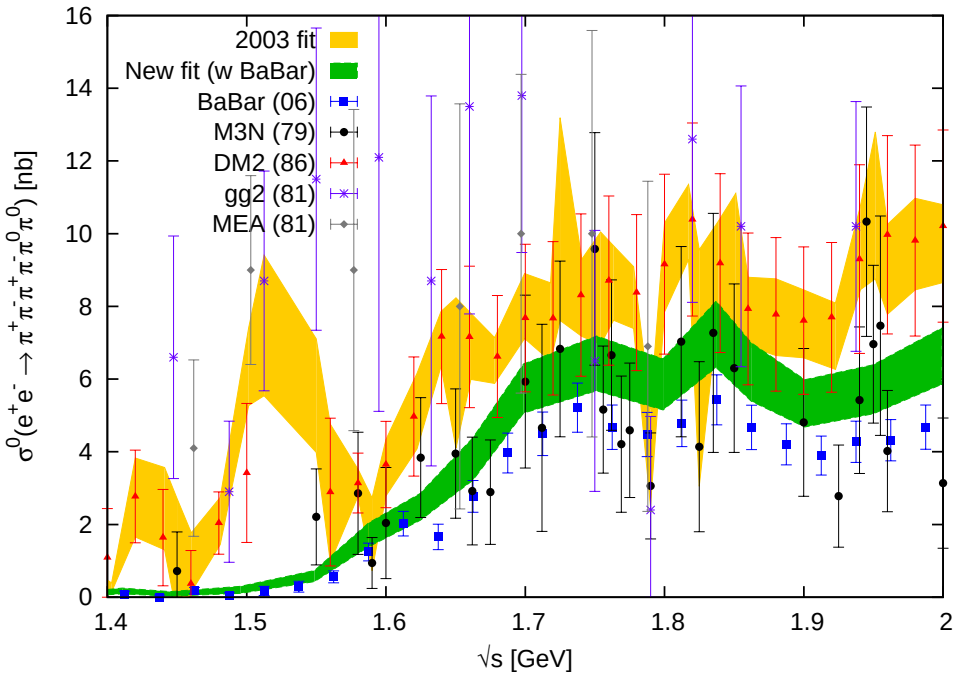
BaBar Radiative Return data lower than less precise older data in most channels.

Region below 2 GeV: impact of recent BaBar Rad. Ret. analyses (contd.)

$2\pi^+2\pi^-\pi^0$ channel



$2\pi^+2\pi^-2\pi^0$ channel



→ Error 'inflation' needed when data inconsistent,

e.g. BaBar lower than previous measurements in $2\pi^+2\pi^-2\pi^0$ channel:

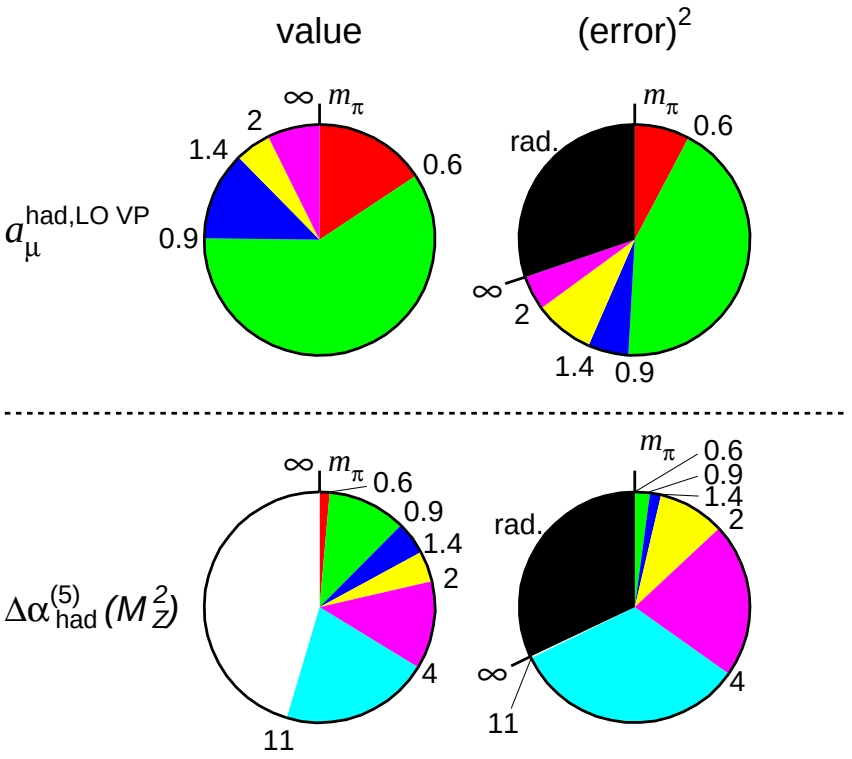
⇝ HLMNT: Errors for $g-2$ inflated by local $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}}$ [global $\chi^2_{\rm min}/{\rm d.o.f.} = 4$]

Future improvements: energy regions; experiments

- New $g-2$ experiments at Fermilab and J-PARC.
- Will $a_\mu^{\rm SM}$ match the planned accuracy? $\rightsquigarrow$ L-by-L may become the limiting factor.
  But at present Hadronic VP still contributes the biggest error in $a_\mu^{\rm SM}$.
- Contributions from energy regions: Pie diagrams for contr. to a_μ and $\alpha(M_Z)$ and their errors^2

$\longrightarrow$ Expected sources for new data:

- More Rad. Ret. in progress at KLOE
- Great opportunity for KLOE-2, BELLE, Super $\tau-c$, in a few years SUPER-Bs, also strong case for DAFNE-HE
- Big improvement envisaged with CMD-3 and SND at VEPP2000
- Higher energies: BES-III at BEPCII in Beijing is on; KEDR at VEPP-4M



Importance of various 'channels'

[Numbers from HLMNT, 'local error infl.', $\cdot 10^{-10}$]

- Errors contributions to a_μ from leading and subleading channels (ordered) up to 2 GeV

Purely from data:

channel	error
$\pi^+\pi^-$	3.09
$\pi^+\pi^-\pi^0\pi^0$	1.26
3π	0.99
$2\pi^+2\pi^-$	0.47
K^+K^-	0.46
$2\pi^+2\pi^-2\pi^0$	0.24
$K^0_SK^0_L$	0.16

'Higher multiplicity' region from 1.4 to 2 GeV with use of isospin relations for some channels:
[Use of old inclusive data disfavoured.]

Channel	contr. $\pm$ error
$K\bar{K}2\pi$	3.31 ± 0.58
$\pi^+\pi^-4\pi^0$	0.28 ± 0.28
$\eta\pi^+\pi^-$	0.98 ± 0.24
$K\bar{K}\pi$	2.77 ± 0.15
$2\pi^+2\pi^-\pi^0$	1.20 ± 0.10

- 'Inclusive' region from 2 to ~ 11 GeV: $\mathbf{41.19 \pm 0.82}$

  Can be 'squeezed' by using pQCD (done by DHMZ from 1.8 GeV);

  region from 2 to 2.6 GeV: $\mathbf{15.69 \pm 0.63} \;\rightarrow\; \mathbf{14.49 \pm 0.13}$, only small changes for higher energies.

- $J/\psi + \psi'$: $\mathbf{7.80 \pm 0.16}$
- Note: All these errors are smaller than the difference 'with vs. w/out Rad. Ret.'
- Differences between HLMNT and DHMZ mostly cancel in the sum of channels, but are sizeable particularly in 2π and 3π. Other analyses?
- Iron out differences via combined effort for RMCL WG's 'best estimate' for a_μ? Feasible or misleading?
- Role of data to better constrain light-by-light?

$\rightarrow$ Need to discuss:

  — how to achieve required improvements;

  — scope for 'inclusive' measurement (at least for higher-multiplicity final states);

  — contributions within RMCL WG?