Complete one loop corrections to $e^+e^- \to \mu^+\mu^-\gamma$ in PHOKHARA generator

Michal Gunia

Working Group 'Radio Monte CarLow',Frascati

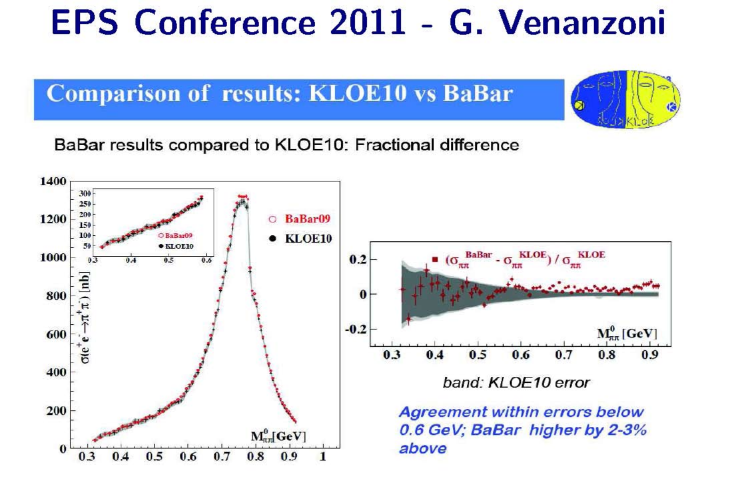
15 April 2012

Collaboration

F. Campanario, (KIT, Karlsruhe)
H. Czyz, (Silesia U.)
J. Gluza, (Silesia U.)
M. Gunia, (Silesia U.)
T. Riemann, (DESY, Zeuthen)
G. Rodrigo, (U. de Valencia)
V. Yundin, (U. of Copenhagen)

Table of contents

Motivation:Measure of the cross section for the process $e^+e^- \to \Pi^+\Pi^-$

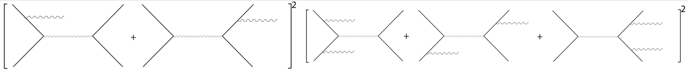


Aim of the work

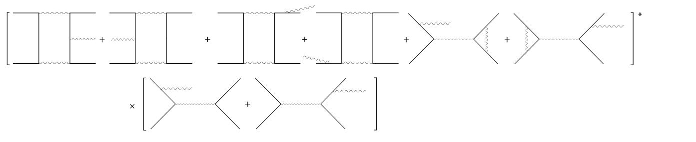
- calculation of the one loop corrections for the process $e^+e^- \to \mu^+\mu^-\gamma$ with PHOKHARA MC generator;
- discussion of the numerical results for energies and with cuts near the real one for KLOE experiment;
- comparison complete calculations with results from previous version of PHOKHARA;
- additional tests of agreement between two independent applications for calculating virtual corrections;

Corrections to $e^+e^- \to \mu^+\mu^-\gamma$ in PHOKHARA generator

- real one photon emission (Born) and two real photon emission (NLO): soft + hard:



- virtual corrections:



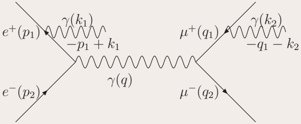
What's new in PHOKHARA for $e^+e^- \to \mu^+\mu^-\gamma$?

- FSR emission of two hard photon, full amplitude for processes with one ISR photon and one FSR, interferences between all diagrams with emission of two photons;
- full soft part when one photon is soft and one is hard;
- all pentagons, box diagrams with radiation from final state and all interferences between box diagrams and Born;
- full corrections with triangles.

Tests in PHOKHARA for $e^+e^- \to \mu^+\mu^-\gamma$

- test for the agreement between trace and helicity method for amplitudes with hard emission of two photons: for 2 FSR photon and 1 FSR and 1 ISR;
- tests for random points from PHOKHARA for soft part for analytical formula and numerical integral;
- tests for soft part for agreement between quad and double precision for analytical formula;
- tests for cut independence between hard and soft part ;
- tests for agreement between two independent subroutines for all virtual corrections;

Soft part



$$\sigma_{soft} = -\frac{\alpha}{4\pi} \int \frac{d^3k_1}{E_{k_1}} \left[\frac{p_1}{p_1 \cdot k_1} - \frac{p_2}{p_2 \cdot k_1} + \frac{q_2}{q_2 \cdot k_1} - \frac{q_1}{q_1 \cdot k_1} \right]^2 \sigma_{BORN} \quad (1)$$

In our frame (incoming particles are in centre of mass frame) it was necessary to calculate soft part independently. Solution is not trivial to present and contains set of dilogarithms.

Solution - example

$$Int1 = \int \frac{|\overline{k_1}|^2 dk_1 d\varphi dcos\phi}{\sqrt{|\overline{k_1}|^2+\lambda^2}} \frac{p_1 \cdot q_2}{(k_1 \cdot p_1)(k_1 \cdot q_2)} = const1 \int_{\hat{t_a}}^{\hat{t_b}} d\hat{t}[\frac{A_1}{\hat{t}-\frac{1}{\hat{t}_3}} + \frac{B_1}{\hat{t}-\frac{1}{\hat{t}_4}} + \frac{C_1}{\hat{t}+\hat{t}_3} + \frac{D_1}{\hat{t}+\hat{t}_4}]$$

$$\cdot log\left(\frac{((E_{q_2}-E_{p_1})-\sqrt{a})(\hat{t}-\frac{1}{\hat{t}_3})(\hat{t}-\frac{1}{\hat{t}_4})}{((E_{q_2}-E_{p_1})+\sqrt{a})(\hat{t}+\hat{t}_3)(\hat{t}+\hat{t}_4)}\right) = \quad (2)$$

$$const1 * \{A_1 * [log(-const2)log(\hat{t}-\frac{1}{\hat{t}_3}) + \frac{1}{2}log^2(\hat{t}-\frac{1}{\hat{t}_3}) + log(\frac{1}{\hat{t}_4}-\frac{1}{\hat{t}_3})log(\hat{t}-\frac{1}{\hat{t}_3}) - Li_2(\frac{-\frac{1}{\hat{t}_3}+\hat{t}}{\frac{1}{\hat{t}_4}-\frac{1}{\hat{t}_3}}) \quad (3)$$

$$-(log(-\hat{t}_3-\frac{1}{\hat{t}_3})log(\hat{t}-\frac{1}{\hat{t}_3}) - Li_2(\frac{-\frac{1}{\hat{t}_3}+\hat{t}}{-\hat{t}_3-\frac{1}{\hat{t}_3}})) - (log(-\hat{t}_4-\frac{1}{\hat{t}_3})log(\hat{t}-\frac{1}{\hat{t}_3}) - Li_2(\frac{-\frac{1}{\hat{t}_3}+\hat{t}}{-\hat{t}_4-\frac{1}{\hat{t}_3}}))] \quad (4)$$

$$+B_1[log(-const2)log(\hat{t}-\frac{1}{\hat{t}_4}) + log(-\frac{1}{\hat{t}_3}+\frac{1}{\hat{t}_4})log(-\frac{1}{\hat{t}_4}+\hat{t}) - Li_2(-\frac{-\frac{1}{\hat{t}_4}+\hat{t}}{-\frac{1}{\hat{t}_3}+\frac{1}{\hat{t}_4}}) \quad (5)$$

$$+\frac{1}{2}log^2(\hat{t}-\frac{1}{\hat{t}_4}) - (log(\hat{t}-\frac{1}{\hat{t}_4})log(\hat{t}+\hat{t}_3)|_{\hat{t_a}}^{\hat{t_b}} - (log(-t_3-\frac{1}{\hat{t}_4})log(\hat{t}+\hat{t}_3)$$

$$-Li_2(-\frac{\hat{t}+\hat{t}_3}{-t_3-\frac{1}{\hat{t}_4}}))) - (log(-\hat{t}_4-\frac{1}{\hat{t}_4})log(-\frac{1}{\hat{t}_4}+\hat{t}) - Li_2(\frac{-\frac{1}{\hat{t}_4}+\hat{t}}{-\hat{t}_4-\frac{1}{\hat{t}_4}}))] + C_1[\ldots] + D_1[\ldots]\}_{\hat{t_a}}^{\hat{t_b}} \quad (6)$$

Tests

Cut independence

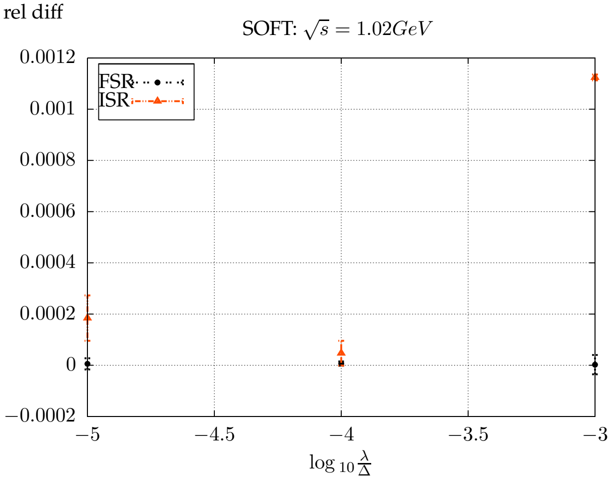
"KLOE cuts":

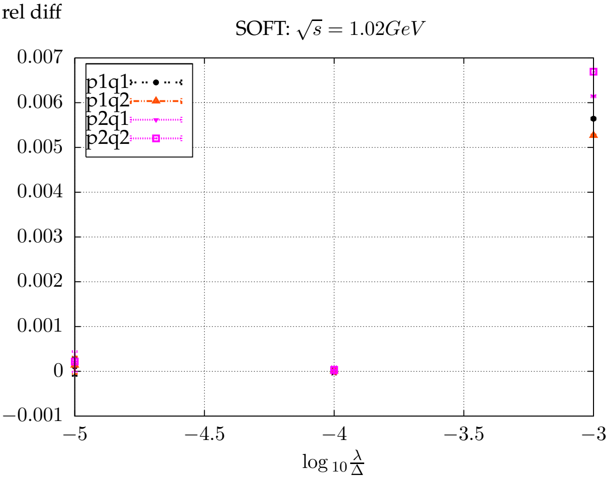
- outcoming tracks between 50 and 130 degree;
- missing photon angle<15 deg (>165 deg)
- 80<mtrk<115 MeV
- minimal photon energy/missing energy 0.01d0 GeV

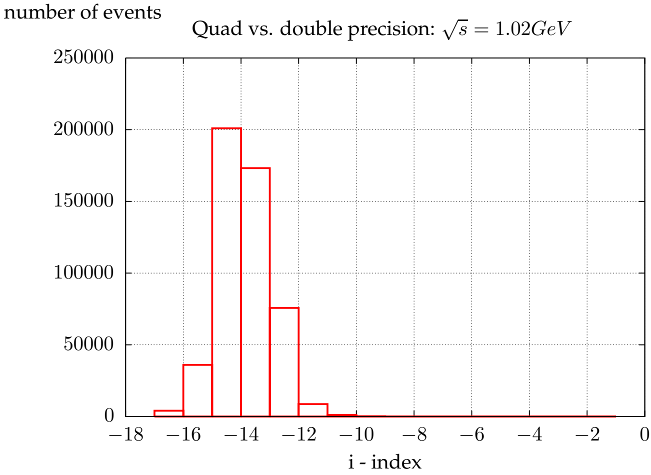
Result:
$\omega = 10^{-4}, \sigma_1 = 5.187(2)$nb
$\omega = 10^{-5}, \sigma_2 = 5.186(2)$nb
$\frac{\sigma_1 - \sigma_2}{\sigma_1} = 0,0002(7)$

Tests

Next slides: Two plots presenting relative difference for random point from PHOKHARA between analytical and numerical expressions ($\frac{f_{analyt} - f_{num}}{f_{analyt}}$) for soft formula as a function of ratio $\frac{\lambda}{\Delta}$.
Third plot shows histogram for relative difference between quad and double precision for analytical soft formula (($\frac{f_{quad} - f_{dble}}{f_{quad}}$)). X (i-index) - axis shows intervals of errors (in logarithm).


rel diff
SOFT: $\sqrt{s} = 1.02 GeV$
FSR
ISR
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
−0.0002
−5
−4.5
−4
−3.5
−3
$\log_{10} \frac{\lambda}{\Delta}$


rel diff
SOFT: $\sqrt{s} = 1.02 GeV$
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
−0.001
−5
−4.5
−4
−3.5
−3
$\log_{10} \frac{\lambda}{\Delta}$
p1q1
p1q2
p2q1
p2q2


number of events
Quad vs. double precision: $\sqrt{s} = 1.02 GeV$
250000
200000
150000
100000
50000
0
−18
−16
−14
−12
−10
−8
−6
−4
−2
0
i - index

Virtual corrections

Two independent versions of subroutines for pentagons and box diagrams were used for tests and preliminary calculations:

- quad precision version (F. Campanario, G. Rodrigo);
- double precision version (V. Yundin, T. Riemann);

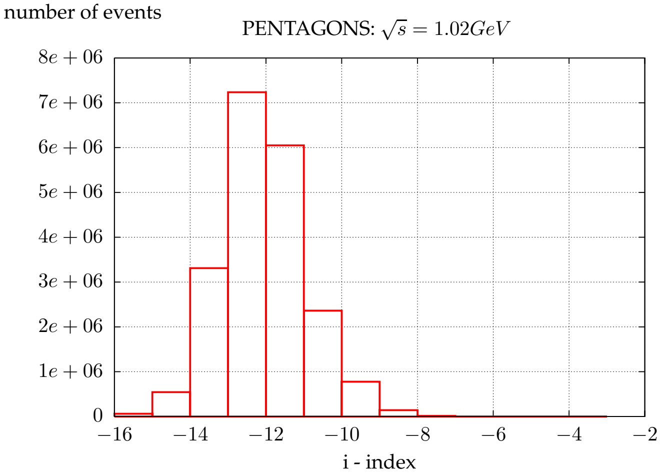
Two independent versions of subroutines for triangle diagrams were checked:

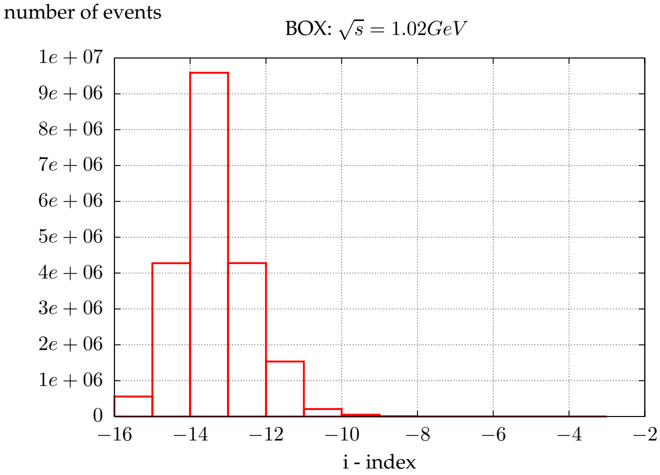
- extended version of PHOKHARA subroutine for triangle diagrams (double precision);
- double precision version (V. Yundin);

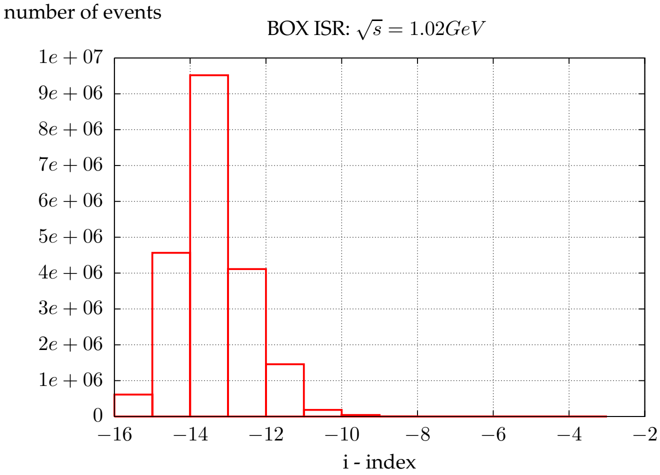
They were compared in points obtained from MC PHOKHARA generator.

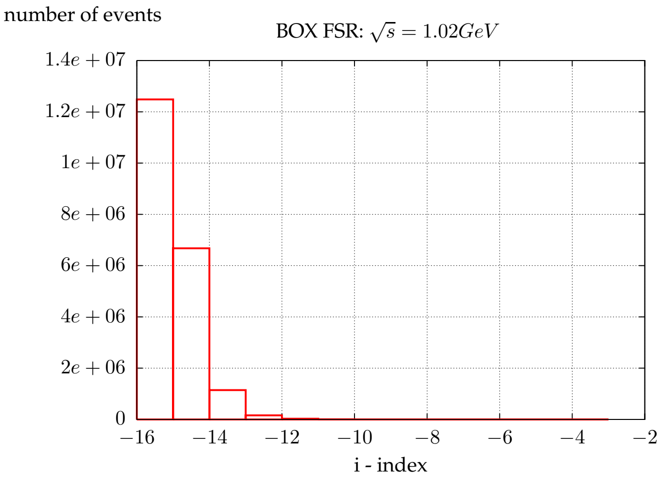
Tests

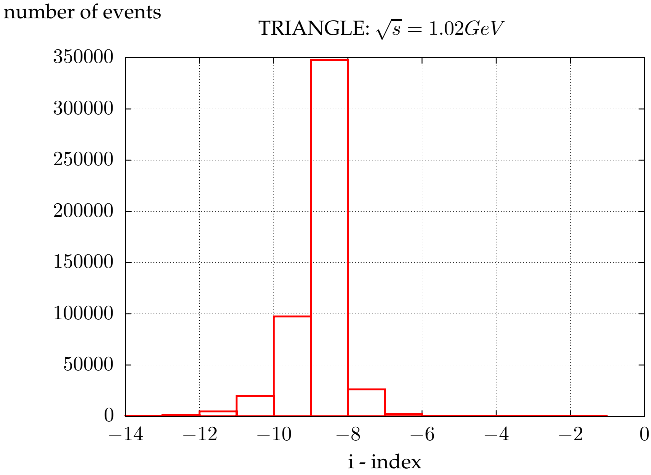
Next slides:
Plots for the pentagons and box diagrams shows histogram for relative difference between two different subroutines $((\frac{f_{Campanario}-f_{Yundin}}{f_{Campanario}}))$. Plot for triangles shows histogram for relative difference between two different subroutines $((\frac{f_{PHOKHARA}-f_{Yundin}}{f_{PHOKHARA}}))$.
X (i-index) - axis shows intervals of errors (in logarithm).


number of events
PENTAGONS: $\sqrt{s} = 1.02 GeV$
8e + 06
7e + 06
6e + 06
5e + 06
4e + 06
3e + 06
2e + 06
1e + 06
0
−16
−14
−12
−10
−8
−6
−4
−2
i - index


number of events
BOX: $\sqrt{s} = 1.02 GeV$
1e + 07
9e + 06
8e + 06
7e + 06
6e + 06
5e + 06
4e + 06
3e + 06
2e + 06
1e + 06
0
−16
−14
−12
−10
−8
−6
−4
−2
i - index


number of events
BOX ISR: $\sqrt{s} = 1.02 GeV$
$1e+07$
$9e+06$
$8e+06$
$7e+06$
$6e+06$
$5e+06$
$4e+06$
$3e+06$
$2e+06$
$1e+06$
0
-16
-14
-12
-10
-8
-6
-4
-2
i - index

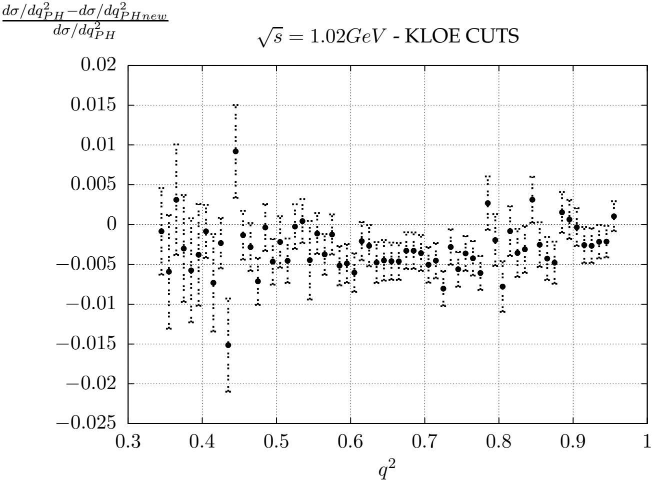

number of events
BOX FSR: $\sqrt{s} = 1.02 GeV$
1.4e + 07
1.2e + 07
1e + 07
8e + 06
6e + 06
4e + 06
2e + 06
0
−16
−14
−12
−10
−8
−6
−4
−2
i - index


number of events
TRIANGLE: $\sqrt{s} = 1.02 GeV$
350000
300000
250000
200000
150000
100000
50000
0
−14
−12
−10
−8
−6
−4
−2
0
i - index

First results

KLOE cuts		
[nb]	PHOKHARA: σ_{PH}	PHOKHARA+new corr.: σ_{PHnew}
1 photon	2.7233(5)	2.708(1)
2 photon	2.4768(5)	2.479(1)
sum	5.2001(6)	5.187(1)
difference		0.013(1)
$\frac{\sigma_{PH}-\sigma_{PHnew}}{\sigma_{PH}}$		2.5(3)‰

$\sqrt{s} = 1.02 GeV$ **no cuts**	
$\frac{\sigma_{PH}-\sigma_{PHnew}}{\sigma_{PH}}$	4.6(9)‰


$\frac{d\sigma/dq^2_{PH} - d\sigma/dq^2_{PHnew}}{d\sigma/dq^2_{PH}}$
$\sqrt{s} = 1.02 GeV$ - KLOE CUTS
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
−0.025
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
q^2

Conclusions

- additional corrections inside PHOKHARA MC generator seems to change result at the level of few ‰;
- better precision is required for physical results;
- further simulations in progress;
- as far all tests look right;
- plans: new public version of generator: PHOKHARA8 (soon).