

Impact of the “untagged” invariant on the single-tag $e^+e^- \rightarrow e^+e^- \mathcal{P}$ cross-section (arXiv:1202.1171 + ...)

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April 17, 2012



Structure of this talk

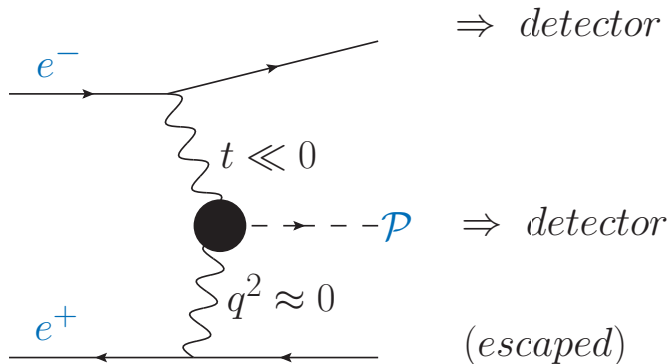
- 1 A few words about “single-tag” experiments
- 2 Is it in fact the $F(Q^2, 0)$ that is measured?
- 3 Can one measure $F(Q^2, q^2)$?

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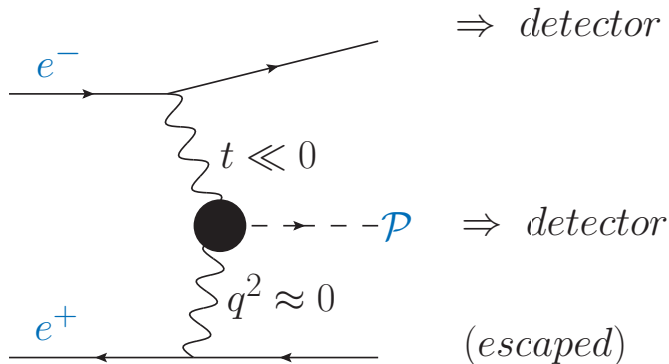
3 Can one measure $F(Q^2, q^2)$?

Single-tag $e^+ e^- \rightarrow e^+ e^- \mathcal{P}$



The first invariant, $Q^2 = -t$, is associated with the detected lepton

Single-tag $e^+ e^- \rightarrow e^+ e^- \mathcal{P}$

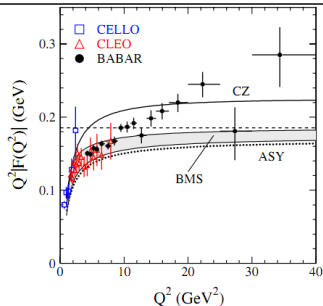


The second invariant, q^2 , is associated with the missing lepton and can be constrained by the event selection

The BaBar's π^0 case

B. AUBERT *et al.*

PHYSICAL REVIEW D **80**, 052002 (2009)



✓ Data/theory discrepancy

⇒ Tons of debates...

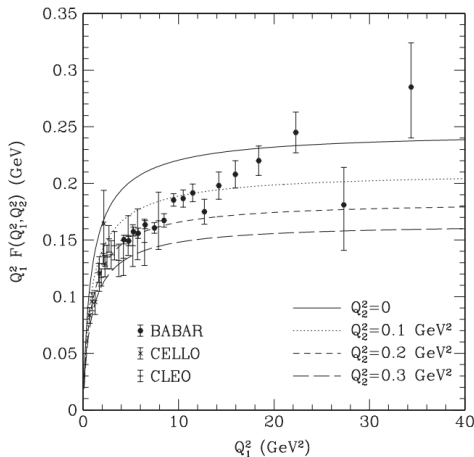
FIG. 23 (color online). The $\gamma\gamma^* \rightarrow \pi^0$ transition form factor multiplied by Q^2 . The dashed line indicates the asymptotic limit for the form factor. The solid and dotted lines show the predictions for the form factor [8] for the CZ [26] and asymptotic (ASY) [27] models for the pion distribution amplitude, respectively. The shaded band represents the prediction for the BMS [28] pion DA model.

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Discussion within Vector meson dominance

Peter Lichard

PHYSICAL REVIEW D **83**, 037503 (2011)



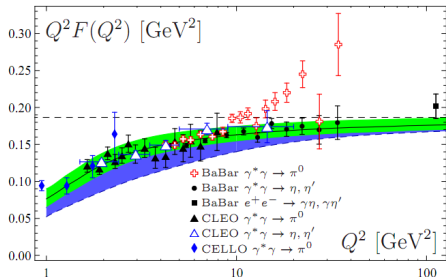
Is it in fact $F(Q^2, 0)$ that is measured?

FIG. 3. Dependence of $Q_1^2 F(Q_1^2, Q_2^2)$ on Q_1^2 for four fixed values of Q_2^2 calculated in the full version of our model. The CELLO [10], CLEO [9], and BABAR [1] data are also shown.

Discussion within Light-cone sum rules

N. G. Stefanis,^{1,†} A. P. Bakulev,^{2,‡} S. V. Mikhailov,^{2,§}
and A. V. Pimikov^{2,3,¶}

arXiv:1202.1781v2 [hep-ph]



Is it in fact $F(Q^2, 0)$ that is measured?

Figure 2: (color online). Theoretical predictions for the scaled $\gamma^* \gamma \pi^0$ transition form factor in comparison with data taken from various experiments, as indicated, using the theoretical framework described in the text. The upper (green) strip shows the results obtained in [5] using the method described in the text. The lower (blue) strip represents the influence of the small virtuality of the quasi-real photon induced by the untagged electron.

The form factor and the cross section

$$d\sigma_{avg}(e^+e^- \rightarrow e^+e^-\pi^0) = \frac{1}{4} \frac{1}{2s} d\text{Lips}_3 \sum |\mathcal{M}|^2$$

$$\begin{aligned} \mathcal{M} = & -\frac{4i\alpha^2}{f_\pi} F(t_1, t_2) \epsilon_{\mu\nu\alpha\beta} \frac{1}{t_1 t_2} (q_1 - p_1)^\alpha (q_2 - p_2)^\beta \\ & \times (\bar{v}(p_1) \gamma^\mu v(q_1)) (\bar{u}(q_2) \gamma^\nu u(p_2)) \end{aligned}$$

- The factor $1/t_2$ strongly suppresses the impact of the t_2 -dependence of $F(t_1, t_2)$ on the cross section

Measured quantity

$d\sigma_{avg}$ integrated within (quite complicated) cuts

The BaBar's comment on the uncertainty

B. AUBERT *et al.*

PHYSICAL REVIEW D **80**, 052002 (2009)

$$F^2(Q^2) = \frac{(d\sigma/dQ^2)_{\text{data}}}{(d\sigma/dQ^2)_{\text{MC}}} F_{\text{MC}}^2. \quad (8)$$

The calculated cross section $(d\sigma/dQ^2)_{\text{MC}}$ has a model-dependent uncertainty due to the unknown dependence on the momentum transfer to the untagged electron. We use a q_2^2 -independent form factor, which corresponds to the QCD-inspired model $F(q_1^2, q_2^2) \propto 1/(q_1^2 + q_2^2) \approx 1/q_1^2$ [23]. Using the vector dominance model with the form factor $F(q_2^2) \propto 1/(1 - q_2^2/m_\rho^2)$, where m_ρ is ρ meson mass, leads to a decrease of the cross section by 3.5%. This difference is considered to be an estimate of model uncertainty due to the unknown q_2^2 dependence. However, it should be noted that this estimate depends strongly on the limit on q_2^2 . The value of 3.5% is obtained with $|q_2^2| < 0.18 \text{ GeV}^2$. For a less stringent q_2^2 constraint, for example $|q_2^2| < 0.6 \text{ GeV}^2$, the difference between the calculated cross sections reaches 7.5%.

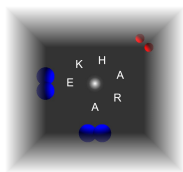
⇒ Yes, it *is indeed* the $F(Q^2, 0)$

✓ The related uncertainty was estimated

⇒ Thus, the fuss about nothing?

- We performed a simulation of model-dependent and q^2 -dependent effects in the cross section
(for details see `arXiv:1202.1171`)

EKHARA 2.1



H. Czyż, S. Ivashyn,
Comp.Phys.Comm., 182, 1338 (2011)

$$e^+ e^- \rightarrow e^+ e^- \pi^0$$

$$e^+ e^- \rightarrow e^+ e^- \eta$$

$$e^+ e^- \rightarrow e^+ e^- \eta'$$



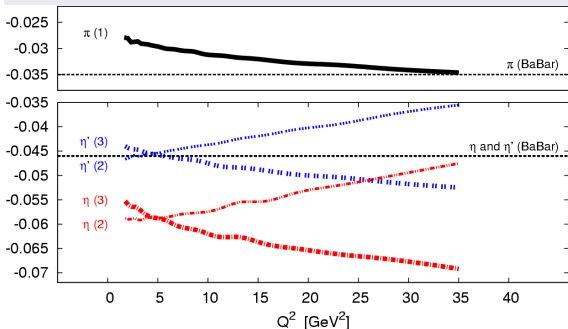
“realistic” form factors

H. Czyż, S. Ivashyn,
A. Korchin, O. Shekhovtsova
arXiv:1202.1171, to appear in Phys.Rev.D

<http://prac.us.edu.pl/%7Eekhara>

The relative difference of the cross sections

$$(d\sigma[full] - d\sigma[approx]) / d\sigma[full]$$

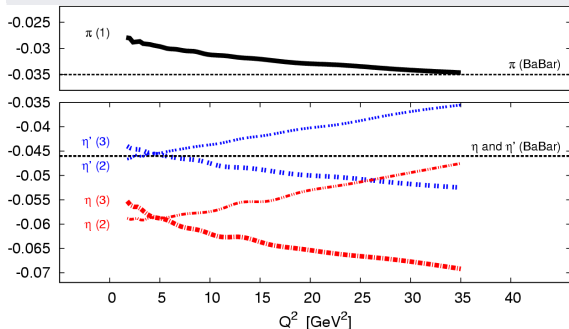


← how good is the approximation
 $F(Q^2, q^2) \approx F(Q^2, 0)$

- $d\sigma[approx]: F(Q^2, q^2) \approx F(Q^2, 0)$
- $d\sigma[full]$ accounts for full $F(Q^2, q^2)$

The relative difference of the cross sections

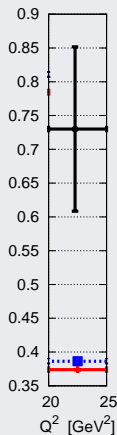
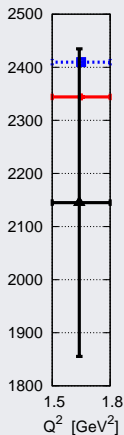
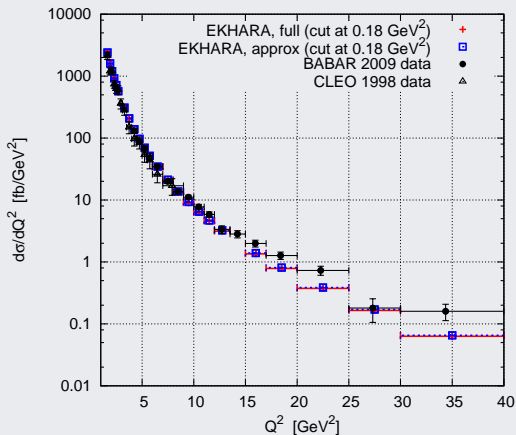
$$(d\sigma[full] - d\sigma[approx])/d\sigma[full]$$



← how good is the approximation
 $F(Q^2, q^2) \approx F(Q^2, 0)$

- Cuts :
- (1) $|q_2^2| < 0.18 \text{ GeV}^2$
 - (2) $|\cos \theta_{eP}^*| > 0.99$ and $|q_2^2| < 0.6 \text{ GeV}^2$
 - (3) $|q_2^2| < 0.38 \text{ GeV}^2$

$\frac{d\sigma}{dQ^2}$: full, approximate and data



Cuts: $|q_2^2| < 0.18 \text{ GeV}^2$

Our remarks (summary)

- The impact of the approximation $F(Q^2, q^2) \approx F(Q^2, 0)$ on the cross section was estimated and accounted for by BaBar
- our simulation of π^0 cross section leads to a similar estimate of uncertainty

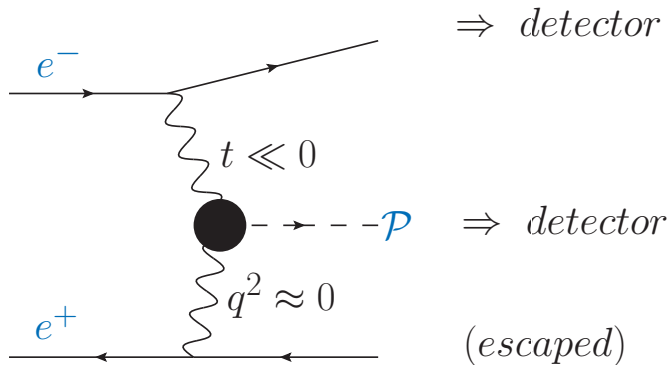
[for details see [arXiv:1202.1171](https://arxiv.org/abs/1202.1171)]

- The statistical error at BaBar was bigger than the above uncertainty

The issue of $F(Q^2, q^2) \approx F(Q^2, 0)$ is **not a reason** for the data/theory discrepancy

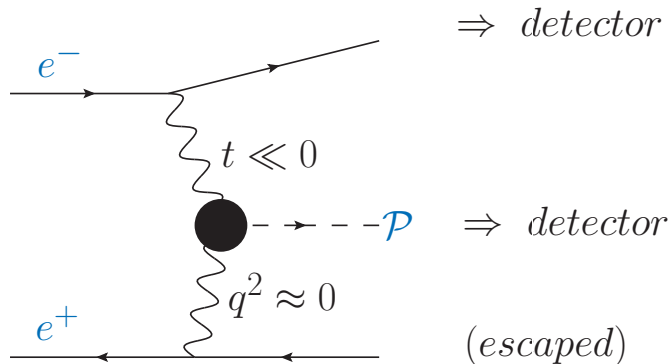
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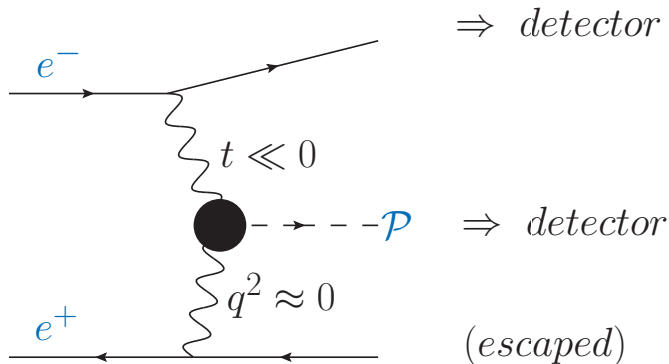
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Let's vary the q^2 cut \Rightarrow a couple of bins in q^2

Single-tag $e^+ e^- \rightarrow e^+ e^- \mathcal{P}$



Let's vary the q^2 cut \Rightarrow a couple of bins in q^2

- **How many events one could expect?**

- $d\sigma$ drops down rapidly with both Q^2 and q^2
- Q^2 is “scanned” by the detected lepton
 - ▶ the bulk of the events are in the first bins
 - ▶ \Rightarrow gives the Q^2 range
- q^2 is cut by the the event selection (“missing lepton angle, etc.”)

BaBar: $|q^2| < 0.18 \text{ GeV}^2$ BES-III: ¹ $|q^2| < 0.07 \text{ GeV}^2$

 - ▶ let’s have a couple of bins in q^2 around this cut
 - \Rightarrow “split” the existing cut
 - \Rightarrow “expand” the existing cut

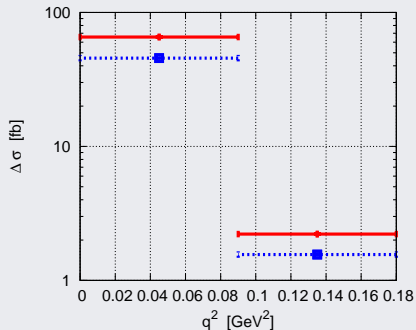
¹Our guesstimate

BaBar energy. Integrated cross section

$$4.0 \text{ GeV}^2 < Q^2 < 4.5 \text{ GeV}^2 \quad \color{red}\blacklozenge$$

$$4.5 \text{ GeV}^2 < Q^2 < 5.0 \text{ GeV}^2 \quad \color{blue}\blacksquare$$

EKHARA simulation ($e^+e^- \rightarrow e^+e^-\pi^0$) at $\sqrt{s}=10.60$ GeV

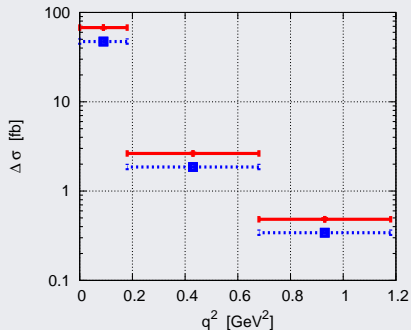


“split”

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“expand”

BaBar energy. Number of events

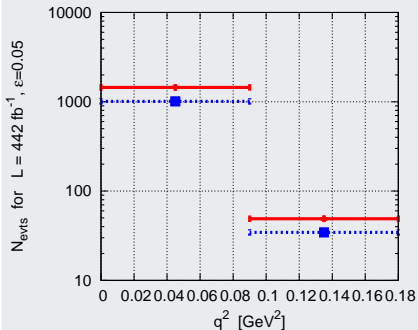
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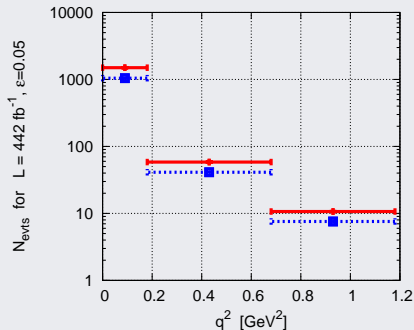
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“expand”

BES-III energy. Integrated cross section

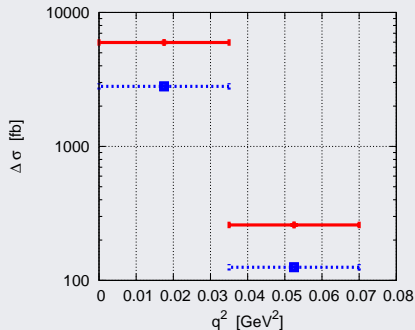
$$0.5 \text{ GeV}^2 < Q^2 < 0.7 \text{ GeV}^2 \quad \color{red}{\blacklozenge}$$

$$0.7 \text{ GeV}^2 < Q^2 < 0.9 \text{ GeV}^2 \quad \color{blue}{\blacksquare}$$

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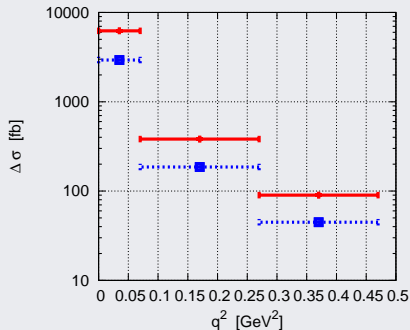
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EKHARA simulation ($e^+e^- \rightarrow e^+e^-\pi^0$) at $\sqrt{s}=3.77 \text{ GeV}$



“split”

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“expand”

BES-III energy. Number of events

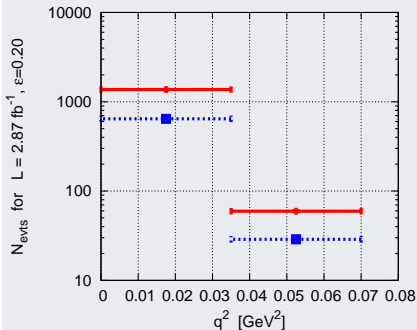
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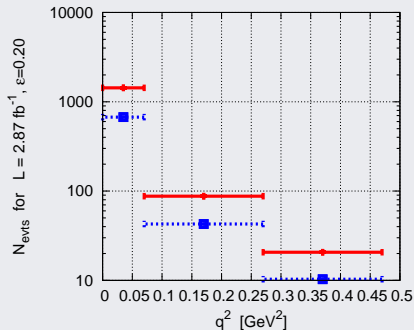
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EKHARA simulation ($e^+e^- \rightarrow e^+e^-\pi^0$) at $\sqrt{s}=3.77 \text{ GeV}$



“split”

EKHARA simulation ($e^+e^- \rightarrow e^+e^-\pi^0$) at $\sqrt{s}=3.77 \text{ GeV}$



“expand”

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- . . . but it indicates a reasonable statistics with the already existing integrated luminosity at BaBar and BES-III. . .

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- Our simulation is simplified . . .
- . . . but it indicates a reasonable statistics with the already existing integrated luminosity at BaBar and BES-III. . .
- . . . for a first couple of bins in Q^2 and q^2
- But this would already be a great achievement!

We suggest the experimentalists to study how the existing q^2 cuts could be “split” or “expanded”

Spare slides



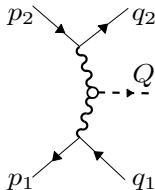
The form factor and the cross section

$$d \sigma_{avg}(e^+ e^- \rightarrow e^+ e^- \pi^0) = \frac{1}{4} \frac{1}{2s} d Lips_3 \sum |\mathcal{M}|^2$$

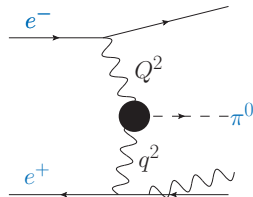
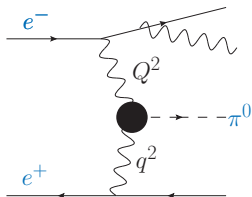
$d Lips_3$ is the differential 3-body Lorentz-invariant phase space

$$\begin{aligned} \mathcal{M} = & - \frac{4 i \alpha^2}{f_\pi} F(t_1, t_2) \epsilon_{\mu\nu\alpha\beta} \frac{1}{t_1 t_2} (q_1 - p_1)^\alpha (q_2 - p_2)^\beta \\ & \times (\bar{v}(p_1) \gamma^\mu v(q_1)) (\bar{u}(q_2) \gamma^\nu u(p_2)) \end{aligned}$$

The normalization is $F(0, 0) = 1$



- Experimental cuts on the the missing lepton are based on the assumption of 3-body final state in the signal process (please correct me, if I am wrong)
- A part of the radiative corrections — hard photon emission \Rightarrow 4-body final state



- How much does this radiative corrections “shift” the Q^2 , q^2 and other distributions?

At present time we are working on the implementation of the radiative corrections in EKHARA