Theoretical basis of the new form factors in TAUOLA and discussion on the errors

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In collaboration with T. Przedzinski, O. Shekhovtsova and Z. Was

e-Print: **arXiv:1203.3955** [hep-ph]

11th Radio MonteCarLow Meeting: Frascati, 16-17 April

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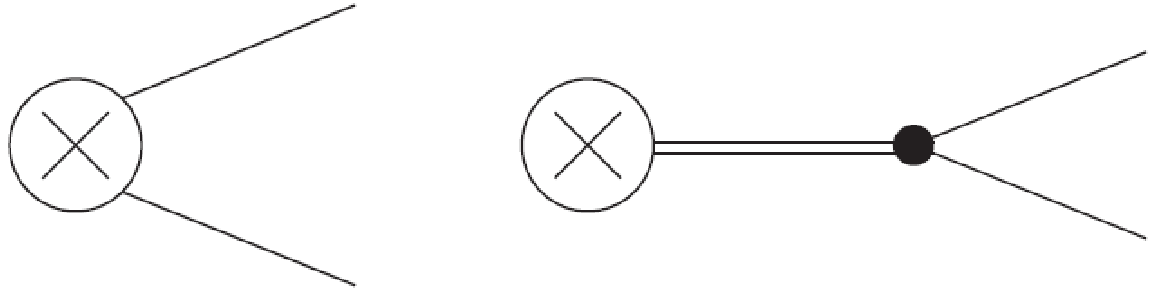
- Form factors in two meson τ decays
- Form factors in three meson τ decays
- Discussion on the errors
- Ongoing/Future improvements

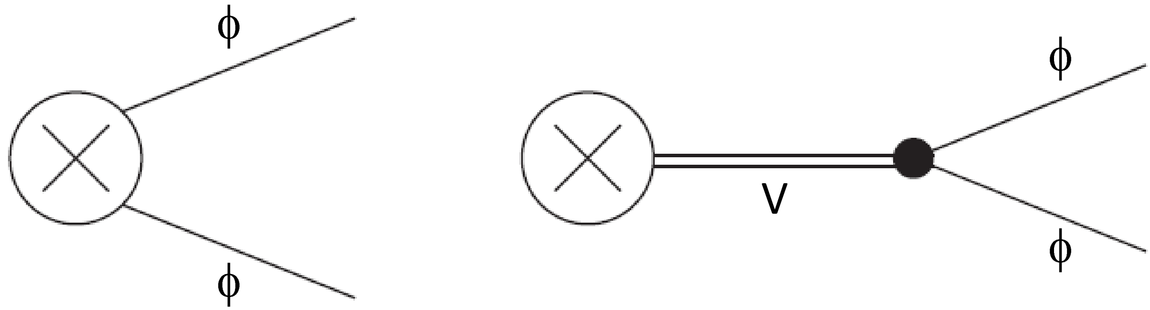
Form factors in two meson τ decays



Form factors in two meson τ decays

$\phi = \pi, K$

$V = \rho, K^*$



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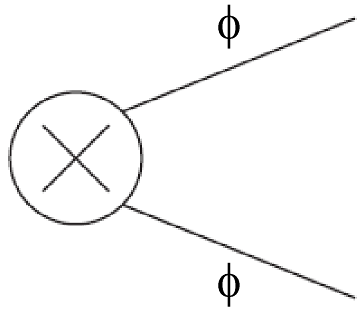
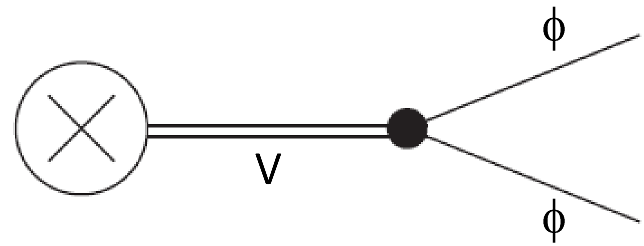


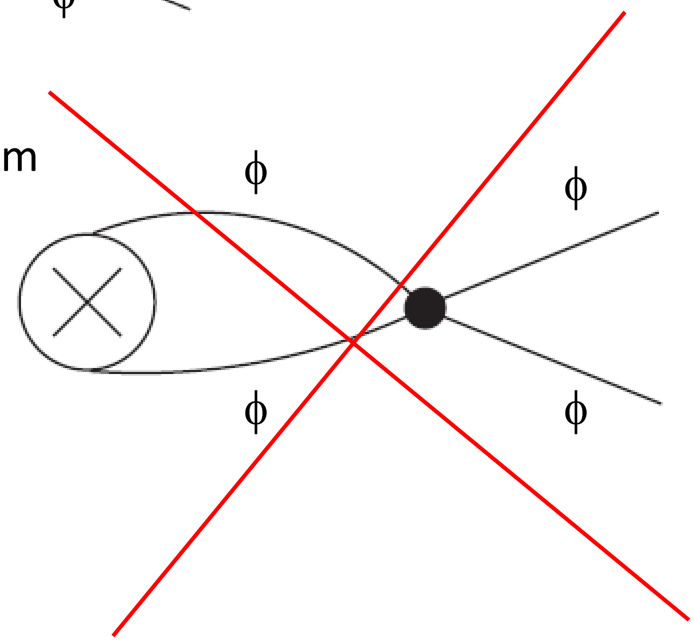


Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989



To avoid double counting

Form factors in two meson τ decays

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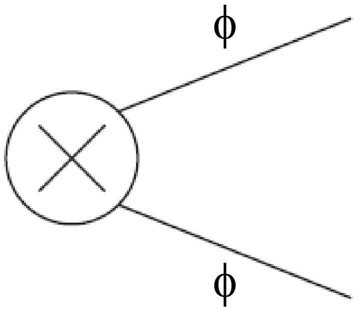

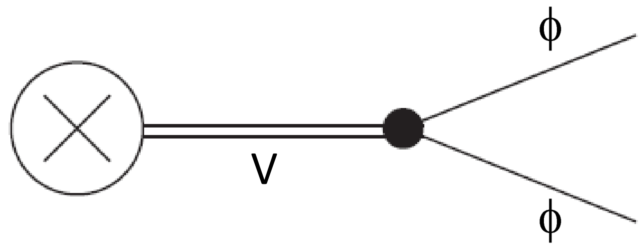

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$f_\pm^{\mu\nu} = u\, F_L^{\mu\nu}\, u^\dagger \pm u^\dagger\, F_R^{\mu\nu}\, u$$

$$u_\mu = i\,\{u^\dagger(\partial_\mu - ir_\mu)\,u - u\,(\partial_\mu - il_\mu)\,u^\dagger\}$$

$$u(\varphi) = \exp\left\{i\frac{\Phi}{\sqrt{2}\,F}\right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda_a\varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

Form factors in two meson τ decays

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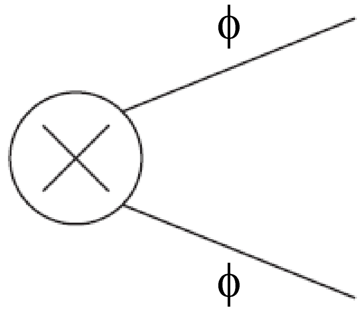

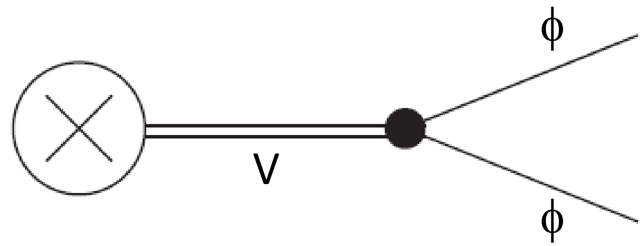

Ecker *et al.*

Phys.Lett.B223:425,1989

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$$F(s)^V = 1 + \frac{F_V G_V}{F^2}\frac{s}{M_V^2 - s}$$

Short-distance constraints

F(s) $\to 0$, for s$\to\infty$ $\Rightarrow$ $F_V G_V / f_\pi^2 = 1$ $\Rightarrow$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s}$$

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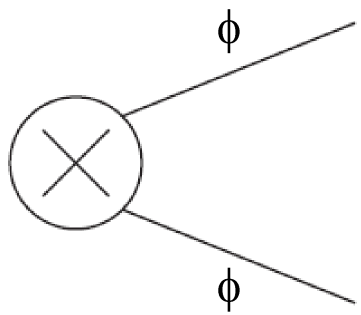

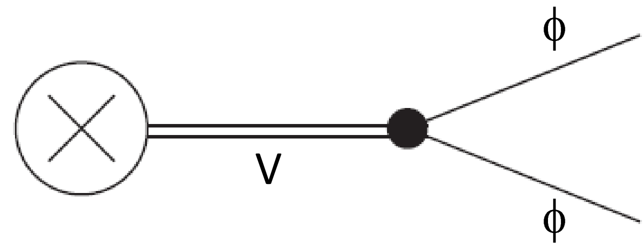

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$$\frac{1}{M_V^2 - s} \to \frac{1}{M_V^2 \left[1 + \sum_{P,Q} N_{loop}^{PQ} \frac{s}{96\pi^2 F^2} A_{PQ}(s)\right] - s}$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P + 1}{\sigma_P - 1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

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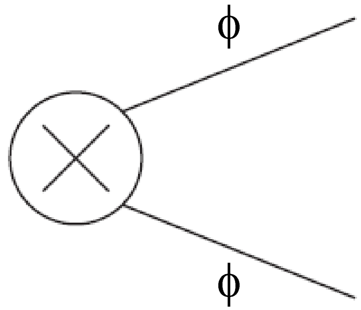

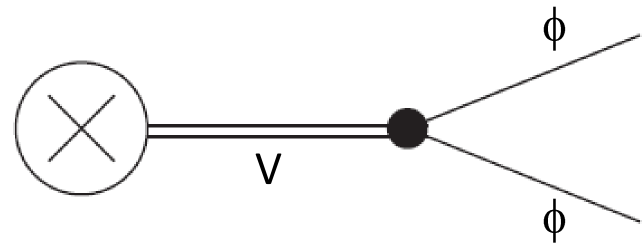

Short-distance constraints

$$F(s)^V = 1 + \frac{F_V G_V}{F^2} \frac{s}{M_V^2 - s}$$

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$$\sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

Phys.Lett.B412:382-388,1997

Guerrero, Pich

→ There are several ways of **resum**ming FSI: Omnès exponentiation of Re $A_{PQ}(s)$, dispersion relations, etc…

Nuovo Cim.8:316-326,1958

Form factors in two meson τ decays

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)^{\mathrm{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

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ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

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ChPT+VMD Guerrero, Pich '97

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Unitarity+Analiticity Omnés, '58

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O(p^2) result for δ^1_1 (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s}\exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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ChPT+VMD Guerrero, Pich '97

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Guerrero, Pich '97

Gómez-Dumm, Pich, Portolés '00
Phys.Rev.D62:054014,2000

$$\begin{aligned}\Gamma_\rho(s) &= \frac{M_\rho s}{96\pi f_\pi^2}\left\{\theta(s-4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s-4m_K^2)\,\sigma_K^3\right\}\\ &= -\frac{M_\rho s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\end{aligned}$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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Phys.Rev.D57:4136-4141,1998

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)

Phys.Lett.B640:176-181,2006

Idea: Follow the approach of Jamin, Pich, Portolés '06 including excited resonances while retaining (some of) these nice properties

Form factors in two meson τ decays

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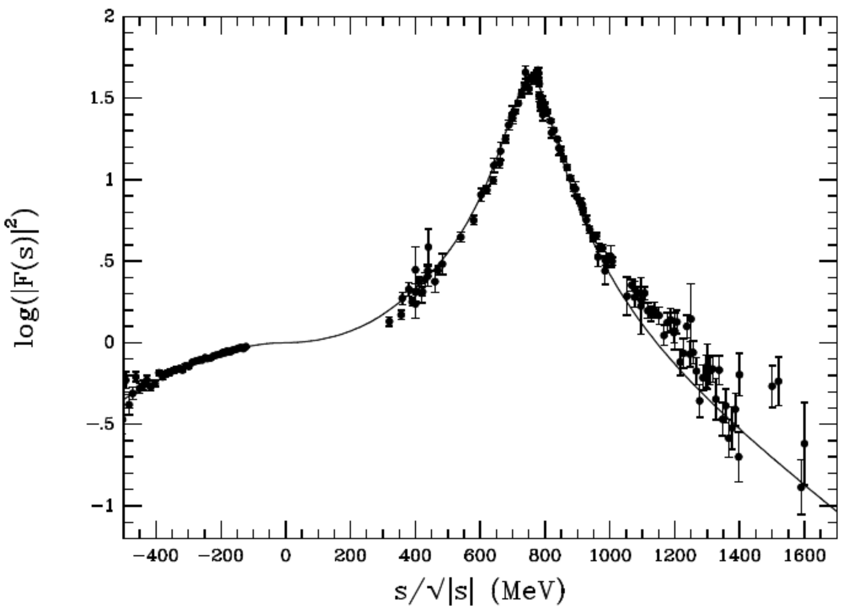

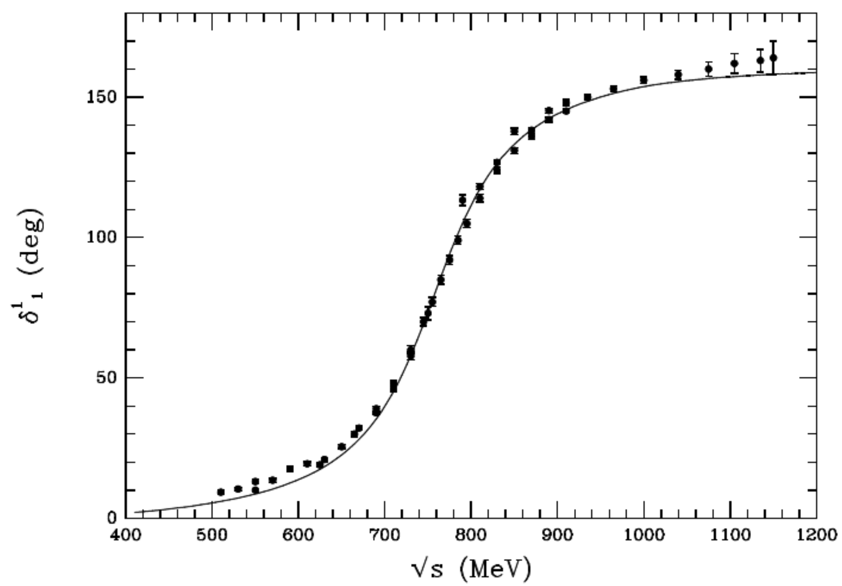

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Our formula

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right] - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right] - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right].$$

Roig '11

e-Print: **arXiv:1112.0962**

- χPT up to O(p^4) and leading O(p^6) contributions Guerrero '98
- Right fall-off at high energies
- SU(2)

- Analiticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of $\rho' + \rho''$

Form factors in two meson τ decays

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Match χPT results to VMD using an Omnés solution for dispersion relation

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Roig ‘11

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- (Phenomenological) contribution of ρ’ + ρ’’

This is what is included in TAUOLA right now

Form factors in two meson τ decays

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-}-p_{\pi^0})^\mu F_V^\pi(s)$$

On the inclusion of **excited resonances**

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1}+\delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$-\frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\Re e A_\pi(s)\right]$$
$$-\frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\Re e A_\pi(s)\right].$$

$$\gamma \equiv -F_V' G_V'/F^2 \qquad \delta \equiv -F_V'' G_V''/F^2 \qquad F_V G_V + F_V' G_V' + F_V'' G_V'' + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

Form factors in two meson τ decays

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-}-p_{\pi^0})^\mu F_V^\pi(s)$$

On the inclusion of **excited resonances**

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right] - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right] - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right].$$

$$\gamma \equiv -F_V' G_V'/F^2 \qquad \delta \equiv -F_V'' G_V''/F^2 \qquad F_V G_V + F_V' G_V' + F_V'' G_V'' + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

→ Easy to implement for two meson modes. For three meson modes a number of new couplings (involving new operator structures) appear. At which stage shall we include them?

Form factors in two meson τ decays

• The procedure for **other two meson decay channels** is analogous (Although SFF relevant in some decay channels, F(0)≠1 due to SU(3) breaking).

•An **alternative** procedure for **resummation** is described at 'Ongoing/Future improvements'

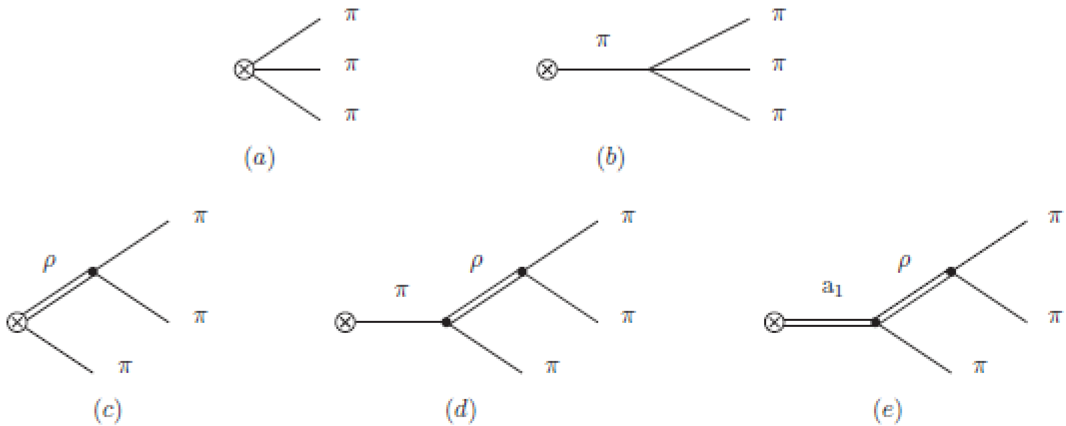
$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)}$$

$$F_{PQ}^V(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N_{loop}^{PQ}\frac{-s}{96\pi^2F^2}ReA_{PQ}(s)\right]$$

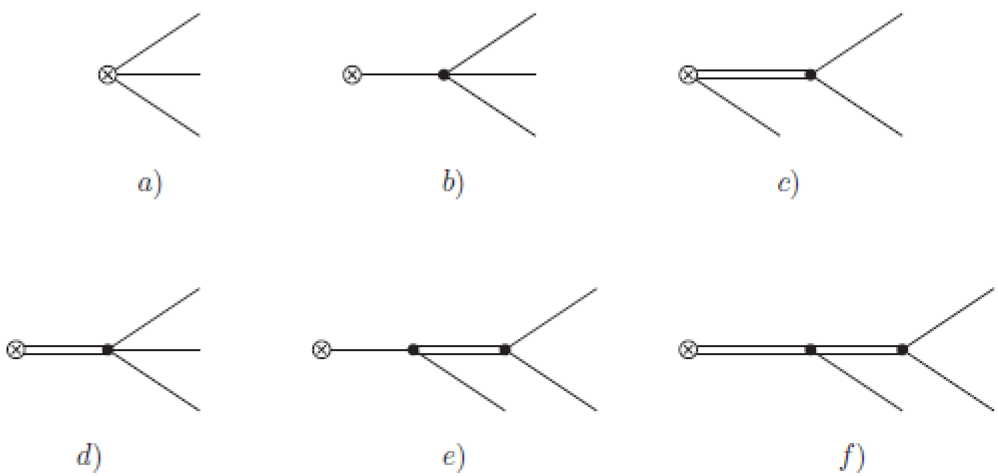
$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3\,\ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right) \qquad \sigma_P \equiv \sqrt{1-4m_P^2/s}$$

Form factors in three meson τ decays

$\tau \to \pi\pi\pi\nu_\tau$



$\tau \to KK\pi\nu_\tau$

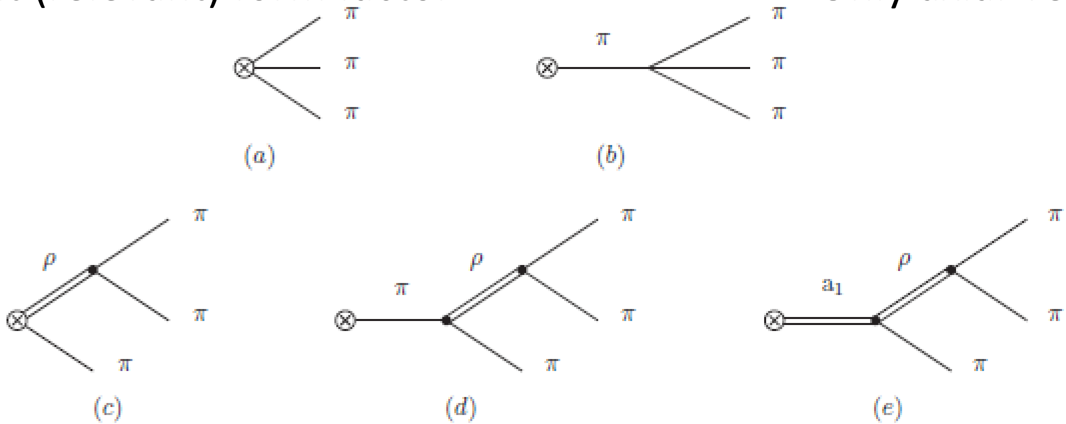


Form factors in three meson τ decays

Only one independent (relevant) form factor

$\tau \to \pi\pi\pi\nu_\tau$

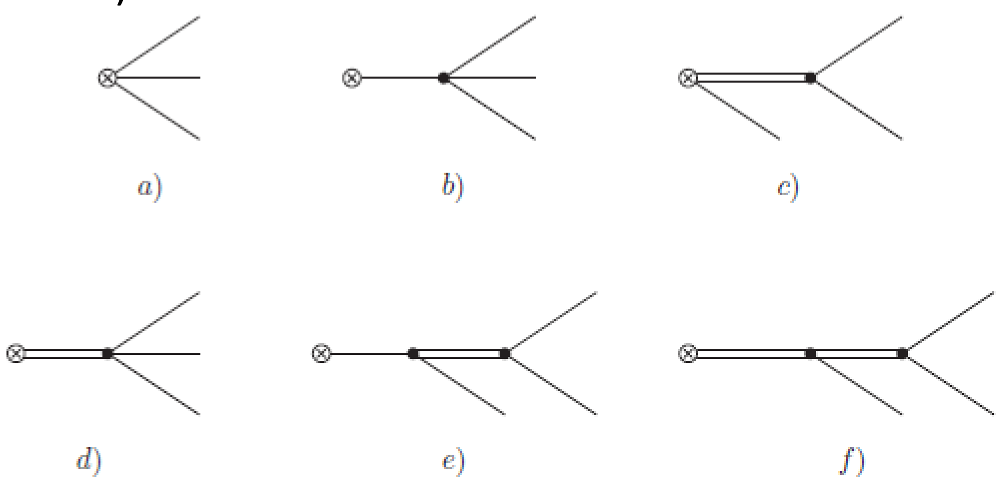
Only axial-vector current



Three independent (relevant) FFs

$\tau \to KK\pi\nu_\tau$

Both axial-vector and vector current



Form factors in three meson τ decays

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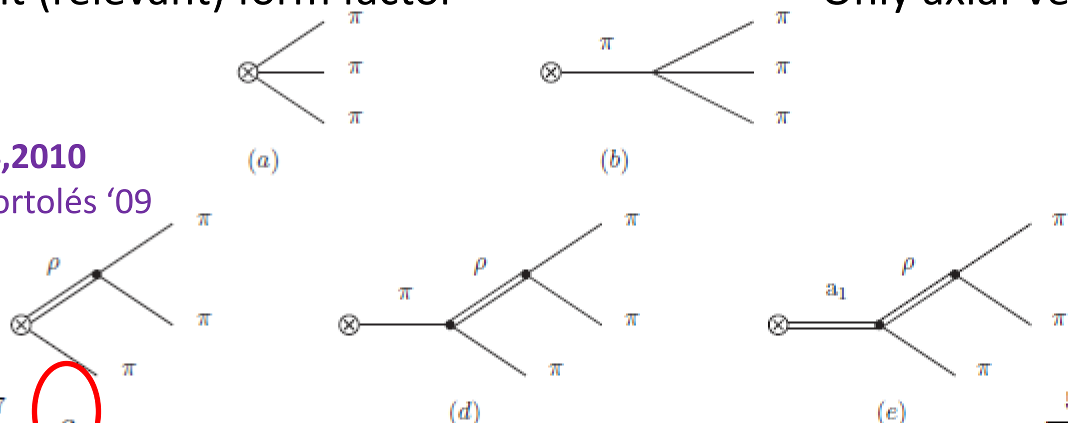
Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés ‘09



$$\mathcal{L}_4^{\mathrm{V}} = \sum_{i=1}^{5} \frac{g_i}{M_V} \mathcal{O}^i_{\mathrm{VPPP}} + \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}^i_{\mathrm{VJP}}$$

$$\mathcal{L}_2^{\mathrm{RR}} = \sum_{i=1}^{5} \lambda_i \mathcal{O}^i_{\mathrm{VAP}} + \sum_{i=1}^{4} d_i \mathcal{O}^i_{\mathrm{VVP}}$$

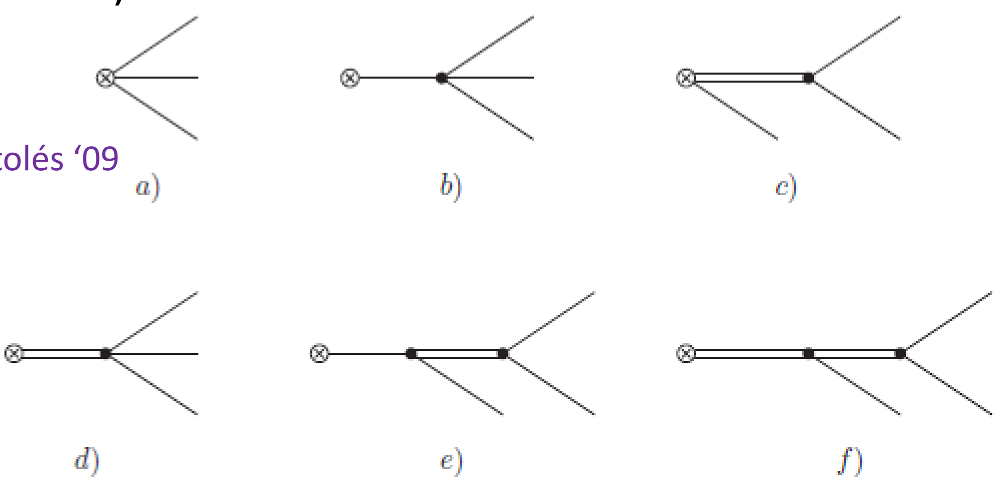
$\tau \to KK\pi\nu_\tau$

Three independent (relevant) FFs

Both axial-vector and vector current

Gómez-Dumm, Roig, Pich, Portolés ‘09

Phys.Rev.D81:034031,2010



Form factors in three meson τ decays

$\tau \to \pi\pi\pi\nu_\tau$

Phys.Rev.D69:073002,2004
Gómez-Dumm, Pich, Portolés '03

Phys.Lett.B685:158-164,2010
Gómez-Dumm, Roig, Pich, Portolés '09

$$F_{\pm i} = \pm(F_i^\chi + F_i^{\mathrm{R}} + F_i^{\mathrm{RR}}) , \qquad i = 1,2 \qquad\qquad F_2(Q^2,s,t) = F_1(Q^2,t,s)$$

$$F_1^\chi(Q^2,s,t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^{\mathrm{R}}(Q^2,s,t) = \frac{\sqrt{2}\,F_V\,G_V}{3\,F^3}\left[\frac{3\,s}{s-M_V^2} - \left(\frac{2G_V}{F_V}-1\right)\left(\frac{2Q^2-2s-u}{s-M_V^2}+\frac{u-s}{t-M_V^2}\right)\right]$$

$$F_1^{\mathrm{RR}}(Q^2,s,t) = \frac{4\,F_A\,G_V}{3\,F^3}\,\frac{Q^2}{Q^2-M_A^2}\left[-(\lambda'+\lambda'')\frac{3\,s}{s-M_V^2} + H(Q^2,s)\,\frac{2Q^2+s-u}{s-M_V^2} + H(Q^2,t)\,\frac{u-s}{t-M_V^2}\right]$$

$$H(Q^2,x) = -\lambda_0\frac{m_\pi^2}{Q^2} + \lambda'\frac{x}{Q^2} + \lambda''$$

Relations from short-distance **QCD**:

$$\begin{cases} F_V\,G_V = F^2 \\ F_V^2 - F_A^2 = F^2 \\ F_V^2\,M_V^2 = F_A^2\,M_A^2 \end{cases}$$

$$\lambda' = \frac{F^2}{2\sqrt{2}\,F_A\,G_V} = \frac{M_A}{2\sqrt{2}\,M_V},$$

$$\lambda'' = \frac{2\,G_V - F_V}{2\sqrt{2}\,F_A} = \frac{M_A^2-2M_V^2}{2\sqrt{2}\,M_V\,M_A},$$

$$4\,\lambda_0 = \lambda'+\lambda'' = \frac{M_A^2-M_V^2}{\sqrt{2}\,M_V\,M_A},$$

Form factors in three meson τ decays

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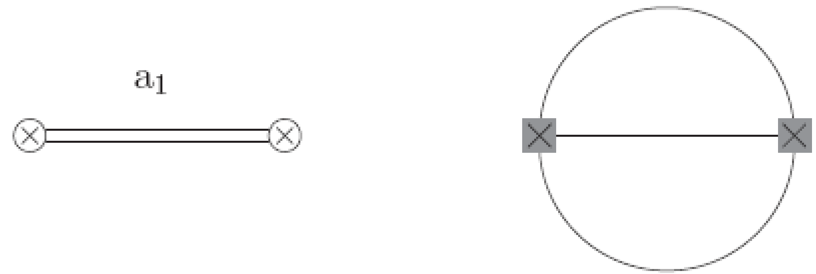
$\Gamma a_1(Q^2)$:



$$\Gamma_{a_1}(Q^2) \; = \; \Gamma^{\pi}_{a_1}(Q^2)\,\theta(Q^2 - 9m_\pi^2) \; + \; \Gamma^{K}_{a_1}(Q^2)\,\theta(Q^2 - (2m_K + m_\pi)^2) \; ,$$

$$\Gamma^{\pi,K}_{a_1}(Q^2) \; = \; \frac{-S}{192(2\pi)^3 F_A^2 M_{a_1}} \left(\frac{M_{a_1}^2}{Q^2} - 1 \right)^2 \int \mathrm{d}s \, \mathrm{d}t \; T^{\pi,K\mu}_{1+} \, T^{\pi,K*}_{1+\mu} \; .$$

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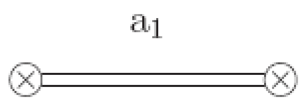

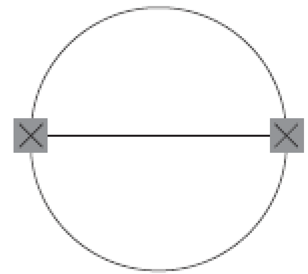

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$$\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)} \longrightarrow \frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)}+\frac{\beta_{\rho'}}{M_{\rho'}^2-q^2-iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

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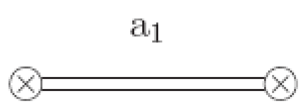

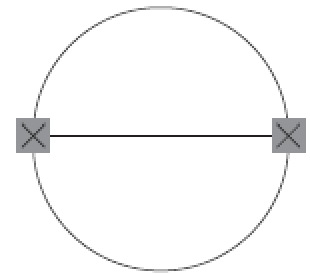

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→ Inclusion from a Lagrangian would imply 3 coups. instead of $\beta_{\rho'}$

Discussion on the errors

- $\varepsilon(1/N_c) \cong 1/3$?

‘t Hooft ‘74, Witten ‘79

Nucl.Phys.B72:461,1974 **Nucl.Phys.B160:57,1979**

Nucl.Phys.B75:461,1974

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Ecker *et al.* '88, '89

QED: $\alpha=e^2/(4\pi)^2$; χPT: $(p,m)^2/(4\pi F,M_V)^2$; RχT: ($\cong 1/N_c$)

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Pich, Rosell, Sanz Cillero '04,'06,'08,'10 Portolés, Rosell, Ruiz-Femenía '06

JHEP 0408:042,2004 **JHEP 0701:039,2007** **JHEP 0807:014,2008** **JHEP 1102:109,2011** **Phys.Rev.D75:114011,2007**

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→ Good description of the data

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07 Gómez-Dumm, Roig, Pich, Portolés '09

Phys.Lett.B664:78-83,2008 **Eur.Phys.J.C59:821-829,2009** **Phys.Lett.B685:158-164,2010**

Discussion on the errors

- $\varepsilon(1/N_c) \cong 1/3$?
- Resummation in the two meson modes

$$F_{PQ}^{V}(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N_{loop}^{PQ}\frac{-s}{96\pi^2F^2}Re A_{PQ}(s)\right]$$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)}$$

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Alternatively:

Exact Unitarity

$$F_V(s) = \frac{M_V^2}{M_V^2\left[1+\sum_{P,Q} N^{PQ}_{loop}\frac{s}{96\pi^2F^2}A_{PQ}(s)\right] - s}$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right] / Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

Tiny differences in observables between both approaches

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07

Discussion on the errors

- $\varepsilon(1/N_c) \cong 1/3$?

- Resummation in the two meson modes. ε below 3%.

- FSI in three meson modes. Relevant in $d\Gamma/ds$ ($\cong$ 10%).

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- Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, …

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Jamin, Oller, Pich '01,'06

Nucl.Phys.B622:279-308,2002 Phys.Rev.D74:074009,2006

Ongoing/Future improvements

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Moussallam **Eur.Phys.J.C53:401-412,2008**

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Bijnens, Colangelo, Talavera **JHEP 9805:014,1998**

López Castro et. al. **Phys.Rev.D74:071301,2006**

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Cirigliano, Ecker, Neufeld **Phys.Lett.B513:361-370,2001** **JHEP 0208:002,2002**

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Jamin, Oller, Pich '01,'06

- Remaining two meson modes: $\pi\eta^{(\prime)}, K\eta^{(\prime)}$

Ongoing/Future improvements

- Res

•F

Let's hope we can make it and discuss it together

THANK YOU!!

RADIO MONTECARlow

gelo, Talavera

14,1998

er, Neufeld

3:361-370,2001

08:002,2002

•Some importa

er, Pich '01,'06

•Remaining two meson modes: $\pi\eta^{(\prime)}, K\eta^{(\prime)}$